Drell-Yan lepton angular distribution at small transverse momentum

Daniël Boer*
Department of Physics and Astronomy, Vrije Universiteit Amsterdam, De Boelelaan 1081, 1081 HV Amsterdam, The Netherlands

Werner Vogelsang†
Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA
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We investigate the dependence of the Drell-Yan cross section on lepton polar and azimuthal angles, as generated by the lowest-order QCD annihilation and Compton processes. We focus, in particular, on the azimuthal-angular distributions, which are of the form $\cos\phi$ and $\cos 2\phi$. At small transverse momentum q_T of the lepton pair, $q_T \ll Q$, with Q the pair mass, these terms are known to be suppressed relative to the ϕ-independent part of the Drell-Yan cross section by one or two powers of the transverse momentum. Nonetheless, as we show, like the ϕ-independent part they are subject to large logarithmic corrections, whose precise form, however, depends on the reference frame chosen. These logarithmic contributions ultimately require resummation to all orders in the strong coupling. We discuss the potential effects of resummation on the various angular terms in the cross section and on the Lam-Tung relation.

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I. INTRODUCTION

It is well known that the angular distribution of the leptons in the Drell-Yan process $P_1 P_2 \to \ell\bar{\ell}X$ may possess azimuthal asymmetries. These correspond to angular-dependent terms in the ratio of differential cross sections,

$$\frac{dN}{d\Omega} \equiv \left(\frac{d\sigma}{d^4q}\right)^{-1} \frac{d\sigma}{d\Omega d^4q}, \tag{1}$$

where q is the four-momentum of the virtual photon (or Z boson, if the energy is sufficiently high) decaying into the lepton pair, and $d\Omega \equiv d\cos\theta d\phi$ is the solid angle of the lepton ℓ in terms of its polar and azimuthal angles in the center-of-mass system (c.m.s.) of the lepton pair. As we shall review below, an analysis of the general Lorentz structure of the hadronic tensor yields the following angular structure [1,2]:

$$\frac{dN}{d\Omega} = \frac{3}{8\pi} \frac{W_T(1+\cos^2\theta) + W_L(1-\cos^2\theta) + W_\Delta \sin 2\theta \cos\phi + W_{\Delta\Delta}\sin^2\theta\cos 2\phi}{2W_T + W_L}. \tag{2}$$

Here the "structure functions" $W_{T,L,\Delta,\Delta\Delta}$ depend on the virtual photon's invariant mass Q, its transverse momentum Q_T, and its rapidity y. Within the lepton pair c.m.s. there is still freedom to choose the axes of the coordinate system, with respect to which the lepton angles are defined. The $W_{T,L,\Delta,\Delta\Delta}$ also depend on this choice. Equivalently to Eq. (2), one also often writes the lepton angular distribution as

$$\frac{dN}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}\left[1 + \lambda\cos^2\theta + \mu\sin 2\theta\cos\phi + \frac{\nu}{2}\sin^2\theta\cos 2\phi\right]. \tag{3}$$

One obviously has

$$\lambda = \frac{W_T - W_L}{W_T + W_L}, \qquad \mu = \frac{W_\Delta}{W_T + W_L}, \qquad \nu = \frac{2W_{\Delta\Delta}}{W_T + W_L}. \tag{4}$$

A third, again equivalent, parametrization for the lepton angular distribution [3] that is also often employed is

$$\frac{dN}{d\Omega} = \frac{3}{16\pi}\left[1 + \cos^2\theta + \frac{A_0}{2}(1 - 3\cos^2\theta) + A_1\sin 2\theta\cos\phi + \frac{A_2}{2}\sin^2\theta\cos 2\phi\right]. \tag{5}$$

Evidently,

$$\lambda = \frac{2-3A_0}{2+A_0}, \qquad \mu = \frac{2A_1}{2+A_0}, \qquad \nu = \frac{2A_2}{2+A_0}, \tag{6}$$

or

$$A_0 = \frac{2W_L}{2W_T + W_L}, \qquad A_1 = \frac{2W_\Delta}{2W_T + W_L}, \qquad A_2 = \frac{4W_{\Delta\Delta}}{2W_T + W_L}. \tag{7}$$

If Q is large, one may calculate the structure functions using parton model concepts, factorizing them into collinear convolutions of the parton distributions of the two scattering hadrons and partonic hard-scattering cross sec-

*Electronic address: D.Boer@few.vu.nl
†Electronic address: vogelsan@quark.phy.bnl.gov

tions that are amenable to QCD perturbation theory. For nonvanishing transverse momentum Q_T of the virtual photon, the lowest-order (LO) partonic processes are $q\bar{q} \to \gamma^* g$ and $qg \to \gamma^* q$. The contributions to the angular distributions by these processes have been calculated in Refs. [4–8]. Also, the next-to-leading-order (NLO) corrections have been derived [9,10].

Experimentally, for a given lepton pair invariant mass Q, the bulk of the Drell-Yan events is at rather low transverse momenta of the pair, $Q_T \ll Q$. It is this regime that is also most interesting from a theoretical point of view. "Intrinsic" transverse momenta of the initial partons may become relevant. Also, as $Q_T \ll Q$, fixed-order calculations of the partonic cross sections are bound to fail. When $Q_T \to 0$, gluon radiation is inhibited, so that only relatively soft gluons may be emitted into the final state. The cancellation of infrared singularities between real and virtual diagrams in the perturbative series then leaves behind large logarithmic remainders of the form $\alpha_s^k \ln^m(Q^2/Q_T^2)/Q_T^2$ in the cross section $d\sigma/d^4q$ at the kth order of perturbation theory, where $m = 1, \ldots, 2k - 1$. Ultimately, when $Q_T \ll Q$, α_s will not be useful anymore as the expansion parameter in the perturbative series since the logarithms will compensate for the smallness of α_s. Accordingly, in order to obtain a reliable estimate for the cross section, one has to sum up ("resum") the large logarithmic contributions to all orders in α_s. Techniques for this resummation are well established, starting with pioneering work mostly on the Drell-Yan process from the late 1970s to mid 1980s [11–18]. The "Collins-Soper-Sterman" (CSS) formalism [18] has become the standard method for Q_T resummation. It is formulated in impact-parameter (b) space, which guarantees conservation of the transverse momenta of the emitted soft gluons.

The large terms $\alpha_s^k \ln^m(Q^2/Q_T^2)/Q_T^2$ just described occur only in the part proportional to W_T of the cross section $d\sigma/d\Omega d^4q$. The function W_Δ is less singular by one power of Q_T at small Q_T, while W_L and $W_{\Delta\Delta}$ are suppressed even by a factor Q_T^2 relative to W_T. For W_Δ and $W_{\Delta\Delta}$ this can be understood as follows: at $Q_T = 0$ the angle ϕ cannot be defined, hence no azimuthal asymmetry could be observable, and, therefore, the asymmetries should smoothly go to zero in the limit of $Q_T \to 0$.

If one is just interested in the small-Q_T behavior of the cross section $d\sigma/d^4q$, it will be sufficient to take into account the resummation of the large logarithms in W_T. However, this may become different if one considers the parameters λ, μ, ν or $A_{0,1,2}$ defined above, which were the object of dedicated experimental studies in $\pi^- N$ Drell-Yan experiments about 20 years ago [19–21] (experimental evidence for nonzero W_L was already reported in [22]). Superficially, one might think that, in order to improve the theoretical prediction for these coefficients by resummation, it will be sufficient to perform the resummation in W_T alone. This is, in fact, what has been done in the literature so far in [23] for Drell-Yan and in [24–26] for the related "semi-inclusive deeply inelastic scattering" (SIDIS) process and has led, for example, to the claim in Ref. [23] that resummation has a strong effect on the perturbative-QCD results for λ, μ, and ν at small Q_T. However, even though W_L, W_Δ, and $W_{\Delta\Delta}$ are down by powers of Q_T as $Q_T \to 0$, they all individually may receive very similar large logarithmic corrections at small Q_T as W_T does. Therefore, it may well happen that in a ratio such as $A_2 = 4W_{\Delta\Delta}/(2W_T + W_L)$ the resummation effects cancel to a large degree, if not completely. In the present paper, we will investigate the small-Q_T behavior of W_L, W_Δ, and $W_{\Delta\Delta}$, and we will also address some qualitative consequences for the Q_T resummation for these and for the various coefficients constructed from them. A complication arises from the fact that the W_L, W_Δ, and $W_{\Delta\Delta}$ depend on the coordinate frame chosen. In a change of frame, terms proportional to Q_T or Q_T^2 may be redistributed among W_T and the $W_{L,\Delta,\Delta\Delta}$ and, because W_T is more singular at small Q_T, may alter the small-Q_T behavior of $W_{L,\Delta,\Delta\Delta}$. Our aim is to exhibit this feature very explicitly by considering two particular frames commonly used in the literature, in order to eliminate some misconceptions concerning the small-Q_T behavior and the resummation of angular distributions.

As discussed above, the CSS formalism [18] describes the behavior of the structure function W_T at small Q_T. It may be used to predict the Q_T-singular pieces arising at a given perturbative order. To LO, these have a particularly simple structure [17], involving the first-order expansion of the Sudakov form factor and the LO DGLAP [27] splitting functions. This structure is straightforwardly recovered from explicit calculations of the cross sections for the partonic reactions $q\bar{q} \to \gamma^* g$ and $qg \to \gamma^* q$. When we use these processes to determine the small-Q_T behavior of the other structure functions W_L, W_Δ, and $W_{\Delta\Delta}$, we find a closely related, but different, form that involves different "splitting functions." This is likely due to the fact that the *direction* of the observed transverse momentum matters for these structure functions. It is important to emphasize that the CSS formalism was not constructed to treat the directional dependence of the transverse momentum distribution. Even though we will discuss in this paper some features of the resummation of the $W_{L,\Delta,\Delta\Delta}$, we have not been able to organize their small-Q_T behavior beyond the leading logarithms into a resummed form in terms of an "extended" CSS formalism that goes beyond collinear factorization. We therefore hope that the study presented in this paper will serve as a motivation for a more general analysis of resummation in cases where the direction of the observed transverse momentum matters. In this respect the proper starting point will be to consider the factorizations and techniques put forward in Refs. [15,16,28].

The article is organized as follows. In Sec. II we express the azimuthal asymmetries in terms of structure functions in two different frames that are commonly used in the literature. This analysis is exact in Q_T and does not rely on the use of perturbative QCD, just on kinematical considerations. In Sec. III we discuss the LO perturbative-QCD results, and in Sec. IV we investigate their small-Q_T limit. In Sec. V we address some aspects of the resummation of the angular terms in this limit. We present our conclusions in Sec. VI.

II. AZIMUTHAL DEPENDENCES IN TERMS OF STRUCTURE FUNCTIONS

In this section we discuss in detail the ϕ dependence of the differential cross section in terms of the structure functions $W_{T,L,\Delta,\Delta\Delta}$. This is a purely kinematical analysis, which will set the notation to be used later on. We follow (in part) the notation of Refs. [2,29], but use the metric diag$(+---)$. In passing, we will point out some differences with results that appeared in the earlier literature.

We first define invariant structure functions $W_{1,2,3,4}$ through the following parametrization of the hadronic tensor:

$$W^{\mu\nu} = -\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)W_1 + \tilde{P}^\mu \tilde{P}^\nu W_2 - \frac{1}{2}(\tilde{P}^\mu \tilde{p}^\nu + \tilde{p}^\mu \tilde{P}^\nu)W_3 + \tilde{p}^\mu \tilde{p}^\nu W_4, \tag{8}$$

where $P = P_1 + P_2$, $p = P_1 - P_2$, $\tilde{P}^\mu = (P^\mu - q^\mu \times (P\cdot q)/Q^2)/\sqrt{s}$, $\tilde{p}^\mu = (p^\mu - q^\mu(p\cdot q)/Q^2)/\sqrt{s}$, with $P_{1,2}$ the initial hadron momenta and q the momentum of the virtual photon, $Q^2 \equiv q^2$. The hadronic c.m.s. energy is $\sqrt{s} = \sqrt{(P_1 + P_2)^2}$.

As mentioned in the Introduction, we will use two different reference frames: the so-called Collins-Soper (CS) frame [3] and the Gottfried-Jackson (GJ) frame [2] (also sometimes referred to as the "t-channel helicity frame"). Both frames are rest frames of the virtual photon or, equivalently, of the lepton pair. However, this property does not completely specify a frame, as one has a freedom in the choice of the coordinate axes, corresponding to rotations among the frames. In both the CS and GJ frames, a set of orthonormal axes (X, Y, Z, and T) are defined, and the hadronic tensor is reexpressed as

$$W^{\mu\nu} = -(g^{\mu\nu} - T^\mu T^\nu)(W_T + W_{\Delta\Delta}) - 2X^\mu X^\nu W_{\Delta\Delta} + Z^\mu Z^\nu(W_L - W_T - W_{\Delta\Delta}) - (X^\mu Z^\nu + Z^\mu X^\nu)W_\Delta. \tag{9}$$

In this way one has $W^\mu{}_\mu = -(2W_T + W_L)$, which is the quantity that appears in the differential cross section $d\sigma/d^4q$, the denominator of $dN/d\Omega$ in Eq. (1). Contracting with the straightforwardly calculated leptonic tensor, one finds the cross section in terms of the structure functions:

$$\frac{d\sigma}{d\Omega d^4q} = \frac{\alpha^2}{2(2\pi)^4 Q^2 s^2}\{W_T(1+\cos^2\theta) + W_L(1-\cos^2\theta) + W_\Delta \sin 2\theta \cos\phi + W_{\Delta\Delta}\sin^2\theta\cos 2\phi\}, \tag{10}$$

again valid in both frames. Here α is the electromagnetic coupling constant. From Eq. (10) one immediately reproduces Eq. (2). Higher harmonics in $\cos(n\phi)$ do not occur in the angular distribution due to the fact that the kinematics of the process are fully determined by only three momentum vectors, P_1, P_2, and q. In case the polarization of the leptons is not summed over or in case of electroweak corrections $\sin\phi$ and $\sin 2\phi$ terms will also be present, but these will not be considered here. We also note that the structure functions $W_{T,L,\Delta,\Delta\Delta}$ are associated with specific polarizations of the virtual photon [2]: $W_T = W^{1,1}$, $W_L = W^{0,0}$, $W_\Delta = (W^{0,1} + W^{1,0})/\sqrt{2}$, and $W_{\Delta\Delta} = W^{1,-1}$, where the first (second) superscript denotes the photon helicity in the amplitude (its complex conjugate) of the Drell-Yan process. The azimuthal dependence introduced by W_Δ and $W_{\Delta\Delta}$ therefore comes from their single- or double-spin-flip property, respectively.

In both frames, $T^\mu \equiv q^\mu/Q$ and $Y^\mu \equiv \epsilon^{\mu\nu\alpha\beta}X_\nu Z_\alpha T_\beta$. In the CS frame, the Z axis is defined as pointing in the direction that bisects the angle between the three-vectors $\vec{P}_2$ and $-\vec{P}_1$; see Fig. 1. This gives

$$Z^\mu \equiv \frac{1}{\sqrt{Q^2 + Q_T^2}}(q_p \tilde{P}^\mu + q_P \tilde{p}^\mu) = \frac{2}{s\sqrt{Q^2 + Q_T^2}}((P_2\cdot q)\tilde{P}_1^\mu - (P_1\cdot q)\tilde{P}_2^\mu), \tag{11}$$

where $Q_T^2 \equiv \boldsymbol{q}_T^2$ is the square of the transverse momentum $\boldsymbol{q}_T$ of the virtual photon with respect to the two hadron momenta. Furthermore, $q_P \equiv (P\cdot q)/\sqrt{s}$, $q_p \equiv -(p\cdot q)/\sqrt{s}$ (in the hadronic c.m.s. q_P is the energy of the virtual photon, while q_p is its z component). Finally, in Eq. (11) $\tilde{P}_i^\mu \equiv P_i^\mu - q^\mu(P_i\cdot q)/Q^2$ (note there is no factor $1/\sqrt{s}$ in the latter definition, compared to the definitions of $\tilde{P}$ and $\tilde{p}$ above). For the X axis one chooses

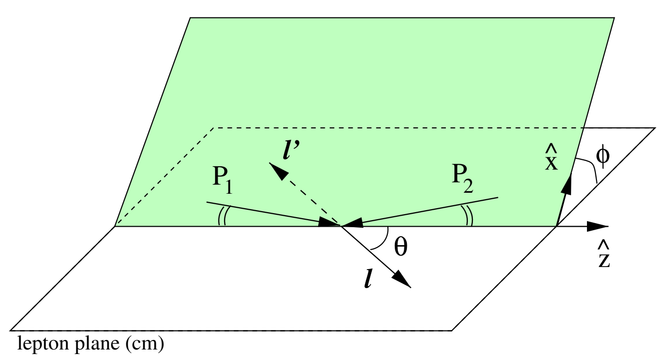

FIG. 1 (color online). The Collins-Soper frame.

$$X^\mu = -\frac{Q}{Q_T\sqrt{Q^2+Q_T^2}}(q_P\tilde{P}^\mu + q_p\tilde{p}^\mu)$$
$$= -\frac{2Q}{sQ_T\sqrt{Q^2+Q_T^2}}((P_2\cdot q)\tilde{P}_1^\mu + (P_1\cdot q)\tilde{P}_2^\mu). \quad (12)$$

In the GJ frame, the Z axis points in the direction of the three-vector $\vec{P}_1$ (see Fig. 2):

$$Z^\mu \equiv \frac{Q}{q_P - q_p}(\tilde{P}^\mu + \tilde{p}^\mu) = \frac{Q}{P_1\cdot q}\tilde{P}_1^\mu. \quad (13)$$

Furthermore,

$$X^\mu = -\frac{1}{Q_T(Q^2+Q_T^2)}((Q^2 q_P - Q_T^2 q_p)\tilde{P}^\mu + (Q^2 q_p - Q_T^2 q_P)\tilde{p}^\mu)$$
$$= -\frac{2Q}{sQ_T}((P_2\cdot Z)\tilde{P}_1^\mu - (P_1\cdot Z)\tilde{P}_2^\mu). \quad (14)$$

One finds that the three-vector components of the X and Z axes of the two frames are related by a rotation:

$$\vec{Z}_{\rm GJ} = \cos\gamma\vec{Z}_{\rm CS} + \sin\gamma\vec{X}_{\rm CS}, \quad (15)$$

$$\vec{X}_{\rm GJ} = -\sin\gamma\vec{Z}_{\rm CS} + \cos\gamma\vec{X}_{\rm CS}, \quad (16)$$

where

$$\cos\gamma = \frac{Q}{\sqrt{Q^2+Q_T^2}}, \qquad \sin\gamma = -\frac{Q_T}{\sqrt{Q^2+Q_T^2}}. \quad (17)$$

Thus, in the limit $Q_T \to 0$ the two sets of X, Z axes coincide.

Inserting the above definitions of the coordinate axes into the hadronic tensor in Eq. (9) and comparing to Eq. (8), one can derive for each frame the relations between the sets $W_{1,2,3,4}$ and $W_{T,L,\Delta,\Delta\Delta}$ of structure functions. For the CS frame this gives [this is Eq. (B2) of Ref. [2]]

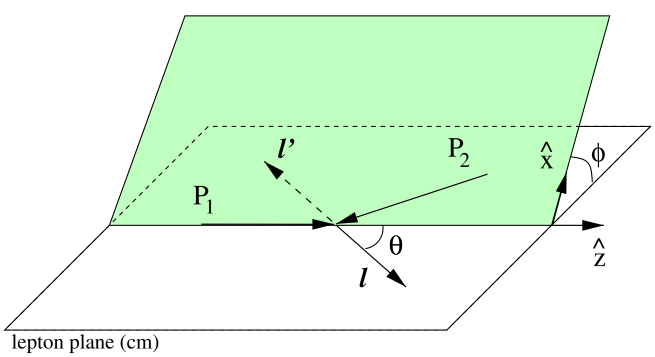

FIG. 2 (color online). The Gottfried-Jackson frame.

$$W_T = W_1 - W_{\Delta\Delta},$$
$$W_L = W_1 + \frac{1}{Q^2+Q_T^2}(q_p^2 W_2 + q_p q_P W_3 + q_P^2 W_4),$$
$$W_\Delta = -\frac{Q_T}{Q(Q^2+Q_T^2)}\left(q_p q_P(W_2+W_4) + \frac{1}{2}(q_P^2+q_p^2)W_3\right),$$
$$W_{\Delta\Delta} = -\frac{Q_T^2}{2Q^2(Q^2+Q_T^2)}(q_P^2 W_2 + q_p q_P W_3 + q_p^2 W_4), \quad (18)$$

while for the GJ frame one finds [this is a corrected version of Eq. (B3) of [2] and also of a corresponding expression in reference-note 5 of [29]]

$$W_T = W_1 - W_{\Delta\Delta},$$
$$W_L = W_1 + (\alpha^2 W_2 + \alpha\beta W_3 + \beta^2 W_4),$$
$$W_\Delta = \frac{Q_T}{q_P - q_p}\left(\alpha\left(W_2 + \frac{1}{2}W_3\right) + \beta\left(W_4 + \frac{1}{2}W_3\right)\right),$$
$$W_{\Delta\Delta} = -\frac{Q_T^2}{2(q_P-q_p)^2}(W_2 + W_3 + W_4), \quad (19)$$

where

$$\alpha \equiv \frac{q_P}{Q} - \frac{Q}{q_P - q_p}, \qquad \beta \equiv \frac{q_p}{Q} - \frac{Q}{q_P - q_p}. \quad (20)$$

The relation between the structure functions in the CS frame and the GJ one is given by the following linear transformation:

$$\begin{pmatrix} W_T \\ W_L \\ W_\Delta \\ W_{\Delta\Delta} \end{pmatrix}_{\rm GJ} = \frac{1}{1+\rho^2}\begin{pmatrix} 1+\frac{1}{2}\rho^2 & \frac{1}{2}\rho^2 & -\rho & \frac{1}{2}\rho^2 \\ \rho^2 & 1 & 2\rho & -\rho^2 \\ \rho & -\rho & 1-\rho^2 & -\rho \\ \frac{1}{2}\rho^2 & -\frac{1}{2}\rho^2 & \rho & 1+\frac{1}{2}\rho^2 \end{pmatrix}\begin{pmatrix} W_T \\ W_L \\ W_\Delta \\ W_{\Delta\Delta} \end{pmatrix}_{\rm CS}, \quad (21)$$

where $\rho = Q_T/Q$. Expressed in terms of the angle γ defined above, the matrix in the above equation is identical to the one presented in [29] [their Eq. (4)], apart from a sign in the third entry of the first row.

For future reference, we also give the transformation between the sets of coefficients λ, μ, ν in the two frames, which were defined in Eq. (4). From Eq. (21) one finds

$$\begin{pmatrix} \lambda \\ \mu \\ \nu \end{pmatrix}_{\rm GJ} = \frac{1}{\Delta_{\rm CS}}\begin{pmatrix} 1-\frac{1}{2}\rho^2 & -3\rho & \frac{3}{4}\rho^2 \\ \rho & 1-\rho^2 & -\frac{1}{2}\rho \\ \rho^2 & 2\rho & 1+\frac{1}{2}\rho^2 \end{pmatrix}\begin{pmatrix} \lambda \\ \mu \\ \nu \end{pmatrix}_{\rm CS}, \quad (22)$$

where

$$\Delta = 1 + \rho^2 + \tfrac{1}{2}\rho^2\lambda + \rho\mu - \tfrac{1}{4}\rho^2\nu. \quad (23)$$

The reverse transformation from the GJ frame to the CS frame is the same upon replacement of $\rho \to -\rho$ (and exchange of the labels CS and GJ). This is in agreement with Ref. [19] [note that what is referred to as the "u-channel" (UC) frame in that reference corresponds to the GJ frame used here, which accounts for the $\rho \to -\rho$ difference with respect to our Eq. (22)]. One notes that, if $\rho \to 0$, the rotation matrix becomes the unit matrix, as expected. All results presented so far are exact in ρ.

III. AZIMUTHAL ASYMMETRIES FROM LO PERTURBATIVE-QCD PROCESSES

The structure functions can be calculated employing collinear factorization and QCD perturbation theory. One writes down an expression analogous to (8) or (9) for the *partonic* tensor, in terms of partonic structure functions $\hat{W}_i$. The hadronic structure functions are obtained as convolutions of the partonic ones with the appropriate parton distribution functions. The $\hat{W}_i$ may be evaluated directly from partonic hard-scattering processes, which at LO are the annihilation (or gluon bremsstrahlung) reaction $q\bar{q} \to \gamma^* g$ and the (QCD) Compton process $qg \to \gamma^* q$. The annihilation process is the dominant process in $\pi^- p$ Drell-Yan scattering (likewise in the $\bar{p}p$ Drell-Yan process). The Compton process is dominant in the Drell-Yan process in pp scattering at large Q_T.

In the following, we will first focus on the annihilation process, which for our purposes is most relevant, since the Compton process is subleading in the region of $Q_T \ll Q$ we are mostly interested in. For the process $q\bar{q} \to \gamma^* g$ one has

$$\frac{W_i}{x_1 x_2} = \int d\xi_1 \int d\xi_2 \delta((\xi_1 - x_1)(\xi_2 - x_2) - Q_T^2/s) \times \sum_a e_a^2 q_a(\xi_1, \mu)\bar{q}_a(\xi_2, \mu) \frac{\hat{W}_i}{\xi_1 \xi_2}, \tag{24}$$

where we define x_1 and x_2 by writing $q = x_1 P_1^+ + x_2 P_2^- + q_T$, with the light-cone components of any four-vector v given by $v^\pm \equiv (v^0 \pm v^3)/\sqrt{2}$. We have $x_1 = (q_P + q_p)/\sqrt{s}$, $x_2 = (q_P - q_p)/\sqrt{s}$ with q_P, q_p as introduced in the previous section [note that in the hadronic c.m.s. one has $q_P = (s + Q^2)/2\sqrt{s}$, $q_p = \sqrt{q_P^2 - Q^2 - Q_T^2} = \sqrt{(s - Q^2)^2/4s - Q_T^2}$]. For the partonic collinear momenta we define $p_1^+ = \xi_1 P_1^+$ and $p_2^- = \xi_2 P_2^-$. The delta function in (24) expresses the on-mass-shell condition for the outgoing "unobserved" gluon in the process $q\bar{q} \to \gamma^* g$. Equation (24) contains the appropriate sum over all quark and antiquark flavors a, each with their corresponding parton distributions $q_a(x, \mu)$ and $q_{\bar{a}}(x, \mu) = \bar{q}_a(x, \mu)$, respectively, where $\mu \sim Q$ is the factorization scale; e_a^2 is the quark's squared electromagnetic charge. Note that, since we are only interested in the lepton angular distribution $dN/d\Omega$, which is a ratio of cross sections, we are free to adjust the overall normalization of the tensors, which we do in such a way as to simplify the formulas. The annihilation process then has the following partonic structure functions [5,6]:

$$\begin{aligned} \hat{W}_1 &= \frac{\alpha_s}{2\pi} \frac{C_F s}{\xi_1 \xi_2 Q_T^2} \left[\frac{Q^4}{s^2} - \frac{2Q_T^2}{s} \xi_1 \xi_2 + \xi_1^2 \xi_2^2 \right] \\ &= \frac{\alpha_s}{2\pi} \frac{C_F}{\hat{t}\hat{u}} [(\hat{t} - Q^2)^2 + (\hat{u} - Q^2)^2], \\ \hat{W}_2 &= -\frac{\alpha_s}{2\pi} \frac{C_F Q^2}{\xi_1 \xi_2 Q_T^2} [\xi_1^2 + \xi_2^2] = -\frac{\alpha_s}{2\pi} \frac{C_F s Q^2}{\hat{t}\hat{u}} [\xi_1^2 + \xi_2^2], \\ \hat{W}_3 &= \frac{C_F \alpha_s}{\pi} \frac{Q^2}{\xi_1 \xi_2 Q_T^2} [\xi_1^2 - \xi_2^2] = \frac{\alpha_s}{2\pi} \frac{2C_F s Q^2}{\hat{t}\hat{u}} [\xi_1^2 - \xi_2^2], \\ \hat{W}_4 &= \hat{W}_2, \end{aligned} \tag{25}$$

where $C_F = 4/3$, and where $\hat{s} = (\xi_1 P_1 + \xi_2 P_2)^2 = \xi_1 \xi_2 s$, $\hat{t} = (q - \xi_1 P_1)^2$, and $\hat{u} = (q - \xi_2 P_2)^2$. Constructing via Eq. (18) the structure functions $W_{T,L,\Delta,\Delta\Delta}$ and inserting these into Eq. (2), one finds [6–8] *in the CS frame*

$$\begin{aligned} \frac{dN}{d\Omega} = \frac{3}{16\pi} \bigg[&\frac{Q^2 + \frac{3}{2}Q_T^2}{Q^2 + Q_T^2} + \frac{Q^2 - \frac{1}{2}Q_T^2}{Q^2 + Q_T^2} \cos^2\theta_{\mathrm{CS}} \\ &+ \frac{Q_T Q}{Q^2 + Q_T^2} K(x_1, x_2, Q_T/s) \sin 2\theta_{\mathrm{CS}} \cos\phi_{\mathrm{CS}} \\ &+ \frac{1}{2} \frac{Q_T^2}{Q^2 + Q_T^2} \sin^2\theta_{\mathrm{CS}} \cos 2\phi_{\mathrm{CS}} \bigg], \end{aligned} \tag{26}$$

where the function $K(x_1, x_2, Q_T/s)$ is given by

$$K(x_1, x_2, Q_T/s) = \frac{\int d\xi_1 \int d\xi_2 \delta((\xi_1 - x_1)(\xi_2 - x_2) - Q_T^2/s) \sum_a e_a^2 q_a(\xi_1, \mu)\bar{q}_a(\xi_2, \mu)(x_1^2/\xi_1^2 - x_2^2/\xi_2^2)}{\int d\xi_1 \int d\xi_2 \delta((\xi_1 - x_1)(\xi_2 - x_2) - Q_T^2/s) \sum_a e_a^2 q_a(\xi_1, \mu)\bar{q}_a(\xi_2, \mu)(x_1^2/\xi_1^2 + x_2^2/\xi_2^2)}. \tag{27}$$

As one can see, for the annihilation contribution in the CS frame all effects of the partonic light-cone momentum fractions and the parton densities cancel in $dN/d\Omega$, except for the term involving $\sin 2\theta_{\mathrm{CS}} \cos\phi_{\mathrm{CS}}$ associated with the ratio $W_\Delta/(2W_T + W_L)$. As a consequence, the coefficients λ and ν defined in Eq. (4) are also free of any dependence on the parton distributions to this order [6]. One reads off

$$\lambda_{\mathrm{CS}} = \frac{Q^2 - \frac{1}{2}Q_T^2}{Q^2 + \frac{3}{2}Q_T^2}, \qquad \nu_{\mathrm{CS}} = \frac{Q_T^2}{Q^2 + \frac{3}{2}Q_T^2}, \tag{28}$$

so that

$$1 - \lambda_{\mathrm{CS}} - 2\nu_{\mathrm{CS}} = 0, \tag{29}$$

which is the well-known Lam-Tung (LT) relation [2,30]. It is equivalent to $W_L = 2W_{\Delta\Delta}$ and $A_0 = A_2$; its origin is the equality of $\hat{W}_2$ and $\hat{W}_4$ in Eq. (25).

In the Gottfried-Jackson frame one finds that all terms in $dN/d\Omega$ depend on the parton densities and do not have particularly simple expressions. Remarkably, the LT relation continues to hold in this frame, however. This may be seen from Eq. (22) which readily shows that the LT relation indeed holds in the GJ frame if it holds in the CS frame, and vice versa. In fact, it turns out that the LT relation holds for any definition of the lepton pair c.m.s. frame to this order.

Next, we consider the $qg \to \gamma^* q$ subprocess. The contributions to the partonic structure functions for $q(\xi_1)g(\xi_2)$ are [5]:

$$\begin{aligned}
\hat{W}_1 &= -\frac{\alpha_s}{2\pi}\frac{2T_R}{\hat{s}\,\hat{t}}[(\hat{s}-Q^2)^2 + (\hat{t}-Q^2)^2],\\
\hat{W}_2 &= \frac{\alpha_s}{2\pi}\frac{2T_R s Q^2}{\hat{s}\,\hat{t}}[2\xi_1(\xi_1+\xi_2)+\xi_2^2],\\
\hat{W}_3 &= \frac{\alpha_s}{2\pi}\frac{4T_R s Q^2}{\hat{s}\,\hat{t}}[\xi_2^2 - 2\xi_1^2],\\
\hat{W}_4 &= \frac{\alpha_s}{2\pi}\frac{2T_R s Q^2}{\hat{s}\,\hat{t}}[2\xi_1(\xi_1-\xi_2)+\xi_2^2],
\end{aligned} \tag{30}$$

where $T_R = 1/2$. For the contribution from $g(\xi_1)q(\xi_2)$ one has to interchange ξ_1 and ξ_2, $\hat{t}$ and $\hat{u}$, and, in addition, change the sign of $\hat{W}_3$.

From the expressions in (30) one finds that for the Compton process the ratios λ, μ, ν all depend on the parton distribution functions. As discussed in Ref. [8], there is, in fact, no frame in which any of the azimuthal distributions for this process become independent of the parton densities (only for very large Q_T does this become approximately the case in the CS frame). Even if there were a frame in which this happened, the result would have no practical relevance, since one would always need to add the contribution by the annihilation process in the numerator and the denominator of the azimuthal ratios, which would spoil the cancellation of the parton distributions anyway. In fact, even the cancellation of the parton densities in $\lambda_{\rm CS}$ and $\nu_{\rm CS}$ in Eq. (28) for the annihilation process is of limited use, except in a pure flavor nonsinglet situation where the Compton process is absent.

However, despite the fact that $\hat{W}_2 \neq \hat{W}_4$, the LT relation does hold for the Compton process as well, and therefore *for the complete LO Drell-Yan cross section* [5,30]. In the next section we will verify the LT relation very explicitly in the small-Q_T limit. However, it is valid at LO regardless of the value of Q_T. It is known to be (mildly) broken by NLO corrections [9,10]. Experimentally the LT relation was found to be rather strongly violated in the $\pi^- p$ Drell-Yan process [19–21], in disagreement (both in magnitude and in sign) with the slight violation predicted at NLO. This has prompted much theoretical work [31–39], offering explanations that go beyond the framework of collinear factorization and perturbative QCD to which we restrict ourselves in this paper.

As a final point in this discussion of the LO contributions, we would like to stress that the above discussion is not specific to the Drell-Yan process, but also applies to similar processes like SIDIS [24–26,40] and back-to-back hadron production in two-jet events in electron-positron annihilation [41], when appropriate definitions of the frames and coordinate axes are used.

IV. SMALL-Q_T LIMIT OF THE STRUCTURE FUNCTIONS W_T, W_L, W_Δ, $W_{\Delta\Delta}$ AT LO

The LO results presented in the previous section should provide good approximations for the angular distributions in the Drell-Yan process at large transverse momentum, $Q_T \sim Q$. We shall now investigate their behavior at small Q_T. This is extracted most conveniently by using the expansion [24]

$$\delta((1-z_1)(1-z_2) - Q_T^2/\hat{s}) = \frac{\delta(1-z_1)}{(1-z_2)_+} + \frac{\delta(1-z_2)}{(1-z_1)_+} - \delta(1-z_1)\delta(1-z_2)\ln\rho^2 + \mathcal{O}(\rho^2) \tag{31}$$

for the delta function in Eq. (24), where $z_i \equiv x_i/\xi_i$. Here, the "plus" distributions are defined as usual for an integral from x to 1 as

$$\int_x^1 \frac{dz}{z}\frac{f(z)}{(1-z)_+} = \int_x^1 \frac{dz}{z}\frac{f(z)-f(1)}{1-z} + f(1)\ln\frac{1-x}{x}, \tag{32}$$

for any suitably regular function f. Inserting the structure functions $\hat{W}_i$ of Eqs. (25) and (30) into Eqs. (18), and expanding for small Q_T with the help of (31), one finds in the CS frame

$$\begin{aligned}
W_{T,\rm CS} &= \frac{\alpha_s}{2\pi}\frac{1}{\rho^2}[-C_F(2\ln\rho^2+3)q(x_1)\bar{q}(x_2) + q(x_1)(P_{qq}\otimes\bar{q})(x_2) + (P_{qq}\otimes q)(x_1)\bar{q}(x_2) + q(x_1)(P_{qg}\otimes g)(x_2)\\
&\quad + (P_{qg}\otimes g)(x_1)\bar{q}(x_2) + \mathcal{O}(\rho^2)],\\
W_{L,\rm CS} &= 2W_{\Delta\Delta,\rm CS} = \frac{\alpha_s}{2\pi}[-C_F(2\ln\rho^2+3)q(x_1)\bar{q}(x_2) + q(x_1)(P_{qq}\otimes\bar{q})(x_2) + (P_{qq}\otimes q)(x_1)\bar{q}(x_2)\\
&\quad + q(x_1)(P'_{qg}\otimes g)(x_2) + (P'_{qg}\otimes g)(x_1)\bar{q}(x_2) + \mathcal{O}(\rho^2)],\\
W_{\Delta,\rm CS} &= \frac{\alpha_s}{2\pi}\frac{1}{\rho}[q(x_1)(\tilde{P}_{qq}\otimes\bar{q})(x_2) - (\tilde{P}_{qq}\otimes q)(x_1)\bar{q}(x_2) + q(x_1)(\tilde{P}_{qg}\otimes g)(x_2) - (\tilde{P}_{qg}\otimes g)(x_1)\bar{q}(x_2) + \mathcal{O}(\rho^2)],
\end{aligned} \tag{33}$$

where in each case we have kept the terms with the leading-power behavior at small Q_T. Furthermore, we have for notational simplicity suppressed the factorization scale $\mu \sim Q$ in the parton distributions, as well as the sums over flavors, and we have defined the usual convolutions

$$(\mathcal{P} \otimes f)(x_1) \equiv \int_{x_1}^{1} \frac{dx}{x} \mathcal{P}(x) f\left(\frac{x_1}{x}\right), \tag{34}$$

with $\mathcal{P}$ variously one of the well-known [27] LO splitting functions

$$\begin{aligned} P_{qq}(x) &= C_F\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right], \\ P_{qg}(x) &= T_R[x^2 + (1-x)^2], \end{aligned} \tag{35}$$

or one of

$$\begin{aligned} P'_{qg}(x) &\equiv P_{qg}(-x), \qquad \tilde{P}_{qq}(x) \equiv C_F(1+x), \\ \tilde{P}_{qg}(x) &\equiv T_R(1-2x^2). \end{aligned} \tag{36}$$

Note that one could cancel the term $-3C_F q(x_1)\bar{q}(x_2)$ in $W_{T,\mathrm{CS}}$ against the contributions by the $3C_F\delta(1-x)/2$ term in the splitting function P_{qq} in Eq. (35). We have, however, kept the term in order to have the full splitting function in (35), and also because the term $-C_F(2\ln\rho^2 + 3)$ is the well-known first-order contribution to the Sudakov form factor. We also note that we could have first taken the small-Q_T limit of the structure functions $W_{1,2,3,4}$ in Eqs. (24) and (25) and then inserted the result into (18). In that case, since all the W_i have the overall power Q_T^{-2} and may be multiplied by powers of Q_T in the transformation (18), it would have been crucial to keep the first subleading term proportional to ρ^0 in the W_i. Otherwise, one would obtain an incorrect result.

As can be seen in Eqs. (33), the annihilation process makes a logarithmic contribution to each of the structure functions W_T, W_L, and $W_{\Delta\Delta}$, on top of their nominal power in Q_T. The Compton process, on the other hand, does not produce this leading-logarithmic behavior. It is also interesting to note that, unlike the other structure functions, $W_{\Delta,\mathrm{CS}}$ does not receive a logarithmic contribution at all. This turns out to be a result specific to the CS frame. In case of W_T, the structure at small Q_T is well understood in terms of the CSS formalism [18], as we shall briefly review in the next section. The LO small-Q_T expressions for the other structure functions are new. We note that the Lam-Tung relation $W_L = 2W_{\Delta\Delta}$ of course still holds in (33).

The small-Q_T expressions for the structure functions $W_{T,L,\Delta,\Delta\Delta}$ in the GJ frame can be obtained in the same way, by using Eqs. (19), or alternatively Eqs. (21), and we find

$$\begin{aligned} W_{T,\mathrm{GJ}} &= W_{T,\mathrm{CS}}, \\ W_{L,\mathrm{GJ}} &= 2W_{\Delta\Delta,\mathrm{GJ}} = 2\frac{\alpha_s}{2\pi}[-C_F(2\ln\rho^2+3)q(x_1)\bar{q}(x_2) + q(x_1)(P^+_{qq}\otimes\bar{q})(x_2) + (P^-_{qq}\otimes q)(x_1)\bar{q}(x_2) \\ &\quad + q(x_1)(P'^+_{qg}\otimes g)(x_2) + (P'^-_{qg}\otimes g)(x_1)\bar{q}(x_2) + \mathcal{O}(\rho^2)], \\ W_{\Delta,\mathrm{GJ}} &= \frac{\alpha_s}{2\pi}\frac{1}{\rho}[-C_F(2\ln\rho^2+3)q(x_1)\bar{q}(x_2) + q(x_1)(P^+_{qq}\otimes\bar{q})(x_2) + (P^-_{qq}\otimes q)(x_1)\bar{q}(x_2) + q(x_1)(\tilde{P}^+_{qg}\otimes g)(x_2) \\ &\quad + (\tilde{P}^-_{qg}\otimes g)(x_1)\bar{q}(x_2) + \mathcal{O}(\rho^2)], \end{aligned} \tag{37}$$

where

$$\begin{aligned} P^\pm_{qq}(x) &\equiv P_{qq}(x) \pm \tilde{P}_{qq}(x), \qquad P'^\pm_{qg}(x) \equiv P_{qg}(x) + P'_{qg}(x) \pm 2\tilde{P}_{qg}(x) = \begin{cases} 4T_R \\ 8T_R x^2 \end{cases}, \\ \tilde{P}^\pm_{qg}(x) &\equiv P_{qg}(x) \pm \tilde{P}_{qg}(x) = \begin{cases} 2T_R(1-x) \\ 2T_R x(2x-1) \end{cases}, \end{aligned} \tag{38}$$

with $P_{qq}(x)$, $\tilde{P}_{qq}(x)$, $P_{qg}(x)$, $\tilde{P}_{qg}(x)$, $P'_{qg}(x)$ as given above. As one can see, in the GJ frame, all structure functions receive logarithmic contributions at small Q_T. We observe that apart from $W_{T,\mathrm{GJ}}$ none of the functions contains subleading terms (i.e., terms nonlogarithmic in ρ) that involve only the usual splitting functions P_{qq} and P_{qg} of Eq. (35). Therefore, W_L and $W_{\Delta\Delta}$ are not proportional to W_T at subleading order for small ρ, not even if one restricts to the annihilation process. The LT relation holds as before.

Comparing the small-Q_T behavior in the CS and GJ frames we find that the rotation between the two frames simply "reshuffles" the various splitting functions appearing in the $W_{T,L,\Delta,\Delta\Delta}$. For example, in the annihilation contribution the functions contributing to the subleading terms in $W_{L,\Delta,\Delta\Delta}$ are P_{qq} and $\pm\tilde{P}_{qq}$ in the CS frame, but $P_{qq} \pm \tilde{P}_{qq}$ in the GJ one. This pattern, which also extends to the Compton part, can be verified by inspection of Eq. (21).

We finally note that the nonstandard splitting functions in the above expressions for the small-Q_T limit are associated with the polarization states of the virtual photon contributing to the various structure functions. Only if the photons in the amplitude and its complex conjugate both have the same, and physical (transverse), polarizations does one recover the ordinary DGLAP splitting functions at small Q_T. This is only the case for W_T, cf. Sec. II.

V. EFFECTS OF RESUMMATION AT SMALL Q_T

The small-Q_T behavior of W_T is predicted by the CSS formalism [18] which resums its singular behavior at $Q_T/Q \to 0$ to all orders in the strong coupling constant. The CSS resummation is formulated in impact-parameter space. Schematically, keeping only those terms in the formalism that play a role for our present study, one has

$$W_T = \int \frac{d^2b}{4\pi} e^{i\vec{q}_T\cdot\vec{b}} \sum_a e_a^2 q_a(x_1, b_0/b)\bar{q}_a(x_2, b_0/b)e^{S(b,Q)}. \tag{39}$$

Here, $S(b, Q)$ is the Sudakov form factor, given by

$$S(b, Q) = -\int_{b_0^2/b^2}^{Q^2} \frac{dk_T^2}{k_T^2}\left[A(\alpha_s(k_T))\ln\left(\frac{Q^2}{k_T^2}\right) + B(\alpha_s(k_T))\right], \tag{40}$$

where $b_0 = 2e^{-\gamma_E}$ with γ_E the Euler constant, and where the functions A and B have perturbative expansions of the form

$$A(\alpha_s) = \sum_{k=1}^{\infty} A_k\left(\frac{\alpha_s}{\pi}\right)^k, \qquad B(\alpha_s) = \sum_{k=1}^{\infty} B_k\left(\frac{\alpha_s}{\pi}\right)^k. \tag{41}$$

Here we step over the fact that the b integration also may require the introduction of a nonperturbative Sudakov form factor, and simply focus on the perturbative part.

For our purposes we only need the coefficients A_1 and B_1, which read

$$A_1 = C_F, \qquad B_1 = -\tfrac{3}{2}C_F. \tag{42}$$

The term $\propto A_1$ in the Sudakov exponent generates the leading logarithms, which in b space are of the form $\alpha_s^k \ln^{2k}(bQ)$, corresponding to $\alpha_s^k \ln^{2k-1}(Q^2/Q_T^2)/Q_T^2$ in Q_T space. Next-to-leading logarithms (NLL) are generated by the term $\propto B_1$, and by the running of the strong coupling and of the parton distribution functions in (39). In the CSS-resummed expression, the factorization scale is b_0/b. Using the customary DGLAP evolution equation, the parton densities at this scale may be expressed by their values at the scale $\mu \sim Q$. Expanding the corresponding evolution matrix for the parton densities and the Sudakov exponential in α_s and considering only the first-order term, we recover the expression for $W_{T,\mathrm{CS}}$ given in Eq. (33) [or (37)]. The terms of $W_{T,\mathrm{CS}}$ in (33) that involve the gluon distribution are generated by the singlet mixing in the evolution of the parton distributions between scales b_0/b and Q. Clearly, for $W_{T,\mathrm{CS}}$ in Eq. (33) to be reproduced by the CSS formalism, it is crucial that the splitting functions in its expression are the usual DGLAP splitting functions [27] P_{qq} and P_{qg} that are associated with the evolution of quark and antiquark distributions.

This implies that the resummation of the structure functions W_L, W_Δ, and $W_{\Delta\Delta}$ will not precisely follow the CSS formalism. Taking their expressions (33) in the CS frame as an example, we notice that the first-order expansion of the Sudakov form factor appears in $W_{L,\mathrm{CS}}$ and $W_{\Delta\Delta,\mathrm{CS}}$, indicating that the resummed expressions for these will contain the exponential of $S(b, Q)$, as the one for W_T does, and hence have the same leading logarithms. However, the splitting functions involved in the nonlogarithmic pieces are not the usual ones, as we already observed in the previous section. For $W_{\Delta,\mathrm{CS}}$, even the Sudakov part is absent, and the splitting functions are different again. All this means that beyond NLL the resummed expressions for $W_{L,\mathrm{CS}}$, $W_{\Delta,\mathrm{CS}}$, and $W_{\Delta\Delta,\mathrm{CS}}$ will not organize into the structure given in Eq. (39). They likely will have a similar structure, but contain additional terms depending on the direction of $\boldsymbol{b}$. To be clear, this is not a problem: while the presence of the true spitting functions in the leading term W_T at small Q_T is *required* by (collinear) factorization, this is not the case for the other structure functions. In other words, the splitting functions in these are not associated with standard parton evolution or collinear singularities. Therefore, the structure we find only means that the resummation of the $W_{L,\mathrm{CS}}$, $W_{\Delta,\mathrm{CS}}$, and $W_{\Delta\Delta,\mathrm{CS}}$ (and likewise in the GJ frame) is more complicated beyond the leading logarithms. We do not address their full NLL resummation in this work, but note that the techniques of Refs. [15,16,28] that go beyond collinear factorization should prove useful for this purpose.

Even without performing the full resummation, we can make some qualitative observations regarding the effects of resummation. The first observation from Eqs. (33) and (37) is that the LT relation will not be affected by resummation. Next, we focus on the angular coefficients λ, μ, ν or A_0, A_1, A_2 introduced in Eqs. (3) and (5), respectively, which are ratios of the structure functions. Because of the structure of the leading logarithms, we expect that in the CS frame λ and ν, and A_0 and A_2, will be rather insensitive to resummation effects. Note that, if the annihilation process alone contributes (which could, in principle, be realized by considering a flavor nonsinglet combination of cross sections), resummation effects cancel identically in λ_{CS} and ν_{CS}, and they retain their LO forms given in Eq. (28) even after resummation. On the other hand, μ and A_1, for which there are no leading logarithms in the numerator because of their absence in $W_{\Delta,\mathrm{CS}}$, will be subject to substantial modification by resummation. In the GJ frame all ratios λ, μ, ν will be affected by NLL resummation effects. These need not be small, in particular, if the overall effects of resummation on W_T are themselves large.

Next, we confront our findings with results of the previous literature. Chiappetta and Le Bellac [23] were the first to study the effects of resummation on the azimuthal asymmetries in the Drell-Yan process. Working in the CS frame, they argued that, since the structure functions W_L, W_Δ, and $W_{\Delta\Delta}$ are less singular than W_T, only the latter requires resummation. Therefore, they took into account resummation only in the denominators of the A_i, while for the numerators they employed the LO expressions. In this way they found large effects of resummation on the A_i. In light of our discussion above, the neglect of resummation in the numerators of the A_i is not justified.

Similarly, in studies [24–26] of resummation effects in semi-inclusive DIS only the resummation of the ϕ-independent terms was taken into account. Specifically, resummation was not applied in studies of the ratio $\langle\cos 2\phi\rangle/\langle\cos\phi\rangle$. Here, a Gottfried-Jackson type of frame (one of the hadron momenta defined the Z axis) was used. Our results above show that the numerator and denominator of this quantity will have a somewhat different resummation, even though it may well turn out to be the case that the residual effects on the ratio are small. It would be desirable to revisit this quantity in the framework of a full NLL resummation study. This also applies to azimuthal asymmetries in polarized scattering [42].

VI. SUMMARY

We conclude by summarizing our main observations. We have studied the structure functions $W_{T,L,\Delta,\Delta\Delta}$ in the Drell-Yan process, on the basis of the contributions by the annihilation process $q\bar{q} \to \gamma^* g$ and the Compton channel $qg \to \gamma^* q$. The structure functions W_Δ and $W_{\Delta\Delta}$, in particular, generate azimuthal asymmetries in the angular distribution of the produced leptons. The structure functions depend, in general, on the choice of coordinate axes, and we have investigated the results in the Collins-Soper and the Gottfried-Jackson frames.

We have focused on the behavior of the structure functions at small transverse momentum Q_T of the lepton pair. We have recovered the known small-Q_T behavior of W_T, in terms of the leading-order DGLAP splitting functions and of the first-order expansion of the Sudakov form factor. For the other structure functions, which are all nominally suppressed by one or two powers of Q_T/Q with respect to W_T, we have found that they, too, in general, have large logarithmic terms at small Q_T, whose form depends on the frame chosen. We are not aware that this feature was pointed out previously in the literature. The small-Q_T structure we find for $W_{L,\Delta,\Delta\Delta}$ differs from that of W_T, however. In the CS frame, W_L and $W_{\Delta\Delta}$ receive large leading-logarithmic corrections identical to the ones in W_T. By contrast, these are absent in W_Δ. In the GJ frame, all structure functions have leading-logarithmic terms. Both frames have in common that the subleading terms in $W_{L,\Delta,\Delta\Delta}$ are different from those in W_T. This implies that the next-to-leading-logarithmic resummation at small Q_T must proceed differently from that for W_T given by the CSS formalism which uses a collinear expansion.

Without actually deriving the NLL resummation, we have discussed some generic features that we expect from it. The Lam-Tung relation, which is an exact property of the full leading-order contributions to the structure functions, independent of the coordinate frame, is not affected by resummation, even though the individual terms entering in it are affected. Furthermore, in the CS frame, the ratios $\lambda = (W_T - W_L)/(W_T + W_L)$ and $\nu = 2W_{\Delta\Delta}/(W_T + W_L)$ will *not* receive large corrections from resummation at small Q_T. In particular, when restricting to the annihilation process, resummation has no effect on the $\cos 2\phi$ asymmetry. This observation is especially relevant for the Drell-Yan process in πp or $p\bar{p}$ scattering. The ratio $\mu = W_\Delta/(W_T + W_L)$, on the other hand, will be subject to considerable resummation effects, due to the lack of the leading-logarithmic terms in its numerator. In the GJ frame all ratios λ, μ, ν will be affected by NLL resummation effects, which are not necessarily small. Similar conclusions were drawn for the ratios A_i.

We finally emphasize again that we hope that our study will provide motivation for a development of the full NLL resummation for the structure functions $W_{L,\Delta,\Delta\Delta}$. Given the experimental information available on the Drell-Yan process, on SIDIS, and on e^+e^- annihilation, this would also have clear phenomenological relevance. Of further interest would be studies of the angular distributions integrated over Q_T [43]. Here, "threshold-type" logarithms may emerge [44,45], whose structure and resummation in azimuthal distributions have so far not been investigated.

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