

## Fixing the solar neutrino parameters with sterile neutrinos

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The neutrino mixing matrix appears to be close to bimaximal mixing, but for the solar mixing angle which is definitively smaller than 45 degrees. Whereas it seems quite easy to understand bimaximal mixing with the use of new global symmetries, as in models using  $L_e - L_\mu - L_\tau$ , understanding the about to 11 degrees of deviation in the observed solar angle seems less simple. We suggest that such a deviation could be due to a light sterile neutrino that mixes with the active sector. The mass scale needed to produce the effect has to be smaller than atmospheric scale, and it would introduce a new mass squared difference which should be smaller than the solar scale. We present a toy model that exemplifies these features.

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### I. INTRODUCTION

Convincing evidence that neutrinos have mass and oscillate has been provided along recent years by Kamiokande, Super-Kamiokande, MACRO, and Soudan results on atmospheric neutrinos; by Chlorine, Kamio-kande, Super-Kamiokande, SAGE, GALLEX and most recently the SNO experiment on solar neutrinos; as well as by KamLAND, K2K, and CHOOZ-PALO-Verde, base-line neutrino experiments [1]. KamLAND independent confirmation of solar oscillation parameters observed by SNO data, indicates that the observed solar mixing is due to a large mixing angle oscillations enhanced by the MSW matter effect [2].

In the standard framework, only three weak neutrino species,  $\nu_e$ ;  $\nu_\mu$ , and  $\nu_\tau$ , are needed to consistently describe

the mentioned experimental results, with the only addition of neutrino masses and mixings as new parameters to the standard model. Central idea in the oscillation phenomena is that, as it happens in the quark sector, neutrino mass eigenstates,  $\nu_{1,2,3}$ , and weak eigenstates are different, but they can be written as linear combinations of each other by using a complex unitary matrix,  $U$ , as  $\nu_\alpha = \sum_i U_{\alpha i} \nu_i$ , for  $\alpha = e, \mu, \tau$  and  $i = 1, 2, 3$ , where we refer only to left handed states. A common parameterization for Majorana neutrinos of the  $U$  matrix is given in terms of three angles and three  $CP$  phases, such that  $U = U_{\text{PMNS}} K$ , where  $K = \text{diag}\{1, e^{i\phi_1}, e^{i\phi_2}\}$ , with  $\phi_1, \phi_2$  the physical  $CP$ -odd Majorana phases. The elements of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [3] are then

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\varphi} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\varphi} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\varphi} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\varphi} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\varphi} & c_{23}c_{13} \end{pmatrix}, \quad (1.1)$$

where  $c_{ij}$  and  $s_{ij}$  stand for  $\cos\theta_{ij}$  and  $\sin\theta_{ij}$  respectively. They represent the observable mixing angles in the basis where the charged lepton masses are diagonal. The Dirac  $CP$  phase,  $\varphi$ , is the only phase involved in neutrino oscillations. Most analysis of neutrino data are usually done in the hypothesis that  $\varphi$  is negligible. This is particularly correct in the case for solar and reactor oscillations data. We will assume so for simplicity hereafter. Finally the kinematical scales for the oscillation are given by the two mass squared differences: (i) the solar/KamLAND scale  $\Delta m_{\text{sol}}^2 = \Delta m_{12}^2$ ; and (ii) the atmospheric scale  $\Delta m_{\text{ATM}}^2 = \Delta m_{23}^2$ .

Combined analysis of all data indicates that at two sigma level [1]

$$\begin{aligned} \Delta m_{\text{sol}}^2 &= (7.92 \pm 0.71) \times 10^{-5} \text{ eV}^2; \\ \Delta m_{\text{ATM}}^2 &= (2.4^{+0.5}_{-0.62}) \times 10^{-3} \text{ eV}^2 \end{aligned} \quad (1.2)$$

for the absolute scales, and

$$\sin^2\theta_{12} = 0.314^{+0.057}_{-0.047}, \quad \sin^2\theta_{23} = 0.44^{+0.18}_{-0.096}. \quad (1.3)$$

CHOOZ-Palo Verde data provide the stringent constraint on  $\theta_{13}$ . Again, the analysis in Ref. [1] gives

$$\sin^2\theta_{13} < 0.9(+0.207) \times 10^{-2}, \quad (1.4)$$

where the number within parenthesis stands for the two sigmas upper uncertainty. It is particularly interesting to notice that two of the above mixing angles are rather large. Also, the fact that  $\theta_{13}$  is consistent with zero indicates that solar/KamLAND experiments are mainly sensible to  $\theta_{12}$ , which means that  $\sin^2\theta_{\text{sol}} = \sin^2\theta_{12}$ ; whereas the mixing observed in atmospheric neutrinos is basically given by

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$\sin^2\theta_{\text{ATM}} = \sin^2\theta_{23}$ . Thus, atmospheric muon neutrino deficit is due to maximal (or almost maximal) mixing among muon and tau neutrinos; whereas solar deficit is due to a large, but not maximal, mixing of electron to other active neutrino species. Indeed, by taking central values of the mixing angles one sees that

$$\theta_{12} \approx 34.08^\circ; \quad \text{and} \quad \theta_{23} \approx 41.55^\circ. \quad (1.5)$$

Thus, solar mixing is far from being maximal by at least 11 degrees.

A complete understanding of the value of neutrino oscillation parameters from a theoretical point of view is yet more challenging. On one hand side, it is tempting to believe that  $U_{\text{PMNS}}$  could be the result of a relatively small perturbation around the bimaximal mixing matrix

$$U_{\text{BM}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{\sqrt{2}} \end{pmatrix}, \quad (1.6)$$

for which two angles are exactly maximal and the third is null. This at least seems to be a very good approximation for atmospheric and  $\theta_{13}$  angles within one sigma level. One could then assume that this matrix arises as the zero order of a theory for neutrino masses and mixings that contains some global (flavor) symmetry  $G$ , which is actually broken in a way that the amount of breaking of  $G$  would provide the 11 degrees of deviation in solar angle. One would then write

$$U_{\text{PMNS}} = U_{\text{BM}} \cdot U_A, \quad (1.7)$$

where  $U_A$  parameterizes the additional rotations induced by the breaking of the flavor symmetry. Indeed there are simple models already in the literature that realize bimaximal mixing. This happens for instance in models that use  $L' = L_e - L_\mu - L_\tau$  as a global symmetry for the neutrino mass matrix [4,5]. On the other hand, it is a typical feature of those models to induce the desired correction in the neutrino mixings via loop effects, which are usually suppressed to the level of being smaller than what is required for an understanding of the solar mixing.

Recently it has also been suggested that  $U_A$  in last equation could be the very same CKM matrix  $U_{\text{CKM}}$  of the quark sector [6]. This quark-lepton complementarity is indeed an intriguing possibility since the Cabbibo angle is just about 12 degrees. Nevertheless, its realization seems to require that quark and lepton masses be somehow correlated in a non trivial way, and so far there are no complete models that may satisfactorily realize it [7].

In this paper we will take a different perspective to the problem, and suggest that the  $U_A$  correction may rather come due to the couplings of the active neutrinos to a fourth (sterile) neutrino which is lighter than the atmospheric scale, and thus it is not constrained by the LSND nor Bugey/CHOOZ nor KARMEN data. Our hypothesis may

rather be constrained by solar data, however, with the currently allowed range of active-sterile mixing at one sigma level,  $\sin^2\eta < 0.09$  [8], there may still be enough room as to provide the desired corrections. We should mention that light sterile neutrinos were suggested earlier in Ref. [9] as a way to fix a small deviation of Homestake Ar production rate results from the generic LMA prediction, and the apparent absence of the upturn of the energy spectrum at low energies in Super-Kamiokande and SNO. We do not analyze this possible effects here, since our main goal for the moment is to present an additional possible theoretical use of the light sterile neutrinos. To be more specific in the discussion we will analyze the particular case of models for inverted hierarchy that use a global  $L' = L_e - L_\mu - L_\tau$  symmetry.

The paper is organized as follows. In Sec. II we discuss the generalities of the  $L'$  models that provide bimaximal mixing. For completeness we show that the squared mass spectrum is inverted, with the two heavier states degenerated at the limit of the exact  $L'$  symmetry. We then show that a generic diagonal correction on the mass matrix, which explicitly breaks the  $L'$  symmetry, does not provide enough freedom to simultaneously generate the solar mass splitting and the 11 degrees of corrections to the solar mixing. Either one of them comes out to be larger than the observed values. In Sec. III we address the question of whether the breaking of the  $L'$  symmetry by the coupling with a sterile neutrino may do the job. As the sterile is expected to alter the solar parameters without substantially affecting the atmospheric ones, we will work in the hypothesis that the sterile mixes preferentially to a single active state after a bimaximal rotation. Thus, we show that within this hypothesis the sterile couplings by themselves are also unlikely to provide both the solar parameters, however, both the effects, the  $L'$  diagonal breaking mass terms on the active sector and the sterile coupling, may compensate each other to provide the solar mass scale and, at the same time, to give the right solar mixing. We present the results of a numerical analysis to identify the narrow region on the parameter space where our mechanism may work for the particular texture that we introduce. Finally, some concluding remarks are presented.

## II. THE SOLAR MIXING PROBLEM IN $L_e - L_\mu - L_\tau$ MODELS

We start by assuming that the Majorana neutrino mass matrix in the basis  $(\nu_e, \nu_\mu, \nu_\tau)$  has the form

$$M_0 = m \begin{pmatrix} 0 & \cos\theta & \sin\theta \\ \cos\theta & 0 & 0 \\ \sin\theta & 0 & 0 \end{pmatrix}, \quad (2.1)$$

where the overall scale sets the atmospheric scale,  $m = \sqrt{\Delta m_{\text{ATM}}^2}$ , and  $\theta$  shall correspond to the atmospheric angle,

which for the present analysis will be treated as general although in our final estimations it would be taken to be exactly  $\pi/4$ . There are many models nowadays in the literature that provide the above given mass texture, see for instance Refs. [4,5]. An attractive way to motivate such mass terms is by assuming that the tree level Majorana mass terms obey a global  $L' = L_e - L_\mu - L_\tau$ . Consider for instance a simple model with total lepton number and  $L'$  as global symmetries, where, besides the Standard Higgs doublet  $H(L = 0, L' = 0)$ , there is a scalar triplet  $\Delta(L = 2, L' = 0)$ . Since our triplet has no  $L'$  charge, the only allowed Yukawa couplings involved in neutrino masses are

$$\Delta \bar{L}_e^c (a L_\mu + b L_\tau) + \text{H.c.},$$

where  $a$  and  $b$  stand for the corresponding Yukawa couplings. Note that we choose to work in the basis where Dirac Yukawa couplings and thus charged lepton masses are diagonal for this field content. After symmetry breaking, assuming that  $\langle \Delta \rangle \neq 0$  and small, one gets the Type II seesaw neutrino masses that are given in the mass matrix of Eq. (2.1), with  $m = \langle \Delta \rangle \sqrt{a^2 + b^2}$  and  $\tan \theta = a/b$ .

It is easy to see that the more general form of the bimaximal mixing matrix

$$U_{\text{BM}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{\cos \theta}{\sqrt{2}} & -\frac{\cos \theta}{\sqrt{2}} & \sin \theta \\ \frac{\sin \theta}{\sqrt{2}} & -\frac{\sin \theta}{\sqrt{2}} & -\cos \theta \end{pmatrix}, \quad (2.2)$$

which for  $\theta = \pi/4$  reduces to that given in Eq. (1.6); diagonalizes  $M_0$ . Indeed,

$$U_{\text{BM}}^\dagger M_0 U_{\text{BM}} = m \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (2.3)$$

Thus, the spectrum is inverted and the heavier squared masses are degenerated. The gap  $\Delta m_{13}^2 = \Delta m_{23}^2 = m^2$  is identified with the atmospheric scale as already mentioned, and thus,  $\theta$  becomes the atmospheric mixing angle. At this level, the model does not provide any explanation for the solar scale, since  $\Delta m_{12}^2$  is exactly zero. Moreover, the mixing angle calculated from the standard formula for the solar mixing:  $\sin^2 2\theta_{\text{sol}} = 4(U_{e1}U_{e2})^2$ ; exactly gives  $\theta_{\text{sol}} = \pi/4$ . Of course, strictly speaking this simple model predicts no solar neutrino oscillations, and the mentioned angle is only for reference proposes.

The problem is relieved when one realizes that the zeros that appeared on the  $M_0$  texture are actually representing some small numbers, which would be introduced by the global symmetry breaking effects that we have neglected so far. In many models this small terms may come from radiative corrections [4] or even from non renormalizable operators [5]. To be specific let us consider the model presented in Ref. [10]. Apart from the standard lepton content, we introduce three Higgs doublets  $\phi_{0,1,2}$ , two

triplets,  $\Delta_{1,2}(Y = 2)$  and one isosinglet,  $\eta^+(Y = 2)$ . We impose the symmetry  $L' \times S_3$ , where  $S_3$  is the permutation group of three elements, under which  $2_L = (L_\mu, L_\tau)$ ,  $2_R = (\mu_R, \tau_R)$ ,  $2_\Phi = (\phi_1, \phi_2)$  and  $2_\Delta = (\Delta_1, \Delta_2)$  are doublets, with all other fields as  $S_3$  singlets, but with  $\eta$  an odd (pseudo) singlet. The allowed Yukawa couplings can be written in compact notation as

$$\begin{aligned} \mathcal{L}_Y = & h_1 \bar{2}_L \cdot 2_R \phi_0 + h_2 \bar{2}_L \times 2_R \cdot 2_\Phi + h_e \bar{L}_e e_R \phi_0 \\ & + f L_e 2_L \cdot 2_\Delta + f' L_\mu L_\tau \eta + \text{H.c.}, \end{aligned}$$

where the indicated doublet products represent the  $S_3$  invariants obtained as follows: given the  $S_3$  doublets  $2_x = (x_1, x_2)$  and  $2_y = (y_1, y_2)$ , we built the even singlet  $1_{xy} = 2_x \cdot 2_y = x_1 y_1 + x_2 y_2$  that gives the first and fourth terms in  $\mathcal{L}_Y$ ; the odd singlet  $1'_{xy} = x_1 y_2 - x_2 y_1$  that gives the coupling to  $\eta$  field, and the new doublet  $2_{xy} = 2_x \times 2_y = (x_1 y_1 - x_2 y_2, x_1 y_2 + x_2 y_1)$  involved in the second Yukawa term above.

To one loop order, one gets the neutrino mass matrix

$$M_1 = M_0 + M_\epsilon = m \begin{pmatrix} 0 & \cos \theta & \sin \theta \\ \cos \theta & \epsilon & 0 \\ \sin \theta & 0 & \epsilon \end{pmatrix}, \quad (2.4)$$

where the diagonal terms come from a one loop graph as the one shown in Fig. 1. They are about same order, hence we assume them equal. The model gives no charged lepton mixing at tree level, thus, from the diagram is easy to see why there are no off-diagonal mass terms generated at one loop. Also,  $\eta$  does not couple to  $L_e$ , which explains why  $m_{ee} = 0$  at the same order. Hereafter, for our analysis we will only required to assume the above texture, in the understanding that other models could certainly fulfill the same neutrino mass structure. Notice however, that this is certainly not the most general texture, but it serves very well to our propose of motivating the possible sterile corrections.

After introducing the bimaximal rotation, one may write the perturbation as

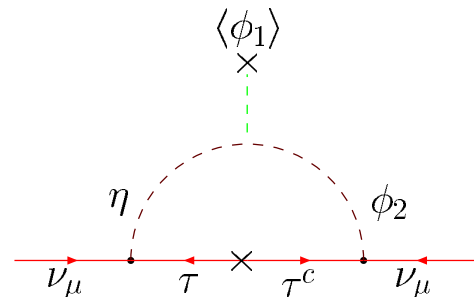


FIG. 1 (color online). One loop diagram that generates the  $m_{\mu\mu}$  term. A similar diagram would provide  $m_{\tau\tau}$ .

$$U_{\text{BM}}^\dagger M_\epsilon U_{\text{BM}} = \epsilon m \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (2.5)$$

This expression already shows what it is desired from the  $L'$  breaking effects: they should mainly modify  $\theta_{12}$ , and split the squared masses of the heavier states whereas leaving almost unaffected the other two mixing angles  $\theta_{23}$  and  $\theta_{13}$ . As a plus, the lighter neutrino gets a small mass  $m_3 = \epsilon m$ . However, since this mass is not constrained by observations it can not be used to fix the new parameter. Clearly, only a new rotation on the 1–2 plane is needed to compensate for the effect of the perturbation. An exact calculation shows that the rotation angle satisfies

$$\tan \alpha = \frac{\epsilon/2}{1 + \sqrt{1 + \epsilon^2/4}} \approx \frac{\epsilon}{4}.$$

Thus, the final mixing matrix would be

$$U_{\text{mix}} = U_{\text{BM}} \cdot U_\alpha = \begin{pmatrix} \frac{c_\alpha - s_\alpha}{\sqrt{2}} & \frac{c_\alpha + s_\alpha}{\sqrt{2}} & 0 \\ \frac{c_\alpha + s_\alpha}{\sqrt{2}} c_\theta & \frac{s_\alpha - c_\alpha}{\sqrt{2}} c_\theta & s_\theta \\ \frac{c_\alpha + s_\alpha}{\sqrt{2}} s_\theta & \frac{s_\alpha - c_\alpha}{\sqrt{2}} s_\theta & -c_\theta \end{pmatrix}, \quad (2.6)$$

where, as before, we have written  $c_\alpha$  ( $s_\alpha$ ) for  $\cos \alpha$  ( $\sin \alpha$ ) and  $c_\theta$  ( $s_\theta$ ) for  $\cos \theta$  ( $\sin \theta$ ) to simplify the expression. Therefore, the solar mixing angle would now be given by

$$\sin^2 2\theta_{\text{sol}} = (\cos^2 \alpha - \sin^2 \alpha)^2 = \cos^2 2\alpha, \quad (2.7)$$

or equivalently

$$\sin^2 2\theta_{\text{sol}} = \left[ \frac{1 + \sqrt{1 + \epsilon^2/4}}{1 + \epsilon^2/4 + \sqrt{1 + \epsilon^2/4}} \right]^2 \approx 1 - \frac{\epsilon^2}{4}. \quad (2.8)$$

It is interesting to note that, since  $\cos 2\alpha = \sin(\pi/2 - 2\alpha)$ , Eq. (2.7) means that

$$\theta_{\text{sol}} = \frac{\pi}{4} - \alpha.$$

This explicitly shows that the rotation introduced due to the  $\epsilon$  corrections works in the right direction, by reducing the value of the solar angle down from maximal. Next question would be whether such a correction would be enough to provide the right observed values. To answer this question one has to consider the heavier mass eigenvalues:  $m_1 = m(\epsilon/2 + \sqrt{1 + \epsilon^2/4})$ ; and  $m_2 = m(\epsilon/2 - \sqrt{1 + \epsilon^2/4})$ ; from which, without approximations, one gets

$$\Delta m_{\text{sol}}^2 = \epsilon m^2 \sqrt{4 + \epsilon^2}. \quad (2.9)$$

Thus,

$$\epsilon \approx \frac{1}{2} \frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{ATM}}^2}, \quad (2.10)$$

which indicates that  $\epsilon$ , and thus  $\alpha$ , are of the order of 1 part in a hundred at most, and so they are rather much smaller than what is needed to give the solar mixing, according to Eq. (2.8). Indeed, such a small  $\epsilon$  would mean a correction on  $\sin^2 2\theta_{\text{sol}}$  on just about one part on ten thousands. Using central values for the solar and atmospheric scales, and the exact expressions (2.8) and (2.9) given above, by fixing  $\epsilon$  with the solar scale, one gets  $\sin^2 2\theta_{\text{sol}} = 0.9999$ , that means  $\sin^2 \theta_{\text{sol}} = 0.496$  which is far larger than what it is actually desired. Conversely, if one takes  $\epsilon$  large enough as to fix the solar mixing to the appropriate level, one gets a too large  $\Delta m_{12}^2$ .

### III. SOLAR MIXING AND THE STERILE NEUTRINO

From the analysis of previous section one can see that the simple correction we considered is unable to provide both the solar mass scale and mixing angle. A complete understanding of these parameters, along this line of thought, needs to consider a more complicated correction of the original texture, which may be an indication of a quite complicated way in which the associated  $L'$  symmetry is broken [11]. Another possibility is that the desired corrections may come from some other source. This external source could be the coupling with a fourth neutrino. Such couplings usually violate the global  $L'$  symmetry and could be as large as the tree level masses without any fundamental contradiction. Another question is, of course, the reason why a singlet field, as a fourth neutrino, comes with mass couplings as light as those of the active sector, since they are not protected by any standard model symmetry. We will not address this question in here, nor give a complete model, but rather just assume that all masses involved in our fourth neutrino scheme are of the same order. However, we would like to mention that there are indeed models where light sterile neutrinos do appear [5,12].

The idea of using the sterile neutrino to improve the active neutrino mixings was discussed earlier in Ref. [13], although in that paper the aim was rather to use a heavy sterile neutrino whose couplings to the active sector could be tuned to provide as much large mixing as possible, so one could start with small mixings (as in the quark sector) right before considering the sterile contributions. Here the idea we shall explore is quite different. We are suggesting that the sterile neutrino could rather be light, and yet have such couplings as to substantially contribute to pull down an initially maximal solar mixing, and to generate the mass splitting that provides the solar mass scale, particularly considering a class of models with the  $L'$  symmetry. The idea may have a realization for models with normal hierarchy too, but we are not exploring that in here.

To exemplify our idea, let us introduce a simple toy model for the sterile couplings. We first consider the general form of the Majorana mass matrix in the basis

$(\nu_\alpha, \nu_s)$ ,

$$M = \begin{pmatrix} \mathcal{M}^{(0)} & \vec{\delta} m \\ \vec{\delta}^\dagger m & m_s \end{pmatrix}, \quad (3.1)$$

where  $\vec{\delta}$  represent three parameters, smaller than unity, that set the scale of the active-sterile mass couplings. In general, these couplings would break  $L'$ , since the sterile carries no standard lepton number. As we expect this couplings to play an important role mainly in fixing the solar mixing, we will introduce the hypothesis that, after bimaximal mixing, the sterile mainly couples to one of the heaviest states. Thus, one can take for instance  $\vec{\delta}^\dagger \sim (\delta/\sqrt{2})(1, c_\theta, s_\theta)$ . Some small deviations of our choice will not affect our conclusions. Moreover, to simplify our calculations even more we will assume that  $m_s \ll m$ , so it can be neglected in the analysis. Any further model realization of the mechanism we present in here will have to provide an explanation for these assumptions. Also, in last equation  $\mathcal{M}^{(0)}$  stands for the active mass terms. Since we continue assuming they are generated in models with the  $L_e - L_\mu - L_\tau$  symmetry, it is natural to use the same form considered in the previous section. Therefore, at first order, we will again take  $\mathcal{M}^{(0)} = M_1$  as in Eq. (2.4).

Within this approximations, and after performing a first rotation with the bimaximal mixing matrix, the four by four mass matrix we have described gets the form

$$\tilde{M} = m \begin{pmatrix} 1 + \frac{\epsilon}{2} & -\frac{\epsilon}{2} & 0 & \delta \\ -\frac{\epsilon}{2} & -1 + \frac{\epsilon}{2} & 0 & 0 \\ 0 & 0 & \epsilon & 0 \\ \delta & 0 & 0 & 0 \end{pmatrix}. \quad (3.2)$$

At zero order, by taking  $\delta = \epsilon = 0$ , it is clear that the mass matrix (3.1) is diagonal. The mass spectrum is of a  $2 + 2$  type, containing two heavy neutrinos, with masses  $\pm m$  as before, and now two massless neutrinos, one of them the sterile neutrino which decouples in this limit. The gap among both the sectors sets the atmospheric scale as usual.

### A. Sterile contributions

To better understand the role the sterile would play, let us first consider the special case where the breaking of the  $L'$  symmetry is dominated by the sterile contributions, that is when  $|\epsilon| \ll |\delta|$ . Thus, we will set  $\epsilon = 0$  for the moment. After bimaximal rotation, the third state remains massless and decoupled from the other neutrinos as well as the second massive state. Hence, only the first and fourth neutrino states remain mixed, with the mass terms

$$m \begin{pmatrix} 1 & \delta \\ \delta & 0 \end{pmatrix}. \quad (3.3)$$

Now, the two eigenvalues of this matrix are  $m_1 = \frac{m}{2} \times (1 + \sqrt{1 + 4\delta^2}) \approx m(1 + \delta^2)$  and  $m_4 = \frac{m}{2} \times (1 - \sqrt{1 + 4\delta^2}) \approx -m\delta^2$ . Therefore, at the lower order

the sterile gets a seesaw type mass, setting a new scale in the oscillation theory which is associated to  $\Delta m_{34}^2 = -m_4^2$ . All other squared mass differences are  $\Delta m_{13}^2 \approx \Delta m_{14}^2 \approx \Delta m_{23}^2 \approx \Delta m_{24}^2 \approx \Delta m_{\text{ATM}}^2$ ; and for the solar scale we get

$$\Delta m_{\text{sol}}^2 = \frac{1}{2} m^2 (\sqrt{1 + 4\delta^2} - 1 + 2\delta^2) \approx 2m^2 \delta^2, \quad (3.4)$$

and thus, to get the right scale one needs

$$\delta^2 \approx \frac{1}{2} \frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{ATM}}^2}. \quad (3.5)$$

This equation implies that  $\delta \sim 0.12$ . Notice that above formula is similar to that in Eq. (2.10). This seems to indicate that once we include a non zero  $\epsilon$ , the solar scale should very likely come due to the compensation or cancellation among both the effects. The hierarchy  $|\delta| > |\epsilon|$  is also suggested.

On the other hand, the 1–4 mixing angle is

$$\tan \beta = \frac{2\delta}{1 + \sqrt{1 + 4\delta^2}} \approx \delta, \quad (3.6)$$

and therefore the total neutrino mixing matrix has the form

$$U_{\text{mix}} = U_{\text{BM}} \cdot U_\beta = \begin{pmatrix} \frac{c_\beta}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & -\frac{s_\beta}{\sqrt{2}} \\ \frac{c_\beta c_\theta}{\sqrt{2}} & -\frac{c_\theta}{\sqrt{2}} & s_\theta & -\frac{s_\beta c_\theta}{\sqrt{2}} \\ \frac{c_\beta s_\theta}{\sqrt{2}} & -\frac{s_\theta}{\sqrt{2}} & -c_\theta & -\frac{s_\beta s_\theta}{\sqrt{2}} \\ s_\beta & 0 & 0 & c_\beta \end{pmatrix}. \quad (3.7)$$

Notice that  $U_{e3}$  remains zero due to the decoupling of the third massive state. This also cancels any contribution of the new scale  $\Delta m_{34}^2$  into the electron neutrino survival probability,  $P_{ee}$ , since the corresponding mixing angle would be  $4(U_{e3}U_{e4})^2 = 0$ . Corrections work in the right direction only via the atmospheric scale. Indeed from above mixings one gets  $P_{es} = \frac{1}{2} \sin^2 2\beta \sin^2(\Delta m_{\text{ATM}}^2 L/4E)$ . However, the corresponding mixing angle is small at this limit, of order  $\delta^2 \sim 10^{-2}$ , so we will not discuss it further.

Solar mixing angle now becomes

$$\sin^2 2\theta_{\text{sol}} = \cos^2 \beta = \frac{1 + \sqrt{1 + 4\delta^2}}{2\sqrt{1 + 4\delta^2}} \approx 1 - \delta^2. \quad (3.8)$$

From here we can write  $\sin 2\theta_{\text{sol}} = \sin(\frac{\pi}{2} - \beta)$ . Hence

$$\theta_{\text{sol}} = \frac{\pi}{4} - \frac{\beta}{2},$$

and so we notice the sterile corrections also work in the right direction, they reduce the solar mixing from its maximal value. However, as before, one can see that the effect is not yet enough to pull  $\theta_{\text{sol}}$  down to the desired values. Indeed for central values of the squared mass differences one gets  $\sin^2 2\theta_{\text{sol}} = 0.984$ , which corresponds to  $\sin^2 \theta_{\text{sol}} = 0.437$ .

### B. Active-sterile compensation: The fall of the solar mixing

From previous sections one sees that neither the active nor the sterile corrections we assumed are capable enough by themselves to provide the right corrections for both the solar parameters. Therefore, we will now consider both scenarios together to show how the correct values may arise when both mechanisms are at work. Next we reinsert the  $\epsilon$  parameter on  $\tilde{M}$ . Motivated by the suggestive hierarchy  $|\delta| > |\epsilon|$ , discussed in previous section, we will proceed as follows. First we write

$$\tilde{M} = \tilde{M}_0 + \tilde{M}_\delta + \tilde{M}_\epsilon, \quad (3.9)$$

where  $\tilde{M}_{\delta,\epsilon}$  only contains the  $\delta$  and  $\epsilon$  contributions, respectively.  $\tilde{M}_0$  is our diagonal zero order mass matrix, as before. Next we change our basis into that where  $\tilde{M}_0 + \tilde{M}_\delta$  is diagonal, and treat  $\tilde{M}_\epsilon$  as a perturbation. Clearly this amounts to simply rotate  $\tilde{M}$  on Eq. (3.2) by the same 1–4 mixing we calculated in previous section [Eq. (3.7)], without fixing  $\delta$  yet. We get  $U_\beta^\dagger \cdot (\tilde{M}_0 + \tilde{M}_\delta) \cdot U_\beta = m \cdot \text{diag}\{\lambda_+, -1, 0, \lambda_-\}$ ; for  $\lambda_\pm = \frac{1}{2}(1 \pm \sqrt{1 + 4\delta^2})$ ; and for the perturbation matrix:

$$U_\beta^\dagger \cdot \tilde{M}_\epsilon \cdot U_\beta = m \frac{\epsilon}{2} \begin{pmatrix} c_\beta^2 & -c_\beta & 0 & -s_\beta c_\beta \\ -c_\beta & 1 & 0 & s_\beta \\ 0 & 0 & 2 & 0 \\ -s_\beta c_\beta & s_\beta & 0 & s_\beta^2 \end{pmatrix}. \quad (3.10)$$

Of course, in the limit  $\beta = 0$  ( $\delta = 0$ ) last expression becomes Eq. (2.5), indicating that at the leading order the effect is effectively to rotate the 1–2 sector. Also, it is worth noticing that the third state always decouples, which means that  $U_{e3} = 0$ . Thus, although there is a new scale in our final model,  $\Delta m_{34}^2$ , it would not contribute to  $P_{ee}$  either.

The first order mass eigenvalues one gets are

$$m_1 \approx \frac{m}{2}(1 + \sqrt{1 + 4\delta^2}) + m \frac{\epsilon}{2} \cos^2 \beta; \quad (3.11)$$

$$m_2 \approx m \left( -1 + \frac{\epsilon}{2} \right); \quad (3.12)$$

$$m_3 = \epsilon m; \quad (3.13)$$

$$m_4 \approx \frac{m}{2}(1 - \sqrt{1 + 4\delta^2}) + m \frac{\epsilon}{2} \sin^2 \beta. \quad (3.14)$$

From here, the splitting among the first two states is

$$\Delta m_{12}^2 \approx [\delta^2 - \lambda_- + \epsilon(1 + \lambda_+ \cos^2 \beta)]m^2. \quad (3.15)$$

We can approximate last equation at first order in both parameters to get the more simple expression

$$2\delta^2 + \epsilon(1 + \cos^2 \beta) \approx \frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{ATM}}^2}; \quad (3.16)$$

from where it is easy to see that for a relatively large  $\delta$  (say of order 0.3),  $\epsilon$  has to be negative in order to compensate the effect.  $\delta$  can well be positive or negative, without any modification on the results. We will assume it positive. This way solar scale would arise as due to the cancellation among both corrections. On the other hand, this would allow the solar mixing to get larger corrections from  $\delta^2$ , as it is suggested from Eqs. (2.8) and (3.8) since quadratic  $\delta$  corrections are expected to dominate.

It is worth noticing that for other mass squared differences we get  $\Delta m_{13}^2 \approx \Delta m_{14}^2 \approx \Delta m_{23}^2 \approx \Delta m_{24}^2 \approx \Delta m_{\text{ATM}}^2$ ; and

$$|\Delta m_{34}^2| \approx \left| \epsilon^2 - \left( \lambda_- + \frac{\epsilon}{2} \sin^2 \beta \right)^2 \right| m^2 \ll \Delta m_{\text{sol}}^2. \quad (3.17)$$

Next, final mixing is given by  $U_{\text{mix}} = U_{\text{BM}} \cdot U_\beta \cdot U_\epsilon$ , where  $U_\epsilon$  is the extra rotation needed to completely diagonalize  $\tilde{M}$  in Eq. (3.2), which at first order in  $\epsilon$  is given by

$$U_\epsilon \approx \begin{pmatrix} 1 & \frac{c_\beta \epsilon}{2(1+\lambda_+)} & 0 & \frac{c_\beta s_\beta \epsilon}{2(\lambda_+ - \lambda_-)} \\ -\frac{c_\beta \epsilon}{2(\lambda_+ + 1)} & 1 & 0 & \frac{s_\beta \epsilon}{2(1+\lambda_-)} \\ 0 & 0 & 1 & 0 \\ -\frac{c_\beta s_\beta \epsilon}{2(\lambda_+ - \lambda_-)} & -\frac{s_\beta \epsilon}{2(1+\lambda_-)} & 0 & 1 \end{pmatrix}. \quad (3.18)$$

From here, the solar mixing is calculated to first order and gives

$$\sin^2 2\theta_{\text{sol}} \approx \cos^2 \beta \left[ 1 + \frac{3}{2} \epsilon \sin^2 \beta \left( \frac{1 + \delta^2}{(2 - \delta^2)\sqrt{1 + 4\delta^2}} \right) \right]^2. \quad (3.19)$$

Leading correction is of order  $\delta^2$ , as expected.

Unlike the previous results, where independent effects were studied, now the contribution of both, active  $L'$  breaking effects and sterile couplings give us enough freedom to account for both the solar parameters. We use Eqs. (3.15) and (3.19) to get a rough estimation of the required values for our parameters in order to get central values for solar scale,  $\Delta m_{12}^2 = 7.92 \times 10^{-5} \text{ eV}^2$ ; and solar mixing,  $\sin^2 \theta_{\text{sol}} = 0.314$  and get  $\delta = 0.412$  and  $\epsilon = -0.141$ , which nicely validate our approximations. Moreover, with this values we get for the new scale about  $|\Delta m_{34}^2| = 9.5 \times 10^{-6} \text{ eV}^2$ , which is a factor of 8 smaller than the solar scale.

A more precise calculation can be done starting from the mass matrix in Eq. (3.2), by performing a numerical diagonalization for given values of the pair  $(\delta, \epsilon)$ . This allows in general to identify the range of parameters that would give acceptable results for our model. We present our results in Fig. 2, where we plot the bands for which solar scale and mixing are obtained within two sigma deviations. Precise results substantially reduce the required value for  $\epsilon$ , as it can be seen from the plot. Notice that whereas the mixing angle,  $\sin^2 \theta_{\text{sol}}$ , is little sensitive to  $\epsilon$ , and corresponds to a wide region on the parameter space, there are

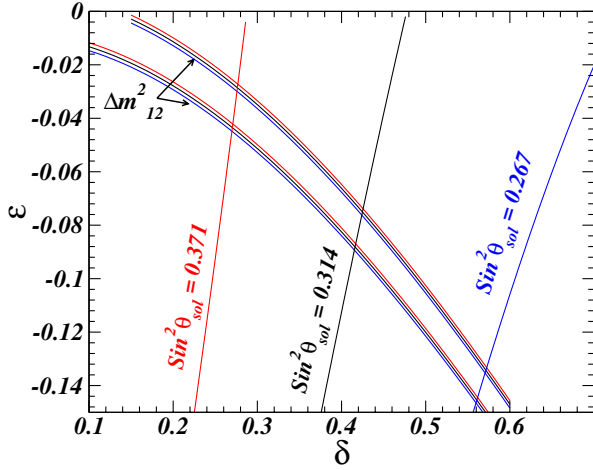


FIG. 2 (color online). Allowed values of  $\epsilon$  and  $\delta$  which give the solar parameters,  $\Delta m_{12}^2$  and  $\sin^2 \theta_{\text{sol}}$ ; within two sigma deviations respect to observed central values. The two narrow regions for the squared mass difference are depicted.

two allowed regions for the solar scale,  $\Delta m_{12}^2$ , that are very narrow and clearly sensitive to both  $\epsilon$  and  $\delta$  parameters. This indicates that some level of fine tuning may be required to get the proper scale.

#### IV. CONFRONTING THE STERILE WITH EXPERIMENTAL DATA

Most studies we are aware of on the constrains for sterile neutrinos are based in the hypothesis that these are heavier than any active scale (see for instance Ref. [8]). However, sterile mixing to electron neutrino at a scale somewhat smaller than solar scale was consider in Ref. [9], and there it was shown that a weak mixing is yet allowed. An extended and detailed study for light sterile neutrinos is still required and will be presented in a forthcoming paper. Here we will only make some remarks on what the potential constrains would be, based on the simple model we have presented, and simply extrapolating the results of the somehow general analysis given in Ref. [8].

First of all, the appearance of a new light scale,  $\Delta m_{34}^2$ , of order  $10^{-5}$  eV<sup>2</sup> may suggest a new level crossing in the resonant conversion of electron neutrinos within the Sun. However, by looking at the general survival probability for electron neutrinos (assuming no  $CP$  violation),

$$P_{ee} = 1 - 4 \sum_{a>b}^4 \sum_{b=1}^4 (U_{ea} U_{eb})^2 \sin^2 \left( \frac{\Delta m_{ab}^2 L}{4E} \right), \quad (4.1)$$

where we have used  $U$  in place of  $U_{\text{mix}}$  for simplicity; it is easy to see that the mixing angle associated to  $\Delta m_{34}^2$  is  $4[U_{e3} U_{e4}]^2 = 0$ , since  $U_{e3}$  is exactly zero in above results. Therefore, as we already mentioned, only solar and atmospheric scales would contribute to the oscillations. This does not mean, however, the absence of electron to sterile

conversion, which indeed appears as indicated by the oscillation probability formula

$$P_{es} = 4 \sum_{a>b}^4 \sum_{b=1}^4 U_{ea} U_{eb} U_{sa} U_{sb} \sin^2 \left( \frac{\Delta m_{ab}^2 L}{4E} \right). \quad (4.2)$$

From here, it is again clear that since  $U_{e3} = U_{s3} = 0$ , there is no contribution from the  $\Delta m_{34}^2$  scale. Resonant conversion may only occur at solar scale, however, the associated mixing

$$4U_{e1} U_{e2} U_{s1} U_{s2} \approx -\frac{1}{2} \sin 2\theta_{\text{sol}} c_{\beta} s_{\beta}^2 \epsilon \approx 10^{-2}, \quad (4.3)$$

corresponds to a point on parameter space, with  $\tan^2 \theta_s \approx 10^{-3}$ , that is not at all excluded, but marginally accepted, as it can be seen from the analysis presented in Ref. [8], if we naively extrapolate their results. Point is that the dip of the survival electron probability produced for the resonance is dominated by the mixing to active neutrinos. We believe, nevertheless, that a detailed study for our case is required to be conclusive. Just to get a naive idea of the effect, we can roughly estimate the fraction of sterile neutrinos in the solar flux using an ‘‘in vacuum’’ approximation assuming  $4E/L$  at the solar scale, to get

$$\eta_s = \frac{P_{es}}{1 - P_{ee}} \approx 7.1 \times 10^{-2}; \quad (4.4)$$

which is just below current limits (see for instance Ref. [8]). Although somehow unjustified due to the matter effects, we believe this calculation should give a close result to the correct one, and provides an indication of the marginal acceptance of our model.

Non resonant conversion will also occur associated to atmospheric scale, but this contribution is expected to be less relevant at solar neutrino energies.

The analysis done by Cirelli *et al.* in Ref. [8] also serves very well for a first check of consistency of our model with Supernova and cosmology constrains, by taking a naive extrapolation of their results to our case. Supernova (SN) produces neutrinos and antineutrinos in roughly a similar amount, however, present experiments focus mainly in antineutrinos, for which there is no resonance matter effects. This is reflected in almost no restrictions for the above small  $\nu_e - \nu_s$  mixing, also relevant at SN neutrino energies, as it can be checked in the mentioned reference. Main cosmological bounds on sterile neutrinos come from big bang nucleosynthesis, which probes the total energy density at  $T \sim 0.1$ – $1$  MeV, constraining the number of relativistic species at that energy,  $N_{\nu}$  (for a review see [14]). A larger effective number contribution of  $N_{\nu}$  than three would affect <sup>4</sup>He and Deuterium abundance, in a rather mild way. Conservative estimates give  $N_{\nu}^{\text{He}} \approx 2.4 \pm 0.7$  and  $N_{\nu}^D \approx 3 \pm 2$ . Currently CMB does not give strong bounds [15,16]:  $N_{\nu} \approx 3 \pm 2$ . Finally, structure formation imposes a constrain on the total energy density in

neutrinos,  $\Omega_\nu h^2$ , from where cosmological bound to absolute neutrino mass is obtained. However, as one can check from Cirelli *et al.* results, light sterile neutrinos with small mixings contribute little to  $N_\nu$ . In fact one gets  $N_\nu$  just around 3.2. Also, no important bounds come from structure formation either due to the small sterile mass.

## V. CONCLUDING REMARKS

We have suggested the possibility that light sterile neutrinos may be the missing ingredient that transforms a maximal mixing into the observed large mixing angle in solar neutrino oscillations. To elaborate the idea we have presented a simple toy model that realizes this possibility with a sizable effect that may accommodate the observed values, in models with inverted hierarchy provided by a broken global  $L_e - L_\mu - L_\tau$  symmetry. Models with this symmetry usually face the problem of predicting a too large solar mixing angle. With the introduction of appropriate sterile couplings the models gain enough freedom to arrange both solar parameters to the proper order of magnitude. We believe our mechanism may also work for models with normal hierarchy which we have not explored here.

The light sterile neutrino should come with a new mass scale,  $\Delta m_{34}^2$ , which in order to produce the desired corrections comes out to be lighter than solar scale. However, in the toy model we have explored the new scale does not contribute to electron neutrino survival probability in solar

oscillations, since it is attached to  $U_{e3}$  which is exactly zero. Thus, no resonant conversion of solar neutrinos is expected for these scale. Moreover, the effective fraction of sterile neutrinos in the solar flux seems to be just below the current limits. Indeed, a very rough calculation using central experimental values and vacuum oscillations gives the prediction  $\eta_s \approx 7.1 \times 10^{-2}$ . Cosmology and Supernova bounds seem to marginally allow the light sterile neutrino with the parameters we have used. A cautionary word should be given. We are not presenting a complete model yet, but rather exploring just the basics of the idea. A more detailed analysis of atmospheric and solar neutrino oscillations, including matter effects is desirable to precise the constraints and determine whether future experiments would be sensible to the new scale. Those results may even rule out the present scenario, but perhaps may suggest a more realistic one. A light sterile neutrino seems to be anyway an interesting possibility that deserves further exploration.

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