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Interacting dark energy, holographic principle, and coincidence problem

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The interacting and holographic dark energy models involve two important quantities. One is the characteristic size of the holographic bound and the other is the coupling term of the interaction between dark energy and dark matter. Rather than fixing either of them, we present a detailed study of theoretical relationships among these quantities and cosmological parameters as well as observational constraints in a general formalism. In particular, we argue that the ratio of dark matter to dark energy density depends on the choice of these two quantities, thus providing a mechanism to change the evolution history of the ratio from that in standard cosmology such that the coincidence problem may be solved. We investigate this problem in detail and construct explicit models to demonstrate that it may be alleviated provided that the interacting term and the characteristic size of holographic bound are appropriately specified. Furthermore, these models are well fitted with the current observation at least in the low redshift region.

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I. INTRODUCTION

Recent astronomical observations indicate that our Universe is currently undergoing an epoch of accelerated expansion [1–6]. Following the standard Friedmann-Robertson-Walker cosmology, such an expansion implies the existence of a dark energy (DE) component to the mass-energy density of the Universe.[1] At present it is fair to say that disclosing the nature of DE is one of the central problems in the research of both cosmology and theoretical physics (for recent reviews, see [11–13]). In this direction we are faced with many fundamental and difficult issues, among which the following two open questions are of particular importance. The first one is on the nature and dynamical properties of the dark energy. Currently it is not clear yet whether DE can be described by a cosmological constant which is independent of time, or by dynamical fields such as quintessence, K essence, tachyon fields, or phantom fields. The second is the coincidence problem, dubbed as "why are the densities of matter and dark energy of precisely the same order today?" [14].

To shed light on these two open questions, some interesting DE models were proposed recently. Those models can be divided into two categories, i.e. the holographic dark energy (HDE) models and the interacting DE models. The former stems from the holographic hypothesis [15–17] and can provide an intriguing way to interpret the dynamics of DE, while it is suggested that the latter can help to understand the coincidence problem by considering the possible interaction between dark energy and cold dark matter [18–22].

Let us first start with a close look on the holographic dark energy model motivated from the holographic hypothesis, which has gradually been believed to be a fundamental principle in the quantum theory of gravity. According to this principle, the number of degrees of freedom for a system within a finite region should be finite and is bounded roughly by the area of its boundary. While in a cosmological setting, the challenge is to put a reasonable and well-defined upper bound on the entropy of the Universe. Motivated by a Bekenstein entropy bound, it seems plausible to require that for an effective quantum field theory in a box of size L with UV cutoff Λ, the total entropy should satisfy the relation

$$L^3\Lambda^3 \leq S_{\rm BH} = \pi L^2 M_p^2, \tag{1}$$

where $S_{\rm BH}$ is the entropy of a black hole with the same size L. but further consideration indicates that to saturate this inequality some states with Schwarzschild radius much larger than the box size have to be counted in. As a result, a stronger entropy bound has been proposed in [15], requiring that the total energy of a system with size L should not exceed the mass of a black hole with the same radius, namely,

$$L^3\Lambda^4 = L^3\rho_\Lambda \leq LM_p^2. \tag{2}$$

While saturating this inequality by choosing the largest L it gives rise to a holographic energy density

$$\rho_\Lambda = 3c^2M_p^2L^{-2}, \tag{3}$$

where c is a dimensionless constant. Then the key issue is what possible physical scale one can choose as the cutoff L constrained by the fact of the current acceleration of the universe. Originally, the natural choice is to identify the Hubble horizon as L, however, as pointed out in [16], this will lead to a wrong equation of state for dark matter which conflicts with the ordinary one in standard cosmology. As a

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[1]For recent discussion on the possibility of constructing an accelerating universe without dark energy, see for instance [7–10].

result, in [17] Li proposed to take the future event horizon as the largest size L, which gives rise to desired results, and then stimulate a lot of interests and discussions in this subject [23]. However, there are some unsatisfactory points in this conjecture. First, it still remains puzzling how the current evolution of dark energy density can be determined by the *future* event horizon. Second, the coincidence problem can hardly be solved in this context.

As pointed out in [22], the reason that one is forced to take the future event horizon is based on the assumption that the energy densities of dark energy and dark matter evolve independently. However, if there exists interaction between DE and dark matter (DM), then the cutoff L is not necessarily identified as the future event horizon. As a matter of fact, the interaction between DE and DM is proposed to solve the coincidence problem and has been discussed in many recent works [24]. This can be accomplished by introducing the coupling terms in the equations of state for matter and dark energy densities, which can bring the ratio of these two ingredients into a constant at late times. From the theoretical point of view this sort of coupling is completely possible due to the unknown nature of DM and DE. In addition, this proposal is compatible with the current observations such as the SNIa and Wilkinson Microwave Anisotropy Probe (WMAP) data [19] and even favored in some circumstances as suggested in [25]. But until now, only certain special interacting terms have been considered in existing literatures.

Now based on the above discussion, it is natural to ask if we could combine these two theoretical proposals together so as to improve our understanding of dark energy. This is the main purpose of our paper. Although there are many existing works in both directions, most discussions in those works only considered specific characteristic sizes or interacting terms. For instance, the characteristic size is usually assumed to be the future event horizon after the work [17] and the interaction term is assumed to take a form as $3b^2H\rho$ where b is a coupling constant. However, there is no strong theoretical motivations for these choices. In this paper, rather than fixing either the interacting term or the holographic characteristic size, we intend to investigate the nature of interacting and holographic DE and the coincidence problem in a more general formalism.

Our paper is organized as follows: We first present brief reviews on interacting dark energy and holographic dark energy in Secs. II and III, respectively. In particular, given the conditions that our Universe is currently accelerating and the ratio of dark matter density to dark energy density deceases, we derive general constraints on the relations among the interacting term, holographic size, and the equation of state parameter of dark energy ω. Then we turn to the coincidence problem in Sec. IV, under the simplest requirement that the ratio of dark matter to dark energy density be constant. In Sec. V we consider the case that the ratio can vary with time slowly and demonstrate how the coincidence problem can be alleviated through some specific examples by providing appropriate interacting terms and holographic sizes.

II. INTERACTING DARK ENERGY

We start with the standard Friedmann equations in which DE and DM are assumed to be independent and there is no interaction between them. Provided our current Universe is dominated by dark energy with the state equations $p = \omega\rho_\Lambda$ and cold dark matter with $p = 0$, these equations read as

$$\rho_\Lambda + \rho_M = 3M_p^2H^2, \tag{4}$$

$$\dot{H} = -\tfrac{1}{2}M_p^{-2}[\rho_\Lambda(1+\omega) + \rho_M], \tag{5}$$

$$\dot{\rho}_\Lambda + \dot{\rho}_M = -3H[\rho_\Lambda(1+\omega) + \rho_M], \tag{6}$$

where $H = \dot{a}/a$ is the Hubble factor. It is well known that Eqs. (4)–(6) are not independent and any one of them can be derived from the other two. By introducing $\Omega_\Lambda = \rho_\Lambda/(3M_p^2H^2)$ and $\Omega_M = \rho_M/(3M_p^2H^2)$, the first Friedmann equation also can be written as $\Omega_\Lambda + \Omega_M = 1$.

Now we proceed to interacting dark energy models in which dark matter and dark energy are postulated to be coupled such that dark energy can decay into cold dark matter. As a result, the last equation can be written as the combination of the following two evolving equations,

$$\dot{\rho}_\Lambda = -3H\rho_\Lambda(1+\omega) - Q, \tag{7}$$

$$\dot{\rho}_M = -3H\rho_M + Q, \tag{8}$$

where Q denotes the interacting term. To be a realistic model, the interacting DE model should satisfy the observational constraints. First, we consider the constraint on Q by the observation that our Universe is currently accelerating. Since the ratio of dark matter to dark energy plays a special role and its dynamics is a major subject of this paper, for convenience we denote the ratio ρ_M/ρ_Λ by r which is related to Ω_Λ by $1 + r = 1/\Omega_\Lambda$. Then from (4) and (5) we have

$$\dot{H} = -\frac{3}{2}\left(1 + \frac{\omega}{1+r}\right)H^2. \tag{9}$$

Notice that Eq. (9) always holds no matter whether the interaction is taken into account or not. The solution to this equation can be formally written as

$$H = H_0 e^{-3/2\int_0^x (1+[\omega/(1+r)])dx}, \tag{10}$$

where $x \equiv \ln a$. As a result, the requirement $\ddot{a} > 0$ leads to

$$1 + r + 3\omega < 0. \tag{11}$$

On the other hand, from (7) and (8) we find that the interacting term has the following general form:

$$\tilde{Q} \equiv \frac{Q}{H\rho_\Lambda} = \frac{\dot{r}}{(1+r)H} - \frac{r}{1+r}3\omega, \tag{12}$$

where H can be absorbed by redefining $\dot{r}/H = dr/dx \equiv r'$, such that

$$\tilde{Q} = \frac{1}{1+r}(r' - 3\omega r). \tag{13}$$

Then from (11) one finds that

$$\tilde{Q} > r + \frac{r'}{1+r}. \tag{14}$$

Furthermore, it is expected that the ratio of dark matter density to dark energy density decreases with the evolution of the Universe, namely, $r' < 0$. This requirement together with the previous one (11) implies that the interacting term should satisfy the following constraint:

$$r + \frac{r'}{1+r} < \tilde{Q} < \frac{-3\omega r}{1+r}. \tag{15}$$

III. HOLOGRAPHIC DARK ENERGY

Now we turn to the holographic dark energy models. As introduced in Sec. I, in this context the dark energy density is assumed to be saturated in the region with size L,

$$\rho_\Lambda = 3c^2 M_p^2 L^{-2}. \tag{16}$$

Comparing this bound with the first Friedmann equation we easily obtain a relation between the characteristic size L and the Hubble factor H as

$$LH = \sqrt{1+r}c. \tag{17}$$

In general the characteristic size L needs to be chosen in such a way that the dark energy can be responsible for the acceleration of the Universe. Furthermore, such a choice should not conflict with the state equations of dark energy and dark matter. As a result, when there is no interaction between DE and DM, i.e. $\tilde{Q} = 0$, L is conventionally taken as the future event horizon [17,23] so as to fit the observational data. However, as explained in Sec. V, the coincidence problem is hardly solved by pure HDE alone. Therefore, in the following discussion on HDE models, we will consider the case with nonvanishing $\tilde{Q}$. Moreover, instead of fixing L or the interacting term $\tilde{Q}$ at the beginning, we will take a more phenomenological view and consider them to be free dynamical quantities but constrained by observations.

Now, as in the previous section, from the requirements of an accelerating universe and decreasing ratio of DM density to DE density, one can derive the constraints on those quantities. Taking the derivative with respect to time on both sides of Eq. (17) and using (13) leads to a relation between L and $\tilde{Q}$ as

$$\tilde{Q} = r' - 2r\left(\frac{L'}{L} - \frac{3}{2}\right). \tag{18}$$

Now due to the constraint (15) we find the size L should satisfy

$$\frac{3}{2}\left(\frac{r'}{3r} + \frac{\omega}{1+r} + 1\right) < \frac{L'}{L} < \frac{r'}{2(1+r)} + 1. \tag{19}$$

Thus we find in a general formalism of interacting holographic dark energy, the interacting term and the characteristic size of holographic bound are constrained by the inequalities (15) and (19), respectively.

Alternatively from (13) and (18) we may have a relation between L and ω as

$$2\frac{L'}{L} = 3\left(1 + \frac{\omega}{1+r}\right) + \frac{r'}{1+r}. \tag{20}$$

Thus $r' < 0$ leads to

$$\frac{L'}{L} < \frac{3}{2}\left(1 + \frac{\omega}{1+r}\right). \tag{21}$$

Furthermore, replacing the parameter ω appearing in (15) by L we may find the following inequality:

$$-2r\left(\frac{L'}{L} - \frac{3}{2}\right) > \tilde{Q} > 2\left(\frac{L'}{L} + \frac{r}{2} - 1\right). \tag{22}$$

In summary, we find for interacting holographic dark energy models, L, $\tilde{Q}$, ω, and r are not independent quantities but related by Eqs. (18) and (20) or (13) and (20) since among these three equations only two of them are independent. Thus, given any two of them, the dyanamics of the other two can be determined. For instance, if we specify the interaction term $\tilde{Q}$ and the characteristic size L, then the dynamics of ω and r may be determined and vice versa. However if only one of them is specified, then the dynamics of the other three cannot be uniquely fixed. For example, in an interacting HDE model with the future event horizon as the characteristic scale L, i.e.

$$L = a(t)\int_t^\infty \frac{1}{a(t')}dt', \tag{23}$$

which gives rise to a relation $\dot{L} = LH - 1$. Then from (20) it is easy to derive the following relation:

$$\frac{1}{2}(1 + r + 3\omega) + \frac{\sqrt{1+r}}{c} + \frac{r'}{2} = 0. \tag{24}$$

In this case $r' < 0$ requires that

$$1 + r + 3\omega > -\frac{2\sqrt{1+r}}{c}. \tag{25}$$

If we further specify the interaction term, e.g. $\tilde{Q} = 3b^2(1+r)$ as in Refs. [24], then the dynamics of ω and r can be determined uniquely, as discussed in [26]. However, since our goal is to investigate the coincidence problem, we intend to put constraints on the evolution of r and then explore what expressions the other quantities including L, $\tilde{Q}$, and ω may take. This is what we are going to do in the next two sections.

IV. COINCIDENCE

Before we proceed, we first demonstrate how the coincidence problem arises in the standard cosmology with a cosmological constant. Setting $Q = 0$ in (7) and (8) will lead to $r' = 3\omega r$ [see also (13)]. Now from (11) one finds that

$$\frac{d\ln r}{dx} = 3\omega < -r - 1 < -1,$$

which means during acceleration r decreases faster than a^{-1}. Furthermore, from Friedmann equations, one finds that

$$r = r_0(a/a_0)^{-3}, \tag{26}$$

and

$$a/a_0 = C(e^{\lambda t/t_0} - e^{-\lambda t/t_0})^{2/3}, \tag{27}$$

where C and λ are $O(1)$ constants which can be related to the current values of Ω_Λ. Then it is easy to see when $t \ll t_0$, $r \propto t^{-2}$ and r decreases quadratically as expected for a matter dominated universe and when $t \gg t_0$, r deceases exponentially as expected in a dark energy dominated universe. Then it is only when t is around t_0 that $r \sim O(1)$.

It is expected that adding interaction may change the dynamics of r greatly. In this section we consider the simplest possibility with $\dot{r} = 0$, which implies $\rho_\Lambda \propto \rho_M \propto H^2$. It is worthwhile to stress that this situation only occurs at late times. Suppose the ratio r is a constant, i.e. $r = r_0$. We immediately obtain the following equations:

$$\tilde{Q} = \frac{-3\omega r_0}{1 + r_0}, \tag{28}$$

$$\dot{L} = \frac{3c}{2}\sqrt{1 + r_0}\left(1 + \frac{\omega}{1 + r_0}\right). \tag{29}$$

In addition, the Hubble factor is inversely proportional to the characteristic size L as $H = \sqrt{1 + r_0}cL^{-1}$. Therefore, specifying any one of the quantities $\tilde{Q}$, L, ω, the dynamics of the other two can be uniquely determined from the above equations. We classify some possibilities in the following subsections.

A. $\dot{r} = 0$ with only interaction term specified

One possible choice for the interaction term is setting $\tilde{Q} = 3b^2(1 + r_0)$ as in previous references, where b is a constant. Then from (28) we find that ω is fixed as

$$\omega = -b^2\frac{(1 + r_0)^2}{r_0}. \tag{30}$$

Consequently, the solutions to H and L can be obtained from (29) as

$$H = H_0 a^{-3/2(1-b^2-(b^2/r_0))}. \tag{31}$$

In addition, from (15) one finds that

$$3b^2 > \frac{r_0}{(1 + r_0)} = \Omega_M. \tag{32}$$

Therefore, it is obvious that in noninteracting models (i.e. $b = 0$) $\dot{r} = 0$ and $\ddot{a} > 0$ cannot be achieved simultaneously. This can be considered as an important hint for the need of interacting dark energy.

B. $\dot{r} = 0$ with only holographic characteristic size specified

As shown in the previous section, specifying the holographic characteristic size will determine $\tilde{Q}$ and ω since r has already been fixed to be r_0. Here we consider the HDE model with L being the future event horizon. From (28) and (29) we find correspondingly that the parameters ω and $\tilde{Q}$ have to be constants as well:

$$\tilde{Q} = \tilde{Q}_0 = r_0\left(1 + \frac{2}{\sqrt{1 + r_0}c}\right). \tag{33}$$

$$\omega = \omega_0 = -\frac{1}{3}\left(1 + r_0 + \frac{2\sqrt{1 + r_0}}{c}\right). \tag{34}$$

Using the current data $\rho_{M0} \simeq 0.25$, $\rho_{\Lambda 0} \simeq 0.72$ and setting $c = 1$, we find

$$r_0 \simeq 0.35, \qquad \omega_0 \simeq -1.22, \tag{35}$$

which is a phantom-preferred model. From (9) we find that

$$a \sim t^{2(1+r_0)/3(1+r_0+\omega_0)}, \tag{36}$$

while

$$\rho_\Lambda \sim \rho_M \sim t^{-2}. \tag{37}$$

C. $\dot{r} = 0$ with both interaction term and holographic characteristic size specified

If both L and $\tilde{Q}$ are specified as in previous subsections, then from (30) and (34) it is easy to see $\dot{r} = 0$ can be reached only when the constant b^2 takes the value as

$$b^2 = \frac{-\omega_0 r_0}{(1 + r_0)^2} \simeq 0.24. \tag{38}$$

V. SOFT COINCIDENCE

The above discussion shows that the interaction between DE and DM can lead to a constant r. Although it is not clear how to obtain a r of $O(1)$ size at early times, this simple strategy can be used to account for particular situations (e.g. late time evolution of the Universe). Nevertheless, there is no strong motivation for setting r

to be a constant. It is worthwhile to explore some more realistic models in which r varies slowly with time. We discuss this possibility in detail here.

Advocated by the above discussion, one expects that a certain amount of interaction can alleviate the coincidence problem. One possibility which has been proposed in [19] is to allow the ratio of two energy densities to vary slowly but require that there are two positive solutions $r_\pm$ to $\dot{r} = 0$. Then the coincidence can be alleviated if r_- is close to $O(1)$ as the ratio r evolves from the unstable but finite maximum r_+ to a stable minimum r_- at late time, instead of from ∞ to 0. To demonstrate this possibility we consider again the model with an interaction term $\tilde{Q} = 3b^2(1 + r)$ and a constant ω. From Eq. (13), we have

$$r' = 3b^2(1 + r)^2 + 3\omega r. \tag{39}$$

Obviously, setting $r' = 0$ the equation has two positive solutions with a relation $r_+ r_- = 1$. It is also possible to show that the ratio will run from an unstable but finite maximum r_+ to a stable minimum r_- at late time [19].

However, this does not occur in the context of pure holographic dark energy if one chooses the future event horizon as the characteristic size L. In the absence of interaction, the dynamics of r is described by

$$r' = -r\left(1 + \frac{2}{c\sqrt{1 + r}}\right). \tag{40}$$

Defining $\sqrt{1 + r} = y$ ($y \geq 1$) leads to

$$2cy^2 y' = (1 - y^2)(cy + 2). \tag{41}$$

There is only one positive solution to $r' = 0$ with $y = 1$ or $r = 0$. Thus, as in standard cosmology r runs from infinity to zero and consequently the coincidence problem still exits in this setting.

Next we intend to propose an alternative way to alleviate the coincidence problem. That is, it might not be necessary to have both an unstable finite maximum and a stable minimum close to $O(1)$. The later is more important in the coincidence problem and presumably is determined by the physics effective at the current evolution of the Universe. The former is more related to the early evolution of the Universe, and whether or not an $O(1)$ initial condition can be obtained is determined by physics beyond the scope of this work. Therefore, we would rather leave the question concerning the existence of a positive maximum open and concentrate on the models with a positive stable minimum at late time. In particular, if we find this stable value is not quite far from the current observation, then the coincidence problem may be alleviated as the Universe has a long time to stay at this stage with a similar ratio. Now, as an example, consider adding the interaction term $\tilde{Q} = 3b^2(1 + r)$ into the holographic dark energy model presented above. Then we find Eq. (41) is changed to

$$cy^2 y' = \frac{c}{2}(3b^2 - 1)y^3 - y^2 + \frac{c}{2}y + 1. \tag{42}$$

Provided $c > 0$ and $3b^2 < 1$, it still has only one positive solution to $y' = 0$ but *not* at $r = 0$. The exact position of the minimum depends on the values of c and b. For explicitness we illustrate the evolution of r in Fig. 3 with $c = 1$ and $b^2 = 0.12$, which is described by the dotted-dashed curve.

Moreover, in the discussion of (39) from which two positive solutions are obtained for $r' = 0$, we have made two assumptions, i.e. the coefficients b^2 and ω are constants. A time dependent b^2 or ω might change this situation. In addition, as mentioned in Sec. III, specifying any two of L (or ρ_Λ), $\tilde{Q}$, ω, and r will determine the other two uniquely. In principle, ω might be different from those assumed in the discussion of (39) and consequently will lead to different results. Below we will discuss this in more detail through the following models.

A. Model 1: Given $\tilde{Q}$ with time dependent b^2

In this model we assume that

$$b^2 = b_c^2 e^{-r/R},$$

i.e.

$$\tilde{Q} = 3b_c^2(1 + r)e^{-r/R}, \tag{43}$$

where $b_c^2 =$ constant. The interaction given by (43) decreases exponentially as r increases and consequently at early times when $r \gg R$, $\tilde{Q}$ is very small and thus can be ignored. Therefore, in this model the early age of the Universe can be described by the standard Friedmann equations without interaction. The interaction becomes important only at late time and will lead to a stable minimum which can mitigate the coincidence problem. As (13), one finds that in this case

$$r' = 3b_c^2 e^{-r/R}(1 + r)^2 + 3\omega r, \tag{44}$$

and subsequently from (18) one has

$$\frac{L'}{L} = \frac{3}{2}[1 + \omega + b_c^2(1 + r)e^{-r/R}]. \tag{45}$$

Now to obtain more explicit results we have two options. One is to set ω to be a constant, for example $\omega = -1$. This is completely possible in the presence of holographic dark energy and the corresponding size of holographic bound is determined by

$$\frac{L'}{L} = \frac{3}{2}b_c^2(1 + r)e^{-r/R}. \tag{46}$$

With this choice the Eq. (44) has only one solution to $r' = 0$ for small R. For example, when $b_c^2 = 0.3$, $R = 0.25$, the solution to $r' = 0$ is $r_f = 0.196$. The second option, instead of specifying ω, is setting an appropriate characteristic size L which can also lead to the same results obtained above. To show this, we consider the HDE model with L being the future horizon. From (45) one has

$$3\omega = -1 - 3b_c^2(1+r)e^{-r/R} - \frac{2}{\sqrt{1+rc}}, \quad (47)$$

which leads to

$$\frac{d\ln r}{dx} = \frac{r'}{r} = -1 + 3b_c^2 e^{-r/R}\frac{1+r}{r} - \frac{2}{\sqrt{1+rc}}. \quad (48)$$

For $b_c^2 = 0.3$, $R = 0.5$, and $c = 1$, the solution to $r' = 0$ is $r_f = 0.246$. In addition, one can check that for $r > r_f$ one always has $\dot{r} < 0$. In addition, in this case ω also can across -1 as in many DE models. For instance, for $b_c^2 = 0.1$, $c = 1$ and $R = 1$, $r_f = 0.102$, one finds that ω across -1 at $r = 0.35$.

B. Model 2: Given $\tilde{Q}$ with time-independent b^2

In this subsection we will consider the situation where the dynamics of r at late time can be approximated by a power law dependence on a, i.e.

$$r = r_f + (r_0 - r_f)a^{-k}, \quad (49)$$

under the assumption that $\tilde{Q} = 3b_c^2(1+r)$ where $k = 3b_c^2/r_f$ and b_c^2 is a constant. As in Sec. III, now one can solve for ω and ρ_Λ. From (7) and (8) it is easy to find that

$$\omega = -b_c^2 r - \left(2 + \frac{1}{r_f}\right)b_c^2. \quad (50)$$

Then from (39), one finds that the only solution to $r' = 0$ is $r = r_f$. Note that this result does not dependent on the choice of k, as what can be obtained from (49) directly. The DE density is found to be

$$\rho_\Lambda = \rho_\Lambda^0 a^{-3(1-b_c^2-b_c^2/r_f)}. \quad (51)$$

Then from (10) one finds that

$$H^2 = H_0^2 a^{-3(1-b_c^2-b_c^2/r_f)}\frac{1+r}{1+r_0}.$$

From (11) one finds that the condition $\ddot{a} > 0$ requires that

$$b_c^2 > \frac{1+r_0}{3(r_0 + 2 + 1/r_f)}.$$

As an example, $r_f = 0.2$ leads to $b_c^2 > 0.06$. To compare the predictions of this model with low redshift observations, the distance moduli vs redshift are plotted in Fig. 1.

As shown in Sec. III, given $\tilde{Q}$ and r, the characteristic size L and the dynamics of ω can be determined uniquely. In fact, from (51) one finds immediately that in HDE models

$$L \propto a^{3/2(1-b_c^2-b_c^2/r_f)}. \quad (52)$$

Moreover, from another point of view, (49) and (50) can also be considered as the consequences of the characteristic size given by (52).

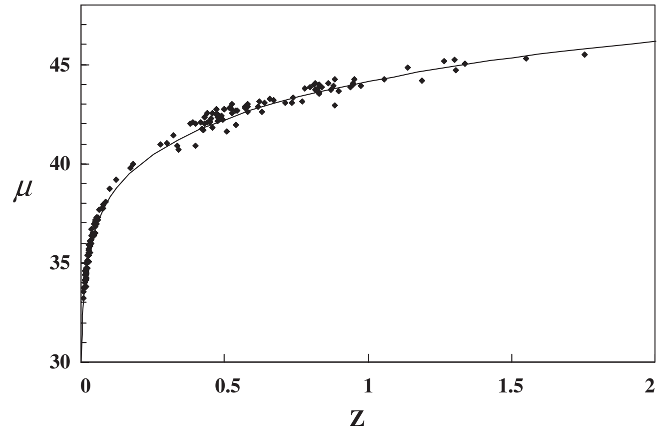

FIG. 1. Distance moduli vs redshift plot for model 2 ($b_c^2 = 0.12$, $r_f = 0.2$). The data points are from the gold sample of type Ia supernovae of [1].

C. Model 3: Given the characteristic scale of holographic bound L

Similar to the previous subsection, here we consider the situation where the late time evolution of r to r_f can be approximated by an exponential function of a, i.e. $r = r_f(1 + \gamma e^{-\lambda a})$ where both λ and γ are constants. For simplicity, in the following discussion we set $\gamma = 1$ and thus we have

$$r = r_f(1 + e^{-\lambda a}), \quad (53)$$

which leads to $r_f = r_0/(1 + e^{-\lambda})$. Nevertheless, rather than fixing the interaction term $\tilde{Q}$, we consider in this subsection the HDE model with the characteristic size L being the future event horizon. As discussed in Sec. III, from (13) and (24), the interaction $\tilde{Q}$ and ω are found to be

$$\begin{aligned} \tilde{Q} &= r - \lambda a(r - r_f) + \frac{2r}{c\sqrt{1+r}}, \\ \omega &= -\frac{1}{3}\left(1 + r + \frac{2\sqrt{1+r}}{c} - \frac{\lambda a e^{-\lambda a}}{1+e^{-\lambda}}r_0\right). \end{aligned} \quad (54)$$

From (11) and (25) one finds that $\ddot{a} > 0$ and $r' < 0$ require that

$$-\frac{2\sqrt{1+r}}{c} < 1 + r + 3\omega < 0.$$

Then from (54) one has

$$-\frac{2\sqrt{1+r}}{c} < -\frac{2\sqrt{1+r}}{c} + \frac{\lambda a e^{-\lambda a}}{1+e^{-\lambda}}r_0 < 0.$$

Since $\lambda a e^{-\lambda a} \le e^{-1}$,

$$\frac{\lambda a e^{-\lambda a}}{1+e^{-\lambda}} < \frac{e^{-1}}{1+e^{-\lambda}} < e^{-1}.$$

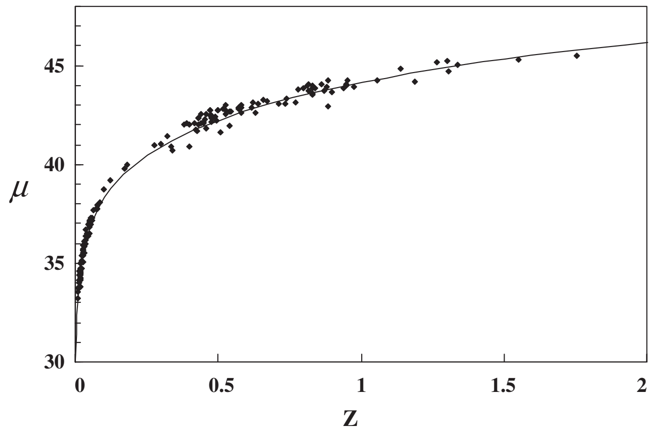

FIG. 2. Distance moduli vs redshift plot for model 3 ($c = 3$, $\lambda = 0.5$). The data points are from the gold sample of type Ia supernovae of [1].

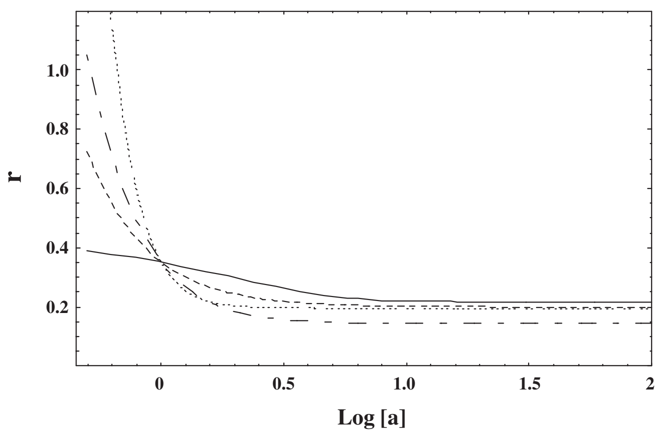

FIG. 3. r vs $\log[a]$ (see text for explanations and parameters used for different curves).

It is easy to check that the above requirement can be satisfied for any λ if $c < 2e/r_0 \simeq 15$. Again we can compare the predictions of this model with low redshift observations, as shown in Fig. 2.

Moreover, to compare the above three models and the interacting HDE model discussed at the beginning of this section [see (42)], the dynamics of r in these models are plotted in Fig. 3 in which the dotted curve corresponds to model 1, the dashed curve to model 2, the solid curve to model 3 and the dotted-dashed curve to (42). The parameters used for models 2 and 3 are the same as those for Fig. 2 and 3. For model 1, the parameters are given in the sentence following (44). For the curve corresponding to (42), c and b^2 in (42) are taken to be 1 and 0.12, respectively.

VI. DISCUSSION

We have presented a general formalism of interacting and holographic dark energy in this paper. Let us summarize the main results as follows. First we pointed out that in this general formalism both the characteristic size of holographic bound L and the coupling term of interaction Q for dark energy are not necessarily fixed as in previous references where these two sorts of models are separately investigated. Given the conditions that our Universe is currently accelerating and the ratio of dark matter to dark energy decreases, we derived the general relations among the quantities of L, Q, ω, and r as well as the constraints on the possible range of these quantities. In particular, the dynamics of parameters ω and r are determined by the choice of L and Q, thus providing a mechanism to change the evolution of r from that in standard cosmology such that the coincidence problem may be solved. This is the main feature of our formalism. Then we proposed three kinds of strategies to show how the coincidence problem can be alleviated in this context. One possibility is to have a constant ratio throughout the evolution of the Universe. The second is to have two constant solutions to the ratio r such that it will run from the maximum constant to the minimum stable one, while the third and perhaps the most practical one is to have a stable constant solution at the late time but this value is not quite far from our current observation. Focusing on the third strategy we constructed some models explicitly and show how this can be implemented by appropriately choosing the quantities of L and Q. In particular, our results show that, at least in the low redshift region, these models are well fitted with the current observation.

In this paper we assume our Universe is spatially flat but it is completely possible to show that the parallel analysis could be extended to the spatially closed and hyperbolic universe. We also expect that further investigation will provide us a more exact picture of the dark matter and dark energy by strictly fitting the observations in a high redshift region.

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