

Stability of strange star crusts and strangeletsMark G. Alford,¹ Krishna Rajagopal,^{2,3} Sanjay Reddy,⁴ and Andrew W. Steiner⁴¹*Department of Physics, Washington University, St. Louis, Missouri 63130, USA*²*Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*³*Nuclear Science Division, MS 70R319, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*⁴*Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*

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We construct strangelets, taking into account electrostatic effects, including Debye screening, and arbitrary surface tension σ of the interface between vacuum and quark matter. We find that there is a critical surface tension σ_{crit} below which large strangelets are unstable to fragmentation and below which quark star surfaces will fragment into a crystalline crust made of charged strangelets immersed in an electron gas. We derive a model-independent relationship between σ_{crit} and two parameters that characterize any quark matter equation of state. For reasonable model equations of state, we find σ_{crit} typically of order a few MeV/fm². If $\sigma \leq \sigma_{\text{crit}}$, the size-distribution of strangelets in cosmic rays could feature a peak corresponding to the stable strangelets that we construct.

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I. INTRODUCTION

Ordinary matter consists of atoms, whose nuclei are ultimately composed of up and down quarks. One can think of nuclei as droplets of nuclear matter, which is observed to be very stable: the most stable nuclei have lifetimes longer than the age of the universe. However, it has been hypothesized [1–3] that nuclear matter may actually be metastable, and the true ground state of matter consists of a combination of up, down, and strange quarks known as “strange matter.” Small nuggets of such matter are called “strangelets.” If this “strange matter hypothesis” is true, then many if not all compact stars are not neutron stars but “strange stars”: large balls of strange matter (for a review see Ref. [4]). Moreover, strangelets may be produced in compact star collisions, and hence contribute to the cosmic ray background, or in heavy-ion collisions, where they could be observed in terrestrial experiments.

Until recently, it has been assumed that the boundary between strange matter and the vacuum is a simple surface, with a layer of positively charged quark matter beneath it and with electrons floating above it, sustained by an electric field which could also support a small normal nuclear crust in suspension above the quark matter [5], as long as the strange star is not too hot [6].

This was recently questioned by Jaikumar, Reddy, and Steiner [7], who showed that if Debye screening and surface tension were neglected then the surface must actually fragment into a charge-separated mixture, involving positively-charged strangelets immersed in a negatively charged sea of electrons, presumably forming a crystalline solid crust. At the surface of this strangelet + electron crust, there would be no electric field. This gives a radically different picture of the strange star surface, making it much more similar to that of an ordinary neutron star, and casts doubt on all of the previous work on the phenomenology of strange star surfaces.

In this paper we address the stability of electrically neutral bulk quark matter with respect to fragmentation into a charge-separated strangelet + electron crust, and the almost precisely equivalent question of whether large strangelets are stable with respect to fission into smaller strangelets. We shall include energy costs due to Debye screening, neglected in Ref. [7], and surface tension. Our treatment of Debye screening follows that of Heiselberg [8] except that we include the energy gained by charge separation which was neglected in Ref. [8] and whose importance was discovered in Ref. [7].

We identify the three microscopic parameters that play a crucial role in the stability analysis: (1) the charge density of quark matter at zero electric charge chemical potential denoted as n_Q ; (2) the electric charge susceptibility of quark matter denoted as χ_Q ; and (3) the surface tension between quark matter and vacuum, denoted as σ . In Secs. II and III we derive all of our results in terms of these parameters. In Sec. II we give a parametric estimate of the critical surface tension below which neutral quark matter and large strangelets are unstable to fragmentation, and in Sec. III we construct explicit strangelet profiles and thus determine the dimensionless quantities left unspecified in Sec. II.

The quantities n_Q and χ_Q can be evaluated for any proposed quark matter equation of state, and in Sec. IV we shall do so in bag models for unpaired quark matter and quark matter in the 2SC color superconducting state [9–12]. We find that even if the strange quark is so heavy that no BCS pairing occurs in bulk quark matter, if such bulk matter is unstable to fission then the strangelets that result are likely to be in the 2SC phase. This occurs because charged 2SC matter may have higher pressure than charged unpaired quark matter, even when neutral 2SC matter has lower pressure than neutral unpaired quark matter.

The color-flavor locked (CFL) phase [12,13] is the favored phase of neutral quark matter if the density is

high enough, the pairing interactions are strong enough, or the strange quark is light enough. The CFL phase is an insulator, so both n_Q and χ_Q vanish, because there is pairing between all three flavors, and no electrons are needed in order to maintain neutrality. This makes CFL matter stable against fission even if σ is arbitrarily small. Our goal in this paper is to understand what happens if the quark matter at the surface of a strange star is not in the CFL phase, although this form of quark matter may occur deeper within.

We close in Sec. V with comments on phenomenological implications for the strange star surface and experimental strangelet searches.

II. STABILITY OF STRANGELETS

The stability of strangelets is determined by the equation of state (EoS) for quark matter (QM), which will be discussed below, and the EoS of the vacuum with electrons, which is well known. The equation of state expresses the pressure $p(\mu, \mu_e)$ as a function of the chemical potentials for the two conserved quantities, quark number (μ) and negative electric charge (μ_e). We work at zero temperature throughout. The quark density n and the electric charge density q (in units of the positron charge e) are

$$n = \frac{\partial p}{\partial \mu}, \quad q = \frac{\rho}{e} = -\frac{\partial p}{\partial \mu_e}. \quad (1)$$

The chemical potential μ_e is the energy cost of adding an electron, so it is the positron charge e multiplied by the electrostatic potential ϕ . Hence, $\mu_e q = \phi \rho$. The energy densities follow from the usual thermodynamic relation

$$\varepsilon = \mu n - \mu_e q - p. \quad (2)$$

The second term in (2) and the final term in (1) are negative because μ_e is the chemical potential for *negative* electric charge. As written, ε and p are understood to include the electric field energy and pressure. Henceforth, however, we shall interpret p as only the kinetic pressure of the quark matter and electrons, in which case the electric field energy $\frac{1}{2} \int d^3r \phi \rho$ must be added to $\int d^3r \varepsilon$. This can be implemented by writing

$$\varepsilon = \mu n - \frac{1}{2} \mu_e q - p_{\text{QM}+e}. \quad (3)$$

We make the strange quark matter hypothesis that at some μ_{crit} , uniform neutral quark matter with some non-zero n has the same pressure as the vacuum. (The hypothesis further requires that $\mu_{\text{crit}} < \mu_{\text{nuclear}}$, where $\mu_{\text{nuclear}} \approx 310$ MeV is the chemical potential of uniform neutral nuclear matter at zero pressure, and that the chemical potential for two-flavor quark matter with zero pressure is greater than μ_{nuclear} .) However, if only volume contributions to the energy are taken into account, then low pressure quark matter is unstable against fragmentation into positively charged strangelets embedded in a gas of electrons [7]. The surface and Coulomb energy cost are

then crucial to determine the stability of the uniform phase. To evaluate these costs consistently Debye screening effects need to be incorporated [14]. In this paper we evaluate the surface tension required to stabilize neutral quark matter at zero pressure against such fragmentation, taking into account Debye screening.

Debye screening causes the positive charge density in large strangelets to migrate towards the surface, resulting in a charged skin whose thickness is of order the Debye length λ_D and a neutral interior. So, for large strangelets ($R \gg \lambda_D$) the volume energy benefit of charge separation is reduced to a surface energy. Typically, λ_D is of order 5 fm in quark matter. Whereas Debye screening penalizes large strangelets, surface tension penalizes small strangelets most, as their surface to volume ratio is the greatest. Both effects make fragmentation into positively charged strangelets less favorable. We can expect their combined effect to be least for droplets with $R \sim \lambda_D$, meaning that if bulk quark matter proves unstable, it will fragment into strangelets of this size.

Jaikumar *et al.* proposed that the outer layers of a strange star could be constructed from positively charged strangelets with some radius R embedded in a gas of electrons, forming a Wigner-Seitz lattice [7]. The local pressure p will vary considerably within a single Wigner-Seitz cell, but the external pressure p_{ext} on a Wigner-Seitz cell from its neighbors will be zero at the surface of the star, and increase smoothly with depth. To evaluate the stability of such a crust, we must evaluate the Gibbs free energy per quark

$$g(R) = \frac{E(R) + p_{\text{ext}}V(R)}{N(R)}, \quad (4)$$

where E , V and N are the energy, volume and quark number of a Wigner-Seitz cell containing a strangelet of radius R . We shall calculate the energy $E(R)$ by solving the Poisson equation upon making a Thomas-Fermi approximation to obtain the distribution of charge, integrating ε as defined in (3), and then adding the surface tension energy $4\pi R^2 \sigma$. We must then compare $g(R)$ with the Gibbs free energy per quark g_{QM} of uniform neutral quark matter at the same pressure p_{ext} . We shall work throughout at $p_{\text{ext}} = 0$, meaning that we analyze the stability of the outer surface of the mixed phase crust proposed in Ref. [7] and at the same time that of an isolated strangelet with radius R . We shall be comparing them to neutral bulk quark matter with $\mu = \mu_{\text{crit}}$, which has zero pressure and $g_{\text{QM}} = \mu_{\text{crit}}$. So, at $p_{\text{ext}} = 0$, the free energy per quark of a strangelet of radius R , relative to infinite neutral quark matter, is

$$\Delta g(R) = g(R) - g_{\text{QM}} = \frac{E(R)}{N(R)} - \mu_{\text{crit}}. \quad (5)$$

For any σ , $\Delta g(R) \rightarrow 0$ for $R \rightarrow \infty$. For large enough values of σ , $\Delta g(R)$ will be dominated by its surface energy contribution $3\sigma/(nR)$, making $\Delta g(R) > 0$ for all R . This

means that for large enough σ neutral bulk matter is stable with respect to fragmentation. For large enough σ , furthermore, $\Delta g(R)$ decreases monotonically with increasing R , meaning that isolated strangelets of any size are stable, but can lower their energy by fusing with other strangelets should they encounter them. At the other extreme, with $\sigma = 0$ we know from the work of Ref. [7] that $\Delta g(R) < 0$ for small enough R . We therefore expect that for small enough values of σ we shall find a range of R in the vicinity of $R \sim \lambda_D$ for which $\Delta g(R) < 0$. This means that, for small enough σ , neutral bulk quark matter and large strangelets are unstable to fragmentation, with the stable strangelets being those having the size $R = R_*$ that minimizes $\Delta g(R)$.

The equation of state of quark matter at phenomenologically interesting densities and $T \ll \mu$ cannot currently be calculated by lattice gauge theory or by other methods, so we can either obtain an approximate EoS from some model, or make a general parametrization. We will find that $\mu_e \ll \mu$ for all strangelets in all models that we consider, so a general parametrization of the EoS can be obtained by expanding in powers of μ_e/μ [7],

$$p_{\text{QM}} = p_0(\mu, m_s) - n_Q(\mu, m_s)\mu_e + \frac{1}{2}\chi_Q(\mu, m_s)\mu_e^2 + \dots \quad (6)$$

This second-order expansion, which neglects the electron pressure $p_e \sim \mu_e^4$, can be used for any model EoS or for that predicted by QCD. We will see below that it is an excellent approximation for the analysis of strangelets. It reduces the EoS-dependence to specifying the three functions p_0 , n_Q and χ_Q . Moreover, only n_Q and χ_Q occur in the Poisson equation.

Neutral bulk quark matter has $q = 0$, so its electron chemical potential (1) is

$$\mu_e^{\text{neutral}} \equiv \frac{n_Q}{\chi_Q}. \quad (7)$$

The quark chemical potential μ_{crit} at which it has zero pressure is determined by solving

$$p_0(\mu_{\text{crit}}, m_s) = \frac{n_Q^2(\mu_{\text{crit}}, m_s)}{2\chi_Q(\mu_{\text{crit}}, m_s)}. \quad (8)$$

The Debye screening length in quark matter is controlled by χ_Q , and is given by

$$\lambda_D = \frac{1}{\sqrt{4\pi\alpha\chi_Q}}, \quad (9)$$

with $\alpha = 1/137$.

We close this section with a parametric estimate of the critical surface tension below which strangelets with $R \sim \lambda_D$ have $\Delta g(R) < 0$, making neutral bulk quark matter (and larger strangelets) unstable to fragmentation. In Sec. III we will quantitatively determine $\mu_e(r)$ and the radius R_* of the most stable strangelet. For now, we write $x_* \equiv R_*/\lambda_D$ and take μ_e in the strangelet to be a constant

with the value fn_Q/χ_Q . Presuming the dimensionless quantities x_* and f to be of order unity, we use Eqs. (1), (3), and (5)–(9) to obtain

$$\Delta g(R_*) = \frac{3\sigma}{nR_*} - \frac{n_Q^2(1-f)}{2n\chi_Q} \quad (10)$$

which is negative if $\sigma < \sigma_{\text{crit}}$ where

$$\sigma_{\text{crit}} = \frac{(1-f)R_*n_Q^2}{6\chi_Q} = \frac{(1-f)x_*n_Q^2}{12\sqrt{\pi\alpha}\chi_Q^{3/2}}. \quad (11)$$

In the next section we shall construct strangelets by solving the Poisson equation and show that this parametric estimate is valid, with $x_* \approx 1.61$ and $f \approx 0.49$.

III. CONSTRUCTING STRANGELETS

We assume that the lowest energy state of a strangelet of radius R is spherically symmetric. The quark chemical potential μ is independent of r because the only net force on a given part of the strangelet is the electrostatic force. (We assume our strangelets are small enough that their self-gravity is unimportant, and that color is screened so strong interactions do not occur across distances greater than about 1 fm.) The value of μ inside the droplet is a little higher than μ_{crit} because the surface tension compresses the droplet slightly. To determine the value of μ , we require the pressure discontinuity across the surface of the strangelet to be balanced by the surface tension:

$$\lim_{\delta r \rightarrow 0} (p(R - \delta r) - p(R + \delta r)) = \frac{2\sigma}{R}. \quad (12)$$

The larger σ becomes, the more compressed the strangelet is, and the higher the value of μ inside the strangelet.

To calculate $\mu_e(r) = e\phi(r)$ in the Thomas-Fermi approximation we solve the Poisson equation, which in Heaviside-Lorentz units with $\hbar = c = 1$ takes the form

$$\nabla^2 \phi(r) = -\rho(r), \quad \text{i.e.} \quad \nabla^2 \mu_e(r) = -4\pi\alpha q(r), \quad (13)$$

subject to the boundary conditions

$$\begin{aligned} \lim_{r \rightarrow \infty} r\phi(r) &= Z_\infty e, & \frac{d\phi}{dr}(r=0) &= 0, \\ \frac{d\phi}{dr}(R^+) - \frac{d\phi}{dr}(R^-) &= 0. \end{aligned} \quad (14)$$

The second and third boundary conditions follow from the fact that there are no delta-functions of localized charge, so $\phi(r)$ and $d\phi/dr$ are continuous everywhere. The first boundary condition states that the net charge of the strangelet, including any electrons inside or around it, is $Z_\infty e$. The Fermi wavelength of the quarks is $2\pi/\mu$ which is around 4 fm, so it is reasonable to use the Thomas-Fermi approximation to describe the charge distribution due to quarks inside a strangelet with diameter of order $3\lambda_D \sim$

15 fm, which is the approximate size of the stable strangelets that we will find.

Strangelets of this size are too small to have electrons localized within them, meaning that the charge of the strangelet itself is given by that of the quark matter,

$$Z = \int_0^R d^3r q(r) = \int_0^R d^3r (n_Q - \chi_Q \mu_e(r)). \quad (15)$$

To analyze an isolated strangelet, not surrounded by an atom-sized cloud of electrons, we must find solutions to the Poisson equation with $Z_\infty = Z$. To analyze a strangelet located within the Wigner-Seitz cell of a strange star crust, however, we must include a cloud of electrons around the strangelet such that the strangelet and electrons together satisfy the Poisson equation with $Z_\infty = 0$. (Each Wigner-Seitz cell contains one strangelet and Z electrons, ensuring that the crust is neutral on macroscopic length scales.) In our numerical work, we have included the electrons outside the strangelet, taking $q(r) = -\mu_e^3/3\pi^2$ for $r > R$, and found solutions to the Poisson equation with $Z_\infty = 0$. As in atomic physics, the Thomas-Fermi description of the cloud of electrons around a strangelet with charge Z becomes more accurate with increasing Z . Fortunately, it turns out that the electrons do not play an important role in the stability of the strangelet, so we can ignore them, setting $q(r) = 0$ for $r > R$ and finding solutions to the Poisson equation with the boundary condition at infinity given simply by $Z_\infty = Z$. We have done all our analytic calculations with this boundary condition, and have confirmed numerically that adding electrons to the system in order to find solutions with $Z_\infty = 0$ makes a negligible difference to the energy E and to $\mu_e(r)$ inside the strangelet. This simplification arises because, as mentioned in Sec. II, the electron contribution to the EoS is subleading in μ_e/μ .

We obtain $E(R)$ and $N(R)$, the quantities which according to (5) we need in order to determine the stability of a strangelet with some specified radius R , as follows. For a given μ we solve (13) subject to the boundary conditions (14). We repeat this for different values of μ until we find the one that obeys (12). We then have μ and $\mu_e(r)$, so from (1) and (3) we can obtain the energy density $\varepsilon(r)$ and quark number density $n(r)$. We integrate these, and add in the surface energy, to obtain the total energy E and quark number N and hence the energy per quark E/N .

The solution to the Poisson equation satisfying the boundary conditions (14) with $Z_\infty = Z$ given by (15) is [8]

$$\begin{aligned} r < R: \mu_e(r) &= \frac{n_Q}{\chi_Q} \left(1 - \frac{\lambda_D}{r} \frac{\sinh(r/\lambda_D)}{\cosh(R/\lambda_D)} \right) \\ r > R: \mu_e(r) &= \frac{Z\alpha}{r} \end{aligned} \quad (16)$$

where

$$Z = 4\pi n_Q \lambda_D^3 (x - \tanh x), \quad (17)$$

where we have defined $x \equiv R/\lambda_D$. We see that $\mu_e(r) <$

$\mu_e^{\text{neutral}} = n_Q/\chi_Q$ throughout the strangelet, with $\mu_e(r)$ closer to zero for smaller strangelets and closer to μ_e^{neutral} for larger ones.

To this point we have not determined μ . (And, recall that both n_Q and χ_Q depend on μ .) Because we have no electrons and hence $p = 0$ outside the strangelet, the condition (12) requires that just inside the surface of the strangelet the pressure is given by $2\sigma/R$. Recalling that $n = \partial p/\partial \mu$ and that $p(\mu_{\text{crit}}) = 0$ and using Eqs. (6) and (16) we see that this requires that the quark chemical potential inside the strangelet (which is constant throughout its interior) is given by

$$\mu = \mu_{\text{crit}} - \frac{n_Q^2}{2n\chi_Q} \frac{\tanh^2 x}{x^2} + \frac{2\sigma}{nR}. \quad (18)$$

The negative term present even when $\sigma = 0$ arises because $\mu_e(R) < \mu_e^{\text{neutral}}$, making $p(\mu_{\text{crit}}, \mu_e(R)) > 0$. To achieve $p = 0$, as required just inside the edge of the droplet if $\sigma = 0$, μ must be less than μ_{crit} . The σ -dependent positive term then enforces the pressure discontinuity (12) required at nonzero σ . (The derivation requires that each of the corrections to μ are separately much smaller than μ_{crit} , which is well satisfied in all results we show.)

We can now construct the profiles of $\mu_e(r)$, $q(r)$, the electric field and $p(r)$ for strangelets with radius R . An example is shown in Fig. 1. We show the profiles in terms of n_Q and χ_Q . Expressed this way, they are model-

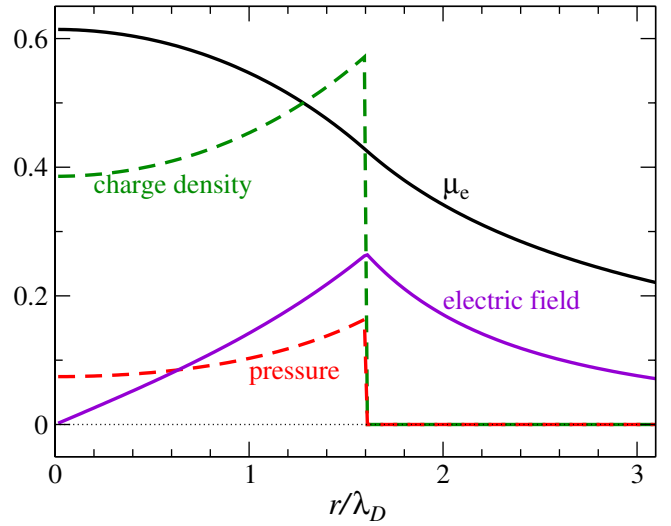


FIG. 1 (color online). A strangelet profile with radius $R = 1.6\lambda_D$, and $\sigma = 0.1325n_Q^2\lambda_D/\chi_Q$. (This corresponds to the barely-stable strangelet at the critical surface tension—see Sec. III.) The horizontal axis is $x = r/\lambda_D$. The charge density q is plotted in units of n_Q , μ_e is plotted in units of n_Q/χ_Q , the electric field is plotted in units of $n_Q/\sqrt{\chi_Q}$ and the pressure p_{QM} is plotted in units of n_Q^2/χ_Q . Because of the electric field, the charge density in the strangelet is pushed towards the outer edge. The pressure gradient within the strangelet is balanced by the electric force on the charged matter.

independent. Given a model equation of state, n_Q and χ_Q must be evaluated at the μ given in (18), which itself depends on n_Q and χ_Q . This means that the explicit determination of μ in a model must be done numerically, although in practice we find that the critical surface tension is very small, so for $\sigma \approx \sigma_{\text{crit}}$ we can use $n_Q(\mu_{\text{crit}})$ and $\chi_Q(\mu_{\text{crit}})$ without making significant errors. We have compared the profiles obtained analytically in terms of n_Q and χ_Q as we have described with those obtained numerically, with n_Q and χ_Q specified by the bag model expressions for unpaired quark matter given in Sec. IV and with electrons included in the solution of the Poisson equation. The agreement between analytical and numerical profiles is excellent.

We now evaluate the Gibbs free energy per quark of a strangelet of radius R relative to that of bulk neutral quark matter, namely $\Delta g(R)$ of (5). Integrating the energy density from (3) and adding the surface contribution, $E(R)$ is given by

$$\begin{aligned} E(R) &= \int_0^R d^3r \left[-p(r) + \mu n - \frac{1}{2} q(r) \mu_e(r) \right] + 4\pi R^2 \sigma \\ &= \int_0^R d^3r \left[-p_0 + \mu n + \frac{1}{2} n_Q \mu_e(r) \right] + 4\pi R^2 \sigma, \end{aligned} \quad (19)$$

where we have used (1) and (6) to replace $p(r)$ and $q(r)$. We see now that the only property of the profile $\mu_e(r)$ that we need in order to evaluate $E(R)$ is the volume average

$$\frac{3}{R^3} \int_0^R d^3r \mu_e(r) = f \frac{n_Q}{\chi_Q} = \frac{n_Q}{\chi_Q} \left(1 - 3 \frac{x - \tanh x}{x^3} \right) \quad (20)$$

where f is the parameter we introduced in making the estimates (10) and (11), and which we have now evaluated using the profile $\mu_e(r)$ in (16). Thus, by solving the Poisson equation we have learned that

$$1 - f = 3 \frac{x - \tanh x}{x^3}. \quad (21)$$

From (19) and (20) we find

$$\frac{E(R)}{N(R)} = \left(-\frac{p_0}{n} + \mu \right) + \frac{n_Q^2 f}{2n\chi_Q} + \frac{3\sigma}{nR} \quad (22)$$

and hence, using (5) and (8),

$$\begin{aligned} n\Delta g(R) &= \left(\frac{n_Q^2}{2\chi_Q} - p_0 \right) + n(\mu - \mu_{\text{crit}}) - \frac{n_Q^2(1-f)}{2\chi_Q} \\ &\quad + \frac{3\sigma}{R}. \end{aligned} \quad (23)$$

Notice, however, that the first term is zero when evaluated at $\mu = \mu_{\text{crit}}$ and is given by $n(\mu_{\text{crit}} - \mu)$ to lowest order in $(\mu - \mu_{\text{crit}})$. Neglecting fractional errors of order $(\mu - \mu_{\text{crit}})/\mu_{\text{crit}}$ in Δg (which Fig. 2 shows are negligible) we

obtain

$$n\Delta g(R) = -\frac{3n_Q^2}{2\chi_Q} \frac{x - \tanh x}{x^3} + \frac{3\sigma}{R}, \quad (24)$$

which is (10) with $(1-f)$ now known but R^* still to be determined. [By comparing to solutions obtained numerically, we have confirmed that at large values of σ/R the fractional error introduced in Δg is $\sim \sigma/(nR\mu_{\text{crit}})$, as in (18).] It is convenient to write (24) in terms of a dimensionless function $\overline{\Delta g}(x)$ of the dimensionless radius $x = R/\lambda_D$,

$$\overline{\Delta g}(x) = -\frac{3}{2} \frac{x - \tanh x}{x^3} + \frac{3\bar{\sigma}}{x}, \quad (25)$$

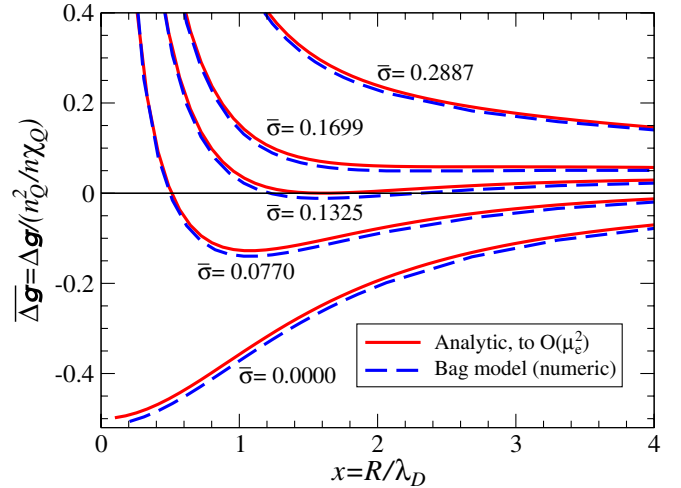


FIG. 2 (color online). The Gibbs free energy (i.e. energy per quark) relative to neutral uniform quark matter with $p = 0$ for strangelets of various sizes and surface tensions. We plot $\overline{\Delta g}(x)$, which is Δg in units of $n_Q^2/(n\chi_Q)$, as a function of the radius $x = R/\lambda_D$, for various values of the surface tension ($\bar{\sigma}$ is σ in units of $n_Q^2\lambda_D/\chi_Q$). The solid lines (red online) are obtained from (24); the dashed lines (blue online) are obtained from a numerical solution of the Poisson equation, including the electrons and not making any approximations in the evaluation of $E(R)$. The numerical results confirm the validity of the approximations made in deriving (24) and (25). (The numerical analysis is for a bag model of unpaired quark matter with $\mu_{\text{crit}} = 305$ MeV and $m_s = 200$ MeV. The n_Q and χ_Q for this model are given in Sec. IV.) At the critical surface tension $\bar{\sigma}_{\text{crit}} = 0.1325$ one can just barely construct a strangelet that is favored over uniform quark matter. The radius of this critical strangelet is $R_* = 1.606\lambda_D$. For $\bar{\sigma} < \bar{\sigma}_{\text{crit}}$, large strangelets and bulk neutral quark matter are unstable to fragmentation. For $\bar{\sigma} > \bar{\sigma}_{\text{no-barrier}} = 0.1699$, there is no fusion barrier: strangelets of any size can lower their free energy by fusing should they encounter each other. For $\bar{\sigma}_{\text{crit}} < \bar{\sigma} < \bar{\sigma}_{\text{no-barrier}}$, there are “metastable” droplets, whose free energy is greater than that of bulk quark matter but which must overcome a barrier in order to fuse.

where

$$\Delta g(R) \equiv \frac{n_Q^2}{n\chi_Q} \overline{\Delta g}(x), \quad \bar{\sigma} \equiv \frac{\chi_Q \sigma}{\lambda_D n_Q^2}. \quad (26)$$

In Fig. 2 we plot Δg versus R in dimensionless units for several values of σ ; this is equivalent to plotting $\overline{\Delta g}(x)$ for several values of $\bar{\sigma}$. The function $\overline{\Delta g}(x)$ has a stationary zero at $x_* = 1.606$ for $\bar{\sigma}_{\text{crit}} = 0.1325$: this corresponds to the marginally stable strangelet at the critical surface tension. For $\bar{\sigma} < \bar{\sigma}_{\text{crit}}$, the minimum of $\overline{\Delta g}$ occurs at smaller x , and the value of $\overline{\Delta g}$ at the minimum is negative, corresponding to a smaller stable strangelet. For $\bar{\sigma} > \bar{\sigma}_{\text{crit}}$, neutral bulk quark matter and large strangelets can lower their free energy by fissioning, although for large enough strangelets there is always an energy barrier to fission. In the marginally stable droplet with $\bar{\sigma} = \bar{\sigma}_{\text{crit}}$ and $x = x_*$, (18) simplifies to $\mu = \mu_{\text{crit}}$, with the latter two terms in this equation cancelling. This means that the quark chemical potential in such a droplet is the same as that in bulk neutral quark matter. In a droplet with size $x = x_*$, the volume average of μ_e is reduced from its value in neutral bulk quark matter by a factor $f = 0.5051$. (Note that we plotted Fig. 1 for $\bar{\sigma} = \bar{\sigma}_{\text{crit}}$ and $x = x_*$.) The parameters in the estimate (11) are now fully determined.

We see in Fig. 2 that at $\bar{\sigma}_{\text{no-barrier}} = 0.1699$, the minimum in $\overline{\Delta g}(x)$ becomes a stationary inflection point at $x = 2.772$. For $\bar{\sigma}_{\text{crit}} < \bar{\sigma} < \bar{\sigma}_{\text{no-barrier}}$, then, there exist metastable strangelets with sizes ranging from $R = 1.606\lambda_D$ to $R = 2.772\lambda_D$. These are stable against fragmentation, and if two of them encounter each other there is an energy barrier to their fusion. However, they do have higher energy per quark than neutral bulk quark matter. For $\bar{\sigma} > \bar{\sigma}_{\text{no-barrier}}$ there is no local minimum, and all strangelets can lower their free energy by fusing with other strangelets should they encounter them.

Converting back to dimensionful quantities, we find

$$\sigma_{\text{crit}} = 0.1325 \frac{n_Q^2 \lambda_D}{\chi_Q} = 0.1325 \frac{n_Q^2}{\sqrt{4\pi\alpha} \chi_Q^{3/2}} \quad (27)$$

$$\sigma_{\text{no-barrier}} = 0.1699 \frac{n_Q^2 \lambda_D}{\chi_Q}.$$

If the strange matter hypothesis holds (e.g. if $\mu_{\text{crit}} < 310$ MeV) and if σ in QCD takes on any value less than $\sigma_{\text{no-barrier}}$, there will be a favored size for the strangelets that could be found in cosmic rays. If $\sigma < \sigma_{\text{crit}}$, if any strangelets were found they would *all* have sizes peaked around a single value. If σ is somewhat larger, in the regime where there are metastable strangelets, the distribution of strangelet size in the universe would include a peak at the size corresponding to the metastable strangelets, and a continuous distribution of larger strangelets, big enough that they sit beyond the local maximum in the $\Delta g(R)$ curve, where $\Delta g(R)$ is a decreasing function of R .

There would be no energy barrier for these larger strangelets to fuse with one another.

IV. EVALUATING THE STABILITY OF STRANGELETS MADE FROM UNPAIRED AND 2SC STRANGE QUARK MATTER

To this point, we have not needed to specify a model for the quark matter equation of state. Given the values of n_Q and χ_Q in any such model (or, ultimately, in the equation of state of QCD) our results can be used to evaluate the critical surface tension σ_{crit} . If $\sigma < \sigma_{\text{crit}}$ in QCD, strange stars will have a solid crust formed from positive strangelets immersed in an electron gas. And, large strangelets will be unstable to fission.

In this section, we provide two bag model examples to illustrate how our model-independent results translate into estimates of σ_{crit} in MeV/fm². In bag models one writes $p_{\text{QM}}(\mu, \mu_e) = p_{\text{quarks}}(\mu, \mu_e) - B$ where B is the bag constant, and p_{quarks} is obtained by making assumptions, such as treating the quarks in the bag as noninteracting (“unpaired quark matter”), or as being in a color superconducting phase, or perhaps as having some weak residual QCD interactions, although we shall not include the last effect here. We begin with a bag model in which the quarks inside each bag are noninteracting, and then discuss the consequences of BCS pairing, which would lead to color superconductivity in infinite quark matter. We shall set the up and down quark masses to zero, and treat the strange quark mass m_s as a parameter. One extension of our work would be to evaluate n_Q and χ_Q in models in which the (strange) quark mass(es) are solved for self-consistently [15–18], rather than taken as a parameter.

A. Unpaired quark matter

A derivation of the pressure in the bag model for unpaired, noninteracting, quark matter can be found, for example, in Ref. [19]. Expanding in powers of μ_e according to (6) yields

$$p_0(\mu, m_s) = -B + \frac{\mu^4}{2\pi^2} + \frac{1}{8\pi^2} (2\mu^3 - 5\mu m_s^2) \sqrt{\mu^2 - m_s^2} + \frac{3m_s^4}{8\pi^2} \log \frac{\mu + \sqrt{\mu^2 - m_s^2}}{m_s} \quad (28)$$

$$n_Q(\mu, m_s) = \frac{1}{3\pi^2} (\mu^3 - (\mu^2 - m_s^2)^{3/2}) \sim \frac{m_s^2 \mu}{2\pi^2} \quad (29)$$

$$\chi_Q(\mu, m_s) = \frac{1}{3\pi^2} (5\mu^2 + \mu \sqrt{\mu^2 - m_s^2}) \sim \frac{2\mu^2}{\pi^2} \quad (30)$$

In the second expressions for n_Q and χ_Q , we have further assumed that $m_s \ll \mu$. The resulting simplified expressions yield results that are familiar in the literature, for example, the fact that $\mu_e^{\text{neutral}} = n_Q/\chi_Q$ gives the well-

known lowest-order result $\mu_e^{\text{neutral}} = m_s^2/(4\mu)$ for unpaired quark matter. We shall not actually use these approximate expressions in the following, because with $\mu \sim 300$ MeV as in a strangelet or at the surface of a strange star we cannot necessarily assume that $m_s \ll \mu$.

Instead of fixing the bag constant B and varying other parameters, we shall fix the more physical quantity μ_{crit} . At each value of μ_{crit} and m_s we use (8) to fix B .

In Fig. 3 we show how σ_{crit} varies with m_s in the bag model for unpaired quark matter for $\mu_{\text{crit}} = 305$ MeV. (The curves for the other allowed values of μ_{crit} , which vary from 283 MeV to 310 MeV, are indistinguishable from this one except that, as we discuss below, the m_s at which they terminate is μ_{crit} -dependent.) We see that the maximum value of the critical surface tension is less than 2.7 MeV/fm². The solid curve in the figure is given by (27) with n_Q and χ_Q now specified for the unpaired quark matter bag model by (29) and (30). The dashed curve was obtained by solving the Poisson equation numerically, including the electrons, and then evaluating Δg without making the approximations that went into deriving (24).

The curves in Fig. 3 end at $m_s = 240$ MeV because beyond this value nuclear matter becomes unstable relative to two-flavor quark matter. Unlike the conversion to three-flavor quark matter, which must surmount a barrier of many weak interactions, this would allow rapid conversion of nuclei to quark matter. For $\mu_{\text{crit}} = 305$ MeV, requiring that the pressure of two-flavor quark matter at $\mu = 310$ MeV be negative requires $m_s \leq 240$ MeV. Note that we can consider values of m_s all the way up to μ , at the expense of tuning μ_{crit} closer and closer to $\mu_{\text{nuclear}} = 310$ MeV as we take $m_s \rightarrow \mu$ in such a way as to keep nuclear matter (just) stable with respect to two-flavor

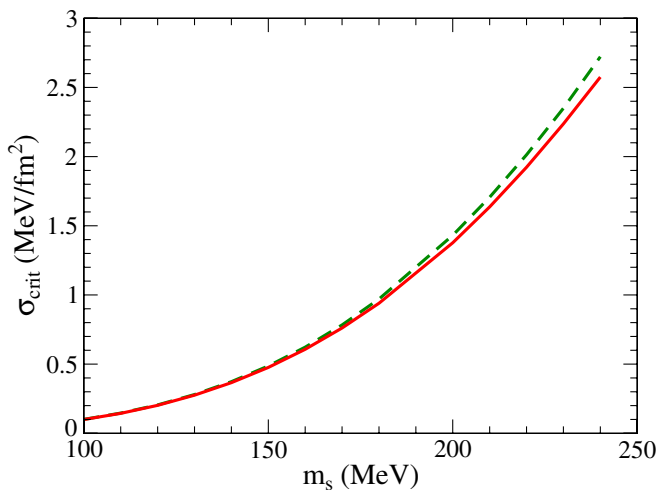


FIG. 3 (color online). The critical surface tension for strangelets made of unpaired quark matter, as a function of the strange quark mass, at $\mu_{\text{crit}} = 305$ MeV. Other allowed values of μ_{crit} give almost indistinguishable curves. The solid curve is given by (27) whereas the dashed curve was obtained by solving the Poisson equation numerically.

quark matter. In this fine-tuned limit, we can get strange quark matter with an arbitrarily small strange quark density, with n_Q and χ_Q arbitrarily close to their two-flavor values $\mu^3/(3\pi^2)$ and $5\mu^2/(3\pi^2)$ respectively. From (27) we see that in this limit, $\sigma_{\text{crit}} \rightarrow 0.01185\mu_{\text{nuclear}}^3/\sqrt{12\pi^3\alpha} = 5.502$ MeV/fm². This is the absolute limit to how large σ_{crit} can be pushed in the bag model for unpaired quark matter. Figure 3 shows the limit if one requires μ_{crit} to be 5 MeV below μ_{nuclear} .

To give a sense of the scales that characterize the critical strangelets, let us take noninteracting quark matter with $\mu_{\text{crit}} = 305$ MeV and $m_s = 200$ MeV as an example. We find $n_Q = 0.07104$ fm⁻³, $\chi_Q = 0.4644$ fm⁻², and $\lambda_D = 4.845$ fm. Bulk neutral quark matter has $\mu_e^{\text{neutral}} = n_Q/\chi_Q = 30.1$ MeV and has quark number density $n = 0.9093$ fm⁻³. If $\sigma = \sigma_{\text{crit}} = 1.377$ MeV, the critical strangelets with radii $1.606\lambda_D = 7.782$ fm have baryon number $A \approx 598$ and charge $Z \approx 69$, consistent with the result $Z \approx 0.1A$ from the literature [20]. If A were significantly smaller, as, for example, would be the case for σ significantly below σ_{crit} , it would be important to include curvature energy and shell effects in the calculation of E/N . The analysis of Ref. [20] indicates that these can reasonably be neglected for strangelets with A 's greater than a few hundred as long as the surface tension is not atypically small. We defer a study of the effects of the curvature energy to future work.

Finally, let us discuss the effect of interactions. Already in the earliest bag model analyses of strange stars and strangelets [3], the perturbative QCD interactions between quarks inside a bag were taken into account. To zeroth order in m_s , these just introduce modifications to the relationship between μ and n which are small if α_s is small. We leave the inclusion of such perturbative effects to future work, and turn to the possibility of color superconducting quark matter, which can introduce changes to the values of n_Q and χ_Q that are qualitative, in the sense that they do not become arbitrarily small when some parameter of the EoS is varied.

B. Color superconducting quark matter

The strong interaction between quarks is attractive in the color-antisymmetric channel, and this leads to BCS pairing and color superconductivity in cold quark matter [12]. The critical temperature for color superconductivity is expected to be in the range of tens of MeV, which is far above the temperature of strangelets or neutron stars. We therefore work at $T = 0$ throughout.

Unlike perturbative interactions, color superconductivity can have dramatic qualitative consequences for the properties of quark matter, because those quarks which undergo BCS pairing have their number densities “locked” into being equal, as the pairing energy gained in so doing overcompensates for the free energy cost of

maintaining number densities that would not minimize the free-particle free energy. In this section we explore the consequences of color superconductivity for the stability of strangelets.

To sketch the context for our discussion, let us describe the neutral color-superconducting phases that may occur in quark matter with $\mu \approx 310$ MeV, depending on the strength of the attractive quark-quark coupling that leads to BCS pairing. We expect the color-flavor locked (CFL) phase at the strongest values of the coupling, with some other, probably nonisotropic, phase occurring at intermediate coupling, and unpaired quark matter at lower couplings still. As we will see, the nonisotropic phases are expected to yield stable strangelets similar to those arising from unpaired quark matter, whereas the analysis of the stability of CFL quark matter strangelets is qualitatively different. We will see that the two-flavor-paired “2SC” phase is not expected to be the favored state of neutral quark matter at $\mu \approx 310$ MeV, but it may well play a role in strangelets, which consist of charged quark matter.

The CFL phase [12,13] is the ground state of three-flavor quark matter when the attractive quark-quark coupling is sufficiently strong that its gap parameter satisfies $\Delta_{\text{CFL}} > m_s^2/(2\mu)$ [21,22]. In the CFL phase, quarks of all three colors and all three flavors undergo BCS pairing [13], and the resultant locking of the Fermi surfaces makes it an electromagnetic insulator, neutral in the absence of electrons [19,23], with $n_Q = \chi_Q = 0$ and $\mu_e^{\text{neutral}} = n_Q/\chi_Q = 0$. If bulk quark matter at μ_{crit} is in the CFL phase, strange stars will be neutral with $\mu_e = 0$ in their interiors [24], meaning that there is no possibility of charge separation and fragmentation, no possibility of a solid crust, and no reason for large strangelets to fission into smaller ones. This can be seen from our analysis by noting that in the CFL phase $\sigma_{\text{crit}} = 0$.

At intermediate values of the quark-quark coupling, we know that a different color-superconducting phase must occur. For $m_s^2/(5.2\mu) \leq \Delta_{\text{CFL}} < m_s^2/(2\mu)$, model analyses that are restricted to isotropic phases predict a gapless CFL (gCFL) phase [21,22]. Depending on the value of $\Delta_{\text{CFL}}\mu/m_s^2$, the values of n_Q , χ_Q and μ_{neutral} in gCFL quark matter can fall anywhere between those characterizing the CFL and unpaired phases, and, in particular, can be arbitrarily small but nonzero. This would make the investigation of strangelets in some gCFL-like phase very interesting, as the Debye length could be arbitrarily large. However, the gCFL phase *per se* is unstable to the formation of current-carrying condensates [25–29] and so it cannot be the ground state. The nature of the ground state at intermediate coupling is not yet established, but one possibility is a three-flavor crystalline color-superconducting phase [29–32], with a nontrivial crystal structure like that favored in the simpler two-flavor case [33,34]. Such phases do *not* involve the locking of Fermi surfaces: this is in large part why they are well-motivated candidates to replace the

gapless CFL phase. This means that, unlike in the CFL or gCFL phases, their values of n_Q , χ_Q and hence n_Q/χ_Q are likely similar to those of unpaired three-flavor quark matter [30,32], so that strangelets made of such matter will have similar stability properties to ones made of unpaired quark matter.

The final possible color-superconducting phase we consider is the “2SC phase” in which BCS pairing occurs only between up and down quarks of two colors [9–12]. Previous studies indicate that we should not expect this phase to occur in bulk *neutral* quark matter near the transition to the vacuum (which we are assuming occurs at $\mu_{\text{crit}} < 310$ MeV). In bag models in which m_s is a parameter, the neutrality constraint ensures that either unpaired, CFL or gCFL quark matter is always favored over 2SC at $T = 0$ [19,22]. In models that solve for m_s self-consistently there is sometimes a small 2SC window, but only if the interaction and parameters are chosen such that m_s is much too large for three-flavor quark matter to exist with $\mu = \mu_{\text{crit}} < 310$ MeV [15–18].

However, even if the 2SC phase is not favored in bulk matter, which must be neutral, it may occur in the charged quark matter of the stable (or metastable) strangelets that we have constructed. We see in Fig. 1 that μ_e inside these droplets is significantly less than μ_e^{neutral} . This means that the unpaired u and d Fermi surfaces are closer to each other, and farther from the s Fermi surface, than in neutral unpaired bulk matter. This should favor the 2SC phase. To investigate this, we take results from Ref. [19] and use them to evaluate the parameters of the generic quark matter EoS (6),

$$p_0^{2\text{SC}}(\mu, m_s) = p_0^{\text{unpaired}} + \frac{\Delta_{2\text{SC}}^2 \mu^2}{\pi^2} \quad (31)$$

$$n_Q^{2\text{SC}}(\mu, m_s) = n_Q^{\text{unpaired}} + \frac{\Delta_{2\text{SC}}^2 \mu}{3\pi^2} \quad (32)$$

$$\begin{aligned} \chi_Q^{2\text{SC}}(\mu, m_s) &= \frac{1}{18\pi^2} (12\mu^2 + 6\mu\sqrt{\mu^2 - m_s^2} + \Delta_{2\text{SC}}^2) \\ &\sim \frac{\mu^2}{\pi^2}. \end{aligned} \quad (33)$$

(These expressions have all been derived assuming $\Delta_{2\text{SC}} \ll \mu$, which implies $\Delta_{2\text{SC}} = 2^{1/3}\Delta_{\text{CFL}}$ [12].) Note the change in χ_Q : it is approximately half as large as in unpaired quark matter, meaning that the Debye length in 2SC quark matter is larger than that in unpaired quark matter by a factor of $\sqrt{2}$. This change once again originates in the locking of those Fermi surfaces which pair, so it is qualitative, in the sense that it is independent of the value of $\Delta_{2\text{SC}}$. To see how big an effect this could have on the stability of strangelets, imagine giving the unpaired phase a larger bag constant than the 2SC phase, so that bulk neutral quark matter is in the 2SC phase. The results of Sec. III show that the size of the critical strangelet would be increased by a factor of approximately $\sqrt{2}$, and σ_{crit} would be increased by a factor of approximately $2\sqrt{2}$. However, if

we keep the same bag constant in unpaired and 2SC quark matter then the effect is smaller. For example, let us analyze the case where $m_s = 200$ MeV, $\Delta_{2SC} = 25$ MeV and $\mu_{\text{crit}}^{\text{unpaired}} = 305$ MeV. We find $n_Q^{2SC} = 1.012n_Q^{\text{unpaired}}$ and $\chi_Q^{2SC} = 0.479\chi_Q^{\text{unpaired}}$. In this case we cannot naively use the results of Sec. III (yielding σ_{crit} enhanced by the factor of about $2\sqrt{2}$ described above) because μ_{crit}^{2SC} is 305.507 MeV, larger than $\mu_{\text{crit}}^{\text{unpaired}}$, and consequently the critical strangelet occurs when the $\Delta g(R)$ curve constructed for the 2SC phase has a local minimum with $\Delta g(R) = \mu_{\text{crit}}^{\text{unpaired}} - \mu_{\text{crit}}^{2SC} < 0$, corresponding to 2SC strangelets with the same E/N as bulk neutral unpaired quark matter. We then find $\sigma_{\text{crit}} = 2.79$ MeV/fm² and $R_* = 1.16\lambda_D^{2SC} = 8.1$ fm. Comparing this to the results for unpaired quark matter strangelets at the same μ_{crit} and m_s (which gave $\sigma_{\text{crit}} = 1.38$ MeV/fm² and $R_* = 7.78$ fm), we see that the occurrence of 2SC matter has changed σ_{crit} by a factor of about 2. The increase in λ_D has been cancelled by a decrease in x_* with the result that R_* hardly changes. Evaluating A and Z for the critically stable 2SC droplet, we find $A \approx 677$ and $Z \approx 105$.

It should be noted that the 2SC Cooper pairs, whose size is $\sim 1/\Delta_{2SC}$, fit within these strangelets, but not by much. This means that for a specified interaction strength between quarks, the parameter Δ_{2SC} occurring in our analysis, which describes the effect of ud -pairing on the E/N of our strangelets, will not have exactly the same value as that describing 2SC pairing in bulk matter. Had we taken Δ_{2SC} significantly smaller, our analysis of 2SC pairing would not even be qualitatively reliable.

We conclude that if the attraction between quarks is strong enough that the critical strangelets are in the 2SC phase (but not so strong as to favor the CFL phase for bulk neutral quark matter) then σ_{crit} is increased by a factor which could be as large as $\approx 2\sqrt{2}$ if Δ_{2SC} were tuned such that $\mu_{\text{crit}}^{2SC} \rightarrow \mu_{\text{crit}}^{\text{unpaired}}$, but which is more typically smaller, of order 2.

V. DISCUSSION

If the surface tension in QCD of an interface between quark matter and vacuum is below a critical value σ_{crit} , and if the strange quark matter hypothesis holds, astrophysicists may observe (or may be observing) strange quark stars that have crusts made of positively charged strangelets, with size of order the Debye length λ_D , immersed in an electron gas. And, larger strangelets will be unstable to fission. In Secs. II and III, we have developed a model-independent evaluation of σ_{crit} , along the way constructing profiles of strangelets taking into account both the energy benefit of charge separation and the energy costs introduced by the surface tension and Debye screening. Our result is given in (27), in terms of two parameters n_Q (the charge density of quark matter with $\mu_e = 0$) and χ_Q (the charge susceptibility of quark matter with $\mu_e = 0$) which

are not currently known from first principles. Given any equation of state, whether from a model or from a future full QCD calculation, n_Q and χ_Q can be evaluated in terms of the quark number chemical potential μ and the strange quark mass m_s .

Our purpose in Sec. IV was to get a sense of the size of σ_{crit} , in MeV/fm², using as a guide bag models for unpaired quark matter and quark matter in the 2SC color-superconducting phase. A new twist in this section was the possibility that neutral bulk quark matter and stable strangelets could be in different phases (unpaired and 2SC, respectively). We conclude from the model-dependent investigation in this section that it is easy to find parameters for which $\sigma_{\text{crit}} \sim 1\text{--}3$ MeV/fm². And, by fine-tuning both m_s and Δ_{2SC} , it is possible to push σ_{crit} to as large as 5–7 MeV/fm². Although these conclusions are model-dependent, our results phrased in terms of n_Q and χ_Q as in Sec. III can be applied in future to any model, for any phase, as our understanding of dense quark matter continues to improve.

The surface tension σ is not known in QCD, but it has been calculated in the bag model for unpaired quark matter [35]. It ranges from about 10 MeV/fm² for $m_s = 100\text{--}120$ MeV to about 4 MeV/fm² for $m_s = 250$ MeV. If the interface has a thickness of order 1 fm, rather than being infinitely sharp as assumed in the bag model, the surface tension is almost certainly larger. It seems most likely that $\sigma > \sigma_{\text{crit}}$, making large strangelets stable against fission and giving strange quark stars, if they exist, fluid surfaces as in Ref. [5]. However, the combination of the bag model estimate for σ from Ref. [35] and our model-dependent investigation in Sec. IV leave open the possibility of the opposite conclusion.

Further investigation of the properties of the strange star crusts that result from fragmentation and charge separation at strange star surfaces, as proposed in Ref. [7], is warranted. We have only analyzed the surface of the crust, where $p_{\text{ext}} = 0$ and strangelets are spherical. If σ is less than σ_{crit} , we must then analyze the deeper layers of the crust. Preliminary numerical work indicates that Δg becomes less negative with increasing p_{ext} , but to date this ignores the alternative geometrical shapes of the strangelets that will occur below the surface of the crust. Furthermore, in the inner half of the crust, characterized by electron-filled “voids” embedded in quark matter rather than by strangelets embedded in an electron-gas, the curvature energy which we have neglected must be included [36]. The curvature energy is positive for strangelets, acting, like the surface tension, to suppress small strangelets. For voids, however, the curvature energy is negative, acting in opposition to the surface tension.

If charge-separated crusts do occur on strange stars, it seems likely that they will prove more similar to conventional neutron star crusts than to the fluid surface previously considered for strange stars. The strange nugget crust differs from the conventional neutron star crust in two

respects: (i) the nugget crust has a spatial extent of the order of 50–100 m [7] while the conventional crust has a radius ~ 1 km; and (ii) the typical expected charge of the nugget $Z_{\text{nugget}} \sim 50\text{--}100$ is several times larger than for nuclei where $Z_{\text{nuclear}} \sim 25$. In this context, it is relevant to note that there have been recent observations of quasiperiodic oscillations in the radiation following the giant flares observed in SGR 1806 – 20 and SGR 1900 + 14 (which are thought to be highly magnetized compact objects called magnetars). These have been suggested to originate from toroidal oscillations in the compact star crust excited during the “starquake” which is thought to have triggered the flare [37]. If this interpretation of the observed data is correct, it would imply that compact star seismology would be able to constrain both the shear modulus (which depends on the typical Z in the crust) and the spatial extent of the crust. Other burst phenomena such as X-ray bursts and superbursts observed in compact stars also probe the composition and structure of the crust. Clearly, further study of the observational consequences for the strange star crust is needed to deduce *how* similar to the conventional neutron star crusts they prove to be, and, in particular, whether the rich X-ray burst phenomenology observed in accreting compact stars can be consistent with a strange star crust.

It remains unlikely that strange star crusts can give rise to pulsar glitches, since glitches require a crust within which charged nuggets (nuclei or strangelets would be equivalent) are immersed in both an electron gas and a superfluid, and there is no superfluid in the strange star crust proposed in Ref. [7]. However, if quark matter in the crystalline color-superconducting phase occurs somewhere within the core of a strange star, this could be the locus in which pulsar glitches originate [33,34].

Another way in which observations could settle some of the questions we have raised here would be the discovery of strangelets in cosmic rays, with a peak in the distribution of A and Z corresponding to the baryon number A_{stable} of

the most favored stable (or metastable) strangelet. If $\sigma < \sigma_{\text{crit}}$, and if strangelets occur in cosmic rays, any strangelets whose initial baryon number (in whatever astrophysical collision produced them) was much larger than A_{stable} would fission into strangelets with $A \geq A_{\text{stable}}$. Strangelets with A only slightly bigger than A_{stable} will not fission because of the high energy per quark of the smallest strangelet produced. We therefore expect a peak in the strangelet distribution at $A \approx A_{\text{stable}}$, tailing off to no strangelets at all over some range of larger values.

If the estimates of Ref. [38] of the flux of strangelets in cosmic rays incident on the earth are correct, the single-strangelet detection capability of the AMS-02 detector, scheduled to go into operation on the International Space Station in a few years, extends to strangelets with $A \leq 3000[(Z/A)/0.1]^{-0.36}$ [39]. For two particular choices of model and model parameters in Sec. IV, we found critical strangelets with $A \approx 600$ and $Z \approx 70$ in one case and $A \approx 680$ and $Z \approx 105$ in the other case. More generally, we expect that if $\sigma < \sigma_{\text{crit}}$ the stable strangelets will have values of A lying between many hundred and a few thousand. If quark matter has such a small surface tension, then both strangelets and a peak in their size-distribution should be within the discovery reach of AMS-02.

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