Closing the $SU(3)_L \otimes U(1)_X$ symmetry at the electroweak scale

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We show that some models with $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge symmetry can be realized at the electroweak scale and that this is a consequence of an approximate global $SU(2)_{L+R}$ symmetry. This symmetry implies a condition among the vacuum expectation value of one of the neutral Higgs scalars, the $U(1)_X$'s coupling constant, g_X, the sine of the weak mixing angle $\sin\theta_W$, and the mass of the W boson, M_W. In the limit in which this symmetry is valid it avoids the tree level mixing of the Z boson of the standard model with the extra Z' boson. We have verified that the oblique T parameter is within the allowed range indicating that the radiative corrections that induce such a mixing at the 1-loop level are small. We also show that a $SU(3)_{L+R}$ custodial symmetry implies that in some of the models we have to include sterile (singlets of the 3-3-1 symmetry) right-handed neutrinos with Majorana masses, since the seesaw mechanism is mandatory to obtain light active neutrinos. Moreover, the approximate $SU(2)_{L+R} \subset SU(3)_{L+R}$ symmetry implies that the extra nonstandard particles of these 3-3-1 models can be considerably lighter than it had been thought before so that new physics can be really just around the corner.

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I. INTRODUCTION

It is usually assumed that any physics beyond the standard model (SM) must have this model as a good approximation at energies of the order of hundred GeVs or, in practice, up to the LEP energies. This implies that the new degrees of freedom should be related to a sufficiently high energy scale. In particular, in many of the extensions of the SM there is at least one additional neutral gauge boson, generically denoted by Z', whose mass has to be of the order of few TeV in order to keep consistency with present phenomenology. This is the case, for instance, of the left-right models [1], models with an extra $U(1)$ factor [2–5], and in grand unified theories with symmetries larger than $SU(5)$ as $SO(10)$ and E_6 [6], little Higgs scenarios [7], and models with extra dimensions [8]. In all these cases, the existence of additional real neutral vector bosons yields deviations from the condition $M_W = \cos\theta_W M_Z$ because of the mixing between the Z-boson of the SM and the new neutral vector boson Z'. There are deviations also in the neutral current parameters of the fermion i in the vector and axial-vector interactions with Z, denoted by $g^i_{V,A}$. These parameters only coincide within certain approximation with those of the Z. In general these deviations are proportional to $(v_W/\Lambda)^2$ or higher power of this ratio, where Λ is an energy scale, say a vacuum expectation value (VEV), related to the breaking of the hidden extra symmetry. So, it is thought to be necessary that $\Lambda \gg v_W \approx 246$ GeV in order to make the models compatible with the present phenomenology. This makes the search for extra neutral gauge bosons one of the main goals of the next high energy collider experiments [9].

Usually, the interactions involving Z' are parameterized (besides the pure kinetic term) as [2,3]

$$\mathcal{L}^{\mathrm{NC}(Z')} = -\frac{\sin\xi}{2} F'_{\mu\nu}F^{\mu\nu} + M^2_{Z'} Z'_\mu Z'^\mu + \delta M^2 Z'_\mu Z^\mu - \frac{g}{2c_W}\sum_i \bar\psi_i \gamma^\mu (f^i_V - f^i_A \gamma^5)\psi_i Z'_\mu, \quad (1)$$

where Z, which is the would be neutral vector boson of the SM, and Z' are not yet mass eigenstates, having a mixing defined by the angle

$$\tan 2\phi = \frac{\delta M^2}{M^2_{Z'} - M^2_Z}, \quad (2)$$

and where $c_W \equiv \cos\theta_W$ (and for future use $s_W \equiv \sin\theta_W$). If Z_1 and Z_2 denote the mass eigenstates, then in most of the models we have $M_{Z_2} \gg M_{Z_1} \approx M_Z$, and hence $\phi \ll 1$. In this situation the vector and the axial-vector neutral current parameters, $g^{i(\mathrm{SM})}_V$ and $g^{i(\mathrm{SM})}_A$, respectively, of the Z-boson with the known fermions are shifted, at tree level, as follows:

$$g^i_V = g^{i(\mathrm{SM})}_V c_\phi + f^i_V s_\phi, \qquad g^i_A = g^{i(\mathrm{SM})}_A c_\phi + f^i_A s_\phi, \quad (3)$$

where $g^{i(\mathrm{SM})}_V = T^i_3 - 2Q_i s^2_W$ and $g^{i(\mathrm{SM})}_A = T^i_3$, being $T^i_3 = \pm 1/2$ and Q_i the electric charge of the fermion i; we have used the notation $c_\phi(s_\phi) = \cos\phi(\sin\phi)$. The parameters $f^i_{V,A}$ in Eq. (3) are not in general the same for all particles of the same electric charge, thus, we have flavor changing neutral currents (FCNC) coupled to Z which imply strong

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constraints coming from experimental data such as ΔM_K and other $|\Delta S| = 2$ processes. These constraints imply a small value for the mixing angle ϕ or, similarly, a large value for the energy scale Λ. If $s_\phi = 0$ is imposed such constraints could be avoided, however in most of the models with Z' this usually implies a fine tuning among $U(1)$ charges and vacuum expectation values that is far from being natural [5].

In principle, a shift as in Eq. (3) occurs in 3-3-1 models [10–12]. These models have a rich scalar sector that implies, in general, a mixing of Z, the vector boson of $SU(2)_L \subset SU(3)_L$, and Z', the gauge boson related to the $SU(3)_L$ symmetry. Working in the Z, Z' basis the condition $\sin\phi \ll 1$ can be obtained if the energy scale Λ (which in these models is identified with the VEV that breaks the $SU(3)_L$ symmetry, v_χ) is above the TeV scale. Hence, it is usually believed that only approximately we can have that $Z_1 \approx Z$, even at the tree level. The same happens with the neutral current parameters, $g^i_{V,A}$, which only approximately coincide with $g^{i(\mathrm{SM})}_{V,A}$. This is true since the corrections to the Z mass and $g^i_{V,A}$ in these models, assuming $v_\chi \gg v_W \simeq 246$ GeV, are proportional to $(v_W/v_\chi)^2$ and for $v_\chi \to \infty$ we recover exactly the SM with all its degrees of freedom, with the heavier ones introduced by the $SU(3)_L$ symmetry decoupled. However, we expect that v_χ should not be extremely large if new physics is predicted to show up in the near future experiments. In practice, measurements of the ρ_0 parameter, and FCNC processes like ΔM_K, should impose constraints upon the v_χ scale at which the $SU(3)_L$ symmetry arises.

However, it was pointed out recently in Ref. [13] that in 3-3-1 models, at the tree level, it is possible that: (i) there is no mixing between Z and Z', and the latter boson may have a mass even below the TeV scale; (ii) $\rho_0 = 1$ since $M_{Z_1} = M_Z$, and (iii) the vector and axial-vector parameters in the neutral currents coupled to Z_1, $g^i_{V,A}$, being exactly those of the SM, $g^{i(\mathrm{SM})}_{V,A}$, independently of the v_χ value. This is implied not by a fine tuning but by a condition which can be verified experimentally involving the parameters of the model, g, M_W, s_W and one of the VEVs. Such condition is a consequence of an approximate global $SU(2)_{L+R}$ symmetry. In the limit of the exact symmetry we have $\sin\phi = 0$, avoiding in this way the shift as in Eq. (3), and $\sin\theta_W = 0$ as in the SM. Remarkably, when $\sin\phi = 0$ but $\sin\theta_W \neq 0$ the parameters in the neutral currents coupled to the heavy boson Z_2 depend only on the weak mixing angle θ_W, meaning that they are not free parameters anymore. Moreover the couplings of Z_2 with leptons are suppressed by the leptophobic factor $(1 - 4\sin^2\theta_W)^{1/2}$ [14].

The outline of this paper is as follows. In Sec. II we review briefly the three 3-3-1 models that will be considered here. We give the representation content of the model with bileptons (Sec. II A), with heavy leptons (Sec. II B) and with right-handed neutrinos in (Sec. II C). Next, in Sec. III we give exact expressions for the gauge vector boson eigenstates and their respective masses for the three models. In Sec. IV A, IV B, and IV C we give what we call the "SM limit" for each model. In Sec. V we show the exact expressions for the parameters $g^i_{V,A}$, $f^i_{V,A}$ appearing in the neutral currents coupled to Z_1 and Z_2: Sec. V A for the case of the model with bileptons; Sec. V B for the heavy lepton models and, Sec. V C for the model with right-handed neutrinos. We also show in Secs. V A, V B, and V C that if we impose that there is no mixing between Z and Z', then $g^i_{V,A}$ coincide exactly with the respective parameters of the SM's Z boson for all the known particles. It means that there is no flavor changing neutral current in the known sector and that $f^i_{V,A}$ depend only on s_W. In Sec. VI we explain the small value for ϕ as consequence of a global approximate custodial symmetry. These results are interpreted in the last section, Sec. VII. In the appendix we show that there is also an approximate global $SU(3)_{L+R}$ symmetry which, although badly broken, is useful for obtaining a realistic mass spectra in the scalar sectors. Moreover we also discuss that this extended custodial symmetry implies that it is mandatory to include right-handed sterile (with respect to the 3-3-1 symmetry) neutrinos and to consider the seesaw mechanism to obtain light active neutrinos.

II. THE MODELS

Models with $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge symmetry (called 3-3-1 models for short) are interesting possibilities for the electroweak interactions at the TeV scale [10–12,15]. At low energies it is expected that, like any other extension of the SM, they must coincide with this model. By choosing appropriately the representation content they give at least partial explanations to some fundamental questions that are accommodated but not explained by the SM [16].

The 3-3-1 models which embed the standard model are those of Refs. [10–12,15]. Here we will consider only: (1) the minimal model which has charged bileptons [10], (2) the model with heavy leptons [11], and (3) the model with right-handed neutrinos [12]. These models have a electric charge operator that can be written as

$$\frac{Q}{e} = T_3 - bT_8 + X, \tag{4}$$

where T_i, $i = 3, 8$ are the diagonal generators of SU(3) and $b = \sqrt{3}$ for the minimal model [10] and also for the model with heavy leptons [11], while $b = 1/\sqrt{3}$ for the model with right-handed neutrinos transforming nontrivially under $SU(3)_L$ [12] or heavy neutral leptons [15].

A. The minimal model

Let us consider first the minimal 3-3-1 model [10] in which $b = \sqrt{3}$ and the known leptons transform as triplets:

$$\Psi_{aL} = (\nu_a, l_a, l_a^c)_L^T \sim (\mathbf{1}, \mathbf{3}, 0), \qquad \nu_{aR} \sim (\mathbf{1}, \mathbf{1}, 0), \quad (5)$$

here T means transpose and $a = e, \mu, \tau$.

In the quark sector we have two antitriplets and one triplet:

$$\begin{aligned} Q_{mL} &= (d_m, u_m, j_m)_L^T \sim (\mathbf{3}, \mathbf{3}^*, -1/3), \\ Q_{3L} &= (u_3, d_3, J)_L^T \sim (\mathbf{3}, \mathbf{3}, 2/3), \end{aligned} \quad (6)$$

with $m = 1, 2$, and the right-handed components transforming as singlets:

$$\begin{aligned} &u_{\alpha R} \sim (\mathbf{3}, \mathbf{1}, 2/3), \quad d_{\alpha R} \sim (\mathbf{3}, \mathbf{1}, -1/3), \quad \alpha = 1, 2, 3; \\ &J_R \sim (\mathbf{3}, \mathbf{1}, 5/3), \quad j_{mR} \sim (\mathbf{3}, \mathbf{1}, -4/3). \end{aligned} \quad (7)$$

The minimal scalar sector of the model consists of three triplets:

$$\begin{aligned} \eta &= (\eta^0, \eta_1^-, \eta_2^+)^T \sim (\mathbf{1}, \mathbf{3}, 0), \\ \rho &= (\rho^+, \rho^0, \rho^{++})^T \sim (\mathbf{1}, \mathbf{3}, 1), \\ \chi &= (\chi^-, \chi^{--}, \chi^0)^T \sim (\mathbf{1}, \mathbf{3}, -1). \end{aligned} \quad (8)$$

and the sextet

$$S = \begin{pmatrix} \sigma_1^0 & \frac{h_2^+}{\sqrt{2}} & \frac{h_1^-}{\sqrt{2}} \\ \frac{h_2^+}{\sqrt{2}} & H_1^{++} & \frac{\sigma_2^0}{\sqrt{2}} \\ \frac{h_1^-}{\sqrt{2}} & \frac{\sigma_2^0}{\sqrt{2}} & H_2^{--} \end{pmatrix} \sim (\mathbf{6}^*, 0). \quad (9)$$

The VEVs for the scalar Higgs multiplets are denoted by $\langle \eta^0 \rangle = v_\eta/\sqrt{2}$, $\langle \rho \rangle = v_\rho/\sqrt{2}$, $\langle \chi^0 \rangle = v_\chi/\sqrt{2}$, for the triplets, and $\langle \sigma_2^0 \rangle = v_s$ and $\langle \sigma_1^0 \rangle = 0$, in the scalar sextet, i.e., we are neglecting left-handed neutrino Majorana masses.

Since the extra quarks have all exotic electric charges, the mixing in the known quark sectors are exactly as in the SM. In the neutrino sector, the presence of sterile neutrinos allows a general mass matrix with both, Dirac and Majorana masses.

B. The model with heavy leptons

In the model of Ref. [11], which also has $b = \sqrt{3}$, it is introduced, in each lepton triplet, a heavy charged field E^+:

$$\Psi_{aL} = (\nu_a, l_a^-, E_a^+)_L^T \sim (\mathbf{1}, \mathbf{3}, 0), \quad (10)$$

and the right-handed components of the leptons transforming as

$$\nu_{aR} \sim (\mathbf{1}, \mathbf{1}, 0); \qquad l_{aR}^- \sim (\mathbf{1}, \mathbf{1}, -1); \qquad E_{aR}^+ \sim (\mathbf{1}, \mathbf{1}, 1). \quad (11)$$

The quark sector is the same of the previous model and here only the triplets in Eq. (8) are needed for given to fermions and gauge bosons appropriate masses.

The mixing in the quark sector is as in the previous model. However, in the charged lepton sector it is possible to have a general mixing between the known charged leptons, l_a^-, and the heavy ones, E_a^-. For instance, interactions such as $\epsilon\overline{(\Psi_{aL})^c}\Psi_{bL}\eta$ and $\bar{l}_{aL}^c E_{bR}$ induce such a mixture. This can be avoided by introducing an appropriate discrete symmetry. On the other hand, neutrinos have Dirac masses and the right-handed sterile ones can get a Majorana mass term. Although in this model the scalar sextet is not necessary, it can be introduced to generate Majorana mass terms for active left-handed neutrinos.

C. Model with right-handed neutrinos

If right-handed sterile neutrinos do exist then it is possible that they transform nontrivially under a larger gauge symmetry group, for instance the 3-3-1 symmetry with $b = 1/\sqrt{3}$ [12]. This model is probably the more economical one to incorporate sterile neutrinos with respect to the SM interactions [17].

In this case the representation content is as follows [12]:

$$\psi_{aL} = (\nu_a, e_a, \nu_a^c)_L^T \sim (\mathbf{1}, \mathbf{3}, -1/3), \quad (12)$$

and the right-handed components for the charged leptons,

$$e_{aR} \sim (\mathbf{1}, \mathbf{1}, -1). \quad (13)$$

The quark sector consists in the following representations:

$$\begin{aligned} Q_{mL} &= (d_m, u_m, D_m)_L^T \sim (\mathbf{3}, \mathbf{3}^*, 0), \\ Q_{3L} &= (u_3, d_3, U)_L^T \sim (\mathbf{3}, \mathbf{3}, 1/3), \end{aligned} \quad (14)$$

with $m = 1, 2$. And the respective right-handed components:

$$\begin{aligned} u_{\alpha R} &\sim (\mathbf{3}, \mathbf{1}, 2/3), \qquad d_{\alpha R} \sim (\mathbf{3}, \mathbf{1}, -1/3), \\ U_R &\sim (\mathbf{3}, \mathbf{1}, 2/3), \qquad D_{mR} \sim (\mathbf{3}, \mathbf{1}, -1/3), \end{aligned} \quad (15)$$

with $\alpha = 1, 2, 3$.

The scalar sector of the model is

$$\begin{aligned} \eta &= (\eta^0, \eta^-, \eta'^0)^T \sim (\mathbf{1}, \mathbf{3}, -1/3), \\ \rho &= (\rho^+, \rho^0, \rho'^+)^T \sim (\mathbf{1}, \mathbf{3}, 2/3), \\ \chi &= (\chi^0, \chi^-, \chi'^0)^T \sim (\mathbf{1}, \mathbf{3}, -1/3). \end{aligned} \quad (16)$$

The only nonzero VEVs are $\langle \eta^0 \rangle = v_\eta/\sqrt{2}$, $\langle \rho^0 \rangle = v_\rho/\sqrt{2}$, and $\langle \chi'^0 \rangle = v_{\chi'}/\sqrt{2}$.

In order to avoid favor changing neutral currents (FCNC) this model has been considered with three scalar triplets of the sort showed in Eq. (16), see Refs. [12]. Only two of them are necessary to give mass to all fermions and to implement the spontaneous breaking of the gauge symmetry [18]. A version of the model with four scalar triplets generates fermion masses without a hierarchy in the Yukawa couplings [19]. However, in this case the equivalent to the Cabibbo-Kobayashi-Maskawa mixing matrix is not unitary and there is also FCNC mediated by the Z vector boson of the SM. In general depending on: the number of scalar triplets, on discrete symmetries and on the VEV structure, the model could have, or not, FCNC in all charged sectors, and the mixing matrix in the couplings with $W^{\pm}$ could be, or not, exactly the same as in the SM. An scalar sextet can also be introduced in order to have more space to generate neutrino masses [15,20].

III. GAUGE BOSON MASSES AND EIGENSTATES

From the kinetic terms for the scalar fields, constructed with the covariant derivatives

$$\mathcal{D}_{\mu}\varphi = \partial_{\mu}\varphi - ig\mathcal{M}_{\mu}\varphi - ig_X X\varphi B_{\mu}, \quad \mathcal{D}_{\mu}S = \partial_{\mu}S - ig[\mathcal{M}_{\mu}S + S^T\mathcal{M}^T_{\mu}], \tag{17}$$

where g_X denotes the $U(1)_X$ gauge coupling constant and $\varphi = \eta, \rho, \chi$, we can obtain the mass matrices for the vector bosons.

Defining $\mathcal{M}_{\mu} \equiv \vec{W}_{\mu}\cdot\vec{T}$, we have

$$\mathcal{M}_{\mu} = \begin{pmatrix} W^3_{\mu} + \frac{1}{\sqrt{3}}W^8_{\mu} + 2tX_{\varphi}B_{\mu} & \sqrt{2}W^+_{\mu} & \sqrt{2}(V^{1/2(\sqrt{3}b-1)}_{\mu})^* \\ \sqrt{2}W^-_{\mu} & -W^3_{\mu} + \frac{1}{\sqrt{3}}W^8_{\mu} + 2tX_{\varphi}B_{\mu} & \sqrt{2}(U^{1/2(\sqrt{3}b+1)}_{\mu})^* \\ \sqrt{2}V^{1/2(\sqrt{3}b-1)}_{\mu} & \sqrt{2}U^{1/2(\sqrt{3}b+1)}_{\mu} & -\frac{2}{\sqrt{3}}W^8_{\mu} + 2tX_{\varphi}B_{\mu} \end{pmatrix}, \tag{18}$$

with $t \equiv g_X/g$ and where the non-Hermitian gauge bosons are defined as

$$W^+_{\mu} = (W^1_{\mu} - iW^2_{\mu})/\sqrt{2}, \quad V^{1/2(\sqrt{3}b-1)}_{\mu} = (W^4_{\mu} + iW^5_{\mu})/\sqrt{2}, \quad U^{1/2(\sqrt{3}b+1)}_{\mu} = (W^6_{\mu} + iW^7_{\mu})/\sqrt{2}, \tag{19}$$

with $\pm\frac{1}{2}(\sqrt{3}b \pm 1)$ denoting the electric charge in units of the $|e|$ of the heavy gauge bosons V and U. In 3-3-1 models with $b = \sqrt{3}$ both heavy vector bosons are charged, $V^{\pm}$ and $U^{\pm\pm}$. In 3-3-1 models with $b = 1/\sqrt{3}$ we have $U^{\pm}$ and a non-Hermitian V^0 neutral vector bosons.

In the minimal model, the mass square of the non-Hermitian vector bosons are given by

$$M^2_W = \tfrac{1}{4}g^2v^2_W, \qquad M^2_V = \tfrac{1}{4}g^2(v^2_{\eta} + 2v^2_s + v^2_{\chi}), \qquad M^2_U = \tfrac{1}{4}g^2(v^2_{\rho} + 2v^2_s + v^2_{\chi}), \tag{20}$$

where $v^2_W \equiv v^2_{\eta} + v^2_{\rho} + 2v^2_s$. Notice that as $v_W \approx 246$ GeV, the usual VEV of the Higgs in the SM, then v_{χ} must be, in principle, large enough in order to keep the new gauge bosons, as V and U, sufficiently heavy to be consistent with the present experimental data. In models where the sextet is not necessary those expressions in Eq. (20) are still valid puting $v_s = 0$. In all these models there is no mixing between Z and Z' in the kinetic term at the tree level, thus $\sin\xi = 0$ in Eq. (1).

Insofar the analysis is valid for both $b = \sqrt{3}$ and $b = 1/\sqrt{3}$ models, however in order to clarify, when considering the real neutral gauge bosons we will study both cases separately.

A. Neutral gauge bosons in the minimal 3-3-1 model

The mass matrix for the real neutral vector bosons in the $(W^3_{\mu}, W^8_{\mu}, B_{\mu})$ basis is

$$M^2_{(b=\sqrt{3})} = \frac{g^2}{4}v^2_{\chi}\begin{pmatrix} \bar{v}^2_W & \frac{1}{\sqrt{3}}(\bar{v}^2_W - 2\bar{v}^2_{\rho}) & -2t\bar{v}^2_{\rho} \\ \frac{1}{\sqrt{3}}(\bar{v}^2_W - 2\bar{v}^2_{\rho}) & \frac{1}{3}(\bar{v}^2_W + 4) & \frac{2}{\sqrt{3}}t(\bar{v}^2_{\rho} + 2) \\ -2t\bar{v}^2_{\rho} & \frac{2}{\sqrt{3}}t(\bar{v}^2_{\rho} + 2) & 4t^2(\bar{v}^2_{\rho} + 1) \end{pmatrix}, \tag{21}$$

and we have introduced the dimensionless ratios $\bar{v}_{\rho} = v_{\rho}/v_{\chi}$ and $\bar{v}_W = v_W/v_{\chi}$. The matrix in Eq. (21) has a vanishing eigenvalue corresponding to the photon. The other two eigenvalues, M_{Z_1} and M_{Z_2}, can be written exactly, introducing two dimensionless parameters m_1 and m_2, as

$$m^2_1 \equiv \frac{2M^2_{Z_1}}{g^2v^2_{\chi}} = A(1 - R); \qquad m^2_2 \equiv \frac{2M^2_{Z_2}}{g^2v^2_{\chi}} = A(1 + R), \tag{22}$$

where we have defined

$$A = \tfrac{1}{3}[3t^2(\bar{v}^2_{\rho} + 1) + \bar{v}^2_W + 1], \tag{23}$$

and

$$R = \left[1 - \frac{1}{3A^2}(4t^2 + 1)[\bar{v}^2_W(\bar{v}^2_{\rho} + 1) - \bar{v}^4_{\rho}]\right]^{1/2}, \tag{24}$$

with t given by $t^2 = s^2_W/(1 - 4s^2_W)$ (see below). Notice that instead of introducing a mixing angle between the Z and the Z' bosons, as in Refs. [21], we have diagonalized directly the mass square matrix in Eq. (21).

The eigenstates of the symmetry W^3_{μ}, W^8_{μ}, and B_{μ} can be written in terms of the mass eigenstates A_{μ}, $Z_{1\mu}$, and $Z_{2\mu}$ in an exact form as

$$\begin{aligned} W^3_\mu &= \frac{t}{\sqrt{4t^2+1}} A_\mu - N_1(3m_2^2 + \bar{v}_\rho^2 - 2\bar{v}_W^2) Z_{1\mu} \\ &\quad - N_2(3m_1^2 + \bar{v}_\rho^2 - 2\bar{v}_W^2) Z_{2\mu}, \\ \frac{W^8_\mu}{\sqrt{3}} &= -\frac{t}{\sqrt{4t^2+1}} A_\mu - N_1\left(m_2^2 + \bar{v}_\rho^2 - \frac{2}{3}\bar{v}_W^2 - \frac{2}{3}\right) Z_{1\mu} \\ &\quad - N_2\left(m_1^2 + \bar{v}_\rho^2 - \frac{2}{3}\bar{v}_W^2 - \frac{2}{3}\right) Z_{2\mu}, \\ B_\mu &= \frac{1}{\sqrt{4t^2+1}} A_\mu + 2t(1-\bar{v}_\rho^2) N_1 Z_{1\mu} \\ &\quad + 2t(1-\bar{v}_\rho^2) N_2 Z_{2\mu}, \end{aligned} \tag{25}$$

with the normalization factors

$$\begin{aligned} N_1^{-2} &= 3\left(2m_2^2 + \bar{v}_\rho^2 - \frac{4}{3}\bar{v}_W^2 - \frac{1}{3}\right)^2 + (\bar{v}_\rho^2 - 1)^2(4t^2+1), \\ N_2^{-2} &= 3\left(2m_1^2 + \bar{v}_\rho^2 - \frac{4}{3}\bar{v}_W^2 - \frac{1}{3}\right)^2 + (\bar{v}_\rho^2 - 1)^2(4t^2+1). \end{aligned} \tag{26}$$

Only the components in A_μ do not depend on the VEVs but the others in $Z_{1,2}$ do. The interaction of the photon with leptons is therefore

$$g\frac{t}{\sqrt{4t^2+1}} \bar{l}_a \gamma^\mu l_a A_\mu = e \bar{l}_a \gamma^\mu l_a A_\mu = g s_W \bar{l}_a \gamma^\mu l_a A_\mu, \tag{27}$$

where e is the electric charge of the positron. We can identify $e = g s_W$ since in 3-3-1 models $SU(2)_L \subset SU(3)_L$, i.e., $g_{SU(3)_L} \equiv g_{SU(2)_L}$. On the other hand, the condition $1/e^2 = 4/g^2 + 1/g_X^2$ for 3-3-1 models with $b = \sqrt{3}$ [10] implies

$$t^2 \equiv \frac{\alpha_X}{\alpha_L} = \frac{s_W^2}{1 - 4s_W^2}, \tag{28}$$

with $\alpha_i = g_i^2/4\pi$, $i = X, L$, where we have introduced the notation $g \equiv g_L$. Notice, for future use in Sec. VI and in the appendix, that $s_W = 0$ implies $g_X = 0$.

B. Neutral gauge bosons in the model with right-handed neutrinos

The masses of the charged vector bosons are as in Eqs. (20) but with $v_s = 0$. In this case the mass square matrix for the real neutral bosons in the $(W^3_\mu, W^8_\mu, B_\mu)$ basis is

$$M^2_{(b=1/\sqrt{3})} = \frac{g^2}{2} v_\chi^2 \begin{pmatrix} \frac{1}{2}\bar{v}_W^2 & \frac{1}{2\sqrt{3}}(\bar{v}_W^2 - 2\bar{v}_\rho^2) & -\frac{1}{3}t(\bar{v}_W^2 + \bar{v}_\rho^2) \\ \frac{1}{2\sqrt{3}}(\bar{v}_W^2 - 2\bar{v}_\rho^2) & \frac{1}{6}(\bar{v}_W^2 + 4) & \frac{1}{3\sqrt{3}}t(3\bar{v}_\rho^2 - \bar{v}_W^2 + 2) \\ -\frac{1}{3}t(\bar{v}_W^2 + \bar{v}_\rho^2) & \frac{1}{3\sqrt{3}}t(3\bar{v}_\rho^2 - \bar{v}_W^2 + 2) & \frac{2}{9}t^2(3\bar{v}_\rho^2 + \bar{v}_W^2 + 1) \end{pmatrix}. \tag{29}$$

As in the previous section we define the dimensionless parameters for this model m_1^2 and m_2^2 as in Eq. (22) but now the VEV appearing in them is $v_{\chi'}$, and A and R are given by

$$A = \tfrac{1}{9}[t^2(\bar{v}_W^2 + 3\bar{v}_\rho^2 + 1) + 3(\bar{v}_W^2 + 1)], \tag{30}$$

and

$$R = \left[1 - \frac{1}{9A^2}(4t^2+3)[\bar{v}_W^2(\bar{v}_\rho^2 + 1) - \bar{v}_\rho^4]\right]^{1/2}. \tag{31}$$

The symmetry eigenstates W^3_μ, W^8_μ, and B_μ can be written in terms of the mass eigenstates A_μ, $Z_{1\mu}$, and $Z_{2\mu}$:

$$\begin{aligned} W^3_\mu &= \frac{\sqrt{3}t}{\sqrt{4t^2+3}} A_\mu - N_1[3m_2^2 - 3(\bar{v}_W^2 - \bar{v}_\rho^2)] Z_{1\mu} \\ &\quad - N_2[3m_1^2 - 3(\bar{v}_W^2 - \bar{v}_\rho^2)] Z_{2\mu}, \\ \frac{W^8_\mu}{\sqrt{3}} &= -\frac{1}{3}\frac{\sqrt{3}t}{\sqrt{4t^2+3}} A_\mu - N_1[3m_2^2 - 3(\bar{v}_W^2 - \bar{v}_\rho^2)] Z_{1\mu} \\ &\quad - N_2[3m_1^2 - 3(\bar{v}_W^2 - \bar{v}_\rho^2)] Z_{2\mu}, \\ B_\mu &= \frac{\sqrt{3}}{\sqrt{4t^2+3}} A_\mu + 2t(1 + \bar{v}_\rho^2 - \bar{v}_W^2) N_1 Z_{1\mu} \\ &\quad + 2t(1 + \bar{v}_\rho^2 - \bar{v}_W^2) N_2 Z_{2\mu}, \end{aligned} \tag{32}$$

with the normalization factors

$$\begin{aligned} N_1^{-2} &= 9(2m_2^2 + \bar{v}_\rho^2 - \bar{v}_W^2 - 1)^2 + (1 + \bar{v}_\rho^2 - v_W^2)^2(4t^2+3), \\ N_2^{-2} &= 9(2m_1^2 + \bar{v}_\rho^2 - \bar{v}_W^2 - 1)^2 + (1 + \bar{v}_\rho^2 - v_W^2)^2(4t^2+3). \end{aligned} \tag{33}$$

As in the previous case only the components on A_μ do not depend on the VEVs. In this sort of models we have

$$t^2 = \frac{\alpha_X}{\alpha_L} = \frac{s_W^2}{1 - \frac{4}{3}s_W^2}. \tag{34}$$

As in the previous models $s_W = 0$ implies $g_X = 0$.

IV. SM LIMIT FROM 3-3-1 MODELS

Let us introduce the parameters $\rho_0 = c_W^2 M_Z^2/M_W^2$ and ρ_1 defined, at the tree level, as

$$\rho_1 \equiv \frac{c_W^2 M_{Z_1}^2}{M_W^2} = \frac{2c_W^2}{\bar{v}_W^2} A(1-R), \tag{35}$$

where A and R are defined in Eqs. (23) and (24), respectively. Notice that we are using the inverse of the standard model ρ_0 definition [3]. At the tree level $\rho_0 = 1$ is a prediction of the SM. Thus we call the SM limit of the 3-3-1 model the condition $\rho_1 = 1$. It means that we are looking for a relation among the parameters of this model such that $\rho_1 = 1$. We have verified that in general $\rho_1 \leq 1$ and that $\rho_1 = 1$ is obtained only under two conditions: (i) when $v_\chi \to \infty$, in practice when it is very large, as we have mentioned in the introduction; or (ii) the less trivial condition when $\bar{v}_\rho$ has a particular value that we denote $\tilde{v}_\rho$.

On the other hand, we saw that the mass eigenstates A_μ, $Z_{1\mu}$, and $Z_{2\mu}$ obtained by inverting the Eq. (25) have a complex structure that depends on the VEVs for the cases of $Z_{1\mu}$ and $Z_{2\mu}$. It means that $Z_{1\mu}$ and $Z_{2\mu}$ have, in general, couplings with fermions that are also functions of the VEVs, of the electric charges and of the weak mixing angle θ_W. In the so called SM limit, we will obtain that neither $Z_{1\mu}$ nor $Z_{2\mu}$ depend on the VEVs and, consequently, the neutral current parameters of $Z_{1\mu}$ are the same as those of the SM for all known fermions.

A. SM limit in the minimal model

For the minimal model using A and R given by Eqs. (23) and (24), respectively, the nontrivial solution for obtaining $\rho_1 = 1$ in Eq. (35) is

$$\tilde{\bar{v}}_\rho^{\,2} = \frac{1 - 4s_W^2}{2c_W^2}\bar{v}_W^2, \tag{36}$$

and, since v_i, the VEVs of the scalars transforming as doublets under $SU(2)$, must satisfy the condition $\sum_i v_i^2 = (246\ \text{GeV})^2$, Eq. (36) also implies

$$\tilde{\bar{v}}_\eta^{\,2} + 2\tilde{\bar{v}}_s^2 = \frac{1 + 2s_W^2}{2c_W^2}\bar{v}_W^2. \tag{37}$$

Notice that these relations do not depend on the v_χ scale. With the present value of $s_W^2 = 0.2312$ [3] we obtain from Eq. (36) and (37), respectively, that $\tilde{v}_\rho \approx 54$ GeV and $\sqrt{\tilde{v}_\eta^2 + 2\tilde{v}_s^2} \approx 240$ GeV.

Using Eq. (36) in the exact eigenvalues of Eq. (25) we obtain *exactly*, in the basis (W^3, W^8, B), the mass eigenstates

$$\begin{aligned} \tilde{A}_\mu &= (s_W, -\sqrt{3}s_W, \sqrt{1 - 4s_W^2}) \equiv A_\mu, \\ \tilde{Z}_{1\mu} &= (-c_W, -\sqrt{3}t_W s_W, \sqrt{1 - 4s_W^2}t_W) \equiv Z_\mu, \\ \tilde{Z}_{2\mu} &= (0, \sqrt{1 - 4s_W^2}/c_W, \sqrt{3}t_W) \equiv Z'_\mu, \end{aligned} \tag{38}$$

where $t_W = s_W/c_W$. Hereafter, the tilde in a quantity x i.e., $\tilde{x}$, indicates that we are using Eq. (36) in the exact expression of x. If we substitute the condition in Eq. (36) and (37) in Eqs. (22) we obtain

$$\begin{aligned} \tilde{M}_{Z_1}^2 &= \frac{g^2}{4c_W^2}v_W^2 \equiv M_Z^2, \\ \tilde{M}_{Z_2}^2 &= \frac{g^2}{2}\frac{(1 - 2s_W^2)(4 + \bar{v}_W^2) + s_W^4(4 - \bar{v}_W^4)}{6c_W^2(1 - 4s_W^2)}v_\chi^2 \equiv M_{Z'}^2. \end{aligned} \tag{39}$$

Moreover, the mass of Z_2 can be large even if v_χ is of the order of the electroweak scale. In fact, from Eq. (20) and (39) we have

$$\frac{\tilde{M}_{Z_2}^2}{M_W^2} = \frac{(1 - 2s_W^2)(4 + \bar{v}_W^2) + 4s_W^4(1 - 2\bar{v}_W^4)}{3c_W^2(1 - 4s_W^2)}\frac{1}{\bar{v}_W^2}, \tag{40}$$

and we see that for $\bar{v}_W = 1$ (the 3-3-1 scale is equal to the electroweak scale) we obtain $M_{Z_2} = 3.77M_W$. Of course for lower values of $\bar{v}_W$, Z_2 is heavier, for instance for $\bar{v}_W = 0.25$ we have $M_{Z_2} = 18.36M_W$. We recall that since v_χ does not contribute to the W mass it is not constrained by the 246 GeV upper bound. Thus, independently if $\bar{v}_W^2$ is larger, smaller or equal to 1, we see that the charged vector boson V is heavier than U. Using Eqs. (20) we see that in general $M_V^2 - M_U^2 = (g^2/4)(v_\eta^2 - v_\rho^2)$ and, after using Eqs. (36) and (37) we can write

$$\frac{\Delta M}{M_W} = \frac{(\tilde{M}_V^2 - \tilde{M}_U^2)^{1/2}}{M_W} = \left(3\tan^2\theta_W - \frac{2v_s^2}{v_W^2}\right)^{1/2} \leq \sqrt{3}\tan\theta_W, \tag{41}$$

with $\Delta M/M_W \approx 0.94$ for $v_s = 0$, as is the case in the model with heavy leptons [11]. Notice that in this minimal model the scalar sextet is introduced only to give the correct mass to the charged leptons, so it is not necessarily a large VEV, $v_s \approx 2$ GeV may be enough.

We see that, very impressing, the solution to the condition $\rho_1 = 1$ given in Eq. (36), implies that the Z has *exactly* the same mass than the respective vector boson of the SM. Thus, unlike the case when $v_\chi \to \infty$ this is far from being a trivial condition. It implies that the expressions for neutral current parameters with Z_1 and Z_2, that usually have been considered only approximately (valid when terms of the order v_W^2/v_χ^2 are neglected), are now exact expressions that depend only on the weak mixing angle θ_W. We will show below that the condition in Eq. (36) is protected by an accidental symmetry and when it is used the VEVs are not arbitrary anymore, up to the sum in Eq. (37).

Notice also that, from Eq. (38), at a high energy μ when $s_W^2(\mu) = 1/4$, the photon A_μ and $Z_{1\mu}$ are the gauge bosons of an $SU(2)_L \otimes U(1)_Y$ symmetry while $Z_{2\mu} \equiv B_\mu$, where B_μ is the gauge boson of the Abelian factor $U(1)_X$. It means that at high energies the product $SU(2) \otimes U(1) \subset SU(3)$ decouples from $U(1)_X$ and this may happen even at the electroweak scale.

B. SM limit in the model with heavy leptons

In this model all happens in the same way that in the previous one, except that $v_s = 0$ and then the condition in Eq. (36) is valid, and which implies also, instead of (37), $v_\eta^2 = [(1 + 2s_W^2)/2c_W^2]v_W^2$. By using $s_W^2 = 0.2312$ [3] we obtain that $v_\rho \approx 54$ GeV and $v_\eta \approx 240$ GeV. It means that in this case there is only a free VEV: v_χ. In particular, in this model without sextet we have $\Delta M/M_W \approx 0.94$ as shown in the previous subsection.

C. SM limit in the model with right-handed neutrinos

In this case we have the definition

$$\rho_1 \equiv \frac{c_W^2 M_{Z_1}^2}{M_W^2} = \frac{2c_W^2}{\bar{v}_W^2} A(1-R), \tag{42}$$

with A and R given in Eqs. (30) and (31), respectively. Thus, in this model the condition $\rho_1 = 1$ implies

$$\tilde{\bar{v}}_\rho^2 = \frac{(1-2s_W^2)}{2c_W^2}\bar{v}_W^2, \tag{43}$$

which gives the numerical values of $\tilde{v}_\rho = 145.5$ GeV and $\tilde{v}_\eta = 198.4$ GeV. We have verified that when this relation is used in the exact expressions for the mass eigenstates, in basis W_μ^3, W_μ^8, B_μ given in Eq. (32), do not depend on the VEVs structure, only on s_W, as in Eq. (38).

V. VECTOR AND AXIAL NEUTRAL CURRENT PARAMETERS

Next, we will study the effect of the relations in Eqs. (36) and (43) on the neutral current parameters of these models. In this vain we will parameterize the neutral currents in the 3-3-1 models considered above, as follows:

$$\mathcal{L}_{331}^{\rm NC} = -\frac{g}{2c_W}\sum_i \bar{\psi}_i\gamma^\mu[(g_V^i - g_A^i\gamma_5)Z_{1\mu} + (f_V^i - f_A^i\gamma_5)Z_{2\mu}]\psi_i, \tag{44}$$

when the exact forms for Z_1 and Z_2, obtained by inverting Eqs. (25) and (32), are used in defining $g_{V,A}^i$ and $f_{V,A}^i$. We recall that the fermions ψ_i are all still symmetry eigenstates.

Below we will write the analytical exact expressions for the neutral current parameters $g_{V,A}^i$ and $f_{V,A}^i$, showing explicitly that they depend on the VEVs in a complicated way. But when the conditions in Eqs. (36) and (43) are used in these expressions, in the respective model, we obtain for the case of the known fermions $g_{V,A}^i \equiv g_{V,A}^{i({\rm SM})}$, and $f_{V,A}^i = f_{V,A}^i(s_W)$, i.e., these parameters depend only on the electroweak mixing angle.

A. Neutral current parameters in the minimal 3-3-1 model

The reduction of the complicated expression, depending on the VEVs for the eigenstates, in Eq. (25) to those in Eq. (38) which depend only on s_W, is not a trivial result. Moreover, we shall calculate the neutral current parameters using the full expressions in (25) and then use the condition (36). The result is that, independently of the value of v_χ, we obtain *exactly* the parameters of the SM Z-boson for those particles that are common with the 3-3-1 models, and that the extra nonstandard particles have vector and axial-vector neutral current parameters that do not depend on the VEVs, but only on θ_W. This implies that the effects of the extra neutral currents due to Z_2 are only constrained by the Z_2 mass. Moreover, in the first two models the $Z' \equiv Z_2$ boson is leptophobic [14].

First consider the leptonic sector. The coupling of the neutrinos are:

$$\begin{aligned} g_V^\nu &= g_A^\nu = -N_1\frac{c_W}{3}(1-6m_2^2-3\bar{v}_\rho^2+4\bar{v}_W^2),\\ f_V^\nu &= f_A^\nu = -N_2\frac{c_W}{3}(1-6m_1^2-3\bar{v}_\rho^2+4\bar{v}_W^2), \end{aligned} \tag{45}$$

where m_1^2 and m_2^2 are defined in Eq. (22), and N_1 and N_2 are defined in Eq. (26).

For the case of the known charged leptons:

$$\begin{aligned} g_V^l &= -N_1 c_W(1-\bar{v}_\rho^2),\\ g_A^l &= N_1\frac{c_W}{3}(1-6m_2^2-3\bar{v}_W^2),\\ f_V^l &= -N_2 c_W(1-\bar{v}_\rho^2),\\ f_A^l &= N_2\frac{c_W}{3}(1-6m_1^2-3\bar{v}_W^2+4\bar{v}_\rho^2). \end{aligned} \tag{46}$$

In the known quark sector we have the exact $g_{V,A}^q$ parameters given by

$$\begin{aligned} g_V^{u_m} &= \frac{N_1}{h^2(s_W^2)}\frac{c_W}{3}[1+(3m_2^2-\bar{v}_W^2)h^2(s_W^2)+2s_W^2(\bar{v}_\rho^2-3)],\\ g_A^{u_m} &= \frac{N_1}{h^2(s_W^2)}\frac{c_W}{3}[1+(3m_2^2-2\bar{v}_W^2)h^2(s_W^2) + 2s_W^2(1+3\bar{v}_\rho^2)],\\ g_V^{u_3} &= -\frac{N_1}{h^2(s_W^2)}\frac{c_W}{3}[1-6m_2^2h^2(s_W^2)-3\bar{v}_\rho^2 + 4s_W^2(1+\bar{v}_\rho^2-4\bar{v}_W^2)],\\ g_A^{u_3} &= -N_1\frac{c_W}{3}(1-6m_2^2-3\bar{v}_\rho^2+4\bar{v}_W^2),\\ g_V^{d_m} &= \frac{N_1}{h^2(s_W^2)}\frac{c_W}{3}[1+(4\bar{v}_W^2-6m_2^2)h^2(s_W^2)-(3-8s_W^2)\bar{v}_\rho^2],\\ g_A^{d_m} &= -g_A^{u_3},\\ g_V^{d_3} &= -\frac{N_1}{h^2(s_W^2)}\frac{c_W}{3}[1+(4\bar{v}_W^2+3m_2^2)h^2(s_W^2) - 2s_W^2(1+\bar{v}_\rho^2)],\\ g_A^{d_3} &= -\frac{N_1}{h^2(s_W^2)}\frac{c_W}{3}[1+(3m_2^2-2\bar{v}_W^2)h^2(s_W^2) + 2s_W^2(1-3\bar{v}_\rho^2)]. \end{aligned} \tag{47}$$

Finally, in the exotic quarks sector we have that the exact $g_{V,A}^j$ parameters are

$$
\begin{aligned}
g_V^{j_m} &= -\frac{N_1}{h^2(s_W^2)}\frac{c_W}{3}[2+(2\bar{v}_W^2-3m_2^2)h^2(s_W^2)-3\bar{v}_\rho^2-2s_W^2(9-11\bar{v}_\rho^2)],\\
g_A^{j_m} &= -\frac{N_1}{h^2(s_W^2)}\frac{c_W}{3}[2+(2\bar{v}_W^2-3m_2^2)h^2(s_W^2)-3\bar{v}_\rho^2-2s_W^2(1-3\bar{v}_\rho^2)],\\
g_V^{J} &= \frac{N_1}{h^2(s_W^2)}\frac{c_W}{3}[2+(2\bar{v}_W^2-3m_2^2)h^2(s_W^2)-3\bar{v}_\rho^2-2s_W^2(11-13\bar{v}_\rho^2)],\\
g_A^{J} &= \frac{N_1}{h^2(s_W^2)}\frac{c_W}{3}[2+(2\bar{v}_W^2-3m_2^2)h^2(s_W^2)-3\bar{v}_\rho^2-2s_W^2(1-3\bar{v}_\rho^2)],
\end{aligned} \tag{48}
$$

where we have denoted $h(s_W^2) = +[1-4s_W^2]^{1/2}$. The parameters of quarks to $Z_{2\mu}$, $f_{V,A}^q$, are obtained from those in Eqs. (47) and (48) by replacing $N_1 \to N_2$ and $m_2^2 \to m_1^2$ and we will not write them explicitly.

When the relation in Eq. (36) is used in the $g_{V,A}^i$ and $f_{V,A}^i$ parameters in Eqs. (46)–(48), we obtain (here we omitted the tilde $\tilde{()}$ in all the expressions on the left side)

$$
g_V^\nu = g_A^\nu = \frac{1}{2}, \qquad f_V^\nu = f_A^\nu = -\frac{\sqrt{3}}{6}h(s_W^2), \qquad g_V^l = -\frac{1}{2}+2s_W^2, \qquad g_A^l = -\frac{1}{2}, \qquad f_V^l = -f_A^l = -\frac{\sqrt{3}}{6}h(s_W^2), \tag{49}
$$

in the lepton sector, while in the known quark sector we have:

$$
\begin{aligned}
&g_V^u = \frac{1}{2}-\frac{4}{3}s_W^2, \quad g_A^u = \frac{1}{2}, \quad u = u_1, u_2, u_3; \quad f_V^{u_m} = \frac{1}{2\sqrt{3}}\frac{1-6s_W^2}{h(s_W^2)}, \quad f_A^{u_m} = \frac{1}{2\sqrt{3}}\frac{1+2s_W^2}{h(s_W^2)}, \quad m = 1, 2;\\
&f_V^{u_3} = -\frac{1}{2\sqrt{3}}\frac{1+4s_W^2}{h(s_W^2)}, \quad f_A^{u_3} = -\frac{1}{\sqrt{3}}h(s_W^2), \quad g_V^d = -\frac{1}{2}+\frac{2}{3}s_W^2, \quad g_A^d = -\frac{1}{2}, \quad d = d_1, d_2, d_3;\\
&f_V^{d_m} = \frac{1}{2\sqrt{3}h(s_W^2)}, \quad f_A^{d_m} = \frac{h(s_W^2)}{2\sqrt{3}}, \quad m = 1, 2; \quad f_V^{d_3} = -\frac{1}{2\sqrt{3}}\frac{1-2s_W^2}{h(s_W^2)}, \quad f_A^{d_3} = -\frac{1}{2\sqrt{3}}\frac{1+2s_W^2}{h(s_W^2)},
\end{aligned} \tag{50}
$$

and, finally in the exotic quark sector:

$$
\begin{aligned}
&g_V^{j_m} = \frac{8}{3}s_W^2, \quad g_A^{j_m} = 0, \quad f_V^{j_m} = -\frac{1}{\sqrt{3}}\frac{1-9s_W^2}{h(s_W^2)}, \quad f_A^{j_m} = -\frac{1}{\sqrt{3}}\frac{c_W^2}{h(s_W^2)}, \quad m = 1, 2;\\
&g_V^J = -\frac{10}{3}s_W^2, \quad g_A^J = 0, \quad f_V^J = \frac{1}{\sqrt{3}}\frac{1-11s_W^2}{h(s_W^2)}, \quad f_A^J = \frac{1}{\sqrt{3}}\frac{c_W^2}{h(s_W^2)}.
\end{aligned} \tag{51}
$$

Notice that, since all fields in Eq. (44) are symmetry eigenstates, from the parameters in Eqs. (49)–(51) we see that in the leptonic sector there is not FCNC neither with $Z_{1\mu}$ nor with $Z_{2\mu}$ and in the quark sector there are FCNC only coupled to $Z_{2\mu}$ [22]. Notice also that the exotic quarks have pure vectorial couplings with $Z_{1\mu}$.

B. Neutral current parameters in the model with heavy leptons

Let us consider the parameters in the neutral currents coupled with $Z_{1\mu}$ and $Z_{2\mu}$. For the neutrinos they are:

$$
\begin{aligned}
g_V^\nu &= g_A^\nu = N_1 c_W(2m_2^2+\bar{v}_\rho^2-\tfrac{4}{3}\bar{v}_W^2-\tfrac{1}{3}),\\
f_V^\nu &= f_A^\nu = N_2 c_W(2m_1^2+\bar{v}_\rho^2-\tfrac{4}{3}\bar{v}_W^2-\tfrac{1}{3}).
\end{aligned} \tag{52}
$$

For the case of the known charged leptons:

$$
\begin{aligned}
g_V^l &= -N_1 c_W\Big[m_2^2+\frac{1}{3}(1-2\bar{v}_W^2)-\frac{2s_W^2}{h^2(s_W^2)}(1-\bar{v}_\rho^2)\Big],\\
g_A^l &= -N_1 c_W\Big[m_2^2+\frac{1}{3}(1-2\bar{v}_W^2)+\frac{2s_W^2}{h^2(s_W^2)}(1-\bar{v}_\rho^2)\Big],\\
f_V^l &= -N_2 c_W\Big[m_1^2+\frac{1}{3}(1-2\bar{v}_W^2)-\frac{2s_W^2}{h^2(s_W^2)}(1-\bar{v}_\rho^2)\Big],\\
f_A^l &= -N_2 c_W\Big[m_1^2+\frac{1}{3}(1-2\bar{v}_W^2)+\frac{2s_W^2}{h^2(s_W^2)}(1-\bar{v}_\rho^2)\Big].
\end{aligned} \tag{53}
$$

For the exotic heavy charged leptons we have:

$$g_V^E = -N_1 c_W\left[m_2^2 + \bar{v}_\rho^2 - \frac{2}{3}(1+\bar{v}_W^2) + \frac{2s_W^2}{h^2(s_W^2)}(1-\bar{v}_\rho^2)\right],$$
$$g_A^E = -N_1 c_W\left[m_2^2 + \bar{v}_\rho^2 - \frac{2}{3}(1+\bar{v}_W^2) - \frac{2s_W^2}{h^2(s_W^2)}(1-\bar{v}_\rho^2)\right],$$
$$f_V^E = -N_2 c_W\left[m_1^2 + \bar{v}_\rho^2 - \frac{2}{3}(1+\bar{v}_W^2) + \frac{2s_W^2}{h^2(s_W^2)}(1-\bar{v}_\rho^2)\right],$$
$$f_A^E = -N_2 c_W\left[m_1^2 + \bar{v}_\rho^2 - \frac{2}{3}(1+\bar{v}_W^2) - \frac{2s_W^2}{h^2(s_W^2)}(1-\bar{v}_\rho^2)\right]. \quad (54)$$

Finally, in the quark sector the parameters are the same as in Eqs. (47) and (48) or, after using (36), in (50) and (51).

If we substitute the condition in Eq. (36) in all the $g^i_{V,A}$ and $f^i_{V,A}$ parameters, given in Eqs. (52)–(54), we obtain (here also we are omitting the tilde in all the expressions on the left side):

$$g_V^\nu = g_A^\nu = \frac{1}{2}, \quad f_V^\nu = f_A^\nu = -\frac{\sqrt{3}}{6}h(s_W^2), \quad g_V^l = -\frac{1}{2} + 2s_W^2,$$
$$g_A^l = -\frac{1}{2}, \quad f_V^l = -\frac{\sqrt{3}}{6}\frac{(1-10s_W^2)}{h(s_W^2)},$$
$$f_A^l = -\frac{\sqrt{3}}{6}\frac{(1+2s_W^2)}{h(s_W^2)}, \quad g_V^E = -2s_W^2, \quad g_A^E = 0, \quad (55)$$
$$f_V^E = \frac{\sqrt{3}}{3}\frac{(1-7s_W^2)}{h(s_W^2)}, \quad f_A^E = \frac{\sqrt{3}}{3}\frac{c_W^2}{h(s_W^2)},$$

and we see that for neutrinos and the usual known charged leptons the neutral current parameters in the $Z_{1\mu}$ interactions are exactly the same as those in the SM at the tree level. Notice also that only the neutrinos have leptophobic interactions with $Z_{2\mu}$ in this model.

C. Neutral current parameters in the model with right-handed neutrinos

In this model we have also obtained the exact neutral current parameters in both sectors $Z_{1\mu}$ and $Z_{2\mu}$, which as in the previous models are denoted by $g^i_{V,A}$ and $f^i_{V,A}$, respectively. For the neutrinos we obtain:

$$g_V^\nu = N_1 c_W[6m_2^2 - 3(\bar{v}_W^2 - \bar{v}_\rho^2) - 3],$$
$$g_A^\nu = \frac{N_1 c_W}{k^2(s_W^2)}[1 + \bar{v}_\rho^2 - \bar{v}_W^2],$$
$$f_V^\nu = N_2 c_W[6m_1^2 - 3(\bar{v}_W^2 - \bar{v}_\rho^2) - 3], \quad (56)$$
$$f_A^\nu = \frac{N_2 c_W}{k^2(s_W^2)}[1 + \bar{v}_\rho^2 - \bar{v}_W^2],$$

where we have denoted $k(s_W^2) = +[1 - (4/3)s_W^2]^{1/2}$. And, in the charged lepton sector:

$$g_V^l = -\frac{N_1 c_W}{k^2(s_W^2)}[1 + \bar{v}_\rho^2 - \bar{v}_W^2]h^2(s_W^2),$$
$$g_A^l = -\frac{N_1 c_W}{k^2(s_W^2)}[1 + \bar{v}_\rho^2 - \bar{v}_W^2],$$
$$f_V^l = -\frac{N_2 c_W}{k^2(s_W^2)}[1 + \bar{v}_\rho^2 - \bar{v}_W^2]h^2(s_W^2), \quad (57)$$
$$f_A^l = -\frac{N_2 c_W}{k^2(s_W^2)}[1 + \bar{v}_\rho^2 - \bar{v}_W^2].$$

Using the condition (43) in Eqs. (56) and (57) we obtain (here again we have omitted the tilde () in all the expressions on the left side)

$$g_V^\nu = g_A^\nu = \frac{1}{2}, \qquad f_V^\nu = -\frac{\sqrt{3}}{2}k(s_W^2),$$
$$f_A^\nu = \frac{\sqrt{3}}{6}\frac{1}{k(s_W^2)}, \qquad g_V^l = -\frac{1}{2} + 2s_W^2, \qquad g_A^l = -\frac{1}{2},$$
$$f_V^l = \frac{\sqrt{3}}{6}\frac{h^2(s_W^2)}{k(s_W^2)}, \qquad f_A^l = -\frac{\sqrt{3}}{6}\frac{1}{k(s_W^2)}, \quad (58)$$

and we see that, again, the parameters in the $Z_{1\mu}$ currents coincide in an exactly way with those of the Z in the SM. Notice also that in this model $Z_{2\mu}$ is leptophobic only with charged leptons. The reduction of the exact parameters of the known fermions with $Z_{1\mu}$ to those of the SM, when the condition (43) is used, also occurs in the quark sector but here we will not write them explicitly.

VI. CUSTODIAL SYMMETRY

We can rewrite Eqs. (36) which is valid for the models of Secs. II A and II B, as

$$g\frac{\tilde{v}_\rho}{\sqrt{2}} = \frac{\sqrt{1 - 4s_W^2}}{c_W}M_W, \quad (59)$$

and Eq. (43) which is valid for the model of Sec. II C, as

$$g\frac{\tilde{v}_\rho}{\sqrt{2}} = \frac{\sqrt{1 - 2s_W^2}}{c_W}M_W. \quad (60)$$

These are like the Goldberger-Treiman relation [23] in the sense that their validity imply, as we will show below, an approximate global symmetry of the models and all quantities appearing in these relations can be measured independently: the W mass M_W, the sine of the weak mixing angle $\sin\theta_W$, the $SU(3)_L$ coupling constant, g, and the VEV of one of the triplets, say, v_ρ. In fact, all but $\tilde{v}_\rho$, are already well known. However, cross sections of several processes, for instance $e^+e^- \to ZH$ where H is a neutral Higgs scalar transforming as doublet of SU(2), are sensitive to the value of v_η (or v_ρ) [24]. So, in principle it is possible to verify if

Eq. (59), or Eq. (60), is satisfied and if the 3-3-1 symmetry can be implemented near the electroweak scale.

We can study the "stability" of the full expressions for ρ_1 in Eq. (35) using the full expression for M_{Z_1} given in Eqs. (22), with Eqs. (23) and (24) for the case of the minimal and heavy lepton models, and Eqs. (30) and (31) for the case of the minimal model and for the model with right-handed neutrinos. We have analyzed how the condition $\rho_1 = 1$ varies when we change arbitrarily v_ρ. We expand the value of v_ρ as $\bar{v}'_\rho = (1+x)\tilde{\bar{v}}_\rho$ and substituting $\bar{v}'_\rho$ in Eq. (35) and expanding in $x \lesssim 1$ we obtain

$$\rho_1 \approx 1 - 0.0025x^2 + 0.00012x^3 + \cdots, \quad \text{for} \quad \bar{v}_W = 1; \tag{61}$$

$$\rho_1 \approx 1 - 0.00024x^2 + 1.8 \times 10^{-6}x^3 + \cdots, \quad \text{for} \quad \bar{v}_W = 0.1, \tag{62}$$

for the minimal model, and

$$\rho_1 \approx 1 - 0.3497x^2 + 0.1051x^3 + \cdots, \quad \text{for} \quad \bar{v}_W = 1; \tag{63}$$

$$\rho_1 \approx 1 - 0.0131x^2 + 0.0001x^3 + \cdots, \quad \text{for} \quad \bar{v}_W = 0.1, \tag{64}$$

for the model with right-handed neutrinos. Thus, we see that the minimal model is more stable, in the sense discussed above, than the model with right-handed neutrinos with respect to the departure of v_ρ from $\tilde{v}_\rho$, i.e., from the condition (59) or (60), respectively. For instance, $\bar{v}_W = 1$, for the minimal model we have that a 20% ($x = 0.2$) depart from the condition (59) the value of ρ_1 is only affected by 0.01%, while for the model with right-handed neutrinos and (60), for the same value of x, the respective ρ_1 changes 1.28%. This suggests that, when both 3-3-1 models were embedded in a $SU(4)_L \otimes U(1)_N$ model [25] which has three real neutral vector bosons, the SU(3) subgroup which contains the SM's Z should be the minimal 3-3-1 model considered in Sec. II A.

In the SM the fact that $\rho_0 = 1$ is a consequence of an approximate (accidental) $SU(2)_{L+R}$ global symmetry named "custodial symmetry" [26,27]. This custodial symmetry is exact when $g' = 0$ ($\sin^2\theta_W = 0$) which implies $M_W = M_Z$ since in this limit W^+, W^-, Z form a triplet of this unbroken global symmetry. Also, due to the unbroken $SU(2)_{L+R}$ in the $g' \to 0$ limit, radiative corrections to the ρ_0 parameter due to gauge and Higgs bosons must be proportional to g'^2 [27]. We try to understand this situation in the context of 3-3-1 models, by showing that these models have also an approximate global $SU(2)_{L+R}$.

Let us consider the model of Sec. II B. We can decompose the triplets as $\mathbf{3} = \mathbf{2} + \mathbf{1}$ under $SU(2)_L \otimes U(1)_Y$. In particular, the scalar triplets we have used can be written as

$$\varphi = H_\varphi + s_\varphi, \tag{65}$$

where $\varphi = \eta, \rho, \chi$. Under $SU(2)_L \otimes U(1)_Y$, H_η, H_ρ, H_χ transform as $(\mathbf{2}, -1/2)$, $(\mathbf{2}, 1/2)$, $(\mathbf{2}, -3)$, respectively, and s_η, s_ρ, s_χ as $(\mathbf{1}, +2)$, $(\mathbf{1}, +4)$, $(\mathbf{1}, 0)$, respectively. Next, we define four 2-doublet

$$\Phi_{\zeta\zeta} = \frac{1}{\sqrt{2}}(\tilde{H}_\zeta H_\zeta), \qquad \Phi_{\rho\eta} = \frac{1}{\sqrt{2}}(H_\rho H_\eta), \tag{66}$$

where $\zeta = \eta, \rho, \chi$, and $\tilde{H} = \epsilon H^*$. We can write the more general scalar potential invariant under 3-3-1 as

$$V(\eta, \rho, \chi) = V(\Phi_{\varphi\varphi'}, s_\varphi) + f\eta\rho\chi + \text{H.c.}, \tag{67}$$

where

$$\begin{aligned} V(\Phi_{\varphi\varphi'}, s_\varphi) = {} & \mu_\eta^2[\mathrm{Tr}(\Phi^\dagger_{\eta\eta}\Phi_{\eta\eta}) + s^\dagger_\eta s_\eta] + \mu_\rho^2[\mathrm{Tr}(\Phi^\dagger_{\rho\rho}\Phi_{\rho\rho}) + s^\dagger_\rho s_\rho] + \mu_\chi^2[\mathrm{Tr}(\Phi^\dagger_{\chi\chi}\Phi_{\chi\chi}) + s^\dagger_\chi s_\chi] + \lambda_1[\mathrm{Tr}(\Phi^\dagger_{\eta\eta}\Phi_{\eta\eta}) \\ & + s^\dagger_\eta s_\eta]^2 + \lambda_2[\mathrm{Tr}(\Phi^\dagger_{\rho\rho}\Phi_{\rho\rho}) + s^\dagger_\rho s_\rho]^2 + \lambda_3[\mathrm{Tr}(\Phi^\dagger_{\chi\chi}\Phi_{\chi\chi}) + s^\dagger_\chi s_\chi]^2 + \lambda_4[\mathrm{Tr}(\Phi^\dagger_{\chi\chi}\Phi_{\chi\chi}) + s^\dagger_\chi s_\chi] \\ & \times [\mathrm{Tr}(\Phi^\dagger_{\eta\eta}\Phi_{\eta\eta}) + s^\dagger_\eta s_\eta] + \lambda_5[\mathrm{Tr}(\Phi^\dagger_{\chi\chi}\Phi_{\chi\chi}) + s^\dagger_\chi s_\chi][\mathrm{Tr}(\Phi^\dagger_{\rho\rho}\Phi_{\rho\rho}) + s^\dagger_\rho s_\rho] + \lambda_6[\mathrm{Tr}(\Phi^\dagger_{\rho\rho}\Phi_{\rho\rho}) + s^\dagger_\rho s_\rho] \\ & \times [\mathrm{Tr}(\Phi^\dagger_{\eta\eta}\Phi_{\eta\eta}) + s^\dagger_\eta s_\eta] + \{\lambda_7[\mathrm{Tr}(\Phi^\dagger_{\chi\chi}\Phi_{\eta\eta}) + s^\dagger_\chi s_\eta] \cdot [\mathrm{Tr}(\Phi^\dagger_{\eta\eta}\Phi_{\chi\chi}) + s^\dagger_\eta s_\chi] + \lambda_8[\mathrm{Tr}(\Phi^\dagger_{\chi\chi}\Phi_{\rho\rho}) \\ & + s^\dagger_\chi s_\rho][\mathrm{Tr}(\Phi^\dagger_{\rho\rho}\Phi_{\chi\chi}) + s^\dagger_\rho s_\chi] + \lambda_9[\mathrm{Tr}(\Phi^\dagger_{\rho\rho}\Phi_{\eta\eta}) + s^\dagger_\rho s_\eta][\mathrm{Tr}(\Phi^\dagger_{\eta\eta}\Phi_{\rho\rho}) + s^\dagger_\eta s_\rho] + \lambda_{10}[\mathrm{Tr}(\Phi^\dagger_{\chi\chi}\Phi_{\eta\eta}) \\ & + s^\dagger_\chi s_\eta][\mathrm{Tr}(\Phi^\dagger_{\rho\rho}\Phi_{\eta\eta}) + s^\dagger_\rho s_\eta] + \text{H.c.}\}. \end{aligned} \tag{68}$$

The full scalar potential in Eq. (67) is invariant under the 3-3-1 symmetry, but the part $V(\Phi_{\varphi\varphi'}, s_\varphi)$ in Eq. (68) is also invariant under $\Phi_{\varphi\varphi'} \to L\Phi_{\varphi\varphi'}$, $\Phi_{\varphi\varphi'} \to \Phi_{\varphi\varphi'}R^\dagger$, and $s_\varphi \to s_\varphi$ i.e., $SU(2)_L \otimes SU(2)_R$ as in the standard electroweak model, when $\sin\theta_W = 0$ ($g' = 0$ in $SU(2)_L \otimes U(1)_Y$ models). Notwithstanding, the trilinear term in Eq. (67), $f(\eta\rho\chi)$, breaks softly this global custodial symmetry. It means that this symmetry is realized only in the limit $f \to 0$. When the Higgs fields acquire vacuum expectation values we have

$$\langle\Phi_{\eta\rho}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_\eta & 0 \\ 0 & v_\rho \end{pmatrix}, \tag{69}$$

breaking $SU(2)_L \otimes SU(2)_R \to SU(2)_{L+R}$, if $v_\eta = v_\rho$ in

Eq. (68). Notice also that $SU(3)_L$ transformations mix the components of the bi-doublets with the singlets, thus breaking also the custodial symmetry. Moreover, the covariant derivative implies the mixing between Z and Z' which breaks explicitly the custodial symmetry unless $\sin\phi = 0$.

This 3-3-1 model has an approximate $SU(2)_{L+R}$ symmetry as in the standard model. This in fact does happen when Eq. (59) (or $\sin\phi = 0$) holds and also $g_X = 0$ (or $\sin\theta_W = 0$) then $v_\eta = v_\rho = v_W/\sqrt{2}$ and we really have the global $SU(2)_{L+R}$ symmetry. This explains why $\sin\phi$ should be small and may be generated by radiactive corrections only. In general, the oblique parameters T and S constraint the mixing angle between Z and Z' [3] with $\rho_0 - 1 \simeq \alpha T$. The T parameter in 3-3-1 models has been calculated in Ref. [28]. Using their expressions but without the mixing between Z and Z' at the tree level (i.e., $\phi = 0$ in Eq. (4.1) of [28]) we obtain, for example, $T = -0.1225$ for $\bar{v}_W = 1$, and $T = -0.012$ for $\bar{v}_W = 0.25$. We have also verified that $T \to 0$ when $v_\chi \to \infty$ and all the values for T with $\bar{v}_W \leq 1$ are within the allowed interval [3]. This implies that the solution in Eq. (59) is not too much disturbed by radiative corrections.

As a consistent verification, we note that if we substitute Eq. (36) in the mass matrix of Eq. (21) we obtain

$$\hat{\mathcal{M}}^2_{(b=\sqrt{3})} = \frac{g^2}{4} v_\chi^2 \begin{pmatrix} \bar{v}_W^2 & \sqrt{3}t_W^2\bar{v}_W^2 & -\frac{t_W h(s_W^2)\bar{v}_W^2}{c_W} \\ \sqrt{3}t_W^2\bar{v}_W^2 & \frac{1}{3}(4+\bar{v}_W^2) & \frac{t_W[4c_W^2 + h^2(s_W^2)\bar{v}_W^2]}{\sqrt{3}c_W h(s_W^2)} \\ -\frac{t_W h(s_W^2)\bar{v}_W^2}{c_W} & \frac{t_W[4c_W^2 + h^2(s_W^2)\bar{v}_W^2]}{\sqrt{3}c_W h(s_W^2)} & \frac{2t_W^2}{h^2(s_W^2)}[2c_W^2 + h^2(s_W^2)\bar{v}_W^2] \end{pmatrix}, \tag{70}$$

where t_W and $h(s_W^2)$ have already been defined in Sec. IV A and V A, respectively. The states $(W_{3\mu}, W_{8\mu}, B_\mu)$ are given in terms of the mass eigenstates (A_μ, Z_μ, Z'_μ), omitting the tilde in the latter fields, as follows

$$\begin{aligned} \tilde{W}^3_\mu &= (s_W, -c_W, 0), \\ \tilde{W}^8_\mu &= \left(-\sqrt{3}s_W, -\sqrt{3}t_W s_W, \frac{h(s_W^2)}{c_W}\right), \\ \tilde{B}_\mu &= (h(s_W^2), t_W h(4s_W^2), \sqrt{3}t_W), \end{aligned} \tag{71}$$

which coincide with the inverse of Eq. (38). Notice that $\tilde{B}_\mu$ is almost Z'_μ, and when $s_W^2 = 1/4$ then $\tilde{B}_\mu = Z'_\mu$. The expressions in Eq. (71) is consistent with those in Eq. (38) after using the equation Eq. (36).

In the limit $\sin\theta_W = 0$ the mass square matrix in Eq. (70) reduces to

$$\tilde{\mathcal{M}}^2_{(b=\sqrt{3})} = \frac{g^2}{4} v_\chi^2 \begin{pmatrix} \bar{v}_W^2 & 0 & 0 \\ 0 & \frac{1}{3}(4+\bar{v}_W^2) & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{72}$$

and we see that, using the relation (59) in Eq. (20), in this limit $M_W = M_{Z_1} \equiv M_1$; $M_U = M_V \equiv M_2$, where we have defined $M_1^2 = g^2 v_W^2/4$ and $M_2^2 = g^2(v_\chi^2 + v_W^2/2)/4$. The photon of course continues massless. These mass relations, valid in the limit $\sin\phi = 0$ and $\sin\theta_W = 0$, are consequences of the custodial $SU(2)_{L+R}$ discussed above. Notice that in this limit, directly from (72), or from (39) with $s_W^2 = 0$, we have $M_{Z'}^2 = g^2(4v_\chi^2 + v_W^2)/12 > M_2^2$. The custodial symmetry appears also in the Yukawa sector as is shown in the Appendix.

These models have also an approximate global $SU(3)_{L+R}$ symmetry which, although badly broken, it may be useful for obtaining an approximate but realistic mass spectra in the scalar sector as can be seen in Appendix A.

VII. CONCLUSIONS AND DISCUSSIONS

Once the v_χ scale is arbitrary when Eq. (36) or (43) are satisfied, we can ask ourselves what about the experimental limit upon the extra particles that appear in the models. After all they depend mainly on v_χ, the scale at which the $SU(3)_L$ symmetry is supposed to be valid. Here we will be concerned only with the minimal model of Sec. II A. Firstly, let us consider the Z' vector boson. It contributes to the ΔM_K at the tree level [22,29]. If this would be the only contribution to this parameter, its experimental measurements constraint the quantity

$$(\mathcal{O}_L^d)_{3d}(\mathcal{O}_L^d)_{3s}\frac{M_Z}{M_{Z'}}, \tag{73}$$

which must be of the order of 10^{-4} to have compatibility with the measured ΔM_K. This can be achieved with $M_{Z'} \sim 4$ TeV if we assume that the mixing matrix have a Fritzsch-structure $\mathcal{O}^d_{Lij} = \sqrt{m_j/m_i}$ [30] or, it is possible that the product of the mixing angles saturates the value 10^{-4} [22,29], in this case Z' it is not too much constrained and may have a mass near the electroweak scale. More important is the fact that there is also in this model FCNC mediated by neutral Higgs scalars which imply new contributions to ΔM_K proportional to

$$(\mathcal{O}_L^d)_{d3}\Gamma^d_{3\beta}(\mathcal{O}_R)_{\beta s}\frac{M_Z}{M_H}, \tag{74}$$

that involves the mass of the scalar M_H, the unknown $\mathcal{O}_R^d$ matrix elements and also the Yukawa coupling Γ^d, so that their contributions to ΔM_K may have opposite sing relative to that of the Z' contribution. Thus, a realistic calculation of the ΔM_K in the context of 3-3-1 models has to take into account these extra scalar contributions as well. Hence, there is not strong constraints on the value the Z' mass in context of 3-3-1 models.

The model has also a doubly charged vector and four doubly charged scalars. Muonium-antimuonium transitions would imply a lower bound of 850 GeV on the mass of the doubly charged gauge bilepton, U_μ^{--} [31]. However this bound depends on assumptions on the mixing matrix in the lepton charged currents coupled to U_μ^{--}, and also it does not take into account that there are in the model doubly charged scalar bileptons which also contribute to that transition [32]. Concerning these doubly charged scalars, model independent lower limits for their masses are of the order of 100 GeV [33]. From fermion pair production at LEP and lepton flavor violating effects suggest a lower bound of 750 GeV for the U_μ^{--} mass, but again it depends on assumptions on the mixing matrix [34]. Other phenomenological analysis in e^+e^-, $e\gamma$, and $\gamma\gamma$ colliders assume bileptons with masses between 500 GeV and 1 TeV [35–37]. The fine structure of muonium only implies $M_U/g >$ 215 GeV [38] but also ignores the contributions of the doubly charged scalars. Concerning the exotic quark masses there is no lower limit for them but if they are in the range of 200–600 GeV they may be discovered at the LHC [39]. Similarly, most of the searches for extra neutral gauge bosons are based on models that do not have the same neutral current couplings as those of the 3-3-1 models [2]. Anyway we have seen that even if $v_\chi = v_W$ the Z'_μ has a mass of the order of 303 GeV. Of course, a value for v_χ of the order of 1 TeV could be safer.

In view of this, we may conclude that there are not yet definitive bounds on the masses of the extra degrees of freedom of the 3-3-1 models. Moreover, the Z'_μ of the minimal 3-3-1 model has interesting features that distinguishes this model from others having also this sort of vector boson, as models with extra dimensions and Little Higgs models. However because of its leptophobic character it is not clear if it could be discovered at the International Linear Collider [40], probably the LHC may be more efficient for searching it.

Finally, some remarks concerning 3-3-1 models in general. (1) The existence of leptophobic neutral vector bosons were proposed in the past to solve what would be anomalies in the weak precision data at LEP, as $R_{b,c}$, see for example [41]. Unlike other sorte of models, the leptophobic boson is a prediction of the 3-3-1 models which already have interesting features. (2) The scalar sector of the latter models has not deserved much attention in literature and we think that phenomenological analysis as that in Ref. [42] should take into account these sort of models. (3) Usually in literature two models are mainly considered. The so called "minimal 3-3-1 model" in which the already known leptons $(\nu_l l^- l^+)_L^T$ transform as $(\mathbf{3}, 0)$ under $SU(3)_L \otimes U(1)_X$ [10], and also the "3-3-1 model with right-handed neutrinos" in which the leptons $(\nu_l l^- \nu_l^c)_L^T$ transform as $(\mathbf{3}, -1/3)$. If there are not right-handed neutrinos in nature the later model should be ruled out. On the other hand, if these neutrinos do really exist it suggests that the larger symmetry among neutral and singly charged leptons could be $SU(4)_L \otimes U(1)_N$, transforming like $(\nu_l l^- \nu_l^c e^+)_L^T \sim (\mathbf{4}, 0)$ [25]. There exist other models which include leptons that are not of the known lepton species but include heavy neutrinos (they have right-handed singlets associated to them) $(\nu_l l^- \nu'_l)_L^T$ [15] or heavy charged leptons $(\nu_l l^- E_l^+)_L^T$ [11] and $(\nu_l l^- E_l^-)_L^T$ [43]. Some of these models have a more economic scalar sector and could serve as a laboratory to explore ideas and mechanism in the context of a 3-3-1 gauge symmetry. For instance, if in a given model with only three triplets, as in the model of Ref. [11], it is possible to implement soft CP violation through three complex VEVs and a complex trilinear term in the scalar potential [16], then it is certain that that mechanism of CP violation will also work in the minimal model which has four complex VEVs and two complex trilinear coupling constants (the opposite is not necessarily true) [10]. We would like to stress that some models do not have the same SM weak isospin attribution [44] and can be phenomenologically ruled out. 4) It is interesting that the minimal model can be embedded in a Pati-Salam-like model with $SU(4)_{\rm PS} \otimes SU(4)_{L+R}$ gauge symmetry [45], where the $SU(3)_L$ subgroup which contains the vector bosons of the SM should be the minimal 3-3-1 model of Sec. II A. This may indicate the route toward a grand unification theory of three family 3-3-1 models.

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APPENDIX: GLOBAL APPROXIMATE $SU(3)_{L+R}$ SYMMETRY

Let us consider the model showed in Sec. II B which has the minimal scalar content: only the three triplets η, ρ, and χ given in Eq. (8). With them we can define the 3-triplet

$$\Phi = \frac{1}{\sqrt{3}}(\rho \chi \eta). \tag{A1}$$

The gauge-covariant derivative is

$$D_\mu \Phi = \partial_\mu \Phi + ig\mathcal{M}_\mu \Phi + ig_X \Phi \hat{X}, \tag{A2}$$

where $\hat{X} = \mathrm{diag}(+1, -1, 0)$ and the matrix $\mathcal{M}_\mu$ is defined in Eq. (18) with $b = \sqrt{3}$.

The scalar Lagrangian is written as

$$\mathcal{L}(\Phi) = \mathrm{Tr}[(D_\mu\Phi)^\dagger D^\mu\Phi] - \mu^2\,\mathrm{Tr}(\Phi^\dagger\Phi) - \bar{\lambda}_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \bar{\lambda}_2\,\mathrm{Tr}(\Phi^\dagger\Phi)^2 - \frac{\sqrt{3}}{2} f\epsilon_{ijk}\epsilon_{mnl}\Phi_{im}\Phi_{jn}\Phi_{kl} + \mathrm{H.c.}, \quad \text{(A3)}$$

which is invariant under global and local $SU(3)_L \otimes U(1)_X$ gauge transformations: respectively

$$\Phi \to L\Phi, \qquad \Phi \to \Phi e^{i\hat{X}\theta}. \quad \text{(A4)}$$

Since it is the $U(1)_X$ charge that distinguishes the triplets η, ρ and χ, in the limit $g_X = 0$, which is the same that $\sin\theta_W = 0$ by Eq. (28), the scalar Lagrangian has an additional $SU(3)_R$ global symmetry under which we have

$$\Phi \to \Phi R^\dagger, \quad \text{(A5)}$$

and we see that in this limit the Higgs sector of the 3-3-1 model has an accidental global symmetry:

$$SU(3)_L \otimes SU(3)_R, \qquad \Phi \to L\Phi R^\dagger, \quad \text{(A6)}$$

where $SU(3)_L$ is the global version of $SU(3)_L$ gauge symmetry, and $SU(3)_R$ is an approximate accidental global symmetry. This symmetry implies in Eq. (68) $\mu_\eta^2 = \mu_\rho^2 = \mu_\chi^2 \equiv \mu^2/3$, and relations between $\lambda_1, \cdots, \lambda_{10}$ in Eq. (68) and $\bar{\lambda}_1$, $\bar{\lambda}_2$ in Eq. (A3).

When the Higgs fields acquire vacuum expectation values we have

$$\langle\Phi\rangle = \frac{1}{\sqrt{6}}\begin{pmatrix} v_\rho & 0 & 0 \\ 0 & v_\chi & 0 \\ 0 & 0 & v_\eta \end{pmatrix}, \quad \text{(A7)}$$

breaking in this way both $SU(3)_L$ and $SU(3)_R$. When the condition in Eq. (59) and $s_W = 0$ are used $v_\eta = v_\rho = v_W/\sqrt{2}$ and we have again the $SU(2)_{L+R}$ global symmetry.

We have verified that the scalar potential defined in Eq. (A3) has the appropriate number of Goldstone bosons and the mass spectra of all the charged and neutral sectors have realistic values. For instance, the physical singly charged scalar fields have square masses given by

$$M_{1^+}^2 = \frac{1}{18}\left[\bar{\lambda}_2(v_\eta^2 + v_\rho^2) - \frac{9f}{\sqrt{2}}\left(\frac{v_\eta}{v_\rho} + \frac{v_\rho}{v_\eta}\right)v_\chi\right],$$
$$M_{2^+}^2 = \frac{1}{18}\left[\bar{\lambda}_2(v_\eta^2 + v_\chi^2) - \frac{9f}{\sqrt{2}}\left(\frac{v_\eta}{v_\chi} + \frac{v_\chi}{v_\eta}\right)v_\rho\right], \quad \text{(A8)}$$

while the double charged scalar has a square mass

$$M_{++}^2 = \frac{1}{18}\left[\bar{\lambda}_2(v_\rho^2 + v_\chi^2) - \frac{9f}{\sqrt{2}}\left(\frac{v_\rho}{v_\chi} + \frac{v_\chi}{v_\rho}\right)v_\eta\right]. \quad \text{(A9)}$$

In the pseudoscalar sector we have

$$M_A^2 = -\frac{f}{\sqrt{2}}\left(\frac{v_\eta v_\rho}{v_\chi} + \frac{v_\eta v_\chi}{v_\rho} + \frac{v_\rho v_\chi}{v_\eta}\right), \quad \text{(A10)}$$

which implies that $f < 0$. The real neutral Higgs scalars have square masses which depend on $\bar{\lambda}_1$ and $\bar{\lambda}_2$ but we will not write them here, given only numerical results.

The constraint equations from the stationary condition of the scalar potential are:

$$3\mu^2 + (\bar{\lambda}_1 + \bar{\lambda}_2)v_\eta^2 + \bar{\lambda}_1(v_\rho^2 + v_\chi^2) + \frac{9}{\sqrt{2}}\frac{f v_\rho v_\chi}{v_\eta} = 0 \;(a),$$
$$3\mu^2 + (\bar{\lambda}_1 + \bar{\lambda}_2)v_\rho^2 + \bar{\lambda}_1(v_\eta^2 + v_\chi^2) + \frac{9}{\sqrt{2}}\frac{f v_\eta v_\chi}{v_\rho} = 0 \;(b),$$
$$3\mu^2 + (\bar{\lambda}_1 + \bar{\lambda}_2)v_\chi^2 + \bar{\lambda}_1(v_\eta^2 + v_\rho^2) + \frac{9}{\sqrt{2}}\frac{f v_\eta v_\rho}{v_\chi} = 0, \;(c). \quad \text{(A11)}$$

These equations should be solved for the parameters $\bar{\lambda}_1$, $\bar{\lambda}_2$, and f, in terms of μ^2 and the VEVs v_η, v_ρ, and v_χ. If the VEVs are left free we get $\bar{\lambda}_1 = -3\mu^2/(v_\eta^2 + v_\rho^2 + v_\chi^2)$, $\bar{\lambda}_2 = 0$ and $f = 0$. Since this is not a realistic scenario, we will impose some constraints on the VEVs. One of the more interesting possibility that we have found is: assuming $\bar{\lambda}_2 = 0$ and $v_\eta = v_\rho = v_\chi \equiv v_W/\sqrt{2}$, in this $\langle\Phi\rangle$ in Eq. (A7) implies a global $SU(3)_{L+R}$ symmetry. It also implies $\bar{\lambda}_1 = -3\sqrt{2}f/(v_W - 4\mu^2/v_W)$ and, using $f = -120$ GeV and $\mu = 80i$ GeV implies $\lambda_1 = 2.49$ (which is within the perturbative regime). With these input parameters we get that all the charged scalar masses are of the order of 102 GeV. The pseudoscalar A has a mass of the order of 144 GeV and in the real neutral scalar sector we obtain an scalar with mass of the order of 121 GeV and two others mass degenerate states with 144 GeV. This is just an illustration, the important point is the fact that with the potential (A3) it is possible to obtain realistic values for the physical scalar masses.

The $SU(3)_{L+R}$ symmetry occurs in the Yukawa sector as well. We can see this by defining, using the quark representation in Eqs. (6) and (7), the 3-triplet $F = (f_1 f_2 f_3)/\sqrt{3}$ with

$$f_1 = Y'_{3\alpha}\bar{Q}_{3L}d_{\alpha R}, \qquad f_2 = Y\bar{Q}_{3L}J_R, \qquad f_3 = Y''_{3\alpha}\bar{Q}_{3L}u_{\alpha R}, \quad \text{(A12)}$$

which transform under $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ as $(\mathbf{1}, \mathbf{3}^*, -1)$, $(\mathbf{1}, \mathbf{3}^*, +1)$ and $(\mathbf{1}, \mathbf{3}^*, 0)$, respectively, and Y, Y' and Y'' are Yukawa couplings. Similarly we define another 3-triplet $G = (g_1 g_2 g_3)/\sqrt{3}$ with

$$g_1 = G_{m\alpha}\bar{Q}_{mL}u_{\alpha R}, \qquad g_2 = G'_{m\alpha}\bar{Q}_{mL}j_{mR}, \qquad g_3 = G''_{m\alpha}\bar{Q}_{mL}d_{\alpha R}, \quad \text{(A13)}$$

transforming as $(\mathbf{1}, \mathbf{3}, +1)$, $(\mathbf{1}, \mathbf{3}, -1)$, and $(\mathbf{1}, \mathbf{3}, 0)$, respectively. With (A12) and (A13) and the 3-triplet Φ defined in Eq. (A1) we can rewrite the usual Yukawa couplings in the following way

$$-\mathcal{L} = \mathrm{Tr}(F\Phi^T) + \mathrm{Tr}(G\Phi^\dagger), \quad \text{(A14)}$$

which is manifestly $SU(3)_{L+R}$ invariant if G and F transform as Φ in Eq. (A6) in the limit $g_X = 0$. Moreover, if we want that the custodial be also manifest in the lepton sector we see that it is mandatory to add right-handed neutrinos. We can then define the 3-triplet, using the leptons in Eqs. (10) and (11), $\psi_{ab} = (\bar{\Psi}_{aL} l_{bR} \bar{\Psi}_{aL} E_{bR} \bar{\Psi}_a \nu_{bR})/\sqrt{3}$, so that the Yukawa interactions can be written as

$$-\mathcal{L} = h_{ab}\,\mathrm{Tr}(\psi_{ab}\Phi^T). \tag{A15}$$

Notice that the Dirac mass of the neutrinos are equal to the masses of the charged leptons. This demand the introduction of Majorana mass terms for the right-handed components, which are singlets of the $SU(3)_{L+R}$, in such a way that we can implement the seesaw mechanism for generating small neutrino masses. This also happens in the SM: it is necessary to add right-handed neutrinos if we want to implement a custodial symmetry in the lepton sector by defining the 2-doublet $D_{ab} = (\bar{L}_a l_{bR} \bar{L}_a \nu_{bR})$. Thus, we can write the Yukawa coupling as $-\mathcal{L} = h_{ab}\,\mathrm{Tr}(D_{ab}\varphi)$, where $\varphi = (H, \tilde{H})$ with H the usual Higgs scalar doublet and $\tilde{H} = \epsilon H^*$.

In the model with the sextet, the sextet is just a symmetrized 3-triplet and we can write down the scalar potential invariant under $SU(3)_L \otimes SU(3)_R$ as in Eq. (A3), using Φ defined in Eq. (A1) and the sextet S. However, notice that, if $\langle\sigma_1^0\rangle \neq 0$ this breaks also the $SU(2)_{L+R}$ global symmetry imposing a strong constraint on this VEV [46]. The model with right-handed neutrinos may be considered in the same way.

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