

Consistency of LEP event excesses with an $h \rightarrow aa$ decay scenario and low-fine-tuning next-to-minimal supersymmetric standard models

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We examine the LEP limits for the $Zh \rightarrow Z + b\bar{b}$ final state and find that the excess of observed events for $m_h \sim 100$ GeV correlates well with there being an $m_h \sim 100$ GeV Higgs boson with SM-like ZZh coupling that decays partly via $h \rightarrow b\bar{b} + \tau^+\tau^-$ [with $B(h \rightarrow b\bar{b}) \sim 0.08$] but dominantly via $h \rightarrow aa$ [with $B(h \rightarrow aa) \sim 0.9$], where $m_a < 2m_b$ so that $a \rightarrow \tau^+\tau^-$ (or light quarks and gluons) decays are dominant. This type of scenario is precisely that predicted in the Next-to-Minimal Supersymmetric Model for parameter choices yielding the lowest possible fine-tuning.

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In the standard model (SM), electroweak symmetry breaking, whereby the W and Z bosons and the quarks and leptons acquire mass, gives rise to a Higgs boson, h_{SM} . Supersymmetric (SUSY) models, such as the Minimal Supersymmetric Model (MSSM), cure the naturalness/hierarchy problem associated with quadratically divergent 1-loop corrections to $m_{h_{\text{SM}}}^2$ present in the SM and predict at least one relatively light Higgs boson. The most natural Higgs mass for SUSY models is closely related to m_Z and lies in the range $\lesssim 105$ GeV, with an upper bound, for example, of $\lesssim 135$ GeV in the MSSM for SUSY-breaking scale $m_{\text{SUSY}} \lesssim 1$ TeV. Understanding the constraints on a light Higgs boson, generically h , and how to search for it at colliders are crucial issues for the progress of high energy physics.

Precision W and Z measurements at LEP, combined with the known top-quark mass, prefer an h with SM-like WW , ZZ couplings and $m_h \sim 100$ GeV. However, the SM and the MSSM predict that $h \rightarrow b\bar{b}$ decays are dominant and LEP has placed strong constraints on $Zh \rightarrow Zb\bar{b}$. The limits on $C_{\text{eff}}^{2b} = [g_{ZZh}^2/g_{ZZh_{\text{SM}}}^2]B(h \rightarrow b\bar{b})$ are shown in Fig. 1 [1]. From this plot, one concludes that $m_h < 114$ GeV is excluded for a SM-like h that decays primarily to $b\bar{b}$. For $m_{\text{SUSY}} \lesssim 1$ TeV, most of CP-conserving MSSM parameter space is ruled out by this LEP limit. The remaining part of MSSM parameter space is very fine-tuned (see later definition).

As we shall discuss, the Next-to-Minimal Supersymmetric Model (NMSSM), which has a more complicated Higgs sector, can evade the LEP limit by virtue of extra h decays and yields a preferred value of $m_h \sim 100$ GeV purely on the basis of minimizing fine-tuning. This value of m_h agrees perfectly with both the preferred precision electroweak value and the location of the well-known excess of observed vs expected C_{eff}^{2b} limits for a test Higgs mass of $m_h \sim 100$ GeV, apparent in Fig. 1. This excess is particularly apparent in the $1 - CL_b$ result (Fig. 7 of [1]) obtained after combining all four LEP experiments.

Various interpretations of this excess in terms of a non-SM Higgs sector have been suggested [2,3]. In this letter, we point out that this excess is consistent with a scenario in which the Higgs boson has SM-like ZZh coupling, but has reduced $B(h \rightarrow b\bar{b})$ by virtue of the presence of h decays to a pair of lighter Higgs bosons, $h \rightarrow aa$, where $B(a \rightarrow b\bar{b})$ is small, as is automatic if $m_a < 2m_b$ so that $a \rightarrow \tau^+\tau^-$ or light quarks and gluons [4]. In more detail, if the ZZh coupling is full SM strength, then $m_h \sim 100$ GeV with $B(h \rightarrow b\bar{b}) \sim 0.08$ and $B(h \rightarrow aa) \sim 0.9$ fits the observed $Z2b$ excess nicely. Meanwhile, there are no current limits on the $Zh \rightarrow Zaa \rightarrow Z\tau^+\tau^-\tau^+\tau^-$ final state for $m_h \geq 87$ GeV [2]. As already stressed and as described below in more detail, we are particularly led to the above interpretation of LEP data since fine-tuning within the NMSSM is absent for model parameters that yield precisely this kind of scenario [6].

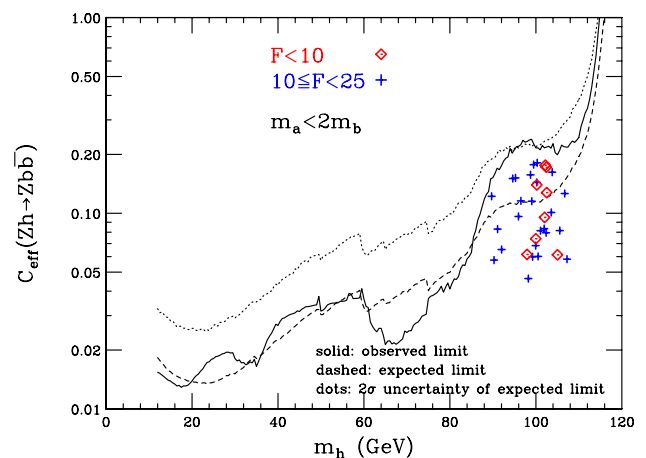


FIG. 1 (color online). Expected and observed 95% CL limits on $C_{\text{eff}}^{2b} = [g_{ZZh}^2/g_{ZZh_{\text{SM}}}^2]B(h \rightarrow b\bar{b})$ from Ref. [1] are shown vs m_h . Also plotted are the predictions for NMSSM parameter choices in our fixed $\tan\beta = 10$, $M_{1,2,3}(m_Z) = 100, 200, 300$ GeV scan that give fine-tuning measure $F < 25$ and $m_{a_1} < 2m_b$ and that are consistent with the preliminary LHWG analysis code [5].

The NMSSM is an extremely attractive model [7]. First, it provides a very elegant solution to the μ problem of the MSSM via the introduction of a singlet superfield \hat{S} . For the simplest possible scale invariant form of the superpotential, the scalar component of \hat{S} naturally acquires a vacuum expectation value of the order of the SUSY-breaking scale, giving rise to a value of μ of order the electroweak scale. The NMSSM is the simplest supersymmetric extension of the standard model in which the electroweak scale originates from the SUSY-breaking scale only. Hence, the NMSSM deserves very serious consideration.

Apart from the usual quark and lepton Yukawa couplings, the scale invariant superpotential of the NMSSM is $W = \lambda \hat{S} \hat{H}_u \hat{H}_d + \frac{\kappa}{3} \hat{S}^3$ depending on two dimensionless couplings λ , κ beyond the MSSM. [Hatted (unhatted) capital letters denote superfields (scalar superfield components).] The associated trilinear soft terms are $\lambda A_\lambda S H_u H_d + \frac{\kappa}{3} A_\kappa S^3$. The final two input parameters are $\tan\beta = h_u/h_d$ and $\mu_{\text{eff}} = \lambda s$, where $h_u \equiv \langle H_u \rangle$, $h_d \equiv \langle H_d \rangle$ and $s \equiv \langle S \rangle$. The Higgs sector of the NMSSM is thus described by the six parameters λ , κ , A_λ , A_κ , $\tan\beta$, μ_{eff} . In addition, values must be input for the gaugino masses and for the soft terms related to the (third generation) squarks and sleptons that contribute to the radiative corrections in the Higgs sector and to the Higgs decay widths.

The particle content of the NMSSM differs from the MSSM by the addition of one CP-even and one CP-odd state in the neutral Higgs sector (assuming CP conservation), and one additional neutralino. The result is three CP-even Higgs bosons ($h_{1,2,3}$) two CP-odd Higgs bosons ($a_{1,2}$) and a total of five neutralinos $\tilde{\chi}_{1,2,3,4,5}^0$. The NMHDECAY program [8], which includes most LEP constraints, allows easy exploration of Higgs phenomenology in the NMSSM.

In [6], we found that the NMSSM can avoid the fine-tuning and hierarchy problems of the MSSM. Defining the fine-tuning measure to be

$$F = \max_p F_p \equiv \max_p \left| \frac{d \log m_Z}{d \log p} \right|, \quad (1)$$

where the parameters p comprise all GUT-scale soft-SUSY-breaking parameters, we found that $F < 10$ could be achieved. In fact, it is very remarkable that in an unbiased (i.e. before applying experimental constraints on the Higgs boson sector) scan over the part of parameter space consistent with experimental bounds on gluino and squark masses, we find a clear minimum for F of $F \sim 6$ precisely at $m_{h_1} \simeq 100$ GeV for $\tan\beta = 10$, where h_1 is very SM-like as regards its gauge and fermionic couplings. (For large $\tan\beta$, this minimum increases by about 2 GeV, while for $\tan\beta = 3$ the minimum is at ~ 90 GeV.) A significant fraction of the very lowest F scenarios are such that h_1 decays primarily into a pair of the lightest CP-odd Higgs bosons of the model, $h_1 \rightarrow a_1 a_1$. (The importance of such decays was first emphasized in [9],

and later in [10], followed by extensive work in [11–13].) And, for a large fraction of the $h_1 \rightarrow a_1 a_1$ scenarios with lowest F , the a_1 is mostly singlet in nature and $m_{a_1} < 2m_b$, implying that $a_1 \rightarrow \tau^+ \tau^-$ (or $q\bar{q} + gg$ if $m_{a_1} < 2m_\tau$) thereby allowing consistency with LEP constraints and, in many cases, the LEP excess in the $h_1 \rightarrow b\bar{b}$ channel for Higgs mass of order 100 GeV.

We note that a light a_1 is natural in the NMSSM in the κA_κ , $\lambda A_\lambda \rightarrow 0$ limit. This can be understood as a consequence of a global $U(1)_R$ symmetry of the scalar potential (in the limit κA_κ , $\lambda A_\lambda \rightarrow 0$) which is spontaneously broken by the vevs, resulting in a Nambu-Goldstone boson in the spectrum [10]. This symmetry is explicitly broken by the trilinear soft terms so that for small κA_κ , λA_λ the lightest CP-odd Higgs boson is naturally much lighter than other Higgs bosons. For the $F < 10$ scenarios, $\lambda(m_Z) \sim 0.15 \div 0.25$, $\kappa(m_Z) \sim 0.15 \div 0.3$, $|A_\kappa(m_Z)| < 4$ GeV and $|A_\lambda(m_Z)| < 200$ GeV, implying small κA_κ and moderate λA_λ . The effect of λA_λ on m_{a_1} is further suppressed when the a_1 is largely singlet in nature. Therefore, a light mostly singlet a_1 is very natural in the NMSSM. We note that the above magnitudes for the κ , λ , A_κ and A_λ parameters are very natural in many SUSY-breaking scenarios that might yield the NMSSM as an effective theory below the unification scale. In particular, small $A_\kappa(m_Z)$ and moderate $A_\lambda(m_Z)$ are natural from the renormalization group equations assuming small values for both at the GUT scale, and the above $\lambda(m_Z)$ values are such that λ will remain perturbative when evolved up to the GUT scale.

We will now discuss in more detail results for the NMSSM using the representative fixed values of $\tan\beta = 10$ and $M_{1,2,3}(m_Z) = 100, 200, 300$ GeV while varying all other model parameters. Similar results are obtained for other choices of $\tan\beta$ and $M_{1,2,3}(m_Z)$. The points plotted for the NMSSM in Fig. 1 show the C_{eff}^{2b} predictions for all parameter choices in our scan that had $F < 25$ and $m_{a_1} < 2m_b$ and that are consistent with the experimental and theoretical constraints built into NMHDECAY as well as with limits from the preliminary LHWG full analysis code [5]. The eight $F < 10$ points are singled out. From Fig. 1 we see that these latter points cluster near $m_{h_1} \sim 98 \div 105$ GeV (see also Fig. 3 of [6]). We will see that most are such that m_{h_1} and $B(h_1 \rightarrow b\bar{b})$ are appropriate for explaining the C_{eff}^{2b} excess. The other primary h_1 decay mode for all the plotted points is $h_1 \rightarrow a_1 a_1$ with $a_1 \rightarrow \tau^+ \tau^-$ or light quarks and gluons (when $m_{a_1} < 2m_\tau$). In Table I, we give the precise masses and branching ratios of the h_1 and a_1 for all the $F < 10$ points. We also give the number of standard deviations, n_{obs} (n_{exp}), by which the observed rate (expected rate obtained for the predicted signal + background) exceeds the predicted background. These are derived from $(1 - CL_b)_{\text{observed}}$ and $(1 - CL_b)_{\text{expected}}$ using the usual tables: e.g. $(1 - CL_b) = 0.32, 0.045, 0.0027$ correspond to $1\sigma, 2\sigma, 3\sigma$ excesses, respectively. The quantity $s95$ is the factor by which the

TABLE I. Some properties of the h_1 and a_1 for the eight allowed points with $F < 10$ and $m_{a_1} < 2m_b$ from our $\tan\beta = 10$, $M_{1,2,3}(m_Z) = 100, 200, 300$ GeV NMSSM scan. The n_{obs} , n_{exp} and $s95$ values are obtained after full processing of all Zh final states using the preliminary LHWG analysis code (thanks to P. Bechtle). See text for details. $N_{\text{SD}}^{\text{LHC}}$ is the statistical significance of the best standard LHC Higgs detection channel for integrated luminosity of $L = 300 \text{ fb}^{-1}$.

m_{h_1}/m_{a_1} (GeV)	Branching Ratios			$n_{\text{obs}}/n_{\text{exp}}$ units of 1σ	$s95$	$N_{\text{SD}}^{\text{LHC}}$
	$h_1 \rightarrow b\bar{b}$	$h_1 \rightarrow a_1 a_1$	$a_1 \rightarrow \tau\bar{\tau}$			
98.0/2.6	0.062	0.926	0.000	2.25/1.72	2.79	1.2
100.0/9.3	0.075	0.910	0.852	1.98/1.88	2.40	1.5
100.2/3.1	0.141	0.832	0.000	2.26/2.78	1.31	2.5
102.0/7.3	0.095	0.887	0.923	1.44/2.08	1.58	1.6
102.2/3.6	0.177	0.789	0.814	1.80/3.12	1.03	3.3
102.4/9.0	0.173	0.793	0.875	1.79/3.03	1.07	3.6
102.5/5.4	0.128	0.848	0.938	1.64/2.46	1.24	2.4
105.0/5.3	0.062	0.926	0.938	1.11/1.52	2.74	1.2

signal predicted in a given case would have to be multiplied in order to exceed the 95% CL limit. All these quantities are obtained by processing each scenario through the full preliminary LHWG confidence level/likelihood analysis. If n_{exp} is larger than n_{obs} then the excess predicted by the signal plus background Monte Carlo is larger than the excess actually observed and vice versa. The points with $m_{h_1} \lesssim 100$ GeV have the largest n_{obs} . Point 2 gives the best consistency between n_{obs} and n_{exp} , with a predicted excess only slightly smaller than that observed. Points 1 and 3 also show substantial consistency. For the 4th and 7th points, the predicted excess is only modestly larger (roughly within 1σ) compared to that observed. The 5th and 6th points are very close to the 95% CL borderline and have a predicted signal that is significantly larger than the excess observed. LEP is not very sensitive to point 8. Thus, a significant fraction of the $F < 10$ points are very consistent with the observed event excess.

We wish to emphasize that in our scan there are many, many points that satisfy all constraints and have $m_{a_1} < 2m_b$. The remarkable result is that those with $F < 10$ have a substantial probability that they predict the Higgs boson properties that would imply a LEP $Zh \rightarrow Z + b$'s excess of the sort seen. The $F < 10$ points with m_{a_1} substantially above $2m_b$ all predict a net $Z + b$'s signal that is ruled out at better than 99% CL by LEP data. Indeed, all such $F < 25$ points have a net $h \rightarrow b$'s branching ratio, $B(h_1 \rightarrow b\bar{b}) + B(h_1 \rightarrow a_1 a_1 \rightarrow b\bar{b}b\bar{b}) \gtrsim 0.85$, which is too large for LEP consistency.

An important question is the extent to which the type of $h \rightarrow aa$ Higgs scenario (whether NMSSM or other) described here can be explored at the Tevatron, the LHC and a future e^+e^- linear collider. This has been examined in the case of the NMSSM in [9,11,13], with the conclusion that observation of any of the NMSSM Higgs bosons may be difficult at hadron colliders. At a naive level, the $h_1 \rightarrow a_1 a_1$ decay mode renders inadequate the usual Higgs

search modes that might allow h_1 discovery at the LHC. Since the other NMSSM Higgs bosons are rather heavy and have couplings to b quarks that are not greatly enhanced, they too cannot be detected at the LHC. The last column of Table I shows the statistical significance of the most significant signal for *any* of the NMSSM Higgs bosons in the "standard" SM/MSSM search channels for the eight $F < 10$ NMSSM parameter choices. For the h_1 and a_1 , the most important detection channels are $h_1 \rightarrow \gamma\gamma$, $Wh_1 + t\bar{t}h_1 \rightarrow \gamma\gamma\ell^\pm X$, $t\bar{t}h_1$, $t\bar{t}a_1 \rightarrow t\bar{t}\gamma\gamma$, $t\bar{t}h_1 \rightarrow t\bar{t}b\bar{b}$, $t\bar{t}a_1 \rightarrow t\bar{t}\tau^+\tau^-$ and $WW \rightarrow h_1 \rightarrow \tau^+\tau^-$ -see [13]. Even after $L = 300 \text{ fb}^{-1}$ of accumulated luminosity, the typical maximal signal strength is at best 3.5σ . For the eight points of Table I, this largest signal derives from the $Wh_1 + t\bar{t}h_1 \rightarrow \gamma\gamma\ell^\pm X$ channel. There is a clear need to develop detection modes sensitive to the dominant $h_1 \rightarrow a_1 a_1 \rightarrow \tau^+\tau^-\tau^+\tau^-$ decay channel.

Let us consider the possibilities. Two detection modes that can be considered are $gg \rightarrow t\bar{t}h_1 \rightarrow t\bar{t}a_1 a_1 \rightarrow t\bar{t}4\tau$ and $WW \rightarrow h_1 \rightarrow a_1 a_1 \rightarrow 4\tau$. Next, recall that the $\tilde{\chi}_2^0 \rightarrow h_1 \tilde{\chi}_1^0$ channel provides a signal in the MSSM when $h_1 \rightarrow b\bar{b}$ decays are dominant. See, for example, [14]. It has not been studied for $h_1 \rightarrow a_1 a_1 \rightarrow 4\tau$ decays. If a light $\tilde{\chi}_1^0$ provides the dark matter of the universe (as possible because of the $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow a_1 \rightarrow X$ annihilation channels for a light a_1 , see [15,16] and references therein), the $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$ mass difference might be large enough to allow such decays. Diffractive production [17], $pp \rightarrow pp h_1 \rightarrow pp X$, where the mass M_X can be reconstructed with roughly a 1–2 GeV resolution, can potentially reveal a Higgs peak, independent of the decay of the Higgs. A study [18] is underway to see if this discovery mode works for the $h_1 \rightarrow a_1 a_1 \rightarrow 4\tau$ decay mode as well as it appears to work for the simpler SM $h_{\text{SM}} \rightarrow b\bar{b}$ case. The main issue may be whether events can be triggered despite the soft nature of the decay products of the τ 's present in X when $h_1 \rightarrow a_1 a_1 \rightarrow 4\tau$ as compared to $h_{\text{SM}} \rightarrow b\bar{b}$. We note that

SUSY particle masses are modest in size for the NMSSM scenarios with low F , implying that even if the light Higgs boson is present but not directly detected, the LHC *would* observe numerous supersymmetry signals and *would* confirm that $WW \rightarrow WW$ scattering is perturbative.

At the Tevatron it is possible that Zh_1 and Wh_1 production, with $h_1 \rightarrow a_1 a_1 \rightarrow 4\tau$, will provide the most favorable channels. If backgrounds are small, one must simply accumulate enough events. However, efficiencies for triggering on and isolating the 4τ final state will not be large. Perhaps one could also consider $gg \rightarrow h_1 \rightarrow a_1 a_1 \rightarrow 4\tau$ which would have substantially larger rate. Studies are needed. If supersymmetry is detected at the Tevatron, but no Higgs is seen, and if LHC discovery of the h_1 remains uncertain, Tevatron studies of the 4τ final state might be essential. However, rates imply that the h_1 signal could only be seen if Tevatron running is extended until $L > 10 \text{ fb}^{-1}$ has been accumulated.

Of course, discovery of the h_1 will be straightforward at an e^+e^- linear collider via the inclusive $Zh \rightarrow \ell^+ \ell^- X$ reconstructed M_X approach (which allows Higgs discovery independent of the Higgs decay mode). Direct detection in the $Zh \rightarrow Z4\tau$ mode will also be possible. At a $\gamma\gamma$ collider, the $\gamma\gamma \rightarrow h \rightarrow 4\tau$ signal will be easily seen [19].

In contrast, since (as already noted) the a_1 in these low- F NMSSM scenarios is fairly singlet in nature, its *direct* (i.e. not in h_1 decays) detection will be very challenging even at the ILC. Further, the low- F points are all such that the other Higgs bosons are fairly heavy, typically above 400 GeV in mass, and essentially inaccessible at both the LHC and all but a ≥ 1 TeV ILC.

We should note that much of the discussion above regarding Higgs discovery is quite generic. Whether the

a is truly the NMSSM CP-odd a_1 or just a lighter Higgs boson into which the SM-like h pair-decays, hadron collider detection of the h in its $h \rightarrow aa$ decay mode will be very challenging—only an e^+e^- linear collider can currently guarantee its discovery.

In conclusion, we reemphasize that the LEP event excess in the $Z + b$'s channel for reconstructed Higgs mass of $m_h \sim 100$ GeV is consistent with a scenario in which the ZZh coupling is SM-like but the h decays mainly via $h \rightarrow aa \rightarrow \tau^+ \tau^- \tau^+ \tau^-$ (for $2m_\tau < m_a < 2m_b$) or 4 jets (for $m_a < 2m_\tau$), leaving an appropriately reduced rate for $h \rightarrow b\bar{b}$. We strongly encourage the LEP groups to push the analysis of the $Z4\tau$ channel in the hope of either ruling out the $h \rightarrow aa \rightarrow 4\tau$ scenario, (for m_h above the current 87 GeV limit of their analysis), or finding an excess consistent with it. Either a positive or negative result would have very important implications for Higgs searches at the Tevatron and LHC. Further, we have emphasized that the NMSSM models with the smallest fine-tuning typically predict precisely the above scenario with $h = h_1$ and $a = a_1$. We speculate that similar results could emerge in other supersymmetric models with a Higgs sector that is more complicated than that of the CP-conserving MSSM.

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- [1] R. Barate *et al.* (LEP Working Group for Higgs boson searches), Phys. Lett. B **565**, 61 (2003).
 - [2] LEP Working Group for Higgs Boson Searches, Search for Neutral MSSM Higgs Bosons at LEP, Report No. LHWG-Note 2005-01 (unpublished).
 - [3] M. Drees, Phys. Rev. D **71**, 115006 (2005). See also references therein to earlier ideas.
 - [4] If $a \rightarrow b\bar{b}$ is dominant, as occurs for $m_a > 2m_b$, then $m_h \geq 110$ GeV is required by LEP data. This is because the combined $Z2b$ and $Z4b$ rates give too large a net $Z + b$'s event rate [5].
 - [5] We thank P. Bechtle for processing our low- F points through the full preliminary LHWG analysis package. In an earlier version of this paper, the $m_a > 2m_b$ points were treated as allowed at 95% CL since they are not excluded when using the less sensitive model-independent limits on the $Z2b$ and $Z4b$ channels independently.
 - [6] R. Dermisek and J. F. Gunion, Phys. Rev. Lett. **95**, 041801 (2005).
 - [7] H. P. Nilles, M. Srednicki, and D. Wyler, Phys. Lett. B **120**, 346 (1983); J. M. Frere, D. R. T. Jones, and S. Raby, Nucl. Phys. **B222**, 11 (1983); J. P. Derendinger and C. A. Savoy, Nucl. Phys. **B237**, 307 (1984); J. R. Ellis, J. F. Gunion, H. E. Haber, L. Roszkowski, and F. Zwirner, Phys. Rev. D **39**, 844 (1989); M. Drees, Int. J. Mod. Phys. A **4**, 3635 (1989); U. Ellwanger, M. Rausch de Traubenberg, and C. A. Savoy, Phys. Lett. B **315**, 331 (1993); Nucl. Phys. **B492**, 21 (1997); S. F. King and P. L. White, Phys. Rev. D **52**, 4183 (1995); F. Franke and H. Fraas, Int. J. Mod. Phys. A **12**, 479 (1997).
 - [8] U. Ellwanger, J. F. Gunion, and C. Hugonie, J. High Energy Phys. 05 (2005) 066.
 - [9] J. F. Gunion, H. E. Haber, and T. Moroi, hep-ph/9610337.
 - [10] B. A. Dobrescu, G. Landsberg, and K. T. Matchev, Phys. Rev. D **63**, 075003 (2001); B. A. Dobrescu and K. T. Matchev, J. High Energy Phys. 09 (2000) 031.
 - [11] U. Ellwanger, J. F. Gunion, and C. Hugonie, hep-ph/0111179; U. Ellwanger, J. F. Gunion, C. Hugonie, and S.

- Moretti, hep-ph/0305109; hep-ph/0401228.
- [12] D.J. Miller and S. Moretti, hep-ph/0403137.
- [13] U. Ellwanger, J.F. Gunion, and C. Hugonie, J. High Energy Phys. **07** (2005) 041.
- [14] N. Marinelli (CMS Collaboration), Report No. CMS-CR-2004-037 (unpublished).
- [15] J.F. Gunion, D. Hooper, and B. McElrath, Phys. Rev. D **73**, 015011 (2006).
- [16] G. Belanger, F. Boudjema, C. Hugonie, A. Pukhov, and A. Semenov, J. Cosmol. Astropart. Phys. **09** (2005) 001.
- [17] A.D. Martin, V.A. Khoze, and M.G. Ryskin, hep-ph/0507305; J.R. Forshaw, hep-ph/0508274; A.B. Kaidalov, V.A. Khoze, A.D. Martin, and M.G. Ryskin, Eur. Phys. J. C **33**, 261 (2004).
- [18] J.F. Gunion, V. Khoze, A. deRoeck, and M. Ryskin (work in progress).
- [19] J.F. Gunion and M. Szleper, hep-ph/0409208.