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Super-Higgs mechanism in string theory

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We exhibit the super-Higgs effect in heterotic string theory by turning on a background antisymmetric tensor B field and deforming the Becchi-Rouet-Stora-Tyutin operator consistent with superconformal invariance. The B field spontaneously breaks spacetime supersymmetry. We show how the gravitini and the physical dilatini gain mass by eating the would-be Goldstone fermions.

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I. INTRODUCTION

String theory is the most ambitious and most promising attempt to incorporate gravity into quantum mechanics. The theory possesses a large number of symmetries, including supersymmetry, gauge symmetry, and coordinate invariance. At low energies, many of the symmetries are broken by the vacuum. At ultrahigh energies, beyond the Planck scale, the results of [1] suggest that the full symmetry group is infinite dimensional.

How can we investigate the symmetry structure of the theory? Evans and Ovrut developed an elegant approach in which string symmetries are generated by inner automorphisms of the superconformal operator algebra [2]. The method treats unbroken and spontaneously broken symmetries on exactly the same footing. In recent work, we used this formalism to illustrate the Higgs mechanism in string theory [3].

In this paper we extend these results to the case of spontaneously broken supersymmetry. For concreteness, we focus on heterotic string theory, but our general results apply to type II string theory as well. In Secs. II and III we consider heterotic string propagation in flat Minkowski space, in the presence of a nontrivial but infinitesimal antisymmetric tensor B field background. We assume that the B field satisfies the string-theory equations of motion, and derive the string-theory equations of motion for the gravitino and the dilatino fields. We also find the spacetime supersymmetry generator in this background. We then use the supersymmetry generator to derive the spacetime supersymmetry transformations of the gravitino and the dilatino fields.

In Sec. IV we use these results to illustrate the super-Higgs mechanism in string theory. We study a simple model in which spacetime is compactified on $M^7 \times T^3$, with a constant $H = dB$ flux in the compact dimensions. For zero flux, the seven-dimensional theory has $N = 2$ spacetime supersymmetry, with two massless gravitini and eight massless dilatini. The nonzero H spontaneously breaks the supersymmetry. The gravitini and the dilatini obey coupled equations of motion. We show that the would-be Goldstone fermions can be eliminated by a supersymmetry transformation, and that in the unitary gauge, the gravitini and the remaining six dilatini obey massive equations of motion. Aspects of supersymmetry breaking in string theory were discussed previously in [4].

II. NILPOTENT DEFORMATIONS AND EQUATIONS OF MOTION

A. Heterotic string in Minkowski space

To fix notation, we first describe the heterotic string in flat Minkowski space. We start with the left- and right-moving Becchi-Rouet-Stora-Tyutin (BRST) operators Q and $\overline{Q}$, which are given by

$$Q = \int d\sigma\left(c(T + \partial c b) - \frac{1}{2}\gamma T_F - \frac{1}{4}b\gamma^2\right),$$
$$\overline{Q} = \int d\sigma\, \overline{c}(\overline{T} + \overline{\partial}\,\overline{c}\,\overline{b}), \tag{1}$$

where the world sheet stress-energy tensor is

$$T = \frac{1}{2}\eta_{\mu\nu}\partial X^\mu \partial X^\nu + \frac{1}{2}\eta_{\mu\nu}\psi^\mu \partial\psi^\nu - \frac{3}{2}\partial\beta\gamma - \frac{1}{2}\beta\partial\gamma,$$
$$\overline{T} = \frac{1}{2}\eta_{\mu\nu}\overline{\partial}X^\mu\overline{\partial}X^\nu, \tag{2}$$

the world sheet supercurrent is

$$T_F = \frac{1}{2}\eta_{\mu\nu}\psi^\mu \partial X^\nu, \tag{3}$$

and b, c and β, γ are the world sheet ghosts (together with their conjugates). The bosonic ghosts β and γ can be bosonized as follows:

$$\beta = e^{-\phi}\partial\xi, \qquad \gamma = e^{\phi}\eta, \tag{4}$$

where ξ and η are conjugate fermions of dimension 0 and 1, and ϕ is a chiral boson.

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An infinitesimal gravitino excitation deforms the BRST operators as follows:

$$Q \to Q + \delta Q, \qquad \overline{Q} \to \overline{Q} + \delta\overline{Q}, \tag{5}$$

where

$$\begin{aligned}\delta Q &= \int d\sigma\, c(\Psi^\alpha_\mu S_\alpha e^{-(\phi/2)}\overline{\partial}X^\mu),\\ \delta\overline{Q} &= \int d\sigma\, \overline{c}(\Psi^\alpha_\mu S_\alpha e^{-(\phi/2)}\overline{\partial}X^\mu),\end{aligned} \tag{6}$$

and S_α is a world sheet spin field. Nilpotency requires

$$\{Q, \delta Q\} = \{\overline{Q}, \delta\overline{Q}\} = \{Q, \delta\overline{Q}\} + \{\overline{Q}, \delta Q\} = 0, \tag{7}$$

which in turn imposes the following restrictions on the field Ψ^α_μ,

$$(\gamma^\mu)_{\alpha\beta}\partial_\mu\Psi^\beta_\nu = 0, \qquad \partial^\mu\Psi^\alpha_\mu = 0. \tag{8}$$

The relations (8) contain an equation of motion and a gauge condition. However, the gauge condition does not separate the physical degrees of freedom. As written, Ψ^α_μ contains a spin-$\frac{3}{2}$ gravitino and a spin-$\frac{1}{2}$ dilatino. To separate the fields, we write $\Psi^\alpha_\mu = \chi^\alpha_\mu + (\gamma_\mu)^{\alpha\beta}\lambda_\beta$, where $(\gamma^\mu)_{\alpha\beta}\chi^\beta_\mu = 0$. Equations (8) then become

$$\begin{aligned}(\gamma^\mu)_{\alpha\beta}\partial_\mu\chi^\beta_\nu = 2\partial_\nu\lambda_\alpha, &\qquad (\gamma^\mu)_{\alpha\beta}\chi^\beta_\mu = 0,\\ \partial^\mu\chi^\alpha_\mu = 0, &\qquad (\gamma^\mu)_{\alpha\beta}\partial_\mu\lambda^\beta = 0.\end{aligned} \tag{9}$$

These are coupled equations of motion for a massless spin-$\frac{3}{2}$ gravitino and a massless spin-$\frac{1}{2}$ dilatino, in a generalized Lorentz gauge.

B. Heterotic string in a B field background

To study supersymmetry breaking, we will work in a background with an infinitesimal gauge field $B_{\mu\nu}$ [5]. This deforms the BRST operators as follows:

$$Q \to Q + \delta Q', \qquad \overline{Q} \to \overline{Q} + \delta\overline{Q}', \tag{10}$$

where

$$\begin{aligned}\delta Q' &= \int d\sigma\Big(c(B_{\mu\nu}\partial X^\mu\overline{\partial}X^\nu + \partial_\kappa B_{\mu\nu}\psi^\kappa\psi^\mu\overline{\partial}X^\nu)\\ &\quad - \frac{1}{2}\gamma B_{\mu\nu}\psi^\mu\overline{\partial}X^\nu\Big),\\ \delta\overline{Q}' &= \int d\sigma\,\overline{c}(B_{\mu\nu}\partial X^\mu\overline{\partial}X^\nu + \partial_\kappa B_{\mu\nu}\psi^\kappa\psi^\mu\overline{\partial}X^\nu).\end{aligned} \tag{11}$$

Nilpotency requires

$$\{Q, \delta Q'\} = \{\overline{Q}, \delta\overline{Q}'\} = \{Q, \delta\overline{Q}'\} + \{\overline{Q}, \delta Q'\} = 0, \tag{12}$$

which imposes an equation of motion and a gauge condition for the $B_{\mu\nu}$ field,

$$\Box B_{\mu\nu} = 0, \qquad \partial^\mu B_{\mu\nu} = 0. \tag{13}$$

As in flat space, a gravitino excitation also deforms the BRST operators,

$$Q \to Q + \delta Q' + \delta Q'', \qquad \overline{Q} \to \overline{Q} + \delta\overline{Q}' + \delta\overline{Q}'', \tag{14}$$

where

$$\begin{aligned}\delta Q'' &= \int d\sigma\, c\Psi^\alpha_\mu\Big(S_\alpha e^{-(\phi/2)}\overline{\partial}X^\mu + \frac{1}{4}(\gamma^{\kappa\nu})^\beta_\alpha B_{\kappa\nu}S_\beta e^{-(\phi/2)}\overline{\partial}X^\mu - \frac{1}{2}B^{\mu\nu}S_\alpha e^{-(\phi/2)}(\partial X_\nu - \overline{\partial}X_\nu)\\ &\quad - \frac{1}{2}\partial^\kappa B^{\mu\nu}S_\alpha e^{-(\phi/2)}\psi_\kappa\psi_\nu\Big),\\ \delta\overline{Q}'' &= \int d\sigma\, \overline{c}\Psi^\alpha_\mu\Big(S_\alpha e^{-(\phi/2)}\overline{\partial}X^\mu + \frac{1}{4}(\gamma^{\kappa\nu})^\beta_\alpha B_{\kappa\nu}S_\beta e^{-(\phi/2)}\overline{\partial}X^\mu - \frac{1}{2}B^{\mu\nu}S_\alpha e^{-(\phi/2)}(\partial X_\nu - \overline{\partial}X_\nu)\\ &\quad - \frac{1}{2}\partial^\kappa B^{\mu\nu}S_\alpha e^{-(\phi/2)}\psi_\kappa\psi_\nu\Big),\end{aligned} \tag{15}$$

and we work to first order in $B_{\mu\nu}$ and Ψ^α_μ. Nilpotency imposes an additional equation of motion

$$(\gamma^\mu)_{\alpha\beta}\partial_\mu\Psi^\beta_\nu - (\gamma^\mu)_{\alpha\beta}H^\kappa_{\mu\nu}\Psi^\beta_\kappa + \frac{1}{6}(\gamma^{\mu\kappa\rho})_{\alpha\beta}H_{\mu\kappa\rho}\Psi^\beta_\nu = 0, \tag{16}$$

and gauge condition,

$$\partial^\mu\Psi^\alpha_\mu + \frac{1}{2}(\gamma_{\kappa\nu})^\alpha_\beta H^{\mu\kappa\nu}\Psi^\beta_\mu = 0, \tag{17}$$

where $H_{\mu\kappa\nu} = \frac{1}{2}(\partial_\nu B_{\mu\kappa} + \partial_\mu B_{\kappa\nu} + \partial_\kappa B_{\nu\mu})$. Writing Ψ^α_μ in terms of its trace and traceless part, and substituting into (16) and (17), we find equations of motion for the gravitino χ^α_μ,

$$\begin{aligned}&(\gamma^\mu)_{\alpha\beta}\partial_\mu\chi^\beta_\nu - (\gamma^\mu)_{\alpha\beta}H_{\mu\nu}{}^\kappa\chi^\beta_\kappa + \frac{1}{6}(\gamma^{\mu\kappa\rho})_{\alpha\beta}H_{\mu\kappa\rho}\chi^\beta_\nu\\ &\quad + \frac{2}{D}(\gamma^{\kappa\mu}_\nu)_{\alpha\beta}H_{\kappa\mu}{}^\rho\chi^\beta_\rho + \frac{4}{D}(\gamma^\mu)_{\alpha\beta}H_{\mu\nu}{}^\rho\chi^\beta_\rho\\ &\quad = \partial_\nu\lambda_\alpha - \frac{3}{2}(\gamma^{\kappa\mu})^\beta_\alpha H_{\kappa\mu\nu}\lambda_\beta - \frac{2}{D}(\gamma_\nu\gamma^{\mu\kappa\rho})^\beta_\alpha H_{\mu\kappa\rho}\lambda_\beta,\end{aligned} \tag{18}$$

the dilatino λ_α,

$$(\gamma^\mu)^{\alpha\beta}\partial_\mu\lambda_\beta + \frac{1}{6}(\gamma^{\mu\kappa\rho})^{\alpha\beta}H_{\mu\kappa\rho}\lambda_\beta = -\frac{2}{D}(\gamma^{\kappa\rho})^\alpha_\beta H_{\kappa\rho}{}^\mu\chi^\beta_\mu + \frac{2}{D}(\gamma^{\kappa\rho}\gamma_\mu)^{\alpha\beta}H_{\kappa\rho}{}^\mu\lambda_\beta, \quad (19)$$

together with the gauge condition

$$\partial^\mu\chi^\alpha_\mu = \frac{4+D}{2D}(\gamma^{\kappa\rho})^\alpha_\beta H^\mu_{\kappa\rho}\chi^\beta_\mu + \frac{6+D}{3D} \times (\gamma^{\mu\kappa\rho})^{\alpha\beta}H_{\mu\kappa\rho}\lambda_\beta, \quad (20)$$

where D is the dimension of spacetime. These are the generalizations of (9) in the $B_{\mu\nu}$ background.

III. SPACETIME SUPERSYMMETRY

In string theory, spacetime symmetries correspond to inner automorphisms of the operator algebra. They are generated by infinitesimal operators h,

$$i[h, \mathcal{O}] = \delta\mathcal{O}, \quad (21)$$

where $\mathcal{O}$ is any operator in the theory. When $\mathcal{O}$ is Q or $\overline{Q}$, the deformed BRST charges δQ and $\delta\overline{Q}$ automatically satisfy the deformation Eqs. (7) because of the Bianchi identity.

Following Evans and Ovrut [2], we define a *canonical deformation* to be generated by an infinitesimal operator h that is the sum of zero modes of $(1, 0)$ and $(0, 1)$ primary fields. Such a deformation preserves the gauge of the spacetime fields. For the heterotic string, the operator that generates a supersymmetry transformation about flat spacetime is [6]

$$h = \int d\sigma\, \epsilon^\alpha S_\alpha e^{-(\phi/2)}. \quad (22)$$

The integrand is of dimension $(1, 0)$ provided the transformation parameter ϵ^α satisfies

$$(\gamma^\mu)_{\alpha\beta}\partial_\mu\epsilon^\beta = 0. \quad (23)$$

In this case, h generates a canonical deformation.

The supersymmetry transformation of the gravitino can be found by commuting h with the BRST operator Q. For the case at hand, we find

$$i[h, Q] = \delta Q = \int d\sigma\, c(\partial_\mu\epsilon^\alpha S_\alpha e^{-(\phi/2)}\overline{\partial}X^\mu), \quad (24)$$

and likewise for $\delta\overline{Q}$. Comparing with (6), we see that the commutator (24) describes a deformation of the Ψ^α_λ field. In this way Q and $\overline{Q}$ generate a flat-space supersymmetry transformation, $\delta\Psi^\alpha_\lambda = \partial_\lambda\epsilon^\alpha$.

The supersymmetry generator h deforms in the $B_{\mu\nu}$ background,

$$h \to h + \delta h = \int d\sigma\, \epsilon^\alpha\Big(S_\alpha e^{-(\phi/2)} + \frac{1}{4}(\gamma^{\mu\nu})^\beta_\alpha B_{\mu\nu}S_\beta e^{-(\phi/2)}\Big). \quad (25)$$

The corresponding deformation is canonical if ϵ^α satisfies the following constraint,

$$(\gamma^\mu)_{\alpha\beta}\partial_\mu\epsilon^\alpha + \frac{1}{6}(\gamma^{\mu\nu\kappa})_{\alpha\beta}H_{\mu\nu\kappa}\epsilon^\alpha = 0. \quad (26)$$

This is the Dirac equation for ϵ^α in the $B_{\mu\nu}$ background.

To find the gravitino transformation in this background, we compute the commutator of $h + \delta h$ with $Q + \delta Q'$. This gives

$$\begin{aligned} i[h+\delta h, Q+\delta Q'] = \int d\sigma\, c\Big(\partial_\mu\epsilon^\alpha + \frac{1}{2}(\gamma^{\rho\lambda})^\alpha_\beta H_{\mu\rho\lambda}\epsilon^\beta\Big) &\times \Big(S_\alpha e^{-(\phi/2)}\overline{\partial}X^\mu \\ &+ \frac{1}{4}(\gamma^{\nu\kappa})^\beta_\alpha B_{\nu\kappa}S_\beta e^{-(\phi/2)}\overline{\partial}X^\mu \\ &- \frac{1}{2}B^{\mu\kappa}S_\alpha e^{-(\phi/2)}(\partial X_\kappa - \overline{\partial}X_\kappa) \\ &- \frac{1}{2}\partial^\rho B^{\mu\kappa}S_\alpha e^{-(\phi/2)}\psi_\rho\psi_\kappa\Big). \end{aligned} \quad (27)$$

Comparing with (15), we can read off the Ψ^α_λ transformation in the $B_{\mu\nu}$ background,

$$\delta\Psi^\alpha_\mu = \partial_\mu\epsilon^\alpha + \frac{1}{2}(\gamma^{\nu\kappa})^\alpha_\beta H_{\mu\nu\kappa}\epsilon^\beta. \quad (28)$$

Decomposing $\Psi^\alpha_\mu = \chi^\alpha_\mu + (\gamma_\mu)^{\alpha\beta}\lambda_\beta$, we find the transformation properties of the gravitino and the dilatino,

$$\begin{aligned} \delta\chi^\alpha_\mu &= \partial_\mu\epsilon^\alpha + \frac{1}{2}(\gamma^{\nu\kappa})^\alpha_\beta H_{\mu\nu\kappa}\epsilon^\beta - \frac{2}{3D}(\gamma_\mu\gamma^{\nu\kappa\rho})^\alpha_\beta H_{\nu\kappa\rho}\epsilon^\beta, \\ \delta\lambda_\alpha &= \frac{2}{3D}(\gamma^{\mu\nu\kappa})_{\alpha\beta}H_{\mu\nu\kappa}\epsilon^\beta. \end{aligned} \quad (29)$$

There are precisely the transformations of ten-dimensional supergravity, derived directly from string theory.

IV. THE SUPER-HIGGS MECHANISM

In the previous sections we studied string theory in the presence of a $B_{\mu\nu}$ field that pervades all of spacetime. In this section we focus on string propagation on $M^7 \times T^3$, where the $B_{\mu\nu}$ field is restricted to T^3. We will see that the $B_{\mu\nu}$ field spontaneously breaks the supersymmetry on M^7.

We start by fixing the notation. We take the spacetime coordinates to be $\{X^\mu, X^i\}$, where $\mu = 0, \cdots 6$ and $i = 7, 8, 9$. We decompose the ten-dimensional gamma matrices in direct product fashion,

$$\Gamma^\mu = \frac{i}{\sqrt{2}}(\gamma^\mu \otimes 1 \otimes \sigma^1), \qquad \Gamma^i = \frac{i}{\sqrt{2}}(1 \otimes \sigma^i \otimes \sigma^2), \tag{30}$$

where the γ^μ satisfy the Clifford algebra in seven dimensions and the σ^i are ordinary Pauli matrices. With these conventions, the ten-dimensional gravitino splits into two seven-dimensional gravitini and two seven-dimensional dilatini, $\Psi_\mu^{\alpha a} = \chi_\mu^{\alpha a} + (\gamma_\mu)^{\alpha\beta}\lambda_\beta^a$, together with six additional seven-dimensional dilatini, $\Psi_i^{\alpha a} = -i\lambda_i^{\alpha a}$, where $\alpha = 1, \cdots 8$ and $a = 1, 2$.

Let us focus on the background in which $H_{ijk} = 2m\epsilon_{ijk}$. Nilpotency of the BRST operators implies that the dilatini $\lambda_i^{\alpha a}$ obey the following equations of motion,

$$(\gamma^\mu)_{\alpha\beta}\partial_\mu\lambda_i^{\beta a} + 2im\epsilon_{ij}{}^k(\sigma^j)^{ab}\lambda_{\alpha k}^b + m\lambda_{\alpha i}^a = 0. \tag{31}$$

It also imposes equations of motion

$$(\gamma^\mu)_{\alpha\beta}\partial_\mu\Psi_\lambda^{\beta a} + m\Psi_{\alpha\lambda}^a = 0, \tag{32}$$

and gauge conditions

$$\partial^\mu\Psi_\mu^{\alpha a} - im(\sigma^i)^{ab}\Psi_i^{\alpha b} = 0 \tag{33}$$

on the $\Psi_\lambda^{\alpha a}$. In terms of gravitini and dilatini parts, Eqs. (32) and (33) can be written as follows,

$$\begin{aligned}(\gamma^\mu)_{\alpha\beta}\partial_\mu\chi_\nu^{\beta a} + m\chi_{\alpha\nu}^a &= -2\partial_\nu\lambda_\beta^a + \frac{2m}{7}(\gamma_\nu)_{\alpha\beta}(\sigma^i)^{ab}\lambda_i^{\beta b},\\ (\gamma^\mu)_{\alpha\beta}\partial_\mu\lambda^{\beta a} - m\lambda_\alpha^a &= \frac{2m}{7}(\sigma^i)^{ab}\lambda_{\alpha i}^b,\\ \partial^\mu\chi_\mu^{\alpha a} + m\lambda^{\alpha a} &= \frac{5m}{7}(\sigma^i)^{ab}\lambda_i^{\alpha b}.\end{aligned} \tag{34}$$

The gravitini and dilatini obey coupled equations of motion. Multiplying Eq. (31) by σ^i and using Eqs. (34), we find

$$\begin{aligned}(\gamma^\mu)_{\alpha\beta}\partial_\mu\Big(\chi_\nu^{\beta a} + \frac{1}{7}(\gamma_\nu)^{\beta\gamma}(\sigma^i)^{ab}\lambda_{\gamma i}^b\Big) + m\Big(\chi_\nu^{\alpha a} + \frac{1}{7}(\gamma_\nu)^{\alpha\beta}(\sigma^i)^{ab}\lambda_{\beta i}^b\Big) &= -2\partial_\nu\Big(\lambda_\alpha^a - \frac{1}{7}(\sigma^i)^{ab}\lambda_{\alpha i}^b\Big),\\ (\gamma^\mu)_{\alpha\beta}\partial_\mu\Big(\lambda^{\beta a} - \frac{1}{7}(\sigma^i)^{ab}\lambda_i^{\beta b}\Big) - m\Big(\lambda_\alpha^a - \frac{1}{7}(\sigma^i)^{ab}\lambda_{\alpha i}^b\Big) &= 0,\\ \partial^\mu\Big(\chi_\mu^{\alpha a} + \frac{1}{7}(\gamma_\mu)^{\alpha\beta}(\sigma^i)^{ab}\lambda_{\beta i}^b\Big) + m\Big(\lambda^{\alpha a} - \frac{1}{7}(\sigma^i)^{ab}\lambda_i^{\alpha b}\Big) - m(\sigma^i)^{ab}\lambda_i^{\alpha b} &= 0.\end{aligned} \tag{35}$$

The form of these equations suggests the following change of variables:

$$\begin{aligned}\chi_\mu'^{\alpha a} &= \chi_\mu^{\alpha a} + \frac{1}{7}(\gamma_\mu)^{\alpha\beta}(\sigma^i)^{ab}\lambda_{\beta i}^b,\\ \lambda_i'^{\alpha a} &= \lambda_i^{\alpha a} - \frac{1}{3}(\sigma_i)^{ab}(\sigma^j)^{bc}\lambda_j^{\alpha c},\\ \lambda_{1\alpha}'^a &= \lambda_\alpha^a - \frac{1}{7}(\sigma^i)^{ab}\lambda_{\alpha i}^b,\\ \lambda_{2\alpha}'^a &= \lambda_\alpha^a - \frac{1}{21}(\sigma^i)^{ab}\lambda_{\alpha i}^b.\end{aligned} \tag{36}$$

In terms of the primed variables, the equations and gauge conditions become

$$\begin{aligned}(\gamma^\mu)_{\alpha\beta}\partial_\mu\chi_\nu'^{\beta a} + m\chi_\nu'^{\alpha a} &= -2\partial_\nu\lambda_{1\alpha}'^a,\\ \partial^\mu\chi_\mu'^{\alpha a} + m\lambda_1'^{\alpha a} - m(\gamma^\mu)^{\alpha\beta}\chi_{\beta\mu}'^a &= 0,\\ (\gamma^\mu)_{\alpha\beta}\partial_\mu\lambda_i'^{\beta a} - 3m\lambda_{\alpha i}'^a &= 0,\\ (\gamma^\mu)_{\alpha\beta}\partial_\mu\lambda_1'^{\beta a} - m\lambda_{1\alpha}'^a &= 0,\\ (\gamma^\mu)_{\alpha\beta}\partial_\mu\lambda_2'^{\beta a} - 3m\lambda_{2\alpha}'^a + 2m\lambda_{1\alpha}'^a &= 0.\end{aligned} \tag{37}$$

To properly interpret these relations, we need to find the supersymmetry transformations in this background. The supersymmetry generator is

$$h = \int d\sigma\, \epsilon^{\alpha a}\Big(S_\alpha^a e^{-(\phi/2)} + \frac{1}{4}(\sigma^{ij})^{ab}B_{ij}S_\alpha^b e^{-(\phi/2)}\Big), \tag{38}$$

where the transformation parameters $\epsilon^{\alpha a}$ obey the following conditions,

$$(\gamma^\mu)_{\alpha\beta}\partial_\mu\epsilon^{\beta a} + m\epsilon_\alpha^a = 0, \qquad \Box\epsilon_\alpha^a = 0. \tag{39}$$

Following the arguments of Sec. III, we can derive the supersymmetry transformations of the gravitini and dilatini,

$$\begin{aligned}\delta\chi_\mu^{\alpha a} &= \partial_\mu\epsilon^{\alpha a} - \frac{m}{7}(\gamma_\mu)^{\alpha\beta}\epsilon_\beta^a, \qquad \delta\lambda^{\alpha a} = \frac{m}{7}\epsilon^{\alpha a},\\ \delta\lambda_i^{\alpha a} &= m(\sigma_i)^{ab}\epsilon^{\alpha b}.\end{aligned} \tag{40}$$

The dilatini transform nonlinearly under supersymmetry transformations.

We now have what we need to interpret Eqs. (37). We first note that $\lambda_1'^{\alpha a}$ shifts under supersymmetry, while $\lambda_2'^{\alpha a}$ and $\lambda_i'^{\alpha a}$ do not. Therefore $\lambda_1'^{\alpha 1}$ and $\lambda_1'^{\alpha 2}$ are the would-be Goldstone fermions that arise from the supersymmetry breaking. In unitary gauge, these fields vanish, and Eqs. (37) become

$$\begin{aligned}
&(\gamma^\mu)_{\alpha\beta}\partial_\mu \chi_\nu'^{\beta a} + m\chi_\nu'^{\alpha a} = 0,\\
&\partial^\mu \chi_\mu'^{\alpha a} - m(\gamma^\mu)^{\alpha\beta}\chi_{\beta\mu}'^{a} = 0,\\
&(\gamma^\mu)_{\alpha\beta}\partial_\mu \lambda_i'^{\beta a} - 3m\lambda_{\alpha i}'^{a} = 0,\\
&(\gamma^\mu)_{\alpha\beta}\partial_\mu \lambda_2'^{\beta a} - 3m\lambda_{2\alpha}'^{a} = 0.
\end{aligned} \tag{41}$$

These are nothing but the equations of motion for two massive gravitini and six massive dilitini. The two gravitini have eaten the two would-be Goldstone fermions, as required by the super-Higgs effect.

V. CONCLUSIONS

In this paper we illustrated the super-Higgs effect in heterotic string theory. We first turned on a background tensor field $B_{\mu\nu}$ and deformed the BRST operator consistent with superconformal invariance. We then derived the string-theory equations of motion for the background, as well as for the gravitino and the dilatino fields. We found the spacetime supersymmetry generator and used it to derive the supersymmetry transformations of the spacetime fields.

We then studied a model in which spacetime is compactified on $M^7 \times T^3$, with a constant flux in the compact dimensions. We showed that the nonzero $B_{\mu\nu}$ field spontaneously breaks supersymmetry. We demonstrated that the would-be Goldstone fermions can be eliminated by a supersymmetry transformation and that in the unitary gauge, the gravitini and the remaining six dilatini obey massive equations of motion. In this way we illustrated the super-Higgs effect in the full string theory, and not just in the effective field theory that arises at low energy.

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