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Determination of the initial flavor composition of ultrahigh-energy neutrino fluxes with neutrino telescopes

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We propose a simple but useful parametrization of the flavor composition of ultrahigh-energy neutrino fluxes produced from distant astrophysical sources: $\phi_e:\phi_\mu:\phi_\tau = \sin^2\xi\cos^2\zeta:\cos^2\xi\cos^2\zeta:\sin^2\zeta$. We show that it is possible to determine or constrain ξ and ζ by observing two independent neutrino flux ratios at the second-generation neutrino telescopes, provided three neutrino mixing angles and the Dirac *CP*-violating phase have been well measured in neutrino oscillations. Any deviation of ζ from zero will signify the existence of cosmic ν_τ and $\bar{\nu}_\tau$ neutrinos at the source, and an accurate value of ξ can be used to test both the conventional mechanism and the postulated scenarios for cosmic neutrino production.

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I. INTRODUCTION

High-energy neutrino telescopes are going to open a new window on the Universe, since they can be used to probe and characterize very distant astrophysical sources [1]. The promising IceCube neutrino telescope [2], which has a kilometer-scale detector, is now under construction. If the relative fluxes of ultrahigh-energy ν_e ($\bar{\nu}_e$), ν_μ ($\bar{\nu}_\mu$), and ν_τ ($\bar{\nu}_\tau$) neutrinos are successfully measured at IceCube and other neutrino telescopes, it will be possible to diagnose the relevant cosmic accelerators (e.g., their locations and characteristics) and examine the properties of neutrinos themselves (e.g., neutrino mixing and leptonic *CP* violation).

Indeed, robust evidence for neutrino masses and lepton flavor mixing has been achieved from the recent solar [3], atmospheric [4], reactor [5], and accelerator [6] neutrino oscillation experiments. Because of neutrino oscillations, the neutrino fluxes observed at the detector $\Phi^{\rm D} = \{\phi_e^{\rm D}, \phi_\mu^{\rm D}, \phi_\tau^{\rm D}\}$ are in general different from the source fluxes $\Phi = \{\phi_e, \phi_\mu, \phi_\tau\}$. Note that our notation is $\phi_\alpha^{\rm (D)} \equiv \phi_{\nu_\alpha}^{\rm (D)} + \phi_{\bar{\nu}_\alpha}^{\rm (D)}$ (for $\alpha = e, \mu, \tau$), where $\phi_{\nu_\alpha}^{\rm (D)}$ and $\phi_{\bar{\nu}_\alpha}^{\rm (D)}$ denote the ν_α-neutrino and $\bar{\nu}_\alpha$-antineutrino fluxes, respectively. The relation between ϕ_{ν_α} (or $\phi_{\bar{\nu}_\alpha}$) and $\phi_{\nu_\beta}^{\rm D}$ (or $\phi_{\bar{\nu}_\beta}^{\rm D}$) is given by

$$\phi_{\nu_\beta}^{\rm D} = \sum_\alpha (\phi_{\nu_\alpha} P_{\alpha\beta}), \qquad \phi_{\bar{\nu}_\beta}^{\rm D} = \sum_\alpha (\phi_{\bar{\nu}_\alpha} \bar{P}_{\alpha\beta}), \tag{1}$$

in which $P_{\alpha\beta}$ and $\bar{P}_{\alpha\beta}$ stand, respectively, for the oscillation probabilities $P(\nu_\alpha \to \nu_\beta)$ and $P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$. As the galactic distances far exceed the observed neutrino oscillation lengths, $P_{\alpha\beta}$ and $\bar{P}_{\alpha\beta}$ are actually averaged over many oscillations and take a very simple form:

$$P_{\alpha\beta} = \bar{P}_{\alpha\beta} = \sum_{i=1}^{3} |V_{\alpha i}|^2 |V_{\beta i}|^2, \tag{2}$$

where $V_{\alpha i}$ and $V_{\beta i}$ (for $\alpha, \beta = e, \mu, \tau$, and $i = 1, 2, 3$) are just the elements of the 3×3 neutrino mixing matrix V. Equations (1) and (2) lead us to a straightforward relation between ϕ_α and $\phi_\beta^{\rm D}$:

$$\phi_\beta^{\rm D} = \sum_\alpha (\phi_\alpha P_{\alpha\beta}). \tag{3}$$

This relation indicates that the observation of $\Phi^{\rm D}$ at a neutrino telescope can *at least* help[1]

(i) to determine or constrain the flavor composition of cosmic neutrino fluxes [8,9], if three neutrino mixing angles and the Dirac *CP*-violating phase hidden in $P_{\alpha\beta}$ have been measured to a good degree of accuracy (e.g., a precision of 10% or smaller relative error bars [10]);

or

(ii) to determine or constrain one or two of three neutrino mixing angles and the Dirac *CP*-violating phase [11,12], provided the production mechanism of ultrahigh-energy neutrinos at an astrophysical source (e.g., the conventional source to be mentioned below) is really understood.

Hence neutrino telescopes will serve as a very useful tool to probe both the high-energy astrophysical processes and the intrinsic properties of massive neutrinos.

This paper aims at a determination of the flavor composition of cosmic neutrino fluxes at the source with the help of neutrino telescopes. Different from the previous works (see, e.g., Refs. [7–9,11,12]), our present study starts from a generic parametrization of the initial neutrino fluxes:

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[1]One may also use neutrino telescopes to test the stability of neutrinos [7,8], possible violation of *CPT* symmetry [8], and other exotic scenarios of particle physics and cosmology.

$$\{\phi_e, \phi_\mu, \phi_\tau\} = \{\sin^2\xi\cos^2\zeta, \cos^2\xi\cos^2\zeta, \sin^2\zeta\}\phi_0, \quad (4)$$

where $\xi \in [0, \pi/2]$ and $\zeta \in [0, \pi/2]$, and ϕ_0 denotes the total flux (i.e., the sum of three neutrino fluxes). Provided ultrahigh-energy neutrinos are produced by certain astrophysical sources (e.g., active galactic nuclei or AGN) via the decay of pions created from pp and $p\gamma$ collisions, for instance, their flavor content is expected to be

$$\{\phi_e:\phi_\mu:\phi_\tau\} = \{\tfrac{1}{3}:\tfrac{2}{3}:0\}. \quad (5)$$

This "standard" neutrino flux ratio corresponds to $\zeta = 0$ and $\tan\xi = 1/\sqrt{2}$ (or equivalently $\xi \approx 35.3°$) in our parametrization. It turns out that any small departure of ζ from zero will measure the existence of cosmic ν_τ and $\bar{\nu}_\tau$ neutrinos, which might come from the decays of D_s and $B\bar{B}$ mesons produced at the source [13]. On the other hand, any small deviation of $\tan^2\xi$ from $1/2$ will imply that the conventional mechanism for ultrahigh-energy neutrino production from the AGN has to be modified. Similar arguments can be put forward for the neutrino fluxes from some other astrophysical sources, such as the postulated neutron beam source [14] with

$$\{\phi_e:\phi_\mu:\phi_\tau\} = \{1:0:0\} \quad (6)$$

(or equivalently $\{\xi, \zeta\} = \{\pi/2, 0\}$) and the possible muon-damped source [15] with

$$\{\phi_e:\phi_\mu:\phi_\tau\} = \{0:1:0\} \quad (7)$$

(or equivalently $\{\xi, \zeta\} = \{0, 0\}$). Therefore, we are well motivated to investigate how the *true* values of ξ and ζ for a specific astrophysical source can be determined or constrained by use of the second-generation neutrino telescopes and with the help of more precise data from the upcoming long-baseline neutrino oscillation experiments. This goal is indeed reachable, as we shall explicitly demonstrate in the remaining part of this paper.

The remaining part of this paper is organized as follows. In Sec. II, we derive the analytical relations between the neutrino flavor parameters (ξ and ζ) at an astrophysical source and the typical observables of neutrino fluxes at a terrestrial detector. Section III is devoted to a detailed numerical analysis of the dependence of those observables on ξ and ζ. A brief summary of our main results is given in Sec. IV.

II. OBSERVABLES

Because of neutrino oscillations and the ν_τ "regeneration" in the Earth [2], it is especially important to detect all three flavors of the cosmic neutrinos at a neutrino telescope. The sum of $\phi_e^{\rm D}$, $\phi_\mu^{\rm D}$, and $\phi_\tau^{\rm D}$ is equal to that of ϕ_e, ϕ_μ, and ϕ_τ,

$$\phi_0 \equiv \phi_e + \phi_\mu + \phi_\tau = \phi_e^{\rm D} + \phi_\mu^{\rm D} + \phi_\tau^{\rm D}, \quad (8)$$

as one may easily see from Eqs. (2) and (3). A measurement of ϕ_0 may involve large systematic uncertainties, but the latter can be largely canceled out in the ratio of two neutrino fluxes. Therefore, let us follow Ref. [12] to define

$$\{R_e, R_\mu, R_\tau\} \equiv \left\{\frac{\phi_e^{\rm D}}{\phi_\mu^{\rm D} + \phi_\tau^{\rm D}}, \frac{\phi_\mu^{\rm D}}{\phi_\tau^{\rm D} + \phi_e^{\rm D}}, \frac{\phi_\tau^{\rm D}}{\phi_e^{\rm D} + \phi_\mu^{\rm D}}\right\} \quad (9)$$

as our *working* observables. We remark that these ratios are largely free from the systematic uncertainties associated with the measurements of $\phi_e^{\rm D}$, $\phi_\mu^{\rm D}$, and $\phi_\tau^{\rm D}$. In particular, it is relatively easy to extract R_μ from the ratio of muon tracks to showers at IceCube [16], even if those electron and tau events may not well be disentangled. Since R_e, R_μ, and R_τ satisfy

$$\frac{R_e}{1 + R_e} + \frac{R_\mu}{1 + R_\mu} + \frac{R_\tau}{1 + R_\tau} = 1, \quad (10)$$

only two of them are independent.

Note that Eqs. (6) and (7) represent two peculiar (nonstandard) scenarios of cosmic neutrino production. In principle, one may also assume an exotic astrophysical source which only produces ν_τ and $\bar{\nu}_\tau$ neutrinos; i.e., $\{\phi_e:\phi_\mu:\phi_\tau\} = \{0:0:1\}$ or equivalently $\zeta = \pi/2$ with unspecified ξ in our parametrization. The expression of R_α (for $\alpha = e, \mu, \tau$) can then be simplified in such special cases:

$$R_\alpha = \begin{cases} \frac{P_{e\alpha}}{1-P_{e\alpha}}, & \text{for } \{\xi, \zeta\} = \{\pi/2, 0\}, \\ \frac{P_{\mu\alpha}}{1-P_{\mu\alpha}}, & \text{for } \{\xi, \zeta\} = \{0, 0\}, \\ \frac{P_{\tau\alpha}}{1-P_{\tau\alpha}}, & \text{for } \{\xi, \zeta\} = \{*, \pi/2\}. \end{cases} \quad (11)$$

Even if the third case is completely unrealistic, it could serve as an example to illustrate the salient feature of R_α defined above.

Without loss of generality, we choose R_e and R_μ as two typical observables and derive their explicit relations with ξ and ζ. By using Eqs. (4), (8), and (9), we obtain

$$R_e = \frac{P_{ee}\sin^2\xi + P_{\mu e}\cos^2\xi + P_{\tau e}\tan^2\zeta}{\sec^2\zeta - [P_{ee}\sin^2\xi + P_{\mu e}\cos^2\xi + P_{\tau e}\tan^2\zeta]},$$
$$R_\mu = \frac{P_{e\mu}\sin^2\xi + P_{\mu\mu}\cos^2\xi + P_{\tau\mu}\tan^2\zeta}{\sec^2\zeta - [P_{e\mu}\sin^2\xi + P_{\mu\mu}\cos^2\xi + P_{\tau\mu}\tan^2\zeta]}. \quad (12)$$

The source flavor parameters ξ and ζ can then be figured out in terms of R_e and R_μ:

$$\sin^2\xi = \frac{r_e(P_{\tau\mu} - P_{\mu\mu}) - r_\mu(P_{\tau e} - P_{\mu e}) + (P_{\mu\mu}P_{\tau e} - P_{\mu e}P_{\tau\mu})}{(r_e - P_{\tau e})(P_{e\mu} - P_{\mu\mu}) - (r_\mu - P_{\tau\mu})(P_{ee} - P_{\mu e})},$$
$$\tan^2\zeta = \frac{r_e(P_{\mu\mu} - P_{e\mu}) - r_\mu(P_{\mu e} - P_{ee}) + (P_{e\mu}P_{\mu e} - P_{ee}P_{\mu\mu})}{(r_e - P_{\tau e})(P_{e\mu} - P_{\mu\mu}) - (r_\mu - P_{\tau\mu})(P_{ee} - P_{\mu e})}, \quad (13)$$

where the notations $r_e \equiv R_e/(1 + R_e)$ and $r_\mu \equiv R_\mu/(1 + R_\mu)$ have been used to simplify the expressions. Indeed, $r_e = \phi_e^{\rm D}/\phi_0$ and $r_\mu = \phi_\mu^{\rm D}/\phi_0$ hold. One may in principle choose either (R_e, R_μ) or (r_e, r_μ) as a set of working observables to inversely determine ξ and ζ. The first set has been chosen by a few authors (see, e.g., Refs. [10,12]) and will also be adopted in this paper. Note that the averaged neutrino oscillation probabilities $P_{\alpha\beta}$ depend on three mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$ and the Dirac CP-violating phase (δ) in the standard parametrization of the 3×3 neutrino mixing matrix V [17]. Taking account of $\theta_{13} < 10°$ [18] but $\theta_{12} \approx 34°$ and $\theta_{23} \approx 45°$ [19], we express $P_{\alpha\beta}$ as the first-order expansion of the small parameter $\sin\theta_{13}$:

$$\begin{aligned}
P_{ee} &= 1 - \tfrac{1}{2}\sin^2 2\theta_{12},\\
P_{e\mu} &= \tfrac{1}{2}\sin^2 2\theta_{12}\cos^2\theta_{23} + \tfrac{1}{4}\sin 4\theta_{12}\sin 2\theta_{23}\sin\theta_{13}\cos\delta,\\
P_{e\tau} &= \tfrac{1}{2}\sin^2 2\theta_{12}\sin^2\theta_{23} - \tfrac{1}{4}\sin 4\theta_{12}\sin 2\theta_{23}\sin\theta_{13}\cos\delta,\\
P_{\mu\mu} &= 1 - \tfrac{1}{2}\sin^2 2\theta_{23} - \tfrac{1}{2}\sin^2 2\theta_{12}\cos^4\theta_{23} - \tfrac{1}{2}\sin 4\theta_{12}\sin 2\theta_{23}\cos^2\theta_{23}\sin\theta_{13}\cos\delta,\\
P_{\mu\tau} &= \tfrac{1}{2}\sin^2 2\theta_{23} - \tfrac{1}{8}\sin^2 2\theta_{12}\sin^2 2\theta_{23} + \tfrac{1}{8}\sin 4\theta_{12}\sin 4\theta_{23}\sin\theta_{13}\cos\delta,\\
P_{\tau\tau} &= 1 - \tfrac{1}{2}\sin^2 2\theta_{23} - \tfrac{1}{2}\sin^2 2\theta_{12}\sin^4\theta_{23} + \tfrac{1}{2}\sin 4\theta_{12}\sin 2\theta_{23}\sin^2\theta_{23}\sin\theta_{13}\cos\delta,
\end{aligned} \quad (14)$$

in which the higher-order terms, such as $\mathcal{O}(\sin^2\theta_{13}) < 3\%$, have been safely neglected. One can see that the sensitivity of $P_{\alpha\beta}$ to δ is suppressed due to the smallness of $\sin\theta_{13}$. Hence the dependence of R_α on δ is expected to be insignificant. In addition, it is hard to distinguish between the cases of $\theta_{13} = 0°$ and $\delta = 90°$, because it is the product of $\sin\theta_{13}$ and $\cos\delta$ that appears in the analytical approximations of $P_{\alpha\beta}$.

A global analysis of current neutrino oscillation data [19] yields

$$30° < \theta_{12} < 38°, \qquad 36° < \theta_{23} < 54°, \qquad 0° \le \theta_{13} < 10°, \quad (15)$$

at the 99% confidence level. The best-fit values of three neutrino mixing angles are $\theta_{12} \approx 34°$, $\theta_{23} \approx 45°$, and $\theta_{13} \approx 0°$ [19], but the CP-violating phase δ is entirely unrestricted. Although the present experimental data remain unsatisfactory, they can be used to constrain the correlation between the parameters (ξ, ζ) and the observables (R_α, R_β). We shall illustrate our analytical results by taking a few typical numerical examples in the subsequent section.

III. ILLUSTRATION

First of all, let us follow a rather conservative strategy to scan the reasonable ranges of $(\theta_{12}, \theta_{23}, \theta_{13}, \delta)$ and (ξ, ζ) so as to examine the sensitivities of (R_e, R_μ, R_τ) to these six parameters. We take $\delta \in [0°, 180°]$ in addition to the generous intervals of three mixing angles given in Eq. (15), and allow ξ to vary in the region $\xi \in [0°, 90°]$. As the amount of ν_τ and $\bar{\nu}_\tau$ neutrinos produced at those realistic astrophysical sources is expected to be very small or even vanishing, we restrict the parameter ζ to a very narrow domain $\zeta \in [0°, 18°]$, which corresponds to $\sin^2\zeta \lesssim 0.1$ [13]. It should be noted that we do the numerical computation by using the exact expressions of $P_{\alpha\beta}$ instead of Eq. (14). Our results are shown in Fig. 1. Two comments are in order:

(1) The source parameter ξ is quite sensitive to the values of the neutrino flux ratios R_α (for $\alpha = e, \mu, \tau$). Even if three neutrino mixing angles involve a lot of uncertainties and the Dirac CP-violating phase is entirely unknown, a combined measurement of (R_e, R_μ) or (R_e, R_τ) can constrain the value of ξ to an acceptable degree of accuracy. This encouraging observation assures that the second-generation neutrino telescopes can really be used to probe the initial flavor composition of ultrahigh neutrino fluxes. Note that the standard pion-decay source $\{\phi_e{:}\phi_\mu{:}\phi_\tau\} = \{1/3{:}2/3{:}0\}$ will produce $\{\phi_e^{\rm D}{:}\phi_\mu^{\rm D}{:}\phi_\tau^{\rm D}\} \simeq \{1/3{:}1/3{:}1/3\}$ or equivalently $R_e \approx R_\mu \approx R_\tau \approx 0.5$ at the detector, as shown in Fig. 1, if $\theta_{23} \approx 45°$ is fixed and $\theta_{13} < 10°$ is taken. Such an expectation cannot be true, however, when ξ deviates from its given value $\xi = 35.3°$ and (or) when θ_{23} departs from its best-fit value $\theta_{23} = 45°$. More precise neutrino oscillation data will greatly help to narrow down the (R_α, ξ) parameter space.

(2) In contrast, the source parameter ζ seems to be insensitive to R_α (for $\alpha = e, \mu, \tau$). The reason for this insensitivity is twofold: (a) the values of ζ have been restricted to a very narrow range $(0° \le \zeta \le 18°)$; and (b) the numerical uncertainties of three

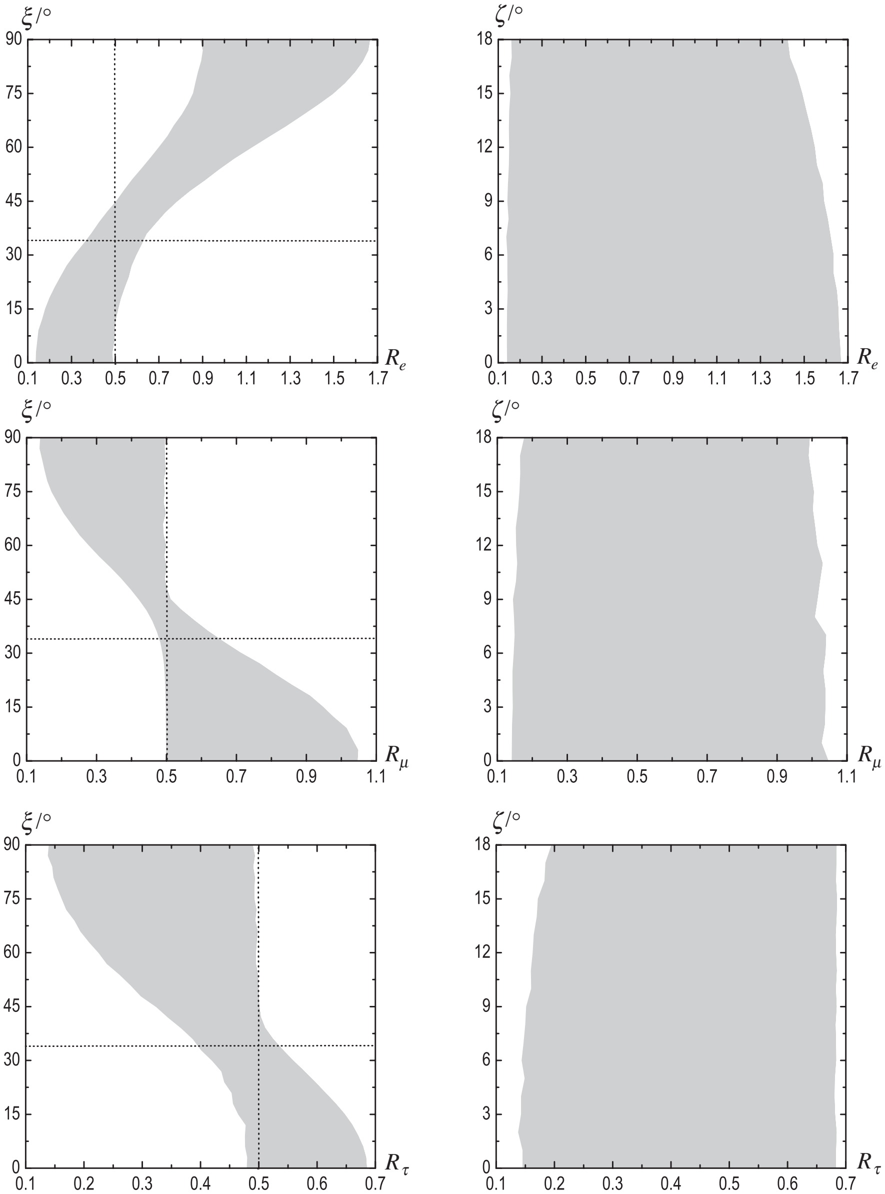

FIG. 1. Allowed regions of the neutrino flux ratios R_α ($\alpha = e, \mu, \tau$) versus the source parameters ξ and ζ, where we have scanned the 99% C.L. intervals of three neutrino mixing angles and taken the Dirac CP-violating phase $\delta \in [0°, 180°]$. The horizontal and vertical lines in the (R_α, ξ) plots correspond to $\xi = 35.3°$ and $R_\alpha = 0.5$, respectively.

neutrino mixing angles and the Dirac CP-violating phase are too large. Provided θ_{12}, θ_{23}, θ_{13}, and δ are all measured to a high degree of accuracy in the near future, it will be possible to find out the definite dependence of R_α on ζ for a given value of ξ.

To be more explicit, we are going to consider three typical scenarios of cosmic neutrino fluxes and illustrate the sensitivities of R_e, R_μ, and R_τ to the source parameters (ξ, ζ) and to the unknown neutrino mixing parameters (θ_{13}, δ).

We argue that the simple flavor content of ultrahigh-energy neutrino fluxes from the standard pion-decay source could somehow be contaminated for certain reasons: e.g., a small amount of ν_e, ν_μ, and ν_τ and their antiparticles might come from the decays of heavier hadrons produced by pp and $p\gamma$ collisions [13]. Similar arguments can also be made for the postulated neutron beam source and the possible muon-damped source, as our present knowledge about the mechanism of cosmic neutrino production remains very poor. Following a phenomenological approach, we slightly modify the scenarios listed in Eqs. (5)–(7) by allowing the relevant ξ and ζ parameters to fluctuate around their given values. Namely, we consider

(i) *Scenario A.*—$30° \leq \xi \leq 40°$ and $0° \leq \zeta \leq 18°$, serving as a modified version of the standard pion-decay source (originally, $\xi = 35.3°$ and $\zeta = 0°$);

(ii) *Scenario B.*—$80° \leq \xi \leq 90°$ and $0° \leq \zeta \leq 18°$, serving as a modified version of the postulated neutron beam source (originally, $\xi = 90°$ and $\zeta = 0°$);

(iii) *Scenario C.*—$0° \leq \xi \leq 10°$ and $0° \leq \zeta \leq 18°$, serving as a modified version of the possible muon-damped source (originally, $\xi = 0°$ and $\zeta = 0°$).

For simplicity, we fix $\theta_{12} = 34°$ and $\theta_{23} = 45°$ in our numerical analysis. We take four typical inputs for the unknown parameters θ_{13} and δ: (a) $\theta_{13} = 0°$ (in this case, δ is not well defined and has no physical significance); (b) $\theta_{13} = 5°$ and $\delta = 0°$; (c) $\theta_{13} = 5°$ and $\delta = 90°$; and (d) $\theta_{13} = 5°$ and $\delta = 180°$. Our numerical results for the sensitivities of R_e, R_μ, and R_τ to ξ and ζ in scenarios A, B, and C are shown in Figs. 2–4, respectively. Some discussions are in order.

(1) *Scenario A in Fig. 2.*—The neutrino flux ratios R_e, R_μ, and R_τ are all sensitive to the small deviation of ξ from its standard value $\xi = 35.3°$. In contrast, the changes of three observables are very small when ζ varies from 0° to 18°. This insensitivity is understandable: about half of the initial ν_μ and $\bar{\nu}_\mu$ neutrinos oscillate into ν_τ and $\bar{\nu}_\tau$ neutrinos, whose amount dominates over the survival amount of initial ν_τ and $\bar{\nu}_\tau$ neutrinos at the detector. In other words, $\phi_\tau^{\rm D} \gg \phi_\tau$ and $\phi_e^{\rm D} \sim \phi_\mu^{\rm D} \sim \phi_\tau^{\rm D}$ hold, implying that R_α must be insensitive to small ϕ_τ or

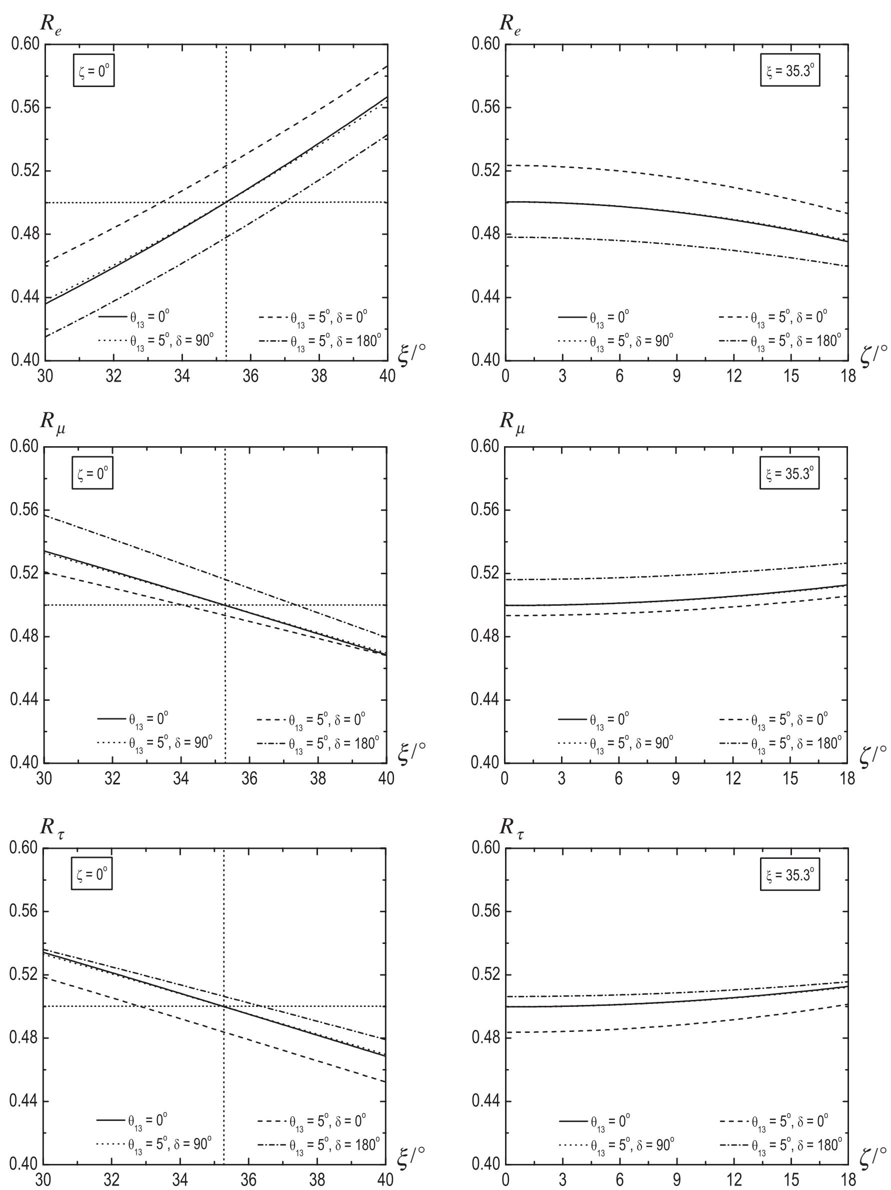

FIG. 2. Numerical illustration of scenario A, where $\theta_{12} = 34°$ and $\theta_{23} = 45°$ have typically been input in our calculations. The horizontal and vertical lines in the (ξ, R_α) plots correspond to $R_\alpha = 0.5$ and $\xi = 35.3°$, respectively.

equivalently to small ζ. It is therefore difficult to pin down the value of ζ from this kind of astrophysical source. As pointed out in the last section, the result of R_α in the $\theta_{13} = 0°$ case (solid curves) is almost indistinguishable from that in the $\delta = 90°$ case (dotted curves). The sensitivity of R_α to δ is insignificant but distinguishable, if the value of θ_{13} is about 5° or larger.

(2) *Scenario B in Fig. 3.*—The neutrino flux ratio R_e is sensitive to the small departures of ξ and ζ from their given values $\xi = 90°$ and $\zeta = 0°$. This salient feature can be understood as follows. Since ν_μ (or $\bar{\nu}_\mu$) and ν_τ (or $\bar{\nu}_\tau$) neutrinos at the detector mainly come from the initial ν_e (or $\bar{\nu}_e$) neutrinos via the oscillation, the sum of $\phi_\mu^{\rm D}$ and $\phi_\tau^{\rm D}$ is expected to be smaller than or comparable with the survival ν_e (or $\bar{\nu}_e$) flux $\phi_e^{\rm D}$. The roles of ϕ_e and ϕ_τ are important in $\phi_e^{\rm D}$ and $\phi_\mu^{\rm D} + \phi_\tau^{\rm D}$, respectively. It turns out that R_e depends, in a relatively sensitive way, on ξ through $\phi_e^{\rm D}$ in its numerator and on ζ through $\phi_\mu^{\rm D} + \phi_\tau^{\rm D}$ in its denominator. Note also that the nearly degenerate results of R_e for $\theta_{13} = 5°$ and $\delta = (0°, 90°, 180°)$ are primarily attributed to the fact that P_{ee} in the numerator of R_e is actually independent of δ. On the other hand, the discrepancy between $R_e(\theta_{13} = 0°)$ and $R_e(\theta_{13} = 5°)$ in Fig. 3 results from the $\mathcal{O}(\sin^2\theta_{13})$ terms of $P_{\alpha\beta}$, which have been neglected in Eq. (14). In comparison with R_e, the neutrino flux ratios R_μ and R_τ are not so sensitive to the small changes of ξ and ζ. But the measurement of R_μ and

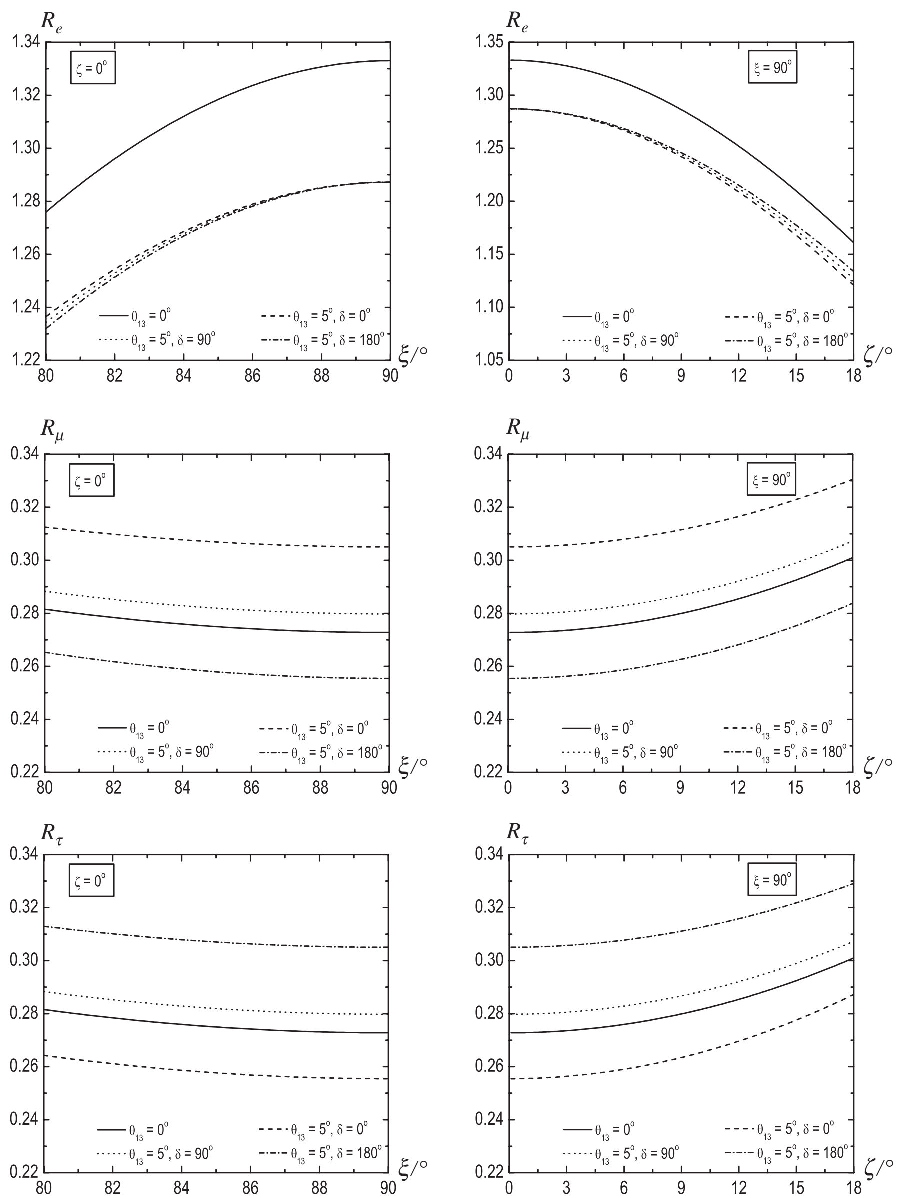

FIG. 3. Numerical illustration of scenario B, where $\theta_{12} = 34°$ and $\theta_{23} = 45°$ have typically been input in our calculations.

R_τ is as important as that of R_e, in order to determine the flavor composition of ultrahigh-energy neutrino fluxes at such an astrophysical source.

(3) *Scenario C in Fig. 4.*—In this case, in which $\phi_e^{\rm D}$ and $\phi_\tau^{\rm D}$ mainly come from the initial ϕ_μ via $\nu_\mu \to \nu_e$ and $\nu_\mu \to \nu_\tau$ oscillations, one can similarly understand the numerical behaviors of R_e, R_μ, and R_τ changing with the small deviation of ξ from its given value $\xi = 0°$. To explain why three observables are almost independent of the fluctuation of ζ, we take $\theta_{23} = 45°$ and $\theta_{13} \to 0°$, which guarantee $P_{\tau\alpha} = P_{\mu\alpha}$ (for $\alpha = e, \mu, \tau$) to hold in the leading-order approximation. We can then simplify Eq. (12) in the $\xi \to 0°$ limit and arrive at the following result:

$$R_\alpha|_{\xi\to 0°} = \frac{P_{\mu\alpha} + P_{\tau\alpha}\tan^2\zeta}{\sec^2\zeta - (P_{\mu\alpha} + P_{\tau\alpha}\tan^2\zeta)} = \frac{P_{\mu\alpha}}{1 - P_{\mu\alpha}}, \tag{16}$$

where ζ is completely cancelled out. Therefore, the tiny dependence of R_α on ζ appearing in Fig. 4 is just a natural consequence of the $\mathcal{O}(\sin\theta_{13})$ or $\mathcal{O}(\sin^2\theta_{13})$ corrections to Eq. (16).

Although the numerical examples taken above can only serve for illustration, they *do* give us a ball-park feeling of the correlation between the source parameters (ξ, ζ) and the neutrino mixing parameters $(\theta_{12}, \theta_{23}, \theta_{13}, \delta)$ via the working observables (R_e, R_μ, R_τ) at a neutrino telescope. This observation is certainly encouraging and remarkable.

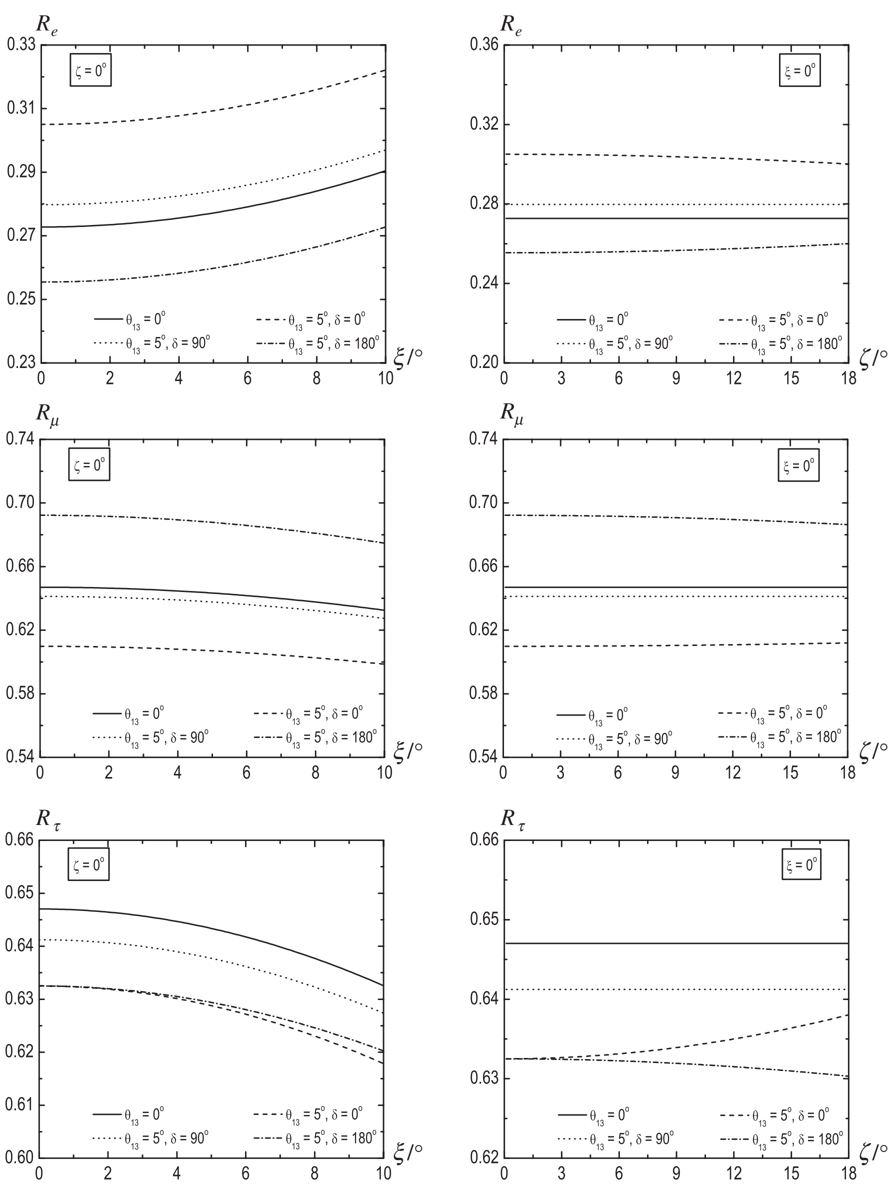

FIG. 4. Numerical illustration of scenario C, where $\theta_{12} = 34°$ and $\theta_{23} = 45°$ have typically been input in our calculations.

IV. SUMMARY

We have proposed a simple parametrization of the initial flavor composition of ultrahigh-energy neutrino fluxes generated from very distant astrophysical sources: $\phi_e{:}\phi_\mu{:}\phi_\tau = \sin^2\xi\cos^2\zeta{:}\cos^2\xi\cos^2\zeta{:}\sin^2\zeta$. The conventional mechanism and the postulated scenarios for cosmic neutrino production can all be reproduced by taking the special values of (ξ, ζ). Of course, such a parametrization is by no means unique. An alternative,

$$\phi_e{:}\phi_\mu{:}\phi_\tau = \frac{x}{1+t}{:}\frac{1-x}{1+t}{:}\frac{t}{1+t} \qquad (17)$$

with $x \in [0, 1]$ and $t \in [0, \infty)$ in general, is also simple and useful. For a realistic astrophysical source, it should be more reasonable to take $0 \leq t \ll 1$. Comparing between Eqs. (4) and (17), one can immediately arrive at $x = \sin^2\xi$ and $t = \tan^2\zeta$. The (x, t) and (ξ, ζ) languages are therefore equivalent to each other.

After defining three neutrino flux ratios R_α (for $\alpha = e$, μ, τ) as our working observables at a neutrino telescope, we have shown that the source parameters ξ and ζ can in principle be determined by the measurement of two independent R_α and with the help of accurate neutrino oscillation data. The standard pion-decay source, the postulated neutrino beam source, and the possible muon-damped source have been slightly modified to illustrate the sensitivities of R_α to small departures of ξ and ζ from their given values. We have also examined the dependence of R_α upon the smallest neutrino mixing angle θ_{13} and upon the Dirac CP-violating phase δ. Our numerical examples indicate that it is quite promising to determine or constrain the initial flavor content of ultrahigh-energy neutrino fluxes with the second-generation neutrino telescopes.

How to measure R_α to an acceptable degree of accuracy is certainly a big challenge to IceCube and other neutrino telescopes. A detailed analysis of the feasibility of our idea for a specific neutrino telescope is desirable, but it is beyond the scope of this paper. Here we remark that our present understanding of the production mechanism of cosmic neutrinos needs the observational and experimental support. We expect that neutrino telescopes may help us to attain this goal in the long run.

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