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Accelerated detector-quantum field correlations: From vacuum fluctuations to radiation flux

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In this paper we analyze the interaction of a uniformly accelerated detector with a quantum field in (3 + 1)D spacetime, aiming at the issue of how kinematics can render vacuum fluctuations the appearance of thermal radiance in the detector (Unruh effect) and how they engender flux of radiation for observers afar. Two basic questions are addressed in this study: (a) How are vacuum fluctuations related to the emitted radiation? (b) Is there emitted radiation with energy flux in the Unruh effect? We adopt a method which places the detector and the field on an equal footing and derive the two-point correlation functions of the detector and of the field separately with full account of their interplay. From the exact solutions, we are able to study the complete process from the initial transient to the final steady state, keeping track of all activities they engage in and the physical effects manifested. We derive a quantum radiation formula for a Minkowski observer. We find that there does exist a positive radiated power of quantum nature emitted by the detector, with a hint of certain features of the Unruh effect. We further verify that the total energy of the dressed detector and a part of the radiated energy from the detector is conserved. However, this part of the radiation ceases in steady state. So the hint of the Unruh effect in radiated power is actually not directly from the energy flux that the detector experiences in Unruh effect. Since all the relevant quantum and statistical information about the detector (atom) and the field can be obtained from the results presented here, they are expected to be useful, when appropriately generalized, for addressing issues of quantum information processing in atomic and optical systems, such as quantum decoherence, entanglement, and teleportation.

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I. INTRODUCTION

Inasmuch as studies of the interaction between a particle and a quantum field are basic to particle physics and field theory, understanding the interaction between an atom and a quantum field is essential to current atomic and optical physics research [1–5]. The interaction of an accelerated charge or detector (an object with some internal degrees of freedom such as an atom or harmonic oscillator) in a quantum field is a simple yet fundamental problem with many implications in quantum field theory [6,7], thermodynamics [8,9], and applications in radiation theory and atomic-optical physics.

It is common knowledge that accelerating charges give rise to radiation [10]. But it is not entirely straightforward to derive the radiation formula from quantum field theory. How are vacuum fluctuations related to the emitted radiation? When an atom or detector moves at constant acceleration, according to Unruh [11], it would experience a thermal bath at temperature $T_U = \hbar a/(2\pi c k_B)$, where a is the proper acceleration. Is there emitted radiation with an energy flux in the Unruh effect?

The Unruh effect, and the related effect for moving mirrors studied by Davies and Fulling [12], were intended originally to mimic Hawking radiation from black holes. Because of this connection, for some time now there has been a speculation that there is real radiation emitted from a uniformly accelerated detector (UAD) under steady-state conditions (i.e., for atoms which have been uniformly accelerated for a time sufficiently long that transient effects have died out), not unlike that of an accelerating charge [10,13]. In light of pending experiments both for electrons in accelerators [14–16] and for accelerated atoms in optical cavities [17] this speculation has acquired some realistic significance. There is a need for more detailed analysis for both the uniform acceleration of charges or detectors and for transient motions because the latter can produce radiation and as explained below, sudden changes in the dynamics can also produce emitted radiation with thermal characteristics.

After Unruh and Wald's [18] earlier explication of what a Minkowski observer sees, Grove [19] questioned whether an accelerated atom actually emits radiated energy. Raine, Sciama, and Grove [20] (RSG) analyzed what an inertial observer placed in the forward light cone of the accelerating detector would measure and concluded that the oscillator does not radiate. Unruh [21], in an independent calculation, basically concurred with the findings of RSG but he also showed the existence of extra terms in the two-point function of the field which would contribute to the excitation of a detector placed in the forward light cone. Massar, Parantani, and Brout [22] (MPB) pointed out that the missing terms in RSG contribute to a "polarization cloud" around the accelerating detector. For a review of earlier work on accelerated detectors, see e.g., [23]. For work after that, see, e.g., Hinterleitner [24], Audretsch, Müller, and Holzmann [25], and Massar and Parantani

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[26]. Our present work follows the vein of Raval, Hu, Anglin (RHA,) and Koks [27–30] on the minimal coupling model and uses some results of Lin [31] on the Unruh-DeWitt model [11,32].

With regard to the question ''Is there a radiation flux emitted from an Unruh detector?'' the findings of RSG, Unruh, MPB, RHA, and others show that, at least in (1 + 1)-dimension model calculations, *there is no emitted radiation from a linear uniformly accelerated oscillator under equilibrium conditions*, even though, as found before, there exists a polarization cloud around it. Hu and Johnson [33] emphasized the difference between an equilibrium condition (steady-state uniform acceleration) where there is no emitted radiation, and nonequilibrium conditions where there could be radiation emitted. Nonequilibrium conditions arise for nonuniformly accelerated atoms (for an example of finite time acceleration, see Raval, Hu, and Koks (RHK) [30]), or during the initial transient time for an atom approaching uniform acceleration, when its internal states have not yet reached equilibrium through interaction with the field. Hu and Raval (HR) [27,28] presented a more complete analysis of the two-point function, calculated for two points lying in arbitrary regions of Minkowski space. This generalizes the results of MPB in that there is no restriction for the two points to lie to the left of the accelerated oscillator trajectory. They show where the extra terms in the two-point function are which were ignored in the RSG analysis. More important to answering the theme question, they show that at least in (1 + 1) dimension the stress-energy tensor vanishes everywhere except on the horizon. This means that there is no net flux of radiation emitted from the uniformly accelerated oscillator in steady state in (1 + 1)D case.

Most prior theoretical work on this topic was done in (1 + 1)-dimensional spacetimes. However since most experimental proposals on the detection of the Unruh effect are designed for the physical four-dimensional spacetime, it is necessary to do a detailed analysis for (3 + 1) dimensions. Although tempting, one cannot assume that all (3 + 1) results are equivalent to those from (1 + 1) calculations. First, there are new divergences in the (3 + 1) case to deal with. Second, the structure of the retarded field in (3 + 1)-dimensional spacetime is much richer: it consists of a bound field (generalized Coulomb field) and a radiation field with a variety of multipole structures, while the (1 + 1) case has only the radiation field in a different form. Third, an earlier work of one of us [31] showed that there is some constant negative monopole radiation emitted from a detector initially in the ground state and uniformly accelerated in (3 + 1)D Minkowski space, and claimed that this signal could be an evidence of the Unruh effect. This contradicts the results reported by HR [27] and others from the (1 + 1)D calculations. We need to clarify this discrepancy and determine the cause of it, by studying the complete process from transient to steady state. In particular, since radiation only exists under nonequilibrium conditions in the (1 + 1) case, it is crucial to understand the transient effects in the (3 + 1) case to gauge our expectation of what could be, against what would be, observed in laboratories.

In conceptual terms, one is tempted to invoke stationarity and thermality conditions for the description of an UAD. This is indeed a simple and powerful way to understand its physics if the detector undergoes uniform acceleration and interacts with the field all throughout (e.g., because of the stationarity of the problem in the Rindler proper time it is guaranteed that the total boost energy operator is conserved). However, this argument is inapplicable for transient epochs during which the physics is quite different (see, e.g., the inertial to uniform acceleration motion treated in [30]). Likewise, one can invoke the thermality condition (i.e., the thermal radiance experienced by a UAD is equivalent to that of an inertial detector in a thermal bath) to obtain results based on simple reasonings. But then we note that the thermality condition does not uniquely arise from uniform acceleration conditions. For example if the motion is rapidly altered [34] the radiation produced can be approximately thermal. This thermality in emitted radiation (e.g., from sudden injection of atoms into a cavity) is similar to those encountered in cosmological particle creation [35], but has a different physical origin from the Unruh effect which is similar to particle creation from black holes (Hawking effect) [9] (see, e.g., [17,36]).

In terms of methodology, instead of using the more sophisticated influence functional method as in the earlier series of papers on accelerated detectors [29,30] and moving charges [37–39], our work here follows more closely the work of HR who used the Heisenberg operator method to calculate the two-point function and the stress-energy tensor of a massless quantum scalar field. In our analysis based on the (3 + 1)D Unruh-DeWitt detector theory we found the full and exact dynamics of the detector and the field in terms of their Heisenberg operator evolution, thus making available the complete quantum and statistical information for this detector-field system, enabling us to address the interplay of thermal radiance in the detector, vacuum polarization cloud around the detector, quantum fluctuations and radiation, and emitted flux of classical radiation.

The paper is organized as follows. In Sec. II we introduce the Unruh-DeWitt detector theory. Then in Sec. III we describe the quantum dynamics of the detector-field system in the Heisenberg picture, yielding the expectation values of the detector two-point function with respect to the Minkowski vacuum and a detector coherent state in Sec. IV. With these results we derive the two-point function of the quantum field and describe what constitutes the ''vacuum polarization'' around the detector in Sec. V. Then in Sec. VI we calculate the quantum expectation values of the stress-energy tensor induced by the uniformly accelerated detector. This allows us to explore the conservation law and derive the quantum radiation formula. A comparison with the results in Ref. [31] follows in Sec. VII. Finally, we summarize our findings in Sec. VIII.

II. UNRUH-DEWITT DETECTOR THEORY

The total action of the detector-field system is given by

$$S = S_Q + S_\Phi + S_I, \tag{1}$$

where Q is the internal degree of freedom of the detector, assumed to be a harmonic oscillator with mass m_0 and a (bare) natural frequency Ω_0:

$$S_Q = \int d\tau \frac{m_0}{2}[(\partial_\tau Q)^2 - \Omega_0^2 Q^2]. \tag{2}$$

Here τ is the detector's proper time. Henceforth we will use an overdot on Q to denote $dQ(\tau)/d\tau$. The scalar field Φ is assumed to be massless,

$$S_\Phi = -\int d^4x \tfrac{1}{2}\partial_\mu \Phi \partial^\mu \Phi. \tag{3}$$

The interaction action S_I for the Unruh-DeWitt (UD) detector theory has the form [11,32],

$$S_I = \lambda_0 \int d\tau \int d^4x Q(\tau)\Phi(x)\delta^4(x^\mu - z^\mu(\tau)), \tag{4}$$

where λ_0 is the coupling constant. This can be regarded as a simplified version of an atom.

Below we will consider the UD detector moving in a prescribed trajectory $z^\mu(\tau)$ in a four-dimensional Minkowski spacetime with metric $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ and line element $ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu$. By "prescribed" we mean the trajectory of the detector is not considered as a dynamical variable, thus we ignore the backreaction effect of the field on the trajectory. (See Ref. [37] for an example where the trajectory and the field are determined self-consistently by each other.) The detector is made (by the act of an external agent) to go along the world line

$$z^\mu(\tau) = (a^{-1}\sinh a\tau, a^{-1}\cosh a\tau, 0, 0), \tag{5}$$

parametrized by its proper time τ. This is the trajectory of a uniformly accelerated detector situated in Rindler wedge R (the portion $t - x^1 < 0$ and $t + x^1 > 0$ of Minkowski space; see Chapter 4 of Ref. [6]).

III. QUANTUM THEORY IN HEISENBERG PICTURE

The conjugate momenta $(P(\tau), \Pi(x))$ of dynamical variables $(Q(\tau), \Phi(x))$ are defined by

$$P(\tau) = \frac{\delta S}{\delta \dot{Q}(\tau)} = m_0 \dot{Q}(\tau), \tag{6}$$

$$\Pi(x) = \frac{\delta S}{\delta \partial_t \Phi(x)} = \partial_t \Phi(x). \tag{7}$$

By treating the above dynamical variables as operators and introducing the equal time commutation relations,

$$[\hat{Q}(\tau), \hat{P}(\tau)] = i\hbar, \tag{8}$$

$$[\hat{\Phi}(t, \mathbf{x}), \hat{\Pi}(t, \mathbf{x}')] = i\hbar\delta^3(\mathbf{x} - \mathbf{x}'), \tag{9}$$

one can write down the Heisenberg equations of motion for the operators and obtains

$$\partial_\tau^2 \hat{Q}(\tau) + \Omega_0^2 \hat{Q}(\tau) = \frac{\lambda_0}{m_0}\hat{\Phi}(\tau, \mathbf{z}(\tau)), \tag{10}$$

$$(\partial_t^2 - \nabla^2)\hat{\Phi}(x) = \lambda_0 \int_0^\infty d\tau \hat{Q}(\tau)\delta^4(x - z(\tau)), \tag{11}$$

which have the same form as the classical Euler-Lagrange equations.

Suppose the system is prepared before $\tau = 0$, and the coupling S_I is turned on precisely at the moment $\tau = 0$ when we allow all the dynamical variables to begin to interact and evolve under the influence of each other. (The consequences of this sudden switch-on and the assumption of a factorizable initial state for the combined system a quantum Brownian oscillator plus oscillator bath is described in some details in [40]). By virtue of the linear coupling (4), the time evolution of $\hat{\Phi}(\mathbf{x})$ is simply a linear transformation in the phase space spanned by the orthonormal basis $(\hat{\Phi}(\mathbf{x}), \hat{\Pi}(\mathbf{x}), \hat{Q}, \hat{P})$, that is, $\hat{\Phi}(x)$ can be expressed in the form

$$\hat{\Phi}(t, \mathbf{x}) = \int d^3x'[f^\Phi(t, \mathbf{x}, \mathbf{x}')\hat{\Phi}(0, \mathbf{x}') + f^\Pi(t, \mathbf{x}, \mathbf{x}')\hat{\Pi}(0, \mathbf{x}')] + f^Q(x)\hat{Q}(0) + f^P(x)\hat{P}(0). \tag{12}$$

Here $f^\Phi(x, \mathbf{x}')$, $f^\Pi(x, \mathbf{x}')$, $f^Q(x)$, and $f^P(x)$ are c-number functions of spacetime. Similarly, the operator $\hat{Q}(\tau)$ can be written as

$$\hat{Q}(\tau) = \int d^3x'[q^\Phi(\tau, \mathbf{x}')\hat{\Phi}(0, \mathbf{x}') + q^\Pi(\tau, \mathbf{x}')\hat{\Pi}(0, \mathbf{x}')] + q^Q(\tau)\hat{Q}(0) + q^P(\tau)\hat{P}(0), \tag{13}$$

with c-number functions $q^Q(\tau)$, $q^P(\tau)$, $q^\Phi(\tau, \mathbf{x}')$, and $q^\Pi(\tau, \mathbf{x}')$.

For the case with initial operators being the free field operators, namely, $\hat{\Phi}(0, \mathbf{x}) = \hat{\Phi}_0(\mathbf{x})$, $\hat{\Phi}(0, \mathbf{x}) = \hat{\Pi}_0(\mathbf{x})$, $\hat{Q}(0) = \hat{Q}_0$, and $\hat{P}(0) = \hat{P}_0$, one can go further by introducing the complex operators $\hat{b}_\mathbf{k}$ and $\hat{a}$:

$$\hat{\Phi}_0(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3}\sqrt{\frac{\hbar}{2\omega}}[e^{i\mathbf{k}\cdot\mathbf{x}}\hat{b}_\mathbf{k} + e^{-i\mathbf{k}\cdot\mathbf{x}}\hat{b}_\mathbf{k}^\dagger], \tag{14}$$

$$\hat{\Pi}_0(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3}\sqrt{\frac{\hbar}{2\omega}}(-i\omega)[e^{i\mathbf{k}\cdot x}\hat{b}_{\mathbf{k}} - e^{-i\mathbf{k}\cdot x}\hat{b}^\dagger_{\mathbf{k}}], \tag{15}$$

with $\omega \equiv |\mathbf{k}|$, and

$$\hat{Q}_0 = \sqrt{\frac{\hbar}{2\Omega_r m_0}}(\hat{a} + \hat{a}^\dagger), \qquad \hat{P}_0 = -i\sqrt{\frac{\hbar\Omega_r m_0}{2}}(\hat{a} - \hat{a}^\dagger). \tag{16}$$

Note that, instead of Ω_0, we use the renormalizd natural frequency Ω_r (to be defined in (40)) in the definition of $\hat{a}$. Then the commutation relations (8) and (9) give

$$[\hat{a}, \hat{a}^\dagger] = 1, \qquad [\hat{b}_{\mathbf{k}}, \hat{b}^\dagger_{\mathbf{k}'}] = (2\pi)^3\delta^3(\mathbf{k} - \mathbf{k}'), \tag{17}$$

and the expressions (12) and (13) can be rewritten as

$$\hat{\Phi}(t, \mathbf{x}) = \hat{\Phi}_b(x) + \hat{\Phi}_a(x), \tag{18}$$

$$\hat{Q}(\tau) = \hat{Q}_b(\tau) + \hat{Q}_a(\tau), \tag{19}$$

where

$$\hat{\Phi}_b(x) = \int \frac{d^3k}{(2\pi)^3}\sqrt{\frac{\hbar}{2\omega}}[f^{(+)}(t, \mathbf{x}; \mathbf{k})\hat{b}_{\mathbf{k}} + f^{(-)}(t, \mathbf{x}; \mathbf{k})\hat{b}^\dagger_{\mathbf{k}}], \tag{20}$$

$$\hat{\Phi}_a(x) = \sqrt{\frac{\hbar}{2\Omega_r m_0}}[f^a(t, \mathbf{x})\hat{a} + f^{a*}(t, \mathbf{x})\hat{a}^\dagger], \tag{21}$$

$$\hat{Q}_b(\tau) = \int \frac{d^3k}{(2\pi)^3}\sqrt{\frac{\hbar}{2\omega}}[q^{(+)}(\tau, \mathbf{k})\hat{b}_{\mathbf{k}} + q^{(-)}(\tau, \mathbf{k})\hat{b}^\dagger_{\mathbf{k}}], \tag{22}$$

$$\hat{Q}_a(\tau) = \sqrt{\frac{\hbar}{2\Omega_r m_0}}[q^a(\tau)\hat{a} + q^{a*}(\tau)\hat{a}^\dagger]. \tag{23}$$

The whole problem therefore can be transformed to solving c-number functions $f^s(x)$ and $q^s(\tau)$ from (10) and (11) with suitable initial conditions. Since $\hat{Q}$ and $\hat{\Phi}$ are Hermitian, one has $f^{(-)} = (f^{(+)})^*$ and $q^{(-)} = (q^{(+)})^*$. Hence it is sufficient to solve the c-number functions $f^{(+)}(t, \mathbf{x}; \mathbf{k})$, $q^{(+)}(\tau, \mathbf{k})$, $f^a(t, \mathbf{x})$, and $q^a(\tau)$. To place this in a more general setting, let us perform a Lorentz transformation shifting $\tau = 0$ to $\tau = \tau_0$, and define

$$\eta \equiv \tau - \tau_0. \tag{24}$$

This does not add any complication to our calculation. Now the coupling between the detector and the field would be turned on at $\tau = \tau_0$. We are looking for solutions with the initial conditions

$$\begin{aligned} f^{(+)}(t(\tau_0), \mathbf{x}; \mathbf{k}) &= e^{i\mathbf{k}\cdot\mathbf{x}}, \\ \partial_t f^{(+)}(t(\tau_0), \mathbf{x}; \mathbf{k}) &= -i\omega e^{i\mathbf{k}\cdot\mathbf{x}}, \\ q^{(+)}(\tau_0; \mathbf{k}) &= \dot{q}^{(+)}(\tau_0; \mathbf{k}) = 0, \end{aligned} \tag{25}$$

$$\begin{aligned} f^a(t(\tau_0), \mathbf{x}) &= \partial_t f^a(t(\tau_0), \mathbf{x}) = 0, \\ q^a(\tau_0) &= 1, \\ \dot{q}^a(\tau_0) &= -i\Omega_r. \end{aligned} \tag{26}$$

A. Solving for $f^{(+)}$ and $q^{(+)}$

The method to solve for f and q are analogous to what we did in classical field theory. First, we find an expression relating the harmonic oscillator and the field amplitude right at the detector. Then substituting this relation into the equation of motion for the oscillator, we obtain a complete equation of motion for q with full information of the field. Last, we solve this complete equation of motion for q, and from its solution determine the field f consistently.

Equation (11) implies that

$$(\partial_t^2 - \nabla^2)f^{(+)}(x; \mathbf{k}) = \lambda_0\int_{\tau_0}^{\infty} d\tau\delta^4(x - z(\tau))q^{(+)}(\tau; \mathbf{k}). \tag{27}$$

The general solution for $f^{(+)}$ reads

$$f^{(+)}(x; \mathbf{k}) = f_0^{(+)}(x; \mathbf{k}) + f_1^{(+)}(x; \mathbf{k}), \tag{28}$$

where

$$f_0^{(+)}(x; \mathbf{k}) \equiv e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}} \tag{29}$$

is the free field solution, and

$$f_1^{(+)}(x; \mathbf{k}) \equiv \lambda_0\int_{\tau_0}^{\infty} d\tau G_{\rm ret}(x; z(\tau))q^{(+)}(\tau; \mathbf{k}) \tag{30}$$

is the retarded solution, which looks like the retarded field in classical field theory. Here $\omega = |\mathbf{k}|$ and the retarded Green's function $G_{\rm ret}$ in Minkowski space is given by

$$G_{\rm ret}(x, x') = \frac{1}{4\pi}\delta(\sigma)\theta(t - t') \tag{31}$$

with $\sigma \equiv -(x_\mu - x'_\mu)(x^\mu - x'^\mu)/2$. Applying the explicit form of the retarded Green's function, one can go further to write

$$f_1^{(+)}(x; \mathbf{k}) = \frac{\lambda_0\theta(\eta_-)}{2\pi a X}q^{(+)}(\tau_-; \mathbf{k}), \tag{32}$$

where

$$X \equiv \sqrt{(-UV + \rho^2 + a^{-2})^2 + 4a^{-2}UV}, \tag{33}$$

$$\tau_- \equiv -\frac{1}{a}\ln\frac{a}{2|V|}(X - UV + \rho^2 + a^{-2}), \tag{34}$$

$$\eta_- \equiv \tau_- - \tau_0, \tag{35}$$

with $\rho \equiv \sqrt{x_2{}^2 + x_3{}^2}$, $U \equiv t - x^1$, and $V \equiv t + x^1$.

The formal retarded solution (32) is singular on the trajectory of the detector. To deal with the singularity, note that the UD detector here is a quantum mechanical object, and the detector number would always be one. This means that at the energy threshold of detector creations, there is a natural cutoff on frequency, which sets an upper bound on the resolution to be explored in our theory. Thus it is justified to assume here that the detector has a finite extent $O(\Lambda^{-1})$, which will introduce the backreaction on the detector.

Let us regularize the retarded Green's function by invoking the essence of effective field theory:

$$G^{\Lambda}_{\rm ret}(x, x') = \frac{1}{4\pi}\sqrt{\frac{8}{\pi}}\Lambda^2 e^{-2\Lambda^4\sigma^2}\theta(t - t'). \tag{36}$$

(For more details on this regularization scheme, see Refs. [37,39]). With this, right on the trajectory, the retarded solution for large Λ is

$$f_1^{(+)}(z(\tau); \mathbf{k}) = \frac{\lambda_0}{4\pi}[\Lambda\zeta q^{(+)}(\tau; \mathbf{k}) - \partial_\tau q^{(+)}(\tau; \mathbf{k}) + O(\Lambda^{-1})], \tag{37}$$

where $\zeta = 2^{7/4}\Gamma(5/4)/\sqrt{\pi}$. Substituting the above expansion into (10) and neglecting $O(\Lambda^{-1})$ terms, one obtains the equation of motion for $q^{(+)}$ with backreaction,

$$(\partial_\tau^2 + 2\gamma\partial_\tau + \Omega_r^2)q^{(+)}(\tau; \mathbf{k}) = \frac{\lambda_0}{m_0}f_0^{(+)}(z(\tau); \mathbf{k}). \tag{38}$$

Fortunately, there is no higher derivatives of q present in the above equation of motion. Now $q^{(+)}$ behaves like a damped harmonic oscillator driven by the vacuum fluctuations of the scalar field, with the damping constant

$$\gamma \equiv \frac{\lambda_0^2}{8\pi m_0}, \tag{39}$$

and the renormalized natural frequency

$$\Omega_r^2 \equiv \Omega_0^2 - \frac{\lambda_0^2\Lambda\zeta}{4\pi m_0}. \tag{40}$$

In (38), the solution for $q^{(+)}$ compatible with the initial conditions $q^{(+)}(\tau_0; \mathbf{k}) = \dot{q}^{(+)}(\tau_0; \mathbf{k}) = 0$ is

$$q^{(+)}(\tau; \mathbf{k}) = \frac{\lambda_0}{m_0}\sum_{j=+,-}\int_{\tau_0}^{\tau} d\tau' c_j e^{w_j(\tau-\tau')}f_0^{(+)}(z(\tau'); \mathbf{k}), \tag{41}$$

where $f_0^{(+)}$ has been given in (29), $c_\pm$ and $w_\pm$ are defined as

$$c_\pm = \pm\frac{1}{2i\Omega}, \qquad w_\pm = -\gamma \pm i\Omega, \tag{42}$$

with

$$\Omega \equiv \sqrt{\Omega_r^2 - \gamma^2}. \tag{43}$$

Throughout this paper we consider only the underdamped case with $\gamma^2 < \Omega_r^2$, so Ω is always real.

B. Solving for f^a and q^a

Similarly, from (10), (11), (18), and (19), the equations of motion for f^a and q^a read

$$(\partial_t^2 - \nabla^2)f^a(x) = \lambda_0 \int d\tau\delta^4(x - z(\tau))q^a(\tau), \tag{44}$$

$$(\partial_\tau^2 + \Omega_0^2)q^a(\tau) = \frac{\lambda_0}{m_0}f^a(z(\tau)). \tag{45}$$

The general solution for f^a, similar to (28), is

$$f^a(x) = f_0^a(x) + \lambda_0 \int_{\tau_0}^{\infty} d\tau G_{\rm ret}(x; z(\tau))q^a(\tau_-). \tag{46}$$

However, according to the initial condition (26), one has $f_0^a = 0$, hence

$$f^a(x) = \frac{\lambda_0\theta(\eta_-)}{2\pi a X}q^a(\tau_-). \tag{47}$$

Again, the value of f^a is singular right at the position of the detector. Performing the same regularization as those for $q^{(+)}$, (45) becomes (cf. (38))

$$(\partial_\tau^2 + 2\gamma\partial_\tau + \Omega_r^2)q^a(\tau) = 0, \tag{48}$$

which describes a damped harmonic oscillator free of driving force. The solution consistent with the initial condition $q^a(\tau_0) = 1$ and $\dot{q}^b(\tau_0) = -i\Omega_r$ reads

$$q^a(\tau) = \frac{1}{2}\theta(\eta)e^{-\gamma\eta}\left[\left(1 - \frac{\Omega_r + i\gamma}{\Omega}\right)e^{i\Omega\eta} + \left(1 + \frac{\Omega_r + i\gamma}{\Omega}\right)e^{-i\Omega\eta}\right]. \tag{49}$$

IV. TWO-POINT FUNCTIONS OF THE DETECTOR

As shown in the previous section, as $\hat{Q}$ evolves, some nonzero terms proportional to $\hat{\Phi}$ and $\hat{\Pi}$ will be generated. Suppose the detector is initially prepared in a state that can be factorized into the quantum state $|q\rangle$ for Q and the Minkowski vacuum $|0_M\rangle$ for the scalar field Φ, that is,

$$|\tau_0\rangle = |q\rangle|0_M\rangle \tag{50}$$

then the two-point function of Q will split into two parts,

$$\begin{aligned}\langle Q(\tau)Q(\tau')\rangle &= \langle 0_M|\langle q|[\hat{Q}_b(\tau)+\hat{Q}_a(\tau)] \\ &\quad \times [\hat{Q}_b(\tau)+\hat{Q}_a(\tau)]|q\rangle|0_M\rangle \\ &= \langle q|q\rangle\langle Q(\tau)Q(\tau')\rangle_{\rm v} + \langle Q(\tau)Q(\tau')\rangle_{\rm a}\langle 0_M|0_M\rangle. \end{aligned} \quad (51)$$

where, from (19),

$$\langle Q(\tau)Q(\tau')\rangle_{\rm v} = \langle 0_M|\hat{Q}_b(\tau)\hat{Q}_b(\tau')|0_M\rangle, \quad (52)$$

$$\langle Q(\tau)Q(\tau')\rangle_{\rm a} = \langle q|\hat{Q}_a(\tau)\hat{Q}_a(\tau)|q\rangle. \quad (53)$$

Similar splitting happens for every two-point function of $\hat{\Phi}(x)$ as well as for the stress-energy tensor.

Observe that $\langle Q(\tau)Q(\tau')\rangle_{\rm v}$ depends on the initial state of the field, or the Minkowski vacuum, while $\langle Q(\tau)Q(\tau')\rangle_{\rm a}$ depends on the initial state of the detector only. One can thus interpret $\langle Q(\tau)Q(\tau')\rangle_{\rm v}$ as accounting for the response to the vacuum fluctuations, while $\langle Q(\tau)Q(\tau')\rangle_{\rm a}$ corresponds to the intrinsic quantum fluctuations in the detector.

In the following, we will demonstrate the explicit forms of some two-point functions we have obtained and analyze their behavior. To distinguish the quantum or classical natures of these quantities, the initial quantum state $|q\rangle$ will be taken to be the coherent state [5],

$$|q\rangle = e^{-\alpha^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle, \quad (54)$$

where $|n\rangle$ is the nth excited state for the free detector, and $\alpha = q_0\sqrt{\Omega_r/2\hbar}$ with a constant q_0. The representation of $|q\rangle$ in Q-space reads

$$\psi(Q,\tau_0) = \left(\frac{\Omega_r}{\pi\hbar}\right)^{1/4} e^{-\Omega_r(Q-q_0)^2/2\hbar}, \quad (55)$$

which is a wave packet centered at q_0 with the spread identical to the one for the ground state.

A. Expectation value of the detector two-point function with respect to the Minkowski vacuum

Along the trajectory $z^\mu(\tau)$ in (5), performing a Fourier transformation with respect to τ on (29), one has

$$f_0^{(+)}(z(\tau);\mathbf{k}) \equiv \int d\kappa e^{-i\kappa\tau}\varphi(\kappa,\mathbf{k}), \quad (56)$$

where the frequency spectrum of the Minkowski mode from the viewpoint of the UAD,

$$\varphi(\kappa,\mathbf{k}) = \frac{e^{-\pi\kappa/2a}}{\pi a}\left(\frac{\omega-k_1}{\omega+k_1}\right)^{-(i\kappa/2a)} K_{-(i\kappa/a)}(\sqrt{k_2^2+k_3^2}/a), \quad (57)$$

is not trivial anymore. Given the result of the integration,

$$\int\frac{\hbar d^3k}{(2\pi)^3 2\omega}\varphi(\kappa,\mathbf{k})\varphi^*(\kappa',\mathbf{k}) = \frac{\hbar}{(2\pi)^2}\frac{\kappa}{1-e^{-2\pi\kappa/a}} \times\delta(\kappa-\kappa'), \quad (58)$$

a Planck factor with the Unruh temperature $a/2\pi$ emerges. Then from (41), (52), and (56), one has

$$\langle Q(\eta)Q(\eta')\rangle_{\rm v} = \hbar\int\frac{d^3k}{(2\pi)^3 2\omega}q^{(+)}(\tau;\mathbf{k})q^{(-)}(\tau;\mathbf{k}) \quad (59)$$

$$= \frac{\lambda_0^2\hbar}{(2\pi)^2 m_0^2}\sum_{j,j'}\int\frac{\kappa d\kappa}{1-e^{-2\pi\kappa/a}}\frac{c_j c_{j'}^* e^{-i\kappa(\tau_0-\tau_0')}}{(w_j+i\kappa)(w_{j'}^*-i\kappa)} \times[e^{w_j(\tau-\tau_0)}-e^{-i\kappa(\tau-\tau_0)}][e^{w_{j'}^*(\tau'-\tau_0')}-e^{i\kappa(\tau'-\tau_0')}], \quad (60)$$

where the integrand has poles at $\kappa = \pm\Omega - i\gamma$ and $\kappa = \pm i\,n\,a$, $n\in N$. Let $\tau_0' < \tau_0 < \tau < \tau'$ and taking the coincidence limit, one obtains

$$\begin{aligned}\langle Q(\eta)^2\rangle_{\rm v} &= \lim_{\eta'\to\eta}\frac{1}{2}\langle\{Q(\eta),Q(\eta')\}\rangle_{\rm v} \\ &= \frac{\hbar\lambda_0^2}{(2\pi m_0\Omega)^2}\theta(\eta)\,{\rm Re}\bigg\{(\Lambda_0-\ln a)e^{-2\gamma\eta}\sin^2\Omega\eta + \frac{a}{2}e^{-(\gamma+a)\eta}\bigg[\frac{F_{\gamma+i\Omega}(e^{-a\eta})}{\gamma+i\Omega+a}\left(-\frac{i\Omega}{\gamma}\right)e^{-i\Omega\eta} \\ &\quad + \frac{F_{-\gamma-i\Omega}(e^{-a\eta})}{\gamma+i\Omega-a}\left(\left(1+\frac{i\Omega}{\gamma}\right)e^{i\Omega\eta}-e^{-i\Omega\eta}\right)\bigg] - \frac{1}{4}\bigg[\left(\frac{i\Omega}{\gamma}+e^{-2\gamma\eta}\left(\frac{i\Omega}{\gamma}+1-e^{-2i\Omega\eta}\right)\right)(\psi_{\gamma+i\Omega}+\psi_{-\gamma-i\Omega}) \\ &\quad - \left(-\frac{i\Omega}{\gamma}+e^{-2\gamma\eta}\left(\frac{i\Omega}{\gamma}+1-e^{-2i\Omega\eta}\right)\right)i\pi\coth\frac{\pi}{a}(\Omega-i\gamma)\bigg]\bigg\}. \end{aligned} \quad (61)$$

Here $F_s(t)$ is defined by the hypergeometric function as

$$F_s(t) \equiv {}_2F_1\left(1+\frac{s}{a},1,2+\frac{s}{a};t\right), \quad (62)$$

and

$$\psi_s \equiv \psi\left(1+\frac{s}{a}\right) \quad (63)$$

is the poly-gamma function. The divergent Λ_0-term is produced by the coincidence limit: as $\eta'\to\eta$, $\Lambda_0 \to -\gamma_e - \ln|\tau_0'-\tau_0|$ with the Euler's constant γ_e. Since $|\tau_0'-\tau_0|$ characterizes the time scale that the interaction

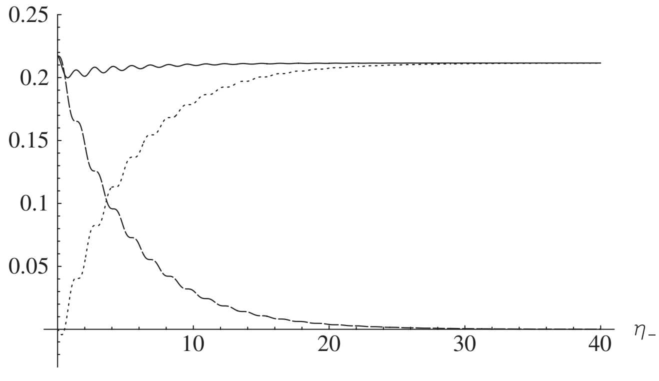

FIG. 1. Plot of $\langle Q(\eta)^2\rangle_{\rm v}$ (dotted line, Eq. (61) with Λ_0-term excluded), $\langle Q(\eta)^2\rangle_{\rm a}^{\rm qm}$ (dashed line, Eq. (75)), and the sum of these two ($\langle \Delta Q(\eta)^2\rangle$, solid line). Here we have taken $a = 1$, $\gamma = 0.1$, $\Omega = 2.3$, and $m_0 = 1$.

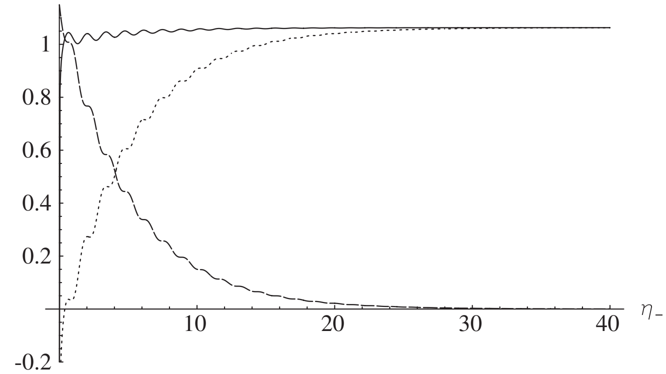

FIG. 2. Plots of $\langle \dot{Q}(\eta)^2\rangle_{\rm v}$ (dotted line, Eq. (68)), $\langle \dot{Q}(\eta)^2\rangle_{\rm a}^{\rm qm}$ (dashed line, Eq. (78)), and their sum ($\langle \Delta \dot{Q}(\eta)^2\rangle$, solid line).

is turned on, Λ_0 could be finite in real processes. In any case, for every finite value of Λ_0, the first line of the result in (61) vanishes as $\eta \to \infty$.

In Fig. 1, we show the $\langle Q(\eta)^2\rangle_{\rm v}$ without Λ_0-term in a dotted line. Roughly speaking the curve saturates exponentially in the detector's proper time. As $\eta \to \infty$, $\langle Q(\eta)^2\rangle_{\rm v}$ saturates to the value

$$\lim_{\eta\to\infty}\langle Q(\eta)^2\rangle_{\rm v} = \frac{\hbar}{2\pi m_0 \Omega}\,{\rm Re}\left[\frac{ia}{\gamma + i\Omega} - 2i\psi_{\gamma+i\Omega}\right]. \tag{64}$$

For $\gamma < a$, the time scale of the rise is about $1/2\gamma$, which can be read off from the $e^{-2\gamma\eta}$ in (61). From there one can also see that the small oscillation around the rising curve has a frequency of $O(\Omega)$.

For $\langle Q(\eta)\dot{Q}(\eta)\rangle_{\rm v}$, it will be clear that what is interesting for the calculation of the flux is the combined quantities like $\langle Q(\eta)\dot{Q}(\eta)\rangle_{\rm v} + \langle \dot{Q}(\eta)Q(\eta)\rangle_{\rm v}$. Notice that

$$\langle Q(\eta)\dot{Q}(\eta)\rangle_{\rm v} + \langle \dot{Q}(\eta)Q(\eta)\rangle_{\rm v} = \partial_\tau \langle Q(\eta)^2\rangle_{\rm v}. \tag{65}$$

With the result of $\langle Q(\eta)^2\rangle_{\rm v}$, this calculation is straightforward. Let us turn to the two-point functions of $\dot{Q}$. Similar to $\langle Q(\eta)^2\rangle_{\rm v}$, one has

$$\langle \dot{Q}(\eta)\dot{Q}(\eta')\rangle_{\rm v} = \int \frac{\hbar d^3k}{(2\pi)^3 2\omega}\dot{q}^{(+)}(\tau;\mathbf{k})\dot{q}^{(-)}(\tau;\mathbf{k}) \tag{66}$$

$$= \frac{\lambda_0^2\hbar}{(2\pi)^2 m_0^2}\sum_{j,j'}\int \frac{\kappa d\kappa}{1-e^{-2\pi\kappa/a}}\frac{c_j c_{j'}^* e^{-i\kappa(\tau_0-\tau_0')}}{(w_j + i\kappa)(w_{j'}^* - i\kappa)} \times [w_j e^{w_j(\tau-\tau_0)} + i\kappa e^{-i\kappa(\tau-\tau_0)}] \times [w_{j'}^* e^{w_{j'}^*(\tau'-\tau_0')} - i\kappa e^{i\kappa(\tau'-\tau_0')}] \tag{67}$$

from (41), (56), and (58). The coincidence limit of the above two-point function reads

$$\begin{aligned}\langle \dot{Q}(\eta)^2\rangle_{\rm v} = \frac{\hbar\lambda_0^2}{(2\pi m_0\Omega)^2}\theta(\eta)\,{\rm Re}\bigg\{&(\Lambda_1 - \ln a)\Omega^2 + (\Lambda_0 - \ln a)e^{-2\gamma\eta}(\Omega\cos\Omega\eta - \gamma\sin\Omega\eta)^2 \\ &+ \frac{a}{2}(\gamma + i\Omega)^2 e^{-(\gamma+a)\eta}\left[\frac{F_{\gamma+i\Omega}(e^{-a\eta})}{\gamma+i\Omega+a}\left(\frac{i\Omega}{\gamma}\right)e^{-i\Omega\eta} + \frac{F_{-\gamma-i\Omega}(e^{-a\eta})}{\gamma+i\Omega-a}\left(\left(1-\frac{i\Omega}{\gamma}\right)e^{i\Omega\eta} - e^{-i\Omega\eta}\right)\right] \\ &+ \frac{1}{4}(\gamma+i\Omega)^2\bigg[\left(\frac{i\Omega}{\gamma} + e^{-2\gamma\eta}\left(\frac{i\Omega}{\gamma} - 1 + e^{-2i\Omega\eta}\right)\right)(\psi_{\gamma+i\Omega} + \psi_{-\gamma-i\Omega}) \\ &- \left(-\frac{i\Omega}{\gamma} + e^{-2\gamma\eta}\left(\frac{i\Omega}{\gamma} - 1 + e^{-2i\Omega\eta}\right)\right)i\pi\coth\frac{\pi}{a}(\Omega - i\gamma)\bigg]\bigg\},\end{aligned} \tag{68}$$

where $\Lambda_1 = -\gamma_e - \lim_{\tau'\to\tau}\ln|\tau - \tau'|$ can be subtracted safely. This will be justified later.

The subtracted $\langle \dot{Q}(\eta)^2\rangle_{\rm v}$ is illustrated in Fig. 2 (in which Λ_0-term has also been excluded). One can immediately recognize that $\langle \dot{Q}(\eta)^2\rangle_{\rm v} \sim \ln\eta$ when η approaches zero; a new divergence occurs at $\eta = 0$. Mathematically, this logarithmic divergence comes about because the divergences in the hypergeometric functions in (68) do not cancel each other, unlike in $\langle Q(\eta)^2\rangle_{\rm v}$. Physically, this divergence at the initial time τ_0 could be another consequence of the sudden switch-on at $\tau = \tau_0$ or $\eta = 0$. We expect that these ill behaviors at the start could be tamed if we turn on the coupling adiabatically. (See [40] for a discussion on this issue.)

For large η, the behavior of the dotted curve in Fig. 2 is quite similar to the one in Fig. 1 for $\langle Q(\tau)^2\rangle_{\rm v}$. It saturates to

$$\lim_{\eta\to\infty}\langle\dot{Q}(\eta)^2\rangle_{\rm v} = \frac{\hbar}{2\pi m_0\Omega}\,{\rm Re}\bigg\{(\Omega - i\gamma)^2\bigg[\frac{ia}{\gamma+i\Omega} - 2i\psi_{\gamma+i\Omega}\bigg] - 4\gamma\Omega\ln a\bigg\}. \tag{69}$$

Comparing (61) and (68), their time scales of saturation ($1/2\gamma$ for $\gamma < a$) and the frequency of the small ripples on the rising curve ($O(\Omega)$) are also the same. Note that when $\gamma \ll 1$ and a is finite, (64) and (69) implies

$$\langle\dot{Q}(\infty)^2\rangle_{\rm v} \approx \Omega^2\langle Q(\infty)^2\rangle_{\rm v}, \tag{70}$$

which justifies the subtraction of Λ_1-term in (68).

B. Expectation values of the detector two-point functions with respect to a coherent state

We now derive the expectation values of the detector two-point functions with respect to the coherent state (54). Substituting (23) into (53) and using (49) and (54), one finds that

$$\langle Q(\tau)Q(\tau')\rangle_{\rm a} = \langle Q(\tau)Q(\tau')\rangle^{\rm qm}_{\rm a} + \langle Q(\tau)Q(\tau')\rangle^{\rm cl}_{\rm a}, \tag{71}$$

where

$$\langle Q(\tau)Q(\tau')\rangle^{\rm qm}_{\rm a} \equiv \frac{\hbar}{2\Omega_r m_0} q^a(\tau)q^{a*}(\tau'), \tag{72}$$

$$\langle Q(\tau)Q(\tau')\rangle^{\rm cl}_{\rm a} \equiv \frac{q_0^2}{m_0}\,{\rm Re}[q^a(\tau)]\,{\rm Re}[q^a(\tau')] = \bar{Q}(\tau)\bar{Q}(\tau'), \tag{73}$$

with the mean value

$$\bar{Q}(\tau) \equiv \langle Q(\tau)\rangle = \frac{q_0}{\sqrt{m_0}}\theta(\eta)e^{-\gamma\eta}\left(\cos\Omega\eta + \frac{\gamma}{\Omega}\sin\Omega\eta\right). \tag{74}$$

While the "qm" term is of purely quantum nature, the "cl" term is of classical nature: $\bar{Q}$ is real and $\langle Q(\tau)Q(\tau')\rangle^{\rm cl}_{\rm a}$ does not involve $\hbar$. Thus the cl term is identified as the semiclassical part of the two-point functions. The coincidence limits of the above two-point functions are

$$\langle Q(\eta)^2\rangle^{\rm qm}_{\rm a} = \frac{\hbar\theta(\eta)}{2\Omega^2\Omega_r m_0}e^{-2\gamma\eta}[\Omega_r^2 - \gamma^2\cos 2\Omega\eta + \gamma\Omega\sin 2\Omega\eta], \tag{75}$$

and $\langle Q(\eta)^2\rangle^{\rm cl}_{\rm a} = \bar{Q}(\eta)^2$.

Similarly, it is easy to find $\langle\dot{Q}(\tau)\dot{Q}(\tau')\rangle_{\rm a} = \langle\dot{Q}(\tau)\dot{Q}(\tau')\rangle^{\rm qm}_{\rm a} + \langle\dot{Q}(\tau)\dot{Q}(\tau')\rangle^{\rm cl}_{\rm a}$:

$$\langle\dot{Q}(\tau)\dot{Q}(\tau')\rangle^{\rm qm}_{\rm a} = \frac{\hbar}{2\Omega_r m_0}\dot{q}^b(\tau)\dot{q}^{b*}(\tau'), \tag{76}$$

$$\langle\dot{Q}(\tau)\dot{Q}(\tau')\rangle^{\rm cl}_{\rm a} = \dot{\bar{Q}}(\tau)\dot{\bar{Q}}(\tau'), \tag{77}$$

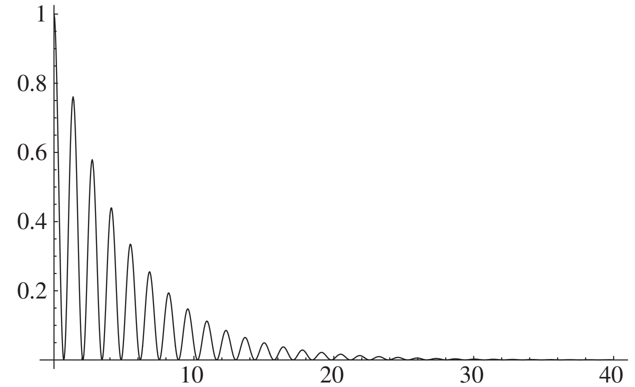

FIG. 3. The semiclassical part of the two-point function, $\langle Q^2\rangle^{\rm cl}_{\rm a} = \bar{Q}^2$ (see Eq. (73) and below). Its behavior is quite different from the quantum part shown in the previous figures. Here we take $q_0 = 1$, with other parameters unchanged.

and their coincidence limit,

$$\langle\dot{Q}(\eta)^2\rangle^{\rm qm}_{\rm a} = \frac{\hbar\Omega_r}{2\Omega^2 m_0}\theta(\eta)e^{-2\gamma\eta}[\Omega_r^2 - \gamma^2\cos 2\Omega\eta - \gamma\Omega\sin 2\Omega\eta], \tag{78}$$

and $\langle\dot{Q}(\eta)^2\rangle^{\rm cl}_{\rm a} = \dot{\bar{Q}}(\eta)^2$. Also one has $\langle Q(\tau)\dot{Q}(\tau)\rangle_{\rm a} + \langle\dot{Q}(\tau)Q(\tau)\rangle_{\rm a} = \partial_\tau\langle Q(\tau)^2\rangle_{\rm a}$.

Note that the above two-point functions with respect to the coherent state are independent of the proper acceleration a. $\langle Q(\eta)^2\rangle^{\rm qm}_{\rm a}$ and the variance (squared uncertainty) of Q,

$$\langle\Delta Q(\eta)^2\rangle \equiv \langle[Q(\eta) - \bar{Q}(\eta)]^2\rangle = \langle Q(\eta)^2\rangle_{\rm v} + \langle Q(\eta)^2\rangle^{\rm qm}_{\rm a} \tag{79}$$

have been shown in Fig. 1. $\langle Q(\eta)^2\rangle^{\rm qm}_{\rm a}$ decays exponentially due to the dissipation of the zero-point energy to the field. As $\langle Q(\eta)^2\rangle^{\rm qm}_{\rm a}$ decays, $\langle Q(\eta)^2\rangle_{\rm v}$ grows and compensates the decrease, then saturates asymptotically. Similar behavior can be found in Fig. 2, in which

$$\langle\Delta\dot{Q}(\eta)^2\rangle \equiv \langle[\dot{Q}(\eta) - \dot{\bar{Q}}(\eta)]^2\rangle = \langle\dot{Q}(\eta)^2\rangle_{\rm v} + \langle\dot{Q}(\eta)^2\rangle^{\rm qm}_{\rm a} \tag{80}$$

is illustrated. In Fig. 3 we show the semiclassical two-point function $\langle Q^2\rangle^{\rm cl}_{\rm a}$. Its behavior is quite different from the quantum part shown in the previous figures.

C. Late-time variances and the proper acceleration

The saturated value $\langle Q(\infty)^2\rangle_{\rm v}$ in Eq. (64) is the late-time variance of Q, namely, $\langle\Delta Q(\infty)^2\rangle = \langle Q(\infty)^2\rangle_{\rm v}$. Its dependence on the proper acceleration a is shown in Fig. 4.

One can see that, when a is large, $\langle Q(\infty)^2\rangle_{\rm v}$ is nearly proportional to a, while in the zero-acceleration limit $a \to 0$ with $\eta \gg (2\gamma)^{-1}\ln|\ln a|$, the saturated value goes to a positive number. From (64) and (75), one finds that

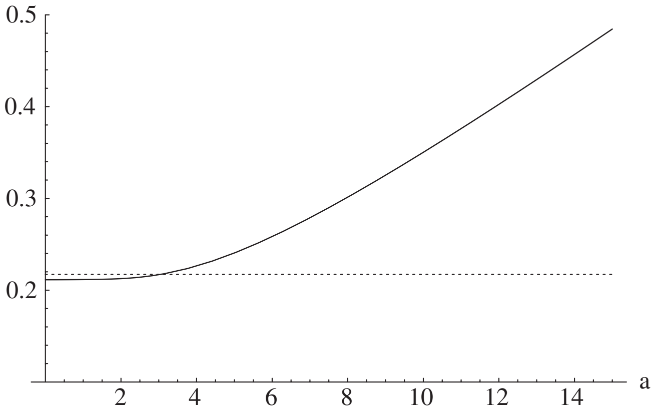

FIG. 4. $\langle\Delta Q(\infty)^2\rangle = \langle Q(\infty)^2\rangle_{\rm v}$ against the proper acceleration a (solid line, Eq. (64)) with other parameters the same as Fig. 1. For small a, the value of $\langle Q(\infty)^2\rangle_{\rm v}$ is less than $\langle\Delta Q(0)^2\rangle = \langle Q(0)^2\rangle_{\rm a}$ (dotted line, see Eq. (75)). For large a, $\langle Q(\infty)^2\rangle_{\rm v}$ is nearly proportional to a.

$$\lim_{a\to 0}\frac{\langle Q(\infty)^2\rangle_{\rm v}}{\langle Q(0)^2\rangle_{\rm a}^{\rm qm}} = \frac{i\Omega_r}{\pi\Omega}\ln\frac{\gamma - i\Omega}{\gamma + i\Omega}, \qquad (81)$$

thus $\langle\Delta Q(\infty)^2\rangle = \langle Q(\infty)^2\rangle_{\rm v}$ is smaller than $\langle\Delta Q(0)^2\rangle = \langle Q(0)^2\rangle_{\rm a}^{\rm qm}$ for every $\gamma > 0$ when $a \to 0$. In other words, for a nonaccelerated detector, whose Unruh temperature is zero, the variance of Q in the detector-field coupled system is still finite and smaller than the one for the ground state in the free theory.

Actually, $\langle\Delta Q(\infty)^2\rangle$ will become smaller than $\langle\Delta Q(0)^2\rangle$ whenever a is small enough. Observing Fig. 4, there is a critical value of a that gives the late-time variance identical to the initial one ($a = a_{\rm cr} \approx 3.0447$ in Fig. 4). Does this mean that the Q-component of the final wave packet with $a_{\rm cr}$ is in the original ground state of the free theory? The answer is no. What happens is that the quantum state of Q has been highly entangled with the quantum state of Φ at late times, and the value of $\sqrt{\langle\Delta Q(\infty)^2\rangle}$ simply represents the width of the projection of the whole wave packet (in the Q-Φ representation of the state) onto the Q-axis. There is actually no factorizable Q-component of the wave packet, and the final configuration of the wave packet in Q-Φ space looks totally different from the initial one. Indeed, with the same critical value of a, $\langle\Delta\dot{Q}(\infty)^2\rangle = \langle\dot{Q}(\infty)^2\rangle_{\rm v}$ is not equal to $\langle\Delta\dot{Q}(0)^2\rangle = \langle\dot{Q}(0)^2\rangle_{\rm a}^{\rm qm}$ for every $\gamma \neq 0$.

But one can still imagine that, at $\eta = 0$, the coherent state for the free detector is an ensemble of particles with a distribution function like $|\langle Q|q\rangle|^2$ in Q-space. $\sqrt{\langle Q(\eta)^2\rangle_{\rm a}^{\rm qm}}$ is the width of this distribution function. When $\eta > 0$, due to the dissipation which comes with the coupling, all particles in the ensemble are going to fall into the bottom of the potential of Q, so $\langle Q(\eta)^2\rangle_{\rm a}^{\rm qm}$ shrinks to zero. On the other hand, the vacuum fluctuations of the field act like a pressure which can push the ensemble of particles outwards, so that the width of the projection of the wave packet in Q-space, $\sqrt{\langle\Delta Q^2\rangle}$, remains finite. A larger a gives a higher Unruh temperature, and a higher outward pressure, so eventually the wave packet reaches equilibrium with a wider projection in the potential well of Q.

D. Shift of the ground-state energy

A natural definition of the energy of the dressed detector (a similar concept is that of a "dressed atom," see e.g., Ref. [4,5]) is

$$E(\eta) \equiv \frac{m_0}{2}[\langle\dot{Q}^2(\eta)\rangle + \Omega_r^2\langle Q^2(\eta)\rangle], \qquad (82)$$

with $\langle Q^2(\eta)\rangle = \langle Q^2(\eta)\rangle_{\rm v} + \langle Q^2(\eta)\rangle_{\rm a}$ and $\langle\dot{Q}^2(\eta)\rangle = \langle\dot{Q}^2(\eta)\rangle_{\rm v} + \langle\dot{Q}^2(\eta)\rangle_{\rm a}$ according to (51). In Figs. 1–3, one can see that $\bar{Q}$, $\langle\dot{Q}^2(\eta)\rangle_{\rm a}^{\rm qm}$ and $\langle Q^2(\eta)\rangle_{\rm a}^{\rm qm}$ eventually die out. So the late-time energy of the dressed detector is

$$E(\infty) = \frac{m_0}{2}[\langle\dot{Q}(\infty)^2\rangle_{\rm v} + \Omega_r^2\langle Q(\infty)^2\rangle_{\rm v}] = \frac{\hbar}{2\pi}\{a - 2\,{\rm Re}[(\gamma + i\Omega)\psi_{\gamma+i\Omega}] - 2\gamma\ln a\} \qquad (83)$$

from (64) and (69). This is actually the true ground-state energy of the dressed detector, with the vacuum fluctuations of the field incorporated. The first term in $E(\infty)$ could be interpreted as the total energy of a harmonic oscillator in thermal bath, $k_B T_U$, with the Unruh temperature $T_U = \hbar a/2\pi k_B$.

The ground-state energy of the dressed detector is not identical to the one for the free detector, $E_0 = \hbar\Omega_r/2$. In particular, if a is small enough, the subtracted $E(\infty)$ is lower than E_0, though there is an ambiguity of a constant in determining the value of the energy. This is analogous to the Lamb shift in atomic physics [3,5,22].

V. TWO-POINT FUNCTIONS OF THE QUANTUM FIELD

Similar to the two-point functions of the detector, for the initial quantum state (50), the two-point function of Φ could be split into two parts,

$$\langle\hat{\Phi}(x)\hat{\Phi}(x')\rangle = \langle 0_M|\langle q|[\Phi_a(x) + \Phi_b(x)] \times [\Phi_a(x') + \Phi_b(x')]|q\rangle|0_M\rangle = G_{\rm v}(x,x') + G_a(x,x'), \qquad (84)$$

where, from (12) and (18),

$$G_{\rm v}(x,x') \equiv \langle q|q\rangle\langle 0_M|\Phi_a(x)\Phi_a(x')|0_M\rangle = \int\frac{\hbar d^3k}{(2\pi)^3 2\omega}f^{(+)}(x;\mathbf{k})f^{(-)}(x';\mathbf{k}), \qquad (85)$$

$$G_{\rm a}(x,x') \equiv \langle 0_M|0_M\rangle\langle q|\Phi_b(x)\Phi_b(x')|q\rangle = \frac{\hbar}{2\Omega_r m_0}f^a(x)f^{a*}(x'). \qquad (86)$$

Equations (28)–(30) and (47) suggest that G_v accounts for the backreaction of the vacuum fluctuations of the scalar field on the field itself, while G_a corresponds to the dissipation of the zero-point energy of the internal degree of freedom of the detector.

Substituting (28) into (85), G_v can be decomposed into four pieces,

$$G_v(x, x') = G_v^{00}(x, x') + G_v^{01}(x, x') + G_v^{10}(x, x') + G_v^{11}(x, x'), \tag{87}$$

in which G_v^{ij} are defined by

$$G_v^{ij}(x, x') \equiv \int \frac{\hbar d^3k}{(2\pi)^3 2\omega} f_i^{(+)}(x; \mathbf{k}) f_j^{(-)}(x', \mathbf{k}), \tag{88}$$

with $i, j = 0, 1$. G_v^{00} is actually the Green's function for free fields, which should be subtracted to obtain the renormalized Green's function for the interacting theory, namely,

$$G_{\rm ren}(x, x') \equiv \langle \hat{\Phi}(x)\hat{\Phi}(x')\rangle - G_v^{00}(x, x'). \tag{89}$$

Since $G_v^{01}(x, x') = [G_v^{10}(x', x)]^*$ by definition, it is sufficient to calculate $G_v^{11}(x, x')$ and $G_v^{10}(x, x')$ in the following.

The structure of G_v^{11} is quite simple. Comparing (32) and (59) and the definition (88), one concludes that

$$G_v^{11}(x, x') = \frac{\lambda_0^2}{(2\pi)^2 a^2 X X'} \langle Q(\tau_-)Q(\tau'_-)\rangle_v. \tag{90}$$

The result (61) can be substituted directly to get the coincidence limit of G_v^{11}.

By definition, G_v^{10} accounts for the interference between the retarded solution $f_1^{(+)}$ and the free solution $f_0^{(+)}$. Since we are interested in the coincidence limit of G_v^{10}, and $f_1^{(+)}$ vanishes in the L-wedge ($U > 0$, $V < 0$) and P-wedge ($U, V < 0$) of Minkowski space, below only the $G_v^{10}(x, x')$ with x and x' in the F-wedge ($U, V > 0$) and R-wedge would be calculated.

It has been given in Ref. [31] that

$$\int \frac{\hbar d^3k}{(2\pi)^3 2\omega} \varphi(\kappa, \mathbf{k}) e^{i\omega t - i\mathbf{k}\cdot\mathbf{x}} = \int \frac{e^{i\kappa\tau} d\tau/(2\pi)^3}{(x^1 - z^1(\tau))^2 + \rho^2 - (t - z^0(\tau) + i\epsilon)^2} = \frac{i}{2\pi a X} \frac{\hbar}{(1 - e^{-2\pi\kappa/a})} [e^{i\kappa\tau_-} - Z(\kappa) e^{i\kappa\tau_+}], \tag{91}$$

where $\epsilon > 0$, $Z(\kappa) = 1$ and $e^{-\pi\kappa/a}$ for x in R and F-wedges [41], respectively, X and τ_- were defined in (33) and (34), and

$$\tau_+ \equiv \frac{1}{a} \ln \frac{a}{2|U|} (X - UV + \rho^2 + a^{-2}). \tag{92}$$

Hence, from (29) and (32), one has

$$G_v^{10}(x, x') = \frac{\hbar \lambda_0^2 \theta(\eta_-)}{(2\pi)^3 m_0 a^2 X X'} \int \frac{d\kappa}{1 - e^{-2\pi\kappa/a}} \sum_j \frac{c_j e^{-i\kappa(\tau_0 - \tau'_0)}}{\kappa - i w_j} [e^{w_j \eta_-} - e^{-i\kappa\eta_-}][e^{i\kappa\eta'_-} - Z(\kappa) e^{i\kappa\eta'_+}] \tag{93}$$

with $\eta_\pm(x) \equiv \tau_\pm(x) - \tau_0$. The coincidence limit of G_v^{10} reads

$$\begin{aligned} G_v^{10}(x, x) &\equiv \lim_{x' \to x} \frac{1}{2} (G_v^{10}(x, x') + G_v^{10}(x', x)) \\ &= \frac{\hbar \lambda_0^2 \theta(\eta_-)}{(2\pi)^3 m_0 \Omega a^2 X^2} \mathrm{Re}\Big\{ i\psi_{\gamma + i\Omega} + \frac{ia}{\gamma + i\Omega + a} [e^{-(\gamma + i\Omega + a)\eta_-} F_{\gamma + i\Omega}(e^{-a\eta_-}) \\ &\quad - (\pm) e^{-(\gamma + i\Omega)\eta_- - a\eta_+} F_{\gamma + i\Omega}(\pm e^{-a\eta_+}) \pm e^{-a(\eta_+ - \eta_-)} F_{\gamma + i\Omega}(\pm e^{-a(\eta_+ - \eta_-)})] \Big\}, \end{aligned} \tag{94}$$

with + and − for x in R and F-wedges, respectively. Near the event horizon $U \to 0$, η_+ diverges, and the last two terms in (94) vanish.

As for $G_a(x, x')$, since $f_0^a = 0$, one has $G_a^{01} = G_a^{10} = 0$, and only G_a^{11} contributes to G_a. Inserting (47) into (86) and comparing with (71), one finds that

$$G_a(x, x') = \frac{\lambda_0^2}{(2\pi)^2 a^2 X X'} \langle Q(\tau_-)Q(\tau'_-)\rangle_a. \tag{95}$$

It can also be divided into a quantum part $G_a^{\rm qm}(x, x')$ and a semiclassical part $G_a^{\rm cl}(x, x')$ according to (71) and below.

A. Effects due to the interfering term

In our (3 + 1)-dimensional UD detector theory, the coincidence limit of the quantum part of $G_{\rm ren}$ reads

$$\begin{aligned} G^{\rm qm}_{\rm ren}(x,x) &\equiv G_{\rm ren}(x,x) - G^{\rm cl}_{\rm a}(x,x) \\ &= G^{\rm qm}_{\rm a}(x,x) + G^{11}_{\rm v}(x,x) + G^{10}_{\rm v}(x,x) \\ &\quad + G^{01}_{\rm v}(x,x), \end{aligned} \tag{96}$$

owing to (84), (87), and (89). Collecting the results in (90), (94), and (95), it is found that $G^{\rm qm}_{\rm ren}(x,x)$ is singular at $x \to y(\tau)$, and one has to be more cautious.

As can be seen from (90) and (95), $G^{11}_{\rm v}(x,x)$ and $G^{\rm qm}_{\rm a}(x,x)$ look like the squares of the retarded field with effective squared scalar charge $\langle Q(\tau)^2\rangle_{\rm v}$ and $\langle Q(\tau)^2\rangle^{\rm qm}_{\rm a}$, respectively. Since the detector is accelerating, these two terms do carry radiated energy (this will be shown explicitly later). The interfering term $G^{10}_{\rm v}(x,x) + G^{01}_{\rm v}(x,x)$, is more intriguing: At first glance, it acts like a polarization in the medium, which screens the radiation field carried by $G^{11}_{\rm v}(x,x)$ and $G^{\rm qm}_{\rm a}(x,x)$. However, the interfering term $G^{10}_{\rm v} + G^{01}_{\rm v}$ does not respond to $G^{\rm qm}_{\rm a}$ at all—it is independent of f^a and impervious to any information about the quantum state of Q. Hence the interfering term cannot be interpreted as the polarization in the medium. The total effect is simply a destructive interference between the field induced by the vacuum fluctuations, and the vacuum fluctuations themselves. For physical interpretations one should group $G^{10}_{\rm v} + G^{01}_{\rm v}$ and $G^{11}_{\rm v}$ together and leave $G^{\rm qm}_{\rm a}$ alone.

These quantities, together with their sum, are illustrated in Figs. 5 and 6. In Fig. 5, one can see that, soon after the coupling is turned on at $V = 1/a$, $G^{10}_{\rm v} + G^{01}_{\rm v}$ build up and pull the solid curve down. Observing (94), the time scale (in proper time of the detector) of this pull-down is about $1/(\gamma + a)$, which is shorter than the time scale $1/2\gamma$ for $\langle Q(\tau)^2\rangle_{\rm v}$ and $\langle Q(\tau)^2\rangle^{\rm qm}_{\rm a}$, since we take $\gamma = 0.1 < a = 1$ here. In Fig. 6, one can also see that $G^{\rm qm}_{\rm a} + G^{11}_{\rm v}$ diverge as X^{-2} around the trajectory, while the divergence of $G^{10}_{\rm v} + G^{01}_{\rm v}$ as $X \to 0$ is a bit weaker than X^{-2}, such that $X^2(G^{10}_{\rm v} + G^{01}_{\rm v})$ goes to zero on the trajectory of the detector.

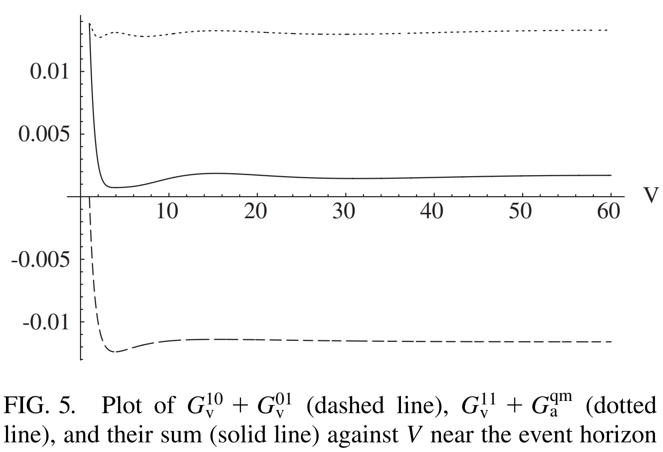

FIG. 5. Plot of $G^{10}_{\rm v} + G^{01}_{\rm v}$ (dashed line), $G^{11}_{\rm v} + G^{\rm qm}_{\rm a}$ (dotted line), and their sum (solid line) against V near the event horizon $U = 0$. Other parameters are the same as those in Fig. 1. One can see the feature that positive $G^{11}_{\rm v} + G^{\rm qm}_{\rm a}$ is screened by negative $G^{10}_{\rm v} + G^{01}_{\rm v}$.

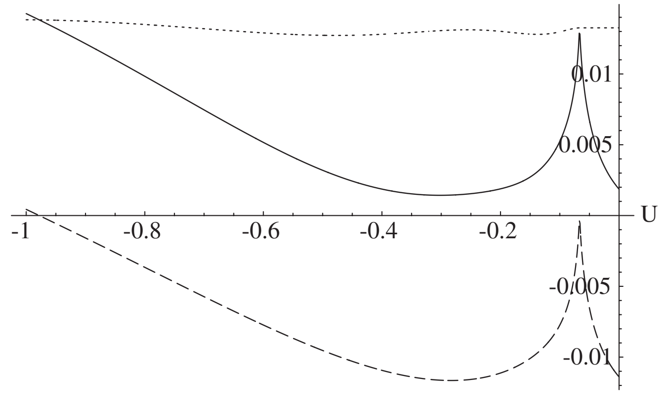

FIG. 6. Plot of $X^2(G^{10}_{\rm v} + G^{01}_{\rm v})$ (dashed line), $X^2(G^{11}_{\rm v} + G^{\rm qm}_{\rm a})$ (dotted line), and their sum (solid line) against U at $V = 15$ and $\rho = 0$. The cusp in the right ($U \approx -0.07$ in this plot) locates at the position of the detector with $X^2(G^{10}_{\rm v} + G^{01}_{\rm v}) = 0$. It is due to the weaker divergence than $1/X^2$ for $G^{10}_{\rm v} + G^{01}_{\rm v}$.

B. What exactly is the "vacuum polarization cloud" around the detector?

In prior work for (1 + 1)D spacetime [20] the counterpart of $G_{\rm ren}(x,x)$ has been considered as evidence for the existence of a "vacuum polarization cloud" around the detector [21,22,29]. This is because $G_{\rm ren}(x,x)$ around the detector does not vanish even after the system reaches equilibrium, it exchanges particles with the detector, and the mean energy it carries is zero. Nevertheless, vacuum polarization is a concept pertinent to field-field quantum interacting systems. In quantum electrodynamics, electrons are described in terms of a field, which distributes in the whole spacetime, so vacuum polarization is pictured as the creation and annihilation of virtual electron-positron pairs everywhere in spacetime. These virtual electron-positron pairs do modify the field strength around the location of a point charge, yielding a nonvanishing variance of the electromagnetic (EM) field. But in the UD detector theory, at the level of precision explored here, the detector-field interaction (hence the virtual processes) only occurs on the trajectory of the detector. There is no virtual detector or scalar charge at any spatial point off the location of the UD detector.

Hence in UD detector theory "vacuum polarization cloud" is not a precise description of $G_{\rm ren}(x,x)$ in steady state. At late times $G_{\rm ren}(x,x)$ simply shows the characteristics of the field in the true vacuum state, in contrast to $\langle Q(\infty)^2\rangle$ for the true ground state of the detector.

VI. RADIATED POWER

In classical theory, the modified stress-energy tensor for a massless scalar field Φ in Minkowski space is [6]

$$T_{\mu\nu}[\Phi(x)] = (1-2\xi)\Phi_{,\mu}\Phi_{,\nu} - 2\xi\Phi\Phi_{;\mu\nu} + \left(2\xi - \frac{1}{2}\right)g_{\mu\nu}\Phi^{,\rho}\Phi_{,\rho} + \frac{\xi}{2}g_{\mu\nu}\Phi\Box\Phi, \tag{97}$$

where ξ is a field coupling parameter, set to zero here. Denote $v^\mu = dz^\mu/d\tau$ as the four velocity of the detector, and define the null distance r and the spacelike unit vector u^μ by $x^\mu - z^\mu(\tau_-) \equiv r(u^\mu + v^\mu(\tau_-))$ [42] with normalization $u_\mu u^\mu = 1$ and $v_\mu u^\mu = 0$ (see Fig. 7). Then the stress-energy tensor for the classical retarded field $\Phi_{\rm ret}$ induced by the UD detector moving along the trajectory $z^\mu(\tau_-)$ can be written as [31]

$$T_{\mu\nu}[\Phi_{\rm ret}(x)]_{\xi=0} = \frac{\lambda_0^2}{(4\pi)^2}\theta(\eta_-)\left\{\frac{1}{r^4}Q^2(\tau_-)\left(-\frac{1}{2}g_{\mu\nu} + u_\mu u_\nu\right) + \frac{1}{r^3}Q(\tau_-)[\dot{Q}(\tau_-) + Q(\tau_-)a_\rho u^\rho] \times (-g_{\mu\nu} + 2u_\mu u_\nu + u_\mu v_\nu + v_\mu u_\nu) + \frac{1}{r^2}[\dot{Q}(\tau_-) + Q(\tau_-)a_\rho u^\rho]^2(u_\mu + v_\mu)(u_\nu + v_\nu)\right\}. \tag{98}$$

The $O(r^{-2})$ term in the above expression corresponds to the radiation field, which carries radiated power given by [42]

$$\frac{dW^{\rm rad}}{d\tau_-} = -\lim_{r\to\infty}\int r^2 d\Omega_{\rm II} u^\mu T_{\mu\nu}v^\nu(\tau_-) = \frac{\lambda_0^2}{(4\pi)^2}\int d\Omega_{\rm II}[\dot{Q}(\tau_-) + Q(\tau_-)a_\rho u^\rho]^2 \tag{99}$$

to the null infinity of Minkowski space. Here the Q-term corresponds to dipole radiation ($l = 1$, $m = 0$ in multipole expansion of the radiation field) with the angular distribution $a_\rho u^\rho = a\cos\theta$ [43], while the $\dot{Q}$-term corresponds to monopole radiation ($l = 0$) isotropic in the rest frame instantaneously for the UD detector at τ_-. The solid angle $d\Omega_{\rm II}$ could be further integrated out, then one obtains the classical radiation formula

$$\frac{dW^{\rm rad}}{d\tau_-} = \frac{\lambda_0^2}{4\pi}\left[\dot{Q}^2(\tau_-) + \frac{a^2}{3}Q^2(\tau_-)\right], \tag{100}$$

which is the counterpart of the Larmor formula for EM radiation. The second term is the usual radiation formula for the massless scalar field emitted by a constant, pointlike scalar charge in acceleration [44].

Naively, one may expect that the quantum version of the radiation formula could look like $(\lambda_0^2/4\pi)[\langle\dot{Q}^2(\tau_-)\rangle + (a^2/3)\langle Q^2(\tau_-)\rangle]$. In the following, we shall calculate the quantum expectation value of the flux $T_{\mu\nu}$, from which we will see that the quantum radiation formula is more complicated than expected.

A. Expectation value of the stress-energy tensor

The expectation value of the renormalized stress-energy tensor $\langle T_{\mu\nu}\rangle_{\rm ren}$ is obtained by calculating

$$\langle T_{\mu\nu}[\Phi(x)]\rangle_{\rm ren} = \lim_{x'\to x}\left[\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x'^\nu} - \frac{1}{2}g_{\mu\nu}g^{\rho\sigma}\frac{\partial}{\partial x^\rho}\frac{\partial}{\partial x'^\sigma}\right]G_{\rm ren}(x,x'), \tag{101}$$

according to (97) with $\xi = 0$. With the results in the previous section, it is straightforward to obtain $\langle T_{\mu\nu}\rangle_{\rm ren}$ induced by the UAD:

$$\langle T_{\mu\nu}[\Phi(x)]\rangle_{\rm ren} = \frac{\lambda_0^2\theta(\eta_-)}{(2\pi)^2a^2X^2}\left[g_\mu{}^\rho g_\nu{}^\sigma - \frac{1}{2}g_{\mu\nu}g^{\rho\sigma}\right]\times\left[\eta_{-,\rho}\eta_{-,\sigma}\langle\dot{Q}(\tau_-)^2\rangle_{\rm tot} + \frac{X_{,\rho}X_{,\sigma}}{X^2}\langle Q(\tau_-)^2\rangle_{\rm tot} - \frac{X_{,\rho}}{X}\eta_{-,\sigma}\langle Q(\tau_-)\dot{Q}(\tau_-)\rangle_{\rm tot} - \eta_{-,\rho}\frac{X_{,\sigma}}{X}\langle\dot{Q}(\tau_-)Q(\tau_-)\rangle_{\rm tot} + (\eta_{-,\rho}\eta_{+,\sigma} + \eta_{+,\rho}\eta_{-,\sigma})\frac{\hbar\Theta_{+-}}{2\pi m_0} - \left(\frac{X_{,\rho}}{X}\eta_{+,\sigma} + \eta_{+,\rho}\frac{X_{,\sigma}}{X}\right)\frac{\hbar\Theta_{+X}}{2\pi m_0}\right]. \tag{102}$$

Upon collecting (A1) and (A2) as well as those from G^a. Here $\langle\dot{Q}^2\rangle_{\rm tot} \equiv \langle\dot{Q}^2\rangle + (\hbar/2\pi m_0)\Theta_{--}$, $\langle Q^2\rangle_{\rm tot} \equiv \langle Q^2\rangle + (\hbar/2\pi m_0)\Theta_{XX}$, and $\langle Q\dot{Q}\rangle_{\rm tot} \equiv \langle Q\dot{Q}\rangle + (\hbar/2\pi m_0)\Theta_{X-}$ with Θ_{ij} defined in (A3)–(A7). To see the properties of quantum nature, we define the total variances by subtracting the semiclassical part from $\langle\cdots\rangle_{\rm tot}$ as

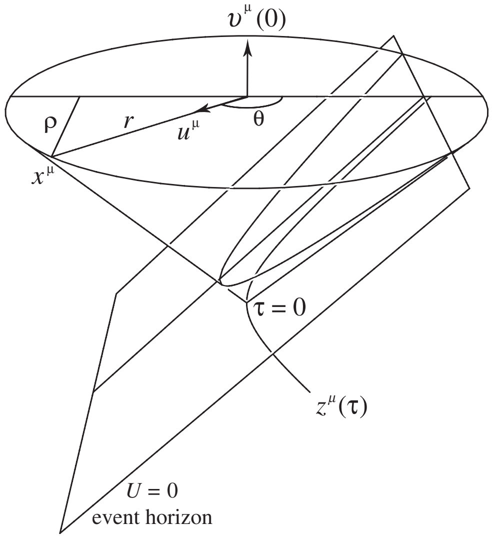

FIG. 7. Definitions of ρ, θ, and r.

$$\langle \Delta Q^2(\tau_-)\rangle_{\rm tot} \equiv \langle Q^2(\tau_-)\rangle_{\rm tot} - \bar{Q}^2(\tau_-) = \langle \Delta Q^2(\tau_-)\rangle + \frac{\hbar\Theta_{XX}(\tau_-)}{2\pi m_0}. \quad (103)$$

$$\langle \Delta \dot{Q}^2(\tau_-)\rangle_{\rm tot} \equiv \langle \dot{Q}^2(\tau_-)\rangle_{\rm tot} - \dot{\bar{Q}}^2(\tau_-) = \langle \Delta \dot{Q}^2(\tau_-)\rangle + \frac{\hbar\Theta_{--}(\tau_-)}{2\pi m_0}, \quad (104)$$

Their evolution against η_- are illustrated in Figs. 8 and 9.

In our case, the Minkowski coordinate (U, V, ρ) of a spacetime point in F and R-wedge can be transformed to the coordinate (r, τ_-, θ) by

$$\rho = r\sin\theta, \quad (105)$$

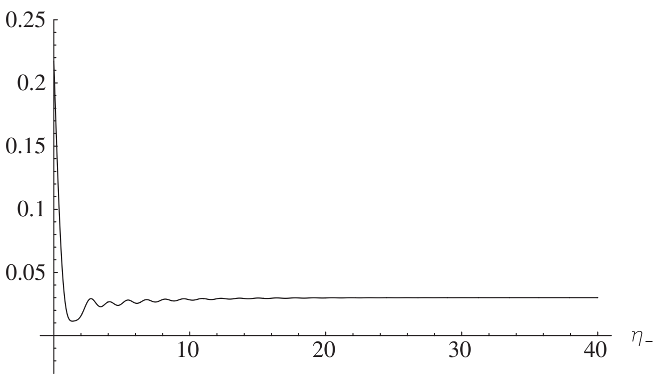

FIG. 8. The total variance $\langle \Delta Q^2\rangle_{\rm tot}$ (Eq. (103)) near the event horizon for the detector ($\eta_+ \to \infty$). This plot is virtually the same as Fig. 5 except the "time" variable is η_- here. The values of parameters are still the same as before. The total variance finally saturates to the value $\hbar a/2\pi m_0\Omega_r^2$. One can compare this plot with Fig. 1 directly and see the suppression.

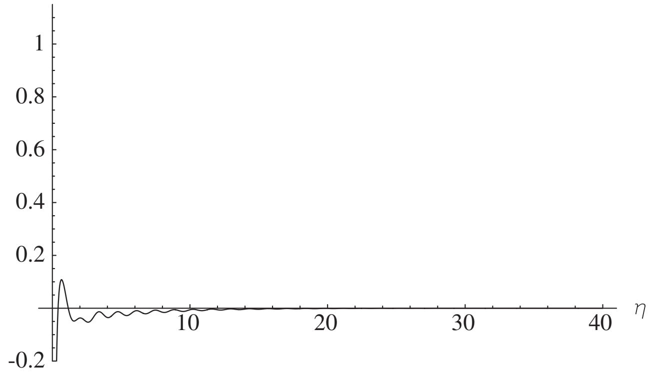

FIG. 9. The total variance $\langle \Delta \dot{Q}^2\rangle_{\rm tot}$ (Eq. (104)) with the same parameters. Note that $\langle \Delta \dot{Q}^2\rangle_{\rm tot}$ is independent of η_+, and this plot is not restricted around the event horizon. One can compare with Fig. 2 and see the suppression.

$$V = re^{a\tau_-}[1 + \cos\theta + (ar)^{-1}], \quad (106)$$

$$U = re^{-a\tau_-}[1 - \cos\theta - (ar)^{-1}], \quad (107)$$

so that $X = 2r/a$. Also one has

$$u^\mu(\tau_-) = \left(r\cos\theta\sinh a\tau_-, r\cos\theta\cosh a\tau_-, \frac{x^2}{r}, \frac{x^3}{r}\right) \quad (108)$$

with $\rho = \sqrt{(x^2)^2 + (x^3)^2}$. Now Eq. (102) can be directly compared with (98) and (99). One can see clearly that the $\langle \dot{Q}(\tau_-)^2\rangle_{\rm tot}$ term has the same angular distribution as the one for the $\dot{Q}^2$ term in (98), hence would be recognized as a monopole radiation by the Minkowski observer. The angular distributions of the remaining terms in (102) are, however, much more complicated because of their dependence on $\eta_+(r, \tau_-, \theta)$.

B. Screening

We have mentioned in the previous section that $G_{\rm a}^{\rm qm}$ and $G_{\rm v}^{11}$ in (90) and (95) carry radiated energy, now this becomes clear. Observing that what corresponds to $\partial_\mu\partial_{\nu'}[G_{\rm a}^{\rm qm}(x, x') + G_{\rm v}^{11}(x, x')]$ are those proportional to $\langle\cdots\rangle$ in $\langle\cdots\rangle_{\rm tot}$ terms of(102). These terms contribute a positive flux. Nevertheless, due to the presence of the interfering terms Θ_{ij}, most of this positive flux of quantum nature will be screened when the system reaches steady state as $\eta_- \to \infty$.

As shown in Fig. 8, the total variance $\langle \Delta Q^2\rangle_{\rm tot}$ near the event horizon $U = 0$ drops exponentially in proper time (power law in the Minkowski time) after the coupling is turned on. Note that $\langle \Delta Q^2(\eta_-(x))\rangle_{\rm tot}$ is proportional to $G_{\rm ren}^{\rm qm}(x, x)$ defined in (96), and X is independent of V on the event horizon, so Fig. 8 is virtually the same plot as Fig. 5 except that the time variable here is η_-. Thus, similar to the behavior of $G_{\rm ren}^{\rm qm}(x, x)$ near the event horizon, $\Theta_{XX}(\sim G_{\rm v}^{10} + G_{\rm v}^{01})$ builds up and the total variance

$\langle \Delta Q(\tau_-)^2 \rangle_{\rm tot}$ is pulled down during the time scale $1/(\gamma + a)$ (for $\gamma < a$) according to (61), (75), and (A4). Then $\langle \Delta Q(\tau_-)^2 \rangle_{\rm tot}$ turns into a tail ($\eta_- > 1$ in Fig. 8) which exponentially approaches the saturated value $\hbar a/2\pi m_0 \Omega_r^2$ with the time scale $1/2\gamma$.

For $\langle \Delta \dot{Q}(\tau_-)^2 \rangle_{\rm tot}$ and $\langle \Delta Q(\tau_-) \Delta \dot{Q}(\tau_-) \rangle_{\rm tot}$, their behaviors are similar (see Fig. 9). In particular, $\langle \Delta \dot{Q}(\tau_-)^2 \rangle_{\rm tot} = \langle \Delta \dot{Q}(\tau_-)^2 \rangle + (\hbar/2\pi m_0)\Theta_{--}$ goes to zero at late times from (68), (78), and (A3) with $\gamma\eta_- \gg 1$, so the corresponding monopole radiation vanishes after the transient.

From the calculations of Ref. [31] based on perturbation theory, it was suggested that the existence of a monopole radiation could be an experimentally distinguishable evidence of the Unruh effect. Here we find from a nonperturbative calculation that, in fact, only the transient of it could be observed. (A comparison of both results will be given in Sec. VII). This appears to agree with the claim that for a UAD in (1 + 1)D, emitted radiation is only associated with the nonequilibrium process [33]. The negative tail of $\langle \Delta \dot{Q}(\tau_-)^2 \rangle_{\rm tot}$ in Fig. 9 and the corresponding quantum radiation could last for a long time with respect to the Minkowski observer ($\sim V^{-2\gamma/a}$), but this is essentially a transient. The interference between the quantum radiation induced by the vacuum fluctuations and the vacuum fluctuations themselves totally screen the information about the Unruh effect in this part of the radiation.

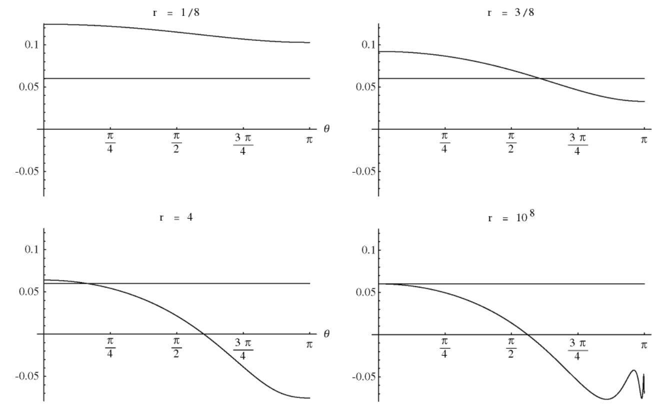

FIG. 10. Angular distributions of $\langle \Delta Q^2 \rangle_{\rm tot}$ at late times (Eq. (103) with $\gamma\tau_- \gg 1$) as the null distance r increases. The values of parameters are the same as before except that here we choose $a = 2$. The horizontal line indicates the saturated value of $\hbar a/2\pi m_0 \Omega_r^2$ shown in Fig. 8. When r starts with 0, $\langle Q^2 \rangle_{\rm tot}$ decreases from $\langle Q^2(\infty) \rangle_{\rm v}$ (top-left). The light cone hits the event horizon at $\theta = \pi$ when $r = 1/(2a) = 1/4$, and $\langle Q^2 \rangle_{\rm tot}$ always has the value $\hbar a/2\pi m_0 \Omega_r^2$ at the event horizon (top-right, the curve and the horizontal line intersect right at the event horizon). As r further increases, $\langle Q^2 \rangle_{\rm tot}$ sinks more and more (bottom-left), and some oscillations begin to develop near $\theta = \pi$. Finally $\langle Q^2 \rangle_{\rm tot}$ is nonvanishing at the null infinity $r \to \infty$, with the value smaller than $\hbar a/2\pi m_0 \Omega_r^2$ whenever $\theta \neq 0$ (bottom-right).

C. Conservation between detector energy and radiation

What is the physics behind the interfering term in $\langle \dot{Q}^2(\eta) \rangle_{\rm tot}$? By inserting our results into (82) and (104), one can show that,

$$E(\eta_i) - E(\eta_f) = \frac{\lambda_0^2}{4\pi} \int_{\eta_i}^{\eta_f} d\eta \langle \dot{Q}^2(\eta) \rangle_{\rm tot} = \frac{\lambda_0^2}{4\pi} \int_{\eta_i}^{\eta_f} d\eta \left[\dot{\bar{Q}}^2(\eta) + \langle \Delta \dot{Q}^2(\eta) \rangle + \frac{\hbar \Theta_{--}(\eta)}{2\pi m_0} \right], \tag{109}$$

for all proper time interval after the interaction is turned on ($\eta_f > \eta_i > 0$). The left-hand side of this equality is the energy loss of the dressed detector from η_i to η_f, while the right-hand side is the radiated energy via the monopole radiation corresponding to $\langle \dot{Q}^2(\eta) \rangle_{\rm tot}$ during the same period. Therefore (109) is simply a statement of energy conservation between the detector and the field *in this channel*, and the interfering terms Θ_{--} must be included so that $\langle \dot{Q}^2(\eta) \rangle_{\rm tot}$ is present on the right-hand side instead of the naively expected $\langle \dot{Q}^2(\eta) \rangle$. A simpler but more general derivation of this relation is given in Appendix B. Equation (109) also justifies that (82) is indeed the correct form of the internal energy of the dressed detector.

With the relation (109) we can make two observations pertaining to results and procedures given before. First, while the Λ_0-terms in $\langle Q^2(\tau) \rangle_{\rm v}$ (Eq. (61)) and $\langle \dot{Q}^2(\tau) \rangle_{\rm v}$ (Eq. (68)) are not included in any figure of this paper, they are consistent with the conservation law (109). Actually the Λ_0-term in (61) satisfies the driving-force-free equation of motion (48), just like the semiclassical $\bar{Q}$ does.

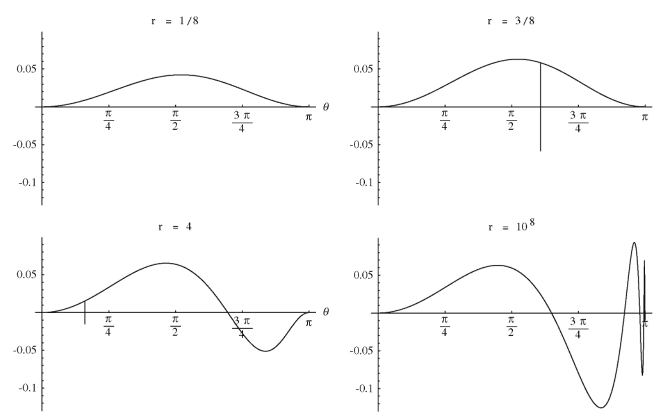

FIG. 11. Angular distributions of $\chi \equiv u^\mu(\eta_-^\mu \eta_+^\nu + \eta_+^\mu \eta_-^\nu) v^\nu \Theta_{+-}$ (Eq. (A7)) at late times as r increases. Parameters are the same as those in Fig. 10. When r starts with 0, χ grows from 0 (top-left). After the light cone hits the event horizon, χ keeps regular on the event horizon (top-right, the vertical line indicates the position of the event horizon). As r further increases, a nodal point enters from the right (bottom-left). Then more and more nodal points can be seen. Finally χ is nonvanishing at the null infinity $r \to \infty$, with infinitely many nodal points close to $\theta = \pi$ (bottom-right).

Second, Eq. (109) implies that all the internal energy of the dressed detector dissipates via a monopole radiation, and the external agent which drives the detector along the trajectory (5) has no additional influence on this channel.

D. Quantum radiation formula

Transforming (102) to the form of (98) by applying (105)–(107), one can calculate the radiation power

$$\left\langle\frac{dW^{\rm rad}}{d\tau_-}\right\rangle = -\lim_{r\to\infty}\int r^2 d\Omega_{\rm II} u^\mu \langle T_{\mu\nu}\rangle_{\rm ren} v^\nu(\tau_-) \qquad (110)$$

following a similar argument in classical theory. Before calculating, let us observe the behavior of the steady state $r^2 u^\mu \langle T_{\mu\nu}\rangle_{\rm ren} v^\nu$ in the forward light cone. As r increases, the developments of two terms in late time $r^2 u^\mu \langle T_{\mu\nu}\rangle_{\rm ren} v^\nu$ are illustrated in Figs. 10 and 11. It turns out that both are regular and nonvanishing at the null infinity of Minkowski space ($r \to \infty$) even in steady state ($\gamma\eta_- \gg 1$). Figures 10 and 11 also indicate that, near the null infinity of Minkowski space, almost all the equal-r surface lies in the F-wedge, except the region around $\theta = 0$ is still in the R-wedge. The contribution to the integral around $\theta = 0$ can be totally neglected because the value of $r^2 u^\mu \langle T_{\mu\nu}\rangle_{\rm ren} v^\nu$ is regular there while the measure for this portion in the angular integral is zero when $r \to \infty$. So the radiation power can be written as

$$\left\langle\frac{dW^{\rm rad}}{d\tau_-}\right\rangle = \frac{\lambda_0^2}{8\pi}\int_0^\pi d\theta \sin\theta\left\{\langle \dot{Q}^2\rangle_{\rm tot} - \frac{\hbar\Theta_{+-}}{\pi m_0} + a^2\cos^2\theta\langle Q^2\rangle_{\rm tot} + a\cos\theta\left[\langle\{Q,\dot{Q}\}\rangle_{\rm tot} - \frac{\hbar\Theta_{+X}}{\pi m_0}\right]\right\}, \qquad (111)$$

by inserting (102) and (108) and $v^\mu(\tau_-) = (\cosh a\tau_-, \sinh a\tau_-, 0, 0)$ into (110). This is the quantum radiation formula for the massless scalar field emitted by the UAD in (3 + 1)D spacetime.

At late times, while $\langle \dot{Q}^2\rangle_{\rm tot}$ ceases, it still remains a positive radiated power flow

$$\left\langle\frac{dW^{\rm rad}}{d\tau_-}\right\rangle \overset{\gamma\tau_-\to\infty}{\longrightarrow} \frac{\hbar\lambda_0^2}{8\pi^2 m_0}\int_0^\pi d\theta\sin\theta\left\{\frac{a}{2\Omega_r^2}a^2\cos^2\theta - a\sin^2\frac{\theta}{2} - \frac{a}{\Omega}\tan^2\frac{\theta}{2}\,{\rm Re}\left[\frac{i(\gamma+i\Omega-a\cos\theta)^2}{\gamma+i\Omega+a}F_{\gamma+i\Omega}\left(-\tan^2\frac{\theta}{2}\right)\right]\right\}$$
$$= \frac{\hbar\lambda_0^2}{8\pi^2 m_0}\left\{\frac{a^3}{3\Omega_r^2} - a - \frac{2}{3}\left[\frac{a^3}{\Omega_r^2} - a + 2\gamma + {\rm Re}\left[\frac{i(\gamma+i\Omega)}{a\Omega}[(\gamma+i\Omega)^2 - a^2]\psi^{(1)}\left(\frac{\gamma+i\Omega}{a}\right)\right]\right]\right\} \qquad (112)$$

to the null infinity of Minkowski space. Thus we conclude that there exists a steady, positive radiated power of quantum nature emitted by the detector even when the detector is in steady state.

For large a, the first term in (112) dominates, and the radiated power is approximately

$$\left\langle\frac{dW^{\rm rad}}{d\tau_-}\right\rangle \approx \frac{\lambda_0^2}{4\pi}\frac{a^2}{3}\frac{\hbar a}{2\pi m_0\Omega_r^2} \propto a^2 T_U, \qquad (113)$$

where T_U is the Unruh temperature. This could be interpreted as a hint of the Unruh effect. Note that it does not originate from the energy flux that the detector experiences in the Unruh effect, since the internal energy of the dressed detector is conserved only in relation to the radiated energy of a monopole radiation corresponding to $\langle \dot{Q}^2\rangle_{\rm tot}$. Learning from the EM radiation emitted by a uniformly accelerated charge [13,42], we expect that the above nonvanishing radiated energy of quantum origin could be supplied by the external agent we introduced in the beginning to drive the motion of the detector. Further analysis on the quantum radiations of the detector involving the dynamics of the trajectory is still ongoing.

VII. COMPARISON WITH EARLIER RESULTS

We can recover the results in Ref. [31], which is obtained by perturbation theory, as follows. In Ref. [31], the first order approximation of the flux $\langle T^{tU}\rangle_{\rm ren}$ through the event horizon $U = 0$ has been calculated. Here, the expectation value of T^{tU} near the event horizon reads, from (102),

$$\langle T^{tU}\rangle_{\rm ren}|_{U\to 0} \equiv \lim_{x'\to x}\left[2\partial_V\partial_{V'} + \frac{1}{2}\partial_\rho\partial_{\rho'}\right]G_{\rm ren}(x,x')|_{U\to 0}$$
$$= \frac{2\lambda_0^2}{(2\pi)^2 a^4(\rho^2+a^{-2})^2}\theta\left[-\frac{1}{a}\ln\frac{a}{V}(\rho^2+a^{-2}) - \tau_0\right]\left\{\frac{1}{V^2}\langle\dot{Q}(\tau_-)^2\rangle_{\rm tot} + \frac{\rho^2}{(\rho^2+a^{-2})^2}\left[a^2\langle Q(\tau_-)^2\rangle_{\rm tot}\right.\right.$$
$$\left.\left. + a\langle\dot{Q}(\tau_-)Q(\tau_-) + Q(\tau_-)\dot{Q}(\tau_-)\rangle_{\rm tot} + \langle\dot{Q}(\tau_-)^2\rangle_{\rm tot}\right]\right\}\Big|_{U\to 0}, \qquad (114)$$

in which Θ_{+-} and Θ_{+X} terms vanish. By letting $\gamma \to 0$ with η_- finite, then taking $\eta_- \to \infty$, the total variance (104) becomes

$$\langle \Delta \dot{Q}(\tau_-)^2 \rangle_{\rm tot} \xrightarrow{\gamma\to 0} \frac{\hbar\Omega_r}{2m_0} - \frac{\hbar}{2\pi m_0}\left\{ a + \frac{2a\cos\Omega_r\eta_-}{e^{a\eta_-}-1} - 2\Omega_r \,{\rm Re}\left[\frac{ae^{(-a+i\Omega_r)\eta_-}}{\Omega_r + ia}F_{i\Omega_r}(e^{-a\eta_-}) - i\psi_{-i\Omega_r}\right]\right\}$$
$$\xrightarrow{\eta_-\to\infty} \frac{\hbar\Omega_r}{m_0(1-e^{2\pi\Omega_r/a})}, \tag{115}$$

owing to (68), (78), and (A3). This is identical to the corresponding part of Eq. (66) in Ref. [31],

$$2\sum_E |\langle E_0|Q(0)|E\rangle|^2 \frac{\varepsilon^2}{1-e^{2\pi\varepsilon}}, \tag{116}$$

by noting that there, $m_0 = 1$, $\sum_E |\langle E_0|Q(0)|E\rangle|^2 = \langle E_0|Q(0)^2|E_0\rangle = \hbar/2\varepsilon$, and ε there is equal to Ω_r/a here.

The monopole radiation corresponding to (116) looks like a constant negative flux since $\varepsilon > 0$. Accordingly it was concluded in Ref. [31] that such a quantum monopole radiation could be experimentally distinguishable from the bremsstrahlung of the detector. At first glance this constant monopole radiation seems to contradict the knowledge gained from (1 + 1)D results. But actually similar results for (1 + 1)D cases were also obtained by Massar and Parentani (MP) [26], who found that a detector initially prepared in the ground state and coupled to a field under a smooth switching function does emit radiation during thermalization. They pointed out that the radiated flux in what they referred to as the "golden rule limit" ($\eta \to \infty$ with $\gamma\eta$ small, while the switching function becomes nearly constant) is approximately a constant negative flux for all $V > 0$ [45]. In spite of the long interaction time and the nearly constant radiated flux, the detector will remain in disequilibrium.

Note that the initial conditions in [31] are similar to those in Sec. II of MP, and the limiting condition for obtaining (115) is exactly what MP assumed there. Hence, the constant negative flux in [31] is essentially a transient effect, which exists only in the period that the above stated condition holds. When the interaction time η exceeds $O(\gamma^{-1})$, this approximation breaks down. To obtain the correct late-time behavior, one should take the limit $\eta_- \to \infty$ before $\gamma \to 0$. Then $\langle \Delta\dot{Q}(\tau_-)^2\rangle_{\rm tot}$ goes to zero.

VIII. SUMMARY

In this paper, we consider the Unruh-DeWitt detector theory in (3 + 1)-dimensional spacetime. A uniformly accelerated detector is modeled by a harmonic oscillator Q linearly coupled with a massless scalar field Φ. The cases with the coupling constant $|\lambda_0|$ less than the renormalized natural frequency $|\Omega_r|$ of the detector are considered.

We solved exactly the evolution equations for the combined system of a moving detector coupled to a quantum field in the Heisenberg picture, and from the evolution of the operators we can obtain complete information on the combined system. For the case that the initial state is a direct product of a coherent state for the detector and the Minkowski vacuum for the field, we worked out the exact two-point functions of the detector and similar functions of the field. By applying the coherent state for the detector, we can distinguish the classical behaviors from others. The quantum part of the coincidence limit of two-point functions, namely, the variances of Q and Φ, are determined by the detector and the field together.

From the exact solutions, we were able to study the complete process from the initial transient to the final steady state. In particular, we can identify the time scales of transient behaviors analytically. When the coupling is turned on, the zero-point fluctuations of the free detector dissipate exponentially, then the vacuum fluctuations take over. The time scales for both processes are the same. Eventually the variance of Q saturates at a finite value, where the dissipation of the detector is balanced by the input from the vacuum fluctuations of the field. Even in the zero-acceleration limit, the variances of Q and $\dot{Q}$, thus the ground-state energy of the interacting detector, shifted from the ones for the free detector. This fluctuations-induced effect shares a similar origin with that of the Lamb shift.

The variance of Q yields an effective squared scalar charge, which induces a positive variance in the scalar field. This variance of the field contributes a positive radiated energy at the quantum level. However, the interference between the vacuum fluctuations and the retarded solution induced by the vacuum fluctuations screens part of the emission of quantum radiation. The time scale of the screening is proportional to $1/(\gamma + a)$ for $a > \gamma$, where a is the proper acceleration and γ is the damping constant proportional to λ^2. After the screening the renormalized Green's functions of the field are still nonzero in steady state.

A quantum radiation formula determined at the null infinity of Minkowski space has been derived. We found that even in steady state there exists a positive radiated power of quantum nature emitted by the uniformly accelerated UD detector. For large a the radiated power is proportional to $a^2 T_U$, where T_U is the Unruh temperature. This could be interpreted as a hint of the Unruh effect. However, the nearly constant negative flux obtained in Ref. [31] for the (3 + 1)D case is essentially a transient effect.

Only part of the radiation is connected to the internal energy of the detector. The total energy of the dressed detector and the radiated energy of a monopole radiation from the detector is conserved for every proper time interval after the coupling is turned on. The external agent which drives the detector's motion has no additional influence on this channel. Since the corresponding monopole radiation of quantum nature ceases in steady state, the hint of the Unruh effect in the late-time radiated power is not directly from the energy flux experienced by the detector in the Unruh effect. This extends the result in Refs. [20,27] that there is no emitted radiation of quantum origin in the Unruh effect in (1 + 1)-dimensional spacetime.

Since all the relevant quantum and statistical information about the detector (atom) and the field can be obtained from the results presented here, when appropriately generalized, they are expected to be useful for addressing issues in atomic and optical schemes of quantum information processing, such as quantum decoherence, entanglement, and teleportation. These investigations are in progress.

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APPENDIX A: DERIVATIVES OF TWO-POINT FUNCTIONS OF THE FIELD

From $G_{\rm v}^{11}$ in (90), it is easy to see that

$$\begin{aligned}\partial_\mu\partial_{\nu'}G_{\rm v}^{11}(x,x') &= \int\frac{\hbar d^3k}{(2\pi)^3 2\omega}\partial_\mu f_1^{(+)}(x)\partial_{\nu'}f_1^{(-)}(x')\\ &= \frac{\lambda_0^2}{(2\pi)^2a^2XX'}\theta(\eta_-)\theta(\eta'_-)\bigg[\frac{X_{,\mu}X'_{,\nu}}{XX'}\langle Q(\tau_-)Q(\tau'_-)\rangle_{\rm v} + \eta_{-,\mu}\eta'_{-,\nu}\langle\dot Q(\tau_-)\dot Q(\tau'_-)\rangle_{\rm v}\\ &\quad -\frac{X_{,\mu}}{X}\eta'_{-,\nu}\langle Q(\tau_-)\dot Q(\tau'_-)\rangle_{\rm v} - \eta_{-,\mu}\frac{X'_{,\nu}}{X'}\langle\dot Q(\tau_-)Q(\tau'_-)\rangle_{\rm v}\bigg].\end{aligned} \tag{A1}$$

Note that the δ-functions at $\eta_- = 0$, coming from the derivative of the step functions, have been neglected here.

With $G_{\rm v}^{10} + G_{\rm v}^{01}$, one can write down in closed form of the interfering terms in the R-wedge of the Rindler space. Under the coincidence limit, it looks like

$$\begin{aligned}\lim_{x'\to x}\frac{1}{2}\{\partial_\mu\partial_{\nu'}[G_{\rm v}^{10}(x,x') + G_{\rm v}^{01}(x,x')] + (x\leftrightarrow x')\} &= \frac{\hbar\lambda_0^2\theta(\eta_-)}{(2\pi)^3 m_0 a^2X^2}\bigg[\eta_{-,\mu}\eta_{-,\nu}\Theta_{--} + \frac{X_{,\mu}X_{,\nu}}{X^2}\Theta_{XX} - \frac{X_{,\mu}}{X}\eta_{-,\nu}\Theta_{X-}\\ &\quad - \eta_{-,\mu}\frac{X_{,\nu}}{X}\Theta_{-X} + \eta_{-,\mu}\eta_{+,\nu}\Theta_{-+} + \eta_{+,\mu}\eta_{-,\nu}\Theta_{+-}\\ &\quad - \frac{X_{,\mu}}{X}\eta_{+,\nu}\Theta_{X+} - \eta_{+,\mu}\frac{X_{,\nu}}{X}\Theta_{+X}\bigg],\end{aligned} \tag{A2}$$

where

$$\begin{aligned}\Theta_{--} &= -4\gamma(\Lambda_1 - \ln a) - a + \frac{2a}{\Omega}\frac{e^{-\gamma\eta_-}}{e^{a\eta_-}-1}\\ &\quad\times(\gamma\sin\Omega\eta_- - \Omega\cos\Omega\eta_-)\\ &\quad -\frac{2}{\Omega}{\rm Re}\bigg[i(\gamma+i\Omega)^2\psi_{\gamma+i\Omega} + \frac{ia(\gamma+i\Omega)^2}{\gamma+i\Omega+a}\\ &\quad\times e^{-(\gamma+i\Omega+a)\eta_-}F_{\gamma+i\Omega}(e^{-a\eta_-})\bigg],\end{aligned} \tag{A3}$$

$$\begin{aligned}\Theta_{XX} &= \frac{2}{\Omega}{\rm Re}\bigg[i\psi_{\gamma+i\Omega} + \frac{ia}{\gamma+i\Omega+a}\\ &\quad\times e^{-(\gamma+i\Omega+a)\eta_-}F_{\gamma+i\Omega}(e^{-a\eta_-})\bigg] + 2\mathcal{F}_0(x),\end{aligned} \tag{A4}$$

$$\Theta_{-X} = \Theta_{X-} = -\frac{a}{\Omega}\frac{e^{-\gamma\eta_-}}{e^{a\eta_-}-1}\sin\Omega\eta_- + \mathcal{F}_1(x), \tag{A5}$$

$$\Theta_{+X} = \Theta_{X+} = \frac{a}{\Omega}\frac{e^{-\gamma\eta_-}}{\pm e^{a\eta_+}-1}\sin\Omega\eta_- - \mathcal{F}_1(x), \tag{A6}$$

$$\begin{aligned}\Theta_{+-} &= \Theta_{-+}\\ &= -\frac{a}{\Omega}\frac{e^{-\gamma\eta_-}}{\pm e^{a\eta_+}-1}(\gamma\sin\Omega\eta_- - \Omega\cos\Omega\eta_-)\\ &\quad - \frac{a}{\pm e^{a(\eta_+-\eta_-)}-1} + \mathcal{F}_2(x),\end{aligned} \tag{A7}$$

with

$$\mathcal{F}_n(x) \equiv \pm \frac{a}{\Omega}\,\mathrm{Re}\Big\{\frac{i(\gamma+i\Omega)^n}{\gamma+i\Omega+a} \times [e^{-a\eta_+-(\gamma+i\Omega)\eta_-}F_{\gamma+i\Omega}(\pm e^{-a\eta_+}) - e^{-a(\eta_+-\eta_-)}F_{\gamma+i\Omega}(\pm e^{-a(\eta_+-\eta_-)})]\Big\}, \quad \text{(A8)}$$

with $+e^{a\eta_+}$ and $-e^{a\eta_+}$ for x in R and F-wedge, respectively. Note that Θ_{--} is independent of η_+. Θ_{XX} is actually proportional to $G^{10}_{\rm v}$ in (94). Another observation is that, combining (A1) and (A2), one finds that the divergent Λ_1 term in Θ_{--} is canceled by the one in $\langle\dot{Q}^2(\eta_-)\rangle_{\rm v}$.

As for G^a, the result is similar to (A1) with $\langle\cdots\rangle_{\rm v}$ being replaced by $\langle\cdots\rangle_a$. Again it splits into the quantum and semiclassical parts as in Sec. IV B.

APPENDIX B: A SIMPLER DERIVATION OF THE CONSERVATION LAW

The conservation law (109) is directly obtained by arranging our somewhat complicated results. It can also be derived in a simpler way as follows.

From the definition of the ground-state energy (82) of the dressed detector, its first derivative of τ is

$$\dot{E}(\tau) = m_0\,\mathrm{Re}[\langle\dot{Q}(\tau)\ddot{Q}(\tau)\rangle + \Omega_r^2\langle Q(\tau)\dot{Q}(\tau)\rangle]. \quad \text{(B1)}$$

Introducing the equations of motion (38) and (45) to eliminate $\ddot{Q}(\tau)$, one has

$$\begin{aligned}\dot{E}(\tau) &\sim m_0\,\mathrm{Re}\Big\{\Big\langle\dot{Q}(\tau)\Big[-2\gamma\dot{Q}(\tau)-\Omega_r^2Q(\tau)+\frac{\lambda_0}{m_0}\Phi_0(y(\tau))\Big]\Big\rangle_{\rm v} + \Omega_r^2\langle Q(\tau)\dot{Q}(\tau)\rangle_{\rm v} \\ &\quad + \langle\dot{Q}(\tau)[-2\gamma\dot{Q}(\tau)-\Omega_r^2Q(\tau)]\rangle_{\rm a} + \Omega_r^2\langle Q(\tau)\dot{Q}(\tau)\rangle_{\rm a}\Big\} \\ &= -2\gamma m_0\langle\dot{Q}^2(\tau)\rangle + \lambda_0\,\mathrm{Re}\langle\dot{Q}(\tau)\Phi_0(y(\tau))\rangle_{\rm v}, \end{aligned} \quad \text{(B2)}$$

where the last term is shorthand for

$$\lambda_0\,\mathrm{Re}\int\frac{\hbar d^3k}{(2\pi)^3 2\omega}\dot{q}^{(+)}(\tau;\mathbf{k})f_0^{(-)}(z(\tau);\mathbf{k}). \quad \text{(B3)}$$

(There is an additional term $\lambda_0\dot{\bar{Q}}(\tau)\Phi_{\rm in}(y(\tau))$ if a background field $\Phi_{\rm in}(x) = \langle\Phi_0(x)\rangle_{\rm v}$ is present.) Substituting (41) and (56) into (B3), integrating out d^3k with the help of Eq. (58), then comparing with $\partial_\mu\partial'_\nu G^{10}(x,x')$ in (A2) where G^{10} has the formal expression (93), one finds that (B3) is actually identical to $-\hbar\gamma\Theta_{--}(\tau)/\pi$. Hence

$$\dot{E}(\tau) = -2\gamma m_0\langle\dot{Q}^2(\tau)\rangle_{\rm tot}. \quad \text{(B4)}$$

Integrating both sides from $\tau = \tau_i$ to τ_f, one ends up with Eq. (109).

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