PHYSICAL REVIEW D **73,** 125016 (2006)

High energy inelastic electron-hadron scattering in peripheral kinematics: Sum rules for hadron form factors

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(Received 2 February 2006; published 19 June 2006)

Relations between differential cross section for inelastic scattering of electrons on hadrons and hadron form factors (sum rules) are derived on the basis of analytical properties of heavy photon forward Compton scattering on hadrons. Sum rules relating the slope of form factors at zero momentum transfer and anomalous magnetic moments of hadrons with some integrals on photoproduction on a hadron are obtained as well. The convergency of these integrals is provided by the difference of individual sum rules for different hadrons. The universal interaction of the Pomeron with nucleons is assumed. We derive the explicit formulas for the processes of electroproduction on proton and light isobar nuclei. Sudakov's parametrization of momenta for peripheral kinematics, relevant here, is used. The light-cone form of differential cross sections is also discussed. The accuracy of sum rules estimated in frames of pointlike hadrons and it is shown to be at the level of precision achievable by experiments. Suggestions and predictions for future experiments are also given.

DOI: 10.1103/PhysRevD.73.125016 PACS numbers: 13.40.−f, 13.40.Gp, 25.30.Bf

I. INTRODUCTION

The idea of construction of sum rules, relating the form factors of electrons with the cross sections of electroproduction processes at e^+e_- high energy collisions, was born in 1974. In a series of papers the cross sections of processes such as

$$e_+e_- \to (2e_-e_+)e_+; (e_-\gamma)e_+; (e_-2\gamma)e_+; (e_-\mu_+\mu_-)e_+$$

were calculated in the so-called peripheral kinematics, when the jets (consisting of particles noted in parentheses) are moving closely to the initial electron direction in the center of mass reference frame. These cross sections do not decrease as a function of the center of mass (CMS) total beam energy $\sqrt{s}$. Moreover, they are enhanced by a logarithmical factor $\ln(s/m^2)$ (where m is the electron mass), which is characteristic for the Weizsacker-Williams approximation. It was obtained for contribution to the process of muon pair production cross section from the so-called bremsstrahlung mechanism (corresponding to the virtual photon conversion into muon–antimuon pairs) [1,2]:

$$\sigma^{e^+e^-\to e^-\mu^+\mu^-e^+} = \frac{\alpha^4}{\pi\mu^2}\left[\left(\ln\frac{s^2}{m^2\mu^2}\right)\left(\frac{77}{18}\xi_2 - \frac{1099}{162}\right) + \text{const.}\right] \quad (1)$$

where $\xi_2 = \frac{\pi^2}{6}$.

In Ref. [3] Barbieri, Mignaco, and Remiddi calculated the slope of the Dirac form factor of the electron for $q^2 \to 0$:

$$F_1'(0) = \frac{\alpha^2}{8\pi^2\mu^2}\left(\frac{77}{18}\xi_2 - \frac{1099}{162}\right). \quad (2)$$

The fact that the same coefficients appear in Eqs. (1) and (2) suggests that a relation exists between the inelastic cross section and elastic form factors. This relation was later on derived in Ref. [2].

Here we suggest an extension of these studies to strong interaction particles, considering QED interactions in the lowest order of perturbation theory. We suggest sum rules which relate the nucleon and light nuclei form factors with the differential cross sections of electron scattering on the corresponding hadrons in peripheral kinematics. This kinematics corresponds to the region of very small values of Bjorken parameter x_B in deep inelastic scattering experiments.

This paper is organized as follows. After describing the relevant processes in peripheral kinematics, in terms of Sudakov variables (Sec. II), we recall the analytical properties of the advanced and the retarded parts of the virtual Compton scattering amplitudes (Sec. II). Then, briefly we discuss how to restore gauge invariance and how to formulate the modified optical theorem.

Following QED analysis in [2], we introduce light-cone projection of the Compton scattering amplitude integrated on a contour in the s_2 plane, where s_2 is the invariant mass squared of the hadronic jet.

Sum rules (Sec. III) arise when the Feynman contour in the s_2 plane is closed to the left singularities of the Compton amplitude and to the right ones on the real axes. Sum rules obtained in such a way contain the left hand cut contribution, which is difficult to be interpreted in terms of cross sections. Moreover, ultraviolet divergencies of contour integral arising from Pomeron Regge pole con-

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tribution are present. Therefore, the final sum rules consist of differences, constructed in such a way to compensate the Pomeron contributions and the left hand cuts, as well.

The applications to different kinds of targets, as proton and neutron, deuteron and light nuclei, are explicitly given. Appendix A is devoted to an estimation of the left cut contribution which is proportional to the cross section of proton photoproduction of a $p\bar{p}$ pair taking into account the effect of identity of protons in final state in the framework of a simple model. In Appendix B the details of kinematics of recoil target particle momentum is investigated in terms of Sudakov's approach.

A. Sudakov parametrization

Let us consider the process presented in Fig. 1, where the inelastic electron—hadron interaction occurs through a virtual photon of momentum q. The particle momenta are indicated in the equation: $p^2 = M^2$, $p_1^2 = p_1'^2 = m^2$. The total energy is $s = (p + p_1)^2$ and the momentum transfer from the initial to the final electron is $t = (p_1 - p_1')^2$.

Let us introduce some useful notations, in order to calculate the differential cross section for the process of Fig. 1, where the hadron is a proton in peripheral kinematics, i.e., $s \gg -t \simeq M^2$. Therefore $s = (p_1 + p)^2 = M^2 + m^2 + 2pp_1 \simeq 2pp_1 = 2ME \gg M^2 \gg m^2$.

The differential cross section can be written as

$$d\sigma = \frac{1}{2 \cdot 2 \cdot 2s} \sum |\mathcal{M}|^2 d\Gamma. \tag{3}$$

Let us define two lightlike vectors:

$$\tilde{p} = p - p_1 \frac{M^2}{s}, \qquad \tilde{p}_1 = p_1 - p\frac{m^2}{s}$$

and a transversal vector, $q_\perp$, such that $\tilde{p}q_\perp = p_1 q_\perp = 0$.

Therefore, $\tilde{p}^2 = \mathcal{O}(\frac{m^2M^4}{s^2})$ and similarly $\tilde{p}^2 = \mathcal{O}(\frac{m^4M^2}{s^2})$. Terms of order $\mathcal{O}(\frac{M^2}{s}, \frac{m^2}{M^2})$ compared to ones of order 1 we will neglect systematically. In the laboratory system, with an appropriate choice of the axis, the four vectors are written, in explicit form, as $\tilde{p}_1 \approx p_1 = E(1, 1, 0, 0)$, $\tilde{p} = \frac{M}{2}(1, -1, 0, 0)$, and $q_\perp = (0, 0, q_x, q_y)$, $q_\perp^2 = -\vec{q}^2 < 0$, which is essentially a two-dimensional vector. Let us express the four momentum of the exchanged photon in the Sudakov parametrization, in an infinite momentum frame, as a function of two (small) parameters α and β (Sudakov parameters):

$$q = \alpha\tilde{p} + \beta\tilde{p}_1 + q_\perp. \tag{4}$$

The on-mass shell condition for the scattered electron can be written as

$$p_1'^2 - m^2 = (p_1 - q)^2 - m^2 = -\vec{q}^2 + s\alpha\beta - \alpha s - \beta m^2 = 0 \tag{5}$$

where we use the relation

$$2p_1\tilde{p}_1 = 2p_1\left(p_1 - p\frac{m^2}{s}\right) = m^2. \tag{6}$$

Similarly, one can find $2p\tilde{p} = M^2$.

From the definition, Eq. (4), and using Eq. (5), the momentum squared of the virtual photon is

$$q^2 = s\alpha\beta - \vec{q}^2 = -\frac{\vec{q}^2 + m^2\beta^2}{1 - \beta} < 0.$$

The variable β is related to the invariant mass of the proton jet, the set of particles moving close to the direction of the initial proton:

$$s_2 = (q + p)^2 - M^2 + \vec{q}^2 \simeq s\beta$$

neglecting small terms as $s\alpha\beta$ and αM^2 (Weizsacker-Williams approximation [4]). We discuss later the consequences of such approximation on the s-dependence of the cross section. In these notations the phase space of the final particle

$$d\Gamma = (2\pi)^4 \frac{d^3p_1'}{2\epsilon_1'(2\pi)^3} \Pi_1^n \frac{d^3q_i}{2\epsilon_i(2\pi)^3} \times \delta^4\left(p_1 + p - p_1' - \sum_i^n q_i\right), \tag{7}$$

introducing an auxiliary integration on the photon transferred momentum $\int d^4q\delta^4(p_1 - q - p_1') = 1$, can be written as

$$d\Gamma = (2\pi)^{-3}\delta^4((p_1 - q)^2 - m^2)d^4q d\Gamma_H, \tag{8}$$

where $d\Gamma_H$ is the hadron phase space:

$$d\Gamma_H = (2\pi)^4\delta^4\left(p + q - \sum_i^n q_i\right)\Pi_1^n \frac{d^3q_i}{2\epsilon_i(2\pi)^3}. \tag{9}$$

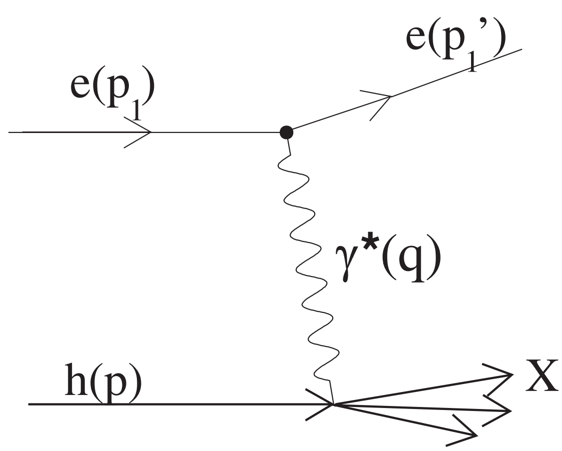

FIG. 1. Feynman diagram for inelastic electron-hadron scattering.

In the Sudakov parametrization:

$$d^4q = \frac{s}{2}d\alpha d\beta d^2q_\perp \simeq \frac{ds_2}{2s}d(s\alpha)d^2\vec{q}, \qquad (10)$$

we obtain

$$d\Gamma = \frac{ds_2}{2s}d^2\vec{q}(2\pi)^{-3}d\Gamma_H. \qquad (11)$$

Let us use the matrix elements to be expressed by the Sudakov parameters. Then it can be rewritten into the form

$$\mathcal{M} = \frac{4\pi\alpha}{q^2}\bar{u}(p_1')\gamma^\mu u(p_1)\mathcal{J}_H^\nu g_{\mu\nu}. \qquad (12)$$

It is convenient to use here the Gribov representation of the numerator of the (exact) Green function in the Feynman gauge for the exchanged photon:

$$g_{\mu\nu} = (g_\perp)_{\mu\nu} + \frac{2}{s}(\tilde{p}_\mu\tilde{p}_{1\nu} + \tilde{p}_\nu\tilde{p}_{1\mu}). \qquad (13)$$

All three terms in the right-hand side of the previous equation give contributions to the matrix element proportional to

$$1:\frac{s}{M^2}:\frac{M}{s}.$$

So, the main contribution (with power accuracy) is

$$\mathcal{M} = \frac{4\pi\alpha}{q^2}\frac{2}{s}\bar{u}(p_1')\hat{p}u(p_1)\mathcal{J}_H^\nu p_{1\nu} = \frac{8\pi\alpha s}{q^2}N\frac{\mathcal{J}_H^\nu p_{1\nu}}{s}, \qquad (14)$$

with $N = \frac{1}{s}\bar{u}(p_1')\hat{p}u(p_1)$. One can see explicitly the proportionality of the matrix element of peripheral processes to s, in the high energy limit, $s \gg -t$. It follows from the relation $\sum_{pol}|N|^2 = \frac{1}{s^2}\mathrm{Tr}\hat{p}_1'\hat{p}\hat{p}_1\hat{p} = 2$ and from the fact, that the quantity $\frac{1}{s}\mathcal{J}_H^\nu p_{1\nu}$ is finite in this limit. Such term can be further transformed using the conservation of the hadron current $\mathcal{J}_H^\nu q_\nu \simeq (\beta p_1 + q_\perp)_\nu \mathcal{J}_H^\nu = 0$. The latter leads to

$$\frac{1}{s}\mathcal{J}_H^\nu p_{1\nu} = \frac{1}{s\beta}\vec{q}\cdot\vec{\mathcal{J}}_H = \frac{|\vec{q}|}{s_2}(\vec{e}\cdot\vec{\mathcal{J}}_H), \qquad (15)$$

where $\vec{e} = \vec{q}/|\vec{q}|$ is the polarization vector of the virtual photon. As a result, one finds:

$$\sum|\mathcal{M}|^2 = \frac{(8\pi\alpha s)^2|\vec{q}|^2}{[(\vec{q})^2 + m^2(\frac{s_2}{s})^2]^2}\frac{2}{s_2^2}(\vec{\mathcal{J}}_H\cdot\vec{e})^2. \qquad (16)$$

With the help of Eqs. (7), (14), and (16) one finds

$$d\sigma^{(e+p\to e+\mathrm{jet})} = \frac{\alpha^2 d^2q\vec{q}^2 ds_2}{\pi[(\vec{q})^2 + m^2(\frac{s_2}{s})^2]^2 s_2^2}(\vec{e}\cdot\vec{\mathcal{J}}_H)^2 d\Gamma_H. \qquad (17)$$

Let us note that the differential cross section at $\vec{q}^2 \neq 0$ does not depend on the CMS energy $\sqrt{s}$. In the logarithmic (Weizsacker-Williams) approximation, the integral over the transverse momentum $\vec{q}$, at small $\vec{q}^2$, gives rise to large logarithm:

$$\sigma_{\mathrm{tot}}^\ell = \frac{\alpha}{\pi}\ln\left(\frac{s^2Q^2}{M^4m^2}\right)\int_{s_{\mathrm{th}}}^\infty \frac{ds_2}{s_2}\sigma_{\mathrm{tot}}^{(\gamma+p\to X)}(s_2), \qquad (18)$$

where $s_{\mathrm{th}} = (M + m_\pi)^2 - M^2$, Q^2 is the characteristic momentum transfer squared, $Q^2 \simeq M^2$, and we introduced the total cross section for real polarized photons interacting with protons:

$$\sigma_{\mathrm{tot}}^{(\gamma^* p\to X)}(s_2, q^2 = 0) = \frac{\alpha\pi}{s_2}\int(\vec{\mathcal{J}}_H\cdot\vec{e})^2 d\Gamma_H. \qquad (19)$$

The differential cross section (18) is closely related (due to the optical theorem) with the s-channel discontinuity of the forward amplitude for electron-proton scattering with the same intermediate state: a single electron and a jet, moving in opposite directions [see Figs. 2(a) and 2(b)] where, by Cutkovsky rule, the denominators of the "cutted" lines in the Feynman graph of Fig. 2(b) must be replaced by:

$$\frac{1}{q^2 - M^2 + i0} \to -2\pi i\delta(q^2 - M^2). \qquad (20)$$

For the spin-averaged forward-scattering amplitude we have

$$\Delta_s A(s) = \frac{4s\alpha}{\pi^2}\int\frac{d^2q_\perp\vec{q}^2}{(q^2)^2}\int\frac{ds_2}{s_2}\sigma^{(\gamma^* p\to X)}(s_2, q) \qquad (21)$$

with

$$\sigma^{(\gamma^* p\to X)}(s_2, q) = \int\frac{4\pi\alpha}{2\cdot 2\cdot 2s_2}(\vec{\mathcal{J}}_H\cdot\vec{e})^2 d\Gamma_H. \qquad (22)$$

From the formulas given above we obtain

$$\frac{d\Delta_s\bar{A}^{eY\to eY}}{d^2q} = 2s\frac{d\sigma^{eY\to eY}}{d^2q}, \qquad (23)$$

where $\bar{A}^{eY\to eY}$ is the averaged on spin states forward-scattering amplitude. This relation is the statement of the optical theorem in differential form.

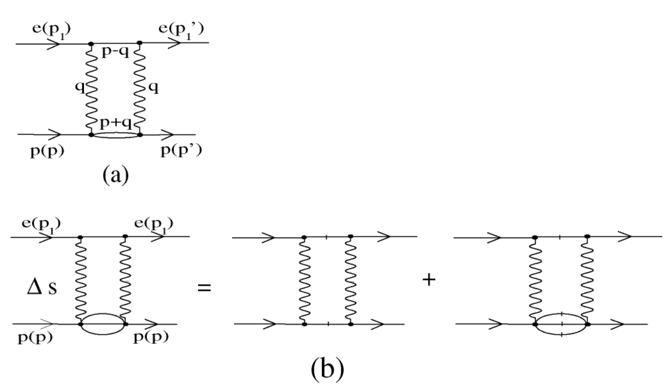

FIG. 2. Feynman diagram for e^+e^- scattering at the order α^3.

Let us now consider the discontinuity of the forward-scattering amplitude with the electron and proton intermediate state, we call it a "pole contribution." For the case of elastic electron-proton scattering we have

$$\frac{d\Delta_s \bar{A}^{ep\to ep}}{d^2q} = \frac{(4\pi\alpha)^2}{(q^2)^2 s(2\pi)^2} Sp, \tag{24}$$

with

$$Sp = Sp(\hat{P} + M)\Gamma(q)(\hat{P}' + M)\Gamma(-q)^*,$$

and

$$\Gamma(q) = F_1\hat{p}_1 - \frac{1}{2M}F_2\hat{q}\hat{p}_1.$$

A simple calculation gives $Sp = 2s^2[(F_1)^2 + \tau(F_2)^2]$, with $\tau = \vec{q}^2/(4M_p^2)$.

For the case of electron-deuteron scattering we use the electromagnetic vertex of deuteron in the form [5]

$$\langle \xi^{\lambda'}(P')|J_\mu^{\rm EM}(q)|\xi^\lambda(P)\rangle = d_\mu\left[F_1(\xi^{\lambda'*}\xi^\lambda) - \frac{F_3}{2M_d^2}(\xi^{\lambda'*}q)(\xi^\lambda q)\right] + F_2[\xi_\mu^\lambda(q\xi^{\lambda'*}) - \xi_\mu^{\lambda'*}(\xi^\lambda q)], \tag{25}$$

where $P^2 = (P')^2 = M_d^2$ (M_d is the deuteron mass),$d_\mu = (P' + P)_\mu$, $q_\mu = (P' - P)_\mu$ and $\xi^\lambda(P)$ is the polarization vector of deuteron in chiral state λ. It has the properties:

$$\xi^2 = -1, \qquad (\xi(P)P) = 0;$$
$$\sum_\lambda \xi^\lambda(P)_\mu \xi^{\lambda*}(P)_\nu = g_{\mu\nu} - \frac{P_\mu P_\nu}{M_d^2}. \tag{26}$$

For the spin averaged forward-scattering amplitude one has

$$\frac{d\Delta_s \bar{A}^{ed\to ed}}{d^2q} = \frac{2s(4\pi\alpha)^2}{3(q^2)^2(2\pi)^2}{\rm Tr}, \tag{27}$$

with

$$Tr = 2(F_1)^2 + (F_1 + 2\tau_d(1+\tau_d)F_3)^2 + 2\tau_d(F_2)^2,$$
$$\tau_d = \frac{\vec{q}^2}{4M_d^2}. \tag{28}$$

We note that the amplitude corresponding to crossed box-type Feynman diagram has a zero s-channel discontinuity.

II. VIRTUAL COMPTON SCATTERING ON PROTON

Let us examine the different contributions to the total amplitude for virtual photon Compton scattering on a proton (hadron). Keeping in mind the baryon number conservation law, we can separate all possible Feynman diagram into four classes. In one, which will be named as a class of retarded diagram (the corresponding amplitude is denoted as A_1), the initial state photon is first absorbed by a nucleon line and then emitted by the scattered proton. Another class (advanced, A_2) corresponds to the diagrams in which the scattered photon is first emitted along the nucleon line and the point of absorption is located later on. The third class corresponds to the case when both photons do not interact with the initial nucleon line. The corresponding amplitude is denoted as A_P. The fourth class contains diagrams in which only one of external photons interacts with the nucleon line. The corresponding notation is $A_{\rm odd}$ (see Fig. 3):

$$A^{\mu\nu}(s,q) = A_1^{\mu\nu}(s,q) + A_2^{\mu\nu}(s,q) + A_P^{\mu\nu}(s,q) + A_{\rm odd}. \tag{29}$$

The amplitude $A_P^{\mu\nu}(s,q)$ corresponds to the Pomeron-type Feynman diagram [Fig. 4(e)] and gives the nonvanishing contribution to the total cross sections in the limit of a large invariant mass squared of initial particles $s_2 \to \infty$. The fourth class amplitude can be relevant in experiments measuring charge-odd effects and it will not be considered here. One can show explicitly that each of the 4 classes amplitudes are gauge invariant. The arguments in favor of it are essentially the same as was used in the QED case [6].

Let us discuss now the analytical properties of the retarded part of the forward Compton scattering of a virtual photon on a proton, $A_1(s_2, q)$ (see Fig. 4) at the s_2-plane. Because of general principles, the singularities—poles and branch points— are situated on the real axis.

These singularities are illustrated in Fig. 5. On the right side, the pole at $s_2 = 0$ corresponds to one nucleon exchange in the s_2 channel [Fig. 5(a)]; the right hand cut starts at the pion-nucleon threshold, $s_2 = (M + m_\pi)^2 - M^2$. The left cut, related with the u-channel 3-nucleon state of the Feynman amplitude, is illustrated in Fig. 4(f). It is situated rather far from the origin at $s_2 = -8M^2$. It can be shown that it is the nearest singularity of the right hand cut. Really the u-channel cut corresponding to the $2\pi N$ state

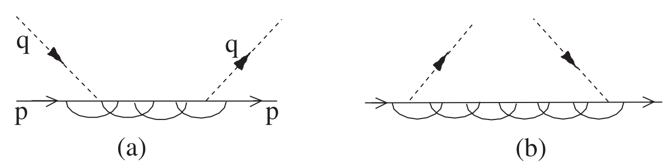

FIG. 3. Illustration of retarded (a) and advanced (b) virtual photon emission and absorption diagrams. The diagram containing Pomeron is not considered here.

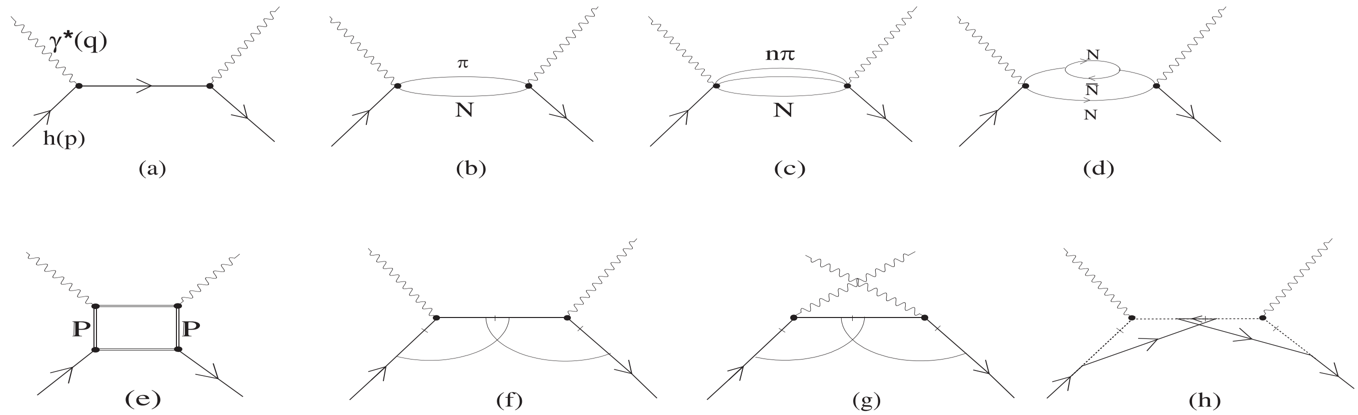

FIG. 4. Feynman diagrams for forward virtual Compton scattering on the proton, contributing to the retarded part of the amplitude: intermediate state in the s_2 channel for a single proton (a), $N\pi$ (b), $Nn\pi$ (c), $\sum N\bar{N}$ (d), two jets s_2 channel state, with two Pomeron t channel state (d), $3N$ intermediate state in u-channel (e), $3N$ intermediate state in u_2 channel (f). The Feynman diagram of the A_2 set which has s-channel discontinuity is illustrated in (g) and an example of the exotic $u$channel state in (h).

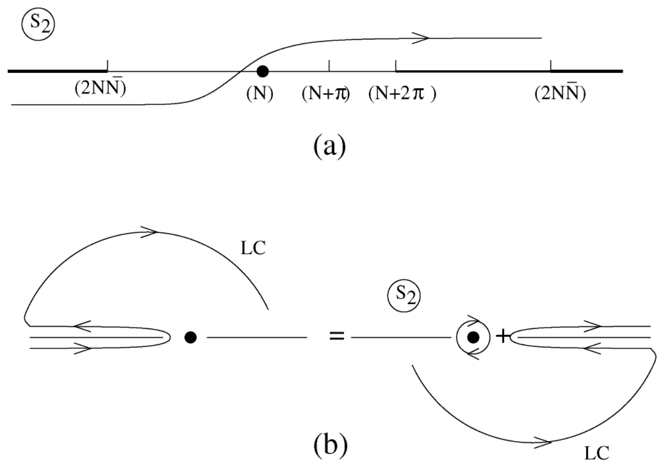

FIG. 5. Illustration of singularities along the s_2 real axis with the open contour C (a), and with the contour C closed (b), corresponding to Fig. 4. LC stands for large circle contribution.

cannot be realized without exotic quantum number states [see Fig. 4(h)].

III. SUM RULES

Following [6] let us introduce the quantity

$$\int_C ds_2 \frac{p_1^\mu p_1^\nu A_{1\mu\nu}^{\gamma^* p \to \gamma^* p}}{s^2(\vec{q}^2)^2} = \frac{dI}{d\vec{q}^2}, \qquad (30)$$

with the Feynman contour C in the s_2 plane as it is shown in Fig. 5(a).

Sum rules appear when one considers the equality of the path integrals along the contours obtained by deforming C such that it is closed to the left and to the right side (Fig. 5). As a result one finds

$$\frac{d\sigma_{\rm left}}{d\vec{q}^2} - \frac{d\sigma_{\rm el} - d\sigma_{\rm el}^B}{d\vec{q}^2} = \frac{d\sigma_{\rm inel}}{d\vec{q}^2}, \qquad (31)$$

where $\frac{d\sigma_{\rm left}}{d\vec{q}^2}$ indicates the contribution of the left cut;[1]

$$\frac{d\sigma_{\rm el}^B}{d\vec{q}^2} = \frac{4\pi\alpha^2 Z^2}{(\vec{q}^2)^2}.$$

The latter is generally the Born cross section of the scattering of an electron on any hadron with charge Z when the strong interaction is switched off, and $\frac{d\sigma_{\rm el}}{d\vec{q}^2}$ is the elastic electron-hadron cross section when the strong interaction is switched on, in the lowest order of QED coupling constant. This quantity can be expressed in terms of electromagnetic form factors of corresponding hadrons.

Using the notation for the generalized square of form factors as Φ^2, we have the following expression for the process of electron scattering on a hadron Y with charge Z:

$$Z^2 - \Phi^2(-\vec{q}^2) = \frac{2(\vec{q}^2)^2}{\pi\alpha^2} \frac{d\sigma^{eY\to eX}}{d\vec{q}^2}. \qquad (32)$$

For the case of the spin-zero target, the quantity Φ^2 coincides with its squared charge form factor.

For the cases of electron scattering on a spin one-half (proton, ^{3}He,^{3}H), which are described by two form factors (Dirac's one F_1 and Pauli one F_2) we have

$$Z_i^2 - F_{1i}^2(-\vec{q}^2) - \tau_i F_{2i}^2(-\vec{q}^2) = \frac{2(\vec{q}^2)^2}{\pi\alpha^2} \frac{d\sigma^{eY_i\to eX_i}}{d\vec{q}^2}, \quad (33)$$

[1] The coincidence of numbers in (1) and (2) is derived from the absence of the left cut contribution, which is known for planar Feynman diagram amplitudes.

with

$$\tau_i = \frac{\vec{q}^2}{4M_i^2}; \qquad Z_p = Z_{^3\mathrm{H}} = 1; \qquad Z_{^3\mathrm{He}} = 2.$$

For scattering of electrons on deuteron we have

$$1 - \frac{1}{3}[2F_1^2(-\vec{q}^2) + [F_1(-\vec{q}^2) + 2\tau_d(1+\tau_d)F_3(-\vec{q}^2)]^2 + 2\tau_d F_2^2(-\vec{q}^2)] = \frac{2(\vec{q}^2)^2}{\pi\alpha^2}\frac{d\sigma^{ed\to eX}}{d\vec{q}^2}. \quad (34)$$

These equations can be tested in experiments with electron-hadron colliders.

Let us consider the differential form of these sum rules applying the operator : $\frac{d}{d\vec{q}^2}(\vec{q}^2)^2 F(\vec{q}^2)|_{\vec{q}^2=0}$ to both sides of Eq. (34), which can be expressed in terms of charge radii, anomalous magnetic moments, etc. and of the total photoproduction cross section.

Considering formally this derivative at $\vec{q}^2 = 0$, we obtain

$$\frac{d}{d\vec{q}^2}\Phi_Y^2|_{\vec{q}^2=0} = \frac{2}{\pi^2\alpha}\int_{s_{\mathrm{th}}}^{\infty}\frac{ds_2}{s_2}\sigma_{\mathrm{tot}}^{\gamma Y\to X}(s_2). \quad (35)$$

Unfortunately, the sum rule in this form cannot be used for experimental verification due to the divergence of the integral in the right-hand side of this equation. Its origin follows from the known fact of increasing of photoproduction cross sections at large values of initial center of mass energies squares s_2. It is commonly known that this fact is the consequence of Pomeron-Regge pole contribution. The universal character of Pomeron interaction with nucleons can be confirmed by the Particle Data Group-2004 (PDG) result [7]

$$[\sigma^{\gamma p}(s_2) - \sigma^{\gamma n}(s_2)]|_{s_2\to\infty} = [2\sigma^{\gamma p}(s_2) - \sigma^{\gamma d}(s_2)]|_{s_2\to\infty} = 0. \quad (36)$$

In Ref. [8] a sum rule which contains the difference between proton and neutron sum photoproduction cross sections was derived

$$\frac{1}{3}\langle r_p^2\rangle + \frac{1}{4M^2}[\kappa_n^2 - \kappa_p^2] = \frac{2}{\pi^2\alpha}\int_{\omega_n}^{\infty}\frac{d\omega}{\omega} \times [\sigma^{\gamma p\to X}(\omega) - \sigma^{\gamma n\to X}(\omega)]. \quad (37)$$

We use here the known relations

$$F_1(-\vec{q}^2) = 1 - \frac{1}{6}\vec{q}^2\langle r^2\rangle + O((\vec{q}^2)^2); \qquad F_2(0) = \kappa,$$

with $\langle r^2\rangle$, κ, are the charge radius squared and the anomalous magnetic moment of nucleon (in units h/Mc).

It was verified in this paper that this sum rule is fulfilled within the experimental errors: both sides of the equation equal 1.925 mb. Here the Pomeron contribution is compensated in the difference of proton and neutron total cross photoproduction cross sections.

In Ref. [9], the similar combination of cross sections was considered for $A = 3$ nuclei:

$$\frac{2}{3}\langle r_{^3\mathrm{He}}^2\rangle - \frac{1}{3}\langle r_{^3\mathrm{H}}^2\rangle - \frac{1}{4M^2}[\kappa_{^3\mathrm{He}}^2 - \kappa_{^3\mathrm{H}}^2] = \frac{2}{\pi^2\alpha}\int_{\omega_{\mathrm{th}}}^{\infty}\frac{d\omega}{\omega}[\sigma^{\gamma^3\mathrm{He}\to X}(\omega) - \sigma^{\gamma^{33}\mathrm{H}\to X}(\omega)]. \quad (38)$$

In a similar way, the combination of cross sections of electron scattering on proton and deuteron leads to the relation

$$\frac{1}{3}\langle r_d^2\rangle - \frac{F_3(0)}{3M_d^2} - \frac{1}{6M_d^2}F_2(0)^2 - 2\left[\frac{1}{3}\langle r_p^2\rangle - \frac{1}{4M_p^2}\kappa_p^2\right] = \frac{2}{\pi^2\alpha}\int_{\omega_{th}}^{\infty}\frac{d\omega}{\omega}[\sigma_{\mathrm{tot}}^{\gamma d\to X}(\omega) - 2\sigma_{\mathrm{tot}}^{\gamma p\to X}(\omega)], \quad (39)$$

with ω_{th} for the deuteron and the proton being different: $(\omega_{\mathrm{th}})_d = 2,2$ MeV, $\qquad (\omega_{\mathrm{th}})_p = m_\pi + \frac{m_\pi^2}{2M_p} \approx 140$ MeV.

We use here the similar expansion for F_1 deuteron form factor $F_1(0) = 1$ and introduce its square charge radius. Other quantities can be found in Ref. [10]:

$$F_2(0) = -\frac{M_d}{M_p}\mu_d; \qquad \mu_d = 0.857;$$

$$2F_3(0) = 1 + F_2(0) - M_d^2 Q_d; \qquad Q_d = 0.2859\ \mathrm{fm}^2.$$

We find for the right side of Eq. (39) by using [7,11]

$$\frac{2}{\pi^2\alpha}\left\{\int_{0.020}^{0.260}\frac{d\omega}{\omega}\sigma_{\mathrm{tot}}^{\gamma d\to X}(\omega) + \int_{0.260}^{16}\frac{d\omega}{\omega}[\sigma_{\mathrm{tot}}^{\gamma d\to X}(\omega) - 2\sigma_{\mathrm{tot}}^{\gamma p\to X}(\omega)]\right\} = 0.8583\ \mathrm{fm}^2 = 8.583\ \mathrm{mb} \quad (40)$$

Using the data for γp and γd cross sections and the parametrization from Ref. [11] we find from Eqs. (39) and (40)

$$\langle r_d\rangle \approx 1.94\ \mathrm{fm}.$$

This quantity is in a satisfactory agreement with prediction of models based dispersion relations [12] where is $\langle r_d\rangle \sim 2$ fm.

The reason for a discrepancy (which does not exceed the errors of 3% inherent to our approach) can be attributed to the lack of data for γp and γd cross sections near the thresholds.

IV. CONCLUSIONS

The left cut contribution has no direct interpretation in terms of cross section. In analogy with the QED case it can be associated with the contribution to the cross section of the process of proton-antiproton pair electroproduction on proton $ep \to e2p\bar{p}$, arising from taking into account the identity of final state protons.

Fortunately, the threshold of this process is located quite far. Using this fact we can estimate its contribution in the framework of QED-like model with nucleons and pions (ρ-mesons), omitting the form-factor effects (so we put them equal to coupling constants of nucleons with pions and vector mesons).

In this paper we applied an optical theorem, which connects the s-channel discontinuity of the forward-scattering amplitude with the total cross section. It is valid for the complete scattering amplitude, whereas we consider only part of it, A_1. We can explicitly point out on the Feynman diagram [see Fig. 4(g)], contributing to A_2, which has 3 nucleon s-channel states. The relevant contribution can be interpreted as the identity effect of proton photoproduction of a $p\bar{p}$ pair.

The explicit calculation in the framework of our approach is given by Appendix A. The corresponding contributions to the derivative on $\vec{q}^2$ at $\vec{q}^2 = 0$ of scattering amplitudes entering the sum rules have an order of magnitude

$$I = \frac{g^4 M^2}{\pi^3 s_{2\,\mathrm{min}}^2}.$$

In order to estimate the strong coupling constant, we use the PDG value for the total cross section of scattering of the pion on the proton $\sigma_{\mathrm{tot}}^{\pi p} = 20$ mb. Keeping in mind the ρ-meson t-channel contribution $\sigma^{\pi p} \sim g^4/(4\pi m_\rho^2)$ and the minimal value of three nucleon invariant mass-squared $s_{2\,\mathrm{min}} = 8M_p^2$, we have $I \approx (1/15)$ mb. Comparing this value with typical values of right- and left-hand sides of sum rules of order of $2mb$, we estimate the error arising by omitting the left cut contribution as well as replacing our incomplete cross sections by the measurable ones on the level of 3%. Gottfried sum rules [15], which are also related to the present question, suffer from ultraviolet divergency due to Pomeron exchange contribution.

ACKNOWLEDGMENTS

The authors are thankful to S. Dubnicka for valuable discussions. We are grateful to A. V. Shebeko, V. V. Burov, M. Mirazita, and P. Rossi for detailed information about deuteron disintegration. One of us (E. A. K.) is grateful to DPNIA/SPhN, Saclay, where part of this work was done. This work was partially supported by the Slovak Grant Agency for Sciences VEGA, Grant No. 2/4099/25 (M. S.).

APPENDIX A: EFFECT OF IDENTITY OF PROTONS TO THE CROSS SECTION OF $2p\bar{p}$ PHOTOPRODUCTION CROSS SECTION

The contribution to the s-channel discontinuity of the part of the scattering amplitude $A_2^{\gamma^* p \to \gamma^* p}$, arising from the interference of the amplitudes for the creation of a proton-antiproton pair of bremsstrahlung type, due to the identity of protons in the final state has the form:

$$\Delta_s A_2(s, q) = -\frac{16g^4}{s^2} \int ds_2 d\Gamma_3 \frac{S}{(q_1^2 - m_\pi^2)(q_2^2 - m_\pi^2)}, \tag{A1}$$

where $q_1 = P_1 - P$, $q_2 = P_2 - P$ and

$$S = \tfrac{1}{4}\mathrm{Sp}(\hat{P} + M)\gamma_5(\hat{P}_1 + M)V_1(\hat{P}_3 - M)V_2(\hat{P}_2 + M)\gamma_5,$$

and $V_1 = \gamma_5 \frac{\hat{q} - \hat{P}_3 + M}{d_3}\hat{p}_1 + \hat{p}_1 \frac{\hat{P}_1 - \hat{q} + M}{d_1}\gamma_5$;
$V_2 = \gamma_5 \frac{-\hat{q} + \hat{P}_2 + M}{d_2}\hat{p}_1 + \hat{p}_1 \frac{-\hat{P}_3 + \hat{q} + M}{d_3}\gamma_5$;
$d_{1,2,3} = (q - P_{1,2,3})^2 - M^2$, $s_2 = (P + q)^2 - M - q^2$.

The elements of phase volume can be written as

$$d\Gamma_3 = (2\pi)^4 \delta^4(P + q - P_1 - P_2 - P_3)\Pi_1^3 \frac{d^3 P_i}{2E_i(2\pi)^3}.$$

Here we consider pions to be interacting with nucleons with a coupling constant g. The similar expression can be written for the case when one or both pions are replaced by the ρ-meson. It can be shown that the corresponding contributions are approximately one order of magnitude smaller than those from pions.

We use Sudakov's parametrization of momenta:

$$q = \alpha\tilde{P} + \beta p_1 + q_\perp; \qquad P = \tilde{P} + \frac{M^2}{s} p_1; \qquad P_i = \alpha_i \tilde{P} + \beta_i p_1 + P_{i\perp}. \tag{A2}$$

Using the formulas given above, all the relevant quantities can be written as

$$\int ds_2 d\Gamma_3 = \frac{1}{4(2\pi)^3} \frac{d^2P_1 d^2P_2 d\alpha_1 d\alpha_2}{\alpha_1\alpha_2\alpha_3}; \qquad \alpha_1 + \alpha_2 + \alpha_3 = 1; \qquad \vec{P}_1 + \vec{P}_2 + \vec{P}_3 = \vec{q} \tag{A3}$$

and

$$s_2 = s\beta = -M^2 + \sum_1^3 \frac{\vec{P}_i^2 + M^2}{\alpha_i}; \qquad q_i^2 = -\frac{\vec{P}_i^2 + (1-\alpha_i)^2}{\alpha_i}, \quad i = 1, 2; \qquad d_i = -s_2\alpha_i + 2\vec{q}\vec{P}_i - \vec{q}^2, \quad i = 1, 2, 3. \tag{A4}$$

Note that the quantities V_i can be written as

$$V_1 = s\gamma_5 A_{13} + \frac{\gamma_5\hat{q}\hat{p}_1}{d_3} - \frac{\hat{p}_1\hat{q}\gamma_5}{d_1}; \quad V_2 = s\gamma_5 A_{23} - \frac{\gamma_5\hat{q}\hat{p}_1}{d_2} + \frac{\hat{p}_1\hat{q}\gamma_5}{d_3}, \tag{A5}$$

with

$$A_{13} = \frac{\alpha_1}{d_1} - \frac{\alpha_3}{d_3}; \qquad A_{23} = \frac{\alpha_2}{d_2} - \frac{\alpha_3}{d_3}.$$

In this form the gauge invariance of the contribution to the forward-scattering amplitude is explicitly seen; namely, this quantity turns out to zero at $\vec{q} \to 0$, (we see that replacements $\hat{q}\hat{p}_1 = \hat{q}_\perp\hat{p}_1$, $\hat{p}_1\hat{q} = \hat{p}_1\hat{q}_\perp$ in $V_{1,2}$ can be done).

The calculation of the trace leads to the result:

$$\frac{S}{s^2} = A_{13}A_{23}S_1 + A_{13}\left[\frac{1}{d_3} + \frac{1}{d_2}\right]S_2 + A_{23}\left[\frac{1}{d_3} + \frac{1}{d_1}\right]S_3 + \left[\frac{1}{d_3} + \frac{1}{d_1}\right]\left[\frac{1}{d_3} + \frac{1}{d_2}\right]S_4, \tag{A6}$$

with

$$\begin{aligned} S_1 &= M^4 + \tfrac{1}{2}M^2\vec{q}^2 + (PP_1)(P_2P_3) + (P_3P_1)(P_2P) - (PP_3)(P_2P_1); \\ S_4 &= -\frac{\alpha_3\vec{q}^2}{2}[M^2\alpha_3 + \alpha_1(PP_2) + \alpha_2(PP_1) - (P_1P_2)]; \\ S_2 &= \frac{M^2}{2}[\alpha_2(\vec{q}\vec{P}_2) - \alpha_3(\vec{q}\vec{P}_3) - (\alpha_1 + 2\alpha_3)(\vec{q}\vec{P}_1)] \\ &\quad + \frac{1}{2}[(\vec{q}\vec{P}_3)[(P_1P_2) - \alpha_2(PP_1) - \alpha_1(PP_2)] \\ &\quad + (\vec{q}\vec{P}_2)[(P_1P_3) + \alpha_3(PP_1) - \alpha_1(PP_3)] \\ &\quad - (\vec{q}\vec{P}_1)[(P_3P_2) - \alpha_2(PP_3) - \alpha_3(PP_2)]]; \\ S_3 &= \frac{M^2}{2}[\alpha_3(\vec{q}\vec{P}_3) - \alpha_1(\vec{q}\vec{P}_1) + (\alpha_2 + 2\alpha_3)(\vec{q}\vec{P}_2)] \\ &\quad + \frac{1}{2}[(\vec{q}\vec{P}_1)[-(P_3P_2) - \alpha_3(PP_2) + \alpha_2(PP_3)] \\ &\quad + (\vec{q}\vec{P}_2)[(P_1P_3) - \alpha_3(PP_1) - \alpha_1(PP_3)] \\ &\quad - (\vec{q}\vec{P}_3)[(P_1P_2) - \alpha_1(PP_2) - \alpha_2(PP_1)]]. \end{aligned} \tag{A7}$$

The invariants entering this expression have a form

$$2PP_i = \frac{\vec{P}_i^2 + M^2(1 + \alpha_i^2)}{\alpha_i}; \quad 2P_iP_j = \frac{(\alpha_i\vec{P}_j - \alpha_j\vec{P}_i)^2 + M^2(\alpha_i^2 + \alpha_j^2)}{\alpha_i\alpha_j}.$$

Numerical integration of this expression confirms the estimate given above within 10%.

APPENDIX B: CORRELATION BETWEEN MOMENTUM AND THE SCATTERING ANGLE OF RECOIL PARTICLE IN LAB FRAME

The idea of expanding four vectors of some relativistic problem using two of them as a basis (Sudakov's parametrization) becomes useful in many regions of quantum field theory. It was crucial in studying the double logarithmical asymptotic of amplitudes of processes with large transversal momenta. Being applied to processes with peripheral kinematics, it essentially coincides with the infinite momentum frame approach.

Here we demonstrate its application to the study of the kinematics of the peripheral process of jet formation on a resting target particle. One of the experimental approaches to studying them is to measure the recoil particle momentum distribution. For instance, this method is used in the process of electron-positron pair production by linearly polarized photon on electrons in a solid target (atomic electrons). Here the correlation between the recoil momentum-initial photon plane and the plane of photon polarization is used to determine the degree of photon polarization [14].

Sudakov's parametrization allows us to give a transparent explanation of correlation between the angle of emission of the recoil target particle of mass M with the recoil momentum value in the laboratory reference frame [14]:

$$\frac{|\vec{P}'|}{M} = \frac{2\cos\theta_p}{\sin^2\theta_p}; \qquad \frac{E'}{M} = \frac{1 + \cos^2\theta_p}{\sin^2\theta_p}, \tag{B1}$$

where $\vec{P}', E'$ are the 3-momentum and energy of recoil particle, $(E')^2 - (P')^2 = M^2$; θ_p is the angle between the beam axes $\vec{k}$ in the rest frame of the target particle.

$$\gamma(k) + P(P) \to \text{jet} + P(P'), \qquad s = 2kP = 2M\omega,$$
$$P - P' = q, \qquad P^2 = (P')^2 = M^2.$$

The kinematics considered here corresponds to the main contribution to the cross section for a jet moving close to projectile direction. Using the Sudakov representation for transfer momentum $q = \alpha\tilde{P} + \beta k + q_\perp$, and the recoil particle on-mass shell condition $(P - q)^2 - M^2 \approx -s\beta - \vec{q}^2 = 0$, we obtain for the ratio of squares of transversal and longitudinal components of the 3-momentum of the recoil particle:

$$\tan^2\theta_p = \frac{\vec{q}^2}{(\beta\omega)^2} = \frac{4M^2}{\vec{q}^2}, \qquad \vec{q}^2 = (\vec{P}')^2\sin^2\theta_p. \tag{B2}$$

The relation noted in the beginning of this section follows immediately.

This correlation was first mentioned in Ref. [13], where the production on electrons from the matter was investigated. It was proven in Ref. [14].

This relation can be applied in experiments with collisions of high energy protons scattered on protons in the matter.

[1] E. A. Kuraev and L. N. Lipatov, Yad. Fiz. **20**, 112 (1974).

[2] E. A. Kuraev, L. N. Lipatov, and N. P. Merenkov, Phys. Lett. B **47B**, 33 (1973).

[3] R. Barbieri, J. A. Mignaco, and E. Remiddi, Nuovo Cimento Soc. Ital. Fis. A **11**, 824 (1972).

[4] C. F. von Weizsacker, Z. Phys. **88**, 612 (1934); E. Williams, Phys. Rev. **45**, 729 (1934).

[5] A. I. Akhiezer and M. P. Rekalo, *Electrodynamics of Hadrons* (Naukova Dumka, Kiev, 1978); S. Dubnička, Acta Phys. Pol. **B27**, 2525 (1996).

[6] V. N. Baier, V. S. Fadin, V. A. Khoze, and E. A. Kuraev, Phys. Rep. **78**, 293 (1981).

[7] S. Eidelman *et al.*, Phys. Lett. B **592**, 1 (2004).

[8] E. Bartoš, S. Dubnička, and E. A. Kuraev Phys. Rev. D **70**, 117901 (2004).

[9] S. Dubnička, E. Bartoš, and E. Kuraev, Nucl. Phys. B, Proc. Suppl. **126**, 100 (2004).

[10] G. I. Gakh, and N. P. Merenkov, JETP **98**, 853 (2004).

[11] P. Rossi *et al.*, Phys. Rev. C **40**, 2412 (1989).

[12] T. Herrmann and R. Rosenfelder, Eur. Phys. J. A **2**, 28 (1998).

[13] D. Benaksas and R. Morrison, Phys. Rev. **160**, 1245 (1967).

[14] E. Vinokurov and E. Kuraev, J. Exp. Theor. Phys. **63**, 1142 (1972).

[15] K. Gottfried, Phys. Rev. Lett. **18**, 1174 (1967).