

**Higgs boson production from black holes at the CERN LHC**

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If the fundamental Planck scale is near a TeV, then TeV scale black holes should be produced in proton-proton collisions at the LHC where  $\sqrt{s} = 14$  TeV. As the temperature of the black holes can be  $\sim 1$  TeV we also expect production of Higgs bosons from them via Hawking radiation. This is a different production mode for the Higgs boson, which would normally be produced via direct pQCD parton fusion processes. In this paper we compare total cross sections and transverse momentum distributions  $d\sigma/dp_T$  for Higgs production from black holes at the LHC with those from direct parton fusion processes at next-to-next-to-leading order and next-to-leading order, respectively. We find that the Higgs production from black holes can be larger or smaller than the direct pQCD production depending upon the Planck mass and black hole mass. We also find that  $d\sigma/dp_T$  of Higgs production from black holes increases as a function of  $p_T$  which is in sharp contrast with the pQCD predictions where  $d\sigma/dp_T$  decreases so we suggest that the measurement of an increase in  $d\sigma/dp_T$  as  $p_T$  increases for Higgs (or any other heavy particle) production can be a useful signature for black holes at the LHC.

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**I. INTRODUCTION**

It is now generally accepted that the scale of quantum gravity *could be* as low as 1 TeV [1] and hence there can be graviton, radion, and black hole production at LHC [2–18]. If such processes occur then LHC collider experiments [19,20] can probe TeV scale quantum gravity. One of the most exciting aspects of this will be the production of black holes in particle accelerators. These “brane-world” black holes will be our first window into the extra dimensions of space predicted by string theory, and required by the several brane-world scenarios that provide for a low energy Planck scale [21]. As the black hole masses at the LHC are relatively small (3–7 TeV) and the temperatures of the black holes are very high ( $\sim 1$  TeV) the black holes can be a source for Higgs boson production via Hawking radiation. In fact there can be an enormous amount of heavy supersymmetric (SUSY) particle production from black holes [22], much more than expected from normal pQCD processes [23]. This comes about from two competing effects as the Planck scale increases: (1) Higgs production from black holes increases because the temperature of the black holes increases as the Planck scale increases for fixed black hole masses (see below) and (2) the cross section for black hole production decreases [24–28]. Recently phenomenological analyses have been made to connect the theoretical models with future data at the LHC [29–31]. Programs have been written to interface the theoretical predictions to Monte Carlo generators for specific detectors such as D0 and CDF at Fermilab and ATLAS at the LHC [32,33]. Reviews of this exciting field are given in [34].

In this paper we compare Higgs production cross sections from TeV scale black hole production at the LHC via

Hawking radiation with the direct pQCD parton fusion processes at next-to-next-to-leading order (NNLO) [35,36]. After all if the Planck and black hole masses are larger than the tentative estimates in the literature then extra dimensional models may not yield any signals whatsoever at the LHC. Or the masses may be so large that the signals from them are very small. Therefore it is necessary to compare the standard pQCD results for Higgs production with the corresponding black hole results. We find that the Higgs production cross sections from black holes at the LHC can be larger or smaller than those from pQCD processes depending on the value of the TeV scale Planck mass and the black hole masses. We find that as long as the temperature of the black holes is of the order of 1 TeV, the Higgs production cross section from the black holes does not depend very much on the Higgs mass ( $M_H$ ). On the other hand the direct pQCD production cross section is sensitive to  $M_H$ . This provides us with an important conclusion: if TeV scale black holes are indeed formed at the LHC, then one signature of this will be an unusually copious production of massive (Higgs and SUSY) particles, which is not possible via pQCD processes. Hence if we observe very high rates for massive particle production at the LHC, this might provide indirect evidence that TeV scale black holes are being produced.

We also study the  $p_T$  differential cross sections for Higgs production from black holes and from the pQCD parton fusion processes, here in next-to-leading order (NLO) [37,38]. One of the interesting results we find is that as long as the temperature of the black holes is very high ( $\sim 1$  TeV) then  $d\sigma/dp_T$  increases as  $p_T$  increases (up to about  $p_T$  equal to 1 TeV then it decreases). This is in sharp contrast to pQCD predictions where  $d\sigma/dp_T$  decreases as  $p_T$  increases for fixed Higgs boson masses. This is not only true for Higgs particles but also for any heavy (SUSY) particles [22] emitted from a black hole via Hawking radiation as long as the temperature of the black

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hole is high ( $\sim 1$  TeV). Therefore if one experimentally observes that the  $d\sigma/dp_T$  for heavy final state particles increases as  $p_T$  increases (up to about 1 TeV or higher) then it might provide a good evidence for black hole production at the LHC.

The paper is organized as follows. In Sec. II we present the computation for the rate of Higgs production and its transverse momentum distribution from black holes via Hawking radiation at the LHC. In Sec. III we sketch the calculation of the pQCD total and  $p_T$  differential cross sections for Higgs production at the LHC. In Sec. IV we present and discuss our results.

## II. HIGGS BOSON PRODUCTION FROM BLACK HOLES AT THE LHC

If black holes are formed at the LHC then they will quickly evaporate by emitting thermal Hawking radiation. The emission rate per unit time for a Higgs particle with momentum  $p = |\vec{p}|$  and energy  $Q = \sqrt{p^2 + M_H^2}$  can be written [14] as

$$\frac{dN}{dt} = \frac{c_s \sigma_s}{8\pi^2} \frac{dp p^2}{(e^{Q/T_{\text{BH}}} - 1)}, \quad (1)$$

where  $\sigma_s$  is the gray body factor and  $T_{\text{BH}}$  is the black hole temperature, which depends on the number of extra dimensions and on the TeV scale Planck mass.  $c_s$  is the multiplicity factor. The temperature of the black hole is given in [3], namely

$$T_{\text{BH}} = \frac{d+1}{4\pi R_S} = \frac{d+1}{4\sqrt{\pi}} M_P \left[ \frac{M_P}{M_{\text{BH}}} \frac{d+2}{8\Gamma(\frac{d+3}{2})} \right]^{1/(1+d)}, \quad (2)$$

where  $R_S$  is the Schwarzschild radius of the black hole,  $M_P$  is the TeV scale Planck mass,  $M_{\text{BH}}$  is the mass of the black hole, and  $d$  is the number of extra dimensions. The gray body factor in the geometrical approximation is given by [24–26]

$$\sigma_s = \Gamma_s 4\pi \left( \frac{d+3}{2} \right)^{2/(d+1)} \frac{d+3}{d+1} R_S^2, \quad (3)$$

where we take  $\Gamma_s = 1$  for scalars. Recent work [33] shows that gray body factors for scalar emission do not vary much with  $d$  in contrast to fermion and gauge boson emission so Eq. (3) is a reasonable ansatz. From Eq. (1) we get:

$$\frac{dN}{dtdp} = \frac{c_s \sigma_s}{8\pi^2} \frac{p^2}{(e^{\sqrt{p^2 + M_H^2}/T_{\text{BH}}} - 1)}. \quad (4)$$

The total number of Higgs particles emitted from the black holes is thus given by:

$$N_{\text{Higgs}} = \int_0^{t_f} dt \int_0^{M_{\text{BH}}} dp \frac{c_s \sigma_s}{8\pi^2} \frac{p^2}{(e^{\sqrt{p^2 + M_H^2}/T_{\text{BH}}} - 1)}, \quad (5)$$

where  $t_f$  is the total time taken by the black hole to completely evaporate, which takes the form [4]:

$$t_f = \frac{C}{M_P} \left( \frac{M_{\text{BH}}}{M_P} \right)^{(d+3)/(d+1)}. \quad (6)$$

$C$  depends on the extra dimensions and on the polarization degrees of freedom, etc. However, the complete determination of  $t_f$  depends on the energy density present outside the black hole which is computed in [27] where the absorption of the quark-gluon plasma [28] by a TeV scale black hole at the LHC is considered (this time is typically about  $10^{-27}$  sec). The value we use throughout this paper is  $t_f = 10^{-3}$  fm which is the inverse of the TeV scale energy.

This result in Eq. (5) is for Higgs particle emission from black holes of temperature  $T_{\text{BH}}$ . To obtain the Higgs production cross section from all black holes produced in proton-proton collisions at the LHC we need to multiply the black hole production cross section with the number of Higgs bosons produced from a single black hole. The black hole production cross section  $\sigma_{\text{BH}}$  in high energy hadronic collisions at zero impact parameter is given in [3,15], namely

$$\begin{aligned} \sigma_{\text{BH}}^{AB \rightarrow \text{BH}+X}(M_{\text{BH}}) &= \sum_{ab} \int_{\tau}^1 dx_a \int_{\tau/x_a}^1 dx_b f_{a/A}(x_a, \mu^2) \\ &\times f_{b/B}(x_b, \mu^2) \hat{\sigma}^{ab \rightarrow \text{BH}}(\hat{s}) \\ &\times \delta(x_a x_b - M_{\text{BH}}^2/s). \end{aligned} \quad (7)$$

In this expression  $x_a(x_b)$  is the longitudinal momentum fraction of the parton inside the hadron A(B) and  $\tau = M_{\text{BH}}^2/s$ , where  $\sqrt{s}$  is the hadronic center-of-mass energy. Energy-momentum conservation implies  $\hat{s} = x_a x_b s = M_{\text{BH}}^2$ . We use  $\mu = M_{\text{BH}}$  as the scale at which the parton distribution functions  $f_{a/A}, f_{b/B}$  are measured.  $\sum_{ab}$  represents the sum over all partonic contributions. The black hole production cross section in a binary partonic collision is given by [3]

$$\hat{\sigma}^{ab \rightarrow \text{BH}}(\hat{s}) = \frac{1}{M_P^2} \left[ \frac{M_{\text{BH}}}{M_P} \left( \frac{8\Gamma(\frac{d+3}{2})}{d+2} \right) \right]^{2/(d+1)}, \quad (8)$$

where  $d$  denotes the number of extra spatial dimensions. Note that  $M_{\text{BH}}$  should be approximately 5 times  $M_P$  for the classical limit to apply and Hawking evaporation to occur. We choose  $M_{\text{BH}} = 3M_P$  and  $M_{\text{BH}} = 5M_P$  for our plots. The total cross section for Higgs production at LHC is then given by

$$\sigma_{\text{Higgs}} = N_{\text{Higgs}} \sigma_{\text{BH}}. \quad (9)$$

We will compare this cross section for Higgs boson production via black hole resonances with the Higgs cross section produced via pQCD processes, as will be explained in the next section. To compare the differential cross sections we decompose the phase-space integration in Eq. (1) as  $d^3\vec{p} = d^2p_t dp_z = d^2p_t m_t \text{cosh}y$  where  $p^\mu = (\sqrt{p_t^2 + M_H^2} \text{cosh}y, p_x, p_y, \sqrt{p_t^2 + M_H^2} \text{sinh}y)$  and integrate over the rapidity  $y$ .

### III. DIRECT HIGGS PRODUCTION IN PP COLLISIONS AT THE LHC

The LEP experiments [39] give a lower mass limit on the mass of the Higgs  $M_H \sim 114 \text{ GeV}/c^2$  and fits to the data using precision calculations in the electroweak sector of the standard model indicate an upper limit  $m_H < 200 \text{ GeV}/c^2$  with 95% confidence level. Therefore we will concentrate on the mass interval  $100 \text{ GeV}/c^2 \leq M_H \leq 300 \text{ GeV}/c^2$ .

At the LHC proton-proton collider the dominant QCD production process involves the gluon-gluon fusion mechanism. (We comment on the weak boson fusion reaction at the end of the paper). In the standard model the Higgs boson couples to the gluons via heavy quark loops. Since the coupling of the scalar Higgs boson H to a fermion loop is proportional to the mass of the fermion (for a review see [40]), the top-quark loop is the most important. In lowest order (LO) the gluon-gluon fusion process  $g + g \rightarrow H$ , represented by the top-quark triangle graph, was computed in [41]. The two-to-two body tree graphs, given by gluon bremsstrahlung  $g + g \rightarrow g + H$ ,  $g + q(\bar{q}) \rightarrow q(\bar{q}) + H$  and  $q + \bar{q} \rightarrow g + H$  were computed in [42]. From these reactions one can derive the transverse momentum ( $p_T$ ) and rapidity ( $y$ ) distributions of the scalar Higgs boson. The total integrated cross section, which also involves the computation of the QCD corrections to the top-quark loop, has been calculated in [43]. This calculation was rather cumbersome since it involved the computation of two-loop triangular graphs with massive quarks. Furthermore also the two-to-three parton reactions have been computed in [44] using helicity methods. From the experience gained from the next-to-leading (NLO) corrections presented in [43] it is clear that it will be very difficult to obtain the exact NLO corrections to one-particle inclusive distributions as well as the NNLO corrections to the total cross section.

Fortunately one can simplify the calculations if one takes the large top-quark mass limit  $m_t \rightarrow \infty$ . In this case the Feynman graphs are obtained from an effective Lagrangian describing the direct coupling of the scalar Higgs boson to the gluons. The LO and NLO contributions to the total cross section in this approximation were computed in [45]. A thorough analysis [43,46] reveals that the error introduced by taking the  $m_t \rightarrow \infty$  limit is less than about 5% provided  $m_H \leq 2m_t$ . The two-to-three body processes were computed with the effective Lagrangian approach for the scalar Higgs bosons in [47] using helicity methods. The one-loop corrections to the two-to-two body reactions above were computed for the scalar Higgs boson in [48]. These matrix elements were used to compute the transverse momentum and rapidity distributions of the scalar Higgs boson up to NLO in [37,49]. The effective Lagrangian method was also applied to obtain the NNLO total cross section for scalar Higgs production by the calculation of the two-loop corrections to the Higgs-

gluon-gluon vertex in [50,51], the soft-plus-virtual gluon corrections in [52], and the computation of the two-to-three body processes in [36,37].

In the large top-quark mass limit, which we use from now on, the Feynman rules for scalar Higgs production can be derived from the following effective Lagrangian density

$$\begin{aligned} \mathcal{L}_{\text{eff}}^H &= G_H \Phi^H(x) O(x) \quad \text{with} \\ O(x) &= -\frac{1}{4} G_{\mu\nu}^a(x) G^{a,\mu\nu}(x), \end{aligned} \quad (10)$$

where  $\Phi^H(x)$  represents the scalar field. Furthermore the gluon field strength is given by  $G_a^{\mu\nu}$ . The factor multiplying the operator is chosen in such a way that the vertices are normalized to the effective coupling constant  $G_H$ . The latter is determined by the top-quark triangular loop graph, which describes the decay process  $H \rightarrow g + g$  including all QCD corrections, taken in the limit that the top-quark mass  $m_t \rightarrow \infty$ . This yields

$$G_H = -2^{5/4} a_s(\mu_r^2) G_F^{1/2} \tau_H F_H(\tau_H) C_H \left( a_s(\mu_r^2), \frac{\mu_r^2}{m_t^2} \right), \quad (11)$$

where  $a_s(\mu_r^2)$  is defined by

$$a_s(\mu_r^2) = \frac{\alpha_s(\mu_r^2)}{4\pi}, \quad (12)$$

with  $\alpha_s(\mu_r^2)$  the running coupling constant and  $\mu_r$  the renormalization scale. Further  $G_F$  represents the Fermi constant and the function  $F_H$  is given by

$$\begin{aligned} F_H(\tau) &= 1 + (1 - \tau)f(\tau), \quad \tau = \frac{4m_t^2}{M_H^2}, \\ f(\tau) &= \arcsin^2 \frac{1}{\sqrt{\tau}} \quad \text{for } \tau \geq 1, \\ f(\tau) &= -\frac{1}{4} \left( \ln \frac{1 - \sqrt{1 - \tau}}{1 + \sqrt{1 - \tau}} + \pi i \right)^2 \quad \text{for } \tau < 1. \end{aligned} \quad (13)$$

In the large  $m_t$ -limit we have

$$\lim_{\tau \rightarrow \infty} F_H(\tau) = \frac{2}{3\tau}. \quad (14)$$

The coefficient function  $C_H$  originates from the corrections to the top-quark triangular graph provided one takes the limit  $m_t \rightarrow \infty$ . The coefficient function has been computed up to order  $\alpha_s^2$  in [46,53] for the Higgs.

Using the effective Lagrangian approach the total cross section of the reaction

$$H_1(P_1) + H_2(P_2) \rightarrow H(-p_3) + ' X', \quad (15)$$

where  $H_1$  and  $H_2$  denote the incoming hadrons and  $X$  represents an inclusive hadronic state has been calculated to NNLO. This total cross section is given by

$$\begin{aligned} \sigma_{\text{tot}} &= \frac{\pi G_{\text{H}}^2}{8(N^2 - 1)} \sum_{a,b=q,\bar{q},g} \int_x^1 dx_1 \int_{x/x_1}^1 dx_2 f_a(x_1, \mu^2) \\ &\quad \times f_b(x_2, \mu^2) \Delta_{ab,\text{H}}\left(\frac{x}{x_1 x_2}, \frac{M_{\text{H}}^2}{\mu^2}\right), \quad \text{with} \\ x &= \frac{M_{\text{H}}^2}{S}, \\ S &= (P_1 + P_2)^2, \quad p_5^2 = M_{\text{H}}^2, \end{aligned} \quad (16)$$

where the factor  $1/(N^2 - 1)$  originates from the color average in the case of the local gauge group  $\text{SU}(N)$ . Further we have assumed that the scalar Higgs boson is mainly produced on shell i.e.  $p_5^2 \sim M_{\text{H}}^2$ . The parton densities denoted by  $f_a(y, \mu^2)$  ( $a, b = q, \bar{q}, g$ ) depend on the mass factorization/renormalization scale  $\mu$ . The same scales also enter the coefficient functions  $\Delta_{ab,\text{H}}$  which are derived from the partonic cross sections.

Up to NNLO we have to compute the following partonic subprocesses. On the Born level we have the reaction

$$g + g \rightarrow \text{H}. \quad (17)$$

In NLO we have in addition to the one-loop virtual corrections to the above reaction the following two-to-two body processes

$$\begin{aligned} g + g &\rightarrow \text{H} + g, & g + q(\bar{q}) &\rightarrow \text{H} + q(\bar{q}), \\ q + \bar{q} &\rightarrow \text{H} + g. \end{aligned} \quad (18)$$

In NNLO we receive contributions from the two-loop virtual corrections to the Born process in Eq. (17) and the one-loop corrections to the reactions in Eq. (18). To these contributions one has to add the results obtained from the following two-to-three body reactions

$$g + g \rightarrow \text{H} + g + g, \quad g + g \rightarrow \text{H} + q_i + \bar{q}_i, \quad (19)$$

$$g + q(\bar{q}) \rightarrow \text{H} + q(\bar{q}) + g, \quad (20)$$

$$q + \bar{q} \rightarrow \text{H} + g + g, \quad q + \bar{q} \rightarrow \text{H} + q_i + \bar{q}_i, \quad (21)$$

$$q_1 + q_2 \rightarrow \text{H} + q_1 + q_2, \quad q_1 + \bar{q}_2 \rightarrow \text{H} + q_1 + \bar{q}_2, \quad (22)$$

$$q + q \rightarrow \text{H} + q + q. \quad (23)$$

The computation of the phase-space integrals has been done in [35], to which we refer for further details (see also [36]). After they have been calculated the partonic cross section is rendered finite by coupling constant renormalization, operator renormalization (see [54],) and the removal of collinear divergences. In the representation of the coefficient functions above we have set the renormalization scale  $\mu_r$  equal to the mass factorization scale  $\mu$ . The final total cross section for Higgs boson production in proton-proton collisions at the LHC can be written as

$$\sigma_{\text{tot}} = \frac{\pi G_{\text{H}}^2}{8(N^2 - 1)} \sum_{a,b=q,\bar{q},g} \int_x^1 dy \Phi_{ab}(y, \mu^2) \Delta_{ab}\left(\frac{x}{y}, \frac{M_{\text{H}}^2}{\mu^2}\right), \quad (24)$$

where  $x = M_{\text{H}}^2/S$  and  $\Phi_{ab}$  is the parton-parton flux defined by

$$\Phi_{ab}(y, \mu^2) = \int_y^1 \frac{du}{u} f_a(u, \mu^2) f_b\left(\frac{y}{u}, \mu^2\right). \quad (25)$$

The coefficient functions  $\Delta_{ab}$  in the effective Lagrangian approach were computed exactly in NNLO and the parton densities are also known to the same order because the exact three-loop splitting functions (anomalous dimensions) have now been calculated [55]. Hence we can check whether the approach of using only a finite number of moments (see [56]) which was used in [57] together with other constraints to approximate the splitting functions is accurate. These approximations are very reliable as long as  $y > 10^{-4}$  in Eq. (24). The approximated splitting functions were used in [58,59] to obtain NNLO parton density sets. For the NLO and NNLO plots we employ the two- and three-loop asymptotic forms of the running coupling constant as given in Eq. (3) of [60]. For our plots we take  $\mu = M_{\text{H}}$  and use the Martin-Robert-Stirling-Thorne (MRST) set above for the NNLO computations. For the computation of the effective coupling constant  $G_{\text{H}}$  in Eq. (11) we choose the top-quark mass  $m_t = 173.4 \text{ GeV}/c^2$  and the Fermi constant  $G_F = 1.16639 \text{ GeV}^{-2} = 4541.68 \text{ pb}$ .

We now discuss briefly the calculation of the  $p_T$  differential cross section for Higgs boson production in proton-proton collisions at the LHC. This was done by a different method since we only integrated analytically over part of final phase space. We still need the 2 to 2 parton fusion processes  $g + g \rightarrow g + \text{H}$ ,  $q + \bar{q} \rightarrow g + \text{H}$ , and  $q(\bar{q}) + g \rightarrow q(\bar{q}) + \text{H}$ , together with all the real (2 to 3) and virtual NLO corrections [37,38]. The  $p_T$  differential cross section for Higgs production in  $\text{H}_1(P_1) + \text{H}_2(P_2) \rightarrow \text{H}(-p_5) + X'$  at NLO is given by

$$\frac{d\sigma}{dp_T} = \int_{-y_{\text{max}}}^{y_{\text{max}}} dy \frac{d^2\sigma^{\text{H}_1\text{H}_2}}{dp_T dy}(S, p_T^2, y, M_{\text{H}}^2), \quad (26)$$

where

$$S \frac{d^2\sigma^{\text{H}_1\text{H}_2}}{dp_T^2 dy}(S, p_T^2, y, M_{\text{H}}^2) = S^2 \frac{d^2\sigma^{\text{H}_1\text{H}_2}}{dT dU}(S, T, U, M_{\text{H}}^2). \quad (27)$$

$M_{\text{H}}$  is the mass of the Higgs boson and  $y$  is its rapidity. The hadronic kinematical variables are defined by

$$\begin{aligned} S &= (P_1 + P_2)^2, & T &= (P_1 + p_5)^2, \\ U &= (P_2 + p_5)^2. \end{aligned} \quad (28)$$

The 2-2 parton momenta satisfy  $p_1 + p_2 + p_3 + p_5 = 0$  in LO. The invariants are given by

$$\begin{aligned}
 T &= M_H^2 - \sqrt{S}\sqrt{p_T^2 + M_H^2} \cosh y + \sqrt{S}\sqrt{p_T^2 + M_H^2} \sinh y, \\
 U &= M_H^2 - \sqrt{S}\sqrt{p_T^2 + M_H^2} \cosh y - \sqrt{S}\sqrt{p_T^2 + M_H^2} \sinh y.
 \end{aligned}
 \tag{29}$$

The rapidity interval is given by  $-y_{\max} \leq y \leq y_{\max}$  where

$$y_{\max} = \frac{1}{2} \ln \frac{1 + \sqrt{1 - sq}}{1 - \sqrt{1 - sq}}, \quad sq = \frac{4S(p_T^2 + M_H^2)}{(S + M_H^2)^2}.
 \tag{30}$$

The hadronic cross sections  $d\sigma^{H_1 H_2}$  are related to the partonic level cross sections  $d\sigma_{ab}$  as follows

$$\begin{aligned}
 &S^2 \frac{d^2\sigma^{H_1 H_2}}{dTdU}(S, T, U, M_H^2) \\
 &= \sum_{a,b=q,g} \int_{x_{1,\min}}^1 \frac{dx_1}{x_1} \int_{x_{2,\min}}^1 \frac{dx_2}{x_2} f_a^{H_1}(x_1, \mu^2) \\
 &\quad \times f_b^{H_2}(x_2, \mu^2) s^2 \frac{d^2\sigma_{ab}}{dtdu}(s, t, u, M_H^2, \mu^2),
 \end{aligned}
 \tag{31}$$

where  $\mu$  is the factorization scale in the parton densities, chosen to satisfy  $\mu^2 = p_T^2 + M_H^2$ . In the case parton  $p_1$  emerges from hadron  $H_1(P_1)$  and parton  $p_2$  emerges from hadron  $H_2(P_2)$

$$\begin{aligned}
 p_1 &= x_1 P_1, \quad p_2 = x_2 P_2, \quad s = x_1 x_2 S, \\
 t &= x_1(T - M_H^2) + M_H^2, \quad u = x_2(U - M_H^2) + M_H^2, \\
 x_{1,\min} &= \frac{-U}{S + T - M_H^2}, \quad x_{2,\min} = \frac{-x_1(T - M_H^2) - M_H^2}{x_1 S + U - M_H^2}.
 \end{aligned}
 \tag{32}$$

Further details on the calculation of the 2 to 2 and 2 to 3 partonic cross sections are available in [37], (see also [38]). We have computed all the differential cross sections using the CTEQ6M NLO parton density set in [61].

#### IV. RESULTS AND DISCUSSIONS

In this section we will compute Higgs production cross sections and  $p_T$  distributions at  $\sqrt{s} = 14$  TeV in pp collisions. We begin with black hole production. We choose the factorization and normalization scales to be equal to the mass of the black hole which is of the order of 1 TeV and use CTEQ6M parton density distributions [61]. The computation of the black hole production cross sections follows from Eqs. (7) and (8) in Sec. II.

In Fig. 1 we plot the black hole production cross section  $\sigma_{\text{BH}}$  in pb at the LHC as a function of the black hole mass  $M_{\text{BH}}$  in TeV. The solid, dashed, and dot-dashed curves are for Planck masses of 1, 2, and 3 TeV, respectively. The number of extra dimensions  $d = 4$ . As can be seen from the figure the cross sections decrease rapidly when both the Planck and black hole masses increase. These black hole production cross sections will be multiplied with the num-

Black hole cross section at the LHC

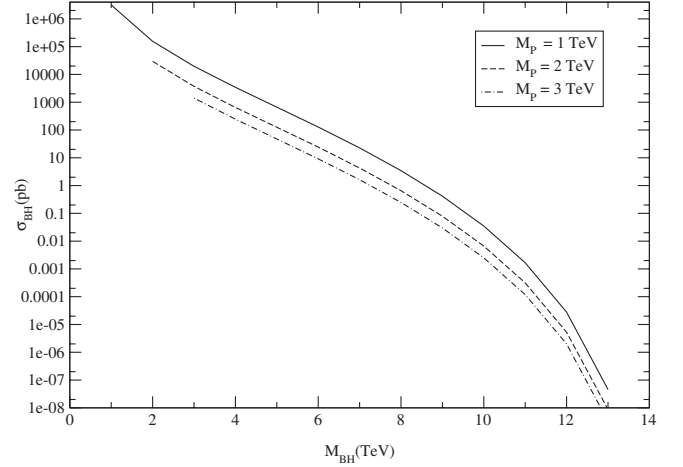


FIG. 1. Total cross sections for black hole production at the LHC.

ber of Higgs bosons produced from a single black hole to obtain the Higgs production cross section from a black hole at the LHC.

As the temperature of the black hole at the LHC is  $\sim 1$  TeV there is not much difference in the Higgs production cross section from black holes if  $M_H$  is increased from 120 to 200 GeV. Hence we will use  $M_H = 200$  GeV when we compute the Higgs production differential and total cross sections from the black holes. In Fig. 2 we present the total Higgs boson production cross section from black hole production (from Eq. (9) in Sec. II) and from pQCD in NNLO. The former is given for two different choices of the Planck mass, each with two choices of the black hole mass, namely  $M_P = 1$  TeV with  $M_{\text{BH}} = 3, 5$  TeV and  $M_P = 2$  TeV with  $M_{\text{BH}} = 6, 10$  TeV. We plot for comparison the NNLO Higgs boson cross section from [35] as a

Total Higgs production cross sections

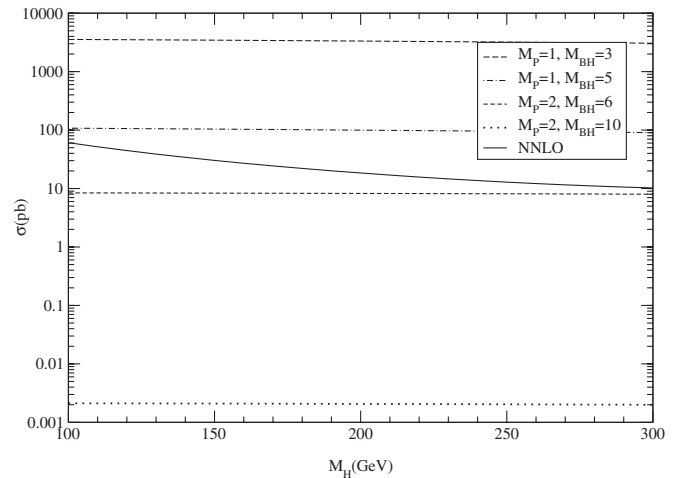


FIG. 2. Total cross sections for Higgs production from black holes and from direct pQCD processes at NNLO.

function of the mass of the Higgs boson in GeV. For the latter curve we use the NNLO MRST parton density set in [58] because there is no NNLO CTEQ set. We concentrate on the mass range  $100 \text{ GeV} \leq M_H \leq 300 \text{ GeV}$ , which is where the bounds from the LEP data indicate that it should be. We see that the Higgs production rate from black holes can be larger or smaller than the direct pQCD production depending upon the values of the Planck mass and the black hole mass.

In Fig. 3 we present results for the  $p_T$  differential cross sections in pb/GeV for Higgs production both via direct parton fusion processes at NLO and indirect Higgs emission through black hole production (we integrate over the rapidity using the transformation described after Eq. (9)). In this figure we use CTEQM6 parton densities in [61]. The three decreasing lines (as  $p_T$  increases) are for NLO parton fusion processes, with the solid, dashed, and dot-dashed lines for Higgs masses equal to 120, 160, and 200 GeV, respectively. The three increasing lines (as  $p_T$  is increased) are from emissions from black hole production, with the solid, dashed, and dot-dashed lines for black hole masses equal to 3, 4, and 5 TeV, respectively, with the Planck mass equal to 1 TeV in each case.

It can be seen from Fig. 3 that, with this choice of masses, the  $p_T$  differential cross section for Higgs production from black hole emission is larger than that from the direct NLO parton fusion processes when  $p_T \geq 50 \text{ GeV}$ . This is interesting because if a black hole is indeed formed and if the Planck scale is around a TeV then we may get more Higgs production from the black hole than that we would have obtained from direct parton fusion processes. This will enhance the chance of detecting Higgs bosons at the LHC.

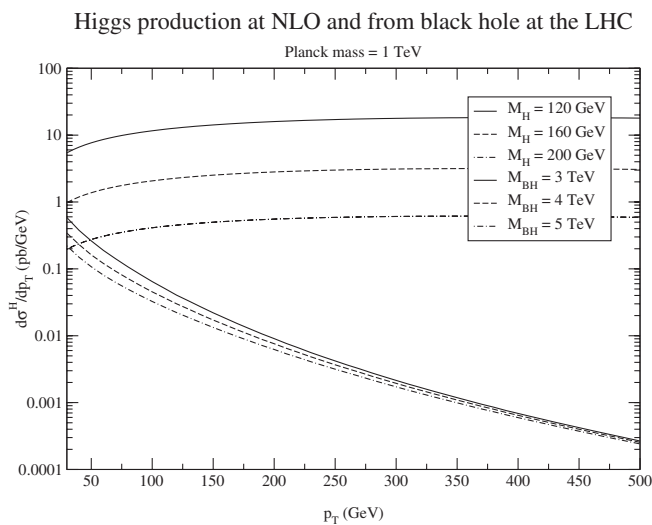


FIG. 3.  $p_T$  differential cross sections for Higgs production from black holes and from direct pQCD processes at NLO. The Planck mass is 1 TeV.

It can also be seen that  $d\sigma/dp_T$  for Higgs production from black holes increases as  $p_T$  increases whereas it decreases in the case of the pQCD NLO calculation. This is a unique feature of black hole production at the LHC. The reason for this is that the mass of the black hole formed is quite small ( $M_{\text{BH}} \sim 5 \text{ TeV}$ ) and hence the temperature of the black hole is very large  $\sim 1 \text{ TeV}$ . For Higgs production with such a high temperature the Bose-Einstein distribution function  $e^{\sqrt{p^2 + M_H^2}/T_{\text{BH}}} - 1$  in Eq. (5) remains almost flat with respect to  $p_T$  as long as  $p_T$  is not much larger than  $T_{\text{BH}}$ . Hence the increase of  $d\sigma/dp_T$  as  $p_T$  increases comes from the increase in the transverse momentum phase space as can be seen from Eq. (5).

In Fig. 4 we present similar results as in the case of Fig. 3 but now for a Planck mass of 2 TeV. The three decreasing lines (as  $p_T$  increases) are from the NLO parton fusion processes, with the solid, dashed, and dot-dashed lines for Higgs masses of 120, 160, and 200 GeV, respectively. The two increasing lines (as  $p_T$  is increased) are from black hole emissions, with the solid and dashed lines for black hole masses equal to 6 and 10 TeV, respectively. Clearly the differential cross section is quite small when  $M_{\text{BH}} = 10 \text{ TeV}$  because the total cm energy is only 14 TeV.

The results in Figs. 3 and 4 show that as the Planck mass increases the Higgs production rate from black hole emission becomes smaller than that from the NLO pQCD Higgs production. However as the  $p_T$  is increased then at some value the Higgs production from black hole emission will be larger than that of the direct pQCD Higgs production. Therefore the cross section from the emission of the black hole via Hawking radiation will dominate over any other standard model processes for large masses and large enough  $p_T$ . Hence a large rate of particle production at

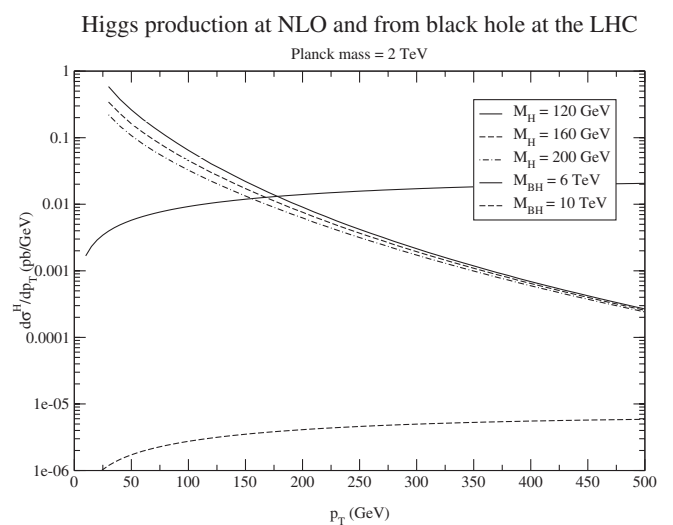


FIG. 4.  $p_T$  differential cross sections for Higgs production from black holes and from direct pQCD processes at NLO. The Planck mass is 2 TeV.

Comparison between pQCD and weak boson fusion

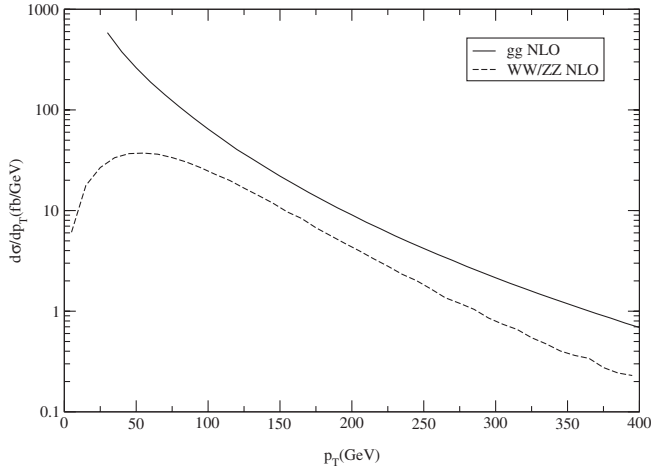


FIG. 5.  $p_T$  differential cross sections for Higgs production at the LHC from direct pQCD processes at NLO and weak boson fusion processes at NLO.

the LHC at high  $p_T$  and high mass can be a possible signature of black hole production.

We have concentrated on Higgs production via QCD reactions and ignored possible contributions from the so-called weak boson fusion reactions involving the couplings of  $W$  and  $Z$  bosons to the Higgs. In the latter case a typical partonic reaction is

$$q + q \rightarrow H + q + q, \quad (33)$$

where two virtual  $Z$ -bosons are exchanged between the quarks and the Higgs. This is a typical  $t$ -channel process. Virtual  $W^+$  and  $W^-$  bosons can also be exchanged. The

reason we have neglected this so-called  $WW/ZZ$  reaction is that it is not the dominant contribution to *inclusive* Higgs production at large  $p_T$ . We show this in Fig. 5 where the contribution from the weak boson fusion reaction was provided by J. Campbell from his NLO calculation in [62]. Here  $M_H = 120$  GeV and CTEQ6 parton densities have been used. Note that the scale is now in fb/GeV. If one identifies the two final state jets and applies  $p_T$  cuts on them then it is possible to enhance the weak boson signal and decrease the QCD signal. However this now involves the study of exclusive processes which is not the subject of this paper.

In summary, we have compared Higgs production total and differential cross sections in proton-proton collisions at the LHC ( $\sqrt{s} = 14$  TeV) via pQCD processes and via Hawking radiation from black holes within the model of TeV scale gravity. As the temperature of the black hole is  $\sim 1$  TeV there is a huge amount of Higgs production from black holes at the LHC if the Planck mass is  $\sim 1$  TeV. We also find that  $d\sigma/dp_T$  for Higgs production increases as a function of  $p_T$  in sharp contrast with the pQCD predictions where it decreases. Hence we suggest that the measurement of an increase in  $d\sigma/dp_T$  for any heavy (Higgs or SUSY) particle production at the LHC as  $p_T$  increases can be a useful signature for black hole production.

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