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Little Higgs model effects at $\gamma\gamma$ collider

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Though the predictions of the standard model (SM) are in excellent agreement with experiments there are still several theoretical problems, such as fine-tuning and the hierarchy problem. These problems are associated with the Higgs sector of the SM, where it is widely believed that some ''new physics'' will take over at the TeV scale. One beyond the SM theory which resolves these problems is the little Higgs model. In this work we shall investigate the effects of the little Higgs model on $\gamma\gamma \to \gamma\gamma$ scattering, where the process $\gamma\gamma \to \gamma\gamma$ at high energies occurs in the SM through diagrams involving W, charged quark, and lepton loops (and is, therefore, particularly sensitive to any new physics).

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I. INTRODUCTION

It has been known for some time that the $\gamma\gamma \to \gamma\gamma$ scattering amplitude at high energies will be a very useful tool in the search for new particles and interactions in an e^+e^- linear collider operated in the $\gamma\gamma$ mode. In particular, as according to present ideas, this scattering can be achieved by colliding $e^\pm$ beams at a future linear collider, such as the International Linear Collider, with laser photons (which are subsequently backscattered, through the Compton effect) to produce very energetic photons of high luminosity along the $e^\pm$ direction; while the $e^\pm$ beams would lose most of their energy. As such, these searches may involve either the direct production of new degrees of freedom (for example, charginos, light sleptons, or a light stop in supersymmetry models); or the precise study of the production of SM particles, where the new degrees of freedom contribute virtually in some loop diagrams. In this respect, processes like $\gamma\gamma \to \gamma\gamma$, $\gamma\gamma \to Z\gamma$, $\gamma\gamma \to ZZ$ should all provide very important tools for searching or constraining new physics [1]—particularly as the standard model (SM) contributions in these processes first appear at the one-loop level and should be small.

As a large number of helicity amplitudes can contribute to these processes, due to the presence of spin-one particles in the initial and final states, consideration of symmetries and other invariances is required to reduce this number. Furthermore, in the SM the amplitudes of $\gamma\gamma \to \gamma\gamma$ will have contributions from one-loop diagrams mediated by charged fermions (leptons and quarks) and W bosons. At large energies ($\sqrt{s_{\gamma\gamma}} \geq 250$ GeV) it is know that the W contributions dominate over the fermionic contributions. At these energies it also should be noted that the dominant amplitudes are predominantly imaginary. Therefore we expect that any new physics effects in the $\gamma\gamma \to \gamma\gamma$ process may come from the interference terms between the predominantly imaginary SM amplitudes and new physics effects to these amplitudes.

At this point we would like to point out that though the SM has been very successful in explaining all electroweak interactions probed so far, there is no symmetry or relation which protects the mass of the Higgs boson. In fact, the Higgs mass diverges quadratically when quantum corrections in the SM are taken into account. But precision electroweak data demands the lightest Higgs boson mass be ~200 GeV. In order for this to happen we either need to invoke some symmetries which will protect the Higgs mass to a much higher scale (possibly grand unified theory scale) or assume that the SM is an effective theory valid up to only the electroweak scale. In either of these possibilities it is expected that some new physics should take over from the SM at the TeV scale. As such, supersymmetry has provided one popular example of new physics, where additional symmetries are invoked which help protect the Higgs mass up to the grand unified theory scale. Recently, a new approach to address this problem has been advocated; this approach is popularly known as the ''little Higgs (LH) model,'' which addresses some of the problems in the SM by making the Higgs boson a pseudo-Goldstone boson of a symmetry which is broken at some higher scale Λ. The suggestion of making the Higgs a pseudo-Goldstone boson was proposed some time ago [2] but has been revived recently, where such models have been successfully constructed by Arkani-Hamed, Cohen, and Georgi [3]. The successful little Higgs models are constructed in such a way that *no single* interaction breaks all the symmetries, but the symmetries are broken *collectively*. In these models the scale Λ ($= 4\pi f$) is chosen to be ~10 TeV. The scale Λ acts as a cutoff which separates the weakly interacting

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low energy range from possible strongly interacting sectors at higher energies. The Higgs fields then acquire a mass radiatively at the electroweak scale. Note that in this model the Higgs field remains light, being protected by the approximate global symmetry and free from any one-loop quadratic sensitivity to the cutoff scale Λ. Note also, that in doing this we are required to introduce several new heavy gauge bosons and other new particles, which shall be discussed further in Sec. II.

However, it must be noted that the originally proposed implementations of the LH approach suffered from severe constraints from precision electroweak measurements [4], which could only be satisfied by finely tuning the model parameters. The most serious constraint resulted from the tree-level corrections to precision electroweak observables due to the exchanges of the additional heavy gauge bosons present in the theories (because their masses are much smaller than the cutoff scale), as well as from the small but nonvanishing vacuum expectation value (vev) of an additional weak-triplet scalar field. As a result, masses of new particles had to be raised, and the fine-tuning of the Higgs boson mass is reintroduced. Motivated by these constraints, several new variants of the LH model were proposed [5]. Particularly interesting is the implementation of the Z_2 symmetry, called T-parity, into the model, as proposed in Refs. [6]. T-parity explicitly forbids any tree-level contribution from the new heavy gauge bosons to the observables involving only SM particles as external states. It also forbids the interactions that induced the triplet vev. As a result, in T-parity symmetric LH models, corrections to precision electroweak observables are generated exclusively at loop level. This implies that the constraints are generically weaker than in the tree-level case, and fine-tuning can be avoided [7].

Note that due to T-parity the lightest T-odd particle becomes stable and a good candidate for dark matter. This is an interesting feature of the model, because the existence of dark matter is now established by recent cosmological observations [8]. Since the lightest T-odd particle is electrically and color neutral and has a mass of $\mathcal{O}(100)$ GeV [6,8] in many LH models with T-parity, these models provide a weakly interacting massive particle dark matter candidate [6] and are able to account for the large scale structure of the present universe.

With this in mind, we review the LH model we have used in Sec. II before proceeding to investigate the helicity amplitudes of the scattering process $\gamma\gamma \to \gamma\gamma$ in Sec. III. Finally we conclude with the discussion of the results of our numerical analysis in Sec. IV.

II. LITTLE HIGGS MODELS

In this section we will briefly describe the LH models which we have used in our analysis, in particular the minimal version of the LH model, the so-called *littlest Higgs model* [9].

To begin, let us recall that it is known that the scalar mass in a generic quantum field theory will receive quadratically divergent radiative corrections, all the way up to the cutoff scale. The LH model solves this problem by eliminating the lowest order contributions via the presence of a partially broken global symmetry (where the nonlinear transformation of the Higgs fields under this global symmetry prohibits the existence of a Higgs mass term of the form $m^2|h|^2$). This is done by introducing a new set of heavy gauge bosons (with the same quantum numbers as the SM gauge bosons), where the gauge couplings to the Higgs bosons are patterned in such a way that the quadratic divergences induced in the SM gauge boson loops are canceled by the quadratic divergence induced by the heavy gauge bosons at one-loop level. One also introduces a heavy fermionic state which couples to the Higgs field in a specific way, so that the one-loop quadratic divergence induced by the top-quark Yukawa coupling to the Higgs boson is canceled. Furthermore, extra Higgs fields exist as the Goldstone boson multiplets from the global symmetry breaking. On this framework the littlest Higgs model was introduced, which was based on an $SU(5)/SO(5)$ coset. The phenomenology of this model was discussed in great detail in precision tests [4,9] and low energy measurements [10]. LH models generically also predict the existence of a doubly charged triplet Higgs. The phenomenology of triplet Higgs within the context of the LH model also has been extensively studied in the literature [11]. A detailed review of LH models can be found in [12].

The model we shall use in our analysis, the littlest Higgs model [9,12], is a nonlinear σ model based on a $SU(5)$ global symmetry which contains a gauged $[SU(2) \times U(1)]_1 \bigotimes [SU(2) \times U(1)]_2$ symmetry with couplings g_1, g_2, g_1', and g_2' as its subgroup. Furthermore, the global $SU(5)$ symmetry is broken into $SO(5)$ by the vacuum expectation value of the sigma field

$$\Sigma_0 = \begin{pmatrix} 0 & 0 & \mathbb{1} \\ 0 & 1 & 0 \\ \mathbb{1} & 0 & 0 \end{pmatrix}, \tag{2.1}$$

where $\mathbb{1}$ is the 2×2 identity matrix. This breaking simultaneously breaks the gauge group to a $SU(2) \times U(1)$ subgroup, which is identified with the SM group. The breaking of the global $SU(5) \to SO(5)$ gives rise to 14 goldstone bosons, $\Pi = \pi^a X^a$ which can be written as

$$\Sigma = e^{i\Pi/f}\Sigma_0 e^{i\Pi^T/f} = \Sigma_0 + \frac{2i}{f}\Pi\Sigma_0 + \mathcal{O}(1/f^2), \tag{2.2}$$

where X^a corresponds to the broken $SU(5)$ generators. Four of the 14 Goldstone bosons are absorbed by the broken gauge generators, and the remaining 10 Goldstones are parameterized as

$$\Pi = \begin{pmatrix} & h^\dagger/\sqrt{2} & \Phi^\dagger \\ h/\sqrt{2} & & h^*/\sqrt{2} \\ \Phi & h^T/\sqrt{2} & \end{pmatrix}, \qquad (2.3)$$

where h is the SM Higgs doublet and Φ is a complex $SU(2)$ triplet[1]:

$$\Phi = \begin{pmatrix} \Phi^{++} & \Phi^{+}/\sqrt{2} \\ \Phi^{+}/\sqrt{2} & \Phi^{0} \end{pmatrix}. \qquad (2.4)$$

The kinetic term for the Σ field can be written as

$$\mathcal{L}_{\rm kin} = \frac{f^2}{8}\,{\rm Tr}\{D_\mu\Sigma(D^\mu\Sigma)^\dagger\}, \qquad (2.5)$$

where

$$D_\mu\Sigma = \partial_\mu\Sigma - i\Sigma_j[g_jW_j^a(Q_j^a\Sigma + \Sigma Q_j^{aT}) + g'_jB_j(Y_j\Sigma + \Sigma Y_j)]. \qquad (2.6)$$

In the above $j = 1$, 2, the Q_j and Y_j are the gauged generators, B_j and W_j^a are the $U(1)_j$ and $SU(2)_j$ gauge fields, respectively, and g_j and g'_j are the corresponding coupling constants.

As stated earlier, the vev (Σ_0) given in Eq. (2.1) breaks the gauge group to the diagonal one, which is then identified with the SM group. This generates mass and mixings of the gauge bosons. The heavy gauge boson mass eigenstates are given by

$$W_H^a = -cW_1^a + sW_2^a, \qquad B_H = -c'B_1 + s'B_2, \quad (2.7)$$

where s, s', c, and c' are the mixing angles given by

$$c' = g'/g'_2, \qquad s' = g'/g'_1, \qquad c = g/g_2, \qquad s = g/g_1. \qquad (2.8)$$

These couplings can be related to the SM couplings (g, g') by [9]

$$\frac{1}{g^2} = \frac{1}{g_1^2} + \frac{1}{g_2^2}, \qquad \frac{1}{g'^2} = \frac{1}{g_1'^2} + \frac{1}{g_2'^2}, \qquad (2.9)$$

where the masses of heavy gauge bosons will then be

$$M_{W_H}^2 = \frac{f^2}{4}(g_1^2 + g_2^2), \qquad M_{B_H}^2 = \frac{f^2}{20}(g_1'^2 + g_2'^2). \quad (2.10)$$

The orthogonal combination of these gauge bosons are identified with the SM W and B.

In the SM, the top quark introduces quadratic corrections to the Higgs boson mass. The LH model addresses this problem by the introduction of a new set of heavy fermions which couple to the Higgs such that it cancels the quadratic divergences to the Higgs mass—due to the SM top quark. A vectorlike top quark is usually introduced in the LH model to do this job. The Yukawa interactions in the LH model are chosen to be

$$\mathcal{L}_Y = \tfrac{1}{2}\lambda_1 f\epsilon_{ijk}\epsilon_{xy}\chi_i\Sigma_{jx}\Sigma_{ky}u_3'^c + \lambda_2 f\tilde{t}\tilde{t}'^c + {\rm H.c.} \quad (2.11)$$

with i, j, k summed over 1, 2, 3 and x, y summed over 4, 5. $\chi_i = (b_3, t_3, \tilde{t})$, b_3, and t_3 are the SM bottom and top quarks, $(\tilde{t}, \tilde{t}'^c)$ is the new vectorlike top quark and $u_3'^c$ is the SM right-handed top quark.

Expanding the Σ field and diagonalizing the mass matrix we arrive at the physical states:

$$m_t \sim \frac{v\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}, \qquad m_T \sim f\sqrt{\lambda_1^2 + \lambda_2^2}. \qquad (2.12)$$

These masses are parameterized in terms of x_L, defined as

$$x_L = \frac{\lambda_1^2}{\lambda_1^2 + \lambda_2^2}. \qquad (2.13)$$

In the LH models there is no Higgs potential at tree level; this is generated at the one-loop level via the interactions with gauge bosons and fermions. This is similar to a Coleman-Weinberg type of potential and gives the Higgs masses as

$$M_\Phi^2 = 2m_H^2\frac{f^2}{v^2}\frac{1}{1 - (\frac{4v'f}{v^2})}, \qquad (2.14)$$

where m_H is the SM Higgs boson mass. Therefore, for this kind of LH model [based on $SU(5)/SO(5)$] we have five input parameters, in addition to the SM Higgs mass. Explicitly, these are

$$s, \quad s', \quad x_L, \quad f, \quad v'.$$

The advantage of this model now becomes apparent; by noting that as the gauge generators are embedded in the $SU(5)$ group, in such a way as to commute with a $SU(3)$ subgroup, one pair of gauge couplings must be set to zero. Therefore, the Higgs mass would be an exact Goldstone boson and massless. As such, any diagram renormalizing the Higgs mass will vanish unless it involves at least two of the gauge couplings. Note that at the one-loop level all diagrams satisfying this condition are only logarithmically divergent. Therefore, the symmetry breaking mechanism protects the Higgs mass from quadratic divergences at this level. Generically, the particle spectrum of the LH model, apart from SM particles is

(i) heavy vectorlike top quark (T);
(ii) heavy gauge bosons: charged (W_H), neutral (Z_H, A_H);
(iii) additional triplet Higgs: (Φ^0, Φ^+, Φ^{++}).

As mentioned in the introduction, the original LH models were severely constrained by precision electroweak experiments [4,9]with the main constraints coming from the ρ parameter and the $Z \to b\bar{b}$ [4] vertex contributions. Other models which evade these constraints have been proposed, but all of these enlarge the global or gauge symmetries. Recently, Cheng and Low [6] introduced a

[1]The existence of the $SU(2)$ triplet is a generic feature of LH models

discrete Z_2 symmetry, which we now call "T-parity" to resolve the electroweak precision constraints in the LH models. The advantages of introducing T-parity is twofold. First, it helps relax the precision constraints and second, it provides a dark matter candidate. The new parity is an exchange of the two gauge groups, and the Lagrangian in Eq. (2.5) is invariant under this exchange provided that $g_1 = g_2$ and $g_1' = g_2'$. The implication of this is that the gauge boson mass eigenstates will have the form $W_\pm = \frac{1}{\sqrt{2}}(W_1 \pm W_2)$ and $B_\pm = \frac{1}{\sqrt{2}}(B_1 \pm B_2)$. The SM gauge bosons are even under T-parity and are designated by a + subscript, and the new "T-odd" gauge bosons are designated by a − subscript. The different T-parity states do not mix and after electroweak symmetry breaking, the Weinberg angle is given by the usual SM relation, as are other electroweak observables (thus removing the constraint). Note further that as the transformation law ensures that the complex $SU(2)$ triplet is odd under T-parity, while the Higgs doublet is even, the trilinear coupling $H^\dagger \phi H$ is forbidden. This further relaxes precision electroweak constraints on the model.

Thus, the main implications of the introduction of T-parity are as follows:

(i) All new particles (except one heavy top quark) are odd under T-parity;
(ii) T-parity exchanges $[SU(2) \times U(1)]_1$ and $[SU(2) \times U(1)]_2$;
(iii) T-parity imposes a relationship between the couplings, for example $g_1 = g_2$, $g_1' = g_2'$;
(iv) The fermion sector is extended to include T-odd fermions;
(v) There is no vev to the triplet Higgs—this being assured by the absence of a $H\Phi H$ coupling.

In such a model $[SU(5)/SO(5)$ with T-parity] the input model parameters (apart from SM Higgs mass m_H) are

$$f, \quad \frac{\lambda_1}{\lambda_2}, \quad \kappa.$$

The first two have been defined before and κ is a free parameter whose range is $0.5 \le \kappa \le 1.5$ [7]. As we are interested in the $\gamma\gamma \to \gamma\gamma$ process, where this process occurs at one-loop by the mediation of charged particles, it should be noted that in the SM this process is mediated by charged W and fermions (charged leptons and quarks). This process in the LH model also can be mediated by charged gauge bosons (W_H), charged Higgs (Φ^-, Φ^{--}), and new fermions (T in the LH model without T-parity; note that in the LH model with T-parity, there shall also be T_+, T_-, and heavy T-odd fermions which can mediate the process). To conclude this section, we have given the mass spectrum of these particles in the LH models in Table I.

III. THE $\gamma\gamma \to \gamma\gamma$ CROSS SECTIONS

The process

$$\gamma(p_1, \lambda_1)\gamma(p_2, \lambda_2) \to \gamma(p_3, \lambda_3)\gamma(p_4, \lambda_4) \tag{3.1}$$

can be represented by 16 possible helicity amplitudes $F_{\lambda_1\lambda_2\lambda_3\lambda_4}(\hat{s}, \hat{t}, \hat{u})$, where the p_i and λ_i represent the respective momenta and helicities; the $\hat{s}$, $\hat{t}$, and $\hat{u}$ are the usual Mandelstam variables. By the use of Bose statistics, crossing symmetries and demanding parity and time-invariance, these 16 possible helicity amplitudes can be expressed in terms of just three amplitudes, namely, (the relationship between various helicity amplitudes as given in Appendix A)

$$F_{++++}(\hat{s}, \hat{t}, \hat{u}), \qquad F_{+++-}(\hat{s}, \hat{t}, \hat{u}), \qquad F_{++--}(\hat{s}, \hat{t}, \hat{u}). \tag{3.2}$$

As such, the cross section for this process can be expressed as [13]:

$$\begin{aligned}\frac{d\sigma}{d\tau d\cos\theta^*} = \frac{d\bar{L}_{\gamma\gamma}}{d\tau}\bigg\{&\frac{d\bar{\sigma}_0}{d\cos\theta^*} + \langle\xi_2\xi_2'\rangle\frac{d\bar{\sigma}_{22}}{d\cos\theta^*} \\ &+ [\langle\xi_3\rangle\cos2\phi + \langle\xi_3'\rangle\cos2\phi]\frac{d\bar{\sigma}_3}{d\cos\theta^*} \\ &+ \langle\xi_3\xi_3'\rangle\bigg[\frac{d\bar{\sigma}_{33}}{d\cos\theta^*}\cos2(\phi+\phi') \\ &+ \frac{d\bar{\sigma}_{33}'}{d\cos\theta^*}\cos2(\phi-\phi')\bigg] \\ &+ [\langle\xi_2\xi_3'\rangle\sin2\phi' - \langle\xi_3\xi_2'\rangle\sin2\phi']\frac{d\bar{\sigma}_{23}}{d\cos\theta^*}\bigg\},\end{aligned} \tag{3.3}$$

where $d\bar{L}_{\gamma\gamma}$ describes the photon-photon luminosity in the $\gamma\gamma$ mode and $\tau = s_{\gamma\gamma}/s_{ee}$. Note that ξ_2, ξ_2', ξ_3, and ξ_3' are the Stokes parameters. Furthermore [13,14],

TABLE I. Mass spectrum of the LH models v is the vev of the SM Higgs and m_H is the SM Higgs mass. In the model without T-parity we have only one vectorlike top quark with mass $m_T = \sqrt{\lambda_1^2 + \lambda_2^2} f$. f_H are the T-odd fermions in the LH with T-parity.

Particle	W_H	Φ	m_t	T_-	T_+	f_H
Masses (LH with T-parity)	gf	$\frac{\sqrt{2}m_H f}{v}$	$\frac{\lambda_1\lambda_2 v}{\sqrt{\lambda_1^2+\lambda_2^2}}$	$\lambda_2 f$	$\sqrt{\lambda_1^2+\lambda_2^2} f$	κf
Masses (LH without T-parity)	$\frac{gf}{2sc}$	$\frac{\sqrt{2}m_H f/v}{\sqrt{[1-(4v'f/v^2)^2]}}$	$\frac{\lambda_1\lambda_2 v}{\sqrt{\lambda_1^2+\lambda_2^2}}$	—	—	—

$$\frac{d\bar{\sigma}_0}{d\cos\theta^*} = \left(\frac{1}{128\pi\hat{s}}\right)\sum_{\lambda_3\lambda_4}[|F_{++\lambda_3\lambda_4}|^2 + |F_{+-\lambda_3\lambda_4}|^2],$$
$$\frac{d\bar{\sigma}_{22}}{d\cos\theta^*} = \left(\frac{1}{128\pi\hat{s}}\right)\sum_{\lambda_3\lambda_4}[|F_{++\lambda_3\lambda_4}|^2 - |F_{+-\lambda_3\lambda_4}|^2],$$
$$\frac{d\bar{\sigma}_3}{d\cos\theta^*} = \left(-\frac{1}{64\pi\hat{s}}\right)\sum_{\lambda_3\lambda_4}\mathrm{Re}[F_{++\lambda_3\lambda_4}F^*_{-+\lambda_3\lambda_4}],$$
$$\frac{d\bar{\sigma}_{33}}{d\cos\theta^*} = \left(\frac{1}{128\pi\hat{s}}\right)\sum_{\lambda_3\lambda_4}\mathrm{Re}[F_{+-\lambda_3\lambda_4}F^*_{-+\lambda_3\lambda_4}], \tag{3.4}$$
$$\frac{d\bar{\sigma}'_{33}}{d\cos\theta^*} = \left(\frac{1}{128\pi\hat{s}}\right)\sum_{\lambda_3\lambda_4}\mathrm{Re}[F_{++\lambda_3\lambda_4}F^*_{--\lambda_3\lambda_4}],$$
$$\frac{d\bar{\sigma}_{23}}{d\cos\theta^*} = \left(\frac{1}{64\pi\hat{s}}\right)\sum_{\lambda_3\lambda_4}\mathrm{Im}[F_{++\lambda_3\lambda_4}F^*_{+-\lambda_3\lambda_4}].$$

To obtain the total cross section from the above expressions, the integration over $cos\theta^*$ has to be done in the range $0 \leq \cos\theta^* \leq 1$. However, the whole range of θ^* will not be experimentally observable, hence, for our numerical estimates we will restrict the scattering angle to be $|\cos\theta^*| \leq \cos30°$. It should be noted that of the above mentioned cross sections only $d\bar{\sigma}_0/d\cos\theta^*$ should be positive, where the angle θ^* is the scattering angle of the photons in the $\gamma\gamma$ rest frame. The process $\gamma\gamma \rightarrow \gamma\gamma$ proceeds through the mediation of charged particles. In the SM these charged particles were charged gauge bosons (W), quarks, and charged leptons. In the LH model, in addition to the charged gauge bosons and fermions, we also have charged scalars. The analytical expressions of the contributions from fermions, gauge bosons, and scalars to the helicity amplitudes are given in [13] and are quoted in Appendix A. With these equations in hand we shall, in the next section, analyze what effects the LH models will have on these cross sections.

IV. RESULTS AND CONCLUSIONS

In this section we shall present our numerical analysis of the $\gamma\gamma \rightarrow \gamma\gamma$ scattering in the LH model, with and without T-parity. Note that in the $\gamma\gamma$ scattering process the helicity amplitudes are proportional to the fourth power of the charge of the particle circulating in the box i.e.

$$F_{\lambda_1\lambda_2\lambda_3\lambda_4} \propto Q^4,$$

where Q is the charge of the particle. In the LH models we generically have a triplet of scalar particles, one of which is doubly charged, such as Φ^{--}. This results in a factor of 16 in the amplitude and hence a factor of 256 in the cross section. This should provide noticeable signatures in the cross sections.

In our first set of results, presented in Figs. 1 and 2, we have shown the contribution of LH particles to the various helicity amplitudes introduced earlier. In Fig. 1 we have shown the behavior of both real and imaginary parts of the helicity amplitudes for $\theta^* = 90°$ and in Fig. 2 the results have been plotted for $\theta^* = 30°$. Note that for a scattering angle (θ^*) of 90° we have $\hat{u} = \hat{t}$, which results in $F_{+--+} = F_{+-+-}$. Whereas this relationship is not present for other values of the scattering angle.

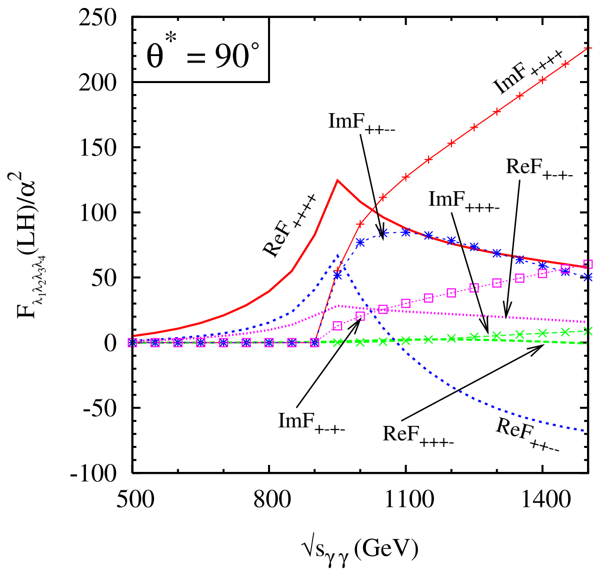

FIG. 1 (color online). Helicity amplitudes of the LH (with T-parity) contributions for $\theta^* = 90°$. For this plot we have taken $f = 700$ GeV. For $\theta^* = 90°$ we have $F_{+--+} = F_{+-+-}$.

We should note, at this point, that the $\gamma\gamma \rightarrow \gamma\gamma$ scattering proceeds through loops, both in the SM and in the LH models. In these loops intermediate particles are pair produced (which is why LH models with T-parity are particularly interesting as precision and cosmological constraints on LH particle masses is much weaker [7,8]). In the SM these are dominated by W loops, leading to a peak in the SM cross sections around the threshold of the W pair production [14]. Similarly, in the LH model (with and without T-parity), the dominant contribution will come from the new heavy W-boson and the Higgs particles (especially those that are doubly charged), once we exceed the threshold for the pair production of these particles. As such, we have plotted the various cross sections for a range of energies ($\sqrt{s_{\gamma\gamma}}$) well above the threshold for the SM W-bosons, but in the vicinity of the pair production energy for the new particles in the LH models; see Figs. 3 and 4. Note further, that we have integrated our differential cross sections in the angular range $30° \leq \theta^* \leq 150°$.

We have plotted the SM value and LH value of the various cross sections introduced in the previous section in Figs. 3 and 4. As expected, the deviation in the SM value of the cross sections becomes visible around the threshold of the pair production of LH particles. At present there are very stringent constraints on the masses of LH particles in models without T-parity [4,9,10], as can be seen from Fig. 3, the deviation from SM values occurs at a very high value of $\sqrt{s_{\gamma\gamma}}$. However, as was noted earlier, in LH

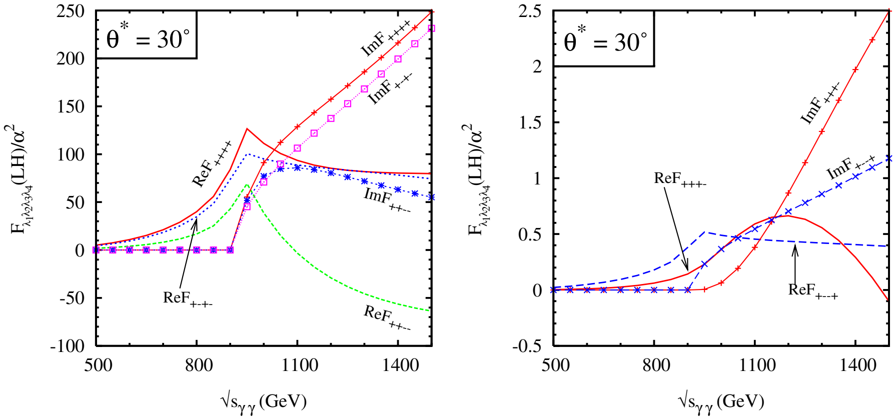

FIG. 2 (color online). Helicity amplitudes of LH (with T-parity) contributions for $\theta^* = 30°$.

models with T-parity a comparatively lower value of the LH particle masses is allowed, which is reflected in the plots in Fig. 4.

For $\gamma\gamma \to \gamma\gamma$ scattering, the LH particles which contribute are the charged gauge bosons (W_H), charged Higgs, and charged fermions. The present constraints on the LH models without T-parity forces the masses of all the new heavy particles to be of the order of TeV. As we are only concerned with charged particles, the only parameters of interest in the LH model without T-parity will be f, x_L, s

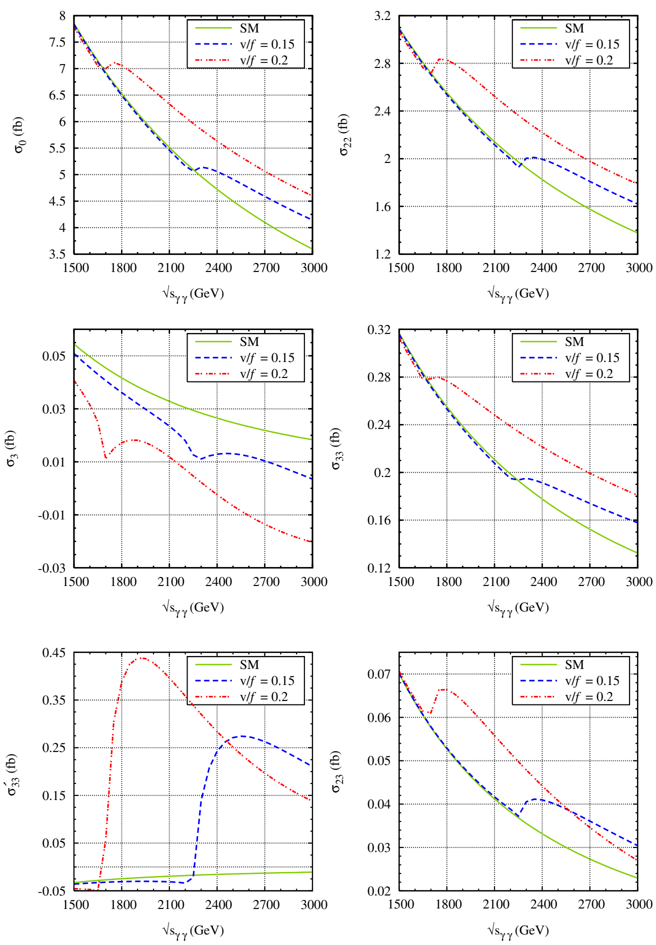

FIG. 3 (color online). Results for the cross sections integrated in the range $30 \leq \theta^* \leq 150$ for various values of v/f. Other LH model parameters are $x_L = 0.2$, $s = s' = 0.6$.

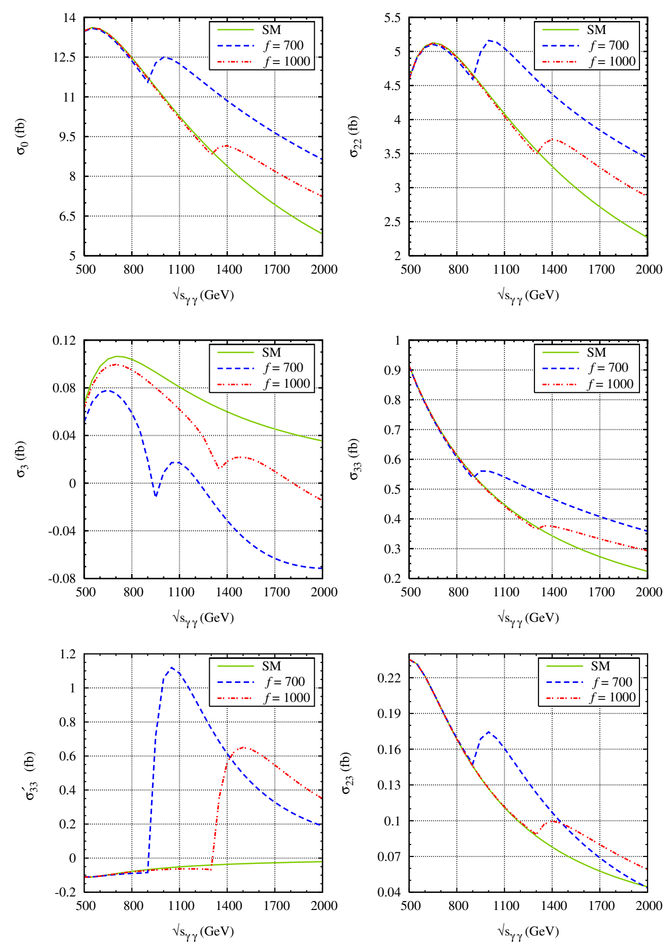

FIG. 4 (color online). Results for the cross sections integrated in the range $30 \leq \theta^* \leq 150$ for various values of f (in GeV) in the LH model with T-parity.

(as defined earlier in Sec. II). The plots for the LH model with T-parity are shown in Fig. 4, where we have chosen λ_1/λ_2 to be 1.

In all cases we can get substantial deviations in the cross sections due to LH effects—these effects being prominent for relatively lower values of $\sqrt{s_{\gamma\gamma}}$ for the models with T-parity, where we have weaker constraints on the model parameters. It should be noted that the σ_3 and σ'_{33} provide the most interesting results, where the σ_3 is the only cross section with pronounced "dips" (these being more pronounced when T-parity is included in the model). The location of these dips being dependent on the model parameters. The other feature of note in these plots are the pronounced peaks in the σ'_{33} cross section. The LH model effects are more pronounced in σ_3 and σ'_{33}. The SM values of the cross sections σ_3 and σ'_{33} are relatively small as compared to the other cross sections; however, the new physics (the LH model here) effects in these two cross sections are very striking. These effects mainly depend upon the LH parameter f (the symmetry breaking scale of the global symmetry). In LH models without T-parity the allowed value of f is high, hence the masses of the new heavy particles are high. This results in the deviations, in LH results from SM results, as manifesting at higher values of the invariant mass. Whereas in the case of T-parity models a much lower value of f is allowed. This now results in lower mass values of T-odd particles—resulting in the onset of LH deviations at a much lower invariant mass.

The results which we have presented for the process $\gamma\gamma \to \gamma\gamma$ are rather generic and can be used as a probe for heavy charged gauge bosons and charged scalars. In our results we have tried to focus ourselves to the range of cm energy ($\sqrt{s_{\gamma\gamma}}$) which is close to the threshold of the pair production of the particles. The deviations from SM results as shown in Figs. 3 and 4 will not be observable in the proposed International Linear Collider but will be easily probed in a multi-TeV e^+e^- compact linear collider where it is proposed to build an e^+e^- linear collider with a center of mass energy from 0.5–3 TeV. Generically, such a mode should lead to $\gamma\gamma$ collisions at cm energies $E_{\rm cm}^{\gamma\gamma} \le 0.8 E_{\rm cm}^{\rm ee}$. Furthermore, the polarized cross sections σ_3 and σ'_{33} can be used to test the spin structure of the particle loops which are responsible for the $\gamma\gamma \to \gamma\gamma$ process [14]. In summary, the $\gamma\gamma \to \gamma\gamma$ process is a very clean process which shall provide a very useful tool for testing LH type models.

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APPENDIX A: HELICITY AMPLITUDES

As noted earlier in this paper, for the process

$$\gamma(p_1, \lambda_1)\gamma(p_2, \lambda_2) \to \gamma(p_3, \lambda_3)\gamma(p_4, \lambda_4), \tag{A1}$$

the helicity amplitudes can be denoted $F_{\lambda_1\lambda_2\lambda_3\lambda_4}(\hat{s}, \hat{t}, \hat{u})$, where the momenta and helicities of the incoming and outgoing photons are as denoted in the above equation, and where we have used the Mandelstam variables $\hat{s} = (p_1 + p_2)^2$, $\hat{t} = (p_1 - p_3)^2$, and $\hat{u} = (p_1 - p_4)^2$. Recall that the use of Bose statistics, crossing symmetry, and parity and time inversion invariance results in the 16 possible helicity amplitudes as being expressible in terms of just three amplitudes. Namely, $F_{+++-}(\hat{s}, \hat{t}, \hat{u})$, $F_{++--}(\hat{s}, \hat{t}, \hat{u})$, and $F_{++++}(\hat{s}, \hat{t}, \hat{u})$ through [13]

$$\begin{aligned} F_{\pm\pm\mp\pm}(\hat{s}, \hat{t}, \hat{u}) &= F_{\pm\mp\pm\pm}(\hat{s}, \hat{t}, \hat{u}) = F_{\pm\mp\mp\mp}(\hat{s}, \hat{t}, \hat{u}) \\ &= F_{---+}(\hat{s}, \hat{t}, \hat{u}) = F_{+++-}(\hat{s}, \hat{t}, \hat{u}), \end{aligned} \tag{A2}$$

$$F_{--++}(\hat{s}, \hat{t}, \hat{u}) = F_{++--}(\hat{s}, \hat{t}, \hat{u}), \tag{A3}$$

$$F_{\pm\mp\pm\mp}(\hat{s}, \hat{t}, \hat{u}) = F_{----}(\hat{u}, \hat{t}, \hat{s}) = F_{++++}(\hat{u}, \hat{t}, \hat{s}), \tag{A4}$$

$$\begin{aligned} F_{\pm\mp\mp\pm}(\hat{s}, \hat{t}, \hat{u}) &= F_{\pm\mp\mp\pm}(\hat{s}, \hat{u}, \hat{t}) = F_{++++}(t, s, u) \\ &= F_{++++}(\hat{t}, \hat{u}, \hat{s}). \end{aligned} \tag{A5}$$

Note that in expressing the SM and LH helicity amplitudes we shall use the notation of B, C, and D functions as given in [15]. The B_0, C_0, and D_0 are the usual one-loop functions first introduced by Passarino and Veltman [16]. The charged gauge boson contributions to the helicity amplitudes can be written as [13]

$$\begin{aligned} \frac{F^W_{++++}(\hat{s}, \hat{t}, \hat{u})}{\alpha^2} &= 12 - 12\left(1 + \frac{2\hat{u}}{\hat{s}}\right)B_0(\hat{u}) - 12\left(1 + \frac{2\hat{t}}{\hat{s}}\right)B_0(\hat{t}) + \frac{24 m_W^2 \hat{t}\,\hat{u}}{\hat{s}} D_0(\hat{u}, \hat{t}) \\ &\quad + 16\left(1 - \frac{3m_W^2}{2\hat{s}} - \frac{3\hat{t}\,\hat{u}}{4\hat{s}^2}\right)[2\hat{t}C_0(\hat{t}) + 2\hat{u}C_0(\hat{u}) - \hat{t}\,\hat{u}\,D_0(\hat{t}, \hat{u})] + 8(\hat{s} - m_W^2)(\hat{s} - 3m_W^2) \\ &\quad \times [D_0(\hat{s}, \hat{t}) + D_0(\hat{s}, \hat{u}) + D_0(\hat{t}, \hat{u})], \end{aligned} \tag{A6}$$

$$\frac{F^W_{+++-}(\hat{s},\hat{t},\hat{u})}{\alpha^2} = -12 + 24m_W^2[D_0(\hat{s},\hat{t}) + D_0(\hat{s},\hat{u}) + D_0(\hat{t},\hat{u})] + 12m_W^2\hat{s}\,\hat{t}\,\hat{u}\left[\frac{D_0(\hat{s},\hat{t})}{\hat{u}^2} + \frac{D_0(\hat{s},\hat{u})}{\hat{t}^2} + \frac{D_0(\hat{t},\hat{u})}{\hat{s}^2}\right] - 24m_W^2\left(\frac{1}{\hat{s}} + \frac{1}{\hat{t}} + \frac{1}{\hat{u}}\right)[\hat{t}C_0(\hat{t}) + \hat{u}C_0(\hat{u}) + \hat{s}C_0(\hat{s})], \tag{A7}$$

$$\frac{F^W_{++--}(\hat{s},\hat{t},\hat{u})}{\alpha^2} = -12 + 24m_W^2[D_0(\hat{s},\hat{t}) + D_0(\hat{s},\hat{u}) + D_0(\hat{t},\hat{u})]. \tag{A8}$$

The contributions from a fermion of charge Q_f and mass m_f to the helicity amplitudes can then be written as [13]

$$\frac{F^f_{++++}(\hat{s},\hat{t},\hat{u})}{\alpha^2 Q_f^4} = -8 + 8\left(1 + \frac{2\hat{u}}{\hat{s}}\right)B_0(\hat{u}) + 8\left(1 + \frac{2\hat{t}}{\hat{s}}\right)B_0(\hat{t}) - 8\left(\frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2} - \frac{4m_f^2}{\hat{s}}\right)[\hat{t}C_0(\hat{t}) + \hat{u}C_0(\hat{u})] + 8m_f^2(\hat{s} - 2m_f^2) \times [D_0(\hat{s},\hat{t}) + D_0(\hat{s},\hat{u})] - 4\left[4m_f^4 - (2\hat{s}m_f^2 + \hat{t}\,\hat{u})\frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2} + \frac{4m_f^2\hat{t}\,\hat{u}}{\hat{s}}\right]D_0(\hat{t},\hat{u}), \tag{A9}$$

$$F^f_{+++-}(\hat{s},\hat{t},\hat{u}) = -\tfrac{2}{3}Q_f^4\{F^W_{+++-}(\hat{s},\hat{t},\hat{u}); m_W \to m_f\}, \tag{A10}$$

$$F^f_{++--}(\hat{s},\hat{t},\hat{u}) = -\tfrac{2}{3}Q_f^4\{F^W_{++--}(\hat{s},\hat{t},\hat{u}); m_W \to m_f\}. \tag{A11}$$

As discussed earlier, the LH model introduces several new particles, including new scalar particles. As such the contribution from new scalar particles of mass m_s and charge Q_s to the helicity amplitudes can be written as [13]

$$\frac{F^s_{++++}(\hat{s},\hat{t},\hat{u})}{\alpha^2 Q_s^4} = 4 - 4\left(1 + \frac{2\hat{u}}{\hat{s}}\right)B_0(\hat{u}) - 4\left(1 + \frac{2\hat{t}}{\hat{s}}\right)B_0(\hat{t}) + \frac{8m_s^2\hat{t}\,\hat{u}}{\hat{s}}D_0(\hat{t},\hat{u}) - \frac{8m_s^2}{\hat{s}}\left(1 + \frac{\hat{u}\,\hat{t}}{2m_s^2\hat{s}}\right)[2\hat{t}C_0(\hat{t}) + 2\hat{u}C_0(\hat{u}) - \hat{t}\,\hat{u}\,D_0(\hat{t},\hat{u})] + 8m_s^4[D_0(\hat{s},\hat{t}) + D_0(\hat{s},\hat{u}) + D_0(\hat{t},\hat{u})], \tag{A12}$$

$$F^s_{+++-}(\hat{s},\hat{t},\hat{u}) = \tfrac{1}{3}Q_s^4\{F^W_{+++-}(\hat{s},\hat{t},\hat{u}); m_W \to m_s\}, \tag{A13}$$

$$F^s_{++--}(\hat{s},\hat{t},\hat{u}) = \tfrac{1}{3}Q_s^4\{F^W_{++--}(\hat{s},\hat{t},\hat{u}); m_W \to m_s\}, \tag{A14}$$

while new fermions and bosons shall be incorporated with helicity amplitudes presented in Eqs. (A6)–(A11).

APPENDIX B: INPUT PARAMETERS

$$m_H = 120\ \text{GeV}, \qquad v = 246\ \text{GeV}, \qquad \alpha = \tfrac{1}{130}, \qquad g^2 = 0.34, \qquad g'^2 = 0.12.$$

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