Radiative decays of decuplet baryons, $\Lambda(1405)$ and $\Lambda(1520)$ hyperons in the chiral quark model

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The chiral quark model gives a reasonably good description of many low-energy observables by incorporating the effective degrees carried by the constituent quarks and Goldstone bosons. We calculate the decuplet to octet transition magnetic moments and the decay widths of several excited hyperons using this model. The various radiative decay widths from the chiral quark roughly agree with experimental data including recent JLAB measurement.

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I. INTRODUCTION

The radiative decays of baryons contribute enormously to our understanding of the underlying structure of baryons. The nonrelativistic quark model (NRQM) of Isgur and Karl [1,2] has been successful in predicting the electromagnetic properties of the ground states of baryons N and their resonances N^*. However, it is unable to give a very good description of radiative decays of all decuplet and other low-lying excited-state hyperons. Therefore, several other theoretical approaches have been proposed to calculate these transitions besides NRQM [3,4], including the relativized constituent-quark model (RCQM) [5], Bonn Constituent-Quark model [6], the MIT bag model [3], the chiral bag model [7], the Skyrme model [8], the soliton model [9], the algebraic model [10], the heavy baryon chiral perturbation theory (HBχPT) [11], the $1/N_c$ expansion of QCD [12] and the lattice calculations [13].

Recently, one JLAB experiment [14] reported some new results of the radiative decays of the $\Sigma^0(1385)$ and $\Lambda(1520)$, suggesting that mesonic effects may play an important role in $\Sigma^0(1385)$ radiative transitions [15]. On the other hand, a series of interesting work about the chiral quark model [16–19] indicates that the constituent quarks and internal Goldstone bosons can offer an adequate description of flavor and spin structure of baryons in the low $Q^2 \lesssim 1$ GeV^2 region. Within the same framework, the octet and decuplet magnetic moments, $\Sigma\Lambda$ transition magnetic moments and the explanation of the violation of the Coleman-Glashow sum rule are in remarkably good agreement with experimental data [20,21]. So it is interesting to explore whether we can make reliable predictions of other important observables using the chiral quark model.

In this paper we calculate radiative decays of decuplet to octet and some excited hyperons within the chiral model incorporating quark sea perturbatively generated by the valence quark's emission of internal Goldstone bosons. We can discern the contributions from sea quarks and pseudoscalar mesons through the results of transition magnetic moments and decay widths.

In Sec. II, we give an essential review of the chiral quark model and the mechanism for the quark sea generation. In Sec. III, we present the formalism of the helicity amplitudes for the baryon radiative decays. In Sec. IV, we present several typical cases of the calculation of decuplet to octet transition magnetic moments. Then we calculate the radiative decay widths of several excited hyperons in Sec. V. The numerical results and conclusions are presented in the final section.

II. MODEL DESCRIPTION

The chiral quark model [16–19] is an effective theory of nonperturbative QCD, which is based on the interaction between constituent quarks and Goldstone bosons. Generally, the two-body electromagnetic (EM) current from the exchange of Goldstone boson is small because it is suppressed by a wave function overlapping factor [22–26]. For example, the correction from the two-body EM from the pion exchange was shown to be less than 5% in Ref. [23]. The exchange of a Goldstone boson between two quarks will also lead to modification of the spin-spin interaction in the quark model. The spin-spin interaction will mainly cause the configuration mixing of the quark wave functions of mesons, and contribute significantly to, e.g. the magnetic moments of the baryons.

Another important effect of the Goldstone bosons in the chiral quark model is that by emission of Goldstone bosons, it can change the quark distributions in baryons. This can also modify the baryon properties. The basic process is the emission of an internal Goldstone boson by a valence quark q:

$$q_+ \to \mathrm{GB} + q'_- \to (q\bar{q}') + q'_-, \tag{1}$$

or,

$$q_+ \to \mathrm{GB} + q'_+ \to (q\bar{q}') + q'_+, \tag{2}$$

where the subscripts indicate the helicity of the quark, and $[\mathrm{GB}, q']$ is in the helicity-flipping state ($\langle l_z\rangle = +1$) in the process (1), and in the helicity-non-flipping state ($\langle l_z\rangle = 0$) in the process (2). The quark may change its helicity and flavor content by emitting a pseudoscalar meson, and $(q\bar{q}') + q'$ constitute the quark sea [19,27–31]. Thus, we consider the valence constituent-quark and the generated quark sea as a CQ-system [31]. Moreover, the probability for the fluctuation of the $q\bar{q}$ pairs is small because of the

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heavy mass of quarks in the $Q^2 \lesssim 1$ GeV^2 range. In other words, the interaction is perturbative [19,27,28,31]. Therefore, the effective Lagrangian describing the interaction between constituent quarks and internal Goldstone bosons can be expressed as follows,

$$\mathcal{L}_I = -g_8 \bar{q} \gamma^5 \Phi q, \tag{3}$$

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \beta\frac{1}{\sqrt{6}}\eta + \zeta\frac{1}{\sqrt{3}}\eta' & \pi^+ & \alpha K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \beta\frac{1}{\sqrt{6}}\eta + \zeta\frac{1}{\sqrt{3}}\eta' & \alpha K^0 \\ \alpha K^- & \alpha \bar{K}^0 & -\beta\frac{2}{\sqrt{6}}\eta + \zeta\frac{1}{\sqrt{3}}\eta' \end{pmatrix}, \tag{4}$$

where $q = (u, d, s)$, g_1 and g_8 denote the coupling constants for the singlet and octet Goldstone bosons, respectively, and $\zeta = g_1/g_8$. Besides, α and β are introduced by considering SU(3) symmetry breaking due to $M_s > M_{u,d}$ [27–30], whereas ζ is introduced by considering the axial U(1) symmetry breaking [19,27–30]. Then, the transition probability for the process $q \to \mathrm{GB} + q'$ can be easily deduced. For example, $P(u \to d + \pi^+) = a (a = | g_8 |^2)$, $P(u \to s + K^+) = \alpha^2 a$, $P(u \to u + \pi^0) = \frac{1}{2} a$, $P(u \to u + \eta) = \frac{1}{6}\beta^2 a$, $P(u \to u + \eta') = \frac{1}{3}\zeta^2 a$ etc. Furthermore, because the total angular momentum space wave function of the [GB, q'] state is

$$\left| J = \frac{1}{2}, J_z = \frac{1}{2} \right\rangle = \sqrt{\frac{2}{3}} |L = 1, L_z = 1\rangle \left| S = \frac{1}{2}, S_z = -\frac{1}{2} \right\rangle - \sqrt{\frac{1}{3}} |L = 1, L_z = 0\rangle \left| S = \frac{1}{2}, S_z = \frac{1}{2} \right\rangle, \tag{5}$$

the transition probability ratio of the process (1) to the process (2) is 2:1. For example, $P(u_+ \to d_- + \pi^+) = \frac{2}{3} a$, and $P(u_+ \to d_+ + \pi^+) = \frac{1}{3} a$.

In order to obtain the spin-flavor structure of the baryon, we write the number operator $\mathcal{N}(j)$ [21,30],

$$\mathcal{N}(j) = \hat{n}_{u_+}(j) u_+ + \hat{n}_{u_-}(j) u_- + \hat{n}_{d_+}(j) d_+ + \hat{n}_{d_-}(j) d_- + \hat{n}_{s_+}(j) s_+ + \hat{n}_{s_-}(j) s_-, \tag{6}$$

where $\mathcal{N}(j)$ operates only on the j-th quark, $\hat{n}_{q_\pm}$ corresponds to the number operator of the $q_\pm$. Thus, the spin-flavor structure of a baryon can be defined as

$$\hat{B} = \sum_{j=1}^{3} \langle B | \mathcal{N}(j) | B \rangle. \tag{7}$$

Using the symmetry of the baryon wave function, we can simplify the equation above,

$$\hat{B} = \sum_{j=1}^{3} \langle B | \mathcal{N}(j) | B \rangle = 3 \langle B | \mathcal{N}(3) | B \rangle, \tag{8}$$

where $|B\rangle$ is the baryon wave function. Taking proton, for example, and making use of the baryon $SU(6) \otimes O(3)$ wave function [32], we get

$$\hat{p} = \frac{5}{3} u_+ + \frac{1}{3} u_- + \frac{1}{3} d_+ + \frac{2}{3} d_-. \tag{9}$$

Then the proton's magnetic moment can be easily obtained

$$\mu(p) = \frac{5}{3}\mu(u_+) + \frac{1}{3}\mu(u_-) + \frac{1}{3}\mu(d_+) + \frac{2}{3}\mu(d_-) = \frac{4}{3}\mu(u_+) - \frac{1}{3}\mu(d_+), \tag{10}$$

where we make use of $\mu(q_-) = -\mu(q_+)$.

Furthermore, by considering the effects of $q \to \mathrm{GB} + q'$, we need modify the baryon's spin-flavor structure above. What we do is to make a replacement of every valence quark $q_\pm$ in the Eq. (7) as follows [21,30],

$$\begin{aligned} q_\pm \to & \Big(1 - \sum_{q'=u,d,s} P_{(q\to q')}\Big) q_\pm + \frac{1}{3} \sum_{q'=u,d,s} P_{(q\to q')} q'_\pm + \frac{2}{3} \sum_{q'=u,d,s} P_{(q\to q')} (q'_\mp + [\mathrm{GB}, q'_\mp]_\pm) \\ \to & \Big(1 - \sum_{q'=u,d,s} P_{(q\to q')}\Big) q_\pm + \frac{1}{3} \sum_{q'=u,d,s} P_{(q\to q')} (q'_\pm + 2q'_\mp) + \frac{2}{3} \sum_{q'=u,d,s} P_{(q\to q')} [\mathrm{GB}, q'_\mp]_\pm \end{aligned} \tag{11}$$

where $P_{(q\to q')}$ denotes the transition probability of the process $q \to \mathrm{GB} + q'$. $l_\pm$ denote the orbit angular momenta between GB and q' with $\langle l_z \rangle = \pm 1$ respectively. Furthermore

$$[\mathrm{GB}, q'_\mp]_\pm = q'_{\langle l^{\mathrm{GB}}_{q'_\mp} \rangle} + \mathrm{GB}_{\langle l^{q'_\mp}_{\mathrm{GB}} \rangle}, \tag{12}$$

where the orbit angular momenta of the q' and GB are $\langle l^{\mathrm{GB}}_{q'_\mp} \rangle = \frac{M_{\mathrm{GB}}}{M_{q'} + M_{\mathrm{GB}}} l_\pm$, $\langle l^{q'_\mp}_{\mathrm{GB}} \rangle = \frac{M_{q'}}{M_{q'} + M_{\mathrm{GB}}} l_\pm$ respectively. In the Eq. (11), the first term still corresponds to the valence quark spin. The latter two terms are both from the contribution of quark sea. The second term corresponds to the sea quark spin and the last term corresponds to the orbit angular momenta between the sea quark and the Goldstone boson.

The spin-flavor structure $\hat{B}$ of the baryon of a given baryon B will be modified accordingly as

$$\hat{B}^{\mathrm{total}} = \hat{B}^{\mathrm{val}} + \hat{B}^{\mathrm{sea}} + \hat{B}^{\mathrm{orbit}}, \tag{13}$$

where $\hat{B}^{\rm val}$ is the contribution from valence quarks

$$\hat{B}^{\rm val} = \sum_{q=u,d,s} n^{\rm val}_{q\pm} q_{\pm}, \tag{14}$$

$\hat{B}^{\rm sea}$ is the contribution from sea quarks

$$\hat{B}^{\rm sea} = \sum_{q=u,d,s} n^{\rm sea}_{q\pm} q_{\pm}, \tag{15}$$

and $\hat{B}^{\rm orbit}$ is the contribution from orbital excitation

$$\hat{B}^{\rm orbit} = \sum_{q=u,d,s} n^{\rm orbit}_{q_{\langle l^{\rm GB}_{q\pm}\rangle}} q_{\langle l^{\rm GB}_{q\pm}\rangle} + \sum_{\rm GB} n^{\rm orbit}_{{\rm GB}_{\langle l^{q\pm}_{\rm GB}\rangle}} {\rm GB}_{\langle l^{q\pm}_{\rm GB}\rangle}. \tag{16}$$

The spin polarizations of the quark, following Refs. [19–21,28,30], are defined as

$$\Delta q = n_{q_+} - n_{q_-}. \tag{17}$$

Similarly, we can define $\Delta q^{\rm orbit}_{\rm GB}$ and $\Delta {\rm GB}^{\rm orbit}_q$ as

$$\Delta q^{\rm orbit}_{\rm GB} = n_{q'_{\langle l^{\rm GB}_{q'_-}\rangle}} - n_{q'_{\langle l^{\rm GB}_{q'_+}\rangle}}, \tag{18}$$

$$\Delta {\rm GB}^{\rm orbit}_q = n_{{\rm GB}_{\langle l^{q'_-}_{\rm GB}\rangle}} - n_{{\rm GB}_{\langle l^{q'_+}_{\rm GB}\rangle}}. \tag{19}$$

Thus, Making use of the Eqs. (17)–(19), we can express the magnetic moment of a given quark B as

$$\mu^{\rm total}_B = \mu^{\rm val}_B + \mu^{\rm sea}_B + \mu^{\rm orbit}_B \tag{20}$$

where

$$\mu^{\rm val}_B = \sum_{q=u,d,s} \Delta q^{\rm val} \mu_q \tag{21}$$

$$\mu^{\rm sea}_B = \sum_{q=u,d,s} \Delta q^{\rm sea} \mu_q \tag{22}$$

$$\mu^{\rm orbit}_B = \sum_{q=u,d,s} \Delta q^{\rm orbit}_{\rm GB} \mu_q \langle l^{\rm GB}_{q-}\rangle + \sum_{\rm GB} \Delta {\rm GB}^{\rm orbit}_q \mu_{\rm GB} \langle l^{q-}_{\rm GB}\rangle. \tag{23}$$

Again we take proton, for example,

$$\begin{aligned}\hat{p} = {}& \frac{4}{3}\Bigg\{\Bigg[1 - a\Big(\frac{9+\beta^2+2\zeta^2}{6} + \alpha^2\Big)\Bigg]u_+ + \frac{1}{3}\Bigg[a\Big(\frac{3+\beta^2+2\zeta^2}{6}\Big)u_- + ad_- + a\alpha^2 s_-\Bigg] + \frac{2}{3}\Bigg[a\Big(\frac{1}{2}\Big)(u_{\langle l^{\pi^0}_{u-}\rangle} + \pi^0_{\langle l^{u-}_{\pi^0}\rangle}) \\ & + a\Big(\frac{\beta^2}{6}\Big)(u_{\langle l^{\eta}_{u-}\rangle} + \eta_{\langle l^{u-}_{\eta}\rangle}) + a\Big(\frac{\zeta^2}{3}\Big)(u_{\langle l^{\eta'}_{u-}\rangle} + \eta_{\langle l^{u-}_{\eta'}\rangle}) + a(d_{\langle l^{\pi^+}_{d-}\rangle} + \pi^+_{\langle l^{d-}_{\pi^+}\rangle}) + a\alpha^2(s_{\langle l^{K^+}_{s-}\rangle} + K^+_{\langle l^{s-}_{K^+}\rangle})\Bigg]\Bigg\} \\ & - \frac{1}{3}\Bigg\{\Bigg[1 - a\Big(\frac{9+\beta^2+2\zeta^2}{6} + \alpha^2\Big)\Bigg]d_+ + \frac{1}{3}\Bigg[a\Big(\frac{3+\beta^2+2\zeta^2}{6}\Big)d_- + au_- + a\alpha^2 s_-\Bigg] + \frac{2}{3}\Bigg[a\Big(\frac{1}{2}\Big)(d_{\langle l^{\pi^0}_{d-}\rangle} + \pi^0_{\langle l^{d-}_{\pi^0}\rangle}) \\ & + a\Big(\frac{\beta^2}{6}\Big)(d_{\langle l^{\eta}_{d-}\rangle} + \eta_{\langle l^{d-}_{\eta}\rangle}) + a\Big(\frac{\zeta^2}{3}\Big)(d_{\langle l^{\eta'}_{d-}\rangle} + \eta_{\langle l^{d-}_{\eta'}\rangle}) + a(u_{\langle l^{\pi^-}_{u-}\rangle} + \pi^-_{\langle l^{u-}_{\pi^-}\rangle}) + a\alpha^2(s_{\langle l^{K^0}_{s-}\rangle} + K^0_{\langle l^{s-}_{K^0}\rangle})\Bigg]\Bigg\}.\end{aligned} \tag{24}$$

In this way, the expressions for proton can be obtained as follows,

$$\Delta u^{\rm val} = \frac{4}{3} \tag{25}$$

$$\Delta d^{\rm val} = -\frac{1}{3} \tag{26}$$

$$\Delta u^{\rm sea} = -a\Big(\frac{57+8\beta^2+16\zeta^2}{27} + \frac{4}{3}\alpha^2\Big), \tag{27}$$

$$\Delta d^{\rm sea} = a\Big(\frac{6+4\beta^2+8\zeta^2}{54} + \frac{1}{3}\alpha^2\Big), \tag{28}$$

$$\Delta s^{\rm sea} = -\frac{a\alpha^2}{3}. \tag{29}$$

For the orbital part

$$\Delta u^{\rm orbit}_{\pi^0} = \frac{4}{9}a, \qquad \Delta \pi^{0\rm orbit}_u = \frac{4}{9}a \tag{30}$$

$$\Delta u^{\rm orbit}_{\eta} = \frac{4\beta^2}{27}a, \qquad \Delta \eta^{\rm orbit}_u = \frac{4\beta^2}{27}a \tag{31}$$

$$\Delta u^{\rm orbit}_{\eta'} = \frac{8\zeta^2}{27}a, \qquad \Delta \eta'^{\rm orbit}_u = \frac{8\zeta^2}{27}a \tag{32}$$

$$\Delta d^{\rm orbit}_{\pi^+} = \frac{8}{9}a, \qquad \Delta \pi^{+\rm orbit}_d = \frac{8}{9}a \tag{33}$$

$$\Delta s^{\rm orbit}_{K^+} = \frac{8\alpha^2}{9}a, \qquad \Delta K^{+\rm orbit}_s = \frac{8\alpha^2}{9}a \tag{34}$$

$$\Delta d^{\rm orbit}_{\pi^0} = -\frac{1}{9}a, \qquad \Delta \pi^{0\rm orbit}_d = -\frac{1}{9}a \tag{35}$$

$$\Delta d_{\eta}^{\mathrm{orbit}} = -\frac{\beta^2}{27}a, \qquad \Delta \eta_d^{\mathrm{orbit}} = -\frac{\beta^2}{27}a \tag{36}$$

$$\Delta d_{\eta'}^{\mathrm{orbit}} = -\frac{2\zeta^2}{27}a, \qquad \Delta \eta_d^{\prime\mathrm{orbit}} = -\frac{2\zeta^2}{27}a \tag{37}$$

$$\Delta u_{\pi^-}^{\mathrm{orbit}} = -\frac{2}{9}a, \qquad \Delta \pi_u^{-\mathrm{orbit}} = -\frac{2}{9}a \tag{38}$$

$$\Delta s_{K^0}^{\mathrm{orbit}} = -\frac{2\alpha^2}{9}a, \qquad \Delta K_s^{0\mathrm{orbit}} = -\frac{2\alpha^2}{9}a. \tag{39}$$

III. HELICITY AMPLITUDES OF RADIATIVE DECAYS

In addition, we give a short review of the helicity amplitude in order to calculate radiative decay widths [32,33]. In case of the process $B_i \to B_f + \gamma$,

$$A_M = -e\sqrt{2\pi/k}\langle B_f, J_z = M-1|\varepsilon^* \cdot \sum_{i=1}^{3} j_{em}(i)|B_i, J_z = M\rangle$$
$$M = \frac{3}{2}, \frac{1}{2} \tag{40}$$

where $\mathbf{k}$ and ε are the momentum and polarization vector of the photon, and $j_{em}(i)$ is the i-th quark current density. Without loss of generality, we take the photon to be right-handed [$\varepsilon = -1/\sqrt{2}(1, i, 0)$] and expand $\varepsilon^* \cdot \sum_{i=1}^{3} j_{em}(i)$,

$$\varepsilon^* \cdot \sum_{i=1}^{3} j_{em}(i) = -\sqrt{2}\sum_{i=1}^{3}\frac{e}{2m_i}[e^{-\mathrm{i}kz(i)}q(i)][k\sigma_-(i) + (p_x(i) - \mathrm{i}p_y(i))]. \tag{41}$$

With the symmetry of the baryon wave function, we can simplify the above equation,

$$\begin{aligned}\varepsilon^* \cdot \sum_{i=1}^{3} j_{em}(i) &= -\sqrt{2}\sum_{i=1}^{3}\frac{e}{2m_i}[e^{-\mathrm{i}kz(i)}q(i)][k\sigma_-(i) + (p_x(i) - \mathrm{i}p_y(i))] \\ &= -\sqrt{2}\frac{3e}{2m_3}[e^{-\mathrm{i}kz(3)}q(3)][k\sigma_-(3) + (p_x(3) - \mathrm{i}p_y(3))]\end{aligned} \tag{42}$$

where the first term contributes to magnetic moments or magnetic-dipole transitions, and the second term contributes to electric-dipole transitions between $L = 1$ orbit excitations and the ground states. Thus, we can obtain the radiative widths in term of $A_{1/2}$ and $A_{3/2}$,

$$\Gamma = \frac{k^2}{2\pi}\frac{1}{2J+1}\frac{m_f}{m_i}\{|A_{3/2}|^2 + |A_{1/2}|^2\}. \tag{43}$$

IV. DECUPLET TO OCTET TRANSITION MAGNETIC MOMENTS

In this section we calculate the decuplet to octet transition magnetic moments. Similarly, we can get the transition magnetic moments of $B_{10} \to B_8 + \gamma$ transitions by using the Eqs. (20)–(23) to calculate $\widehat{B_{10}B_8} = \sum_{j=1}^{3}\langle B_8, J_z = \frac{1}{2}|\mathcal{N}(j)|B_{10}, J_z = \frac{1}{2}\rangle$ [21]. However, there is a form factor from the integral $\langle\psi_{000}^s|e^{-\mathrm{i}kz(3)}|\psi_{000}^s\rangle$ [32,33] for the radiative decays between baryons with $L_i = 0$ and $L_f = 0$. Therefore, we need make slight modification of the $\hat{B}$ and get $\hat{B}(k)$,

$$\begin{aligned}\widehat{B_{10}B_8}(k) &= \sum_{j=1}^{3}\left\langle B_8, J_z = \frac{1}{2}\,\middle|\,\mathcal{N}(j)\cdot e^{-\mathrm{i}kz(j)}\,\middle|\,B_{10}, J_z = \frac{1}{2}\right\rangle \\ &= 3\left\langle B_8, J_z = \frac{1}{2}\,\middle|\,\mathcal{N}(3)\cdot e^{-\mathrm{i}kz(3)}\,\middle|\,B_{10}, J_z = \frac{1}{2}\right\rangle \\ &= \widehat{B_{10}B_8}\cdot\langle\psi_{000}^s|e^{-\mathrm{i}kz(3)}|\psi_{000}^s\rangle,\end{aligned} \tag{44}$$

where $\mathbf{k}$ is the momentum of the photon. Accordingly, we add the form factor to the transition magnetic moments,

$$\mu_{B_{10}B_8}(k) = \mu_{B_{10}B_8}\cdot\langle\psi_{000}^s|e^{-\mathrm{i}kz(3)}|\psi_{000}^s\rangle. \tag{45}$$

Because A_M contains contribution only from magnetic-dipole transitions ($M1$) for the decuplet to octet transitions, we can write A_M in terms of $\mu_{B_{10}B_8}(k)$,

$$A_{3/2} = A_{3/2}^{M1} = -\sqrt{3\pi k}\cdot\mu_{B_{10}B_8}(k) \tag{46}$$

$$A_{1/2} = A_{1/2}^{M1} = -\sqrt{\pi k}\cdot\mu_{B_{10}B_8}(k). \tag{47}$$

In our work, the baryon wave functions are taken from the well-known quark model calculation [32,34] (the details of the wave functions are given in the Appendix).

Next, we list the detailed calculations of $\Sigma^0(1385) \to \Lambda(1116) + \gamma$,

$$\begin{aligned}\widehat{\Sigma^{*,0}\Lambda}(k) &= \widehat{\Sigma^{*,0}\Lambda}\cdot\langle\psi_{000}^s|e^{-\mathrm{i}kz(3)}|\psi_{000}^s\rangle \\ &= \frac{\sqrt{6}}{3}(u^+ - d^+)\cdot e^{-(1/6)k^2R^2}\end{aligned} \tag{48}$$

considering $q \to \mathrm{GB} + q'$,

$$\widehat{\Sigma^{*,0}\Lambda}(k) = \frac{\sqrt{6}}{3}\bigg\{\bigg[1 - a\bigg(\frac{9+\beta^2+2\zeta^2}{6}+\alpha^2\bigg)\bigg]u_+ + \frac{1}{3}\bigg[a\bigg(\frac{3+\beta^2+2\zeta^2}{6}\bigg)u_- + ad_- + a\alpha^2 s_-\bigg]$$
$$+ \frac{2}{3}\bigg[a\bigg(\frac{1}{2}\bigg)(u_{\langle l^{\pi^0}_{u_-}\rangle} + \pi^0_{\langle l^{u_-}_{\pi^0}\rangle}) + a\bigg(\frac{\beta^2}{6}\bigg)(u_{\langle l^{\eta}_{u_-}\rangle} + \eta_{\langle l^{u_-}_{\eta}\rangle}) + a\bigg(\frac{\zeta^2}{3}\bigg)(u_{\langle l^{\eta'}_{u_-}\rangle} + \eta_{\langle l^{u_-}_{\eta'}\rangle}) + a(d_{\langle l^{\pi^+}_{d_-}\rangle} + \pi^+_{\langle l^{d_-}_{\pi^+}\rangle})$$
$$+ a\alpha^2(s_{\langle l^{K^+}_{s_-}\rangle} + K^+_{\langle l^{s_-}_{K^+}\rangle})\bigg] - \bigg[1 - a\bigg(\frac{9+\beta^2+2\zeta^2}{6}+\alpha^2\bigg)\bigg]d_+ - \frac{1}{3}\bigg[a\bigg(\frac{3+\beta^2+2\zeta^2}{6}\bigg)d_- + au_- + a\alpha^2 s_-\bigg]$$
$$- \frac{2}{3}\bigg[a\bigg(\frac{1}{2}\bigg)(d_{\langle l^{\pi^0}_{d_-}\rangle} + \pi^0_{\langle l^{d_-}_{\pi^0}\rangle}) + a\bigg(\frac{\beta^2}{6}\bigg)(d_{\langle l^{\eta}_{d_-}\rangle} + \eta_{\langle l^{d_-}_{\eta}\rangle}) + a\bigg(\frac{\zeta^2}{3}\bigg)(d_{\langle l^{\eta'}_{d_-}\rangle} + \eta_{\langle l^{d_-}_{\eta'}\rangle}) + a(u_{\langle l^{\pi^-}_{u_-}\rangle} + \pi^-_{\langle l^{u_-}_{\pi^-}\rangle})$$
$$+ a\alpha^2(s_{\langle l^{K^0}_{s_-}\rangle} + K^0_{\langle l^{s_-}_{K^0}\rangle})\bigg]\bigg\} \cdot e^{-(1/6)k^2R^2} \quad (49)$$

$$\mu^{\text{total}}_{\Sigma^{*,0}\Lambda}(k) = \mu^{\text{val}}_{\Sigma^{*,0}\Lambda}(k) + \mu^{\text{sea}}_{\Sigma^{*,0}\Lambda}(k) + \mu^{\text{orbit}}_{\Sigma^{*,0}\Lambda}(k) \quad (50)$$

$$\mu^{\text{val}}_{\Sigma^{*,0}\Lambda}(k) = \mu^{\text{val}}_{\Sigma^{*,0}\Lambda} \cdot e^{-(1/6)k^2R^2} = \frac{\sqrt{6}}{3}(\mu_u - \mu_d) \cdot e^{-(1/6)k^2R^2} \quad (51)$$

$$\mu^{\text{sea}}_{\Sigma^{*,0}\Lambda}(k) = \mu^{\text{sea}}_{\Sigma^{*,0}\Lambda} \cdot e^{-(1/6)k^2R^2} = -\frac{\sqrt{6}}{3}a\bigg(\frac{12+2\beta^2+4\zeta^2}{9}+\alpha^2\bigg)(\mu_u - \mu_d) \cdot e^{-(1/6)k^2R^2} \quad (52)$$

$$\mu^{\text{orbit}}_{\Sigma^{*,0}\Lambda}(k) = \mu^{\text{orbit}}_{\Sigma^{*,0}\Lambda} \cdot e^{-(1/6)k^2R^2}$$
$$= \frac{2\sqrt{6}}{9}a\bigg[\frac{1}{2}(\mu_u\langle l^{\pi^0}_{u_-}\rangle + \mu_{\pi^0}\langle l^{u_-}_{\pi^0}\rangle) + \frac{\beta^2}{6}(\mu_u\langle l^{\eta}_{u_-}\rangle + \mu_\eta\langle l^{u_-}_{\eta}\rangle) + \frac{\zeta^2}{3}(\mu_u\langle l^{\eta'}_{u_-}\rangle + \mu_\eta\langle l^{u_-}_{\eta'}\rangle) + (\mu_d\langle l^{\pi^+}_{d_-}\rangle$$
$$+ \mu_{\pi^+}\langle l^{d_-}_{\pi^+}\rangle) + \alpha^2(\mu_s\langle l^{K^+}_{s_-}\rangle + \mu_{K^+}\langle l^{s_-}_{K^+}\rangle) - \frac{1}{2}(\mu_d\langle l^{\pi^0}_{d_-}\rangle + \mu_{\pi^0}\langle l^{d_-}_{\pi^0}\rangle) - \frac{\beta^2}{6}(\mu_d\langle l^{\eta}_{d_+}\rangle + \mu_\eta\langle l^{d_+}_{\eta}\rangle) - \frac{\zeta^2}{3}(\mu_d\langle l^{\eta'}_{d_+}\rangle$$
$$+ \mu_\eta\langle l^{d_+}_{\eta'}\rangle) - (\mu_u\langle l^{\pi^-}_{u_+}\rangle + \mu_{\pi^-}\langle l^{u_+}_{\pi^-}\rangle) - \alpha^2(\mu_s\langle l^{K^0}_{s_+}\rangle + \mu_{K^0}\langle l^{s_+}_{K^0}\rangle)\bigg]e^{-(1/6)k^2R^2} \quad (53)$$

So,

$$A^{\text{total}}_{3/2}(\Sigma^{*,0} \to \Lambda + \gamma) = A^{M1,\text{val}}_{3/2}(\Sigma^{*,0} \to \Lambda + \gamma) + A^{M1,\text{sea}}_{3/2}(\Sigma^{*,0} \to \Lambda + \gamma) + A^{M1,\text{orbit}}_{3/2}(\Sigma^{*,0} \to \Lambda + \gamma) \quad (54)$$

where,

$$A^{M1,\text{val}}_{3/2}(\Sigma^{*,0} \to \Lambda + \gamma) = -\sqrt{3\pi k} \cdot \mu^{\text{val}}_{\Sigma^{*,0}\Lambda}(k) \quad (55)$$

$$A^{M1,\text{sea}}_{3/2}(\Sigma^{*,0} \to \Lambda + \gamma) = -\sqrt{3\pi k} \cdot \mu^{\text{sea}}_{\Sigma^{*,0}\Lambda}(k) \quad (56)$$

$$A^{M1,\text{orbit}}_{3/2}(\Sigma^{*,0} \to \Lambda + \gamma) = -\sqrt{3\pi k} \cdot \mu^{\text{orbit}}_{\Sigma^{*,0}\Lambda}(k) \quad (57)$$

$$A^{\text{total}}_{1/2}(\Sigma^{*,0} \to \Lambda + \gamma) = A^{M1,\text{val}}_{1/2}(\Sigma^{*,0} \to \Lambda + \gamma) + A^{M1,\text{sea}}_{1/2}(\Sigma^{*,0} \to \Lambda + \gamma) + A^{M1,\text{orbit}}_{1/2}(\Sigma^{*,0} \to \Lambda + \gamma) \quad (58)$$

where,

$$A^{M1,\text{val}}_{1/2}(\Sigma^{*,0} \to \Lambda + \gamma) = -\sqrt{\pi k} \cdot \mu^{\text{val}}_{\Sigma^{*,0}\Lambda}(k) \quad (59)$$

$$A^{M1,\text{sea}}_{1/2}(\Sigma^{*,0} \to \Lambda + \gamma) = -\sqrt{\pi k} \cdot \mu^{\text{sea}}_{\Sigma^{*,0}\Lambda}(k) \quad (60)$$

$$A^{M1,\text{orbit}}_{1/2}(\Sigma^{*,0} \to \Lambda + \gamma) = -\sqrt{\pi k} \cdot \mu^{\text{orbit}}_{\Sigma^{*,0}\Lambda}(k) \quad (61)$$

besides,

$$A^{\text{total}}_{3/2}(\Sigma^{*,0} \to \Lambda + \gamma) = -\sqrt{3\pi k} \cdot \mu^{\text{total}}_{\Sigma^{*,0}\Lambda}(k) \quad (62)$$

$$A^{\text{total}}_{1/2}(\Sigma^{*,0} \to \Lambda + \gamma) = -\sqrt{\pi k} \cdot \mu^{\text{total}}_{\Sigma^{*,0}\Lambda}(k) \quad (63)$$

$$\Gamma(\Sigma^{*,0} \to \Lambda + \gamma) = \frac{k^3}{2}\frac{m_\Lambda}{m_{\Sigma^{*,0}}}[\mu^{\text{total}}_{\Sigma^{*,0}\Lambda}(k)]^2. \quad (64)$$

V. RADIATIVE DECAYS OF $\Lambda(1405)$ AND $\Lambda(1520)$

Both the magnetic-dipole transitions and electric-dipole transitions contribute to the radiative decays of low-lying excited ($L = 1$) hyperons.

$$A_M = A_M^{M1} + A_M^{E1}, \qquad M = \frac{1}{2} \quad \text{or} \quad \frac{3}{2} \tag{65}$$

We have to take the calculations of both $\widehat{B_iB}_{f(M)}(k)$ (corresponding to A_M^{M1}) and $\widehat{B_iB}^*_{f(M)}(k)$ (corresponding to A_M^{E1}),

$$\begin{aligned}\widehat{B_iB}_{f(M)}(k) &= \sum_{j=1}^{3}\langle B_f, J_z = M|\mathcal{N}(j)\cdot e^{-\mathrm{i}kz(j)}|B_i, J_z = M\rangle \\ &= 3\langle B_f, J_z = M|\mathcal{N}(3)\cdot e^{-\mathrm{i}kz(3)}|B_i, J_z = M\rangle, \\ M &= -\frac{1}{2} \quad \text{or} \quad \frac{1}{2}\end{aligned} \tag{66}$$

and

$$\begin{aligned}\widehat{B_iB}^*_{f(M)}(k) &= \frac{1}{k}\sum_{j=1}^{3}\langle B_f, J_z = M-1|\mathcal{N}^*(j) \\ &\quad\cdot e^{-\mathrm{i}kz(j)}(p_x(i) - \mathrm{i}p_y(i))|B_i, J_z = M\rangle \\ &= \frac{3}{k}\langle B_f, J_z = M-1|\mathcal{N}^*(3) \\ &\quad\cdot e^{-\mathrm{i}kz(3)}(p_x(3) - \mathrm{i}p_y(3))|B_i, J_z = M\rangle, \\ M &= \frac{1}{2} \quad \text{or} \quad \frac{3}{2}\end{aligned} \tag{67}$$

where $\mathcal{N}^*(j)$ is defined as

$$\mathcal{N}^*(j) = \hat{n}_u(j)u + \hat{n}_d(j)d + \hat{n}_s(j)s. \tag{68}$$

$\widehat{B_iB}_f(k)$ corresponds to the magnetic-dipole transitions, which has been discussed in Sec. IV, while $\widehat{B_iB}_f^*(k)$ corresponds to the electric-dipole transitions, which will be explained below.

From the Eq. (41), the magnetic term flips the spin of the quark and transforms as σ_- [35]. We must ensure that the CQ-system remains to be a spin $\frac{1}{2}$ entity as a single valence quark [31] when considering the effects of $q \to \mathrm{GB} + q'$. Similarly, the electric term flips the L_z of the quark and transforms as L_-. So we calculate spin-flavor structure $\widehat{B_iB}_f^*(k)$ and must ensure that the CQ-system remains the same L_z as a single valence quark. Besides, because the electric term has no spin operators, $\mathcal{N}^*(j)$ does not need spin subscripts.

Therefore, the $\widehat{B_iB}_f^*(k)$ can be calculated as follows: first we calculate the $\widehat{B_iB}_f^*(k)$ without considering $q \to \mathrm{GB} + q'$, then we replace every valence quark as

$$q \to \left(1 - \sum_{q'=u,d,s} P_{(q\to q')}\right)q + \sum_{q'=u,d,s} P_{(q\to q')}\left(q'_{\langle l_{q'}^{\mathrm{GB}}\rangle} + \mathrm{GB}_{\langle l_{\mathrm{GB}}^{q'}\rangle}\right) \tag{69}$$

where $\langle l_{q'}^{\mathrm{GB}}\rangle = \frac{M_{\mathrm{GB}}}{M_{q'}+M_{\mathrm{GB}}}$, and $\langle l_{\mathrm{GB}}^{q'}\rangle = \frac{M_{q'}}{M_{q'}+M_{\mathrm{GB}}}$.

We take $\Lambda(1520) \to \Lambda(1116) + \gamma$ for example.

$$\left|\Lambda(1520), J_z = \frac{3}{2}\right\rangle = \frac{1}{\sqrt{2}}(\phi^a\chi^\lambda_{1/2}\psi^\rho_{111} - \phi^a\chi^\rho_{1/2}\psi^\lambda_{111}), \tag{70}$$

$$\begin{aligned}\left|\Lambda(1520), J_z = \frac{1}{2}\right\rangle &= \frac{1}{\sqrt{6}}(\phi^a\chi^\lambda_{-(1/2)}\psi^\rho_{111} - \phi^a\chi^\rho_{-(1/2)}\psi^\lambda_{111}) \\ &\quad + \frac{1}{\sqrt{3}}(\phi^a\chi^\lambda_{1/2}\psi^\rho_{110} - \phi^a\chi^\rho_{1/2}\psi^\lambda_{110}),\end{aligned} \tag{71}$$

$$\left|\Lambda(1116), J_z = \frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}(\phi^\rho_\Lambda\chi^\rho_{1/2} + \phi^\lambda_\Lambda\chi^\lambda_{1/2})\psi^s_{000}. \tag{72}$$

With these wave functions, we obtain

$$\begin{aligned}\Lambda_{1520}\widehat{\Lambda}_{1116\,(-(1/2))}(k) &= \mathrm{i}\frac{1}{6}(u_+ + d_+ - 2s_+)\cdot kR \\ &\quad\cdot e^{-(1/6)k^2R^2},\end{aligned} \tag{73}$$

in which $\langle\psi^s_{000}|e^{-\mathrm{i}kz(3)}|\psi^\lambda_{110}\rangle = \mathrm{i}\frac{\sqrt{3}}{3}\cdot kR\cdot e^{-(1/6)k^2R^2}$.

$$\begin{aligned}\Lambda_{1520}\widehat{\Lambda}_{1116}{}^*_{(3/2)}(k) &= -\mathrm{i}\frac{\sqrt{3}}{6}(u + d - 2s)\cdot\frac{1}{kR} \\ &\quad\cdot e^{-(1/6)k^2R^2},\end{aligned} \tag{74}$$

$$\Lambda_{1520}\widehat{\Lambda}_{1116}{}^*_{(1/2)}(k) = -\mathrm{i}\frac{1}{6}(u + d - 2s)\cdot\frac{1}{kR}\cdot e^{-(1/6)k^2R^2}, \tag{75}$$

in which $\langle\psi^s_{000}|e^{-\mathrm{i}kz(3)}(p_x(3) - \mathrm{i}p_y(3))|\psi^\lambda_{111}\rangle = -\mathrm{i}\frac{\sqrt{6}}{3}\cdot\frac{1}{R}\cdot e^{-(1/6)k^2R^2}$.

Then, considering $q \to \mathrm{GB} + q'$, we give a modification of the Eqs. (73)–(75).

$$\Lambda_{1520}\widehat{\Lambda}_{1116(-(1/2))}(k) = \bigg\{\bigg[1 - a\bigg(\frac{9+\beta^2+2\zeta^2}{6}+\alpha^2\bigg)\bigg]u_+ + \frac{1}{3}\bigg[a\bigg(\frac{3+\beta^2+2\zeta^2}{6}\bigg)u_- + ad_- + a\alpha^2 s_-\bigg] + \frac{2}{3}\bigg[a\bigg(\frac{1}{2}\bigg)(u_{\langle l^{\pi^0}_{u_-}\rangle} + \pi^0_{\langle l^{u_-}_{\pi^0}\rangle}) + a\bigg(\frac{\beta^2}{6}\bigg)(u_{\langle l^{\eta}_{u_-}\rangle} + \eta_{\langle l^{u_-}_{\eta}\rangle}) + a\bigg(\frac{\zeta^2}{3}\bigg)(u_{\langle l^{\eta'}_{u_-}\rangle} + \eta_{\langle l^{u_-}_{\eta'}\rangle}) + a(d_{\langle l^{\pi^+}_{d_-}\rangle} + \pi^+_{\langle l^{d_-}_{\pi^+}\rangle}) + a\alpha^2(s_{\langle l^{K^+}_{s_-}\rangle} + K^+_{\langle l^{s_-}_{K^+}\rangle})\bigg] + \bigg[1 - a\bigg(\frac{9+\beta^2+2\zeta^2}{6}+\alpha^2\bigg)\bigg]d_+ + \frac{1}{3}\bigg[a\bigg(\frac{3+\beta^2+2\zeta^2}{6}\bigg)d_- + au_- + a\alpha^2 s_-\bigg] + \frac{2}{3}\bigg[a\bigg(\frac{1}{2}\bigg)(d_{\langle l^{\pi^0}_{d_-}\rangle} + \pi^0_{\langle l^{d_-}_{\pi^0}\rangle}) + a\bigg(\frac{\beta^2}{6}\bigg)(d_{\langle l^{\eta}_{d_-}\rangle} + \eta_{\langle l^{d_-}_{\eta}\rangle}) + a\bigg(\frac{\zeta^2}{3}\bigg)(d_{\langle l^{\eta'}_{d_-}\rangle} + \eta_{\langle l^{d_-}_{\eta'}\rangle}) + a(u_{\langle l^{\pi^-}_{u_-}\rangle} + \pi^-_{\langle l^{u_-}_{\pi^-}\rangle}) + a\alpha^2(s_{\langle l^{K^0}_{s_-}\rangle} + K^0_{\langle l^{s_-}_{K^0}\rangle})\bigg] - 2\bigg[1 - a\bigg(\frac{2\beta^2+\zeta^2}{3}+2\alpha^2\bigg)\bigg]s_+ - \frac{1}{3}\bigg[\frac{2a}{3}(2\beta^2+\zeta^2)s_- + 2a\alpha^2 u_- + 2a\alpha^2 d_-\bigg] - \frac{2}{3}\bigg[\frac{4a}{3}\beta^2(s_{\langle l^{\eta}_{s_-}\rangle} + \eta_{\langle l^{s_-}_{\eta}\rangle}) + \frac{2a}{3}\zeta^2(s_{\langle l^{\eta'}_{s_-}\rangle} + \eta'_{\langle l^{s_-}_{\eta'}\rangle}) + 2a\alpha^2(u_{\langle l^{K^-}_{u_-}\rangle} + K^-_{\langle l^{u_-}_{K^-}\rangle}) + 2a\alpha^2(d_{\langle l^{\bar{K}^0}_{d_-}\rangle} + \bar{K}^0_{\langle l^{d_-}_{\bar{K}^0}\rangle})\bigg]\bigg\}\cdot\bigg(\mathrm{i}\frac{1}{6}kR\cdot e^{-(1/6)k^2R^2}\bigg), \tag{76}$$

$$\Lambda_{1520}\widehat{\Lambda}^{*}_{1116(3/2)}(k) = \bigg\{\bigg[1 - a\bigg(\frac{9+\beta^2+2\zeta^2}{6}+\alpha^2\bigg)\bigg]u + a\bigg(\frac{1}{2}\bigg)(u_{\langle l^{\pi^0}_{u}\rangle} + \pi^0_{\langle l^{u}_{\pi^0}\rangle}) + a\bigg(\frac{\beta^2}{6}\bigg)(u_{\langle l^{\eta}_{u}\rangle} + \eta_{\langle l^{u}_{\eta}\rangle}) + a\bigg(\frac{\zeta^2}{3}\bigg)(u_{\langle l^{\eta'}_{u}\rangle} + \eta_{\langle l^{u}_{\eta'}\rangle}) + a(d_{\langle l^{\pi^+}_{d}\rangle} + \pi^+_{\langle l^{d}_{\pi^+}\rangle}) + a\alpha^2(s_{\langle l^{K^+}_{s}\rangle} + K^+_{\langle l^{s}_{K^+}\rangle}) + \bigg[1 - a\bigg(\frac{9+\beta^2+2\zeta^2}{6}+\alpha^2\bigg)\bigg]d + a\bigg(\frac{1}{2}\bigg)(d_{\langle l^{\pi^0}_{d}\rangle} + \pi^0_{\langle l^{d}_{\pi^0}\rangle}) + a\bigg(\frac{\beta^2}{6}\bigg)(d_{\langle l^{\eta}_{d}\rangle} + \eta_{\langle l^{d}_{\eta}\rangle}) + a\bigg(\frac{\zeta^2}{3}\bigg)(d_{\langle l^{\eta'}_{d}\rangle} + \eta_{\langle l^{d}_{\eta'}\rangle}) + a(u_{\langle l^{\pi^-}_{u}\rangle} + \pi^-_{\langle l^{u}_{\pi^-}\rangle}) + a\alpha^2(s_{\langle l^{K^0}_{s}\rangle} + K^0_{\langle l^{s}_{K^0}\rangle}) - 2\bigg[1 - a\bigg(\frac{2\beta^2+\zeta^2}{3}+2\alpha^2\bigg)\bigg]s - \frac{4a}{3}\beta^2(s_{\langle l^{\eta}_{s}\rangle} + \eta_{\langle l^{s}_{\eta}\rangle}) - \frac{2a}{3}\zeta^2(s_{\langle l^{\eta'}_{s}\rangle} + \eta'_{\langle l^{s}_{\eta'}\rangle}) - 2a\alpha^2(u_{\langle l^{K^-}_{u}\rangle} + K^-_{\langle l^{u}_{K^-}\rangle}) - 2a\alpha^2(d_{\langle l^{\bar{K}^0}_{d}\rangle} + \bar{K}^0_{\langle l^{d}_{\bar{K}^0}\rangle})\bigg\}\cdot\bigg(-\mathrm{i}\frac{\sqrt{3}}{6}\frac{1}{kR}\cdot e^{-(1/6)k^2R^2}\bigg), \tag{77}$$

$$\Lambda_{1520}\widehat{\Lambda}^{*}_{1116(1/2)}(k) = \bigg\{\bigg[1 - a\bigg(\frac{9+\beta^2+2\zeta^2}{6}+\alpha^2\bigg)\bigg]u + a\bigg(\frac{1}{2}\bigg)(u_{\langle l^{\pi^0}_{u}\rangle} + \pi^0_{\langle l^{u}_{\pi^0}\rangle}) + a\bigg(\frac{\beta^2}{6}\bigg)(u_{\langle l^{\eta}_{u}\rangle} + \eta_{\langle l^{u}_{\eta}\rangle}) + a\bigg(\frac{\zeta^2}{3}\bigg)(u_{\langle l^{\eta'}_{u}\rangle} + \eta_{\langle l^{u}_{\eta'}\rangle}) + a(d_{\langle l^{\pi^+}_{d}\rangle} + \pi^+_{\langle l^{d}_{\pi^+}\rangle}) + a\alpha^2(s_{\langle l^{K^+}_{s}\rangle} + K^+_{\langle l^{s}_{K^+}\rangle}) + \bigg[1 - a\bigg(\frac{9+\beta^2+2\zeta^2}{6}+\alpha^2\bigg)\bigg]d + a\bigg(\frac{1}{2}\bigg)(d_{\langle l^{\pi^0}_{d}\rangle} + \pi^0_{\langle l^{d}_{\pi^0}\rangle}) + a\bigg(\frac{\beta^2}{6}\bigg)(d_{\langle l^{\eta}_{d}\rangle} + \eta_{\langle l^{d}_{\eta}\rangle}) + a\bigg(\frac{\zeta^2}{3}\bigg)(d_{\langle l^{\eta'}_{d}\rangle} + \eta_{\langle l^{d}_{\eta'}\rangle}) + a(u_{\langle l^{\pi^-}_{u}\rangle} + \pi^-_{\langle l^{u}_{\pi^-}\rangle}) + a\alpha^2(s_{\langle l^{K^0}_{s}\rangle} + K^0_{\langle l^{s}_{K^0}\rangle}) - 2\bigg[1 - a\bigg(\frac{2\beta^2+\zeta^2}{3}+2\alpha^2\bigg)\bigg]s - \frac{4a}{3}\beta^2(s_{\langle l^{\eta}_{s}\rangle} + \eta_{\langle l^{s}_{\eta}\rangle}) - \frac{2a}{3}\zeta^2(s_{\langle l^{\eta'}_{s}\rangle} + \eta'_{\langle l^{s}_{\eta'}\rangle}) - 2a\alpha^2(u_{\langle l^{K^-}_{u}\rangle} + K^-_{\langle l^{u}_{K^-}\rangle}) - 2a\alpha^2(d_{\langle l^{\bar{K}^0}_{d}\rangle} + \bar{K}^0_{\langle l^{d}_{\bar{K}^0}\rangle})\bigg\}\cdot\bigg(-\mathrm{i}\frac{1}{6}\frac{1}{kR}\cdot e^{-(1/6)k^2R^2}\bigg). \tag{78}$$

Using Eqs. (20)–(23), we can calculate $\mu_M^{\text{total}}(B_iB_f)$ and $\mu_M^{\text{total}}(B_iB_f)^*$ corresponding to (66) and (67) respectively,

$$\mu_M^{\text{total}}(\Lambda_{1520}\Lambda_{1116}) = \mu_M^{\text{val}}(\Lambda_{1520}\Lambda_{1116}) + \mu_M^{\text{sea}}(\Lambda_{1520}\Lambda_{1116}) + \mu_M^{\text{orbit}}(\Lambda_{1520}\Lambda_{1116}), \tag{79}$$

$$\mu^{\text{val}}_{(-(1/2))}(\Lambda_{1520}\Lambda_{1116}) = (\mu_u + \mu_d - 2\mu_s)\cdot\bigg(\mathrm{i}\frac{1}{6}kR\cdot e^{-(1/6)k^2R^2}\bigg), \tag{80}$$

$$\mu^{\mathrm{sea}}_{(-(1/2))}(\Lambda_{1520}\Lambda_{1116}) = -a\bigg[\bigg(\frac{18+2\beta^2+4\zeta^2}{9}+\frac{1}{3}\alpha^2\bigg)(\mu_u+\mu_d)-\bigg(\frac{16\beta^2+8\zeta^2}{9}+\frac{10}{3}\alpha^2\bigg)\mu_s\bigg]\cdot\bigg(\mathrm{i}\frac{1}{6}kR\cdot e^{-(1/6)k^2R^2}\bigg), \tag{81}$$

$$\begin{aligned}\mu^{\mathrm{orbit}}_{(-(1/2))}(\Lambda_{1520}\Lambda_{1116}) = \frac{2}{3}a\bigg[&\frac{1}{2}(\mu_u\langle l^{\pi^0}_{u_-}\rangle+\mu_{\pi^0}\langle l^{u_-}_{\pi^0}\rangle)+\frac{\beta^2}{6}(\mu_u\langle l^{\eta}_{u_-}\rangle+\mu_\eta\langle l^{u_-}_{\eta}\rangle)+\frac{\zeta^2}{3}(\mu_u\langle l^{\eta'}_{u_-}\rangle+\mu_\eta\langle l^{u_-}_{\eta'}\rangle)+(\mu_d\langle l^{\pi^+}_{d_-}\rangle\\&+\mu_{\pi^+}\langle l^{d_-}_{\pi^+}\rangle)+\alpha^2(\mu_s\langle l^{K^+}_{s_-}\rangle+\mu_{K^+}\langle l^{s_-}_{K^+}\rangle)+\frac{1}{2}(\mu_d\langle l^{\pi^0}_{d_-}\rangle+\mu_{\pi^0}\langle l^{d_-}_{\pi^0}\rangle)+\frac{\beta^2}{6}(\mu_d\langle l^{\eta}_{d_-}\rangle+\mu_\eta\langle l^{d_-}_{\eta}\rangle)\\&+\frac{\zeta^2}{3}(\mu_d\langle l^{\eta'}_{d_-}\rangle+\mu_\eta\langle l^{d_-}_{\eta'}\rangle)+(\mu_u\langle l^{\pi^-}_{u_-}\rangle+\mu_{\pi^-}\langle l^{u_-}_{\pi^-}\rangle)+\alpha^2(\mu_s\langle l^{K^0}_{s_-}\rangle+\mu_{K^0}\langle l^{s_-}_{K^0}\rangle)-\frac{4a}{3}\beta^2(\mu_s\langle l^{\eta}_{s_-}\rangle\\&+\mu_\eta\langle l^{s_-}_{\eta}\rangle)-\frac{2a}{3}\zeta^2(\mu_s\langle l^{\eta'}_{s_-}\rangle+\mu_{\eta'}\langle l^{s_-}_{\eta'}\rangle)-2a\alpha^2(\mu_u\langle l^{K^-}_{u_-}\rangle+\mu_{K^-}\langle l^{u_-}_{K^-}\rangle)-2a\alpha^2(\mu_d\langle l^{\bar K^0}_{d_-}\rangle\\&+\mu_{\bar K^0}\langle l^{d_-}_{\bar K^0}\rangle)\bigg]\cdot\bigg(\mathrm{i}\frac{1}{6}kR\cdot e^{-(1/6)k^2R^2}\bigg),\end{aligned} \tag{82}$$

$$\mu^{\mathrm{total}}_{M}(\Lambda_{1520}\Lambda_{1116})^* = \mu^{\mathrm{val}}_{M}(\Lambda_{1520}\Lambda_{1116})^*+\mu^{\mathrm{sea}}_{M}(\Lambda_{1520}\Lambda_{1116})^*+\mu^{\mathrm{orbit}}_{M}(\Lambda_{1520}\Lambda_{1116})^*, \tag{83}$$

$$\mu^{\mathrm{val}}_{(3/2)}(\Lambda_{1520}\Lambda_{1116})^* = (\mu_u+\mu_d-2\mu_s)\cdot\bigg(-\mathrm{i}\frac{\sqrt{3}}{6}\frac{1}{kR}\cdot e^{-(1/6)k^2R^2}\bigg), \tag{84}$$

$$\begin{aligned}\mu^{\mathrm{sea}}_{(3/2)}(\Lambda_{1520}\Lambda_{1116})^* = &\bigg[-a\bigg(\frac{9+\beta^2+2\zeta^2}{6}+\alpha^2\bigg)\bigg](\mu_u+\mu_d)\\&-2\bigg[-a\bigg(\frac{2\beta^2+\zeta^2}{3}+2\alpha^2\bigg)\bigg]\mu_s\cdot\bigg(-\mathrm{i}\frac{\sqrt{3}}{6}\frac{1}{kR}\cdot e^{-(1/6)k^2R^2}\bigg),\end{aligned} \tag{85}$$

$$\begin{aligned}\mu^{\mathrm{orbit}}_{(3/2))}(\Lambda_{1520}\Lambda_{1116})^* = a\bigg[&\frac{1}{2}(\mu_u\langle l^{\pi^0}_{u}\rangle+\mu_{\pi^0}\langle l^{u}_{\pi^0}\rangle)+\frac{\beta^2}{6}(\mu_u\langle l^{\eta}_{u}\rangle+\mu_\eta\langle l^{u}_{\eta}\rangle)+\frac{\zeta^2}{3}(\mu_u\langle l^{\eta'}_{u}\rangle+\mu_\eta\langle l^{u}_{\eta'}\rangle)+(\mu_d\langle l^{\pi^+}_{d}\rangle\\&+\mu_{\pi^+}\langle l^{d}_{\pi^+}\rangle)+\alpha^2(\mu_s\langle l^{K^+}_{s}\rangle+\mu_{K^+}\langle l^{s}_{K^+}\rangle)+\frac{1}{2}(\mu_d\langle l^{\pi^0}_{d}\rangle+\mu_{\pi^0}\langle l^{d}_{\pi^0}\rangle)+\frac{\beta^2}{6}(\mu_d\langle l^{\eta}_{d}\rangle+\mu_\eta\langle l^{d}_{\eta}\rangle)\\&+\frac{\zeta^2}{3}(\mu_d\langle l^{\eta'}_{d}\rangle+\mu_\eta\langle l^{d}_{\eta'}\rangle)+(\mu_u\langle l^{\pi^-}_{u}\rangle+\mu_{\pi^-}\langle l^{u}_{\pi^-}\rangle)+\alpha^2(\mu_s\langle l^{K^0}_{s}\rangle+\mu_{K^0}\langle l^{s}_{K^0}\rangle)-\frac{4a}{3}\beta^2(\mu_s\langle l^{\eta}_{s}\rangle\\&+\mu_\eta\langle l^{s}_{\eta}\rangle)-\frac{2a}{3}\zeta^2(\mu_s\langle l^{\eta'}_{s}\rangle+\mu_{\eta'}\langle l^{s}_{\eta'}\rangle)-2a\alpha^2(\mu_u\langle l^{K^-}_{u}\rangle+\mu_{K^-}\langle l^{u}_{K^-}\rangle)-2a\alpha^2(\mu_d\langle l^{\bar K^0}_{d}\rangle\\&+\mu_{\bar K^0}\langle l^{d}_{\bar K^0}\rangle)\bigg]\cdot\bigg(-\mathrm{i}\frac{\sqrt{3}}{6}\frac{1}{kR}\cdot e^{-(1/6)k^2R^2}\bigg),\end{aligned} \tag{86}$$

$$\mu^{\mathrm{val}}_{(1/2)}(\Lambda_{1520}\Lambda_{1116})^* = (\mu_u+\mu_d-2\mu_s)\cdot\bigg(-\mathrm{i}\frac{1}{6}\frac{1}{kR}\cdot e^{-(1/6)k^2R^2}\bigg), \tag{87}$$

$$\mu^{\mathrm{sea}}_{(1/2)}(\Lambda_{1520}\Lambda_{1116})^* = \bigg[-a\bigg(\frac{9+\beta^2+2\zeta^2}{6}+\alpha^2\bigg)\bigg](\mu_u+\mu_d)-2\bigg[-a\bigg(\frac{2\beta^2+\zeta^2}{3}+2\alpha^2\bigg)\bigg]\mu_s\cdot\bigg(-\mathrm{i}\frac{1}{6}\frac{1}{kR}\cdot e^{-(1/6)k^2R^2}\bigg), \tag{88}$$

$$\mu^{\rm orbit}_{(1/2)}(\Lambda_{1520}\Lambda_{1116})^* = a\Big[\frac{1}{2}(\mu_u\langle l_u^{\pi^0}\rangle + \mu_{\pi^0}\langle l^u_{\pi^0}\rangle) + \frac{\beta^2}{6}(\mu_u\langle l_u^{\eta}\rangle + \mu_\eta\langle l^u_{\eta}\rangle) + \frac{\zeta^2}{3}(\mu_u\langle l_u^{\eta'}\rangle + \mu_\eta\langle l^u_{\eta'}\rangle) + (\mu_d\langle l_d^{\pi^+}\rangle + \mu_{\pi^+}\langle l^d_{\pi^+}\rangle) + \alpha^2(\mu_s\langle l_s^{K^+}\rangle + \mu_{K^+}\langle l^s_{K^+}\rangle) + \frac{1}{2}(\mu_d\langle l_d^{\pi^0}\rangle + \mu_{\pi^0}\langle l^d_{\pi^0}\rangle) + \frac{\beta^2}{6}(\mu_d\langle l_d^{\eta}\rangle + \mu_\eta\langle l^d_{\eta}\rangle) + \frac{\zeta^2}{3}(\mu_d\langle l_d^{\eta'}\rangle + \mu_\eta\langle l^d_{\eta'}\rangle) + (\mu_u\langle l_u^{\pi^-}\rangle + \mu_{\pi^-}\langle l^u_{\pi^-}\rangle) + \alpha^2(\mu_s\langle l_s^{K^0}\rangle + \mu_{K^0}\langle l^s_{K^0}\rangle) - \frac{4a}{3}\beta^2(\mu_s\langle l_s^{\eta}\rangle + \mu_\eta\langle l^s_{\eta}\rangle) - \frac{2a}{3}\zeta^2(\mu_s\langle l_s^{\eta'}\rangle + \mu_{\eta'}\langle l^s_{\eta'}\rangle) - 2a\alpha^2(\mu_u\langle l_u^{K^-}\rangle + \mu_{K^-}\langle l^u_{K^-}\rangle) - 2a\alpha^2(\mu_d\langle l_d^{\bar K^0}\rangle + \mu_{\bar K^0}\langle l^d_{\bar K^0}\rangle)\Big]\cdot\Big(-\mathrm{i}\frac{1}{6}\frac{1}{kR}\cdot e^{-(1/6)k^2R^2}\Big). \tag{89}$$

Considering the relationship between $\mu_M(B_iB_f)$, $\mu_M(B_iB_f)^*$ and A_M^{M1}, A_M^{E1} for $\Lambda(1520)\to\Lambda(1116)+\gamma$, we have

$$A^{\rm total}_{3/2}(\Lambda_{1520}\to\Lambda_{1116}+\gamma) = A^{E1,\rm val}_{3/2}(\Lambda_{1520}\to\Lambda_{1116}+\gamma) + A^{E1,\rm sea}_{3/2}(\Lambda_{1520}\to\Lambda_{1116}+\gamma) + A^{E1,\rm orbit}_{3/2}(\Lambda_{1520}\to\Lambda_{1116}+\gamma) \tag{90}$$

$(A^{M1}_{3/2}(\Lambda_{1520}\to\Lambda_{1116}+\gamma)=0)$, where,

$$A^{E1,\rm val}_{3/2}(\Lambda_{1520}\to\Lambda_{1116}+\gamma) = 2\sqrt{\pi k}\cdot[\mu^{\rm val}_{3/2}(\Lambda_{1520}\Lambda_{1116})^*] \tag{91}$$

$$A^{E1,\rm sea}_{3/2}(\Lambda_{1520}\to\Lambda_{1116}+\gamma) = 2\sqrt{\pi k}\cdot[\mu^{\rm sea}_{3/2}(\Lambda_{1520}\Lambda_{1116})^*] \tag{92}$$

$$A^{E1,\rm orbit}_{3/2}(\Lambda_{1520}\to\Lambda_{1116}+\gamma) = 2\sqrt{\pi k}\cdot[\mu^{\rm orbit}_{3/2}(\Lambda_{1520}\Lambda_{1116})^*] \tag{93}$$

$$A^{\rm total}_{1/2}(\Lambda_{1520}\to\Lambda_{1116}+\gamma) = A^{M1,\rm val}_{1/2}(\Lambda_{1520}\to\Lambda_{1116}+\gamma) + A^{M1,\rm sea}_{1/2}(\Lambda_{1520}\to\Lambda_{1116}+\gamma) + A^{M1,\rm orbit}_{1/2}(\Lambda_{1520}\to\Lambda_{1116}+\gamma) + A^{E1,\rm val}_{1/2}(\Lambda_{1520}\to\Lambda_{1116}+\gamma) + A^{E1,\rm sea}_{1/2}(\Lambda_{1520}\to\Lambda_{1116}+\gamma) + A^{E1,\rm orbit}_{1/2}(\Lambda_{1520}\to\Lambda_{1116}+\gamma) \tag{94}$$

where,

$$A^{M1,\rm val}_{1/2}(\Lambda_{1520}\to\Lambda_{1116}+\gamma) = 2\sqrt{\pi k}\cdot[\mu^{\rm val}_{-(1/2)}(\Lambda_{1520}\Lambda_{1116})] \tag{95}$$

$$A^{M1,\rm sea}_{1/2}(\Lambda_{1520}\to\Lambda_{1116}+\gamma) = 2\sqrt{\pi k}\cdot[\mu^{\rm sea}_{-(1/2)}(\Lambda_{1520}\Lambda_{1116})] \tag{96}$$

$$A^{M1,\rm orbit}_{1/2}(\Lambda_{1520}\to\Lambda_{1116}+\gamma) = 2\sqrt{\pi k}\cdot[\mu^{\rm orbit}_{-(1/2)}(\Lambda_{1520}\Lambda_{1116})] \tag{97}$$

$$A^{E1,\rm val}_{1/2}(\Lambda_{1520}\to\Lambda_{1116}+\gamma) = 2\sqrt{\pi k}\cdot[\mu^{\rm val}_{1/2}(\Lambda_{1520}\Lambda_{1116})^*] \tag{98}$$

$$A^{E1,\rm sea}_{1/2}(\Lambda_{1520}\to\Lambda_{1116}+\gamma) = 2\sqrt{\pi k}\cdot[\mu^{\rm sea}_{1/2}(\Lambda_{1520}\Lambda_{1116})^*] \tag{99}$$

$$A^{E1,\rm orbit}_{1/2}(\Lambda_{1520}\to\Lambda_{1116}+\gamma) = 2\sqrt{\pi k}\cdot[\mu^{\rm orbit}_{1/2}(\Lambda_{1520}\Lambda_{1116})^*] \tag{100}$$

besides,

$$A^{\rm total}_{3/2}(\Lambda_{1520}\to\Lambda_{1116}+\gamma) = 2\sqrt{\pi k}\cdot[\mu^{\rm total}_{3/2}(\Lambda_{1520}\Lambda_{1116})^*] \tag{101}$$

$$A^{\rm total}_{1/2}(\Lambda_{1520}\to\Lambda_{1116}+\gamma) = 2\sqrt{\pi k}\cdot[\mu^{\rm total}_{-(1/2)}(\Lambda_{1520}\Lambda_{1116}) + \mu^{\rm total}_{1/2}(\Lambda_{1520}\Lambda_{1116})^*]. \tag{102}$$

VI. RESULTS AND CONCLUSIONS

In our calculation, the wave functions are taken from the quark model calculation [2,34,36,37]. These wave functions include the important effects of configuration mixing due to the spin-spin interactions between two quarks which affects the magnetic moments and decay amplitudes of the baryons. For the octet baryons [2,34,36] (here we omit some terms whose contribution is tiny and negligible),

$$\left|N(939),\frac{1^+}{2}\right\rangle = 0.90|N^2S_S\rangle - 0.34|N^2S'_S\rangle - 0.27|N^2S_M\rangle, \quad (103)$$

$$\left|\Lambda(1116),\frac{1^+}{2}\right\rangle = 0.93|\Lambda_8^2S_S\rangle - 0.30|\Lambda_8^2S'_S\rangle - 0.20|\Lambda_8^2S_M\rangle, \quad (104)$$

$$\left|\Sigma(1193),\frac{1^+}{2}\right\rangle = 0.95|\Sigma_8^2S_S\rangle - 0.18|\Sigma_8^2S'_S\rangle - 0.16|\Sigma_8^2S_M\rangle, \quad (105)$$

$$\left|\Xi(1318),\frac{1^+}{2}\right\rangle = 0.93|\Xi_8^2S_S\rangle - 0.25|\Xi_8^2S'_S\rangle - 0.16|\Xi_8^2S_M\rangle. \quad (106)$$

For the $\Lambda(1520)$ and $\Lambda(1405)$ [2],

$$\left|\Lambda(1520),\frac{3^-}{2}\right\rangle = 0.91|\Lambda_1^2P_M\rangle - 0.40|\Lambda_8^2P_M\rangle - 0.01|\Lambda_8^4P_M\rangle \quad (107)$$

$$\left|\Lambda(1405),\frac{1^-}{2}\right\rangle = 0.90|\Lambda_1^2P_M\rangle - 0.43|\Lambda_8^2P_M\rangle + 0.06|\Lambda_8^4P_M\rangle. \quad (108)$$

Note that the signs of the last two coefficients are different from those in the references because we used a different definition of ρ_+, λ_+ and $|B_1^2L_AJ^-\rangle$ which are given in the Appendix.

We collect the input parameters a, α, β, ζ, R (the harmonic-oscillator radius parameter) and the masses of the quarks and GBs in Table I. The parameters a, α, β and ζ are fixed by fitting the octet baryon magnetic moments [21,27,29,30]. We have used the commonly used values in hadron spectroscopy for R and constituent-quark masses [32,33,36]. And we use physical masses for GBs [21,28].

With the above parameters, the octet magnetic moments in the chiral quark model are listed in Table II. Numerically speaking, the sea quark and orbital contributions to the octet baryon magnetic moments are both quite large in magnitude except for Ξ^- and Λ. However, their contributions cancel each other to a large extent. The sum of the residual sea and the naive valence quark contribution agrees with experimental value quite well as can be seen from Table II.

TABLE I. The values of various inputs used in our calculation.

Input	a	α	β	ζ	R(GeV^{-1})	$M_{u,d}$ (MeV)	M_s (MeV)
Value	0.1	0.4	0.4	−0.4	2.45	330	500

TABLE II. The octet magnetic moments in units of μ_N

Octet	exp.	χQM			
Baryons	[38]	$\mu^{\rm val}$	$\mu^{\rm sea}$	$\mu^{\rm orbit}$	$\mu^{\rm total}$
p	2.794	2.700	−0.460	0.527	2.767
n	−1.913	−1.754	0.252	−0.493	−1.994
Σ^+	2.458	2.689	−0.445	0.452	2.697
Σ^-	−1.160	−1.048	0.153	−0.403	−1.298
Ξ^0	−1.250	−1.409	0.141	−0.115	−1.383
Ξ^-	−0.651	−0.518	−0.002	0.089	−0.431
Λ	−0.613	−0.592	0.025	−0.005	−0.573
$\Sigma^0\Lambda$	1.61	1.548	−0.248	0.354	1.655

With the same parameters, we present the results of decuplet to octet transition magnetic moments in Table III.

We present the helicity amplitudes of various radiative decays in the chiral quark model in Table IV. These helicity amplitudes are decomposed into valence quark contribution, sea contribution and orbital contribution, respectively.

In Table V, we present the radiative decay widths of decuplet baryons and excited Λ hyperons in the chiral quark model. Moreover, we collect all the available calculations of these processes in literature and experimental data in Table V.

With these tables, we compare the contribution of the quark sea with that of the valence quarks, which gives a modification of the NRQM. For example in Table III, the orbital part contributes with the same sign as the sum of the valence quark contribution, while the sea part contributes with the opposite sign. Especially for these two decays: $\Delta^+ \to p + \gamma$ and $\Sigma^{*,0} \to \Lambda + \gamma$, the experimental values are higher than the valence contribution. We find that the total contribution from the quark sea is positive and increases the valence contribution by 10%. Another important observation is the large cancellation between the orbital and sea quark contribution.

For the hyperons in Table IV, the amplitudes of the magnetic-dipole transitions and electric-dipole transitions from the quark sea are not more than 10% of the valence

TABLE III. The decuplet to octet transition magnetic moments in units of μ_N

	χQM				
$B_{10} \to B_8 + \gamma$	$\mu^{\rm val}_{B_{10}B_8}$	$\mu^{\rm sea}_{B_{10}B_8}$	$\mu^{\rm orbit}_{B_{10}B_8}$	$\mu^{\rm total}_{B_{10}B_8}$	$\mu^{\rm total}_{B_{10}B_8}(k)$
$\Delta^+ \to p + \gamma$	2.466	−0.395	0.565	2.637	2.467
$\Delta^0 \to n + \gamma$	2.466	−0.395	0.565	2.637	2.467
$\Sigma^{*,+} \to \Sigma^+ + \gamma$	−2.318	0.331	−0.320	−2.307	−2.235
$\Sigma^{*,0} \to \Sigma^0 + \gamma$	−1.011	0.121	−0.021	−0.910	−0.882
$\Sigma^{*,0} \to \Lambda + \gamma$	2.193	−0.351	0.502	2.344	2.211
$\Sigma^{*,-} \to \Sigma^- + \gamma$	0.296	−0.087	0.278	0.487	0.472
$\Xi^{*,0} \to \Xi^0 + \gamma$	−2.276	0.325	−0.314	−2.266	−2.181
$\Xi^{*,-} \to \Xi^- + \gamma$	0.291	−0.086	0.273	0.478	0.460

TABLE IV. The helicity amplitudes for the radiative transitions (in $\mathrm{GeV}^{-(1/2)}$)

$B_i \to B_f + \gamma$	χQM $A_{3/2}^{M1}$ val	sea	orbit	$A_{3/2}^{E1}$ val	sea	orbit	$A_{3/2}$ total	$A_{1/2}^{M1}$ val	sea	orbit	$A_{1/2}^{E1}$ val	sea	orbit	$A_{1/2}$ total
$\Delta^+ \to p + \gamma$	−0.164	0.026	−0.038	0	0	0	−0.176	−0.095	0.015	−0.022	0	0	0	−0.101
$\Delta^0 \to n + \gamma$	−0.164	0.026	−0.038	0	0	0	−0.176	−0.095	0.015	−0.022	0	0	0	−0.101
$\Sigma^{*,+} \to \Sigma^+ + \gamma$	0.133	−0.019	0.018	0	0	0	0.132	0.077	−0.011	0.010	0	0	0	0.076
$\Sigma^{*,0} \to \Sigma^0 + \gamma$	0.058	−0.007	0.001	0	0	0	0.052	0.033	−0.004	0.001	0	0	0	0.030
$\Sigma^{*,0} \to \Lambda + \gamma$	−0.142	0.023	−0.033	0	0	0	−0.152	−0.082	0.013	−0.019	0	0	0	−0.088
$\Sigma^{*,-} \to \Sigma^- + \gamma$	−0.017	0.005	−0.016	0	0	0	−0.028	−0.010	0.003	−0.009	0	0	0	−0.016
$\Xi^{*,0} \to \Xi^0 + \gamma$	0.136	−0.019	0.019	0	0	0	0.136	0.078	−0.011	0.011	0	0	0	0.078
$\Xi^{*,-} \to \Xi^- + \gamma$	−0.017	0.005	−0.016	0	0	0	−0.028	−0.010	0.003	−0.010	0	0	0	−0.017
$\Lambda_1{}^2P_M\frac{3}{2}^- \to \Lambda_8{}^2S_S$	0	0	0	−0.063	0.007	−0.002	−0.058	0.027	−0.003	−0.001	−0.036	0.004	−0.001	−0.010
$\Lambda_8{}^2P_M\frac{3}{2}^- \to \Lambda_8{}^2S_S$	0	0	0	0.063	−0.007	0.002	0.058	−0.004	−0.002	0	0.036	−0.004	0.001	0.028
$\Lambda_8{}^4P_M\frac{3}{2}^- \to \Lambda_8{}^2S_S$	−0.019	0.004	0	0	0	0	−0.015	−0.004	0.001	0	0	0	0	−0.003
$\Lambda_1{}^2P_M\frac{3}{2}^- \to \Lambda_8{}^2S_S'$	0	0	0	0.041	−0.004	0.001	0.038	0.013	−0.002	0	0.023	−0.002	0.001	0.034
$\Lambda_8{}^2P_M\frac{3}{2}^- \to \Lambda_8{}^2S_S'$	0	0	0	−0.041	0.004	−0.001	−0.038	−0.002	−0.001	0	−0.023	0.002	−0.001	−0.025
$\Lambda_8{}^4P_M\frac{3}{2}^- \to \Lambda_8{}^2S_S'$	−0.005	0.001	0	0	0	0	−0.004	−0.001	0	0	0	0	0	−0.001
$\Lambda_1{}^2P_M\frac{3}{2}^- \to \Lambda_8{}^2S_M$	0	0	0	−0.029	0.003	−0.001	−0.027	−0.09	0.001	0	−0.017	0.002	0	−0.024
$\Lambda_8{}^2P_M\frac{3}{2}^- \to \Lambda_8{}^2S_M$	0	0	0	−0.008	0.003	−0.001	−0.006	0.005	0	0	−0.005	0.002	0	0.002
$\Lambda_8{}^4P_M\frac{3}{2}^- \to \Lambda_8{}^2S_M$	−0.004	0.001	0	0	0	0	−0.003	−0.001	0	0	0	0	0	−0.001
$\Lambda_1{}^2P_M\frac{3}{2}^- \to \Sigma_8^{02}S_S$	0	0	0	−0.160	0.028	−0.055	−0.187	0.047	−0.007	0.011	−0.092	0.016	−0.032	−0.058
$\Lambda_8{}^2P_M\frac{3}{2}^- \to \Sigma_8^{02}S_S$	0	0	0	−0.160	0.028	−0.055	−0.187	0.016	−0.002	0.004	−0.092	0.016	−0.032	−0.091
$\Lambda_8{}^4P_M\frac{3}{2}^- \to \Sigma_8^{02}S_S$	0	0	0	0.026	−0.004	0.006	0.028	0	0	0	0.005	−0.001	0.001	0.005
$\Lambda_1{}^2P_M\frac{3}{2}^- \to \Sigma_8^{02}S_S'$	0	0	0	0.100	0.017	0.034	0.117	0.025	−0.004	0.006	0.058	−0.010	0.020	0.094
$\Lambda_8{}^2P_M\frac{3}{2}^- \to \Sigma_8^{02}S_S'$	0	0	0	0.100	0.017	0.034	0.117	0.008	−0.001	0.002	0.058	−0.010	0.020	0.076
$\Lambda_8{}^4P_M\frac{3}{2}^- \to \Sigma_8^{02}S_S'$	0	0	0	0.008	−0.001	0.002	0.009	0	0	0	0.002	0	0	0.002
$\Lambda_1{}^2P_M\frac{3}{2}^- \to \Sigma_8^{02}S_M$	0	0	0	−0.070	0.012	−0.024	−0.083	−0.017	0.003	−0.004	−0.041	0.007	−0.014	−0.067
$\Lambda_8{}^2P_M\frac{3}{2}^- \to \Sigma_8^{02}S_M$	0	0	0	0	0	0	0	−0.012	0.002	−0.002	0	0	0	−0.012
$\Lambda_8{}^4P_M\frac{3}{2}^- \to \Sigma_8^{02}S_M$	0	0	0	0.006	−0.001	0.001	0.006	0	0	0	0.001	0	0	0.001
$\Lambda_1{}^2P_M\frac{1}{2}^- \to \Lambda_8{}^2S_S$	0	0	0	0	0	0	0	−0.013	0.002	0	−0.063	0.006	−0.002	−0.070
$\Lambda_8{}^2P_M\frac{1}{2}^- \to \Lambda_8{}^2S_S$	0	0	0	0	0	0	0	0.002	0.001	0	0.063	−0.006	0.002	0.062
$\Lambda_8{}^4P_M\frac{1}{2}^- \to \Lambda_8{}^2S_S$	0	0	0	0	0	0	0	−0.005	0.001	0	0	0	0	−0.004
$\Lambda_1{}^2P_M\frac{1}{2}^- \to \Lambda_8{}^2S_S'$	0	0	0	0	0	0	0	−0.007	0.001	0	0.039	−0.004	0.001	0.030
$\Lambda_8{}^2P_M\frac{1}{2}^- \to \Lambda_8{}^2S_S'$	0	0	0	0	0	0	0	0.001	0.001	0	−0.039	0.004	−0.001	−0.035
$\Lambda_8{}^4P_M\frac{1}{2}^- \to \Lambda_8{}^2S_S'$	0	0	0	0	0	0	0	−0.003	0.001	0	0	0	0	−0.002
$\Lambda_1{}^2P_M\frac{1}{2}^- \to \Lambda_8{}^2S_M$	0	0	0	0	0	0	0	0.005	−0.001	0	−0.027	0.003	−0.001	−0.021
$\Lambda_8{}^2P_M\frac{1}{2}^- \to \Lambda_8{}^2S_M$	0	0	0	0	0	0	0	−0.002	0	0	−0.008	0.003	−0.001	−0.008
$\Lambda_8{}^4P_M\frac{1}{2}^- \to \Lambda_8{}^2S_M$	0	0	0	0	0	0	0	−0.002	0	0	0	0	0	−0.002
$\Lambda_1{}^2P_M\frac{1}{2}^- \to \Sigma_8^{02}S_S$	0	0	0	0	0	0	0	−0.019	0.003	−0.004	−0.167	0.029	−0.057	−0.216
$\Lambda_8{}^2P_M\frac{1}{2}^- \to \Sigma_8^{02}S_S$	0	0	0	0	0	0	0	−0.006	0.001	−0.001	−0.167	0.029	−0.057	−0.202
$\Lambda_8{}^4P_M\frac{1}{2}^- \to \Sigma_8^{02}S_S$	0	0	0	0	0	0	0	0.006	−0.001	0.002	0	0	0	0.007
$\Lambda_1{}^2P_M\frac{1}{2}^- \to \Sigma_8^{02}S_S'$	0	0	0	0	0	0	0	−0.011	0.002	−0.012	0.100	−0.017	0.034	0.106
$\Lambda_8{}^2P_M\frac{1}{2}^- \to \Sigma_8^{02}S_S'$	0	0	0	0	0	0	0	−0.007	0.001	−0.002	0.100	−0.017	0.034	0.109
$\Lambda_8{}^4P_M\frac{1}{2}^- \to \Sigma_8^{02}S_S'$	0	0	0	0	0	0	0	0.004	−0.001	0.001	0	0	0	0.004
$\Lambda_1{}^2P_M\frac{1}{2}^- \to \Sigma_8^{02}S_M$	0	0	0	0	0	0	0	0.007	−0.001	0.002	−0.071	0.012	−0.024	−0.074
$\Lambda_8{}^2P_M\frac{1}{2}^- \to \Sigma_8^{02}S_M$	0	0	0	0	0	0	0	0.005	−0.001	0.001	0	0	0	0.005
$\Lambda_8{}^4P_M\frac{1}{2}^- \to \Sigma_8^{02}S_M$	0	0	0	0	0	0	0	0.003	0	0	0	0	0	0.003

contribution in most cases. But for the processes $|\Lambda_1^2 P_M \frac{1}{2}\rangle \to \Sigma^0 + \gamma-$ and $|\Lambda_1^2 P_M \frac{3}{2}\rangle \to \Sigma^0 + \gamma-$, the quark sea contribution is significant, which is around 20% of the valence quark contribution. For these hyperons, configuration mixing effects are also important. We confirm the $\Lambda_8^2 P_M$ and $\Lambda_8^4 P_M$ terms in Eqs. (107) and (108) make an important contribution to the decay of $\Lambda(1520)$ and $\Lambda(1405)$ [2]. For $\Lambda(1520) \to \Lambda(1116)$ and $\Lambda(1405) \to \Lambda(1116)$, their decay widths increase by more than 30% after considering the configuration mixing effects of the hyperon ground states. The radiative decays from the chiral quark model roughly agrees with recent JLAB measurement. Hopefully this model can be further extended to calculate other interesting observables.

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TABLE V. The radiative widths (in keV) of the theoretical predictions and experimental values for the radiative transitions.

$B_i \to B_f + \gamma$	χQM	NRQM	RCQM [5]	BonnCQM [6]	MIT Bag [3]	Chiral Bag [7]	Skyrme [8]
$\Delta^+ \to p + \gamma$	334	360 [34]					309–348
$\Delta^0 \to n + \gamma$	334	360 [34]					309–348
$\Sigma^{*,+} \to \Sigma^+ + \gamma$	102	104 [4]					
$\Sigma^{*,0} \to \Sigma^0 + \gamma$	16	22 [3], 19 [4]	23		15		7.7–16
$\Sigma^{*,0} \to \Lambda + \gamma$	234	273 [3], 232 [4]	267		152		157–209
$\Sigma^{*,-} \to \Sigma^- + \gamma$	4.5	2.5 [4]					
$\Xi^{*,0} \to \Xi^0 + \gamma$	130						
$\Xi^{*,-} \to \Xi^- + \gamma$	5.8						
$\Lambda_{1405} \to \Lambda_{1116} + \gamma$	168	200 [3], 143 [4]	118	912	60, 17	75	
$\Lambda_{1405} \to \Sigma^0_{1193} + \gamma$	103	72 [3], 91 [4]	46	233	18, 2.7	1.9	
$\Lambda_{1520} \to \Lambda_{1116} + \gamma$	134	156 [3], 96 [4]	215	258	46	32	
$\Lambda_{1520} \to \Sigma^0_{1193} + \gamma$	92	55 [3], 74 [4]	293	157	17	51	

$B_i \to B_f + \gamma$	Solition [9]	Algebraic model [10]	HBχPT [11]	$1/N_c$ [12]	Lattice [13]	Previous Exp.	JLAB Exp. [14]
$\Delta^+ \to p + \gamma$		343.7	670–790		430	640–720 [39]	
$\Delta^0 \to n + \gamma$		343.7	670–790		430	640–720	
$\Sigma^{*,+} \to \Sigma^+ + \gamma$					100		
$\Sigma^{*,0} \to \Sigma^0 + \gamma$	19, 11	33.9	1.4–36	24.9 ± 4.1	17	<1750 [40]	
$\Sigma^{*,0} \to \Lambda + \gamma$	243, 170	221.3	290–470	298 ± 25		<2000 [40]	$479 \pm 120^{+81}_{-100}$
$\Sigma^{*,-} \to \Sigma^- + \gamma$					3.3		
$\Xi^{*,0} \to \Xi^0 + \gamma$				129			
$\Xi^{*,-} \to \Xi^- + \gamma$				3.8			
$\Lambda_{1405} \to \Lambda_{1116} + \gamma$	44, 40	116.9				27 ± 8 [41]	
$\Lambda_{1405} \to \Sigma^0_{1193} + \gamma$	13, 17	155.7				10 ± 4 [41], 23 ± 7 [41]	
$\Lambda_{1520} \to \Lambda_{1116} + \gamma$		85.1				33 ± 11 [42], 134 ± 23 [43]	$167 \pm 43^{+26}_{-12}$
$\Lambda_{1520} \to \Sigma^0_{1193} + \gamma$		180.4				47 ± 17 [42]	

APPENDIX: THE WAVE FUNCTION CONVENTIONS

We denote the baryon $SU(6) \otimes O(3)$ wave functions as $|B\rangle = |B^{2S+1}_{N_3} L_\sigma J^P\rangle$ where B is the baryon state, N_3 is $SU(3)$ multiplicity. S, L, J and P are the total spin, total orbital angular momentum, total angular momentum and parity while $\sigma = S, M, A$ denotes the permutation symmetry of $SU(6)$. The total wave functions of baryons are expressed as combinations of $\phi\chi\psi$, where ϕ, χ, and ψ are the flavor, spin, and space wave functions respectively [32,34].

1. Flavor wave functions

The flavor wave functions ϕ are denoted as ϕ^σ_B, where B denotes the baryon state, and $\sigma = s, \rho, \lambda, a$ denotes the permutation symmetry of the wave function.

(i) Octet:

$$\phi^\rho_p = \frac{1}{\sqrt{2}}(udu - duu), \quad \phi^\lambda_p = \frac{1}{\sqrt{6}}(2uud - duu - udu) \quad \phi^\rho_n = \frac{1}{\sqrt{2}}(udd - dud), \quad \phi^\lambda_n = \frac{1}{\sqrt{6}}(dud + udd - 2ddu)$$
$$\phi^\rho_{\Sigma^+} = \frac{1}{\sqrt{2}}(suu - usu), \quad \phi^\lambda_{\Sigma^+} = \frac{1}{\sqrt{6}}(suu + usu - 2uus) \quad \phi^\rho_{\Sigma^0} = \frac{1}{2}(sud + sdu - usd - dsu),$$
$$\phi^\lambda_{\Sigma^0} = \frac{1}{2\sqrt{3}}(sdu + sud + usd + dsu - 2uds - 2dus) \quad \phi^\rho_{\Sigma^-} = \frac{1}{\sqrt{2}}(sdd - dsd), \quad \phi^\lambda_{\Sigma^-} = \frac{1}{\sqrt{6}}(sdd + dsd - 2dds)$$
$$\phi^\rho_\Lambda = \frac{1}{2\sqrt{3}}(usd + sdu - sud - dsu - 2dus + 2uds), \quad \phi^\lambda_\Lambda = \frac{1}{2}(sud + usd - sdu - dsu) \quad \phi^\rho_{\Xi 0} = \frac{1}{\sqrt{2}}(sus - uss),$$
$$\phi^\lambda_{\Xi 0} = \frac{1}{\sqrt{6}}(2ssu - sus - uss) \quad \phi^\rho_{\Xi 0} = \frac{1}{\sqrt{2}}(sds - dss), \quad \phi^\lambda_{\Xi 0} = \frac{1}{\sqrt{6}}(2ssd - sds - dss) \tag{A1}$$

(ii) Decuplet:

$$\phi^s_{\Delta^{++}} = uuu \qquad \phi^s_{\Delta^+} = \frac{1}{\sqrt{3}}(uud + udu + duu) \qquad \phi^s_{\Delta^0} = \frac{1}{\sqrt{3}}(udd + dud + ddu) \qquad \phi^s_{\Delta^-} = ddd$$
$$\phi^s_{\Sigma^+} = \frac{1}{\sqrt{3}}(uus + usu + suu) \qquad \phi^s_{\Sigma^0} = \frac{1}{\sqrt{6}}(uds + usd + dus + dsu + sud + sdu)$$
$$\phi^s_{\Sigma^-} = \frac{1}{\sqrt{3}}(dds + dsd + sdd) \qquad \phi^s_{\Xi^0} = \frac{1}{\sqrt{6}}(uss + sus + ssu) \qquad \phi^s_{\Xi^-} = \frac{1}{\sqrt{3}}(dss + sds + ssd) \tag{A2}$$
$$\phi^s_{\Omega^-} = sss$$

(iii) Singlet:

$$\phi^a_\Lambda = \frac{1}{\sqrt{6}}(uds + dsu + sud - dus - usd - sdu) \tag{A3}$$

2. Spin wave functions

For the spin wave functions, we denotes them as $\chi = \chi^\sigma_{S_z}$, where S_z is the third component of spin, and $\sigma = s, \rho, \lambda$.

$$\chi^s_{3/2} = \uparrow\uparrow\uparrow, \qquad \chi^\rho_{1/2} = \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow),$$
$$\chi^\lambda_{1/2} = \frac{1}{\sqrt{2}}(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \tag{A4}$$

Other values of S_z are obtained by applying the lowering operator in spin space and normalizing to one.

3. Spatial wave functions

The spatial wave function are denoted by $\psi = \psi^\sigma_{NLL_z}$, where N, L, and L_z are the main quantum number, total orbital angular momentum, and the third component of the L, and $\sigma = s, \rho, \lambda, a$.

Since the harmonic-oscillator wave functions are good approximations to the eigenfunctions of low-lying states of a system bound by Coulomb-plus–linear potentials [44], Isgur and Karl [1] employed the harmonic-oscillator wave functions to compute matrix elements in quark model. The thus obtained meson wave functions in the quark model are expressed in the base of the oscillator wave functions.

Using the relative coordinates $\boldsymbol{\rho}$ and $\boldsymbol{\lambda}$,

$$\boldsymbol{\rho} = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_2), \qquad \boldsymbol{\lambda} = \frac{1}{\sqrt{6}}(\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3), \tag{A5}$$

where $\mathbf{r}_i (i = 1, 2, 3)$ denote the coordinate of the ith quark, The harmonic-oscillator wave functions we used in the present work are:

$$\psi^s_{000} = \psi_0(\boldsymbol{\rho}, \boldsymbol{\lambda}), \tag{A6}$$

$$\psi^\rho_{111} = R^{-1}\rho_+\psi_0(\boldsymbol{\rho}, \boldsymbol{\lambda}), \qquad \psi^\lambda_{111} = R^{-1}\lambda_+\psi_0(\boldsymbol{\rho}, \boldsymbol{\lambda}), \tag{A7}$$

$$\psi^s_{200} = \frac{1}{\sqrt{3}}R^{-2}(\boldsymbol{\rho}^2 + \boldsymbol{\lambda}^2 - 3R^2)\psi_0(\boldsymbol{\rho}, \boldsymbol{\lambda}),$$
$$\psi^\rho_{200} = \frac{1}{\sqrt{3}}R^{-2}(2\boldsymbol{\rho}\cdot\boldsymbol{\lambda})\psi_0(\boldsymbol{\rho}, \boldsymbol{\lambda}),$$
$$\psi^\lambda_{200} = \frac{1}{\sqrt{3}}R^{-2}(\boldsymbol{\rho}^2 - \boldsymbol{\lambda}^2)\psi_0(\boldsymbol{\rho}, \boldsymbol{\lambda}),$$
$$\psi^s_{222} = \frac{1}{2}R^{-2}(\rho_+^2 + \lambda_+^2)\psi_0(\boldsymbol{\rho}, \boldsymbol{\lambda}), \tag{A8}$$
$$\psi^\rho_{222} = R^{-2}\rho_+\lambda_+\psi_0(\boldsymbol{\rho}, \boldsymbol{\lambda}),$$
$$\psi^\lambda_{222} = \frac{1}{2}R^{-2}(\rho_+^2 - \lambda_+^2)\psi_0(\boldsymbol{\rho}, \boldsymbol{\lambda}),$$
$$\psi^a_{211} = R^{-2}(\rho_+\lambda_z - \lambda_+\rho_z)\psi_0(\boldsymbol{\rho}, \boldsymbol{\lambda})$$

where $\rho_+ = -(\rho_x + i\rho_y)$, $\lambda_+ = -(\lambda_x + i\lambda_y)$ and $\psi_0(\boldsymbol{\rho}, \boldsymbol{\lambda}) = (R^6\pi^3)^{-(1/2)}\exp(-\frac{\rho^2+\lambda^2}{2R^2})$.

4. $SU(6) \otimes O(3)$ wave functions

Combining the wave functions above in a symmetric way, we can get the total wave functions. For the states of $N = 0$ and $L^P = 0^+$,

$$\left| B^2_8 S_S \frac{1^+}{2} \right\rangle = \frac{1}{\sqrt{2}}(\phi^\rho_B\chi^\rho + \phi^\lambda_B\chi^\lambda)\psi^s_{000} \tag{A9}$$

$$\left| B^4_{10} S_S \frac{3^+}{2} \right\rangle = \phi^s_B\chi^s\psi^s_{000}. \tag{A10}$$

For the states of $N = 1$ and $L^P = 1^-$,

$$|B^2_8 L_M J^-\rangle = \frac{1}{2}[(\phi^\rho_B\chi^\lambda + \phi^\lambda_B\chi^\rho)\psi^\rho_{11L_z} + (\phi^\rho_B\chi^\rho - \phi^\lambda_B\chi^\lambda)\psi^\lambda_{11L_z}]_J \tag{A11}$$

$$|B^4_8 L_M J^-\rangle = \frac{1}{\sqrt{2}}[\phi^\rho\chi^s\psi^\rho_{11L_z} + \phi^\lambda\chi^s\psi^\lambda_{11L_z}]_J \tag{A12}$$

$$|B^2_{10} L_M J^-\rangle = \frac{1}{\sqrt{2}}[\phi^s\chi^\rho\psi^\rho_{11L_z} + \phi^s\chi^\lambda\psi^\lambda_{11L_z}]_J \tag{A13}$$

$$|B^2_1 L_A J^-\rangle = \frac{1}{\sqrt{2}}[\phi^a\chi^\lambda\psi^\rho_{11L_z} - \phi^a\chi^\rho\psi^\lambda_{11L_z}]_J \tag{A14}$$

where the index J in the bracket means that the spatial and spin wave functions are coupled to get a total angular momentum J.

For the states of $N = 2$,

$$\left| B_8^2 S_S' \frac{1^+}{2} \right\rangle = \frac{1}{\sqrt{2}} (\phi_B^\rho \chi^\rho + \phi_B^\lambda \chi^\lambda) \psi_{200}^s \tag{A15}$$

$$\left| B_8^2 S_M \frac{1^+}{2} \right\rangle = \frac{1}{2} [(\phi_B^\rho \chi^\lambda + \phi_B^\lambda \chi^\rho) \psi_{200}^\rho + (\phi_B^\rho \chi^\rho - \phi_B^\lambda \chi^\lambda) \psi_{200}^\lambda]. \tag{A16}$$

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