Reply to "Comment on 'Scalar-tensor gravity coupled to a global monopole and flat rotation curves'"

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(Received 22 December 2005; published 6 June 2006)

In Brans-Dicke theory of gravity, we explain how the extra constant value in the formula for rotation velocities of stars in a galactic halo can be obtained due to the global monopole field. We argue on a few points of the preceding Comment and discuss improvement of our model.

DOI: 10.1103/PhysRevD.73.128502 PACS numbers: 04.50.+h, 14.80.Hv, 95.35.+d, 98.35.Gi

I. INTRODUCTION

In Ref. [1], we considered a global monopole (GM) [2] as a candidate for galactic dark matter in Brans-Dicke (BD) theory of gravity [3]. Within the weak gravity approximation, we solved the equations of metric tensor fields and BD scalar field coupled to the GM, and determined the asymptotic structure of a galactic spacetime at a distance r from the monopole center. A metric component in the so-called physical frame [4] was given by

$$\tilde{g}_{tt}(r) \simeq -1 - 2(v_{\rm rot}^{(0)})^2 \ln(r/r_i) + 2G \int d^3r' \rho(r')/\Delta r, \tag{1}$$

with $\Delta r = | \vec{r} - \vec{r}' |$, $G = G_*(1 + \alpha_0^2)$, and an ordinary matter density $\rho(r)$. From the geodesic equation in the spacetime at the center of which there is the monopole, we obtained a formula for the rotation velocity of stars in the galactic halo, which contains an extra value $v_{\rm rot}^{(0)}$ in addition to the other known term,

$$v_{\rm rot}^2(r) = (v_{\rm rot}^{(0)})^2 + GM_{\rm in}(r)/r, \tag{2}$$

with a mass $M_{\rm in}(r) = 4\pi \int_0^r dr' r'^2 \rho(r')$ of a sphere of radius r, and we discussed its possible relation to flat rotation curve (RC)s in spiral galaxies.

In the preceding Comment [5], Salucci and Gentile suggested that the paradigm of flat RCs of spiral galaxies [6], which was constructed about 20 years ago, has no observational support now and that the notion of flat RCs might be superseded by universal RCs [7]. The universal RC that they advocated seems to be a great improvement on the flat RC in spiral galaxies. However, the notion of flat RCs is supposed to remain valid as a first approximation for large r on RCs of some spiral galaxies, and therefore we cannot agree with their argument that the paradigm of the flat RCs was dismissed.

In Sec. II, we explain how the extra constant value $v_{\rm rot}^{(0)}$ of the rotation velocity in the galactic halo region can be generated by GM in BD theory of gravity, and we argue against the preceding Comment [5]. Section III includes a summary and discussion.

II. BD GRAVITY COUPLED TO GM AND RCs

The nearly flat behavior of spiral galaxy RCs implies the existence of dark matter with the energy density going like $1/r^2$. This could be generated by a cylindrical halo around the galaxy in nonrelativistic Newton's theory of gravity. Since the rotation velocity in Eq. (2) can be written as $v_{\rm rot}^2 \simeq -\vec{r}/2 \cdot \nabla \tilde{g}_{tt}$ with $\nabla \tilde{g}_{tt} = -2\hat{r}((v_{\rm rot}^{(0)})^2/r + GM_{\rm in}(r)/r^2)$, for instance, the singular isothermal sphere [8] with energy density proportional to $1/r^2$ yields a mass $M_{\rm in}(r) \propto r$ and the third term in Eq. (1) gives us a classical potential $\Psi \propto \ln(r/r_i)$. It satisfies

$$\nabla^2 \Psi(r) = (r\Psi')'/r = \Psi'' + \Psi'/r = 0, \tag{3}$$

with $\Psi' \equiv \frac{d\Psi}{dr}, \ldots$ in cylindrical coordinates in flat space.

Such a cylindrical halo seems not to be realistic, but topological defects such as GMs or cosmic strings [9] have energy densities proportional to $1/r^2$ and could be generated when symmetry-breaking phase transitions took place in the early Universe. The defects were thought of as the seeds for galaxy and large-scale structure formation, and their remnants might remain as galactic dark matter. Though Harari and Loustó [10] proposed that the monopole core mass is negative and that there exist no bound orbits in Einstein's theory of gravity, Nucamendi and others [11,12] suggested that the GM could account for some fraction of the galactic dark matter. It was also claimed by Nucamendi *et al.* [11] that there is an attractive region where bound orbits exist, by the introduction of a nonminimal coupling of gravity to the GM, $-\xi R\vec{\Phi}^2$, which seems to play a role similar to the interaction term $A^4(\varphi)\vec{\Phi}^2$ between the monopole and BD field in Eq. (3) of Ref. [1]. We thus suggested [1] that, in relativistic scalar-tensor theories of gravity such as BD theory, GMs with the energy density $\propto 1/r^2$ can yield the logarithmic gravitational potential as the second term in Eq. (1). This can be responsible for flat RCs, while GMs induce only deficit angles in minimal coupling Einstein gravity. It can be explicitly understood as follows.

In the spherically symmetric, static spacetime as $ds^2 \equiv g_{\mu\nu}dX^\mu dX^\nu = -b(r)c(r)dt^2 + dr^2/b(r) + r^2 d\Omega^2$, a (t, t)-component of the Einstein's equation $G_\mu^\nu = \kappa T_\mu^\nu$

leads, up to $\mathcal{O}(\kappa)$,

$$b = 1 + \kappa b_1 = 1 - \kappa\{\eta^2(1 + \delta^2/r^2) + 2M/r\} + \mathcal{O}(\delta^4/r^4),$$

where $G^t_t = -(1 - b)/r^2 + b'/r$ is used. The energy density of the GM, $\rho(r) \equiv -T^t_t$, varies as $1/r^2$, for the vacuum solution [1] $f_0 = \eta(1 - \delta^2/r^2) + \mathcal{O}(\delta^4/r^4)$ of the monopole field $\vec{\Phi} = f(r)\hat{r}$, when we consider the potential $V_M(\vec{\Phi}^2) = \frac{\lambda}{4}(\vec{\Phi}^2 - \eta^2)^2$ with a constant η and the monopole core size $\delta = 1/\sqrt{\lambda}\eta$ [2]. Since a (r, r)-component of the Einstein's equation gives us $c = 1 + \mathcal{O}(\delta^4/r^4)$ up to $\mathcal{O}(\kappa)$, there is no logarithmic potential term generated by the GM in Einstein's theory of gravity.

On the other hand, a generalized Einstein's equation in the physical frame [4] in BD gravity theory is given by $\kappa T^t_t = G^t_t + b(\varphi')^2$, and thus the solution of the BD field is $\varphi = \text{const} + \kappa\varphi_1$ with $\varphi_1 = \alpha_0(\eta^2 \ln(r/r_i) - M/8\pi r) + \mathcal{O}(\delta^4/r^4)$ for $\rho(r) \propto 1/r^2$ [1]. Since $\tilde{g}_{tt} = A^2(\varphi)g_{tt} = 1/(G_*\tilde{\varphi})g_{tt}$, we have the gravitational potential $\tilde{g}_{tt}$ proportional to $\ln(r/r_i)$. The massless BD field satisfies also the field equation,

$$\varphi_1'' + 2\varphi_1'/r = \alpha_0\{f_0^2/r^2 + 1/2(f_0')^2 + 2V_M(f_0)\}, \quad (4)$$

which can be approximated by a cylindrically symmetric equation like Eq. (3), $\varphi_1'' + \varphi_1'/r \simeq 0$, since the second term in the left-hand side of Eq. (4) is twice as large as the first term in the right-hand side of the equation and remaining terms are negligible in the large r limit. The effectively 2-dimensional structure formed due to topological defects including GM (as candidates for dark matter) can be well represented as a $\ln(r/r_i)$-like potential in BD theory of gravity, while it is rather obscure in Einstein's theory of gravity [13]. Other authors [14] also found similar gravitational potentials in various theories of gravity. Note that the constant value $v^{(0)}_{\rm rot}$ originates from the logarithmic term of the massless BD field contribution to $\tilde{g}_{tt}$.

Our model cannot explain all detailed data of various spiral galaxies, but it must be the basic framework on which we will proceed to perform more realistic model building. Salucci and Gentile's Comment [5] will be helpful for the purpose, though we would like to notice a few things not to be misinterpreted.

First, they claimed that in our model the dark matter phenomenon always emerges at outer radius r of a galaxy as a constant threshold value below which the circular velocity $v_{\rm rot}(r)$ cannot decrease, regardless of the distance between r and the location of the bulk of the stellar component. As we discussed previously [1], however, the formula (2) is valid only for the finite range $r_i < r < r_h$ given by the range of global monopole field [11], because the GM field (and BD field) shall vanish at distance larger than the galactic halo radius r_h due to interactions with the nearest topological defect such as antimonopole [15], that is GM field lines can be absorbed into the antimonopole core. We moreover took a weak gravity approximation valid for $r < r_h \ll r_i e^{10^6}$ [16], and to go beyond the weak gravity approximation we need a numerical analysis of the GM field coupled to scalar-tensor gravity as Nucamendi *et al.* [11].

Second, they claimed that we predicted a constant value of $v^{(0)}_{\rm rot} = \sqrt{8\pi G\eta^2\alpha_0^2/(1 + \alpha_0^2)} \sim 300$ km/s, which is in disagreement with their result that the extrapolated asymptotic amplitude $v(\infty)$ varies, according to the galaxy luminosity, between 50 and 250 km/s [7]. From measurements of the RCs in spiral galaxies we estimated the asymptotic value of $v_{\rm rot}$ as between 100 and 300 km/s [6,11], for $\eta \sim 3 \times 10^{16}$ GeV and 10^{17} GeV, respectively, which are the natural scales for grand unified theories (GUT) [2,11]. Since we used an astronomical constraint for α_0 [17], $\alpha_0^2 \lesssim 0.001$, such that the constraint was saturated, our value of $v^{(0)}_{\rm rot}$ was not in conflict with that of Salucci and Gentile [5].

If the unsaturated constraint for α_0 or a more stringent bound on the value α_0 given by experiments around a few AU range [18] is used, then we will get a smaller value of $v^{(0)}_{\rm rot}$ (for instance, $v^{(0)}_{\rm rot} \lesssim 100$ km/s) for such α_0 and a fixed η. To fit various RC data of spiral galaxies, we need more contributions from other (dark) matter, $\Delta_{\rm other}$, besides GM contributions, and in this case Eq. (1) leads to

$$\tilde{g}_{tt}(r) \rightsquigarrow -1 - 2\{(v^{(0)}_{\rm rot})^2 + \Delta_{\rm other}\}\ln(r/r_i) + \cdots. \quad (5)$$

Our model explains, as a universal property, the "bare" flat tendency of RCs in spiral galaxies which might be seeded by GMs formed at the GUT scale η. To comprehend the detailed "dressed" RCs of specific galaxies, we need other contributions $\Delta_{\rm other}$. With those contributions added to the GM effects, ours can be a phenomenologically more improved model of RCs dependent on some properties of spiral galaxies. For example, the potential of scalar fields considered by Matos and others [19] to account for the flat RCs could be adjusted differently for different galaxies, which seems to us to play a subsidiary role, as part of $\Delta_{\rm other}$, though their models themselves might be too specific as noted by Nucamendi *et al.* [11].

The last thing is as follows. From available data of a sample of about 1000 galaxies, Persic and others obtained the expression for the universal RC [7,20], for a galaxy of luminosity L/L_* and normalized radius x,

$$v^2_{\rm urc}(x) = G(M_h(1)F(x, L) + kM_{\rm in}(x)/x), \quad (6)$$

where $M_h(1)$ is the halo mass inside optical radii $R_{\rm opt}$ and k of the order of unity. Then, they claimed that the dark contribution $F(x, L)$ to the RC varies as $x^2/(x^2 + a^2)$ with a constant a in each object, differently from our model and the flat RC paradigm. Even though it was discussed in Ref. [21] for Verheijen [22] to show that one third of 30 spiral galaxies in the Ursa Major cluster have velocity curves which do not conform to the universal curve shape and several studies [23] discussed the inadequacy of the universal RC parametrization, we are going to make our model more applicable to diverse RC models such as those of Persic *et al* [7] and to study the cases with cored distributions of dark matter [24], $\rho(r) \propto (r_i^2 + r^2)^{-m} r^{-n}$, (with the core radius r_i and constants m and n) to be valid for small r, since we can write the energy density of GM as $\rho(r) = \eta^2(\delta^2 + r^2)^{-1}$ up to $\mathcal{O}(\delta^4/r^4)$. In so doing, our formula (2) will be viable as a first approximation ($a \longrightarrow 0$) of various improved models including the universal RC expressed as in Eq. (6).

III. SUMMARY AND DISCUSSION

Though a GM induces only a deficit angle [2,10] in Einstein's theory of gravity, its energy density proportional to $1/r^2$ generates the $\ln(r/r_i)$-term in a metric component $\tilde{g}_{tt}$ in the physical frame [4] in BD theory. The logarithmic potential yields the constant value $v_{\rm rot}^{(0)}$ in the rotation velocity. The flat tendency of RCs can be related with such an effectively 2-dimensional structure formed because of GM, as discussed in the former part of the previous section. If the nearly flat RCs are really generated due to GMs gravitationally coupled to galaxies, then they would be yielded in generalized theories of gravity (such as BD theory, Dilaton gravity theories, and nonminimal gravity theories) rather than in Einstein's theory of gravity. We thus anticipate being able to differentiate generalized theories of gravity from Einstein's, by investigating thoroughly the motion of particles at the galactic level through RC data.

The universal RC model [7] in spiral galaxies looks like an improved one in comparison with the flat RC. However, it was discussed [22,23] that the universal curve shape is not confirmed yet and the universal RC parametrization may not be adequate. Therefore the paradigm of flat RCs seems not to be superseded by the universal RC as Salucci and Gentile [5] argued, and we think that the notion of flat RCs remains still as a foundation of various detailed studies on the spiral galaxy RCs, which is discussed in the latter part of the previous section.

ACKNOWLEDGMENTS

We thank Professors Myungshin Im, Soon-Tae Hong, and Jin Min Kim for useful discussions. This work was supported by the Soongsil University Research Fund.

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