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Holography and D3-branes in Melvin universes

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Recently, Hashimoto and Thomas [J. High Energy Phys., 01 (**2006**) 083] found that noncommutative super Yang-Mills (NCSYM) theory with space-dependent noncommutativity can be formulated as a decoupling limit of open strings ending on D3-branes wrapping a Melvin universe supported by a flux of the NSNS B-field. Under S-duality, we show that this theory turns into a noncommutative open string (NCOS) theory with space-dependent space-time noncommutativity and effective space-dependent string scale. It is an NCOS theory with both space-dependent space-space and space-time noncommutativities under more general $SL(2, \mathbb{Z})$ transformation. These space-dependent noncommutative theories (NCSYM and NCOS) have completely the same thermodynamics as that of ordinary super YM theory, NCSYM and NCOS theories with constant noncommutativity in the dual supergravity description. Starting from black D3-brane solution in the Melvin universe and making a Lorentz boost along one of spatial directions on the worldvolume of D3-branes, we show that the decoupled theory is a lightlike NCSYM theory with space-dependent noncommutativity in a static frame or in an infinite-momentum frame depending on whether there is a gravitational pp-wave on the worldvolume of the D3-branes.

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I. INTRODUCTION

Over the past decade one of the most important progresses in string/M theories is that some super Yang-Mills (SYM) theories can be naturally realized as a decoupling limit of open strings ending on D-branes and they have dual supergravity descriptions [1–4]. As a concrete example, $\mathcal{N} = 4$ SYM theory in four dimensions is the decoupling limit of open string theory on D3-branes, and its dual supergravity geometry is $AdS_5 \times S^5$, which is near horizon geometry of D3-brane solution in Type IIB supergravity. Furthermore, thermodynamics of black D3-branes in the decoupling limit can be identified with that of the $\mathcal{N} = 4$ SYM theory [5,6].

The $\mathcal{N} = 4$ SYM theory can be deformed in various manners. One of them is the one by adding a dimension 6 operator [7], its bosonic part is

$$\mathcal{O}_{\mu\nu} = \frac{1}{2g_{\rm YM}^2}\,\mathrm{Tr}\Big(F_{\mu\sigma}F^{\sigma\rho}F_{\rho\nu} - F_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} + 2F_{\mu\rho}\Sigma_{i=1}^{6}\partial_\nu\phi^i\partial^\rho\phi^i - \frac{1}{2}F_{\mu\nu}\Sigma_{i=1}^{6}\partial_\rho\phi^i\partial^\rho\phi^i\Big), \tag{1.1}$$

where $g_{\rm YM}$ is the SYM coupling constant, $F_{\mu\nu}$ is the $U(N)$ field strength and ϕ^i ($i = 1, 2, \cdots, 6$) are the adjoint scalars. This deformed theory can be extended to a complete theory, noncommutative SYM theory. From the point of view of open string theory on the D3-branes, the deformed theory corresponds to the decoupling limit of open string theory on the D3-branes with nonzero NSNS B-field. Indeed, four-dimensional NCSYM theory with a constant space-space noncommutativity parameter can be realized as a decoupling limit of open strings ending on D3-branes with a constant spacelike NSNS B-field (or constant magnetic field) on the worldvolume [8]. Its dual supergravity description is the near horizon geometry of (D3, D1) bound states [9]. Under S-duality, the bound state (D3, D1) turns out to be (D3, F1) bound state. It was found that under the S-duality, the four-dimensional NCSYM theory with a constant space-space noncommutativity changes to noncommutative open string (NCOS) theory with a constant time-space noncommutativity parameter, rather than an NCSYM theory with a time-space noncommutativity [10]. This NCOS theory is a decoupling limit of open strings ending on D3-branes with a constant timelike NSNS B-field (or constant electric field) on the worldvolume. Note that there is an $SL(2, \mathbb{Z})$ symmetry in type IIB string theory. One can have more general NCOS theory with constant time-space and space-space noncommutativity parameters, and its dual supergravity description is the geometry of the bound state (D3, (D1, F1)) solutions [11].

While NCSYM theories with space-space noncommutativity is a well-defined theory, an NCSYM theory with a time-space noncommutativity will violate the unitarity [12]. However, it is interesting to note that an NCSYM theory with lightlike noncommutativity is well-defined and

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obeys the unitarity [13]. Supergravity descriptions dual to lightlike NCSYM theories are investigated in [14,15]. Note that the background, on which NCSYM and NCOS theories reside, is the flat Minkowski spacetime, and the noncommutativity parameters are constant. One can consider D-branes in time-dependent spacetime, and obtain NCSYM theories in time-dependent background [16,17]. In this case, after an S-duality, the resulting theory is an NCOS theory also in a time-dependent background but now with time-dependent time-space noncommutativity parameter and time-dependent string scale [18].

More recently, considering D3-branes wrapping a Melvin universe supported by a flux of the NSNS B-field, Hashimoto and Thomas [19] have shown that the resulting theory on the worldvolume in a decoupling limit is a four-dimensional NCSYM theory with a space-dependent noncommutativity parameter and the supersymmetry is completely broken. Along the same line, some generalizations to other dimensions and in M theory have been investigated in [20]. These works widen the family of nonlocal field theories and provide more examples to realize the gravity/gauge field correspondence.

In this note we will further study D3-branes in Melvin universes, and generalize the works [19,20] in some aspects. In the next section we will first obtain black D3-brane solutions in Melvin universes and study its thermodynamics, and find that in the dual supergravity description, the thermodynamics of NCSYM with space-dependent noncommutativity is completely identical to that of NCSYM theory with constant noncommutativity and of ordinary SYM theory. In Sec. III we study the S-duality of the NCSYM theory and find that the resulting theory is an NCOS theory with space-dependent time-space noncommutativity and space-dependent string scale. More general $SL(2,\mathbb{Z})$ transformation gives an NCOS theory with both space-dependent time-space and space-space noncommutativities. In Sec. IV we make a Lorentz boost along one of worldvolume directions of D3-branes in Melvin universe, and obtain a lightlike NCSYM theory with a space-dependent lightlike noncommutativity parameter. We end this paper with our conclusion in Sec. V.

II. NCSYM AND BLACK D3-BRANES IN MELVIN UNIVERSE

To construct the black D3-brane solution in the Melvin universe, we follow the steps given by Hashimoto and Thomas [19]: (i) starting from ordinary black D3-brane solution in an asymptotically flat spacetime, and making a T-duality along one of spatial directions, say z, on the worldvolume of D3-branes (The other two spatial coordinates denoted by x and y, or r and φ), one obtains the black D2-brane solution with a smeared transverse coordinate z; (ii) Making a twist, $d\varphi \to d\varphi + \eta dz$ (where η is a constant), and T-dualizing along the coordinate z, we get the black D3-brane solution in a Melvin universe[1]

$$\begin{aligned} ds^2 &= H^{-1/2}[-fdt^2 + dr^2 + h(r^2 d\varphi^2 + dz^2)] \\ &\quad + H^{1/2}(f^{-1}d\rho^2 + \rho^2 d\Omega_5^2), \\ e^{2\phi} &= g_s^2 h, \\ 2\pi\alpha' B &= \frac{r^2\eta}{H + r^2\eta^2} d\varphi \wedge dz, \\ A_2 &= g_s^{-1}H^{-1}\eta r \coth\alpha dt \wedge dr, \\ A_4 &= g_s^{-1}H^{-1} rh \coth\alpha dt \wedge dr \wedge d\varphi \wedge dz, \end{aligned} \tag{2.1}$$

where g_s is the closed string coupling constant at infinity,

$$f = 1 - \frac{\rho_0^4}{\rho^4}, \qquad H = 1 + \frac{\rho_0^4 \sinh^2\alpha}{\rho^4}, \qquad h = \frac{H}{H + \eta^2 r^2}, \tag{2.2}$$

ρ_0 is the Schwarzschild parameter and α is the D3-brane charge parameter. When $\rho_0 \to 0$, $\alpha \to \infty$, but keeping $\rho_0^4 \sinh\alpha \cosh\alpha = \text{const.} = 4\pi g_s N \alpha'^2$ with N being the number of D3-branes, we obtain D3-brane solution in a Melvin universe.

The black D3-brane solution (2.1) has a horizon at $\rho = \rho_0$. The associated Hawking temperature and entropy are

$$T = \frac{1}{\pi\rho_0 \cosh\alpha}, \qquad S = \frac{\pi^3 V_3}{4G_{10}} \rho_0^5 \cosh\alpha, \tag{2.3}$$

respectively, where $G_{10} = 2^3\pi^6 g_s^2 \alpha'^4$ is the Newtonian constant in ten dimensions and V_3 is the spatial volume of D3-brane worldvolume. The solution (2.1) is not asymptotically flat. The usual approach does not work for getting the mass of the solution. Taking the case with $\rho_0 = 0$ as a reference background, we obtain the mass of the solution by using the background subtraction approach

$$M = \frac{5\pi^2\rho_0^4 V_3}{16G_{10}}\left(1 + \frac{4}{5}\sinh^2\alpha\right). \tag{2.4}$$

We note that these thermodynamic quantities are completely the same as those of black D3-brane solutions with/without NSNS B-field [9].

To discuss the low-energy decoupling limit of open string theory in the background of the D3-branes in the Melvin universe (2.1), a good starting point is still the Seiberg-Witten relation connecting the open string and closed string moduli [21]

[1]Here $F_5 = *F_5$ should be understood.

$$\begin{aligned} G_{ij} &= g_{ij} - (2\pi\alpha')^2(Bg^{-1}B)_{ij}, \\ \Theta^{ij} &= 2\pi\alpha'\Big(\frac{1}{g+2\pi\alpha' B}\Big)^{ij}_A, \\ G^{ij} &= \Big(\frac{1}{g+2\pi\alpha' B}\Big)^{ij}_S, \\ G_s &= g_s\Big(\frac{\det G_{ij}}{\det(g_{ij}+2\pi\alpha' B_{ij})}\Big)^{1/2}, \end{aligned} \tag{2.5}$$

where $()_A$ and $()_S$ stand for the antisymmetric and symmetric parts, respectively. The open string moduli occur in the disk correlators on the open string worksheet boundaries

$$\langle X^i(\tau)X^j(0)\rangle = -\alpha' G^{ij}\ln(\tau)^2 + \frac{i}{2}\Theta^{ij}\epsilon(\tau), \tag{2.6}$$

where $\epsilon(\tau)$ is a function that is 1 or -1 for positive or negative τ. For the black D3-brane solution (2.1) in the limit of the Melvin universe, that is, $H = f = 1$, we can examine the open string moduli by using Seiberg-Witten relation (2.5). We find that the open string metric G^{ij} is given by

$$G_{ij}dx^idx^j = -dt^2 + dr^2 + r^2d\varphi^2 + dz^2, \tag{2.7}$$

the nonvanishing noncommutativity parameter is

$$\Theta^{\varphi z} = -2\pi\alpha'\eta, \tag{2.8}$$

and the open string coupling constant $G_s = g_s$. In the decoupling limit

$$\alpha' \to 0\colon\ g_s = \text{const.}, \qquad \eta = \triangle/\alpha', \tag{2.9}$$

where $\triangle$ is constant, one can see from (2.7) and (2.8) that open string moduli are well-defined [19,22]. The decoupling limit of the open string theory is a four-dimensional NCSYM theory residing on the spacetime (2.7) with YM coupling $g^2_{\rm YM} = 2\pi g_s$ and noncommutativity parameter

$$\Theta^{\varphi z} = -2\pi\triangle. \tag{2.10}$$

Though it appears that the noncommutativity parameter is constant in the coordinates (2.7), it is space-dependent in the coordinates $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$:

$$\Theta^{xz} = -2\pi y\triangle, \qquad \Theta^{yz} = 2\pi x\triangle. \tag{2.11}$$

This noncommutativity parameter is divergence free and obeys the Jacobi identity [19]. Since the noncommutativity parameter is space-dependent, to construct a gauge invariant action in such a background, the product between operators has to use the so-called Kontsevich product [23]

$$\begin{aligned} f*g &= fg + i\theta^{ij}\partial_if\partial_jg - \frac{1}{8}\theta^{ij}\theta^{kl}\partial_{ik}f\partial_j\partial_lg \\ &\quad - \frac{1}{12}\theta^{ij}\partial_j\theta^{lm}(\partial_i\partial_lf\partial_mg - \partial_lf\partial_i\partial_mg) + \mathcal{O}(\theta^3). \end{aligned} \tag{2.12}$$

When θ are constants, the Kontsevich product reduces to the Moyal product.

The dual supergravity description of the NCSYM theory with space-dependent noncommutativity parameter can be obtained from the black D3-brane solution (2.1) by taking the decoupling limit (2.9) combined with

$$\rho = \alpha' u, \qquad \rho_0 = \alpha' u_0. \tag{2.13}$$

The supergravity solution becomes

$$\begin{aligned} ds^2 &= \alpha'\Big\{\frac{u^2}{R^2}(-\tilde f dt^2 + dr^2 + \tilde h(r^2d\varphi^2 + dz^2)) \\ &\qquad + \frac{R^2}{u^2}(\tilde f^{-1}du^2 + u^2d\Omega_5^2)\Big\}, \\ e^{2\phi} &= g_s^2\tilde h, \\ 2\pi\alpha' B &= \alpha' r^2\triangle u^4R^4\tilde h d\varphi\wedge dz, \\ A_2 &= \alpha' g_s^{-1}\triangle r\frac{u^4}{R^4}dt\wedge dr, \\ A_4 &= \alpha'^2g_s^{-1}r\tilde h\frac{u^4}{R^4}dt\wedge dr\wedge d\varphi\wedge dz, \end{aligned} \tag{2.14}$$

where

$$\tilde f = 1 - \frac{u_0^4}{u^4}, \qquad \tilde h = \frac{R^4}{R^4 + r^2\triangle^2u^4}, \tag{2.15}$$

and $R^4 = 4\pi g_sN = 2g^2_{\rm YM}N$. When $u_0 = 0$, namely, $\tilde f = 1$, our solution (2.14) reduces to the one given in [19,22]. In other words, our solution (2.14) describes the gravity dual of the NCSYM theory at finite temperature

$$T = \frac{u_0}{\pi R^2}. \tag{2.16}$$

From the solution (2.14) we can see that the solution deviates from the supergravity dual $AdS_5\times S^5$ of the ordinary SYM theory by a factor $\tilde h$. When $u^4 \ll R^4/r^2\triangle^2$, we see from (2.15) that the deviation is small. Since the coordinate u stands for the energy scale of dual YM theory in the AdS/CFT correspondence, the NCSYM theory with space-dependent noncommutativity has only small deviation from the ordinary SYM theory in the low-energy limit as the case of NCSYM with constant noncommutativity parameter [9]. The NCSYM theory at the temperature (2.16) has entropy

$$S = \frac{R^2u_0^3V_3}{2^5\pi^3g_s^2} = \frac{1}{2}\pi^2N^2T^3V_3. \tag{2.17}$$

Once again, the entropy has completely the same expression as the one for the ordinary SYM theory and NCSYM with a constant noncommutativity parameter in the supergravity duals [9]. This indicates that in the large N limit, even for the space-dependent noncommutativity case, the total number of degrees of freedom of NCSYM theories coincides with the one for ordinary SYM theory for any energy scale [8].

III. *S*-DUALITY AND NCOS WITH SPACE-DEPENDENT NONCOMMUTATIVITY

It is shown that the S-duality of NCSYM theory with constant space-space noncommutativity results in an NCOS theory with constant time-space noncommutativity [10]. This also holds for time-dependent background [18]. Therefore one may expect that S-duality for the NCSYM theory with space-dependent noncommutativity will give an NCOS theory with space-dependent time-space noncommutativity. In this section we will show that it is indeed the case. Furthermore more general $SL(2,\mathbb{Z})$ transformation will result in an NCOS theory with both space-dependent time-space and space-space noncommutativities.

Making an S-duality along the coordinate z for the black D3-brane solution (2.1), we obtain[2]

$$\begin{aligned} ds^2 &= H^{-1/2}h^{-1/2}(-fdt^2+dr^2+h(r^2d\varphi^2+dz^2)) \\ &\quad + H^{1/2}h^{-1/2}(f^{-1}d\rho^2+\rho^2 d\Omega_5^2), \\ e^{2\phi} &= g_s^2 h^{-1}, \\ 2\pi\alpha' B &= H^{-1}\eta r\coth\alpha dt\wedge dr, \\ A_2 &= g_s^{-1}\frac{r^2\eta}{H+r^2\eta^2}d\varphi\wedge dz, \\ A_4 &= g_s^{-1}H^{-1}rh\coth\alpha dt\wedge dr\wedge d\varphi\wedge dz. \end{aligned} \tag{3.1}$$

To see what a well-defined decoupled theory can be obtained for open string theory in the background (3.1), we use again the Seiberg-Witten relation (2.5) for the solution (3.1) in the "flat" limit, namely, $f=H=1$. We find that the open string metric is

$$G_{ij}dx^idx^j = h^{1/2}(-dt^2+dr^2+r^2d\varphi^2+dz^2), \tag{3.2}$$

the nonvanishing noncommutativity parameter

$$\Theta^{tr} = 2\pi\alpha'\eta r, \tag{3.3}$$

and the open string coupling constant is still $G_s = g_s$. In the decoupling limit (2.9), a set of well-defined open string moduli can be obtained as follows:

$$\alpha' G^{ij}\partial_i\partial_j = \triangle r(d^2/dt^2+d^2/dr^2+d^2/r^2d\varphi^2+d^2/dz^2), \tag{3.4}$$

$$\Theta^{tr} = 2\pi\triangle r. \tag{3.5}$$

Indeed we see that the resulting theory is an open string theory with coupling constant $G_s = g_s$ and space-dependent time-space noncommutativity parameter Θ^{tr}. The effective open string scale is $\alpha'_{\rm eff} = r\triangle$, having a space dependence. As usual, the supergravity dual of the NCOS theory can be found from the solution (3.1) in the decoupling limit (2.9) and (2.13)

$$\begin{aligned} ds^2 &= \alpha'\tilde h^{-1/2}\Big\{\frac{u^2}{R^2}(-\tilde f dt^2+dr^2+\tilde h(r^2d\varphi^2+dz^2)) \\ &\quad + \frac{R^2}{u^2}(\tilde f^{-1}du^2+u^2d\Omega_5^2)\Big\}, \\ e^{2\phi} &= g_s^2\tilde h^{-1}, \\ A_2 &= \alpha' g_s^{-1}r^2\triangle u^4R^4\tilde h d\varphi\wedge dz, \\ 2\pi\alpha' B &= \alpha'\triangle r\frac{u^4}{R^4}dt\wedge dr, \\ A_4 &= \alpha'^2 g_s^{-1}r\tilde h\frac{u^4}{R^4}dt\wedge dr\wedge d\varphi\wedge dz. \end{aligned} \tag{3.6}$$

It is easy to show that the temperature and entropy of the supergravity dual (3.6) are still given by (2.16) and (2.17), respectively. This implies that the NCOS theory has the same thermodynamics and degrees of freedom as those of the NCSYM theory with space-dependent noncommutativity described by (2.14).

It is well known that type IIB string theory has an $SL(2,\mathbb{Z})$ invariance. Considering the $SL(2,\mathbb{Z})$ transformation to the solution (2.1), we can get more general black D3-brane solution in Melvin universe. Since the axion field vanishes for the solution (2.1), we have the transformed solution

$$\begin{aligned} \tilde\tau &= \frac{a\tau+b}{c\tau+d}, \qquad \tau = ie^{-\phi}, \\ ad-bc &= 1, \qquad a,b,c,d\in\mathbb{Z}, \end{aligned} \tag{3.7}$$

whose imaginary part gives the transformed dilaton field

$$e^{-\tilde\phi} = \frac{e^{-\phi}}{|c\tau+d|^2}, \tag{3.8}$$

and real part gives a nonvanishing axion field $\tilde\chi$. The new configuration has the form[3]

[2]Here we have rescaled the closed string coupling constant as $1/g_s \to g_s$.

[3]Here we have made rescaling of coordinates and η as follows: $(t,r,z,\rho,1/\eta) \to g_s^{1/2}/(c^2+d^2g_s^2)^{1/4}(t,r,z,\rho,1/\eta)$.

$$
\begin{aligned}
ds^2 &= H^{-1/2}h^{-1/2}\sqrt{\frac{c^2+d^2g_s^2h}{c^2+d^2g_s^2}}[-fdt^2+dr^2 \\
&\quad + h(r^2d\varphi^2+dz^2)+H(f^{-1}d\rho^2+\rho^2d\Omega_5^2)], \\
e^{-2\tilde{\phi}} &= g_s^2h(c^2+d^2g_s^2h)^{-2}, \\
\tilde{\chi} &= \frac{ac+dbg_s^2h}{c^2+d^2g_s^2h}, \\
2\pi\alpha'\tilde{B} &= \frac{r^2\eta}{H+r^2\eta^2}\frac{dg_s}{\sqrt{c^2+d^2g_s^2}}d\varphi\wedge dz \\
&\quad - \frac{\eta r\coth\alpha}{H}\frac{c}{\sqrt{c^2+d^2g_s^2}}dt\wedge dr, \\
\tilde{A}_2 &= H^{-1}\eta r\coth\alpha\frac{a}{\sqrt{c^2+d^2g_s^2}}dt\wedge dr \\
&\quad - \frac{r^2\eta}{H+r^2\eta^2}\frac{bg_s}{\sqrt{c^2+d^2g_s^2}}d\varphi\wedge dz.
\end{aligned} \tag{3.9}
$$

Using the Seiberg-Witten relation (2.5), we find the open string moduli in this case: open string metric

$$
G_{ij}dx^idx^j = h^{1/2}\sqrt{\frac{c^2+d^2g_s^2}{c^2+d^2g_s^2h}} \times(-dt^2+dr^2+r^2d\varphi^2+dz^2), \tag{3.10}
$$

nonvanishing noncommutativity parameter

$$
\begin{aligned}
\Theta^{tr} &= -2\pi\alpha'\frac{c\eta r}{\sqrt{c^2+d^2g_s^2}}, \\
\Theta^{\varphi z} &= -2\pi\alpha'\frac{d\eta g_s}{\sqrt{c^2+d^2g_s^2}},
\end{aligned} \tag{3.11}
$$

and open string coupling constant $G_s = \tilde{g}_s = g_s^{-1}(c^2 + d^2g_s^2)$. In the decoupling limit (2.9), we have

$$
\alpha' G^{ij}\partial_i\partial_j = \frac{r\triangle c}{\sqrt{c^2+d^2g_s^2}}\left(-\frac{d^2}{dt^2}+\frac{d^2}{dr^2}+\frac{d^2}{r^2d\varphi^2}+\frac{d^2}{dz^2}\right), \tag{3.12}
$$

and

$$
\Theta^{tr} = -\frac{2\pi c\triangle r}{\sqrt{c^2+d^2g_s^2}}, \qquad \Theta^{\varphi z} = -\frac{2\pi\triangle dg_s}{\sqrt{c^2+d^2g_s^2}}. \tag{3.13}
$$

We see from (3.12) and (3.13) that the decoupled theory is an NCOS theory with both nonconstant time-space and space-space noncommutativities and the effective open string scale is $\alpha'_{\rm eff} = r\triangle c/\sqrt{c^2+d^2g_s^2}$ and open string coupling constant $G_s = g_s^{-1}(c^2+d^2g_s^2)$. The supergravity dual to this NCOS theory is

$$
\begin{aligned}
ds^2 &= \alpha'\tilde{h}^{-1/2}\frac{u^2}{\tilde{R}^2}\sqrt{\frac{c^2+d^2g_s^2\tilde{h}}{c^2+d^2g_s^2}}\Big(-\tilde{f}dt^2+dr^2 \\
&\quad + \tilde{h}(r^2d\varphi^2+dz^2)+\frac{\tilde{R}^4}{u^4}(\tilde{f}^{-1}du^2+u^2d\Omega_5^2)\Big), \\
e^{-2\tilde{\phi}} &= g_s^2\tilde{h}(c^2+d^2g_s^2\tilde{h})^{-2}, \\
\tilde{\chi} &= \frac{ac+dbg_s^2\tilde{h}}{c^2+d^2g_s^2\tilde{h}}, \\
2\pi\alpha' B &= \alpha'\frac{r^2\triangle u^4}{\tilde{R}^4+r^2\triangle^2u^4}\frac{dg_s}{\sqrt{c^2+d^2g_s^2}}d\varphi\wedge dz \\
&\quad - \alpha'\frac{\triangle ru^4}{\tilde{R}^4}\frac{c}{\sqrt{c^2+d^2g_s^2}}dt\wedge dr,
\end{aligned} \tag{3.14}
$$

where $\tilde{R}^4 = 4\pi\tilde{g}_sN$. This supergravity configuration (3.14) has the same Hawking temperature (2.16) and entropy (2.17) as the configuration (2.14) or (3.6). This implies that this set of noncommutative theories (NCOS theory and NCSYM theory) has completely the same thermodynamic properties in the dual supergravity description.

IV. LORENTZ BOOST AND LIGHT-LIKE NCSYM

In [15] it is shown that starting from (spacelike) NCSYM or (timelike) NCOS theories, one can obtain lightlike NCSYM theories by using Lorentz transformation and rescaling some coordinates and variables. In that case, all noncommutativity parameters are constant in those theories. Here we show that the same approach gives us lightlike NCSYM theory with nonconstant noncommutativity parameter starting from (spacelike)NCSYM or (timelike) NCOS theory with nonconstant noncommutativity parameter.

Let us begin with black D3-brane solution (2.1) in the Melvin universe with $f = 1$, namely, D3-brane solution with $H = 1 + 4\pi g_s\alpha'^2/\rho^4$. Making a Lorentz boost along the coordinate z,

$$
z\to z\cosh\beta - t\sinh\beta, \qquad t\to t\cosh\beta - z\sinh\beta, \tag{4.1}
$$

where β is the boost parameter, and taking the limit $\beta\to\infty$ and $\eta\to 0$ but keeping $\eta\cosh\beta = \gamma$ as a constant, we obtain

$$
\begin{aligned}
ds^2 &= H^{-1/2}\left(-dz_+dz_- + dr^2 + r^2d\varphi^2 - \frac{\gamma^2r^2}{H}dz_-^2\right) \\
&\quad + H^{1/2}(d\rho^2+\rho^2d\Omega_5^2), \\
e^{2\phi} &= g_s^2, \\
2\pi\alpha' B &= -\gamma r^2H^{-1}d\varphi\wedge dz_-, \\
A_2 &= -g_s^{-1}H^{-1}\gamma r dz_-\wedge dr, \\
A_4 &= g_s^{-1}H^{-1}r dt\wedge dr\wedge d\varphi\wedge dz,
\end{aligned} \tag{4.2}
$$

where $z_\pm = t \pm z$ are two null coordinates. This is a pp-wave solution propagating along the direction z. But this is not a gravitational wave, instead it is produced by the NSNS B-field. For this background, using the Seiberg-Witten relation (2.5) we obtain the open string moduli: open string metric

$$G_{ij}dx^i dx^j = -dz_+ dz_- + dr^2 + r^2 d\varphi^2, \tag{4.3}$$

nonvanishing noncommutativity parameter

$$\Theta^{z_+\varphi} = 4\pi\alpha'\gamma, \tag{4.4}$$

and open string coupling constant $G_s = g_s$. In the decoupling limit,

$$\alpha' \to 0, \qquad \gamma = \Gamma/\alpha', \tag{4.5}$$

and keeping g_s to be constant, the decoupled theory on the D3-branes is a lightlike NCSYM theory living in (4.3) with the Yang-Mills coupling $g_{\rm YM}^2 = 2\pi g_s$ and nonconstant noncommutativity parameter $\Theta^{z_+\phi} = 4\pi\Gamma$. That the noncommutativity parameter is not a constant can be seen easily in the coordinates (x, y, z):

$$\Theta_{z_-x} = -2\pi\Gamma y, \qquad \Theta_{z_-y} = 2\pi\Gamma x. \tag{4.6}$$

The dual supergravity description can be obtained from the solution (4.2) by taking the decoupling limit (4.5) and $\rho = \alpha' u$. The geometry configuration is

$$\begin{aligned}
ds^2 &= \alpha'\frac{u^2}{R^2}\Big(-dz_+dz_- + dr^2 + r^2 d\varphi^2 - \frac{\Gamma^2 r^2 u^4}{R^4}dz_-^2 \\
&\quad + \frac{R^4}{u^4}(du^2 + u^2 d\Omega_5^2)\Big),\\
e^{2\phi} &= g_s^2,\\
2\pi\alpha' B &= -\alpha'\Gamma r^2\frac{u^4}{R^4}d\varphi\wedge dz_-,\\
A_2 &= -\alpha' g_s^{-1}\Gamma r\frac{u^4}{R^4}dz_-\wedge dr,\\
A_4 &= \alpha'^2 g_s^{-1} r\frac{u^4}{R^4}dt\wedge dr\wedge d\varphi\wedge dz,
\end{aligned} \tag{4.7}$$

where $R^4 = 4\pi g_s N$. From the supergravity configuration we can see that near the origin, $u^4 \ll R^4/\Gamma^2 r^2$, the deviation from the $AdS_5 \times S^5$ is small. This implies that in the low-energy limit, the difference of the lightlike NCSYM theory with nonconstant noncommutativity parameter from $\mathcal{N} = 4$ SYM theory is negligible.

It is easy to see that we can obtain the same lightlike NCSYM theory if we start from the supergravity dual (3.1) or more general (3.9) for NCOS theory with nonconstant noncommutativity parameter. We will not repeat to produce the result. Instead we will consider the black D3-brane solution (2.1), namely, nonextremal case, in the Melvin universe. In this case, $f = 1 - \rho_0^4/\rho^4$. Applying the Lorentz boost (4.1) to the solution (2.1), and taking the limit, $\eta \to 0$, $\alpha \to \infty$, $\beta \to \infty$, but keeping $\eta\cosh\beta = \gamma$, $\rho_0^4\cosh\alpha\sinh\alpha = 4\pi g_s\alpha'^2 N$ and $\rho_0^4\cosh^2\beta = P$ as constant, we have solution

$$\begin{aligned}
ds^2 &= H^{-1/2}\Big(-dz_+dz_- + dr^2 + r^2 d\varphi^2 - \frac{\gamma^2 r^2}{H}dz_-^2 \\
&\quad + \frac{P}{\rho^4}dz_-^2\Big) + H^{1/2}(d\rho^2 + \rho^2 d\Omega_5^2),\\
e^{2\phi} &= g_s^2,\\
2\pi\alpha' B &= -\gamma r^2 H^{-1} d\varphi\wedge dz_-,\\
A_2 &= -g_s^{-1}H^{-1}\gamma r dz_-\wedge dr,\\
A_4 &= g_s^{-1}H^{-1} r dt\wedge dr\wedge d\varphi\wedge dz,
\end{aligned} \tag{4.8}$$

where P has an interpretation as momentum of gravitational pp-wave propagating along direction z. It is easy to show that the decoupling limit of open string ending on D3-branes in the background (4.8) is the same as that of the background (4.2), that is, it is a lightlike NCSYM theory with nonconstant noncommutativity parameter. Because of the appearance of the momentum P, the field theory lives in the background (4.3), but in an infinite-momentum frame [15,24]. The supergravity dual is obtained by taking decoupling limit (4.5) combining with

$$\rho = \alpha' u, \qquad P = \alpha'^4\tilde{P}, \tag{4.9}$$

with $\tilde{P}$ being a constant. The supergravity configuration is

$$\begin{aligned}
ds^2 &= \alpha'\frac{u^2}{R^2}\Big(-dz_+dz_- + dr^2 + r^2 d\varphi^2 - \frac{\Gamma^2 r^2 u^4}{R^4}dz_-^2 \\
&\quad + \frac{\tilde{P}}{u^4}dz_-^2 + \frac{R^4}{u^4}(du^2 + u^2 d\Omega_5^2)\Big),\\
e^{2\phi} &= g_s^2,\\
2\pi\alpha' B &= -\alpha'\Gamma r^2\frac{u^4}{R^4}d\varphi\wedge dz_-,\\
A_2 &= -\alpha' g_s^{-1}\Gamma r\frac{u^4}{R^4}dz_-\wedge dr,\\
A_4 &= \alpha'^2 g_s^{-1} r\frac{u^4}{R^4}dt\wedge dr\wedge d\varphi\wedge dz,
\end{aligned} \tag{4.10}$$

where $R^4 = 4\pi g_s N = 2g_{\rm YM}^2 N$. When $\tilde{P} = 0$, the configuration (4.10) reduces to (4.7), which is the supergravity configuration dual to the lightlike NCSYM theory with nonconstant noncommutativity in a static frame.

V. CONCLUSION

According to the holographic principle, quantum theory of gravity must be a nonlocal quantum theory. Therefore studying nonlocal field theory is of great interest in its own right. More recently Hashimoto and Thomas [19] have shown that the decoupling theory on D3-branes in Melvin universe supported by a spacelike NSNS B-field is an NCSYM theory with space-dependent space-space noncommutativity. This generalized the case of the

NCSYM theory with constant space-space noncommutativity [8]. In this paper we have first extended the supergravity dual to the NCSYM theory with space-dependent noncommutativity to the nonextremal case, which describes the NCSYM theory at finite temperature. We have shown that under S-duality, the NCSYM theory with space-dependent noncommutativity changes to be an NCOS theory with space-dependent space-time noncommutativity, while it is an NCOS theory with both space-dependent space-space and space-time noncommutativities for a general $SL(2, \mathbb{Z})$ transformation. These NCOS theories with space-dependent noncommutativity widen the family of NCOS theories with constant noncommutativity parameter [10]. Furthermore we have found that in the dual supergravity description, these NCSYM and NCOS theories have completely the same thermodynamics as that of ordinary SYM theory, NCSYM and NCOS theories with constant noncommutativity. This implies that these theories have the same degrees of freedom in the dual supergravity description. In addition, starting from black D3-brane solution in Melvin universe and making Lorentz boost along a spatial direction on the worldvolume, we have shown that the decoupled theory is a lightlike NCSYM theory with space-dependent noncommutativity in static frame or infinite-momentum frame, depending on whether there is a gravitational pp-wave on the worldvolume of D3-branes.

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