# Space-time propagation of neutrino wave packets at high temperature and density

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We study the space-time evolution of ''flavor'' neutrino wave packets at finite temperature and density in the early Universe prior to big bang nucleosynthesis (BBN). We implement nonequilibrium field theory methods and linear response to study the space-time evolution directly from the effective Dirac equation in the medium. There is a rich hierarchy of time scales associated with transverse and longitudinal dispersion and coherence. A phenomenon of ''freezing of coherence'' is a result of a competition between longitudinal dispersion and the separation of wave packets of propagating modes in the medium. Near a resonance the coherence and oscillation time scales are enhanced by a factor $1/\sin 2\theta$ compared to the vacuum. Collisional relaxation via charged and neutral currents occurs on time scales much shorter than the coherence time scale and for small vacuum mixing angle, shorter than the oscillation scale. Assuming that the momentum spread of the initial wave packet is determined by the large angle scattering mean free path of charged leptons, we find that the transverse dispersion time scale is the shortest and is responsible for a large suppression in both the survival and transition probabilities on time scales much shorter than the Hubble time. For small mixing angle the oscillation time scale is *longer* than the collisional relaxation scale. The method also yields the evolution of right-handed wave packets. Corrections to the oscillation frequencies emerge from wave-packet structure as well as from the energy dependence of mixing angles in the medium.

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## I. INTRODUCTION

Neutrinos are the bridge between astrophysics, cosmology, particle physics and nuclear physics [1–4]. In recent years, there has been increasing experimental evidence that confirm that neutrinos are massive and oscillate between different flavors [5–9] providing the first indisputable hint of *new physics* beyond the standard model.

Neutrino mixing and oscillations in extreme conditions of high temperature and density play a fundamental role in astrophysics and cosmology [10–16]. Resonant flavor mixing due to Mikheyev-Smirnov-Wolfenstein (MSW) effect can provide a concrete explanation to the solar neutrino problem [17,18]. During big bang nucleosynthesis (BBN) neutrino oscillations may result in corrections to the abundance of electron neutrinos [10]. This in turn changes the neutron-to-proton ratio, affecting the mass fraction of $^4$He (see Ref. [10] and references therein). Neutrino oscillations violate lepton number leading to the possibility that the cosmological baryon asymmetry may originate in the lepton sector [19–21].

Neutrino propagation in a cold medium has been first studied by Wolfenstein [17] who included the refractive index from electron neutrinos. Early studies focused on the neutrino dispersion relations and damping rates at the temperature limit relevant for stellar evolution or BBN [22]. This work has been extended to include charged leptons, neutrinos and nucleons in the thermal medium [23]. The matter effects of neutrino oscillations in the early universe have been investigated in [10,22,24,25]. More Recently, a nonequilibrium field-theoretical description of neutrino oscillations in the early universe in the real-time formulation has been reported in [26].

Kayser [27] first pointed out subtle but important caveats in the vacuum oscillation formula obtained from the standard plane wave treatment, which result from assuming a definite neutrino momentum for different mass eigenstates. He showed that knowledge of momentum allows experiments to distinguish different neutrino mass eigenstates, essentially destroying the oscillation pattern. He then proposed a wave-packet treatment of neutrino oscillations, in which the neutrino momentum is spread out. Since then, the wave-packet approach has been studied by many authors in both quantum mechanical [1,2,28–32] and field-theoretical [33–37] frameworks, including the study of oscillations of neutrinos produced and detected in crystals [38].

The quantum mechanical approach usually refers to the intermediate wave-packet model in which each propagating mass eigenstate of neutrino is associated with a wave packet [29]. This model eliminates some of the problems in the plane wave treatment although several conceptual questions remaining unsettled [28]. See [37] and references therein for detailed descriptions of these issues. A field-theoretical approach is the external wave-packet model [33] in which the oscillating neutrino is represented by an internal line of a Feynman diagram, while the source and the detector are, respectively, described by in-coming and out-going wave packets. A recent review [37] presents the different approaches, summarizes their advantages and caveats and includes the dispersion of wave packets in the study.

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An important physical consequence of the wave-packet description of neutrino evolution is the concept of the coherence length beyond which neutrino oscillations vanish. A "flavor neutrino" wave packet is a linear superposition of wave packets of mass eigenstates. The different mass states entail a difference in the group velocity and an eventual separation of the wave packets associated with mass eigenstates. This separation results in a progressive loss of coherence as overlaps between the wave packets dimishes. See for example [31] for an early explanation. In an actual source-detector experimental setup, the observation time is usually not measured and is commonly integrated out in a wave-packet treatment [31]. This leads to a localization term in the vacuum oscillation formula, which states that neutrino oscillations are suppressed if the spatial uncertainty is much larger than the oscillation length [31].

The coherence of neutrino oscillations in matter has been studied within a geometrical representation in [30], but the medium oscillation formula was not derived. While most of the studies focus on reproducing the standard vacuum oscillation formula, a consistent study of neutrino mixing and propagation in a medium *in real time* has not yet emerged.

While in the vacuum the space-time propagation can be studied in the wave-packet approach by focusing on the space-time evolution of initially prepared single particle "flavor states", the study of the space-time evolution in a medium at finite temperature and density requires a density matrix description.

To the best of our knowledge, a full finite temperature field-theoretical treatment of the space-time propagation of neutrino wave packets in a medium including medium corrections and dispersion dynamics has not yet been offered. We consider this study an important aspect of the program to study the nonequilibrium evolution of neutrinos in the early Universe. Detailed studies have shown that neutrino oscillations and self-synchronization lead to flavor equilibration before BBN [39–43], beginning at a temperature of $T \sim 30$ MeV with complete flavor equilibration among all flavors at $T \sim 2$ MeV [41]. If neutrinos are produced in the form of spatially localized wave packets rather than extended plane waves before BBN, the two mass eigenstates separate progressively during propagation due to the small difference in group velocities. A significant amount of neutrino oscillations, which are crucial for "flavor equalization" requires that the two mass eigenstates overlap appreciably throughout their propagation, namely, the coherence time scale should be sufficiently large to ensure that "flavor equilibration" through oscillations is effective. Therefore, it is important to pursue a full field-theoretical study of neutrino wave-packet propagation in the medium directly in *real time* to determine the relevant time scales for coherence to be maintained and to identify the processes that contribute to its loss.

### A. The main goals of this article

In this article we study the space-time evolution of neutrino wave packets in extreme environments at high temperature and density, conditions that prevail in the early Universe or during supernovae explosions. Our goals are the following:

(i) to provide a consistent and systematic nonequilibrium field-theoretical formulation to study the space-time evolution of initially prepared neutrino wave packets at finite temperature and density. This goal requires a treatment of the space-time evolution in terms of the density matrix, which goes beyond the usual treatment in terms of single particle states. To achieve this goal we implement a recently developed method [26] to study nonequilibrium aspects of neutrino propagation in a medium as an *initial value problem* in linear response. This method yields the effective Dirac equation of motion for the expectation value of the neutrino field induced by an external source. The effective Dirac equation in the medium includes self-energy contributions from charged and neutral currents up to one loop.

(ii) To assess the different time scales associated with wave-packet dispersion, coherence and oscillations including the medium effects, in particular, near possible resonances in the in-medium mixing angles. This is achieved by solving the effective Dirac equation in the medium, which includes self-energy corrections, as an initial value problem. The initial wave packet configuration is "prepared" by an external source term in linear response. This method also allows to assess corrections from the *energy dependence of the mixing angles in the medium* upon the wave-packet dynamics.

(iii) The space-time evolution of the initially prepared wave packet, including dispersive effects allows an assessment of the competition between the progressive loss of coherence in the wave-packet dynamics by the separation of mass eigenstates, collisional decoherence, and cosmological expansion. While our study only includes the self-energy from charged and neutral currents up to one-loop, the final result allows us to include results available in the literature [10,22,24,25] to understand the effects of collisional decoherence and cosmological expansion when there is a separation of time scales.

(iv) We focus our study within the context of early Universe cosmology, in particular, in the temperature regime just prior to BBN where there is a possibility for resonant transitions [10,22,24–26]. Of particular interest are the medium modifications of the dispersion relations, wave-packet dispersion, oscillation and coherence time scales in this temperature and energy regime.

(v) A bonus of this field-theoretical formulation is that it also allows to obtain the space-time evolution of right-handed neutrino wave packets. Although the amplitude for such wave packets is suppressed by $M/k$ with $M$, $k$ the typical mass and energy scale of neutrinos in the medium, and may not be relevant for neutrino processes in the early Universe, the method systematically yields this information.

Since we study the propagation of neutrino wave packets in a medium, aspects associated with the source-detector measurement processes are not well-defined or relevant in this case. Consequently, in contrast to most studies in the literature, *we do not integrate in time* as is the case for a description of experiments in the vacuum [31]. Therefore, our study of propagation is both in *space and time*.

### B. Main results

Our main results are the following:

(i) A systematic field-theoretical formulation of the space-time dynamics of wave packets of massive and mixed neutrinos in terms of the effective Dirac equation including self-energy corrections. The space-time evolution is approached as an initial value problem via linear response with the full density matrix.

(ii) Wave-packet evolution features characteristic time scales associated with transverse and longitudinal dispersion. The ratio of these scales is given by the enormous Lorentz dilation factor in the case of relativistic neutrinos. Neither of these scales receives substantial medium corrections. The shortest scale, associated with the transverse dispersion dominates the suppression of both the survival and transition probabilities. There is an interesting phenomenon of "coherence freezing" which results from the competition between longitudinal dispersion and the separation of the mass eigenstates. We find that there are medium as well as wave-packet modifications to the oscillation formula both for the oscillation frequency as well as the survival and transition probabilities.

(iii) There is a resonance for the mixing angle in the medium just prior to BBN [10,22,24–26] at an energy and temperature $k \sim T \sim 3.6$ MeV for large mixing angle or $k \sim T \sim 7$ MeV for small mixing angle. Both the coherence and the oscillation time scales are enhanced by a factor $1/\sin 2\theta$ near the resonance, where $\theta$ is the *vacuum* mixing angle. This suggests a substantial increase both in the coherence and the oscillation scales for 1–3 mixing, but not an appreciable modification for 1–2 mixing.

(iv) Assuming that the momentum spread of the initial wave packet is determined by the large angle scattering mean free path of charged leptons [1], we find that near the resonance the loss of coherence via charged and neutral current elastic scattering is faster than the loss of coherence by the separation of the mass eigenstates and occurs on a time scale much shorter than the Hubble time. However, the decoherence time scale is many orders of magnitude *larger* than the time scale for transverse dispersion which ultimately determines the suppression of the survival and transition probabilities.

### C. Outline

This article is organized as follows. In Sec. II, we obtain the effective Dirac equation of neutrino in a thermal medium implementing the methods of nonequilibrium field theory and linear response [26]. In this section we obtain the in-medium dispersion relations and mixing angles. In Sec. III, we develop the general formulation to study the space-time propagation of neutrino wave packet. In this section we discuss the different time scales associated with dispersion, oscillations and coherence. In Sec. IV we compare the different time scales with the Hubble and collisional relaxation time scales and discuss the impact of the different scales upon the space-time evolution of the neutrino wave packets, coherence, and oscillations. We present our conclusions in Sec. V.

## II. EFFECTIVE DIRAC EQUATION OF NEUTRINOS IN A MEDIUM AND LINEAR RESPONSE

The study of the evolution of neutrino wave packets in the vacuum typically involves a description of the experimental production and detection of these wave packets. We study the space-time evolution of wave packets in a medium as an *initial value problem*. This is achieved in *linear response* by coupling an external source term which induces an expectation value of the neutrino field in the density matrix, after this source is switched off the expectation value evolves in time. The propagation of this initial state is described by the effective Dirac equation in the medium, which includes the self-energy corrections. This is the familiar linear response approach to studying the evolution out of equilibrium in condensed matter systems. The correct framework to implement this program is the real-time formulation of field theory in terms of the closed-time-path integral [44–46].

We restrict our study to the case of two Dirac flavor neutrinos, while the formulation is general and can treat 1–2 or 1–3 mixing on equal footing, for convenience we will refer to electron and muon neutrino mixing. Neutrino mixing and oscillations is implemented by adding to the standard model a Dirac mass matrix $M_{ab}$ which is off-diagonal in the flavor basis. For our discussion, the relevant part of the Lagrangian is given by

$$\mathcal{L} = \mathcal{L}^0_\nu + \mathcal{L}^0_W + \mathcal{L}^0_Z + \mathcal{L}_{\rm CC} + \mathcal{L}_{\rm NC} + \bar{\eta}_a \nu_a + \bar{\nu}_a \eta_a, \tag{2.1}$$

where $\mathcal{L}^0_\nu$ is the free field neutrino Lagrangian density

$$\mathcal{L}^0_\nu = \bar{\nu}_a (i\not{\partial}\delta_{ab} - M_{ab})\nu_b, \tag{2.2}$$

where $a$, $b$ are flavor indices. $\mathcal{L}^0_{W,Z}$ are the free field vector boson Lagrangian densities in the unitary gauge, namely

$$\mathcal{L}^0_W = -\tfrac{1}{2}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)(\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}) + M_W^2 W^+_\mu W^{-\mu}, \tag{2.3}$$

$$\mathcal{L}^0_Z = -\tfrac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) + \tfrac{1}{2}M_Z^2 Z_\mu Z^\mu, \tag{2.4}$$

and the charged and neutral current interaction Lagrangian densities are given by

$$\mathcal{L}_{\rm CC} = \frac{g}{\sqrt{2}}[\bar{\nu}_a \gamma^\mu L l_a W^+_\mu + \bar{l}_a \gamma^\mu L \nu_a W^-_\mu], \tag{2.5}$$

$$\mathcal{L}_{\rm NC} = \frac{g}{2\cos\theta_w}[\bar{\nu}_a \gamma^\mu L \nu_a Z_\mu + \bar{f}_a \gamma^\mu (g^V_a - g^A_a \gamma^5) f_a Z_\mu], \tag{2.6}$$

where $\theta_w$ is the Weinberg angle, $L = (1-\gamma^5)/2$ is the left-handed chiral projection operator, and $g^{V,A}$ are the vector and axial vector couplings for quarks and leptons. The label $l$ stands for leptons and $f$ generically for the fermion species with neutral current interactions. The external sources $\eta_a$, $\bar{\eta}_a$ which couple to the neutrino fields depend explicitly on space and time and induce an expectation value whose time evolution is studied in linear response.

For two Dirac flavor neutrinos, the mass matrix $M_{ab}$ is parametrized by

$$\mathbb{M} = \begin{pmatrix} m_{ee} & m_{e\mu} \\ m_{e\mu} & m_{\mu\mu} \end{pmatrix}. \tag{2.7}$$

For the vacuum case, the elements $m_{ee}$, $m_{\mu\mu}$, and $m_{e\mu}$ are related to the vacuum mixing angle $\theta$ and masses of the propagating mass eigenstates $M_1$ and $M_2$ as follows:

$$\begin{aligned} m_{ee} &= \cos^2\theta M_1 + \sin^2\theta M_2; \\ m_{\mu\mu} &= \sin^2\theta M_1 + \cos^2\theta M_2; \\ m_{e\mu} &= -(M_1 - M_2)\sin\theta\cos\theta. \end{aligned} \tag{2.8}$$

For later convenience and to establish contact with observable parameters we introduce the following quantities

$$\bar{M} = \frac{M_1 + M_2}{2}; \qquad \delta M^2 = M_1^2 - M_2^2, \tag{2.9}$$

The masses $M_{1,2}$ can be conveniently written in terms of these quantities as

$$M_a = \bar{M}\left[1 + (-1)^{a-1}\frac{\delta M^2}{4\bar{M}^2}\right]; \qquad a = 1, 2. \tag{2.10}$$

The current value for $\bar{M}$ obtained by WMAP [47] and the oscillation parameters from the combined fitting of the solar and KamLAND data are [48] respectively:

$$\begin{aligned} \bar{M} &\approx 0.25\ ({\rm eV}); \\ |\delta M^2_{12}| &\approx 7.9 \times 10^{-5}\ ({\rm eV})^2; \\ \tan^2\theta_{12} &\approx 0.40. \end{aligned} \tag{2.11}$$

For atmospheric neutrinos, analysis from SuperKamiokande, CHOOZ, and atmospheric neutrino data yield,

$$\begin{aligned} |\delta M^2_{13}| &\approx (1.3 - 3.0) \times 10^{-3}\ ({\rm eV})^2; \\ \sin^2\theta_{13} &< 0.067(3\sigma). \end{aligned} \tag{2.12}$$

This implies that $|\delta M^2|/\bar{M}^2 \ll 1$, an almost degenerate hierarchy of neutrino masses.

### A. Linear response

The medium is described by an *ensemble* of states, and the description is in terms of a density matrix. Therefore the question of space-time evolution is more subtle, while in the vacuum one can consider preparing an initial *single particle* state and evolving it in time, such a consideration is not available in a medium, and the question of time evolution *must* be formulated differently, namely, in terms of expectation values of the relevant operators in the density matrix.

In equilibrium the neutrino field *cannot* have an expectation value in the density matrix. The usual method in many body theory to study the nonequilibrium evolution of single quasiparticle states is the method of linear response: an external source is coupled to the field which develops an expectation value in the density matrix induced by the source. The expectation value of this field obeys the equation of motion with medium corrections. Upon switching off the external source, the expectation value evolves in time as a solution of the effective equations of motion in the medium. For a detailed description of this method in nonequilibrium quantum field theory see Refs. [26,45,46]. The external sources $\eta_a$ in the Lagrangian density (2.1) induce an expectation value of the neutrino field

$$\psi_a = \langle \nu_a \rangle = {\rm Tr}\hat{\rho}\nu_a, \tag{2.13}$$

where $\hat{\rho}$ is the full density matrix of the medium. In linear response this expectation value is linear in the external source and obeys the effective Dirac equation of motion in the medium [46]. It is most conveniently written in terms of the spatial Fourier transforms of the fields, sources and self-energies $\psi_a(\vec{k},t)$, $\eta_a(\vec{k},t)$, $\Sigma(\vec{k},t-t')$ respectively.[1] The one-loop self-energies with neutral and charged current contributions had been obtained in refs. [22,25,26], and the effective Dirac equation in the medium up to one loop has been obtained in the real-time formulation in Ref. [26]. It is given by

$$\Big[\Big(i\gamma^0\frac{\partial}{\partial t}-\vec{\gamma}\cdot\vec{k}\Big)\delta_{ab}-M_{ab}+\Sigma^{\rm tad}_{ab}L\Big]\psi_b(\vec{k},t)+\int_{-\infty}^{t}dt'\Sigma_{ab}(\vec{k},t-t')L\psi_b(\vec{k},t')=-\eta_a(\vec{k},t), \quad (2.14)$$

where $L$ is the projector on left-handed states, $\Sigma^{\rm tad}_{ab}L$ is the (local) tadpole contribution from the neutral currents and $\Sigma_{ab}(\vec{k},t-t')$ is the spatial Fourier transform of the (retarded) self-energy which includes both neutral and charged current interactions, and whose spectral representation is given by

$$\Sigma(\vec{k},t-t')=i\int_{-\infty}^{\infty}\frac{dk_0}{\pi}\,{\rm Im}\Sigma(\vec{k},k_0)e^{-ik_0(t-t')};\qquad \Sigma(\vec{k},k_0)=\Sigma_W(\vec{k},k_0)+\Sigma_Z(\vec{k},k_0), \quad (2.15)$$

where we have separated the charged and neutral current contributions, respectively.

The external source term $\eta$ allows to "prepare" a determined initial state, leading to the time evolution of $\psi$ as an initial value problem. This approach is implemented as follows. Consider switching on the source adiabatically from $t=-\infty$ up to $t=0$ and switching it off at $t=0$,

$$\eta_a(\vec{k},t)=\eta_a(\vec{k},0)e^{\epsilon t}\theta(-t);\qquad \epsilon\to0^+. \quad (2.16)$$

It is straightforward to confirm that for $t<0$ the solution of the Dirac Eq. (2.14) is given by

$$\psi_a(\vec{k},t<0)=\psi_a(\vec{k},0)e^{\epsilon t}. \quad (2.17)$$

Inserting this ansatz into the Eq. (2.14) yields a linear relation which determines the initial value $\psi_a(\vec{k},0)$ from $\eta_a(\vec{k},0)$, or equivalently, for a given initial value $\psi(\vec{k},0)$ allows to determine the adiabatic source that yields this initial value problem. The evolution for $t>0$ is determined by the following effective (retarded) Dirac equation,

$$\Big[\Big(i\gamma^0\frac{\partial}{\partial t}-\vec{\gamma}\cdot\vec{k}\Big)\delta_{ab}-M_{ab}+\Sigma^{\rm tad}_{ab}L\Big]\psi_b(\vec{k},t)+\int_0^t dt'\Sigma_{ab}(\vec{k},t-t')L\psi_b(\vec{k},t')=-\psi_a(\vec{k},0)\int_{-\infty}^{\infty}\frac{dk_0}{\pi}\frac{{\rm Im}\Sigma(\vec{k},k_0)}{k_0}e^{-ik_0t}. \quad (2.18)$$

[1]We have kept the same functions to avoid cluttering the notation, but the label $\vec{k}$ makes it clear that these are the spatial Fourier transforms.

This equation can be solved by Laplace transform. Introduce the Laplace transforms

$$\tilde{\psi}_b(\vec{k},s)=\int_0^\infty e^{-st}\psi_b(\vec{k},t);\qquad \tilde{\Sigma}(\vec{k},s)=\int_0^\infty e^{-st}\Sigma(\vec{k},t)=\int_{-\infty}^{\infty}\frac{dk_0}{\pi}\frac{{\rm Im}\Sigma(\vec{k},k_0)}{k_0-is}, \quad (2.19)$$

where we have used Eq. (2.15) to obtain the Laplace transform of the self-energy. At this stage it is convenient to establish contact with the more familiar description of the Dirac equation in the frequency representation (Fourier transform in time) by introducing the time Fourier transform of the retarded self-energy,

$$\Sigma(\vec{k},\omega)=\int\frac{dk_0}{\pi}\frac{{\rm Im}\Sigma(\vec{k},k_0)}{k_0-\omega-i\epsilon}, \quad (2.20)$$

related via analyticity to the Laplace transform, namely

$$\tilde{\Sigma}(\vec{k},s)=\Sigma(\vec{k},\omega=is-i\epsilon). \quad (2.21)$$

In terms of Laplace transforms the equation of motion becomes the following algebraic equation

$$D_{ab}(\vec{k},s)\tilde{\psi}_b(\vec{k},s)=i\Big(\gamma^0\delta_{ab}+\frac{1}{is}[\tilde{\Sigma}_{ab}(\vec{k},s)-\tilde{\Sigma}_{ab}(\vec{k},0)]L\Big)\times\psi_b(\vec{k},0), \quad (2.22)$$

where $D(\vec{k},s)\equiv D(\vec{k},\omega=is-i\epsilon)$ is the analytic continuation of the Dirac operator in frequency and momentum space

$$D_{ab}(\vec{k},\omega)=[(\gamma^0\omega-\vec{\gamma}\cdot\vec{k})\delta_{ab}-M_{ab}+\Sigma^{\rm tad}_{ab}L+\Sigma_{ab}(\vec{k},\omega)L]. \quad (2.23)$$

The full space-time evolution of an initial state is determined by

$$\psi_a(\vec{x},t)=\int d^3ke^{i\vec{k}\cdot\vec{x}}\int_\Gamma\frac{ds}{2\pi i}D^{-1}_{ab}(\vec{k},s)(i\gamma^0)\psi_b(\vec{k},0)e^{st}, \quad (2.24)$$

where $\Gamma$ is the Bromwich contour in the complex $s$ plane running parallel to the imaginary axis to the right of all the singularities of the function $\tilde{\psi}(\vec{k},s)$ and closing on a large semicircle to the left. We have simplified the expression for the Eq. (2.24) by discarding a perturbatively small correction to the amplitude of $\mathcal{O}(G_F)$, given by the self-energy corrections on the right hand side of Eq. (2.22). Therefore the space-time evolution is completely determined by the singularities of the function $\tilde{\psi}(\vec{k},s)$ in the complex $s$-plane. Up to one-loop order and for temperatures well below the mass of the vector bosons, the only singularities are simple poles along the imaginary axis, corresponding to the dispersion relations of the propagating modes in the medium. In this temperature range absorptive processes emerge at the two loop level, consequently these are of $\mathcal{O}(G_F^2)$ and are neglected in the one-loop analysis presented here. The integral along the Bromwich contour in the complex $s$-plane can now be written

$$\int_\Gamma \frac{ds}{2\pi i} D^{-1}_{ab}(\vec{k}, s)(i\gamma^0)\psi_b(\vec{k}, 0)e^{st} = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} D^{-1}_{ab}(\vec{k}, \omega)(i\gamma^0)\psi_b(\vec{k}, 0)e^{-i\omega t}, \tag{2.25}$$

where the frequency integral is performed along a line parallel to but slightly below the real $\omega$ axis closing counterclockwise in the upper half plane.

The one-loop contributions to the self-energy for $\omega$, $k$, $T \ll M_W$ were obtained in Ref. [22,25,26] and found to be of the form [26]

$$\Sigma^{\text{tad}}_{ab} + \Sigma_{ab}(\vec{k}, \omega) = \gamma^0 \mathbb{A}(\omega) - \vec{\gamma}\cdot\hat{\mathbf{k}}\mathbb{B}(k), \tag{2.26}$$

where $\mathbb{A}(\omega)$ and $\mathbb{B}(k)$ are $2\times 2$ diagonal matrices in the neutrino flavor basis.[2] To lowest order in $k/M_W$; $\omega/M_W$ these matrices are found to be [10,22,25,26]

$$\mathbb{A}(\omega) = \begin{pmatrix} A_e(\omega) & 0 \\ 0 & A_\mu(\omega) \end{pmatrix}; \qquad \mathbb{B}(k) = \begin{pmatrix} B_e(k) & 0 \\ 0 & B_\mu(k) \end{pmatrix}. \tag{2.27}$$

Imposing charge neutrality, the results of Ref. [26] (see also [10,22,24,25]) are summarized in the following regimes: high temperature and density, relevant during the early Universe or low temperature and high density, relevant for cold dense matter in supernovae, neutron stars or the sun

(i) $m_e \ll T \ll m_\mu$

In this regime we consider the following degrees of freedom $\nu$, $e$, and $p$, $n$ in nuclear statistical equilibrium. The matrix elements are given by

$$A_e(\omega) = \frac{G_F n_\gamma}{\sqrt{2}}\left[-\mathcal{L}_e + \frac{7\pi^4}{60\zeta(3)}\frac{\omega T}{M_W^2}(2 + \cos^2\theta_w)\right], \tag{2.28}$$

$$A_\mu(\omega) = \frac{G_F n_\gamma}{\sqrt{2}}\left[-\mathcal{L}_\mu + \frac{7\pi^4}{60\zeta(3)}\frac{\omega T}{M_W^2}\cos^2\theta_w\right], \tag{2.29}$$

$$B_e(k) = -\frac{G_F n_\gamma}{\sqrt{2}}\frac{7\pi^4}{180\zeta(3)}\frac{kT}{M_W^2}(2 + \cos^2\theta_w), \tag{2.30}$$

$$B_\mu(k) = -\frac{G_F n_\gamma}{\sqrt{2}}\frac{7\pi^4}{180\zeta(3)}\frac{kT}{M_W^2}\cos^2\theta_w, \tag{2.31}$$

where in terms of the asymmetries $L_f$ with $f = \nu, e, p, n$

$$\begin{aligned} -\mathcal{L}_e &= -\tfrac{1}{2}L_{\nu_e} + L_{\nu_\mu} - 3L_e - L_n; \\ -\mathcal{L}_\mu &= -\tfrac{1}{2}L_{\nu_\mu} + L_{\nu_e} - L_n. \end{aligned} \tag{2.32}$$

(ii) $m_e, m_\mu \ll T \ll M_W$

In this regime the relevant degrees of freedom are the lightest deconfined quarks $u$, $d$ and leptons

$$A_e(\omega) = \frac{G_F n_\gamma}{\sqrt{2}}\left[\widetilde{-\mathcal{L}_e} + \frac{7\pi^4}{60\zeta(3)}\frac{\omega T}{M_W^2}(2 + \cos^2\theta_w)\right], \tag{2.33}$$

$$A_\mu(\omega) = \frac{G_F n_\gamma}{\sqrt{2}}\left[\widetilde{-\mathcal{L}_\mu} + \frac{7\pi^4}{60\zeta(3)}\frac{\omega T}{M_W^2}(2 + \cos^2\theta_w)\right], \tag{2.34}$$

$$B_e(k) = -\frac{G_F n_\gamma}{\sqrt{2}}\frac{7\pi^4}{180\zeta(3)}\frac{kT}{M_W^2}\left(2 + \cos^2\theta_w - \frac{60}{7\pi^2}\left(\frac{m_e}{T}\right)^2\right), \tag{2.35}$$

$$B_\mu(k) = -\frac{G_F n_\gamma}{\sqrt{2}}\frac{7\pi^4}{180\zeta(3)}\frac{kT}{M_W^2}\left(2 + \cos^2\theta_w - \frac{60}{7\pi^2}\left(\frac{m_\mu}{T}\right)^2\right), \tag{2.36}$$

where

$$\begin{aligned} \widetilde{-\mathcal{L}_e} &= -\tfrac{1}{2}L_{\nu_e} + L_{\nu_\mu} - 3L_e \\ &\quad + (1 - 4\sin^2\theta_w)(2L_e - L_\mu) \\ &\quad - (1 - 8\sin^2\theta_w)L_u - 2L_d, \end{aligned} \tag{2.37}$$

$$\begin{aligned} \widetilde{-\mathcal{L}_\mu} &= -\tfrac{1}{2}L_{\nu_\mu} + L_{\nu_e} - 3L_\mu \\ &\quad + (1 - 4\sin^2\theta_w)(2L_e - L_\mu) \\ &\quad - (1 - 8\sin^2\theta_w)L_u - 2L_d. \end{aligned} \tag{2.38}$$

(iii) Cold dense matter with $e$, $\nu$, $\mu$, $p$, $n$:

$$A_e = -\frac{G_F\mathcal{N}_e}{\sqrt{2}}; \qquad A_\mu = -\frac{G_F\mathcal{N}_\mu}{\sqrt{2}}; \qquad B_{e,\mu} = 0; \tag{2.39}$$

$$-\mathcal{N}_e = -\tfrac{1}{2}\mathcal{N}_{\nu_e} + \mathcal{N}_{\nu_\mu} - 3\mathcal{N}_e - \mathcal{N}_n; \tag{2.40}$$

[2] The equivalence with the notation of Ref. [22] is (see Eq. (2) in Ref. [22]): $a_{NR} = \mathbb{B}(k)/k$; $b_{NR} = \mathbb{A}(\omega) - \omega\mathbb{B}(k)/k$.

$$-\mathcal{N}_\mu = -\tfrac{1}{2}\mathcal{N}_{\nu_\mu} + \mathcal{N}_{\nu_e} - \mathcal{N}_n. \qquad (2.41)$$

(iv) Cold dense matter with quarks and leptons:

$$A_e = -\frac{G_F}{\sqrt{2}}\tilde{\mathcal{N}}_e; \qquad A_\mu = -\frac{G_F}{\sqrt{2}}\tilde{\mathcal{N}}_\mu; \qquad B_{e,\mu} = 0, \qquad (2.42)$$

$$-\widetilde{\mathcal{N}}_e = -\tfrac{1}{2}\mathcal{N}_{\nu_e} + \mathcal{N}_{\nu_\mu} - 3\mathcal{N}_e + (1 - 4\sin^2\theta_w)(2\mathcal{N}_e - \mathcal{N}_\mu) - (1 - 8\sin^2\theta_w)\mathcal{N}_u - 2\mathcal{N}_d, \qquad (2.43)$$

$$-\widetilde{\mathcal{N}}_\mu = -\tfrac{1}{2}\mathcal{N}_{\nu_\mu} + \mathcal{N}_{\nu_e} - 3\mathcal{N}_\mu + (1 - 4\sin^2\theta_w)(2\mathcal{N}_e - \mathcal{N}_\mu) - (1 - 8\sin^2\theta_w)\mathcal{N}_u - 2\mathcal{N}_d, \qquad (2.44)$$

where $n_\gamma = 2\zeta(3)T^3/\pi^2$ is the photon density, and

$$L_f = \frac{\mathcal{N}_f}{n_\gamma}; \qquad \mathcal{N}_f = 2\int\frac{d^3p}{(2\pi)^3}[N_f(p) - N_{\bar{f}}(p)], \qquad (2.45)$$

$\mathcal{N}_f$ are the particle-antiparticle asymmetry *densities*.

### B. Dispersion relations and mixing angles in the medium

The simple poles of the integrand in (2.25) are the solutions of the homogeneous Dirac equation

$$[\gamma^0\omega\mathbb{1} - \vec{\gamma}\cdot\hat{\mathbf{k}}k\mathbb{1} - \mathbb{M} + (\gamma^0\mathbb{A}(\omega) - \vec{\gamma}\cdot\hat{\mathbf{k}}\mathbb{B}(k))L] \times \psi(\omega, k) = 0, \qquad (2.46)$$

where $\mathbb{1}$ is the $2\times 2$ identity matrix in the flavor basis in which the field $\psi(\omega, k)$ is given by

$$\psi(\omega, k) = \begin{pmatrix} \nu_e(\omega, k) \\ \nu_\mu(\omega, k) \end{pmatrix}, \qquad (2.47)$$

with $\nu_e(\omega, k)$ and $\nu_\mu(\omega, k)$ each being a 4-component Dirac spinor.

It turns out to be most convenient to work in the chiral basis in which the left-handed and right-handed components of the Dirac doublets are written as [26]

$$\psi_L = \sum_{h=\pm1}\begin{pmatrix} 0 \\ v^{(h)}\otimes\varphi^{(h)} \end{pmatrix}; \qquad \psi_R = \sum_{h=\pm1}\begin{pmatrix} v^{(h)}\otimes\xi^{(h)} \\ 0 \end{pmatrix}, \qquad (2.48)$$

where the two component Weyl spinors $v^{(h)}$ are the eigenstates of the helicity operator $\vec{\sigma}\cdot\hat{\mathbf{k}}$ with eigenvalues $h = \pm 1$.

To the leading order in $G_F$, the left-handed flavor doublet

$$\varphi^{(h)}(\omega, k) = \begin{pmatrix} \nu_e^{(h)}(\omega, k) \\ \nu_\mu^{(h)}(\omega, k) \end{pmatrix}, \qquad (2.49)$$

obeys the following effective Dirac equation [26]

$$[(\omega^2 - k^2)\mathbb{1} + (\omega - hk)(\mathbb{A} + h\mathbb{B}) - \mathbb{M}^2]\varphi^{(h)}(\omega, k) = 0, \qquad (2.50)$$

while the right-handed doublet is determined by the relation [26]

$$\xi^{(h)}(\omega, k) = -\frac{(\omega + hk)}{\omega^2 - k^2}\mathbb{M}\varphi^{(h)}(\omega, k). \qquad (2.51)$$

The propagating modes in the medium are found by diagonalization of the above effective Dirac equation. This can be done by performing a unitary transformation $\varphi^{(h)}(\omega, k) = U_m^{(h)}\chi^{(h)}(\omega, k)$ where

$$U_m^{(h)} = \begin{pmatrix} \cos\theta_m^{(h)} & \sin\theta_m^{(h)} \\ -\sin\theta_m^{(h)} & \cos\theta_m^{(h)} \end{pmatrix}; \qquad \chi^{(h)}(\omega, k) = \begin{pmatrix} \nu_1^{(h)}(\omega, k) \\ \nu_2^{(h)}(\omega, k) \end{pmatrix}, \qquad (2.52)$$

and a similar transformation for the right-handed doublet $\xi^{(h)}(\omega, k)$, with the medium mixing angle $\theta_m^{(h)}$ depending on $h$, $k$, and $\omega$. Upon diagonalization, the eigenvalue equation is given by [26]

$$\left\{\omega^2 - k^2 + \frac{1}{2}S_h(\omega, k) - \frac{1}{2}(M_1^2 + M_2^2) - \frac{1}{2}\delta M^2\Omega_h(\omega, k)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right\}\chi^{(h)}(\omega, k) = 0, \qquad (2.53)$$

where $S_h(\omega, k)$, $\Delta_h(\omega, k)$, and $\Omega_h(\omega, k)$ are, respectively, given by

$$S_h(\omega, k) = (\omega - hk)[A_e(\omega) + A_\mu(\omega) + hB_e(k) + hB_\mu(k)], \qquad (2.54)$$

$$\Delta_h(\omega, k) = (\omega - hk)[A_e(\omega) - A_\mu(\omega) + hB_e(k) - hB_\mu(k)], \qquad (2.55)$$

$$\Omega_h(\omega, k) = \left[\left(\cos 2\theta - \frac{\Delta_h(\omega, k)}{\delta M^2}\right)^2 + \sin^2 2\theta\right]^{1/2}. \qquad (2.56)$$

This requires the matrix elements in $U_m^{(h)}$ to be of the following form

$$\sin 2\theta_m^{(h)}(\omega, k) = \frac{\sin 2\theta}{\Omega_h(\omega, k)}; \qquad \cos 2\theta_m^{(h)}(\omega, k) = \frac{\cos 2\theta - \frac{\Delta_h(\omega,k)}{\delta M^2}}{\Omega_h(\omega, k)}. \tag{2.57}$$

A resonance is available whenever

$$\left(\cos 2\theta - \frac{\Delta_h(\omega, k)}{\delta M^2}\right) \approx 0 \tag{2.58}$$

in which case $\sin 2\theta_m^{(h)}(\omega, k) \approx 1$; $\cos 2\theta_m^{(h)}(\omega, k) \approx 0$.

The solutions of Eq. (2.53) yield the dispersion relations $\omega(k)$ of the "exact" quasiparticle states in the medium and correspond to the "exact poles" of the Dirac propagator. The dispersion relations $\omega_a^{(h)}(k, \lambda)$ for the propagating modes in the medium are found in perturbation theory consistently up to $\mathcal{O}(G_F)$ by writing [26]

$$\omega_a^{(h)}(k, \lambda) = \lambda[E_a(k) + \delta\omega_a^{(h)}(k, \lambda)], \quad a = 1, 2; \quad \lambda = \pm, \tag{2.59}$$

where $E_a(k) = \sqrt{k^2 + M_a^2}$, and $\delta\omega_a^{(h)}(k, \lambda)$ are found to be

$$\delta\omega_a^{(h)}(k, \lambda) = -\frac{1}{4E_a(k)}\{S_h(\lambda E_a(k), k) + (-1)^a \delta M^2(\Omega_h(\lambda E_a(k), k) - 1)\}. \tag{2.60}$$

For relativistic neutrinos with $k \gg M_a$ the dispersion relations $\omega_a(k)$; $a = 1, 2$ for the different cases are given to leading order in $G_F$ by

(i) *Positive energy, negative helicity neutrinos*, $\lambda = +1$, $h = -1$:

$$\omega_a(k) = k + \frac{M_a^2}{2k} - \frac{1}{4k}[S_-(k, k) + (-1)^a \delta M^2(\Omega_-(k, k) - 1)]. \tag{2.61}$$

(ii) *Positive energy, positive helicity neutrinos*, $\lambda = +1$, $h = +1$:

$$\omega_a(k) = k + \frac{M_a^2}{2k} - \frac{1}{4k}[S_+(k, k) + (-1)^a \delta M^2(\Omega_+(k, k) - 1)]; \qquad \omega - hk \approx \frac{\bar{M}^2}{2k}, \tag{2.62}$$

where we have neglected corrections of order $\delta M^2/\bar{M}^2$.

(iii) *Negative energy, negative helicity neutrinos*, $\lambda = -1$, $h = -1$:

$$\omega_a(k) = -k - \frac{M_a^2}{2k} + \frac{1}{4k}[S_-(-k, k) + (-1)^a \delta M^2(\Omega_-(-k, k) - 1)]; \qquad \omega - hk \approx \frac{\bar{M}^2}{2k}, \tag{2.63}$$

where we have again neglected corrections of order $\delta M^2/\bar{M}^2$.

(iv) *Negative energy, positive helicity neutrinos*, $\lambda = -1$, $h = +1$:

$$\omega_a(k) = -k - \frac{M_a^2}{2k} + \frac{1}{4k}[S_+(-k, k) + (-1)^a \delta M^2(\Omega_+(-k, k) - 1)]. \tag{2.64}$$

In the above expressions the $\Omega_\pm$ are given by Eq. (2.56).

The vacuum and medium oscillation time scales are, respectively, defined as

$$T_{\text{vac}} = \frac{2\pi}{E_1 - E_2}; \qquad T_{\text{med}} = \frac{2\pi}{\omega_1^{(h)}(k, \lambda) - \omega_2^{(h)}(k, \lambda)}, \tag{2.65}$$

In the relativistic case when $k \gg M_a$, we find

$$T_{\text{vac}} \approx \frac{4\pi k}{\delta M^2}; \qquad T_{\text{med}} \approx \frac{4\pi k}{\delta M^2 \Omega_h(\lambda k, k)}, \tag{2.66}$$

leading to the relation

$$\frac{T_{\text{med}}}{T_{\text{vac}}} = \frac{\sin 2\theta_m^{(h)}(\omega, k)}{\sin 2\theta}. \tag{2.67}$$

## III. SPACE-TIME PROPAGATION OF A NEUTRINO WAVE PACKET

We now have all the ingredients necessary to study the space-time evolution of a initial wave packet by carrying out the integrals in Eq. (2.24). For this purpose it is convenient to write $\psi(\vec{k}, 0) = \psi_R(\vec{k}, 0) + \psi_L(\vec{k}, 0)$ and expand the right and left-handed components in the helicity basis as in Eq. (2.48), namely

$$\psi_L(\vec{k}, 0) = \sum_{h=\pm 1} \begin{pmatrix} 0 \\ v^{(h)} \otimes \varphi^{(h)}(\vec{k}, 0) \end{pmatrix}; \qquad \psi_R(\vec{k}, 0) = \sum_{h=\pm 1} \begin{pmatrix} v^{(h)} \otimes \xi^{(h)}(\vec{k}, 0) \\ 0 \end{pmatrix}, \tag{3.1}$$

where

$$\varphi^{(h)}(\vec{k}, 0) = \begin{pmatrix} \nu_{eL}^{(h)}(\vec{k}, 0) \\ \nu_{\mu L}^{(h)}(\vec{k}, 0) \end{pmatrix}; \qquad \xi^{(h)}(\vec{k}, 0) = \begin{pmatrix} \nu_{eR}^{(h)}(\vec{k}, 0) \\ \nu_{\mu R}^{(h)}(\vec{k}, 0) \end{pmatrix}. \tag{3.2}$$

The general initial value problem requires to furnish initial conditions for the four components above.

However, an inhomogeneous neutrino state is produced by a weak interaction vertex, which produces left-handed neutrinos, suggesting to set $\nu^{(h)}_{eR}(\vec{k},0)=0$; $\nu^{(h)}_{\mu R}(\vec{k},0)=0$. Without loss of generality let us consider an initial state describing an inhomogeneous wave packet of *electron neutrinos* of arbitrary helicity, thus $\nu^{(h)}_{eL}(\vec{k},0)\neq 0$; $\nu^{(h)}_{\mu L}(\vec{k},0)=0$.

In the cases of interest neutrinos are relativistic with typical momenta $k\gg\bar{M}$. Following the real-time analysis described in detail in Ref. [26] in the relativistic case we find

$$\varphi^{(h)}(\vec{k},t)=\frac{1}{2}\nu^{(h)}_{eL}(\vec{k},0)\bigg[\begin{pmatrix}1+\mathcal{C}^{(h)}_{-h}\\ -\mathcal{S}^{(h)}_{-h}\end{pmatrix}e^{-i\omega^{(h)}_1(k,-h)t}+\begin{pmatrix}1-\mathcal{C}^{(h)}_{-h}\\ \mathcal{S}^{(h)}_{-h}\end{pmatrix}e^{-i\omega^{(h)}_2(k,-h)t}+\mathcal{O}\Big(\frac{\bar{M}^2}{k^2}\Big)\bigg], \tag{3.3}$$

$$\begin{aligned}\xi^{(h)}(\vec{k},t)=\frac{1}{2}\nu^{(h)}_{eL}(\vec{k},0)\Big(\frac{h\bar{M}}{2k}\Big)\bigg\{&\begin{pmatrix}1+\mathcal{C}^{(h)}_{-h}\\ -\mathcal{S}^{(h)}_{-h}\end{pmatrix}e^{-i\omega^{(h)}_1(k,-h)t}+\begin{pmatrix}1-\mathcal{C}^{(h)}_{-h}\\ \mathcal{S}^{(h)}_{-h}\end{pmatrix}e^{-i\omega^{(h)}_2(k,-h)t}\\ &-\begin{pmatrix}1+\cos2\theta\\ -\sin2\theta\end{pmatrix}e^{-i\omega^{(h)}_1(k,h)t}-\begin{pmatrix}1-\cos2\theta\\ \sin2\theta\end{pmatrix}e^{-i\omega^{(h)}_2(k,h)t}+\mathcal{O}\Big(\frac{\bar{M}}{k}\Big)\bigg\},\end{aligned} \tag{3.4}$$

where $\varphi^{(h)}(\vec{k},t)$ and $\xi^{(h)}(\vec{k},t)$ are the flavor doublets corresponding to the left-handed and right-handed neutrinos with helicity $h$ respectively. The upper component corresponds to the electron neutrino $\nu^{(h)}_e(\vec{k},t)$ while the lower component corresponds to the muon neutrinos $\nu^{(h)}_\mu(\vec{k},t)$. The factors $\mathcal{C}^{(h)}_\lambda(k)$ and $\mathcal{S}^{(h)}_\lambda(k)$ are defined as

$$\mathcal{C}^{(h)}_\lambda(k)=\cos[2\theta^{(h)}_m(\lambda k)];\qquad \mathcal{S}^{(h)}_\lambda(k)=\sin[2\theta^{(h)}_m(\lambda k)]. \tag{3.5}$$

The suppression factor $M/k$ in the right-handed component (3.4) is of course a consequence of the chirality flip transition from a mass term in the relativistic limit. For relativistic neutrinos and more specifically for neutrinos in the medium prior to BBN with $k\sim T\sim$ few MeV the right-handed component is negligible as expected.

The one-loop computation of the self-energy performed above does not include absorptive processes such as collisions of neutrinos with leptons (or hadrons) in the medium. Such absorptive part will emerge in a two loops calculation and is of $\mathcal{O}(G_F^2)$. While we have not calculated these contributions it is clear from the analysis what it should be expected: the frequencies $\omega_{1,2}(k)$ are the exact dispersion relations of the single particle poles of the Dirac propagator in the medium. At two loops the self-energy will feature an imaginary part with support on the mass shell of these single particle states. The imaginary part of the self-energy evaluated at these single particle energies yield the *width* of the single quasiparticle states $\Gamma_1(k)$; $\Gamma_2(k)$ and the oscillatory exponentials in the expressions above are replaced as follows

$$e^{-i\omega_a(k)t}\to e^{-\Gamma_a(k)t}e^{-i\omega_a(k)t};\qquad a=1,2. \tag{3.6}$$

While our one-loop calculation does not include the damping rates $\Gamma_a$ we will invoke results available in the literature [10,22,24,25] to estimate the collisional relaxation time scales (see Sec. IV).

The corresponding fields for the left-handed and right-handed component neutrinos in configuration space are obtained by performing the spatial Fourier transform

$$\varphi^{(h)}(\vec{r},t)=\int\frac{d^3\vec{k}}{(2\pi)^3}\varphi^{(h)}(\vec{k},t)e^{i\vec{k}\cdot\vec{r}}, \tag{3.7}$$

$$\xi^{(h)}(\vec{r},t)=\int\frac{d^3\vec{k}}{(2\pi)^3}\xi^{(h)}(\vec{k},t)e^{i\vec{k}\cdot\vec{r}}. \tag{3.8}$$

For an arbitrary initial configuration these integrals must be done numerically, but analytic progress can be made by assuming an initial Gaussian profile, describing a wave packet in momentum space centered at a given momentum, $\vec{k}_0$ with a width $\sigma$. While the width could generally depend on helicity we will consider the simpler case in which it is the same for both helicities. Namely, we consider

$$\nu^{(h)}_{eL}(\vec{k},0)=\nu^{(h)}_{eL}(0)\Big(\frac{\pi}{\sigma^2}\Big)^{3/2}\exp\Big[-\frac{(\vec{k}-\vec{k}_0)^2}{4\sigma^2}\Big], \tag{3.9}$$

where $\nu^{(h)}_{eL}(0)$ is an arbitrary amplitude and assume that wave packet is narrow in the sense that $\sigma\ll k_0$. In the limit $\sigma\to 0$ the above wave packet becomes $\nu^{(h)}_{eL}(0)\delta^3(\vec{k}-\vec{k}_0)$. In the opposite limit of large $\sigma$ the wave packet describes an inhomogeneous distribution spatially localized within a distance $\approx 1/\sigma$. For a narrow wave packet the momentum integral can be carried out by expanding the integrand in a series expansion around $k_0$ keeping up to quadratic terms.

### A. Integrals

The typical integrals are of the form

$$I(\vec{r},t)=\Big(\frac{\pi}{\sigma^2}\Big)^{3/2}\int\frac{d^3k}{(2\pi)^3}\mathcal{A}(k)\exp\Big[-\frac{(\vec{k}-\vec{k}_0)^2}{4\sigma^2}+i\vec{k}\cdot\vec{r}-i\omega(k)t\Big], \tag{3.10}$$

where $\mathcal{A}$ stands for the factors $(1\pm\mathcal{C})$; $\mathcal{S}$ in Eqs. (3.3) and (3.4), and $\omega(k)$ are the general dispersion relations obtained above. The computation of these integrals is simplified by noticing that for any function $F(k)$ the expansion around $\vec{k}_0$ up to quadratic order is given by

$$F(k) = F(k_0) + F'(k_0)\hat{\mathbf{k}}_0 \cdot (\mathbf{k} - \mathbf{k}_0) + \frac{1}{2}\Big(F''(k_0)P^{\parallel}_{ij}(\hat{\mathbf{k}}_0) + \frac{F'(k_0)}{k_0}P^{\perp}_{ij}(\hat{\mathbf{k}}_0)\Big)(\mathbf{k} - \mathbf{k}_0)_i(\mathbf{k} - \mathbf{k}_0)_j + \cdots, \tag{3.11}$$

where

$$P^{\parallel}_{ij}(\hat{\mathbf{k}}) = \hat{\mathbf{k}}_i\hat{\mathbf{k}}_j; \qquad P^{\perp}_{ij}(\hat{\mathbf{k}}) = \delta_{ij} - \hat{\mathbf{k}}_i\hat{\mathbf{k}}_j. \tag{3.12}$$

The result of the integration can be written more compactly by introducing the following quantities

$$\sigma^2_{\parallel}(t) = \sigma^2\frac{(1 - i\frac{t}{\tau_{\parallel}})}{[1 + \frac{t^2}{\tau^2_{\parallel}}]} \equiv \Phi_{\parallel}(t)\Big(1 - i\frac{t}{\tau_{\parallel}}\Big),$$
$$\sigma^2_{\perp}(t) = \sigma^2\frac{(1 - i\frac{t}{\tau_{\perp}})}{[1 + \frac{t^2}{\tau^2_{\perp}}]} \equiv \Phi_{\perp}(t)\Big(1 - i\frac{t}{\tau_{\perp}}\Big), \tag{3.13}$$

where we have introduced the perpendicular and parallel dispersion time scales given, respectively, by

$$\tau_{\perp} = \frac{k_0}{2\sigma^2 v_g}; \qquad \tau_{\parallel} = \frac{1}{2\sigma^2\omega''(k_0)} = \gamma^2\tau_{\perp}. \tag{3.14}$$

It will be seen in detail below that these two time scales are indeed associated with the spreading of the wave packet in the transverse and longitudinal directions.

The group velocity $v_g$ and effective Lorentz factor[3] $\gamma$ are given by

$$\vec{v}_g = \omega'(k_0)\hat{\mathbf{k}}_0; \qquad \gamma^2 = \frac{v_g}{k_0\omega''(k_0)}. \tag{3.15}$$

The transverse and longitudinal coordinates are

$$\vec{X}_{\parallel}(t) = \hat{\mathbf{k}}_0(\vec{r} \cdot \hat{\mathbf{k}}_0 - v_g t); \qquad \vec{X}_{\perp} = \vec{r} - \hat{\mathbf{k}}_0(\vec{r} \cdot \hat{\mathbf{k}}_0), \tag{3.16}$$

and in terms of these variables we finally find

$$I(\vec{r}, t) = \Big[\frac{\sigma_{\parallel}(t)\sigma^2_{\perp}(t)}{\sigma^3}\Big]\mathcal{A}(k_0; \vec{r}, t)e^{i(\vec{k}_0\cdot\vec{r} - \Psi(\vec{r},t)t)} \times e^{-(\Phi_{\perp}(t)\vec{X}^2_{\perp} + \Phi_{\parallel}(t)\vec{X}^2_{\parallel}(t))}, \tag{3.17}$$

where the phase

$$\Psi(\vec{r}, t) = \omega(k_0) + \frac{\Phi_{\perp}(t)}{\tau_{\perp}}X^2_{\perp} + \frac{\Phi_{\parallel}(t)}{\tau_{\parallel}}X^2_{\parallel}(t), \tag{3.18}$$

and

$$\mathcal{A}(k_0; \vec{r}, t) = \mathcal{A}(k_0) + 2i\mathcal{A}'(k_0)\sigma^2_{\parallel}(t)\hat{\mathbf{k}}_0 \cdot \vec{X}_{\parallel}(t) + \mathcal{A}''(k_0)\sigma^2_{\perp}(t)(1 - 2\sigma^2_{\perp}(t)\vec{X}^2_{\perp}) + \frac{\mathcal{A}'(k_0)}{k_0}\sigma^2_{\parallel}(t)(1 - 2\sigma^2_{\parallel}(t)\vec{X}^2_{\parallel}(t)). \tag{3.19}$$

Neglecting the prefactor $\mathcal{A}(k_0; \vec{r}, t)$ we see that

$$|I(\vec{r}, t)|^2 \propto \Big[\Big(1 + \frac{t^2}{\tau^2_{\parallel}}\Big)\Big(1 + \frac{t^2}{\tau^2_{\perp}}\Big)^2\Big]^{-(1/2)} e^{-2(\Phi_{\perp}(t)\vec{X}^2_{\perp} + \Phi_{\parallel}(t)\vec{X}^2_{\parallel}(t))} \tag{3.20}$$

describes a wave packet moving in the direction parallel to the momentum $\vec{k}_0$ with the group velocity $v_g$ and dispersing both along the perpendicular and parallel directions. The expressions for $\Phi_{\perp}(t)$ and $\Phi_{\parallel}(t)$ given by Eq. (3.13) clearly show that the dispersion time scales along the parallel direction and transverse directions are given by $\tau_{\parallel}$, $\tau_{\perp}$ respectively and $\tau_{\parallel}$ displays the time dilation factor $\gamma$. The wave packet is localized in space within a distance of order $1/\sqrt{\Phi(t)} \propto 1/\sqrt{\sigma}$ in either direction. Small $\sigma$ localizes the wave packet in momentum space while large $\sigma$ the wave packet is spatially localized. For large $\sigma$ the integrals must necessarily be performed numerically.

This discussion highlights that the derivative terms in the prefactor $\mathcal{A}(k_0; \vec{r}, t)$, which are a consequence of the *momentum dependence of the mixing angles* correspond to an expansion in the ratio $\sigma/k_0$. This can be understood from the following argument: $\mathcal{A} \sim (1 \pm \mathcal{C})$, $\mathcal{S}$, hence its derivatives with respect to momentum are of the form $f(k)\Delta'$ with $f(k)$ being smooth and bounded functions of $\mathcal{O}(1)$, while $\Delta$ is at most of the form $\Delta_0 k + \Delta_1 k^2$ in the relativistic limit, [see Eq. (2.55)] therefore $\Delta' \approx \Delta/k$. These derivatives multiply powers of $\sigma_{\perp,\parallel}X_{\perp,\parallel}$, and the exponential damping in $I$ restricts these contributions to the range $|\sigma_{\perp,\parallel}X_{\perp,\parallel}| \approx 1$. Therefore in the narrow packet approximation $\sigma \ll k_0$ the higher order derivative terms are suppressed by powers of $\sigma/k_0 \ll 1$. We have invoked this narrow packet approximation to carry out the momentum integral, therefore consistently with this approximation we will only keep the first derivative term, which is of $\mathcal{O}(\sigma/k_0)$ and neglect the higher order derivatives, which are of higher order in this ratio. Namely in the analysis that follows we approximate

$$|\mathcal{A}(k_0; \vec{r}, t)|^2 \approx |\mathcal{A}(k_0)|^2\Big[1 + 4\frac{\mathcal{A}'(k_0)}{\mathcal{A}(k_0)}\Phi_{\parallel}(t)\hat{\mathbf{k}}_0 \cdot \vec{X}_{\parallel}(t)\frac{t}{\tau_{\parallel}}\Big]. \tag{3.21}$$

In this manner we consistently keep the lowest order corrections arising from the *momentum dependence of the mixing angles in the medium.*

[3]For the usual dispersion relation $\omega(k) = \sqrt{k^2 + M^2}$ it is straightforward to confirm that $\gamma^2 = (1 - v_g^2)^{-1}$.

We now have all the ingredients for our analysis of the space-time evolution. The above general expressions for the time evolution of initially prepared wave packets, Eqs. (3.3) and (3.4) combined with the dispersion relations obtained in Sec. II B provide a solution to the most general case. We focus our discussion on the case of the early Universe, in which the typical neutrino energies are $\sim$MeV. With (active) neutrino masses in the range $M_a \sim$ eV and $\delta M^2 \sim 10^{-5} - 10^{-3}$ it is clear from the results above that the amplitude of the right-handed component is suppressed by a factor $\bar{M}/k \sim 10^{-6}$ and the medium corrections to the dispersion relations for positive energy neutrinos with positive helicity and negative energy neutrinos with negative helicity are suppressed by a factor $\bar{M}^2/k^2$ with respect to the opposite helicity assignement. Therefore in what follows we focus our discussion to the case of left-handed negative helicity neutrinos (and positive helicity antineutrinos).

### B. Space-time evolution and oscillations

We now focus on describing the evolution of negative helicity neutrinos or positive helicity antineutrinos.

The initial state considered above corresponds to a wave packet of electron neutrinos at $t = 0$ but no muon neutrinos. The lower component of the flavor spinor in Eq. (3.3) and (3.7) describes the wave packet of the muon neutrino at any arbitrary time. We begin by studying the transition probability from an initial electron neutrino wave packet of negative helicity to a muon neutrino wave packet.

Using the results obtained in the previous section for the integrals in the narrow packet approximation we find the transition probability

$$\begin{aligned}\mathcal{P}_{e\to\mu}(\vec{r},t) &= |\nu^{(h)}_{\mu L}(\vec{r},t)|^2\\ &= \frac{1}{4}|\nu^{(h)}_{eL}(\vec{k},0)|^2|\mathcal{S}(k_0)|^2\left[1 + 4\frac{\mathcal{S}'(k_0)}{\mathcal{S}(k_0)}\bar{\Phi}_{\parallel}(t)\hat{\mathbf{k}}_0\cdot\vec{X}_{\parallel}(t)\frac{t}{\bar{\tau}_{\parallel}}\right][|I_1(\vec{r},t)|^2 + |I_2(\vec{r},t)|^2\\ &\quad - 2|I_1(\vec{r},t)||I_2(\vec{r},t)|\cos[(\Psi_1(\vec{r},t) - \Psi_2(\vec{r},t))t]], \end{aligned} \tag{3.22}$$

where $\mathcal{S} = \sin[2\theta^{(h)}_m(\pm k)]$ and $I_{1,2}(\vec{r},t)$; $\Psi_{1,2}$ correspond to the integrals and phases given by Eq. (3.17) and (3.18) with the frequencies $\omega_{1,2}(k)$ for negative helicity given by Eqs. (2.61). In the expression above we have taken a common *prefactor* by neglecting the differences between the group velocities and the masses, taking $v_g = 1$, and $\bar{\Phi}_{\parallel}$, $\bar{\tau}_{\parallel}$ correspond to $\Phi_{\parallel}$, $\tau_{\parallel}$ with a mass $\bar{M}$. We focus our attention on the interference term which is the space-time manifestation of the oscillation phenomenon and features the oscillatory cosine function. The amplitude of the oscillation, $|I_1(\vec{r},t)I_2(\vec{r},t)|$ describes the product of two wavepackets of the form given by Eq. (3.20).

It is convenient to write the product $|I_1I_2|$ in the following form

$$|I_1(\vec{r},t)I_2(\vec{r},t)| \approx \left[\left(1+\frac{t^2}{\tau^2_{\parallel}}\right)\left(1+\frac{t^2}{\tau^2_{\perp}}\right)^2\right]^{-(1/2)} e^{-(\Phi_{\perp,1}(t)+\Phi_{\perp,2}(t))\vec{X}^2_{\perp}}e^{-\Phi_{\rm CM}(t)\vec{X}^2_{\rm CM}(t)}e^{-\Phi_{\rm R}(t)X^2_{\rm R}(t)}, \tag{3.23}$$

where we have introduced the center of mass (CM) and relative (R) variables

$$\begin{aligned}\vec{X}_{\rm CM} &= \hat{\mathbf{k}}_0(\vec{r}\cdot\hat{\mathbf{k}}_0 - v_{\rm CM}(t)t);\\ v_{\rm CM}(t) &= \frac{\Phi_{\parallel 1}(t)v_{g1} + \Phi_{\parallel 2}(t)v_{g2}}{\Phi_{\parallel 1}(t)+\Phi_{\parallel 2}(t)},\end{aligned} \tag{3.24}$$

$$\vec{X}_R = \vec{X}_{\parallel 1} - \vec{X}_{\parallel 2} = -(\vec{v}_{g1} - \vec{v}_{g2})t, \tag{3.25}$$

$$\Phi_{\rm CM} = \Phi_{\parallel 1} + \Phi_{\parallel 2}; \qquad \Phi_{\rm R} = \frac{\Phi_{\parallel 1}\Phi_{\parallel 2}}{\Phi_{\parallel 1}+\Phi_{\parallel 2}}. \tag{3.26}$$

The integral (3.23) describes the product of two gaussian wave packets spreading in the transverse and longitudinal directions and separating in the longitudinal direction because of the difference in group velocities, made explicit by the term $\Phi_R(t)X^2_R(t)$.

The first two terms in Eq. (3.22) describe the incoherent sum of the probabilities associated with separated wave packets of propagating mode eigenstates, in the third, interference term, the product $|I_1||I_2|$ is the overlap between these two wave packets that are slowly separating because of different group velocities. As discussed above a two loop calculation of the self-energies will lead to a quasiparticle *width* and a damping rate $\Gamma_a$ for the individual quasiparticle modes of frequency $\omega_a(k)$, the discussion leading up to Eq. (3.6) suggests that the integrals

$$|I_a(\vec{r},t)| \to e^{-\Gamma_a(k)t}|I_a(\vec{r},t)|. \tag{3.27}$$

#### *1. Coherence and "freeze-out"*

Since $\vec{X}_R = (\vec{v}_{g2} - \vec{v}_{g1})t$ does not depend on position, the overlap between the separating wave packets becomes vanishingly small for $t \gg t_{\rm coh}$ where the coherence time scale $t_{\rm coh}$ is defined by

$$\Phi_R(t_{\rm coh})(\vec{v}_{g2} - \vec{v}_{g1})^2 t^2_{\rm coh} = 1. \tag{3.28}$$

Before we engage in an analysis of the different cases, it is important to recognize that there are several dimensionless small ratios: (i) $\sigma/k_0 \ll 1$ describes narrow wave packets, this approximation was implemented in the calculation of the integrals, (ii) $\bar{M}/k \ll 1$ in the relativistic limit with $k \sim$ MeV for example in the early Universe near the epoch of BBN or for supernovae, (iii) $\delta M^2/\bar{M}^2 \ll 1$ describes a nearly degenerate hierarchy of neutrino masses. Since in the relativistic limit $v_{1g} - v_{2g} \sim \delta M^2/k^2$ we can neglect the difference in the masses in $\Phi_{\parallel}$ and write $\Phi_{\parallel 1} \sim \Phi_{\parallel 2} \sim \bar{\Phi}_{\parallel}$ where the masses are replaced by the mean mass $\bar{M}$ given by Eq. (2.9), and similarly for $\Phi_{\perp}$. Therefore to leading order in small quantities we can replace $\Phi_R$ above by $\bar{\Phi}_{\parallel}/2$ leading to

$$\Phi_{\rm CM}(t) = 2\bar{\Phi}_{\parallel}(t) = \frac{2\sigma^2}{1 + \frac{t^2}{\bar{\tau}_{\parallel}^2}}; \qquad v_{\rm CM} = \frac{1}{2}(v_{g1} + v_{g2}), \tag{3.29}$$

where $\bar{\tau}_{\parallel}$ is given by Eq. (3.14) for $\bar{M}$, and

$$\frac{1}{2}\bar{\Phi}_{\parallel}(t)X_R^2(t) = \frac{(\frac{t}{t_c})^2}{1 + (\frac{t}{\bar{\tau}_{\parallel}})^2}, \tag{3.30}$$

where we have introduced the time scale

$$t_c = \frac{\sqrt{2}}{\sigma|v_{g2} - v_{g1}|}. \tag{3.31}$$

The coherence time scale is the solution of the equation

$$\frac{(\frac{t_{\rm coh}}{t_c})^2}{1 + (\frac{t_{\rm coh}}{\bar{\tau}_{\parallel}^2})^2} = 1. \tag{3.32}$$

The expression (3.30) reveals a remarkable feature: for $t \gg \bar{\tau}_{\parallel}$ the overlap between the separating wave packets saturates to a *time independent value*

$$\frac{1}{2}\bar{\Phi}_{\parallel}(t)X_R^2(t) \rightarrow \left(\frac{\bar{\tau}_{\parallel}}{t_c}\right)^2. \tag{3.33}$$

This effect has been recognized in Ref. [37] and results from the longitudinal dispersion catching up with the separation of the wave packets. This phenomenon is relevant only in the case when $t_c > \bar{\tau}_{\parallel}$ in which case the overlap of the separating wave packets "freezes" and the packets maintain coherence for the remainder of their evolution. There are two distinct possibilities:

$$t_c \ll \bar{\tau}_{\parallel} : \textbf{(a)}, \tag{3.34}$$

$$t_c \gg \bar{\tau}_{\parallel} : \textbf{(b)}. \tag{3.35}$$

In case (a) we can approximate

$$\frac{1}{2}\bar{\Phi}_{\parallel}(t)X_R^2(t) \approx \left(\frac{t}{t_c}\right)^2, \tag{3.36}$$

since during the time interval in which the separating packets maintain coherence $t \ll t_c \ll \bar{\tau}_{\parallel}$ and in this case the relevant coherence time scale is $t_c$.

In case (b) the "freeze-out" of coherence results and the long time limit of the overlap between the wave packets in the longitudinal direction remains large and determined by Eq. (3.33).

However, while this "freezing of coherence" phenomenon in the longitudinal direction ensues in this regime, by the time when the coherence freezes $t \sim \bar{\tau}_{\parallel}$ the wave packet has spread dramatically in the *transverse* direction. This is because of the enormous Lorentz time dilation factor in the longitudinal direction which ensures that $t \sim \bar{\tau}_{\parallel} \gg \tau_{\perp}$ [see Eq. (3.14)]. The large spreading in the transverse direction entails a large suppression of the transition probability

$$\mathcal{P}_{e\rightarrow\mu}(\vec{r}, t \sim \bar{\tau}_{\parallel}) \propto \left(\frac{\tau_{\perp}}{\bar{\tau}_{\parallel}}\right)^2 \sim \frac{1}{\gamma^4} \sim \left(\frac{\bar{M}}{k_0}\right)^4. \tag{3.37}$$

For $\bar{M} \sim$ eV and $k_0 \sim$ MeV the above ratio is negligible. Therefore while the phenomenon of freezing of coherence is remarkable and fundamentally interesting, it may not lead to important consequences because the transition probability is strongly suppressed in this regime. Therefore in the time scale during which the transition probability is non-negligible, namely $t \ll \bar{\tau}_{\parallel}$ the overlap integral can be simplified to

$$e^{-(1/2)\bar{\Phi}_{\parallel}(t)X_R^2(t)} \approx e^{-(t/t_c)^2}. \tag{3.38}$$

### *2. Effective oscillation frequency*

Another aspect of the interference term is the *effective time dependent oscillation frequency* $\Psi_1(\vec{r}, t) - \Psi_2(\vec{r}, t)$ where the $\Psi_a$ are given by Eq. (3.18) for the frequencies $\omega_a(k)$ of the propagating modes. The spatio-temporal dependence of this effective phase is a consequence of the dispersion of the inhomogeneous configurations, encoded in the functions $\Phi$ and results in a *drift* of the oscillation frequency, a result that confirms a similar finding in the vacuum case in ref. [49]. Because of the exponential fall off of the amplitudes the maximum value of the *drift* contribution is achieved for $\Phi_{\perp,a}X_{\perp}^2 \sim 1$; $\Phi_{\parallel,a}X_{\parallel,a}^2(t) \sim 1$, namely, in front and back of the center of the wave packets, both in the transverse and the longitudinal directions. Furthermore, because of the Lorentz dilation factor, $\tau_{\parallel} \gg \tau_{\perp}$ for relativistic neutrinos. Therefore we can approximate the effective oscillation frequencies as

$$\Psi_1 - \Psi_2 \sim \omega_1(k) - \omega_2(k) + \frac{2\sigma^2}{k_0}(v_{g,1} - v_{g,2}). \tag{3.39}$$

The dispersion relations and mixing angles obtained above along with the results (3.3) and (3.4), yield the complete space-time evolution for wave packets with initial conditions corresponding to an electron neutrino.

Rather than studying the general case, we focus on three different situations which summarize the most general cases, (i) $\Delta_h/\delta M^2 \ll 1$ corresponding to the case of vacuum oscillations, (ii) $\Delta_h/\delta M^2 \sim \cos 2\theta$ corresponding to a resonance in the medium, and (iii) $\Delta_h/\delta M^2 \gg 1$ corresponding to the case a hot and or dense medium in which oscillations are suppressed.

### C. $\Delta_h/\delta M^2 \ll 1$: Vacuum oscillations

We study this case not only to compare to results available in the literature, but also to establish a "benchmark" to compare with the results with medium modifications. Beuthe [37] has studied the propagation of neutrino wave packets in the vacuum case including dispersion and in Ref. [49] an effective frequency similar to (3.18) has been found for wave packets propagating in the vacuum. In this case for positive energy, negative helicity neutrinos with $a = 1, 2$

$$\omega_a(k_0) \sim k_0 + \frac{M_a^2}{2k_0}; \qquad v_{g,a} \sim 1 - \frac{M_a^2}{2k_0^2}; \qquad \omega_a''(k_0) \sim \frac{M_a^2}{k_0^3} \tag{3.40}$$

leading to the vacuum time scales

$$t_{c,v} = \frac{2\sqrt{2}k_0^2}{\sigma|\delta M^2|}, \tag{3.41}$$

$$\bar{\tau}_{\parallel} = \frac{k_0^3}{2\sigma^2\bar{M}^2}. \tag{3.42}$$

In the case when

$$\left| \frac{k_0}{\sigma} \frac{\delta M^2}{4\bar{M}^2} \right| \gg 1 \tag{3.43}$$

the vacuum coherence time is given by

$$t_{c,v} = \bar{\tau}_{\parallel} \left| \frac{\sigma}{k_0} \frac{4\bar{M}^2}{\delta M^2} \right| = \frac{2k_0^2}{\sigma|\delta M^2|} \ll \bar{\tau}_{\parallel} \tag{3.44}$$

and the overlap between the separating wave packets vanishes well before the packets disperse appreciably along the longitudinal direction. On the other hand, in the case when

$$\left| \frac{k_0}{\sigma} \frac{\delta M^2}{4\bar{M}^2} \right| \ll 1 \tag{3.45}$$

the spreading of the wave packets catches up with the separation and the overlap between them freezes when $t \equiv t_{f,v} = \bar{\tau}_{\parallel} = k_0^3/2\sigma^2\bar{M}^2$. With $|\delta M^2|/4\bar{M}^2 \sim 10^{-4}$ for solar or $\sim 10^{-3}$ for atmospheric neutrinos the phenomenon of "freezing" of the overlap and the survival of coherence is available for

$$1 \ll \frac{k_0}{\sigma} \ll \frac{4\bar{M}^2}{|\delta M^2|}, \tag{3.46}$$

which is well within the "narrow wave packet" regime. However, as discussed above, when the coherence freezes the transition probability has been strongly suppressed by transverse dispersion. Therefore during the time scale during which the transition probability is non-negligible we can approximate the exponent in (3.23)

$$\frac{1}{2}\bar{\Phi}_{\parallel}(t)X_R^2(t) \sim \left(\frac{t}{t_{c,v}}\right)^2. \tag{3.47}$$

The effective oscillation frequency is given by Eq. (3.39) which becomes

$$\Psi_1 - \Psi_2 \sim \frac{\delta M^2}{2k}\left(1 - \frac{2\sigma^2}{k_0^2}\right), \tag{3.48}$$

while the corrections tend to diminish the oscillation frequency, these are rather small in the narrow packet approximation.

### D. Medium effects: Near resonance

In Refs. [10,22,24–26] it was established that if the lepton asymmetries are of the order of the baryon asymmetry $\eta \sim 10^{-9}$ there is the possibility of a resonance for the temperature range $m_e \ll T \ll m_\mu$ for positive energy negative helicity neutrinos with $\omega(k) \sim k + \bar{M}^2/2k$; $h = -1$ or positive energy positive helicity antineutrinos with $\omega(k) \sim -k - \bar{M}^2/2k$; $h = 1$ respectively. It is convenient to introduce the following notation

$$\mathcal{L}_9 = 10^9(\mathcal{L}_e - \mathcal{L}_\mu), \tag{3.49}$$

$$\delta_5 = 10^5\left(\frac{\delta M^2}{\mathrm{eV}^2}\right). \tag{3.50}$$

*If* the lepton and neutrino asymmetries are of the same order of the baryon asymmetry, then $0.2 \lesssim |\mathcal{L}_9| \lesssim 0.7$ and the fitting from solar and KamLAND data suggests $|\delta_5| \approx 8$. In this temperature regime we find [26] for positive energy, negative helicity neutrinos

$$\frac{\Delta_-(k,k)}{\delta M^2} \approx \frac{4}{\delta_5}\left(\frac{0.1T}{\mathrm{MeV}}\right)^4 \frac{k}{T}\left[-\mathcal{L}_9 + \left(\frac{2T}{\mathrm{MeV}}\right)^2 \frac{k}{T}\right] \tag{3.51}$$

and for positive energy positive helicity antineutrinos

$$\frac{\Delta_+(-k,k)}{\delta M^2} \approx \frac{4}{\delta_5}\left(\frac{0.1T}{\mathrm{MeV}}\right)^4 \frac{k}{T}\left[\mathcal{L}_9 + \left(\frac{2T}{\mathrm{MeV}}\right)^2 \frac{k}{T}\right]. \tag{3.52}$$

In the above expressions we have neglected terms of order $\frac{\bar{M}^2}{k^2}$. With $k \sim T$ and in the temperature regime just prior to BBN with $T \sim$ few MeV the lepton asymmetry contribution $\mathcal{L}$ is much smaller than the momentum dependent contribution and will be neglected in the analysis that follows, therefore we refer to $\Delta_h(\lambda k, k)$ and $S_h(\lambda k, k)$ as $\Delta(k)$ and $S(k)$ respectively since these are independent of $h$, $\lambda$ in this regime. In this temperature regime we find for both cases (negative helicity neutrinos and positive helicity antineutrinos) the following simple expressions

$$\Delta(k) \approx \frac{56\pi^2}{45\sqrt{2}} \frac{G_F k^2 T^4}{M_W^2}; \qquad S(k) \approx \Delta(k)(1 + \cos^2\theta_w). \tag{3.53}$$

A resonance is available when $\Delta(k_0) \sim \delta M^2 \cos 2\theta$, which may occur in this temperature regime for $k_0 \sim T \sim 3.6$ MeV [10,22,24–26] for large mixing angle ($\theta_{12}$) or $k \sim T \sim 7$ MeV for small mixing angle ($\theta_{13}$). Near the resonance the in-medium dispersion relations and group velocities are given by

$$\omega_a(k) \approx k + \frac{M_a^2}{2k} - \frac{\delta M^2}{4k}\{(1 + \cos^2\theta_w)\cos 2\theta + (-1)^{a-1}(1 - \sin 2\theta)\}, \tag{3.54}$$

$$v_{g,a} \approx 1 - \frac{M_a^2}{2k^2} + \frac{\delta M^2}{4k^2}\{(1 + \cos^2\theta_w)\cos 2\theta + (-1)^{a-1}(1 - \sin 2\theta)\}. \tag{3.55}$$

Again we focus our discussion on the interference terms in the transition probability (3.22), in particular, the medium modifications to the oscillation frequencies and coherence time scales. To assess these we note the following (primes stand for derivatives with respect to $k$):

$$\Omega(k)|_{\rm res} = \sin 2\theta; \qquad \Omega'(k)|_{\rm res} = 0; \qquad \Omega''(k)|_{\rm res} = \frac{4}{k^2}\frac{\cos^2\theta}{\sin 2\theta}, \tag{3.56}$$

which when combined with Eq. (2.61) yield

$$\omega_1(k) - \omega_2(k) = \frac{\delta M^2}{2k}\sin 2\theta; \qquad v_{g,1} - v_{g,2} \approx -\frac{\delta M^2}{2k^2}\sin 2\theta. \tag{3.57}$$

We also note that near the resonance

$$\sin{}' 2\theta_m(k) \propto \cos 2\theta_m \approx 0, \tag{3.58}$$

therefore the corrections arising from the energy dependence of the mixing angle in the transition probability (3.22) become *vanishingly small*. The transverse and longitudinal dispersion time scales are given by

$$\tau_{a\perp} = \frac{k_0}{2\sigma^2 v_{g,a}}; \qquad \tau_{a\|} \approx \bar{\tau}_{\|}\left[1 - (-1)^{a-1}\frac{\delta M^2}{2\bar{M}^2}\frac{1 + \cos^2 2\theta}{\sin 2\theta}\right]; \qquad \bar{\tau}_{\|} = \frac{k_0^3}{2\sigma^2 \bar{M}^2}. \tag{3.59}$$

Therefore in the medium near the resonance, the argument of the exponential that measures the overlap between the separating wave packets is given by

$$\frac{1}{2}\bar{\Phi}_{\|}(t)X_R^2(t) \sim \left(\frac{k_0}{\sigma}\frac{\delta M^2}{4\bar{M}^2}\sin 2\theta\right)^2 \frac{t^2}{\bar{\tau}_{\|}^2 + t^2} = \frac{(\frac{t}{t_{c,m}})^2}{1 + (\frac{t}{\bar{\tau}_{\|}})^2}, \tag{3.60}$$

where

$$t_{c,m} = \frac{2k_0^2}{\sigma|\delta M^2|\sin 2\theta} = \frac{t_{c,v}}{\sin 2\theta}. \tag{3.61}$$

The effective oscillation frequency (3.39) is given by

$$\Psi_1 - \Psi_2 \sim \frac{\delta M^2}{2k}\sin 2\theta\left(1 - \frac{2\sigma^2}{k_0^2}\right), \tag{3.62}$$

which when compared to the vacuum result (3.48) confirms the relation between the vacuum and in-medium oscillation time scales (2.67) since near the resonance $\sin 2\theta_m \sim 1$.

We conclude that the main effects from the medium near the resonance are an increase in the coherence and in the oscillation time scale $T = 2\pi/|\Psi_1 - \Psi_2|$ by a factor $1/\sin 2\theta$. For solar neutrino mixing with $\sin 2\theta_{12} \sim 0.9$ the increase in these time scales is at best a 10% effect, but it becomes much more pronounced in the case of atmospheric neutrino mixing since $\sin 2\theta_{13} \ll 1$.

### $\Delta_h/\delta M^2 \gg 1$ *oscillation suppression by the medium*

In the temperature or momentum regime for which $\Delta_h/\delta M^2 \gg 1$ the expression for the in-medium mixing angles (2.57) reveals that $\cos 2\theta_m \to -1$. In this case the in-medium mixing angle reaches $\theta_m \to \pi/2$ and the transition probability $\mathcal{P}_{e\to\mu}$ vanishes. Equation (3.3) shows that in this case an electron neutrino wave packet of negative helicity propagates as an eigenstate of the effective Dirac Hamiltonian in the medium with a dispersion relation

$$\omega_2(k) \sim vk + \frac{M_2^2}{2k}; \qquad v = \left[1 - \frac{14}{45\sqrt{2}}\frac{G_F T^4}{M_W^2}(1 + \mathrm{sign}(\delta M^2) + \cos^2\theta_w)\right], \tag{3.63}$$

where we have used Eq. (3.53) for the case when the momentum dependent contribution is much larger than the asymmetries. The in-medium correction to the group velocity being proportional to

$$\frac{G_F T^4}{M_W^2} \sim 10^{-21}\left(\frac{T}{\rm MeV}\right)^4 \tag{3.64}$$

is negligible in the temperature regime in which the calculation is reliable, namely, for $T \ll M_W$.

## IV. TIME SCALES IN THE RESONANCE REGIME

There are several important time scales that impact on the dynamics of wave packets in the medium as revealed by the discussions above, but also there are two more relevant time scales that are pertinent to a plasma in an expanding Universe: the Hubble time scale $t_H \sim 1/H$ which is the cooling time scale $T(t)/\dot{T}(t)$ and the collisional relaxation time scale $t_{\rm rel} = 1/\Gamma$ with $\Gamma$ the weak interaction collision rate. Neither $t_H$ nor $t_{\rm rel}$ has been input explicitly in the calculations above which assumed a medium in equilibrium and considered self-energy corrections only up to $\mathcal{O}(G_F)$. The damping factor that leads to the decoherence from neutral and charged current interactions has been studied in detail in Refs. [10,22,24,25] and we take this input from these references in order to compare this time scale for damping and decoherence to the time scales for the space-time evolution of the wave packets obtained above at the one-loop level.

In the temperature regime 1 MeV $\leq T \leq$ 100 MeV the Hubble time scale is [50]

$$t_H \sim 0.6\left(\frac{T}{\rm MeV}\right)^{-2} \text{ s} \tag{4.1}$$

and the collisional rate is estimated to be [10,22,24,25]

$$\Gamma \sim 0.25 G_F^2 T^5 \sim 0.25 \times 10^{-22}\left(\frac{T}{\rm MeV}\right)^5 \text{ MeV}$$
$$\Rightarrow t_{\rm rel} \sim 1.6\left(\frac{T}{\rm MeV}\right)^{-5} \text{ s}. \tag{4.2}$$

In order to determine the relevant time scales an estimate of the momentum spread of the initial wave packet $\sigma$ is needed. For example, for neutrinos in the LSND experiment, the momentum spread of the stopped muon is estimated to be about 0.01 MeV [35]. An estimate of the momentum spread in the medium can be the inverse of the mean free path of the charged lepton associated with the neutrino [1]. This mean free path is determined by the electromagnetic interaction, in particular, large angle scattering, which can be simply estimated from one-photon exchange to be $\lambda_{mf} \sim (\alpha_{\rm em}^2 T)^{-1}$. This estimate yields

$$\sigma \sim \alpha_{\rm em}^2 T \sim 10^{-4}\left(\frac{T}{\rm MeV}\right) \text{ (MeV)}. \tag{4.3}$$

For neutrinos in the neutrinosphere of a core-collapse supernovae, the estimate for $\sigma$ is also $\sim 10^{-2}$ MeV [1]. We will take a value $\sigma \sim 10^{-3}$ MeV in the middle of this range as representative to obtain order of magnitude estimates for the time scales, but it is straightforward to modify the estimates if alternative values of $\sigma$ can be reliably established.

We now consider the large mixing angle (LMA) case to provide an estimate of the different time scales, but a similar analysis holds for the case of small vacuum mixing (SMA) by an appropriate change of $k_0$; $T$. Taking $k_0 \sim T \sim 3.6$ MeV, $\sigma \sim 10^{-3}$ MeV, $|\delta M^2| \sim 8 \times 10^{-5}$ (eV)$^2$, $\bar{M} \sim 0.25$ eV we obtain the following time scales near the resonance region:

(i) *Oscillation time scales:*

$$T_{\rm vac} = \frac{4\pi k_0}{|\delta M^2|} \sim 3.8 \times 10^{-4} \text{ s}; \qquad T_{\rm med} = \frac{T_{\rm vac}}{\sin 2\theta}; \tag{4.4}$$

(ii) *Dispersion and coherence time scales:*

$$\tau_\perp \sim \frac{k_0}{2\sigma^2} \sim 1.2 \times 10^{-15} \text{ s}; \qquad \bar{\tau}_\parallel \sim \frac{k_0^3}{2\sigma^2 \bar{M}^2} \sim 0.25 \text{ s}; \tag{4.5}$$

$$t_{c,v} = \frac{2k_0^2}{\sigma|\delta M^2|} \sim 0.21 \text{ s}; \qquad t_{c,m} = \frac{t_{c,v}}{\sin 2\theta}; \tag{4.6}$$

(iii) *Expansion and collisional relaxation time scales:*

$$t_H \sim 4.6 \times 10^{-2} \text{ s}; \qquad t_{\rm rel} \sim 2.8 \times 10^{-3} \text{ s}; \tag{4.7}$$

For small vacuum mixing angle ($\theta_{13}$) the above results are modified by taking $k_0 \sim T \sim 7$ MeV.

In the resonance region the in-medium coherence time scale is of the same order as the Hubble time (for LMA) or much longer (for SMA) and there is a large temperature variation during the coherence time scale. However, the decoherence of the wave packets occurs on much shorter time scales determined by the collisional relaxation scale and the coherence time scale is not the relevant one in the medium near the resonance.

Decreasing the momentum spread of the initial wave packet $\sigma$ *increases* the dispersion and coherence time scales, with the dispersion scales increasing faster. The medium effects are manifest in an increase in the oscillation and the coherence time scales by a factor $1/\sin 2\theta$. This effect is more pronounced for 1–3 mixing because of a much smaller mixing angle. It is clear from the comparison between the coherence time scale in the medium $t_{c,m}$ and the relaxational (collisional) time scale $t_{\rm rel}$ that unless $\sigma$ is substantially *larger* than the estimate above, by at least 1 order of magnitude in the case of 1–2 mixing, or even more for 1–3 mixing, collisions via neutral and charge currents is the main source of decoherence between the separating wave packets near the resonance. However, increasing $\sigma$ will decrease the transverse dispersion time scale thus leading to greater suppression of the amplitude of the wave packets through dispersion. Furthermore, for large mixing angle $\sin 2\theta \sim 1$ the oscillation scale is *shorter* than the collisional decoherence time scale via the weak interactions $t_{\rm rel}$, therefore allowing several oscillations before the wave packets decohere, and because the oscillation scale is much smaller than the Hubble scale the evolution is adiabatic over the scale $t_{\rm rel}$. But for small mixing angle the opposite situation results and the transition probability is suppressed by collisional decoherence, furthermore for small enough mixing angle there is a breakdown of adiabaticity. However, the strongest suppression of the survival $\mathcal{P}_{e\to e}$ as well as the transition $\mathcal{P}_{e\to\mu}$ probabilities (equally) is the *transverse* dispersion of the wave packets, on a time scale many orders of magnitude *shorter* than the decoherence $t_{c,m}$ and the collisional $t_{\rm rel}$ time scales. Unless $\sigma^2$ is within the same order of magnitude of $|\delta M^2|$ the transverse dispersion occurs on time scales much faster than any of the other relevant time scales and the amplitude of the wave packets is suppressed well before any oscillations or decoherence by any other process can occur. Clearly a better understanding of the initial momentum spread is necessary for a full assessment of the oscillation probability in the medium.

## V. DISCUSSIONS AND CONCLUSIONS

In this article we implemented a nonequilibrium quantum field theory method that allows to study the space-time propagation of neutrino wave packets directly from the effective Dirac equation in the medium. The space-time evolution is studied as an initial value problem with the full density matrix via linear response. The method systematically allows to obtain the space-time evolution of left and right-handed neutrino wave packets.

A "flavor neutrino" wave packet evolves in time as a linear superposition of wave packets of exact (quasi) particle states in the medium, described by the poles of the Dirac propagator *in the medium*. These states propagate in the medium with different group velocities and the slow separation between these packets causes their overlap to diminish leading to a loss of spatial and temporal coherence. However, the time evolution of the packets also features *dispersion* as a result of the momentum spread of the wave packets [37].

The space-time dynamics feature a rich hierarchy of time scales that depend on the initial momentum spread of the wave packet: the transverse and longitudinal dispersion time scales $\tau_\perp \ll \tau_\parallel$ which are widely separated by the enormous Lorentz time dilation factor $\approx (k/\bar{M})^2$ with $\bar{M}$ the average neutrino mass, and a coherence time scale $t_{c,m}$ that determines when the overlap of the wave packets becomes negligible. The dynamics also displays the phenomenon of "freezing of coherence" which results from the competition between the separation and spreading of the wave packet along the direction of motion (longitudinal). For time scales larger than $\tau_\parallel$ the overlap of the wave packets freezes, with a large overlap in the case when $t_{c,m} \gg \tau_\parallel$, which occurs for a wide range of parameters.

We have focused on studying the space-time propagation in the temperature and energy regime in which there is a resonance in the mixing angle in the medium, prior to BBN [22,24–26]. Our main results are summarized as follows:

(i) Both the coherence and oscillation time scales are enhanced in the medium with respect to the vacuum case by a factor $1/\sin 2\theta$ near the resonance, where $\theta$ is the vacuum mixing angle.

(ii) There are small corrections to the oscillation formula from the wave-packet treatment, but these are suppressed by two powers of the ratio of the momentum spread of the initial packet to the main momentum.

(iii) There are also small corrections to the space-time evolution from the energy dependence of the mixing angle, but these are negligible near the resonance region.

(iv) The spreading of the wave packet leads to the phenomenon of "freezing of coherence" which results from the competition between the longitudinal dispersion and coherence time scales. This phenomenon is a result of the longitudinal spreading of the wave packets "catching up" with their separation. Substantial coherence remains frozen for $t_{c,m} \gg \tau_\parallel$.

(v) We have compared the wide range of time scales present in the early Universe when the resonance is available for $T \sim 3.6$ MeV [10,22,24–26] for large mixing angle. *Assuming* that the initial momentum spread of the wave packet is determined by the large angle scattering mean free path of charged leptons in the medium [1], we find the following hierarchy between the transverse dispersion $\tau_\perp$, oscillation $T_{\rm med}$, collisional relaxation $t_{\rm rel}$, Hubble $t_H$, in-medium coherence $t_{c,m}$ and longitudinal dispersion $\tau_\parallel$ time scales respectively: for large vacuum mixing angle $\sin 2\theta \sim 1$:

$$\tau_\perp \ll T_{\rm med} < t_{\rm rel} < t_H \ll t_{c,m} \lesssim \tau_\parallel \tag{5.1}$$

and for small mixing angle $\sin 2\theta \ll 1$

$$\tau_\perp \ll t_{\rm rel} \lesssim T_{\rm med} < t_H \ll t_{c,m} \lesssim \tau_\parallel. \tag{5.2}$$

The rapid transverse dispersion is responsible for the main suppression of both the persistent and transition probabilities making the amplitudes extremely small on scales much shorter than any of the other scales. Only a momentum spread $\sigma \sim \sqrt{|\delta M|^2}$ will make the transverse dispersion time scale comparable with the oscillation and relaxation ones. Clearly a better assessment of the momentum spread of wave packets in the medium is required to provide a more reliable estimate of the wave packet and oscillation dynamics.

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