Testing gravity against the early time integrated Sachs-Wolfe effect

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(Received 8 November 2005; published 2 June 2006)

A generic prediction of general relativity is that the cosmological linear density growth factor D is scale independent. But in general, modified gravities do not preserve this signature. A scale dependent D can cause time variation in gravitational potential at high redshifts and provides a new cosmological test of gravity, through early time integrated Sachs-Wolfe (ISW) effect-large scale structure (LSS) cross correlation. We demonstrate the power of this test for a class of $f(R)$ gravity, with the form $f(R) = -\lambda_1 H_0^2 \exp(-R/\lambda_2 H_0^2)$. Such $f(R)$ gravity, even with degenerate expansion history to ΛCDM, can produce detectable ISW effect at $z \gtrsim 3$ and $l \gtrsim 20$. Null-detection of such effect would constrain λ_2 to be $\lambda_2 > 1000$ at >95% confidence level. On the other hand, robust detection of ISW-LSS cross correlation at high z will severely challenge general relativity.

DOI: 10.1103/PhysRevD.73.123504 PACS numbers: 98.65.Dx, 04.50.+h, 98.70.Vc, 98.80.Es

I. INTRODUCTION

Cosmological observations provide unique tools to study gravity at $\gtrsim$ Mpc scales. General relativity, with the aid of the cosmological constant, or dark energy with equation of state $w \sim -1$, successfully reproduces the accelerated expansion of the Universe, indicated by SN Ia observations [1], along with the flatness of the Universe measured by the cosmic microwave background (CMB) [2] and distance measured by the baryon oscillations [3]. However, these observational evidences mainly constrain the mean expansion history of the Universe and can be reproduced by modified gravity such as brane world DGP theory [4] and generalized $f(R)$ gravity [5]. Essentially, the large scale structure (LSS) of the universe, such as weak gravitational lensing, galaxy clustering and the integrated Sachs-Wolfe (ISW) effect [6–11], is required to break this degeneracy.

General relativity imprints a unique signature in the LSS, which is scale *independent* linear density growth factor D at subhorizon scale after matter-radiation equality epoch [12]. Modifications to general relativity not only changes the amplitude of D, but in general, causes D to be scale dependent. This unique feature of modified gravity has already been noticed in phenomenological theory of modified Newtonian potential [6,9,10]. It can be detected by weak gravitational lensing [6], galaxy clustering [9,10] and late time ISW effect. Counter-intuitively, in this paper, we show that modified gravity can produce a detectable *early time* ISW effect.

We investigate a class of $f(R)$ gravity with action

$$L = \int (R + f(R))\sqrt{g} d^4x + L_{\rm matter}, \qquad (1)$$

and field equation

$$(1 + f_R)R_{uv} - \frac{g_{uv}}{2}(R + f - 2\Box f_R) - f_{R;u;v} = 8\pi G T_{uv}, \qquad (2)$$

where $f_R \equiv df/dR$. We design $f(R) = -\lambda_1 H_0^2 \exp(-R/\lambda_2 H_0^2)$, where $\lambda_{1,2}$ are two positive dimensionless constants and H_0 is the Hubble constant at present. To mimic a ΛCDM universe, $\lambda_1 \sim 1$ is required. To reduce to the general relativity in the solar system and pass the solar system test, $f \ll R$ is required. In this limit, we have $R \to 8\pi G\rho_{\rm solar}$, where $\rho_{\rm solar}$ is the local density where solar system tests are carried out. In this limit, $f(R)/R \sim [\rho_c/\rho_{\rm solar}]\exp[-3\rho_{\rm solar}/\lambda_2\rho_c]$. Since $3\rho_{\rm solar}/\rho_c \gtrsim 10^6$ [13] (ρ_c is the critical density of the Universe), models with $\lambda_2 \ll 10^6$ can survive all solar system tests. For example, For $\lambda_2 = 10^3$, this correction is of the order $\sim 10^{-400}$. Given such tiny $f(R)$, we expect that f, f_R, $\Box f_R$ and $f_{R;\mu;\nu}$ in Eq. (2) can all be safely neglected for any physical purpose.

For the $f(R)$ gravity, the application of Birkhoff theorem to perturbations of a spherically symmetric region leads to scale independent D [16]. We reinvestigate this issue by solving the structure evolution of the fully covariant $f(R)$ gravity to linear order in the metric perturbation. We find that D shows nontrivial scale dependence, consistent with the results based on the Palatini approach [17].

II. THE *H*-*z* RELATION OF THE *f*(*R*) GRAVITY

Cosmological observations prohibit strong deviation of $f(R)$ from the cosmological constant. At the limit that $R(z = 0) \ll \lambda_2 H_0^2$, the H-z relation of $f(R)$ gravity can have the same asymptotic behavior as that of ΛCDM. At low redshift where $R(a) \ll \lambda_2 H_0^2$, $f(R)$ behaves as a cosmological constant and the H-z relation resembles that of ΛCDM. At high redshifts where $R \gg \lambda_2 H_0^2$, $f(R) \to 0$ and $H(z) \to \Omega_0^{1/2}(1 + z)^{3/2}$. Deviation from ΛCDM happens at some intermediate redshifts where $R(a) \sim \lambda_2 H_0^2$ and

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vanishes toward both higher and lower z. We quantify their difference by solving Eq. (2) of a flat universe to zero order

$$H^2 + \frac{f}{6} - \frac{\ddot{a}}{a} f_R + H\dot{f}_R = H_0^2 \Omega_0 a^{-3}. \quad (3)$$

Here, $a \equiv 1/(1+z)$ is the scale factor. This equation can be rewritten as $y = \Omega_0 - C(y(a))$, where $y \equiv a^3 H^2$, $C(y(a)) \equiv [f/6 - \ddot{a} f_R/a + H\dot{f}_R]a^3$ and Ω_0 is the dimensionless matter density at present. Since $C(y(a))$ is completely determined once y as a function of a is given, Eq. (3) can be solved iteratively by the iteration relation $y^{(i+1)} = \Omega_0 - C(y^{(i)})$. To mimic a ΛCDM universe, we fix λ_1 by requiring $f(R(a=1)) = -6H_0^2(1-\Omega_0)$. The iteration converges quickly by taking the initial guess $y^{(0)} = \Omega_0 + (1-\Omega_0)a^3$. For $\lambda_2 \geq 100$, $y^{(1)}$ is accurate to $\sim 1\%$. As expected, for $\lambda_2 \geq 100$, the $H(z)$-z relation is almost identical to the corresponding ΛCDM cosmology (Fig. 1). Such $f(R)$ gravity can not be distinguished from ΛCDM by inflation, big bang nucleosynthesis (BBN), primary CMB, SN Ias and other measures of H-z relation.

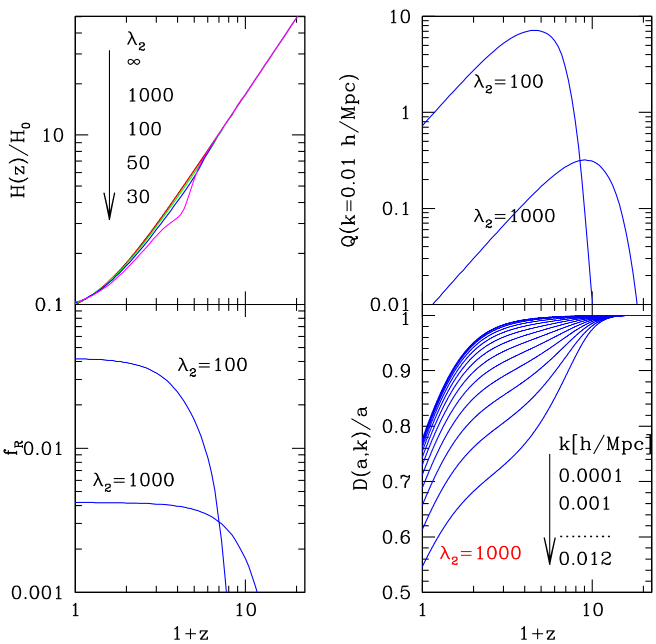

FIG. 1 (color online). The $H(z)$-z relation and structure growth in the exponential $f(R)$ gravity. Top left panel: H-z. $\lambda_2 \to \infty$ corresponds to ΛCDM cosmology. Top right panel: $Q(k,a) \propto k^2$, which describes the main effect of $f(R)$ gravity to structure formation. We plot the result of $k = 0.01$ h/Mpc. Bottom left panel: $f_R(a)$, which determines the effective Newton's constant $G_{\rm eff} = G/(1+f_R)$. For $\lambda_2 \gtrsim 100$, its effect to structure formation can be neglected. Bottom right panel: $D(k,a)/a$ ($\lambda_2 = 1000$), where the linear density growth factor D is normalized such that $D \to a$ when $a \to 0$.

III. THE LARGE SCALE STRUCTURE OF THE $f(R)$ GRAVITY

We will show that, even with this degeneracy in H-z relation and solar system behavior, the LSS of the $f(R)$ gravity could be significantly different to that of ΛCDM. We choose the Newtonian gauge

$$ds^2 = -(1+2\psi)dt^2 + a^2(1+2\phi)\sum_{i=1}^{3}(dx^i)^2. \quad (4)$$

There are four perturbation variables ϕ, ψ, the matter overdensity δ and the (comoving) peculiar velocity convergence θ.

In general relativity, $\phi = -\psi$, as long as there is no anisotropic stress. But in modified gravity, this relation breaks in general. ij ($i \neq j$) component of Eq. (2) provides the relation between ϕ and ψ. For $f(R)$ gravity, due to nonvanishing $f_{R;i;j}$ ($i \neq j$), $\phi - \psi$ relation becomes scale dependent. Throughout this paper, we neglect time derivative terms with respect to spatial derivative terms of corresponding variables. This simplification holds at scales $k \gtrsim aH \lesssim 10^{-3}$ h/Mpc. Since we will focus on the ISW effect at $l \gtrsim 20$ and $z \gtrsim 3$ where the relevant $k \gtrsim 5 \times 10^{-3}$ h/Mpc, this simplification is sufficiently accurate. We then obtain

$$\phi + \psi = \frac{f_{RR}c^2}{1+f_R}\frac{2}{a^2}(\nabla^2\psi + 2\nabla^2\phi). \quad (5)$$

In Fourier space, this reads $\psi = -\phi(1-2Q)/(1-Q)$, where $Q(k,a) \equiv -2f_{RR}c^2k^2/(1+f_R)a^2$ and $f_{RR} \equiv d^2f/dR^2$. For clarity, we explicitly show the speed of light c. We will see that this scale dependent $\phi - \psi$ relation has profound effect on the LSS. Combining Eq. (5) and the tt component of Eq. (2), we obtain the new Poisson equation

$$\nabla^2(\phi - \psi) = -\frac{3H_0^2\Omega_0}{1+f_R}a^{-1}\delta. \quad (6)$$

The energy-momentum tensor is still conserved and provides the remaining two equations:

$$\dot{\delta} + \theta = 0, \qquad \dot{\theta} + 2H\theta + \frac{1}{a^2}\nabla^2\psi = 0. \quad (7)$$

Combining all 4 equations, we obtain the main equation of this paper:

$$\delta'' + \delta'\left(\frac{3}{a} + \frac{H'}{H}\right) - \frac{\delta}{a^2}\frac{1-2Q}{2-3Q}\frac{3H_0^2\Omega_0}{a^3H^2(1+f_R)} = 0, \quad (8)$$

where $' \equiv d/da$. In general relativity, $Q = 0$, so the linear density growth factor $D \propto \delta(a)/\delta(a_i)$ is scale independent at scales $k \gtrsim aH/c$, no matter what the form of dark energy is. Here, a_i is the scale factor at some early epoch and we normalze D such that $D \to a$ when $a \to 0$. But in $f(R)$, the scale dependent $Q(k,a)$ induces nontrivial scale dependence to D. This behavior can not be obtained by a simple change in the effective Newton's constant.

Furthermore, the correction Q has a nontrivial dependence on a. This is hard to realize by simply changing the form of the Newtonian potential (e.g. to Yukawa potential).

Since $f_{RR} < 0$, there exist one *apparent singularity* $Q = 2/3$ in Eq. (8), where only $\delta = 0$ solution is accepted and two at $Q = 1/2, 1$ in the $\phi - \psi$ relation, where only $\psi = \phi = 0$ solution is accepted. We leave this issue alone until the discussion section. For the moment, we take a modest goal by only using regions where $Q < 1/2$ to constrain $f(R)$. For $\lambda_2 = 1000$, this constrains us to region where $k \le 0.012$ h/Mpc.

Hereafter, we fix $\lambda_2 = 1000$. At $z \gg 1$, $H \propto a^{-3/2}$, $D \propto a^{1-\eta}$ when $\eta \equiv 3Q/5(2 - 3Q) \ll 1$. Thus gravitational potential decays at high redshifts with rate $\propto a^{-\eta}$ and causes an observable integrated Sachs-Wolfe (ISW) effect. At later time when $R \lesssim \lambda_2 H_0^2$, $Q \to 0$ (Fig. 1), the evolution of D approaches that of ΛCDM. For the exponential $f(R)$, $Q(a)$ peaks at $z \gg 1$ (Fig. 1), so the resulting ISW effect peaks at $z \gg 1$, as contrast to that of ΛCDM cosmology or dark energy models with $w \sim -1$. This provides us a unique way to test this form of $f(R)$. We solve Eq. (8) numerically. Initial condition is set to normalize $D \to a$ when $a \to 0$.

IV. THE INTEGRATED SACHS-WOLFE EFFECT

Time variation in $\psi - \phi$ causes a fractional CMB temperature variation [18]

$$\frac{\Delta T}{T_{\rm CMB}} = \int [\dot{\psi} - \dot{\phi}] a d\chi. \tag{9}$$

Here, χ is the comoving angular diameter distance. Since both $\psi - \phi$ and the LSS trace the underlying matter distribution, there exists an ISW-LSS cross correlation, with power spectrum

$$\frac{l^2}{2\pi} C_l^{\rm ISW-LSS} = \frac{\pi}{l} \int \Delta^2_{(\dot{\psi}-\dot{\phi})\delta_{\rm LSS}}\left(\frac{l}{\chi}\right) W_{\rm LSS}(\chi) a^2 \chi d\chi. \tag{10}$$

Here, $\delta_{\rm LSS}$ is the density fluctuation of the LSS tracers, $W_{\rm LSS}$ is the corresponding weighting function and $\Delta^2_{(\dot{\psi}-\dot{\phi})\delta_{\rm LSS}}$ is the corresponding 3D power spectrum(variance). The above formula adopts the Limber's approximation, which is sufficiently accurate to serve for our interest at $l \gtrsim 20$. The amplitude and sign of the ISW effect is determined by $A_{\rm ISW} \equiv D/a - dD/da$. Positive $A_{\rm ISW}$ means positive correlation between ISW and LSS. For $k \gtrsim 0.007$ h/Mpc, $A_{\rm ISW}$ has a bump at $z \sim 6$, whose amplitude increases towards small scales (large k). This boosts early time small scale ISW signal (Fig. 2).

The S/N of the ISW-LSS cross correlation measurement of each l mode is

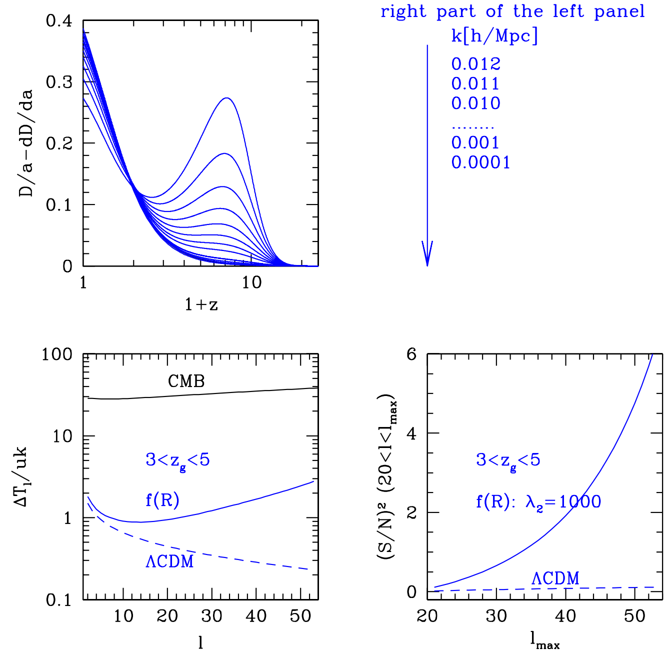

FIG. 2 (color online). The ISW effect. $\lambda_2 = 1000$ is adopted. Top left panel: $D/a - dD/da$, which determines the sign and amplitude of the ISW effect. D is normalized such that $D \to a$ when $a \to 0$. Bottom left panel: the ISW effect. Bottom right panel: Cumulative S/N of the ISW-LSS cross correlation measurements.

$$\left(\frac{S}{N}\right)^2 = \frac{(2l+1) f_{\rm sky} C^2_{\rm ISW-LSS}}{(C_{\rm CMB} + C_{\rm ISW} + C^{\rm shot}_{\rm CMB})(C_g + C^{\rm shot}_g) + C^2_{\rm ISW-LSS}} \tag{11}$$

Here, $C_{\rm CMB}$, $C_{\rm ISW}$, C_g are the power spectra of primary CMB, ISW, and galaxies, respectively, while $C^{\rm shot}_{\rm CMB}$ and $C^{\rm shot}_g$ are the power spectra of associated shot noises, respectively. Since the exponential $f(R)$ does not affect physics at $z \gtrsim 100$, we adopt the same primordial power spectrum with power index $n = 1$, the same transfer function BBKS [19] and the same amplitude at $a_i = 0.01$, as that of the ΛCDM cosmology. The LSS tracers we choose are 21 cm emitting galaxies at $3 < z < 5$, which will be measured by proposed 21 cm experiments such as Square Kilometer Array [20]. Singularities presented in the perturbation equations limit us to $l < 60$, where one can neglect shot noises of CMB. For the estimations of LSS clustering signal and shot noise, biggest uncertainties are (i) HI (*neutral hydrogen*) mass function at $3 < z < 5$, (ii) 21 cm emitting galaxy bias and (iii) specifications of 21 cm experiments. If one adopts HI mass functions calibrated against observations of damped Lyman-α systems and Lyman limit systems, SKA can detect $\gtrsim 10^9$ galaxies at $z > 3$ in five years across the whole sky, for a field of view $\gtrsim 10$ deg^2 at ~300 Mhz (for details of the calculation, see, e.g. [21]). Detection thresholds of HI mass at $z \gtrsim 3$ are $\gtrsim 10^9 M_\odot$, so detected galaxies are likely having biases bigger than one. Then, one can neglect the shot noise term $C_g^{\rm shot}$ with respect to C_g. Taking the fact that $C^{\rm CMB} \gg C^{\rm ISW}$ (Fig. 2), the S/N of each l is simplified to

$$\left(\frac{S}{N}\right)^2 \simeq \frac{(2l+1)f_{\rm sky}r^2}{C_l^{\rm CMB}/C_l^{\rm ISW}}. \quad (12)$$

Here r is the cross correlation coefficient between ISW and LSS. Since r has very weak dependence on galaxy bias, the estimation presented here is weakly model dependent. We disregard signals from $l < 20$, to reduce confusions of ΛCDM cosmology or dark energy models. For sparse galaxy sampling which is sufficient for our purpose, SKA is able to cover the whole sky. So we assume that $f_{\rm sky} = 1$. The cumulative $\sum_{20}^{l_{\rm max}}(S/N)^2$ is shown in Fig. 2.

The ISW signal peaks at $z \gtrsim 3$ and increases toward high l. This is hard to mimic by ΛCDM, dark energy or many forms of modified gravity. (i) For ΛCDM or dark energy models with $w \lesssim -1$, at $z \gtrsim 3$, the ISW effect effectively vanishes. Figure 2 shows that ΛCDM can be distinguished from the $\lambda_2 = 1000$ $f(R)$ gravity with $>2\sigma$ confidence by the ISW-21 cm emitting galaxy cross correlation. (ii) For dark energy models with $w \gtrsim -1$, $A_{\rm ISW}$ does not decrease as fast as that of ΛCDM. But the ISW signal (including contributions from dark energy fluctuations) decreases toward high l [22] and one does not expect a detectable ISW effect. (iii) DGP preserves the property of scale independent D [8,23], so the ISW signal decreases toward high l, like the dark energy case. Therefore we do not expect a detectable signal at $l > 20$ and $z > 3$. (iv) For generalized $f \propto (\alpha R^2 + \beta R_{ab}R^{ab} + \gamma R_{abcd}R^{abcd})^{-n}$ ($n > 0$), the ISW effect vanishes at high z because the f correction decreases much faster than the exponential $f(R)$. So we expect that null detection of ISW-LSS cross correlation at $l \geq 20$ and $z \geq 3$ would constrain λ_2 to $\lambda_2 > 1000$ at $>2\sigma$ confidence level. On the other hand, a detection of such cross correlation would present as a severe challenge to general relativity.

V. DISCUSSION

The scale dependence of D, as an unambiguous signature of modified gravity, can in principle be measured from weak gravitational lensing by the mean of lensing tomography. Since ϕ is no longer equal to $-\psi$, we provide the general form of the lensing transformation matrix A_{ij}

$$A_{ij} - \delta_{ij} = \int_0^{\chi_s} d\chi(\phi - \psi)_{,ij}W(\chi, \chi_s), \quad (13)$$

where $W(\chi, \chi_s) = \chi(1 - \chi/\chi_s)$ is the usual lensing kernel. All basic lensing theorems remain unchanged. For example, lensing shear field is still curl free (*if neglecting second order corrections such as Born correction*). For $f(R)$ gravity, relation between the lensing convergence $\kappa = 1 - (A_{11} + A_{22})/2$ and the matter over-density resembles that of the general relativity, with

$$\kappa = \frac{3}{2}H_0^2\Omega_0 \int \delta a^{-1}W(\chi, \chi_s)(1 + f_R)^{-1}d\chi. \quad (14)$$

It is interesting to see how well weak lensing alone can constrain modified gravity. For the exponential $f(R)$, one complexity is that lensing mainly probes LSS at $z \lesssim 1$, where Q is small and the deviation from a scale independent D is small, so the constraints may be weak. This can be significantly improved by gravitational potential reconstructed from primary CMB. Combining lensing and CMB measurements, it is very promising to measure the evolution of the gravitational potential between $z = 1100$ and $z \sim 0$ robustly. This will put strong constraints on the nature of gravity. Unfortunately, due to singularities in the perturbation equations, we are limited to scales $k \lesssim 0.012$ h/Mpc or $l \lesssim 20$ at $z \lesssim 1$ (for $\lambda_2 = 1000$). Information contained in this region is very limited and could be contaminated by other physics such as dark energy fluctuations. Solving the field equation crossing those singularities consistently is nontrivial. We leave this work for future study.

The $Q = 1/2, 2/3, 1$ singularities may be caused by awkward gauge choice, the neglecting of time derivative terms with respect to corresponding spatial derivative terms, or the failure of the perturbation approach. For example, for $Q = 2/3$, the only solution $\delta = 0$ does not depend on initial conditions. This could be caused by neglecting time derivative terms, which erases some degrees of freedom. These issues require detailed study. But if these singularities in LSS equations are physical, they can be applied to rule out many forms of modified gravities as alternatives to dark energy or general relativity. To produce a similar expansion history as those of dark energy model, (i) R should increase when a decreases and (ii) $f(R(a = 1))$ should be negative in order to mimic positive dark energy. Furthermore, in order not to affect inflation, BBN and primary CMB, $f(R(a \to 0))$ must be sufficiently small. A sufficient (but not necessary) condition satisfying the BBN constraint is that $f(R(a \to 0)) \to 0$. The exponential $f(R)$ and $1/R^n$ $f(R)$ all fall into this class. This results in $f_R > 0$ at least at some early epoch a_+. As we have seen from previous discussions, $f_{RR} < 0$ is a sufficient condition for the existence of singularities. To avoid singularities, $f_{RR} \geq 0$ must be satisfied at all epochs. However, we will see that this requirement contradicts with (i) and (ii). $f_{RR} \geq 0$ results in $f_R(a < a_+) \geq f_R(a_+) > 0$, because $R(a < a_+) > R(a_+)$. So, f increases toward high redshift, crosses over zero at some epoch and then increases more quickly than R. Since when $a \to 0$, $R \propto a^{-3}$, f increases more quickly than a^{-3} and thus more quickly than the matter density. This violates condition (ii). It could have non-negligible effect on BBN and contradicts our expectation. On the other hand, only for those $f(R)$ gravities in which $f(R(a \to 0))$ does not vanish, singularities in LSS equations can be avoided. A log$R$ $f(R)$ gravity is such an example.

To demonstrate the power of LSS to constrain gravity, we adopt a conservative requirement to avoid singularities at $k < k_s$. At the limit that $\lambda_2 \gg 1$, Q peaks at $a = (2\lambda_2/9\Omega_0)^{-1/3}$ and the peak amplitude is $\simeq 12(1 - \Omega_0) \times (2/9\Omega_0 e)^{2/3}\lambda_2^{-4/3}(ck/H_0)^2$, where we show the speed of light c explicitly. To avoid singularities at $k < k_s$,

$$\lambda_2 \geq 2.5 \times 10^5 \left(\frac{k_s}{\mathrm{h/Mpc}}\right)^{3/2} \tag{15}$$

should be satisfied.

ACKNOWLEDGMENTS

I thank Jochen Weller for providing the Runge-Kutta code dverk.F and John Barrow, Salvatore Capozziello, Xuelei Chen, Tomi Koivisto, Xinhe Meng, Sergei Odintsov and Roman Scoccimarro for helpful discussions. This work was supported in part by the One-Hundred-Talent Program of China and by the DOE and the NASA grant No. NAG 5-10842 at Fermilab.

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[12] Neutrino free streaming causes D to be scale dependent, but this effect is small. Fluctuations in dark energy cause D to be scale dependent at $\sim$ horizon scale. But this effect vanishes toward smaller scales.
[13] The condition $3\rho_{\mathrm{solar}}/\rho_c \gtrsim 10^6$ is well met for any experiments carried in the atmosphere of the Earth, since the atmosphere density is $\sim 10^{-3}$ g/cm^3 and $\rho_c \simeq 10^{-29}$ g/cm^3 ($h = 0.7$). Though the matter density in the interplanetary space is not well measured, we have solid evidences that the condition can also be met for experiments carried out in the space. (i) The density of interplanetary dust (IPD) is $\rho_{\mathrm{IPD}} \gtrsim 10^{-24}$ g/cm^3 ([14] and references therein). This gives $3\rho_{\mathrm{solar}}/\rho_c \gtrsim 3 \times 10^6$. (ii) The total mass of the Kuiper Belt objects between 40 and 90 AU is $(0.6, 40) \times 10^{-5} M_\oplus$ [15]. This converts to $3\rho_{\mathrm{solar}}/\rho_c \gtrsim 2 \times 10^6$–$10^8$. (ii) The anomalous Doppler frequency drift experienced by Pioneer 10 and 11, if caused by drag force of interplanetary matter, requires the interplanetary matter density $\sim$(5–30) $\times 10^{-20}$ g/cm^3 at 20–70 AU [14]. This converts to $3\rho_{\mathrm{solar}}/\rho_c \gtrsim 10^{10}$.
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