What fraction of boron-8 solar neutrinos arrive at the Earth as a ν_2 mass eigenstate?

Hiroshi Nunokawa,[1,*] Stephen Parke,[2,†] and Renata Zukanovich Funchal[3,‡]

[1]Departamento de Física, Pontifícia Universidade Católica do Rio de Janeiro, C.P. 38071, 22452-970, Rio de Janeiro, Brazil
[2]Theoretical Physics Department, Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510, USA
[3]Instituto de Física, Universidade de São Paulo, C.P. 66.318, 05315-970 São Paulo, Brazil

(Received 30 January 2006; published 24 July 2006)

We calculate the fraction of ^{8}B solar neutrinos that arrive at the Earth as a ν_2 mass eigenstate as a function of the neutrino energy. Weighting this fraction with the ^{8}B neutrino energy spectrum and the energy dependence of the cross section for the charged current interaction on deuteron with a threshold on the kinetic energy of the recoil electrons of 5.5 MeV, we find the integrated weighted fraction of ν_2's to be $(91 \pm 2)\%$ at the 95% CL. This energy weighting procedure corresponds to the charged current response of the Sudbury Neutrino Observatory (SNO). We have used SNO's current best fit values for the solar mass squared difference and the mixing angle, obtained by combining the data from all solar neutrino experiments and the reactor data from KamLAND. The uncertainty on the ν_2 fraction comes primarily from the uncertainty on the solar δm^2 rather than from the uncertainty on the solar mixing angle or the standard solar model. Similar results for the Super-Kamiokande experiment are also given. We extend this analysis to three neutrinos and discuss how to extract the modulus of the Maki-Nakagawa-Sakata mixing matrix element U_{e2} as well as place a lower bound on the electron number density in the production region for ^{8}B solar neutrinos.

DOI: 10.1103/PhysRevD.74.013006 PACS numbers: 14.60.Pq, 25.30.Pt, 28.41.−i

I. INTRODUCTION

Recently the KamLAND [1] and Sudbury Neutrino Observatory (SNO) [2] experiments have given a precise determination of the neutrino solar mass squared difference and mixing angle responsible for the solar neutrino deficit first observed in the Davis [3] experiment when compared to the theoretical calculations by Bahcall [4]. Subsequently this deficit has been observed by many other experiments [5,6], while the theoretical calculations of the neutrino flux based on the standard solar model (SSM) have been significantly improved [7]. When all of these results are combined in a two neutrino fit as reported by SNO [2], the allowed values for the solar mass squared difference, $\delta m^2_\odot$, and the mixing angle, $\theta_\odot$, are individually (for 1 degree of freedom) restricted to the following range,[1]

$$\delta m^2_\odot = 8.0^{+0.4}_{-0.3} \times 10^{-5}\ \mathrm{eV}^2, \qquad \sin^2\theta_\odot = 0.310 \pm 0.026, \tag{1}$$

at the 68% confidence level. Maximal mixing, $\sin^2\theta_\odot = 0.5$, has been ruled out at greater than 5σ. The solar neutrino data is consistent with $\nu_e \to \nu_\mu$ and/or ν_τ conversion. The precision on $\delta m^2_\odot$ comes primarily from the KamLAND experiment [1] whereas the precision on $\sin^2\theta_\odot$ comes primarily from the SNO experiment [2].

The physics responsible for the reduction in the solar ^{8}B electron neutrino flux is the Wolfenstein matter effect [9] with the electron neutrinos produced above the Mikheyev-Smirnov (MS) resonance [10]. The combination of these two effects in the Mikheyev-Smirnov-Wolfenstein (MSW) large mixing angle (LMA) region, given by Eq. (1), implies that the ^{8}B solar neutrinos are produced and propagate adiabatically to the solar surface, and hence to the Earth, as almost a pure ν_2 mass eigenstate.[2] Since approximately one-third of the ν_2 mass eigenstate is ν_e, this explains the solar neutrino deficit first reported by Davis. If the ^{8}B solar neutrinos arriving at the Earth were 100% ν_2, then the *daytime* charged current (CC) to neutral current (NC) ratio, CC/NC, measured by SNO would be exactly $\sin^2\theta_\odot$, the fraction of ν_e in ν_2 in the two neutrino analysis.

Of course, the ν_2 mass eigenstate purity of the solar ^{8}B neutrinos is not 100%. In fact, in the MSW-LMA region, defined using only solar data [11], the range in the ν_2 fraction varies from 77% at high $\delta m^2_\odot$ ($\sim 14 \times 10^{-5}$ eV^2) to 99% at low $\delta m^2_\odot$ ($\sim 3 \times 10^{-5}$ eV^2), i.e. the ν_1 fraction can be as large as 23% or as small as 1%. It is only because of the recent precision measurement of $\delta m^2_\odot$ from KamLAND [1] that a precise ($\pm$ few %) determination of the ν_2 fraction can be made. The departure of the ν_2 fraction from 100% is related to the difference between SNO's daytime CC/NC ratio and their best fit value of $\sin^2\theta_\odot$. If the electron neutrino has a nonzero component in ν_3 (i.e. nonzero $\sin^2\theta_{13}$) then there will be a small fraction arriving as ν_3's.

A precise determination of the mass eigenstate purity of the ^{8}B solar neutrinos is the main subject of this paper. These mass eigenstates can be considered to be incoherent,

*Electronic address: nunokawa@fis.puc-rio.br
†Electronic address: parke@fnal.gov
‡Electronic address: zukanov@if.usp.br

[1]We use the notation of [8] with the subscript "⊙" reserved for the two neutrino analysis whereas the subscript "12" is reserved for the three neutrino analysis.

[2]Without the matter effect, the fraction of ν_2's would be simply $\sin^2\theta_\odot$, i.e. about 31%, and energy independent.

for all practical solar neutrino experiments, since the kinematic phase, $\delta m^2_\odot L/4E$, is greater than 10^6; for further discussion of this topic see [12]. In the next section we will summarize the important physics of the MSW-LMA solar neutrino solution outlined above and calculate the mass eigenstate purity of ^{8}B neutrinos as a function of the neutrino energy in a two neutrino analysis for both the SNO and Super-Kamiokande (SK) experiments. In Sec. III we will discuss what happens in a full three neutrino analysis. In Sec. IV, as an application of the previous sections, we will discuss the possibility of extracting information about the solar interior independently from the standard solar model. Finally, in Sec. V, we present our summary and conclusions.

II. TWO NEUTRINO ANALYSIS

A. ^{8}B ν_2 fraction

In the two neutrino analysis, let $f_1(E_\nu)$ and $f_2(E_\nu)$ be the fraction of ^{8}B solar neutrinos of energy E_ν which exit the sun and thus arrive at the Earth's surface as either a ν_1 or a ν_2 mass eigenstate, respectively. Following the analytical studies of Ref. [13], these fractions are given by

$$f_1(E_\nu) = \langle \cos^2\theta^N_\odot - P_x \cos 2\theta^N_\odot \rangle_{^8\mathrm{B}}, \tag{2}$$

$$f_2(E_\nu) = \langle \sin^2\theta^N_\odot + P_x \cos 2\theta^N_\odot \rangle_{^8\mathrm{B}}, \tag{3}$$

where $\theta^N_\odot$ is the mixing angle defined at the ν_e production point, P_x is the probability of the neutrino to jump from one mass eigenstate to the other during the MS-resonance crossing, and the sum is constrained to be 1, $f_1 + f_2 = 1$. The average $\langle \cdots \rangle_{^8\mathrm{B}}$ is over the electron density of the ^{8}B ν_e production region in the center of the sun predicted by the standard solar model [14]. The mixing angle, $\theta^N_\odot$, and the mass difference squared, δm^2_N, at the production point are

$$\sin^2\theta^N_\odot = \frac{1}{2}\left\{1 + \frac{(A - \delta m^2_\odot \cos 2\theta_\odot)}{\sqrt{(\delta m^2_\odot \cos 2\theta_\odot - A)^2 + (\delta m^2_\odot \sin 2\theta_\odot)^2}}\right\}, \tag{4}$$

$$\delta m^2_N = \sqrt{(\delta m^2_\odot \cos 2\theta_\odot - A)^2 + (\delta m^2_\odot \sin 2\theta_\odot)^2} \tag{5}$$

where

$$A \equiv 2\sqrt{2} G_F (Y_e \rho / M_n) E_\nu = 1.53 \times 10^{-4}\ \mathrm{eV}^2 \left(\frac{Y_e \rho E_\nu}{\mathrm{kg.cm}^{-3}\ \mathrm{MeV}}\right) \tag{6}$$

is the matter potential, E_ν is the neutrino energy, G_F is the Fermi constant, Y_e is the electron fraction (the number of electrons per nucleon), M_n is the nucleon mass, and ρ is the matter density. The combination $Y_e \rho / M_n$ is just the number density of electrons.

Figure 1 shows, for a wide range of $\delta m^2_\odot$ and $\sin^2\theta_\odot$, the isocontours of

$$f_2 \equiv \langle f_2(E_\nu) \rangle_E, \tag{7}$$

where $\langle \cdots \rangle_E$ is the average over the ^{8}B neutrino energy spectrum [15] convoluted with the energy dependence of the CC interaction $\nu_e + d \to p + p + e^-$ cross section [16] at SNO with the threshold on the recoil electron's kinetic energy of 5.5 MeV. Here we use $\sin^2\theta_\odot$ as the metric for the mixing angle as it is the fraction of ν_e's in the vacuum ν_2 mass eigenstate. In this work, we mainly focus on SNO rather than SK since the former is the unique solar neutrino experiment which can measure the total active ^{8}B neutrino flux as well as ^{8}B electron neutrino flux, independently from the SSM prediction and other experiments. However, we give a brief discussion on SK later in this section.

In the LMA region the propagation of the neutrino inside the sun is highly adiabatic [10,13,17], i.e. $P_x \approx 0$, therefore,

$$f_2(E_\nu) \equiv 1 - f_1(E_\nu) = \langle \sin^2\theta^N_\odot \rangle_{^8\mathrm{B}}. \tag{8}$$

Because of the fact that ^{8}B neutrinos are produced in a region where the density is significantly higher (about a

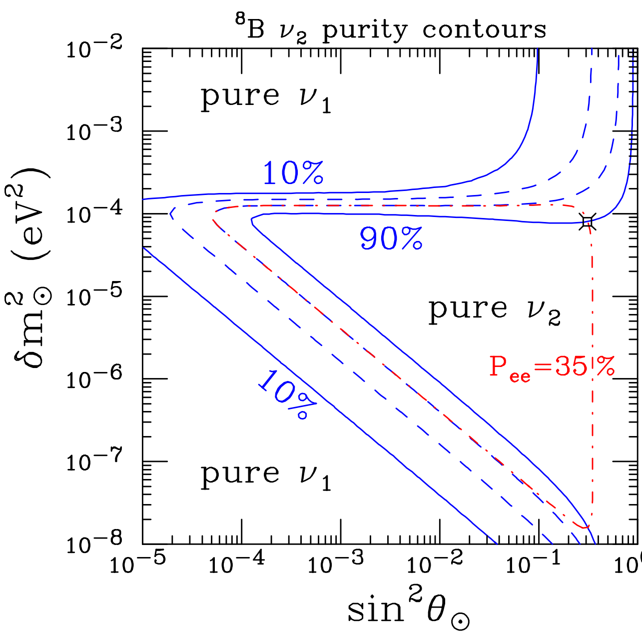

FIG. 1 (color online). The solid and dashed (blue) lines are the 90%, 65%, 35%, and 10% isocontours of the fraction of the solar ^{8}B neutrinos that are ν_2's in the $\delta m^2_\odot$ and $\sin^2\theta_\odot$ plane. The current best fit value, indicated by the open circle with the cross, is close to the 90% contour. The isocontour for an electron neutrino survival probability, P_{ee}, equal to 35% is the dot-dashed (red) "triangle" formed by the 65% ν_2 purity contour for small $\sin^2\theta_\odot$ and a vertical line in the pure ν_2 region at $\sin^2\theta_\odot = 0.35$. Except at the top and bottom right hand corners of this triangle the ν_2 purity is either 65% or 100%.

factor of 4) than that of the MS-resonance value, the average $\langle f_2(E_\nu)\rangle_E$ is close to 90% for the current solar best fit values of the mixing parameters from the recent KamLAND plus SNO analysis [2]. Since $\sin^2\theta_\odot^N \to 1$ when $A/\delta m_\odot^2 \to \infty$ [see Eq. (4)], we can see that at the high energy end of the ^{8}B neutrinos $\langle \sin^2\theta_\odot^N\rangle_{^8\mathrm{B}}$ must be close to 1.

We can check our result using the analysis of SNO with a simple back-of-the-envelope calculation. In terms of the fraction of ν_1 and ν_2 the *daytime* CC/NC of SNO, which is equal to the daytime average ν_e survival probability, $\langle P_{ee}\rangle$, is given by

$$\frac{\mathrm{CC}}{\mathrm{NC}}\bigg|_{\mathrm{day}} = \langle P_{ee}\rangle = f_1\cos^2\theta_\odot + f_2\sin^2\theta_\odot, \tag{9}$$

where f_1 and f_2 are understood to be the ν_1 and ν_2 fractions, respectively, averaged over the ^{8}B neutrino energy weighted with the CC cross section, as mentioned before. Using the central values reported by SNO,[3]

$$\frac{\mathrm{CC}}{\mathrm{NC}}\bigg|_{\mathrm{day}} = 0.347 \pm 0.038, \tag{10}$$

which was obtained from Table XXVI of Ref. [2], and the current best fit value of the mixing angle, we find $f_2 = (1 - f_1) \approx 90\%$, as expected. Because of the correlations in the uncertainties between the CC/NC ratio and $\sin^2\theta_\odot$ we are unable to estimate the uncertainty on f_2 here. Note that if the fraction of ν_2 were 100%, then $\frac{\mathrm{CC}}{\mathrm{NC}} = \sin^2\theta_\odot$.

Alternatively, we can rewrite Eq. (9) as[4]

$$\sin^2\theta_\odot = \frac{1}{1-2f_1}\left(\frac{\mathrm{CC}}{\mathrm{NC}} - f_1\right). \tag{11}$$

Thus, how much CC/NC differs from $\sin^2\theta_\odot$ is determined by how much f_2 differs from 100%, i.e. the size of f_1. In Fig. 2 we have plotted the contours of the daytime CC/NC ratio in the $\sin^2\theta_\odot$ versus $\delta m_\odot^2$ plane for the LMA region. Clearly, at smaller values of $\delta m_\odot^2$ the daytime CC/NC tracks $\sin^2\theta_\odot$ whereas at larger values an appreciable difference appears. This difference is caused by a decrease (increase) in the fraction that is ν_2 (ν_1) as $\delta m_\odot^2$ gets larger.

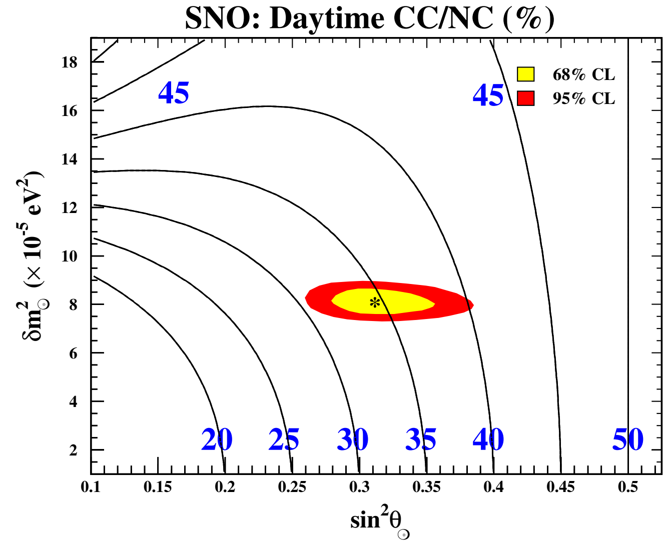

FIG. 2 (color online). SNO's daytime CC/NC ratio in the $\delta m_\odot^2$ versus $\sin^2\theta_\odot$ plane. At small values of $\delta m_\odot^2$, the daytime CC/NC ratio equals $\sin^2\theta_\odot$. The current allowed regions at 68% and 95% CL from the combined fit of KamLAND and solar neutrino data [2] are also shown by the shaded areas with the best fit indicated by the star.

Hence, if we know the ν_1 fraction we can easily calculate $\sin^2\theta_\odot$ from Eq. (11) and the measured value of the daytime CC/NC ratio. In fact, all we need to know is $\frac{\mathrm{CC}}{\mathrm{NC}}$. Using $\sin^2\theta_\odot \simeq \frac{\mathrm{CC}}{\mathrm{NC}}$, which is valid at zeroth order in $\delta m_\odot^2/A$, one can determine the ν_1 fraction from Eq. (4) with sufficient accuracy to make this work, given the current uncertainties on $\frac{\mathrm{CC}}{\mathrm{NC}}$ and $\delta m_\odot^2$. Also, this procedure can be iterated for better precision on the ν_1 fraction as well as $\sin^2\theta_\odot$ when necessary. Of course this is not a substitute for a full global analysis but provides a simple determination of $\sin^2\theta_\odot$ from SNO's $\frac{\mathrm{CC}}{\mathrm{NC}}$ measurement.

A similar analysis can also be performed using the event rate of the elastic scattering (ES) at SK and/or at SNO. In fact, ES is related to the ν_1 and ν_2 fractions, as follows,

$$\frac{\mathrm{ES}}{\mathrm{NC}} = f_1(\cos^2\theta_\odot + r\sin^2\theta_\odot) + f_2(\sin^2\theta_\odot + r\cos^2\theta_\odot) \tag{12}$$

where $r \equiv \langle\sigma_{\nu_{\mu,\tau}e}\rangle/\langle\sigma_{\nu_e e}\rangle \approx 0.155$ is the ratio of the ES cross sections for $\nu_{\mu,\tau}$ and ν_e [19], averaged over the observed neutrino spectrum. Here, $f_2 \equiv 1 - f_1 \equiv \langle f_2(E_\nu)\rangle_E$, where $\langle\cdots\rangle_E$ is the average over the ^{8}B neutrino energy spectrum [15] convoluted with the energy dependence of the ES interaction $\nu + e \to \nu + e$ cross section [19] using the SK threshold. Note that we are normalizing the ES event rate to that of SNO NC such that Eq. (12) is valid, independent of the SSM prediction of the ^{8}B neutrino flux. Equation (12) can be used to give a rough estimate of the f_2 fraction for SK, using the ES flux measured by SK [5] and SNO's result for NC flux and

[3]For the sake of simplicity and transparentness of the discussion, we have avoided the Earth matter effect which causes the so-called regeneration of ν_e during the night, by simply restricting our analysis to the daytime neutrino flux throughout this paper. We note that, due to the large error, the observed night-day asymmetry at SNO is consistent with any value from -8% to 5% [2] whereas the expected night-day asymmetry, $2(N-D)/(N+D)$, is about 2.2%–3.5% for the current allowed solar mixing parameters [18]. Thus the difference between the day and the day plus night average CC/NC is less than 2% and much smaller than SNO's 10% measurement uncertainty on CC/NC. See the Appendix for further discussion on the day-night effect.

[4]The relationship between daytime $\frac{\mathrm{CC}}{\mathrm{NC}}$ and $\theta_\odot$ ($=\arcsin\sqrt{(\frac{\mathrm{CC}}{\mathrm{NC}} - f_1)/(1-2f_1)}$) or $\tan^2\theta_\odot$ ($= (\frac{\mathrm{CC}}{\mathrm{NC}} - f_1)/(1 - f_1 - \frac{\mathrm{CC}}{\mathrm{NC}})$) is not as transparent as $\sin^2\theta_\odot$.

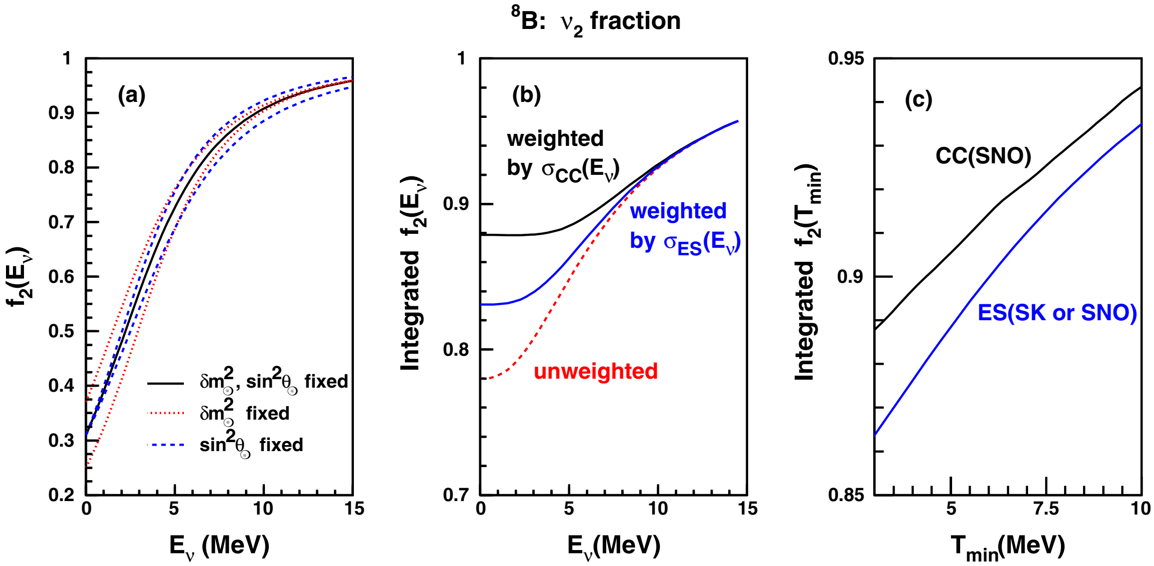

FIG. 3 (color online). (a) The fraction of ν_2, $f_2(E_\nu)$, as a function of the neutrino energy. The solid (black) curve is obtained using the central values for $\delta m^2_\odot = 8.0 \times 10^{-5}$ eV^2 and $\sin^2\theta_\odot = 0.31$ whereas the blue dashed (red dotted) lines are the 90% CL range varying $\delta m^2_\odot$ ($\sin^2\theta_\odot$) but holding $\sin^2\theta_\odot$ ($\delta m^2_\odot$) fixed at the central value, Eq. (1). (b) The integrated fraction of ^{8}B neutrinos which are ν_2's above an energy, E_ν [dashed (red) curve], whereas the solid upper black and middle blue curves are weighted by the energy dependence of the CC cross section [16] and the ES cross section [19], respectively. (c) The integrated fraction of ^{8}B neutrinos as a function of the threshold kinetic energy of the recoil electrons for CC (SNO) and ES (SK or SNO) reactions.

the best fit value for $\sin^2\theta_\odot$. Numerically, we find $f_2 = 82\%$ with errors of the order of 10% which are difficult to estimate due to correlations between the various factors.[5]

In general, in the presence of neutrino flavor transitions, the fraction of ν_1 and ν_2 are not the same for ES and CC because the energy dependence of the cross sections are different. However, in Ref. [20], it was suggested that if we set analysis threshold energies for SK and SNO appropriately as $T_{\rm SNO} = 0.995T_{\rm SK} - 1.71$ (MeV), where $T_{\rm SNO}$ and $T_{\rm SK}$ are the kinetic energy threshold of the resulting electron, the energy responses of these detectors become practically identical [20]. Thus, using such a set of thresholds, even if there is a spectral distortion in the recoil electron energy spectrum, to a good approximation, SK/SNO ES and SNO CC are related as follows,

$$\frac{\rm ES}{\rm NC} = \frac{\rm CC}{\rm NC} + r\left(1 - \frac{\rm CC}{\rm NC}\right), \tag{13}$$

and all the results we obtained for SNO in this paper are equally valid for ES at SK and/or at SNO provided the energy thresholds are set appropriately.[6]

In Fig. 3(a) we show the ν_2 fraction, $f_2(E_\nu)$, versus E_ν. The rapid decrease in the ν_2 fraction below $E_\nu \sim 8$ MeV is responsible for the expected spectral distortion at energies near threshold in both SNO (see Fig. 36 of Ref. [2]) and SK (see Fig. 51 of the last reference in [5]). For a neutrino energy near 10 MeV, the SNO sweet spot, the 90% CL variation in $\delta m^2_\odot$ changes $f_2(E_\nu)$ more than the 90% CL variation in $\sin^2\theta_\odot$, whereas in Fig. 3(b) we give the fraction of ν_2's above a given energy both unweighted and weighted by the energy dependence of the CC interaction and ES cross sections. Note that above a neutrino energy of 7.5 MeV there is little difference between the weighted and unweighted integrated ν_2 fraction. Furthermore, in Fig. 3(c), we show the fraction of ν_2's above a given kinetic energy for the recoil electron for both CC (SNO) and ES (SK or SNO) reactions. We observe that, for the same threshold, f_2 for ES is always smaller than that for CC. This is expected since unweighted f_2 is an increasing function of E_ν and the CC cross section increases more rapidly with energy than that of ES cross section. Hereafter, unless otherwise stated, we focus on the SNO CC reaction, as the results for ES reaction are qualitatively similar and the thresholds can be adjusted to give identical results for all practical purposes.[7]

In Fig. 4 we give the breakdown into ν_1 and ν_2 for the raw ^{8}B spectrum as well as the spectrum weighted by the

[5]This implies that the non-ν_e component of SK's ES event rate comes primarily from the ν_2 component with a very small contribution (less than 10%) from the ν_1 component.

[6]In fact this suggests an alternative to looking for a spectral distortion to test MSW; compare ES to (1-r)CC + rNC for a variety of kinetic energy thresholds.

[7]Since the average ν_e survival probability, $\langle P_{ee}\rangle$, can be equated for SNO and SK with appropriate thresholds [20] and the average ν_2 fraction is related to $\langle P_{ee}\rangle$ by $f_2 = (\cos^2\theta_\odot - \langle P_{ee}\rangle)/\cos 2\theta_\odot$, then for these thresholds the ν_2 fractions are equal.

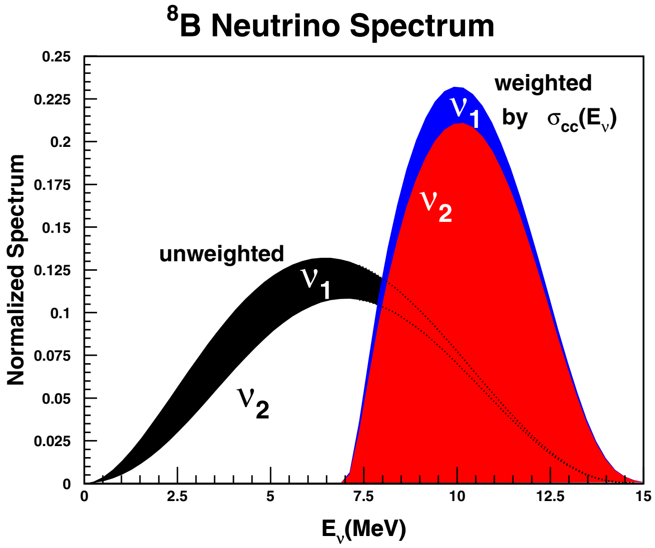

FIG. 4 (color online). The normalized ^{8}B energy spectrum broken into the ν_1 and ν_2 components. The left hand curves (black and white) are unweighted whereas the right hand curves (blue and red) are weighted by the energy dependence of the CC cross section [16] with a threshold of 5.5 MeV for the recoil electron's kinetic energy.

energy dependence of the CC interaction using a threshold of 5.5 MeV for the kinetic energy of the recoil electrons. Here we have used the current best fit values for $\delta m^2_\odot$ and $\sin^2\theta_\odot$.

How does the fraction of ν_2 vary if we allow $\delta m^2_\odot$ and $\sin^2\theta_\odot$ to deviate from their best fit values? In Fig. 5(a) we show the contours of the fraction of ν_2 in the $\delta m^2_\odot$ versus $\sin^2\theta_\odot$ plane where we have weighted the spectrum by the energy dependence of the CC interaction cross section, and we have used a threshold on the kinetic energy of the recoil electrons of 5.5 MeV. This energy dependence mimics the energy dependence of the SNO detector. Because of the strong correlation between $\sin^2\theta_\odot$ and the daytime CC/NC ratio we also give the contours of the fraction of ν_2 in the $\delta m^2_\odot$ versus the daytime CC/NC plane in Fig. 5(b). Thus the ^{8}B energy weighted average fraction of ν_2's observed by SNO is

$$f_2 = 91 \pm 2\% \quad \text{at the 95\%CL.} \tag{14}$$

This is the two neutrino answer to the question posed in the title of this paper. We note, however, that as we showed in Fig. 3(c) the value of f_2 is a function of the threshold energy and also depends on the experiment. We estimate that, for SK with the current 4.5 MeV threshold for the kinetic energy of the recoil electrons,

$$f_2 = 88 \pm 2\% \quad \text{at the 95\%CL.} \tag{15}$$

The uncertainty is dominated by the uncertainty in $\delta m^2_\odot/A$. However, the uncertainty on $\delta m^2_\odot$ is approximately 5% from the KamLAND data whereas the uncertainty on the matter potential, A, in the region of ^{8}B production of the standard solar model is 1%–2%; see [21]. Hence, the uncertainty on $\delta m^2_\odot$ dominates.

For the current allowed values for $\delta m^2_\odot$ and $\sin^2\theta_\odot$, the ratio

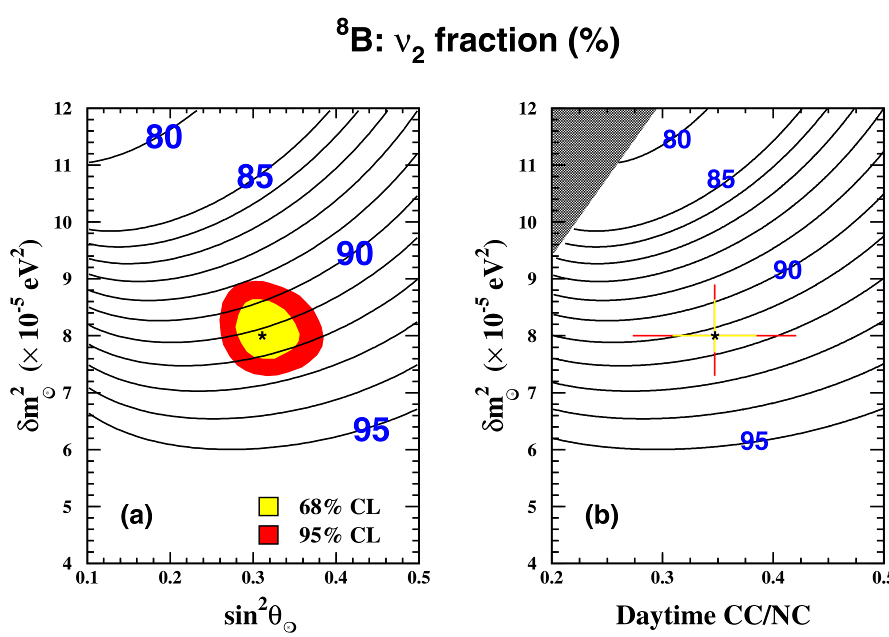

FIG. 5 (color online). (a) The ν_2 fraction (%) in the $\delta m^2_\odot$ versus $\sin^2\theta_\odot$ plane. As in Fig. 2, the current allowed region is also shown. (b) The ν_2 fraction (%) in the $\delta m^2_\odot$ versus the daytime CC/NC ratio of the SNO plane. We have excluded a region in the top left hand corner of this plot which corresponds to $\sin^2\theta_\odot < 0.1$. The current allowed range is indicated by the cross.

$$\frac{\delta m_\odot^2 \sin 2\theta_\odot}{A(^8\mathrm{B}) - \delta m_\odot^2 \cos 2\theta_\odot} \approx \frac{3}{4}, \tag{16}$$

where $A(^8\mathrm{B})$ is obtained using a typical number density of electrons at ^{8}B neutrino production ($Y_e\rho \approx 90$ g.cm^{-3}) and the typical energy of the observed ^{8}B neutrinos ($\approx$ 10 MeV).

For the best fit central values of $\delta m_\odot^2$ and $\sin^2\theta_\odot$, given by Eq. (1), let us define an effective matter potential for the ^{8}B neutrinos, $A_{\mathrm{eff}}^{^8\mathrm{B}}$, such that the left-hand side of Eq. (4) equals our best fit value for the fraction that is ν_2. Thus,

$$A_{\mathrm{eff}}^{^8\mathrm{B}} \equiv \delta m_\odot^2 \sin 2\theta_\odot \left[\cot 2\theta_\odot + \frac{2f_2 - 1}{2\sqrt{f_2(1-f_2)}}\right] = 1.36 \times 10^{-4}\ \mathrm{eV}^2, \tag{17}$$

for $f_2 = 0.910$. This $A_{\mathrm{eff}}^{^8\mathrm{B}}$ corresponds to a $Y_e\rho E_\nu = 0.892$ kg cm^{-3} MeV, the effective mixing angle, $\theta_\odot^N|_{\mathrm{eff}}$, is 73° and the effective $\delta m_N^2|_{\mathrm{eff}} = 12.9 \times 10^{-5}$ eV^2.

We can then use this $A_{\mathrm{eff}}^{^8\mathrm{B}}$ to perform a Taylor series expansion about the best fit point as follows,

$$f_2 = \langle \sin^2\theta_\odot^N \rangle_{^8\mathrm{B}} \approx \frac{9}{10} + \frac{24}{125}\xi + \mathcal{O}(\xi^2)$$
$$\text{with} \quad \xi \equiv \frac{3}{4} - \frac{\delta m_\odot^2 \sin 2\theta_\odot}{(A_{\mathrm{eff}}^{^8\mathrm{B}} - \delta m_\odot^2 \cos 2\theta_\odot)}. \tag{18}$$

This simple expression reproduces the values of f_2 to high precision throughout the 95% allowed region of the KamLAND and the solar neutrino experiments given in Fig. 5(a). In this sense our $A_{\mathrm{eff}}^{^8\mathrm{B}}$ is the effective matter potential for the ^{8}B neutrinos. An expansion in $\delta m_\odot^2/A$ around its typical value of 0.6 could also be used but the coefficients are ever more complex trigonometric functions of $\theta_\odot$, whereas with our ξ expansion the coefficients are small rational numbers.

B. ^{7}Be and pp neutrinos

For ^{7}Be and pp neutrinos the fractions of ν_1 and ν_2 are much closer to the vacuum values of $\cos^2\theta_\odot$ and $\sin^2\theta_\odot$, respectively, as they are produced well below (more than a factor of 2) the MS resonance in the sun, and an expansion in $A/\delta m_\odot^2$ is the natural one. In the third reference in [17], the electron neutrino survival probability was obtained by a similar expansion around the average of the matter potential. Using this expansion, we find that

$$f_2 = 1 - f_1 = \sin^2\theta_\odot^N = \sin^2\theta_\odot + \frac{1}{2}\sin^2 2\theta_\odot\left(\frac{A}{\delta m_\odot^2}\right) + \mathcal{O}\left(\frac{A}{\delta m_\odot^2}\right)^2 \tag{19}$$

$$\text{with} \quad A_{\mathrm{eff}}^{^7\mathrm{Be}} = 1.1 \times 10^{-5}\ \mathrm{eV}^2 \quad \text{and} \quad A_{\mathrm{eff}}^{\mathrm{pp}} = 0.31 \times 10^{-5}\ \mathrm{eV}^2, \tag{20}$$

where the averaged value of the energy (weighted by the cross section) as well as the electron densities used are, respectively, $\langle E_\nu\rangle_{\mathrm{pp}} = 0.33$ MeV and $\langle Y_e\rho\rangle_{\mathrm{pp}} = 62$ g/cm^3 for pp, and $\langle E_\nu\rangle_{^7\mathrm{Be}} = 0.86$ MeV and $\langle Y_e\rho\rangle_{^7\mathrm{Be}} = 81$ g/cm^3 for ^{7}Be. Thus $f_2(^7\mathrm{Be}) = 37 \pm 4(7)\%$ and $f_2(\mathrm{pp}) = 33 \pm 4(7)\%$ at 68% (95%) CL where the uncertainty here is dominated by our knowledge of $\sin^2\theta_\odot$.

C. Two neutrino summary

In Fig. 6 we give the neutrino mass spectrum, the value of the fraction of ν_2's ($\sin^2\theta_\odot^N$) and the fractional flux as a function of the electron number density times neutrino energy, $Y_e\rho E_\nu$, which is proportional to the matter potential, for the ^{8}B, ^{7}Be, and pp neutrinos using the best fit values of $\delta m_\odot^2$ and $\sin^2\theta_\odot$ in Eq. (1). The ^{8}B energy spectrum has been weighted by the energy dependence of the CC interaction of SNO with a 5.5 MeV threshold on the kinetic energy of the recoil electrons whereas the pp energy spectrum has been weighted by the energy dependence of the charged current interaction on gallium with a 0.24 MeV threshold. The vertical dashed lines give the value of $Y_e\rho E_\nu$ which reproduces the average ν_2 fraction using

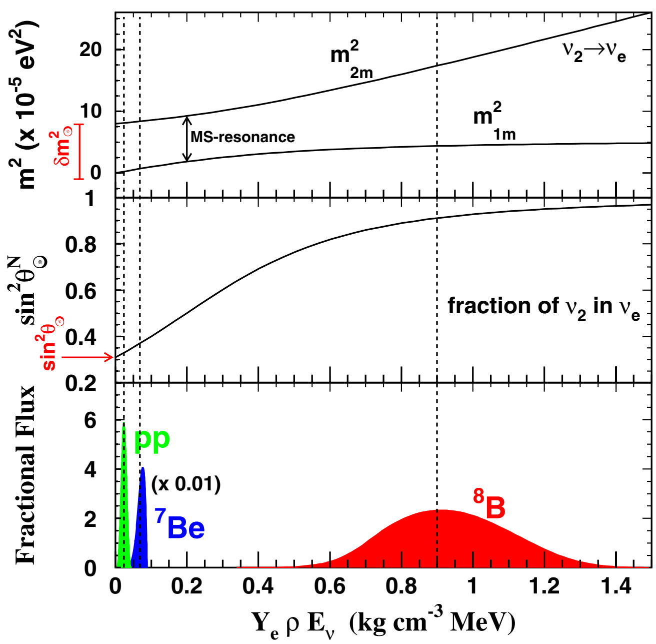

FIG. 6 (color online). The mass spectrum (top panel), the fraction of ν_2's produced, $\sin^2\theta_\odot^N$ (middle panel), and the fractional flux (bottom panel) versus the product of the electron fraction (Y_e), the matter density (ρ), and the neutrino energy (E_ν) for the best fit values $\delta m_\odot^2 = 8.0 \times 10^{-5}$ eV^2 and $\sin^2\theta_\odot = 0.310$. The vertical dashed lines give the value of $Y_e\rho E_\nu$ which reproduces the average ν_2 fractions: 91%, 37%, and 33% for ^{8}B, ^{7}Be, and pp, respectively. This value of $Y_e\rho E_\nu = 0.89$ kg cm^{-3} MeV, for the ^{8}B neutrinos, gives a production mixing angle equal to 73° and a production $\delta m_N^2 = 13 \times 10^{-5}$ eV^2. $Y_e\rho E_\nu = 1$ kg cm^{-3} MeV corresponds, in terms of the matter potential, to 15.3×10^{-5} eV^2; see Eq. (6).

the simple expression in Eq. (4) and are useful for the approximations given in Eqs. (18) and (19).

The energy weighted ν_2 fractions for ^{8}B, ^{7}Be, and pp neutrinos using a two neutrino analysis, at the 95% CL, are

$$f_2(^8\mathrm{B}) = 91 \pm 2\%, \tag{21}$$

$$f_2(^7\mathrm{Be}) = 37 \pm 4\%, \tag{22}$$

$$f_2(\mathrm{pp}) = 33 \pm 4\%, \tag{23}$$

where the uncertainties for ^{7}Be and pp are dominated by the uncertainty on $\sin^2\theta_\odot$ whereas for ^{8}B the uncertainty is dominated by the uncertainty on $\delta m^2_\odot$. The ν_1 fractions, f_1, are simply $1 - f_2$.

III. THREE NEUTRINO ANALYSIS

For the three neutrino analysis we first must discuss the size of the component of ν_3 which is ν_e, i.e. the size of $\sin^2\theta_{13}$. This mixing angle determines the size of the effects on ν_e associated with the atmospheric mass squared difference. The CHOOZ reactor experiment [22] gives a limit of $0 \leq \sin^2\theta_{13} < 0.04$ at the 90% CL for $\delta m^2_{31} = 2.5 \times 10^{-3}$ eV^2. This constraint depends on the precise value of δm^2_{31} with a stronger (weaker) constraint at higher (lower) allowed values of δm^2_{31}. The best constraint on $\sin^2\theta_{13}$ comes from global analyses (see Ref. [18]) which give

$$0 \leq \sin^2\theta_{13} < 0.025 \tag{24}$$

at the 90% CL when marginalized over δm^2_{31}.

So far the inclusion of genuine three flavor effects has not been important because these effects are controlled by the two small parameters

$$\frac{\delta m^2_{21}}{\delta m^2_{32}} \approx 0.03 \quad \text{and/or} \quad \sin^2\theta_{13} \leq 0.025. \tag{25}$$

However as the accuracy of the neutrino data improves it will become inevitable to take into account genuine three flavor effects. See [18,23] for recent studies on the impact of θ_{13} on solar neutrinos.

Suppose that Double CHOOZ [24], T2K [25], NoνA [26], or some other experiment measures a nonzero value for $\sin^2\theta_{13}$. What effect does this have on the previous analysis? How does this change our knowledge of the solar parameters and the relationship between the solar mixing angle and the fraction of ν_2?

Our knowledge of the solar δm^2 comes primarily from the KamLAND experiment where the effects of the atmospheric δm^2 are averaged over many oscillations, thus to high accuracy

$$\delta m^2_{21} = \delta m^2_\odot, \tag{26}$$

i.e. the solar δm^2 remains unaffected. Remember, we are using the notation δm^2_{21} and $\sin^2\theta_{12}$ for the three neutrino analysis to distinguish it from $\delta m^2_\odot$ and $\sin^2\theta_\odot$ used in the two neutrino analysis.

A. ^{8}B three neutrino analysis

For the mixing angle $\sin^2\theta_{12}$ the situation is more complicated in the three neutrino analysis. The ^{8}B electron neutrino survival probability measured by SNO's daytime CC/NC ratio can be written as

$$\frac{\mathrm{CC}}{\mathrm{NC}} = \mathcal{F}_1\cos^2\theta_{13}\cos^2\theta_{12} + \mathcal{F}_2\cos^2\theta_{13}\sin^2\theta_{12} + \mathcal{F}_3\sin^2\theta_{13}, \tag{27}$$

where $\mathcal{F}_1$, $\mathcal{F}_2$, and $\mathcal{F}_3$ are the fraction of ν_1, ν_2, and ν_3, respectively, satisfying $\mathcal{F}_1 + \mathcal{F}_2 + \mathcal{F}_3 = 1$. The ν_3 fraction is given by

$$\mathcal{F}_3 = \left(1 \pm \frac{2A}{|\delta m^2_{31}|}\right)\sin^2\theta_{13} \approx \sin^2\theta_{13}, \tag{28}$$

where the $+(-)$ sign refers to the normal $\delta m^2_{31} > 0$ (inverted $\delta m^2_{31} < 0$) mass hierarchy. The small correction factor $(2A/|\delta m^2_{31}|) \sim 10\%$ comes from matter effects associated with atmospheric δm^2 in the center of the sun. We will ignore this correction since it is small and currently the sign is unknown. Hence, $\mathcal{F}_1 + \mathcal{F}_2 = 1 - \mathcal{F}_3 = \cos^2\theta_{13}$.

With this approximation the ν_1 and ν_2 fractions can be written as

$$\mathcal{F}_1 = \cos^2\theta_{13}\langle\cos^2\theta^N_{12}\rangle_{^8\mathrm{B}}$$
$$\text{and} \quad \mathcal{F}_2 = \cos^2\theta_{13}\langle\sin^2\theta^N_{12}\rangle_{^8\mathrm{B}}, \tag{29}$$

where the average $\langle\cdots\rangle_{^8\mathrm{B}}$ is over the solar production region and the energy of the observed neutrinos. $\sin^2\theta^N_{12}$ is given by Eq. (4) with the replacements $\sin^2\theta_\odot \to \sin^2\theta_{12}$ and $A \to A\cos^2\theta_{13}$ [27].

In going from the two neutrino analysis to the three neutrino analysis the quantity that must remain unchanged is the value of the electron neutrino survival probability, i.e. the CC/NC ratio. This implies that we must adjust the value of $\sin^2\theta_{12}$ and hence the fractions of ν_1 and ν_2 so that the CC/NC ratio remains constant. We have performed this procedure numerically and report the result as a Taylor series expansion in the fraction of ν_1's about $\sin^2\theta_{13} = 0$. If we write

$$\mathcal{F}_1(\sin^2\theta_{13}) = \mathcal{F}_1(0) + \alpha\sin^2\theta_{13} + \mathcal{O}(\sin^4\theta_{13}), \tag{30}$$

$$\text{then} \quad \mathcal{F}_1(0) \equiv f_1, \quad \text{and} \quad \alpha \equiv \left.\frac{d\mathcal{F}_1}{d\sin^2\theta_{13}}\right|_{\sin^2\theta_{13}=0}. \tag{31}$$

In Fig. 7(a) we have plotted the contours of $\alpha \equiv (d\mathcal{F}_1/d\sin^2\theta_{13})|_0$ in the $\delta m^2_\odot$ versus $\sin^2\theta_\odot$ plane. Near the best values this total derivative is close to zero, i.e.

$$\left.\frac{d\mathcal{F}_1}{d\sin^2\theta_{13}}\right|_{\sin^2\theta_{13}=0} = 0.00^{+0.02}_{-0.04} \tag{32}$$

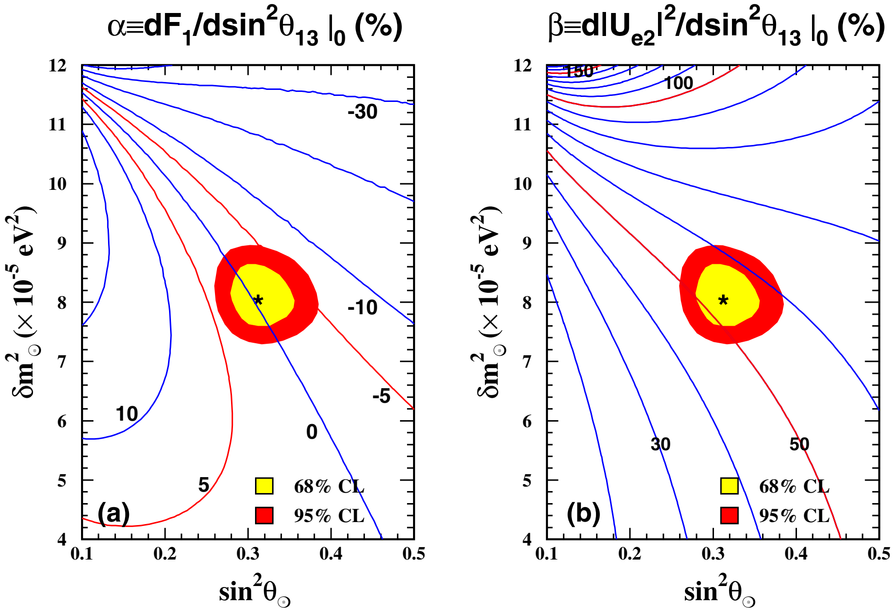

FIG. 7 (color online). Isocontours of the derivatives of $\mathcal{F}_1$ (a) and $|U_{e2}|^2$ (b) with respect to $\sin^2\theta_{13}$ evaluated at $\sin^2\theta_{13} = 0$ in the $\delta m^2_\odot$ versus $\sin^2\theta_\odot$ plane. The contours are labeled as percentages. The 68% and 95% CL allowed regions are also indicated.

at the 68% CL. As $\sin^2\theta_{13}$ grows above zero, the size of $\mathcal{F}_1$ is influenced by a number of effects; the first is the factor of $\cos^2\theta_{13}$ in Eq. (29) which reduces $\mathcal{F}_1$, the second is the matter potential A which is reduced to $A\cos^2\theta_{13}$ raising the fraction $\mathcal{F}_1$, and the third is the value of $\sin^2\theta_{12}$ which changes to hold the CC/NC ratio fixed. By coincidence the sum of these effects approximately cancels at the current best fit values and the fraction of ν_1 remains approximately *unchanged* as $\sin^2\theta_{13}$ gets larger. This implies that the fraction of ν_2 is reduced by $\sim\sin^2\theta_{13}$ since the sum of $\mathcal{F}_1 + \mathcal{F}_2$ is simply $\cos^2\theta_{13}$, thus

$$\mathcal{F}_1 \approx f_1 = 0.09 \mp 0.02, \tag{33}$$

$$\mathcal{F}_2 = f_2 - \sin^2\theta_{13} \approx 0.91 \pm 0.02 - \sin^2\theta_{13}, \tag{34}$$

$$\mathcal{F}_3 = \sin^2\theta_{13}. \tag{35}$$

Remember f_i and $\mathcal{F}_i$ are the fractions of the ith mass eigenstate in the two and three neutrino analyses, respectively. The uncertainty comes primarily from the uncertainty in $\delta m^2_\odot$ measured by KamLAND.

As a use of these fractions one can, for example, evaluate the Maki-Nakagawa-Sakata (MNS) matrix element, $|U_{e2}|^2 = \cos^2\theta_{13}\sin^2\theta_{12}$, by rewriting Eq. (27) as

$$|U_{e2}|^2 = \cos^2\theta_{13}\sin^2\theta_{12} = \frac{(\frac{\mathrm{CC}}{\mathrm{NC}} - \cos^2\theta_{13}\mathcal{F}_1)}{(\cos^2\theta_{13} - 2\mathcal{F}_1)}, \tag{36}$$

where terms of $\mathcal{O}(\sin^4\theta_{13})$ have been dropped. Performing a Taylor series expansion about $\sin^2\theta_{13} = 0$, we find

$$|U_{e2}|^2 = \sin^2\theta_\odot^{8B} + \beta\sin^2\theta_{13} + \mathcal{O}(\sin^4\theta_{13}), \tag{37}$$

$$\text{with } \beta \equiv \frac{d|U_{e2}|^2}{d\sin^2\theta_{13}}\bigg|_0 = \frac{(f_1 - \alpha) + (1 + 2\alpha)\sin^2\theta_\odot}{(1 - 2f_1)}. \tag{38}$$

For the current allowed region of the solar parameters, this implies that

$$|U_{e2}|^2 \approx \sin^2\theta_\odot^{^8\mathrm{B}} + (0.53^{+0.06}_{-0.04})\sin^2\theta_{13} \tag{39}$$

at the 68% CL, i.e. the three neutrino $|U_{e2}|^2$ is approximately equal to the $\sin^2\theta_\odot^{^8\mathrm{B}}$ using a two neutrino analysis of only the ^{8}B electron neutrino survival probability using KamLAND's $\delta m^2_\odot$ constraint plus 53% of $|U_{e3}|^2$ determined, say, by a CHOOZ-like reactor experiment; see Fig. 7(b). Again, this is not a substitute for a full global analysis but provides an approximate expression for the change in $|U_{e2}|^2$ for nonzero values of $\sin^2\theta_{13}$. Even the sign is opposite from naive expectation.

If a similar analysis is performed for the three neutrino sine squared solar mixing angle $\sin^2\theta_{12}$, the total derivative with respect to $\sin^2\theta_{13}$ is simply $(\beta + \sin^2\theta_\odot)$. For $\tan^2\theta_{12}$ the total derivative is $(\beta + \sin^2\theta_\odot)/\cos^4\theta_\odot$. Alternatively we can turn this discussion inside out and write the ^{8}B effective two component $\sin^2\theta_\odot$ in terms of three component quantities as

$$\sin^2\theta_\odot^{^8\mathrm{B}} = \sin^2\theta_{12} - (\beta + \sin^2\theta_{12})\sin^2\theta_{13}. \tag{40}$$

For KamLAND, the equivalent relationship is

$$\sin^2\theta_\odot^{^8\mathrm{Kam}} = \sin^2\theta_{12} - \left(\frac{\sin^2 2\theta_{12}}{2\cos 2\theta_{12}}\right)\sin^2\theta_{13}. \tag{41}$$

For the current best fit values $(\beta + \sin^2\theta_{12}) \approx 0.90$ is close to $\sin^2 2\theta_{12}/2\cos 2\theta_{12} \approx 1.1$, i.e. in a two component analysis the difference between the solar ^{8}B and KamLAND $\sin^2\theta_\odot$'s is approximately $0.2\sin^2\theta_{13}$.

B. ^{7}Be and pp three neutrino analysis

Performing a similar three neutrino analysis for the pp (or ^{7}Be) neutrinos we find that the fraction of neutrino mass eigenstates is

$$\mathcal{F}_1 \approx \cos^2\theta_\odot - \frac{1}{2}\sin^2 2\theta_\odot\left(\frac{A}{\delta m^2_\odot}\right) + \frac{\sin^2\theta_\odot}{\cos 2\theta_\odot}\sin^2\theta_{13} = f_1 + 0.82\sin^2\theta_{13}, \tag{42}$$

$$\mathcal{F}_2 \approx \sin^2\theta_\odot + \frac{1}{2}\sin^2 2\theta_\odot\left(\frac{A}{\delta m^2_\odot}\right) - \frac{\cos^2\theta_\odot}{\cos 2\theta_\odot}\sin^2\theta_{13} = f_2 - 1.8\sin^2\theta_{13}, \tag{43}$$

$$\mathcal{F}_3 \approx \sin^2\theta_{13}, \tag{44}$$

where the $\sin^2\theta_\odot$ here is determined from the pp (or ^{7}Be) neutrinos. Terms of order $\mathcal{O}(A/\delta m^2_\odot)^2$, $\mathcal{O}(\sin^4\theta_{13})$, and $\mathcal{O}(\sin^2\theta_{13}A/\delta m^2_\odot)$ have been dropped here. The two neutrino fractions f_1 and f_2 are given in Eq. (19).

Again we can use these fractions to determine the $|U_{e2}|^2$ element of the MNS matrix,

$$|U_{e2}|^2 = \sin^2\theta_\odot - \left(\frac{\cos^2\theta_\odot}{\cos 2\theta_\odot}\right)\sin^2\theta_{13} \approx \sin^2\theta_\odot - 1.8\sin^2\theta_{13}. \tag{45}$$

This equation appears to be in contradiction with Eq. (39) but this is not so since if $\sin^2\theta_{13} \neq 0$ then the two component analysis of the ^{8}B and pp (or ^{7}Be) neutrinos will lead to different values of $\sin^2\theta_\odot$; in fact,

$$\sin^2\theta^{\rm pp}_\odot - \sin^2\theta^{^8{\rm B}}_\odot \approx 2.3\sin^2\theta_{13}. \tag{46}$$

This difference has been extensively exploited in Ref. [23] to determine $\sin^2\theta_{13}$ using only solar neutrino experiments. Their $\sin^2\theta_{12}$ versus $\sin^2\theta_{13}$ figures, e.g. Fig. 6, demonstrates this point in a clear and useful fashion. Also, the numerical values of our derivatives of $|U_{e2}|^2$ are consistent with the inverse of the slopes of their Fig. 6.

Equations (39) and (45) also imply that the uncertainty in the determination of $|U_{e2}|^2$ from the current unknown value of $\sin^2\theta_{13}$ is smaller for the analysis of ^{8}B neutrinos than pp or ^{7}Be neutrinos. Of course the current uncertainty on the two neutrino $\sin^2\theta_\odot$ dominates.

IV. PROBING THE SOLAR INTERIOR BY ^{8}B NEUTRINOS

In this section, as an application of our analysis, we will invert the discussions found in Ref. [28] where the validity of the MSW physics has been tested assuming the SSM prediction of the electron number density as well as the ^{8}B neutrino production region. Here, we will discuss what can be said about these quantities, assuming the validity of the MSW effect in the LMA region. While there is no strong reason to doubt the correctness of the SSM, which is in good agreement also with the helioseismological data [29], it is nevertheless interesting if we can test it independently.

Since the propagation of ^{8}B neutrinos, in the sun, is highly adiabatic in the LMA region, the fraction of ν_2 and, consequently, the SNO CC/NC ratio are determined only by the effective value of the matter potential, $A^{^8{\rm B}}_{\rm eff}$, defined in Sec. II A. This implies that, if we can measure $\sin^2\theta_\odot$ using an experiment independent of the ^{8}B solar neutrinos, then from the measured value of SNO's CC/NC ratio we can determine the value of $A^{^8{\rm B}}_{\rm eff}$. Note that we cannot extract information on the electron number density distribution or the ^{8}B neutrino production distribution, separately, but only on $A^{^8{\rm B}}_{\rm eff}$ which is a single characteristic of the convolution of these two distributions.

For the two flavor neutrino analysis, if we rewrite the definition of the effective matter potential $A^{^8{\rm B}}_{\rm eff}$ given by Eq. (17) using the relationship between f_2 and SNO's CC/NC ratio, Eq. (9), we obtain

$$A^{^8{\rm B}}_{\rm eff} = \delta m^2_\odot \sin 2\theta_\odot\left[\cot 2\theta_\odot + \frac{1 - 2\frac{\rm CC}{\rm NC}}{2\sqrt{(\cos^2\theta_\odot - \frac{\rm CC}{\rm NC})(\frac{\rm CC}{\rm NC} - \sin^2\theta_\odot)}}\right]. \tag{47}$$

This expression can be used to obtain a value of $A^{^8{\rm B}}_{\rm eff}$ from the global fit of $(\sin^2\theta_\odot, \delta m^2_\odot)$ and SNO's CC/NC ratio. Alternatively, suppose $(\sin^2\theta_\odot, \delta m^2_\odot)$ were measured independent of the ^{8}B solar neutrinos. Then, when combined with SNO's ^{8}B neutrino CC/NC ratio, we would have a solar model independent determination of $A^{^8{\rm B}}_{\rm eff}$. We can convert this into an effective value of the electron number density, $Y_e\rho|^{^8{\rm B}}_{\rm eff}$, in the solar ^{8}B production region, as follows,

$$Y_e\rho|^{^8{\rm B}}_{\rm eff} \equiv \frac{M_n}{2\sqrt{2}G_F}\frac{A^{^8{\rm B}}_{\rm eff}}{\langle E_\nu\rangle_{^8{\rm B}}}, \tag{48}$$

where $\langle E_\nu\rangle_{^8{\rm B}} = 10.5$ MeV is the CC cross-section weighted average energy of neutrinos observed by SNO. For a given solar model, the value of $Y_e\rho|^{^8{\rm B}}_{\rm eff}$ can be calculated for any value of $\sin^2\theta_\odot$ and $\delta m^2_\odot$. The SSM prediction is that $Y_e\rho|^{^8{\rm B}}_{\rm eff} = 85\ {\rm g\,cm^{-3}}$ at the current best fit point.[8] As a comparison, the mean value of $Y_e\rho$ over the ^{8}B production region is 90 ${\rm g\,cm^{-3}}$. The reason that $Y_e\rho|^{^8{\rm B}}_{\rm eff}$ is below the mean value is because values of $Y_e\rho$ below the

[8]Because of the way we have defined $A^{^8{\rm B}}_{\rm eff}$, our $Y_e\rho|^{^8{\rm B}}_{\rm eff}$ has a weak dependence on $\sin^2\theta_\odot$ and $\delta m^2_\odot$ but this variation is less than 2% over the 95% CL allowed region.

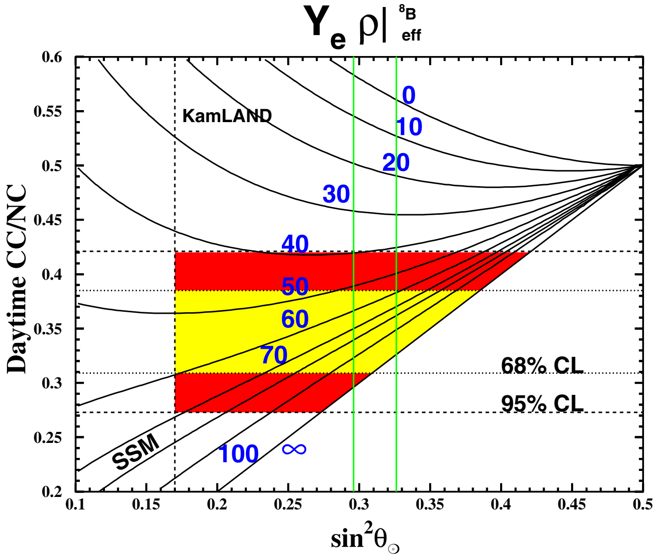

FIG. 8 (color online). The isocontours of $Y_e\rho|_{\rm eff}^{^8\rm B}$ in the $\sin^2\theta_\odot - \mathrm{CC/NC}|_{\rm day}$ plane using $\delta m^2_\odot = 8 \times 10^{-5}\ \mathrm{eV}^2$. The line labeled SSM is the standard solar model prediction for $Y_e\rho|_{\rm eff}^{^8\rm B}$ ($\approx 85\ \mathrm{g/cm^3}$) at the best fit point. The range of observed values of CC/NC is indicated by the shaded horizontal bands. The KamLAND experiment places a lower bound on $\sin^2\theta_\odot$ independent of solar neutrinos at 0.17; see [1]. The vertical band indicates the uncertainty which could be expected by future reactor experiments [30].

mean pull down the ν_2 fraction more than values above the mean raise the ν_2 fraction.

We show in Fig. 8, the isocontours of $Y_e\rho|_{\rm eff}^{^8\rm B}$ in the $\sin^2\theta_\odot - \mathrm{CC/NC}|_{\rm day}$ plane, for the current best fit value of $\delta m^2_\odot$. The observed range of SNO's CC/NC are shown by the horizontal lines.[9] From this plot, we can derive a *solar model independent lower bound* on $Y_e\rho|_{\rm eff}^{^8\rm B}$ which is 40 $\mathrm{g/cm^3}$ for any value of $\theta_\odot$ at 95% CL. Future reactor neutrino oscillation experiments [30] can perform a 2%–3% measurement of $\sin^2\theta_\odot$. The 68% range of $\sin^2\theta_\odot$ is indicated by vertical lines in this figure. However, such precision on $\sin^2\theta_\odot$ will not reduce the allowed values for $Y_e\rho|_{\rm eff}^{^8\rm B}$ unless the error on the measured value of CC/NC is reduced.

A three neutrino analysis is needed if $U_{e3} \neq 0$ and this can be performed using Eq. (47) with the following replacements,

$$\theta_\odot \to \theta_{12}, \qquad \delta m^2_\odot \to \delta m^2_{21}/\cos^2\theta_{13} \quad \text{and} \quad \frac{\mathrm{CC}}{\mathrm{NC}} \to \frac{\mathrm{CC}}{\mathrm{NC}} \frac{1}{\cos^4\theta_{13}}. \tag{49}$$

[9] Another horizontal band could be included by combining the Super-Kamiokande electron scattering measurement with the SNO neutral current measurement. However, since the uncertainty on the NC measurement dominates, this would produce a similar sized band.

A weak upper bound could be derived using a precision measurement of the ^{7}Be and/or pp electron neutrino survival probability in a similar fashion. As $A_{\rm eff}$ gets larger, the fraction of ν_2 gets larger [see Eq. (19)] and hence the electron neutrino survival probability gets smaller for fixed values of the mixing parameters. The upper bound arises when this survival probability is below the measured survival probability at some confidence level, assuming that the mixing parameters have been determined independent of these solar neutrinos.

V. SUMMARY AND CONCLUSIONS

We have performed an extensive analysis of the mass eigenstate fractions of ^{8}B solar neutrinos using only two mass eigenstates ($\sin^2\theta_{13} = 0$) and with three mass eigenstates ($\sin^2\theta_{13} \neq 0$). In the two neutrino analyses the ν_2 fraction is $(91 \pm 2)\%$. The remaining $(9 \mp 2)\%$ is, of course, in the ν_1 mass eigenstate. With these fractions in hand, which are primarily determined by the solar δm^2 measured by the KamLAND experiment, the sine squared of the solar mixing angle is simply related to the CC/NC ratio measured by the SNO experiment. For completeness the mass eigenstate fractions for ^{7}Be and pp are also given.

Allowing for small but nonzero $\sin^2\theta_{13}$, in a full three neutrino analysis, we found very little change in the fraction of ν_1's. This implies, since the ν_3 fraction is $\sin^2\theta_{13}$, that the ν_2 fraction is reduced by $\sin^2\theta_{13}$. That is, the ν_2 fraction is

$$91 \pm 2 - 100\sin^2\theta_{13}\% \quad \text{at the 95\%CL.} \tag{50}$$

Since the CHOOZ experiment constrains the value of $\sin^2\theta_{13} < 0.04$ at the 90% CL this places a lower bound on the ν_2 fraction of ^{8}B solar neutrinos in the mid-eighty percent range making the ^{8}B solar neutrinos the purest mass eigenstate neutrino beam known so far, and it is a ν_2 beam.

As an example of the use of these mass eigenstate fractions, we have shown that, for the ^{8}B neutrinos observed by the SNO experiment, the U_{e2} element of the MNS matrix is given by

$$|U_{e2}|^2 \approx \sin^2\theta_\odot^{^8\rm B} + (0.53^{+0.06}_{-0.04})\sin^2\theta_{13}, \tag{51}$$

where $\sin^2\theta_\odot^{^8\rm B}$ is the sine squared of the solar mixing angle determined by using a two neutrino analysis of the ^{8}B neutrinos plus KamLAND. An analysis for this $\sin^2\theta_\odot^{^8\rm B}$ obtained from the SK, SNO, and KamLAND data [31] gives $\sin^2\theta_\odot^{^8\rm B} = 0.30^{+0.11}_{-0.08}$ at the 95% CL. With the data currently available this is our best estimate of $|U_{e2}|^2$ and it is the most accurately known MNS matrix element.

Finally, we have also demonstrated the possibility of probing the solar interior by ^{8}B neutrinos. We have derived a lower bound on the average electron number density over the region where the solar ^{8}B neutrinos are produced, which is 50% of the standard solar model value.

ACKNOWLEDGMENTS

This work was supported by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) and Conselho Nacional de Ciência e Tecnologia (CNPq). Fermilab is operated under DOE Contract No. DE-AC02-76CH03000. Two of us (H. N. and R. Z. F.) are grateful for the hospitality of the Theoretical Physics Group of the Fermi National Accelerator Laboratory during numerous visits. R. Z. F. is also grateful to the Abdus Salam International Center for Theoretical Physics where the final part of this work was performed. We thank Marc Pinsonneault, Hisakazu Minakata, and Alexei Smirnov for discussions. We would like to thank John Bahcall for many enlightening discussions and encouragement.

APPENDIX: DAY-NIGHT EFFECT

In this appendix we give a sketch of how to use the mass eigenstate fractions to understand the day-night effect caused by the neutrinos traveling through the Earth. For simplicity we will make the approximation that the Earth is constant density with $Y_e\rho \approx 2$ g.cm^{-3}. The *nighttime* CC/NC ratio as measured by SNO can be written as

$$\left.\frac{\mathrm{CC}}{\mathrm{NC}}\right|_{\mathrm{night}} = f_1^{\oplus}\cos^2\theta_{\oplus}^N + f_2^{\oplus}\sin^2\theta_{\oplus}^N, \tag{A1}$$

where $f_i^{\oplus}$'s are the mass eigenstate fractions in the Earth and $\theta_{\oplus}^N$ is the effective mixing angle in the Earth. This expression assumes that all oscillations have been averaged out, which is reasonable, since the oscillation length of a 10 MeV neutrino is 300 km and the neutrinos travel many 1000's of km in the earth. From Eq. (19),

$$\sin^2\theta_{\oplus}^N = \sin^2\theta_{\odot} + \frac{1}{2}\sin^2 2\theta_{\odot}\left(\frac{A_{\oplus}}{\delta m_{\odot}^2}\right) \tag{A2}$$

where $A_{\oplus}$ is the matter potential in the Earth which is about 2% of that at the center of the sun. Also, as we shall show later, the fraction of ν_1 and ν_2 in the Earth are unchanged to leading order in $(A_{\oplus}/\delta m_{\odot}^2)$, i.e.

$$f_2^{\oplus} = 1 - f_1^{\oplus} = f_2 + \mathcal{O}\left(\frac{A_{\oplus}}{\delta m_{\odot}^2}\right)^2. \tag{A3}$$

Therefore

$$\left.\frac{\mathrm{CC}}{\mathrm{NC}}\right|_{\mathrm{night}} - \left.\frac{\mathrm{CC}}{\mathrm{NC}}\right|_{\mathrm{day}} = (f_2 - f_1)(\sin^2\theta_{\oplus}^N - \sin^2\theta_{\odot}) = (f_2 - f_1)\frac{1}{2}\sin^2 2\theta_{\odot}\left(\frac{A_{\oplus}}{\delta m_{\odot}^2}\right). \tag{A4}$$

Using the numerical values for the parameters in this expression gives a 1% difference, hence the asymmetry, $2(N-D)/(N+D)$, is approximately 3%. This is consistent with other calculations; see for example [18].

If we include the coherence of the mass eigenstate neutrinos after they have entered the Earth then (see [32])

$$\begin{aligned}\left.\frac{\mathrm{CC}}{\mathrm{NC}}\right|_{\mathrm{night}} &= f_1|\sqrt{1-\epsilon}\cos\theta_{\oplus}^N e^{-im_1^2L_{\oplus}/2E} \\ &\quad + \sqrt{\epsilon}\sin\theta_{\oplus}^N e^{-im_2^2L_{\oplus}/2E}|^2 \\ &\quad + f_2|\sqrt{1-\epsilon}\sin\theta_{\oplus}^N e^{-im_2^2L_{\oplus}/2E} \\ &\quad - \sqrt{\epsilon}\cos\theta_{\oplus}^N e^{-im_1^2L_{\oplus}/2E}|^2,\end{aligned} \tag{A5}$$

where $L_{\oplus}$ is the distance the neutrino travels in the Earth. When $L_{\oplus} = 0$, this expression must match the *daytime* CC/NC. This matching determines ϵ, the fraction of ν_2 which becomes $\nu_1^{\oplus}$ as the neutrino enters the Earth, and is given by

$$\sqrt{\epsilon} = \frac{\sin^2\theta_{\oplus}^N - \sin^2\theta_{\odot}}{\sin 2\theta_{\odot}} = \frac{1}{2}\sin 2\theta_{\odot}\left(\frac{A_{\oplus}}{\delta m_{\odot}^2}\right). \tag{A6}$$

Note that the fraction that is converted goes as $(A_{\oplus}/\delta m_{\odot}^2)^2$ as assumed earlier.

Hence, the *nighttime* CC/NC is simply

$$\begin{aligned}\left.\frac{\mathrm{CC}}{\mathrm{NC}}\right|_{\mathrm{night}} &= f_1\cos^2\theta_{\odot} + f_2\sin^2\theta_{\odot} + 2(f_2 - f_1) \\ &\quad \times \sqrt{\epsilon}\sqrt{1-\epsilon}\sin 2\theta_{\oplus}^N \sin^2\left(\frac{\delta m_{\odot}^2 L_{\oplus}}{4E}\right),\end{aligned} \tag{A7}$$

and, therefore, the difference is

$$\begin{aligned}\left.\frac{\mathrm{CC}}{\mathrm{NC}}\right|_{\mathrm{night}} - \left.\frac{\mathrm{CC}}{\mathrm{NC}}\right|_{\mathrm{day}} &= 2(f_2 - f_1)(\sin^2\theta_{\oplus}^N - \sin^2\theta_{\odot}) \\ &\quad \times \left\langle \sin^2\left(\frac{\delta m_{\odot}^2 L_{\oplus}}{4E}\right)\right\rangle \\ &= (f_2 - f_1)\sin^2 2\theta_{\odot}\left(\frac{A_{\oplus}}{\delta m_{\odot}^2}\right) \\ &\quad \times \left\langle \sin^2\left(\frac{\delta m_{\odot}^2 L_{\oplus}}{4E}\right)\right\rangle.\end{aligned} \tag{A8}$$

This is identical to our previous result, Eq. (A4), when $\sin^2(\delta m_{\odot}^2 L_{\oplus}/4E)$ is averaged to $\frac{1}{2}$. In this analysis it is clear that to leading order the day-night asymmetry comes from the change in the effective mixing angle in the Earth once the oscillations are averaged out and that the mass eigenstate fractions are nearly identical for both the day and night solar neutrino fluxes.

[1] T. Araki *et al.* (KamLAND Collaboration), Phys. Rev. Lett. **94**, 081801 (2005).
[2] B. Aharmim *et al.* (SNO Collaboration), Phys. Rev. C **72**, 055502 (2005).
[3] R. Davis, Jr., Phys. Rev. Lett. **12**, 303 (1964).
[4] J. N. Bahcall, Phys. Rev. Lett. **12**, 300 (1964).
[5] K. S. Hirata *et al.* (KAMIOKANDE-II Collaboration), Phys. Rev. Lett. **65**, 1297 (1990); M. B. Smy *et al.* (Super-Kamiokande Collaboration), Phys. Rev. D **69**, 011104 (2004); J. Hosaka *et al.* (Super-Kamiokande Collaboration), Phys. Rev. D **73**, 112001 (2006).
[6] B. T. Cleveland *et al.*, Astrophys. J. **496**, 505 (1998); J. N. Abdurashitov *et al.* (SAGE Collaboration), Phys. Rev. C **60**, 055801 (1999); W. Hampel *et al.* (GALLEX Collaboration), Phys. Lett. B **447**, 127 (1999).
[7] J. N. Bahcall and R. K. Ulrich, Rev. Mod. Phys. **60**, 297 (1988); I.-J. Sackman, A. I. Boothroyd, and W. A. Fowler, Astrophys. J. **360**, 727 (1990); J. N. Bahcall and M. H. Pinsonneault, Rev. Mod. Phys. **64**, 885 (1992); V. Castellani, S. Degl'Innocenti, and G. Fiorentini, Astron. Astrophys. **271**, 601 (1993); S. Turck-Chièze and I. Lopes, Astrophys. J. **408**, 347 (1993); G. Berthomieu, J. Provost, P. Morel, and Y. Lebreton, Astron. Astrophys. **268**, 775 (1993); A. Kovetz and G. Shaviv, Astrophys. J. **426**, 787 (1994); V. Castellani, S. Degl'Innocenti, G. Fiorentini, M. Lissia, and B. Ricci, Phys. Lett. B **324**, 425 (1994); X. Shi, D. N. Schramm, and D. S. P. Dearborn, Phys. Rev. D **50**, 2414 (1994); J. N. Bahcall and M. H. Pinsonneault, Rev. Mod. Phys. **67**, 781 (1995); J. Christensen-Dalsgaard *et al.* (GONG Collaboration), Science **272**, 1286 (1996); J. Christensen-Dalsgaard, Europhysics News **25**, 71 (1994); A. Dar and G. Shaviv, Astrophys. J. **468**, 933 (1996); O. Richard, S. Vauclair, C. Charbonnel, and W. A. Dziembowski, Astron. Astrophys. **312**, 1000 (1996); F. Ciacio, S. Degl'Innocenti, and B. Ricci, Astron. Astrophys. Suppl. Ser. **123**, 449 (1997); J. N. Bahcall and M. H. Pinsonneault, Phys. Rev. Lett. **92**, 121301 (2004); S. Turck-Chieze *et al.*, Phys. Rev. Lett. **93**, 211102 (2004).
[8] O. Mena and S. J. Parke, Phys. Rev. D **69**, 117301 (2004).
[9] L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); in *Neutrino '78, Proceedings of 8th International Conference on Neutrino Physics and Astrophysics, Purdue University, West Lafayette, Indiana, 1978*, edited by E. C. Fowler (Purdue University, West Lafayette, 1978), p. C3–C6.
[10] S. P. Mikheev and A. Yu. Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1985)]; Nuovo Cimento Soc. Ital. Fis. C **9**, 17 (1986).
[11] As a definition of the LMA region, from solar experiments only, we used the 95% CL curve in Fig. 35(a) from Ref. [2].
[12] A. S. Dighe, Q. Y. Liu, and A. Y. Smirnov, hep-ph/9903329.
[13] S. J. Parke, Phys. Rev. Lett. **57**, 1275 (1986); S. J. Parke and T. P. Walker, Phys. Rev. Lett. **57**, 2322 (1986).
[14] J. N. Bahcall, A. M. Serenelli, and S. Basu, Astrophys. J. **621**, L85 (2005); see also http://www.sns.ias.edu/~jnb/.
[15] C. E. Ortiz, A. Garcia, R. A. Waltz, M. Bhattacharya, and A. K. Komives, Phys. Rev. Lett. **85**, 2909 (2000).
[16] S. Nakamura, T. Sato, V. Gudkov, and K. Kubodera, Phys. Rev. C **63**, 034617 (2001); **73**, 049904(E) (2006); S. Nakamura, T. Sato, S. Ando, T. S. Park, F. Myhrer, V. Gudkov, and K. Kubodera; see also http://nuc003.psc.sc.edu/ kubodera/NU-D-NSGK/.
[17] W. C. Haxton, Phys. Rev. Lett. **57**, 1271 (1986); S. T. Petcov, Phys. Lett. B **200**, 373 (1988); P. C. de Holanda, W. Liao, and A. Y. Smirnov, Nucl. Phys. **B702**, 307 (2004).
[18] G. L. Fogli, E. Lisi, A. Marrone, and A. Palazzo, hep-ph/0506083; M. Maltoni, T. Schwetz, M. A. Tortola, and J. W. F. Valle, New J. Phys. **6**, 122 (2004).
[19] J. N. Bahcall, M. Kamionkowski, and A. Sirlin, Phys. Rev. D **51**, 6146 (1995).
[20] F. L. Villante, G. Fiorentini, and E. Lisi, Phys. Rev. D **59**, 013006 (1998); see also G. L. Fogli, E. Lisi, A. Palazzo, and F. L. Villante, Phys. Rev. D **63**, 113016 (2001).
[21] J. N. Bahcall, M. H. Pinsonneault, and S. Basu, Astrophys. J. **555**, 990 (2001).
[22] M. Apollonio *et al.* (CHOOZ Collaboration), Phys. Lett. B **420**, 397 (1998); **466**, 415 (1999).
[23] S. Goswami and A. Y. Smirnov, Phys. Rev. D **72**, 053011 (2005).
[24] F. Ardellier *et al.*, hep-ex/0405032.
[25] Y. Itow *et al.*, hep-ex/0106019; for an updated version, see http://neutrino.kek.jp/jhfnu/loi/loi.v2.030528.pdf.
[26] D. Ayres *et al.* (NoνA Collaboration), hep-ex/0210005; hep-ex/0503053.
[27] C. S. Lim, in *Proceedings of the BNL Neutrino Workshop on Opportunities for Neutrino Physics, Upton, New York, 1987* (BNL Report No. BNL-52079, C87/02/05) edited by M. J. Murtagh; T. K. Kuo and J. T. Pantaleone, Phys. Rev. Lett. **57**, 1805 (1986).
[28] G. Fogli and E. Lisi, New J. Phys. **6**, 139 (2004); V. Barger, D. Marfatia, and K. Whisnant, Phys. Lett. B **617**, 78 (2005).
[29] J. N. Bahcall, M. H. Pinsonneault, S. Basu, and J. Christensen-Dalsgaard, Phys. Rev. Lett. **78**, 171 (1997).
[30] H. Minakata, H. Nunokawa, W. J. C. Teves, and R. Zukanovich Funchal, Phys. Rev. D **71**, 013005 (2005); A. Bandyopadhyay, S. Choubey, S. Goswami, and S. T. Petcov, Phys. Rev. D **72**, 033013 (2005).
[31] J. Hosaka *et al.* (Super-Kamiokande Collaboration) (Ref. [5]).
[32] M. Blennow, T. Ohlsson, and H. Snellman, Phys. Rev. D **69**, 073006 (2004); E. K. Akhmedov, M. A. Tortola, and J. W. F. Valle, J. High Energy Phys. 05 (2004) 057.