Populating the landscape: A top-down approach

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We put forward a framework for cosmology that combines the string landscape with no boundary initial conditions. In this framework, amplitudes for alternative histories for the universe are calculated with final boundary conditions only. This leads to a top-down approach to cosmology, in which the histories of the universe depend on the precise question asked. We study the observational consequences of no boundary initial conditions on the landscape, and outline a scheme to test the theory. This is illustrated in a simple model landscape that admits several alternative inflationary histories for the universe. Only a few of the possible vacua in the landscape will be populated. We also discuss in what respect the top-down approach differs from other approaches to cosmology in the string landscape, like eternal inflation.

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I. INTRODUCTION

It seems likely that string theory contains a vast ensemble of stable and metastable vacua, including some with a small positive effective cosmological constant [1] and the low energy effective field theory of the standard model. Recent progress on the construction of metastable de Sitter vacua [2] lends further support to the notion of a string landscape [3], and a statistical analysis gives an idea of the distribution of some properties among the vacua [4]. But it has remained unclear what is the correct framework for cosmology in the string landscape. There are good reasons to believe, however, that a proper understanding of the cosmological dynamics will be essential for the landscape to be predictive [5].

In particle physics, one usually computes S-matrix elements. This is useful to predict the outcome of laboratory experiments, where one prepares the initial state and measures the final state. It could be viewed as a bottom-up approach to physics, in which one evolves forward in time a particular initial state of the system. The predictivity of this approach arises from and relies upon the fact that one has control over the initial state, and that experiments can be repeated many times to gain statistically significant results.

But cosmology poses questions of a very different character. In our past there is an epoch of the early universe when quantum gravity was important. The remnants of this early phase are all around us. The central problem in cosmology is to understand why these remnants are what they are, and how the distinctive features of our universe emerged from the big bang. Clearly it is not an S-matrix that is the relevant observable[1] for these predictions, since we live in the middle of this particular experiment. Furthermore, we have no control over the initial state of the universe, and there is certainly no opportunity for observing multiple copies of the universe.

In fact if one does adopt a bottom-up approach to cosmology, one is immediately led to an essentially classical framework, in which one loses all ability to explain cosmology's central question—why our universe is the way it is. In particular a bottom-up approach to cosmology either requires one to postulate an initial state of the universe that is carefully fine-tuned [10]—as if prescribed by an outside agency—or it requires one to invoke the notion of eternal inflation [11], which prevents one from predicting what a typical observer would see.

Here we put forward a different approach to cosmology in the string landscape, based not on the classical idea of a single history for the universe but on the quantum sum over histories [12]. We argue that the quantum origin of the universe naturally leads to a framework for cosmology where amplitudes for alternative histories of the universe are computed with boundary conditions at late times only. We thus envision a set of alternative universes in the landscape, with amplitudes given by the no boundary path integral [13].

The measure on the landscape provided by no boundary initial conditions allows one to derive predictions for observations. This is done by evaluating probabilities for alternative histories that obey a set of constraints at late times. The constraints provide information that is supplementary to the fundamental laws and act as a selection principle. In particular, they select the subclass of histories that contribute to the amplitude of interest. One then identifies alternatives within this subclass that have probabilities near one. These include, in particular, predictions of future observations. The framework we propose is thus more like a top-down approach to cosmology, where the histories of the universe depend on the precise question asked.

We illustrate our framework in a model landscape that admits several distinct classes of inflationary histories for the universe. In this model, we predict several properties of

[1]See [6–9] for recent work on the existence and the construction of observables in cosmological spacetimes.

the subclass of histories that are three-dimensional, expanding and approximately flat at late times. We also discuss in general terms the predictions of top-down cosmology in more complicated models like the string landscape.

Finally we discuss in what respect the top-down approach differs from other (bottom-up) approaches to cosmology in the string landscape, such as eternal inflation or pre-big bang cosmology.

II. QUANTUM STATE

In cosmology one is generally not concerned with observables at infinity or with properties of the entire four-geometry, but with alternatives in some finite region in the interior of the spacetime. The amplitudes for these more restricted sets of observables are obtained from the amplitudes of four dimensional metric and matter field configurations, by integrating over the unobserved quantities.[2] A particularly important case is the amplitude of finding a compact spacelike surface S with induced three-metric g^3_{ij} and matter field configuration ϕ,

$$\Psi[g^3, \phi] \sim \int_C [\mathcal{D}g][\mathcal{D}\phi] e^{iS[g,\phi]}. \quad (1)$$

Here the path integral is taken over the class C of spacetimes which agree with g^3_{ij} and ϕ on a compact boundary S. The quantum state of the universe is determined by the remaining specification of the class C.

Usually one sums over histories that have an initial and a final boundary. This is useful for the computation of S-matrix elements to predict the outcome of laboratory experiments, where one prepares the initial state and measures the final state. It is far from clear, however, that this is the appropriate setup for cosmology, where one has no control over the initial state, and no opportunity for observing multiple copies of the universe. In fact, if one does apply this approach to cosmology one is naturally led to an essentially classical picture, in which one simply assumes the universe began and evolved in a way that is well-defined and unique.

Pre-big bang cosmologies [10] are examples of models that are based on a bottom-up approach. In these models one specifies an initial state on a surface in the infinite past and evolves this forward in time. A natural choice for the initial state would be flat space, but that would obviously remain flat space. Thus one instead starts with an unstable state in the infinite past, tuned carefully in order for the big crunch/big bang transition to be smooth and the path integral to be peaked around a single semiclassical history. Several explicit solutions of such bouncing cosmologies have been found in various minisuperspace approximations [14]. It has been shown, however, using several different techniques, that solutions of this kind are unstable [15,16]. In particular, one finds that generic small perturbations at early times (or merely taking in account the remaining degrees of freedom) dramatically change the evolution near the transition. Rather than evolving towards an expanding semiclassical universe at late times, one generically produces a strong curvature singularity. Hence the evolution of pre-big bang cosmologies always includes a genuinely quantum gravitational phase, unless the initial state is extremely fine-tuned. It is therefore more appropriate to describe these cosmologies by a path integral in quantum cosmology, and not in terms of a single semiclassical trajectory. The universe will not have a single history but every possible history, each with its own probability.

In fact, the quantum state of the universe at late times is likely to be independent of the state on the initial surface. This is because there are geometries in which the initial surface is in one universe and the final surface in a separate disconnected universe. Such metrics exist in the Euclidean regime, and correspond to the quantum annihilation of one universe and the quantum creation of another. Moreover, because there are so many different possible universes, these geometries dominate the path integral. Therefore even if the path integral had an initial boundary in the infinite past, the state on a surface S at late times would be independent of the state on the initial surface. It would be given by a path integral over all metric and matter field configurations whose only boundary is the final surface S. But this is precisely the no boundary quantum state [13]

$$\Psi[g^3, \phi] \sim \int_C [\mathcal{D}g][\mathcal{D}\phi] e^{-S_E[g,\phi]}, \quad (2)$$

where the integral is taken over all regular geometries bounded only by the compact three-geometry S with induced metric g^3_{ij} and matter field configuration ϕ. The Euclidean action S_E is given by[3]

$$S_E = -\frac{1}{2}\int d^4x\sqrt{g}(R + L(g, \phi)) - \int_S d^3x\sqrt{g^3}K, \quad (3)$$

where $L(g, \phi)$ is the matter Lagrangian.

One expects that the dominant contributions to the path integral will come from saddle points in the action. These correspond to solutions of the Einstein equations with the prescribed final boundary condition. If their curvature is bounded away from the Planck value, the saddle point metric will be in the semiclassical regime and can be regarded as the most probable history of the universe. Saddle point geometries of particular interest include geometries where a Lorentzian metric is rounded off

[2] The precise relation between familiar quasilocal observables and the diffeomorphism-invariant observables of quantum gravity remains an important outstanding issue. See e.g. [9] for recent work on this.

[3] We have set $8\pi G = 1$.

smoothly in the past on a compact Euclidean *instanton*. Well known examples of such geometries are the Hawking-Moss (HM) instanton [17] which matches to Lorentzian de Sitter space, and the Coleman-De Luccia (CdL) instanton [18], which continues to an open FLRW universe. The former occurs generically in models of gravity coupled to scalar fields, while the latter requires a rather fine-tuned potential.

The usual interpretation of these geometries is that they describe the decay of a false vacuum in de Sitter space. However, they have a different interpretation in the no boundary proposal [19]. Here they describe the beginning of a new, independent universe with a completely self-contained "no boundary" description.[4] By this we mean, in particular, that the expectation values of observables that are relevant to local observers within the universe can be unambiguously computed from the no boundary path integral, without the need for assumptions regarding the prebubble era. The original de Sitter universe may continue to exist, but it is irrelevant for observers inside the new universe. The no boundary proposal indicates, therefore, that the prebubble inflating universe is a redundant theoretical construction.

It is appealing that the no boundary quantum state (2) is computed directly from the action governing the dynamical laws. There is thus essentially a single theory of dynamics and of the quantum state. It should be emphasized however that this remains a *proposal* for the wave function of the universe. We have argued it is a natural choice, but the ultimate test is whether its predictions agree with observations.

III. PREDICTION IN QUANTUM COSMOLOGY

Quantum cosmology aims to identify which features of the observed universe follow directly from the fundamental laws, and which features can be understood as consequences of quantum accidents or late time selection effects. In no boundary cosmology, where one specifies boundary conditions at late times only, this program is carried out by evaluating probabilities for alternative histories that obey certain constraints at the present time. The final boundary conditions provide information that is supplementary to the fundamental laws, which selects a subclass of histories and enables one to identify alternatives that (within this subclass) have probabilities near one. In general the probability for an alternative α, given H, Ψ and a set of constraints β, is given by

$$p(\alpha|\beta, H, \Psi) = \frac{p(\alpha, \beta|H, \Psi)}{p(\beta|H, \Psi)}. \tag{4}$$

The conditions β in (4) generally contain environmental selection effects, but they also include features that follow from quantum accidents in the early universe.[5]

A typical example of a condition β is the dimension D of space. For good reasons, one usually considers string compactifications down to three space dimensions. However, there appears to be no dynamical reason for the universe to have precisely four large dimensions. Instead, the no boundary proposal provides a framework to calculate the quantum amplitude for every number of spatial dimensions consistent with string theory. The probability distribution of various dimensions for the universe is of little significance, however, because we have already measured we live in four dimensions. Our observation only gives us a single number, so we cannot tell from this whether the universe was likely to be four dimensional, or whether it was just a lucky chance. Hence as long as the no boundary amplitude for three large spatial dimensions is not exactly zero, the observation that $D = 3$ does not help to prove or disprove the theory. Instead of asking for the probabilities of various dimensions for the universe, therefore, we might as well use our observation as a final boundary condition and consider only amplitudes for surfaces S with three large dimensions. The number of dimensions is thus best used as a constraint to restrict the class of histories that contribute to the path integral for a universe like ours. This restriction allows one to identify definite predictions for future observations.

The situation with the low energy effective theory of particle interactions may well be similar. In string theory this is the effective field theory for the modular parameters that describe the internal space. It is well known that string theory has solutions with many different compact manifolds. The corresponding effective field theories are determined by the topology and the geometry of the internal space, as well as the set of fluxes that wind the 3-cycles. Furthermore, for each effective field theory the potential for the moduli typically has a large number of local minima. Each local minimum of the potential is presumably a valid vacuum of the theory. These form a landscape [3] of possible stable or metastable states for the universe at the present time, each with a different theory of low energy particle physics.

In the bottom-up picture it is thought that the universe begins with a grand unified symmetry, such as $E_8 \times E_8$. As the universe expands and cools the symmetry breaks to the standard model, perhaps through intermediate stages. The idea is that string theory predicts the pattern of breaking, and the masses, couplings and mixing angles of the standard model. However, as with the dimension of space, there seems to be no particular reason why the universe should evolve precisely to the internal space that gives the

[4]The interpretation of these saddle point geometries is in line with their interpretation that follows from holographic reasoning, as described e.g. in [20]. Some of our conclusions, however, differ from [20].

[5]These are quantum accidents that became "frozen", leaving an imprint on the universe at late times.

standard model.[6] It is therefore more useful to compute no boundary amplitudes for a spacelike surface S with a given internal space. This is the top-down approach, where one sums only over the subclass of histories which end up on S with the internal space for the standard model.

We now turn to the predictions α we can expect to derive from amplitudes like (4). We have seen that the relative amplitudes for radically different geometries are often irrelevant. By contrast, the probabilities for neighboring geometries are important. The most powerful predictions are obtained from the relative amplitudes of nearby geometries, conditioned on various discrete features of the universe. This is because these amplitudes are not determined by the selection effects of the final boundary conditions. Rather, they depend on the quantum state $|\Psi\rangle$ itself.

Neighboring geometries correspond to small quantum fluctuations of continuous quantities, like the temperature of the cosmic microwave background (CMB) radiation or the expectation values of the string theory moduli in a given vacuum. In inflationary universes these fluctuations are amplified and stretched, generating a pattern of spatial variations on cosmological scales in those directions of moduli space that are relatively flat.[7] The spectra depend on the quantum state of the universe. Correlators of fluctuations in the no boundary state can be calculated by perturbatively evaluating the path integral around instanton saddle points [19]. In general if $\mathcal{P}(x_1)$ and $\mathcal{Q}(x_2)$ are two observables at x_1 and x_2 on a final surface S, then their correlator is formally given by the following integral over a complete set of observables $\mathcal{O}(x)$ on S [19],

$$\langle \mathcal{P}(x_1)\mathcal{Q}(x_2)\rangle \sim \sum_B \int [\mathcal{D}\mathcal{O}(S)]\Psi_B[\mathcal{O}]^*\Psi_B[\mathcal{O}]\mathcal{P}(x_1)\mathcal{Q}(x_2). \tag{5}$$

Here the sum is taken over backgrounds B that satisfy the prescribed conditions on S. The amplitude Ψ_B for fluctuations about a particular background geometry $(\bar{g}, \bar{\phi})$ is given by

$$\Psi_B[g^3, \phi] \sim e^{-S_0(\bar{g},\bar{\phi})} \int [\mathcal{D}\delta g][\mathcal{D}\delta\phi] e^{-S_2[\delta g, \delta\phi]} \tag{6}$$

where the metric $g = \bar{g} + \delta g$ and the fields $\phi = \bar{\phi} + \delta\phi$. The C_l's of the CMB temperature anisotropies are classic examples of observables that can be calculated from correlators like this. Whilst the full correlator (5) generally involves a sum over several saddle points, for most practical purposes only the lowest action instanton matters.

In no boundary backgrounds like the HM geometry, where a real Euclidean instanton is matched onto a real Lorentzian metric, one can find the correlators by first calculating the 2-point functions in the Euclidean region. The Euclidean correlators are then analytically continued into the Lorentzian region, where they describe the quantum mechanical vacuum fluctuations of the various fields in the state determined by no boundary initial conditions. The path integral unambiguously specifies boundary conditions on the Euclidean fluctuation modes. This essentially determines a reflection amplitude $R(k)$, where k is the wavenumber, which depends on the instanton geometry. The spectra in the Lorentzian, and, in particular, the primordial gravitational wave spectrum [22], depend on the instanton background through $R(k)$.

The relative amplitudes of neighboring geometries can thus be used to predict, from first principles, the precise shape of the primordial fluctuation spectra that we observe. This provides a test of the no boundary proposal and, more generally, an observational discriminant between different proposals for the state of the universe, because the spectra contain a signature of the initial conditions.

Before we illustrate the top-down approach in a simple model in Sec. V, we briefly comment on the role of anthropic selection effects in top-down cosmology.

IV. ANTHROPIC REASONING

In general anthropic reasoning [23] aims to explain certain features of our universe from our existence in it. One possible motivation for this line of reasoning is that the observed values and correlations of certain parameters in particle physics and cosmology appear necessary to ensure life emerges in our universe. If this is indeed the case it seems reasonable to suppose that certain environmental selection effects need to be taken in account in the calculation of probabilities for observations.

It has been pointed out many times, however, that anthropic reasoning is meaningless if it is not implemented in a theoretical framework that determines which parameters can vary and how they vary. Top-down cosmology, by combining the string landscape with the no boundary proposal, provides such a framework.[8] The anthropic principle is implemented in the top-down approach by specifying a set of conditions β in (4) that select the subclass of histories where life is likely to emerge. More specifically, anthropic reasoning in the context of top-down cosmology amounts to the evaluation of conditional probabilities like

$$p(\alpha|O, H, \Psi), \tag{7}$$

where O represents a set of conditions that are required for

[6]An extension of the bottom-up approach invokes the notion of eternal inflation to accommodate the possibility that the position in the moduli space falls into different minima in different places in space, leading to a mosaic structure for the universe. The problem with this approach is that one cannot predict what a typical local observer within such a universe would see. We discuss this in more detail in Sec. VII.

[7]Spatial variations of coupling constants from scalar moduli field fluctuations generate large scale isocurvature fluctuations in the matter and radiation components [21].

[8]Several alternatives to this framework have been proposed, and we comment on some of these in Sec. VII.

the appearance of complex life. The utility and predictivity of anthropic reasoning depends on how sensitive the probabilities (7) are to the inclusion of O. Anthropic reasoning is useful and predictive only if (7) is sharply peaked around the observed value of α, and if the *a priori* theoretical probability $p(\alpha|H, \Psi)$ itself is broadly distributed [24].

Anthropic reasoning, therefore, can be naturally incorporated in the top-down approach. In particular it may provide a qualitative understanding for the origin of certain conditions β that one finds are useful in top-down cosmology. Consider the number of dimensions of space, for example. We have argued that this is best used as a final constraint, but the top-down approach itself does not explain why this particular property of the universe cannot be predicted from first principles. In particular, the top-down argument does not depend on whether four dimensions is the only arena for life. Rather, it is that the probability distribution over dimensions is irrelevant, because we cannot use our observation that $D = 3$ to falsify the theory. But it may turn out that anthropically weighted probabilities (7) are always sharply peaked around $D = 3$. In this case one can essentially interpret the number of dimensions as an anthropic requirement, and it would be an example where anthropic reasoning is useful to understand why one needs to condition on the number of dimensions in top-down cosmology.

We emphasize, however, that the top-down approach developed here goes well beyond conventional anthropic reasoning. Firstly, the top-down approach gives *a priori* probabilities that are more sharply peaked, because it adopts a concrete prescription for the quantum state of the universe—as opposed to the usual assumption that predictions are independent of Ψ. Hence the framework we propose is more predictive than conventional anthropic reasoning.[9]

Top down cosmology is also more general than anthropic reasoning, because there is a wider range of selection effects that can be quantitatively taken in account. In particular the conditions β that are supplied in (4) need not depend on whether they are necessary for life to emerge. The set of conditions generally includes environmental selection effects similar to anthropic requirements, but it also includes chance outcomes of quantum accidents in the early universe that became frozen. The latter need not be relevant to the emergence of life. Furthermore, they cannot be taken in account by simply adding an *a posteriori* selection factor proportional to the number density of some reference object, because they change the entire history of the universe!

[9]Anthropic selection effects have been used to constrain the value of the cosmological constant [25], and the dark matter density [26]. In these studies it is assumed, however, that the *a priori* probability distributions are independent of the state of the universe. This reduces the predictivity of the calculations, and could in fact be misleading.

We illustrate this in the next section, where we derive several predictions of top-down cosmology in a simple toy model.

V. MODELS OF INFLATION

How can one get a nonzero amplitude for the present state of the universe if, as we claim, the metrics in the sum over histories have no boundary apart from the surface S at the present time? We do not have a definitive answer, but one possibility would be if the four dimensional part of the saddle point metric was an inflating universe at early times. Hartle and Hawking [13] have shown that such metrics can be rounded off in the past, without a singular beginning and with curvature bounded well away the Planck value. They give a nonzero value of the no boundary amplitude for almost any universe that arises from an early period of inflation. Thus to illustrate the top-down approach described above, we consider a simple model with a few positive extrema of the effective potential.

We assume the instability of the inflationary phase can be described as the evolution of a scalar order parameter ϕ moving in a double well potential $V(\phi)$, shown in Fig. 1. We take the potential to have a broad flat-topped maximum V_0 at $\phi = 0$ and a minimum at ϕ_1. The value at the bottom is the present small cosmological constant Λ. A concrete example would be gravity coupled to a large number of light matter fields [27]. The trace anomaly generates a potential which has unstable de Sitter space as a self-consistent solution.[10]

We are interested in calculating the no boundary amplitude of an expanding nonempty region of spacetime similar to the one we observe today. In the semiclassical approximation, this will come from one or more saddle points in the action. These correspond to solutions of the Einstein equations. One solution is de Sitter space with the field ϕ sitting at the minimum of the potential $V(\phi)$. This will have a very large amplitude, but will be complete empty and therefore does not contribute to the top-down amplitude for a universe like ours. To obtain an expanding universe with $\Omega_m \sim \mathcal{O}(1)$ and with small perturbations that lead to galaxies, it seems necessary to have a period of inflation.[11]

[10]See [28] for an earlier discussion of trace anomaly inflation with no boundary initial conditions.

[11]One might think it would be more likely for a universe like ours to arise from a fluctuation of the big de Sitter space directly into a hot big bang, rather than from a homogeneous fluctuation up the potential hill that leads to an early period of inflation. The amplitude of a hot big bang fluctuation is much smaller, however, than the amplitude of the inflationary saddle points we discuss below (see also [29]). The latter do not directly connect to the large de Sitter space, but they could be connected with very little cost in action by a thin bridge [30]. We can therefore simplify the analysis by taking the final surface to be one during inflation rather than one at the present time.

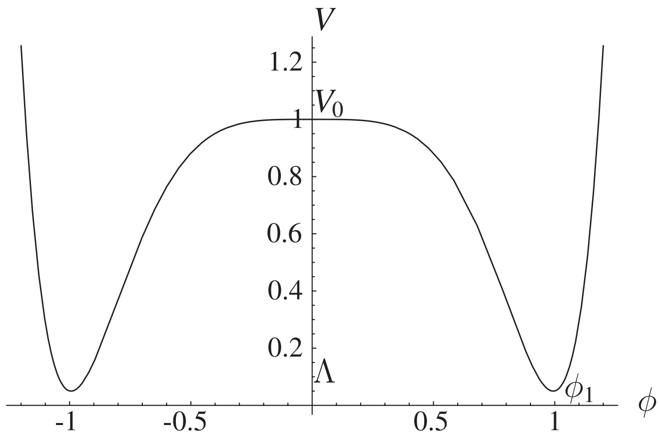

FIG. 1. Inflation occurs naturally in top-down cosmology in theories where the potential has a broad maximum.

We therefore consider the no boundary amplitude[12] $\Phi[\tilde{g}^3, K, \phi]$ for a closed inflating universe bounded by a three-surface S with a large approximately constant Hubble parameter $H = \dot{a}/a$ (and corresponding trace $K = -3\dot{a}/a = -3H$), and a nearly constant field ϕ near the top of V. The value of ϕ on S is chosen sufficiently far away from the minimum of V to ensure there are at least enough efoldings of inflation for the universe at the present time to be approximately flat.

We first calculate the wave function for imaginary K, or real Euclidean $K_e = iK$, and then analytically continue the result to real Lorentzian K. There are two distinct saddle point contributions to the amplitude for an inflating universe in this model [31]. In the first case, the universe is created by the HM instanton with constant $\phi = 0$. Then quantum fluctuations disturb the field, causing it to classically roll down the potential to its prescribed value on S. Histories of this kind thus have a long period of inflation, and lead to a perfectly flat universe today. The action of the HM geometry is given by

$$S^k_{\rm HM}(K) = -\frac{12\pi^2}{V_0}\left(1 - \frac{K_e}{(V_0^2 + K_e^2)^{1/2}}\right) \tag{8}$$

where $K_e = 3b_{,\tau}/b$.

There is, however, a second saddle point contribution which comes from a deformed four sphere, with line element

$$ds^2 = d\tau^2 + b^2(\tau)d\Omega_3^2, \tag{9}$$

where $\phi(\tau)$ varies across the instanton. The Euclidean field equations for $O(4)$-invariant instantons are

$$\phi'' = -K_e\phi' + V_{,\phi}, \qquad K_e' + K_e^2 = -(\phi_{,\tau}^2 + V) \tag{10}$$

where $\phi' = \phi_{,\tau}$. These equations admit a solution, which is part of a Hawking-Turok instanton[13] [32], where ϕ slowly rolls up the potential from some value ϕ_0 at the (regular) South Pole to its prescribed value on the three-surface S. Hence this solution represents a class of histories where the scalar starts as far down the potential as the condition that the present universe be approximately flat allows it to. This naturally leads to fewer efoldings of inflation, and hence a universe that is only approximately flat today. The Euclidean action $S^k_{\rm HT}(K)$ of the deformed four sphere was given in [31] (eq. 4.8), in the approximation that ϕ is reasonably small everywhere.

A comparison of the action of both saddle points shows that the deformed four sphere dominates the path integral for amplitudes with real Euclidean K_e on S. This would seem to suggest that the universe is least likely to start with ϕ at the top of the hill. However, we are interested in the amplitude for an expanding Lorentzian universe, with real Lorentzian K on S. If one analytically continues the action into the complex K_e-plane, one finds the action of the deformed four sphere rapidly increases along the imaginary K_e- axis whereas the real part of $S^K_{\rm HM}$ remains constant, and the dominant contribution to amplitudes for larger K on S actually comes from the HM geometry. The reason for this is that a constant scalar field saves more in gradient energy, than it pays in potential energy for being at the top of the hill. Hence a Lorentzian, expanding universe with large Hubble parameter H is most likely to emerge in an inflationary state, with ϕ constant at the maximum of the potential.[14]

Top down cosmology thus predicts that in models like trace anomaly inflation, expanding universes with small perturbations that lead to galaxies, start with a long period of inflation, and are perfectly flat today. Furthermore, as discussed earlier, the precise shape of the primordial fluctuation spectra can be computed from the Euclidean path integral, by perturbatively evaluating around the HM saddle point.

VI. PREDICTION IN A POTENTIAL LANDSCAPE

The predictions we obtained in the previous section extend in a rather obvious way to models where one has a potential landscape. A generic potential landscape admits a large class of alternative inflationary histories with no boundary initial conditions. There will be HM geometries at all positive saddle points of the potential. For saddle

[12]We work in the K representation of the wave function (see e.g. [30]), where one replaces g^3_{ij} on the three-surface S by $\tilde{g}^3_{ij}$, the three-metric up to a conformal factor, and K, the trace of the second fundamental form. The action S^k_E differs from (3) in that the surface term has a coefficient 1/3.

[13]There is no CdL instanton that straddles the maximum in our model, because we have assumed the potential has a broad flat-topped maximum, $|V''(0)|/H^2 \leq 1$.

[14]The HM instanton has a negative mode in which the scalar field changes the same amount everywhere. However, this mode is removed by the constraint that matter forms, which implies a boundary with large Lorentzian K at large values of the potential. Thus the HM geometry should be an accurate approximation to the no boundary amplitude for an inflating universe.

points with more than one descent direction, there will generally be a lower saddle point with only one descent direction, and with lower action. If this descent direction is sharply curved $|V''(0)|/H^2 > 1$, one would not expect a significant top-down amplitude to come from the saddle point. This is because HM geometries at narrow positive saddle points have several inhomogeneous negative modes, which are not removed by the constraint that matter forms.[15] Thus only broad saddle points with a single descent direction will give rise to amplitudes for universes like our own. Only a few of the saddle points will satisfy the demanding condition that they be broad, because it requires that the scalar field varies by order the Planck value across them.

In an unconstrained path integral, the prefactor $-1/V_0$ of the Euclidean action (8) would strongly favor extrema with small V_0. The requirement that the primordial fluctuations be sufficiently large to form galaxies, however, sets a lower bound on the value of V_0. Furthermore, the fact that we are only interested in internal spaces of the standard model may provide a natural explanation for why the observed primordial amplitude is somewhat larger than what seems required to produce galaxies.

Because the dominant saddle points are in the semiclassical regime, the solutions will evolve from the saddle points to the neighboring minima of V. Thus top-down cosmology predicts that only a few of the possible vacua in the landscape will have significant amplitudes.

VII. ALTERNATIVE PROPOSALS

To conclude, we briefly comment on a number of different approaches to the problem of initial conditions in cosmology, and we clarify in what respect they differ from the top-down approach we have put forward.[16]

We have already discussed the pre-big bang cosmologies [10], where one specifies initial conditions in the infinite past and follows forward in time a single semiclassical history of the universe. pre-big bang cosmology is thus based on a bottom-up approach to cosmology. It requires one to postulate a fine-tuned initial state, in order to have a smooth deterministic transition through the big crunch singularity.

We have also discussed the anthropic principle [23]. This can be implemented in top-down cosmology, through the specification of final boundary conditions that select histories where life emerges. Anthropic reasoning within the top-down approach is reasonably well-defined, and useful to the extent that it provides a qualitative understanding for the origin of certain late time conditions that one finds are needed in top-down cosmology.

A. Eternal inflation

A different approach to string cosmology has been to invoke the phenomenon of eternal inflation [11] to populate the landscape. There are two different mechanisms to drive eternal inflation, which operate in different moduli space regions of the landscape. In regions where the moduli potential monotonically increases away from its minimum, it is argued that inflation can be sustained forever by quantum fluctuations up the potential hill. Other regions of the landscape are said to be populated by the nucleation of bubbles in metastable de Sitter regions. The interior of these bubbles may or may not exit inflation, depending on the shape of the potential across the barrier.

Both mechanisms of eternal inflation lead to a mosaic structure for the universe, where causally disconnected thermalized regions with different values for various effective coupling constants are separated from each other by a variety of inflating patches. It has proven difficult, however, to calculate the probability distributions for the values of the constants that a local observer in an eternally inflating universe would measure.[17] This is because there are typically an infinite number of thermalized regions.

One could also consider the no boundary amplitude for universes with a mosaic structure. However, these amplitudes would be much lower than the amplitudes for final states that are homogeneous and lie entirely within a single minimum, because the gradient energy in a mosaic universe contributes positively to the Euclidean action. Histories in which the universe eternally inflates, therefore, hardly contribute to the no boundary amplitudes we measure. Thus the global structure of the universe that eternal inflation predicts, differs from the global structure predicted by top-down cosmology. Essentially this is because eternal inflation is again based on the classical idea of a unique history of the universe, whereas the top-down approach is based on the quantum sum over histories. The key difference between both cosmologies is that in the proposal based on eternal inflation there is thought to be only one universe with a fractal structure at late times, whereas in top-down cosmology one envisions a set of alternative universes, which are more likely to be homogeneous, but with different values for various effective coupling constants.

It nevertheless remains a challenge to identify predictions that would provide a clear observational discriminant between both proposals.[18] We emphasize, however, that

[15]There exists in general also a (lower action) CdL instanton around sharply curved maxima, but this too presumably has a negative mode [33] that is not removed by the above constraint.

[16]We believe the framework described here addresses the concerns raised in [34] regarding a top-down approach cosmology.

[17]See however [35] for recent progress on this problem.

[18]It has been argued [36] that eternal inflation in the string landscape predicts we live in an open universe. It seems this is not a prediction of no boundary initial conditions on the string landscape; the HM geometries we discussed occur generically, and thus provide a counterexample.

even a precise calculation of conditional probabilities in no boundary cosmology, which takes in account the backreaction of quantum fluctuations, will make no reference to the exterior of our past light cone. Indeed, the top-down framework we have put forward indicates that the mosaic structure of an eternally inflating universe is a redundant theoretical construction, which should be excised by Ockham's razor.[19] It appears unlikely, therefore, that something like a 'volume-weighted' probability distribution—which underlies the idea of eternal inflation—can arise from calculations in top-down cosmology. The implementation of selection effects in both approaches is fundamentally different, and this should ultimately translate into distinct predictions for observations.

VIII. CONCLUDING REMARKS

In conclusion, the bottom-up approach to cosmology would be appropriate, if one knew that the universe was set going in a particular way in either the finite or infinite past. However, in the absence of such knowledge one is required to work from the top down.

In a top-down approach one computes amplitudes for alternative histories of the universe with final boundary conditions only. The boundary conditions act as late time constraints on the alternatives and select the subclass of histories that contribute to the amplitude of interest. This enables one to test the proposal, by searching among the conditional probabilities for predictions of future observations with probabilities near one. In top-down cosmology the histories of the universe thus depend on the precise question asked, i.e. on the set of constraints that one imposes. There are histories in which the universe eternally inflates, or is 11-dimensional, but we have seen they hardly contribute to the amplitudes we measure.

A central idea that underlies the top-down approach is the interplay between the fundamental laws of nature and the operation of chance in a quantum universe. In top-down cosmology, the structure and complexity of alternative universes in the landscape is predictable from first principles to some extent, but also determined by the outcome of quantum accidents over the course of their histories.

We have illustrated our framework in a simple model of gravity coupled to a scalar with a double well potential, and a small fundamental cosmological constant Λ. Imposing constraints that select the subclass of histories that are three-dimensional and approximately flat at late times, with sufficiently large primordial perturbations for structure formation to occur, we made several predictions in this model.

In particular we have shown that universes within this class are likely to emerge in an inflationary state. Furthermore, we were able to identify the dominant inflationary path as the history where the scalar starts all the way at the maximum of its potential, leading to a long period of inflation and a perfectly flat universe today. Moreover, one can calculate the relative amplitudes of neighboring geometries by perturbatively evaluating the path integral around the dominant saddle point. Neighboring geometries correspond to small quantum fluctuations of various continuous quantities, like the temperature of the CMB radiation or the expectation values of moduli fields. In inflationary universes these fluctuations are amplified and stretched, generating a pattern of spatial variations on cosmological scales in those directions of moduli space that are relatively flat. The shape of these primordial spectra depends on the (no) boundary conditions on the dominant geometry and provides a strong test of the no boundary proposal.

When one extends these considerations to a potential that depends on a multidimensional moduli space, one finds that only a few of the minima of the potential will be populated, i.e. will have significant amplitudes.

The top-down approach we have described leads to a profoundly different view of cosmology, and the relation between cause and effect. Top down cosmology is a framework in which one essentially traces the histories backwards, from a spacelike surface at the present time. The no boundary histories of the universe thus depend on what is being observed, contrary to the usual idea that the universe has a unique, observer independent history. In some sense no boundary initial conditions represent a sum over all possible initial states. This is in sharp contrast with the bottom-up approach, where one assumes there is a single history with a well-defined starting point and evolution. Our comparison with eternal inflation provides a clear illustration of this. In a cosmology based on eternal inflation there is only one universe with a fractal structure at late times, whereas in top-down cosmology one envisions a set of alternative universes, which are more likely to be homogeneous, but with different values for various effective coupling constants.

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[19]Or on the basis of holography [37]?

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