PHYSICAL REVIEW D **73,** 104033 (2006)

Strings, black holes, and quantum information

Renata Kallosh and Andrei Linde
Department of Physics, Stanford University, Stanford, California 94305, USA
(Received 22 February 2006; published 25 May 2006)

We find multiple relations between extremal black holes in string theory and 2- and 3-qubit systems in quantum information theory. We show that the entropy of the axion-dilaton extremal black hole is related to the concurrence of a 2-qubit state, whereas the entropy of the STU black holes, Bogomol'nyi-Prasad-Sommerfield (BPS) as well as non-BPS, is related to the 3-tangle of a 3-qubit state. We relate the 3-qubit states with the string theory states with some number of D-branes. We identify a set of large black holes with the maximally entangled Greenberger, Horne, Zeilinger (GHZ) class of states and small black holes with separable, bipartite, and W states. We sort out the relation between 3-qubit states, twistors, octonions, and black holes. We give a simple expression for the entropy and the area of stretched horizon of small black holes in terms of a norm and 2-tangles of a 3-qubit system. Finally, we show that the most general expression for the black hole and black ring entropy in $N = 8$ supergravity/M theory, which is given by the famous quartic Cartan $E_{7(7)}$ invariant, can be reduced to Cayley's hyperdeterminant describing the 3-tangle of a 3-qubit state.

DOI: 10.1103/PhysRevD.73.104033 PACS numbers: 04.70.Dy, 03.67.−a, 04.65.+e, 11.25.−w

I. INTRODUCTION

During the last 15 years there was a significant progress in two different fields of knowledge: a description of black holes in string theory and the theory of quantum information and quantum computing. At the first glance these two subjects may seem quite distant from each other. However, there are some general themes, such as entanglement, information, and entropy, which repeatedly appear both in the theory of black holes and in the theory of quantum information.

Studies of stringy black holes began with a discovery of a broad class of new extremal black hole solutions [1], investigation of their supersymmetry [2], a discovery of the black hole attractor mechanism [3], and the microscopic calculation of black hole entropy [4]. Investigation of stringy black holes resulted in a better understanding of the information loss paradox in the theory of black holes, revealed nonperturbative symmetries between different versions of string theory, and stimulated what is now called "the second string theory revolution" [5–7]. For reviews on stringy black holes see [8]. On the other hand, there were many exciting developments in the theory of quantum computation, quantum cryptography, quantum cloning, quantum teleportation, classification of entangled states, and investigation of a measure of entanglement in the context of the quantum information theory; for a review see e.g. [9]. It would be quite useful to find some links between these different sets of results.

One of the first steps in this direction was made in a recent paper by Michael Duff [10]. He discovered that a complicated expression for the entropy of the so-called extremal STU black holes[1] obtained in [12] can be represented in a very compact way as Cayley's hyperdeterminant [13], which appears in the theory of quantum information in the calculation of the measure of entanglement of the 3-qubit system (3-tangle) [14,15]. The STU black holes represent a broad class of classical solutions of the effective supergravity derived from string theory in [16].

As emphasized in [10], the intriguing relation between STU extremal black holes and 3-qubit systems in quantum information theory may be coincidental. It may be explained, e.g., by the fact that both theories have the same underlying symmetry. At the level of classical supergravity the symmetry of extremal STU black holes is $[SL(2,\mathbb{R})]^3$. This symmetry may be broken down to $[SL(2,\mathbb{Z})]^3$ by quantum corrections or by the requirement that the electric and magnetic charges have to be quantized. In string theory a consistent microscopic description of the extremal black holes requires $[SL(2,\mathbb{Z})]^3$ symmetry. In ABC system the symmetry is $[SL(2,\mathbb{C})]^3$.

But even if the relation between the STU black holes and the 3-qubit system boils down to their underlying symmetry, this fact by itself can be quite useful. It may allow us to obtain new classes of black hole solutions and provide their interpretation based on the general formalism of quantum information. It may also provide us with an extremely

[1]The explicit construction of BPS black holes with four charges and a finite area of the horizon within $D = 4$ $N = 4$ toroidally compactified string theory was obtained in [11]. This solution has an embedding as a generating solution in the STU model.

nontrivial playground for testing the general ideas of the theory of quantum information. It would be very interesting to see how the puzzles and paradoxes associated with black holes may be related to the puzzles and paradoxes of the quantum information theory.

In this paper we will pursue a detailed analysis of the relations between the structures which appear in the theory of extremal black holes and in the theory of quantum information. In Sec. II we will describe some basic facts about general 2- and 3-qubit systems (for the hep-th reader unfamiliar with these concepts). In Sec. III we will discuss the relation between the 2-qubit systems [17] and the axion-dilaton black holes of [18]. We will also describe the relation between the 3-qubit systems and STU black holes represented as string theory states with some number of D0, D2, D4, and D6 branes. This description is known to provide a microscopic entropy via counting of states of string theory [19,20]. This microscopic entropy coincides with the macroscopic Hawking-Bekenstein entropy (quarter of the area of the horizon) of the STU black holes at large values of charges/branes. Section IV gives a dictionary between a particular (S|TU) basis of STU black holes and the twistor geometry used in the description of the 3-qubit system in [21]. In Sec. V we find a one-to-one correspondence between the states of 3-qubit systems classified in [22] and black holes in string theory. In Sec. VI we observe an intriguing relation between the value of the subsystem entanglement and the value of the quantum corrected entropy of the so-called ‘‘small’’ black holes. These black holes in a classical approximation have zero entropy and a singular horizon, but acquire a nonzero entropy and horizon area after quantum corrections [23–25]. We give a simple expression for the entropy of small black holes in terms of 2-tangles of a 3-qubit system and its norm. Finally, in Sec. VII we show that not only the entropy of the STU black holes, but the most general expression for the black hole and black ring entropy in $N = 8$ supergravity/M theory, given by the famous Cartan $E_{7(7)}$ invariant [26], can also be represented as Cayley's hyperdeterminant describing the 3-tangle of a 3-qubit state. This, in turn, provides a natural link between the 3-qubit states and octonions.

II. QUBITS AND A MEASURE OF ENTANGLEMENT

Let us bring up several most important definitions from quantum information theory, which will be required to understand the correspondence between the language of string theory black holes and the language of the quantum information theory.

Quantum entanglement is a quantum mechanical phenomenon in which the quantum states of two or more objects have to be described with reference to each other.

A quantum bit, or *qubit* is a smallest unit of quantum information. That information is described by a state in a 2-level quantum mechanical system. The two basis states are conventionally written as $|0\rangle$ and $|1\rangle$. A pure qubit state is a linear quantum superposition of those two states. This means that each qubit can be represented as a linear combination of $|0\rangle$ and $|1\rangle$:

$$|\Psi\rangle = \psi_0|0\rangle + \psi_1|1\rangle, \tag{2.1}$$

where ψ_0 and ψ_0 are complex probability amplitudes of finding the system in a particular state when one makes measurements. This leads to a normalization condition

$$|\Psi|^2 = \langle\Psi|\Psi\rangle = \sum_i |\psi_i|^2 = |\psi_0|^2 + |\psi_1|^2 = 1. \tag{2.2}$$

A 1-qubit system usually goes by the name A (Alice).

For a *2-qubit* state AB (Alice and Bob) one has

$$|\Psi\rangle = \psi_{00}|00\rangle + \psi_{01}|01\rangle + \psi_{10}|10\rangle + \psi_{11}|11\rangle, \tag{2.3}$$

with the corresponding normalization condition, $\langle\Psi|\Psi\rangle = 1$. One can introduce a partial density matrix, a trace over the subsystem A, $\rho_A = \mathrm{Tr}_B|\psi\rangle\langle\psi|$, and the same for B. For a pure state, entanglement E is defined as the entropy of either of the two subsystems

$$E(\psi) = -\mathrm{Tr}(\rho_A \log_2 \rho_A) = -\mathrm{Tr}(\rho_B \log_2 \rho_B). \tag{2.4}$$

This is von Neumann entropy of a quantum state. The properties of an AB system are also determined by the so-called concurrence $\mathcal{C}$, which is a measure of the entanglement. Concurrence of the 2-qubit AB system in a pure state can be given as

$$\mathcal{C} = \mathcal{C}_{AB} = 2\sqrt{\det\rho_A} = 2\sqrt{\det\rho_B} = 2|\det\psi|. \tag{2.5}$$

These two measures of entanglement are related to each other [17]:

$$E(C(\psi)) = -\frac{1+\sqrt{1-C^2}}{2}\log_2\frac{1+\sqrt{1-C^2}}{2} - \frac{1-\sqrt{1-C^2}}{2}\log_2\frac{1-\sqrt{1-C^2}}{2}. \tag{2.6}$$

The function $E(C)$ is monotonically increasing, and ranges from 0 to 1 as C goes from 0 to 1.

For a mixed state of the AB system concurrence is more complicated. For our purposes we will need to define the concurrence of a particular AB state inside of a pure 3-qubit state.

The *3-qubit* system ABC (Alice, Bob, and Charlie) in turn is given by the normalized wave function

$$\begin{aligned}|\Psi\rangle &= \sum_{ijk=1,0} \psi_{ijk}|ijk\rangle \\ &= \psi_{000}|000\rangle + \psi_{001}|001\rangle + \psi_{010}|010\rangle + \psi_{011}|011\rangle \\ &\quad + \psi_{100}|100\rangle + \psi_{101}|101\rangle + \psi_{110}|110\rangle + \psi_{111}|111\rangle.\end{aligned} \tag{2.7}$$

A 3-dimensional matrix corresponding to the 3-qubit sys-

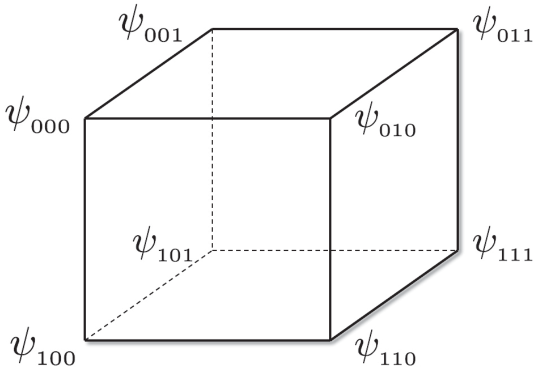

FIG. 1. The $2 \times 2 \times 2$ matrix corresponding to the quantum state (2.7).

tem can be represented as a cube with vertices corresponding to ψ_{ijk}, see Fig. 1.

The 3-qubit system ψ_{ijk} has an invariant, Cayley's hyperdeterminant [13] defined as[2]

$$\begin{aligned}\mathrm{Det}\,\psi &= -\tfrac{1}{2}\epsilon^{ii'}\epsilon^{jj'}\epsilon^{kk'}\epsilon^{mm'}\epsilon^{nn'}\epsilon^{pp'}\psi_{ijk}\psi_{i'j'm}\psi_{npk'}\psi_{n'p'm'}\\ &= \psi_{000}^2\psi_{111}^2 + \psi_{001}^2\psi_{110}^2 + \psi_{010}^2\psi_{101}^2 + \psi_{100}^2\psi_{011}^2\\ &\quad - 2(\psi_{000}\psi_{001}\psi_{110}\psi_{111} + \psi_{000}\psi_{010}\psi_{101}\psi_{111}\\ &\quad + \psi_{000}\psi_{100}\psi_{011}\psi_{111} + \psi_{001}\psi_{010}\psi_{101}\psi_{110}\\ &\quad + \psi_{001}\psi_{100}\psi_{011}\psi_{110} + \psi_{010}\psi_{100}\psi_{011}\psi_{101})\\ &\quad + 4(\psi_{000}\psi_{011}\psi_{101}\psi_{110} + \psi_{001}\psi_{010}\psi_{100}\psi_{111}).\end{aligned} \tag{2.8}$$

The 3-tangle of the ABC system as shown in [14] is given by

$$\tau_{\mathrm{ABC}} = 4|\det\psi|. \tag{2.9}$$

When the wave function is normalized, $\langle\Psi|\Psi\rangle = 1$, the 3-tangle τ_{ABC} is also normalized to take values in the range from 0 to 1.

An important tool in describing 3-qubit states is a reduced density matrix. For example,

$$\rho_{\mathrm{A}} = \mathrm{Tr}_{\mathrm{BC}}|\Psi\rangle\langle\Psi|, \qquad \mathrm{S_A} = 4\det\rho_{\mathrm{A}} \equiv \tau_{\mathrm{A(BC)}}, \tag{2.10}$$

where ρ_A is a 2×2 matrix. $\mathrm{S_A}$ is sometimes called local entropy, it is a measure of how entangled A is with the pair (BC). The threeway tangle τ_{ABC} consists of three contributions [14]:

$$\tau_{\mathrm{ABC}} = \tau_{\mathrm{A(BC)}} - \tau_{\mathrm{AB}} - \tau_{\mathrm{AC}}. \tag{2.11}$$

Each term in Eq. (2.11) is a particular contraction of the 4 terms ψ_{ijk} with each other and with some number of totally antisymmetric 2-component ϵ^{ij} tensor. It was shown in [14] that the first term $\tau_{\mathrm{A(BC)}}$, which is a tangle between Alice with Bob-and-Charlie system, is a square of the concurrence in A(BC) system: $\tau_{\mathrm{A(BC)}} = \mathcal{C}^2_{\mathrm{A(BC)}}$. The second term, $\tau_{\mathrm{AB}} = \mathcal{C}^2_{\mathrm{AB}}$, which is called a 2-tangle between Alice and Bob in the 3-cubit system ABC, is a square of the concurrence in AB system inside the ABC, $\mathcal{C}_{\mathrm{AB}}$ will be defined below in Eq. (2.16). Finally, the third one $\tau_{\mathrm{AC}} = \mathcal{C}^2_{\mathrm{AC}}$ is the 2-tangle between Alice and Charlie in ABC; it is a square of the concurrence of the AC system inside ABC, $\mathcal{C}_{\mathrm{AC}}$ will be defined below in Eq. (2.17). Equation (2.11) and its analogues obtained by permutations of A, B, and C, can be represented in the form [14]

$$\tau_{\mathrm{ABC}} = \mathcal{C}^2_{\mathrm{A(BC)}} - \mathcal{C}^2_{\mathrm{AB}} - \mathcal{C}^2_{\mathrm{AC}}, \tag{2.12}$$

$$\tau_{\mathrm{ABC}} = \mathcal{C}^2_{\mathrm{B(CA)}} - \mathcal{C}^2_{\mathrm{BC}} - \mathcal{C}^2_{\mathrm{BA}}, \tag{2.13}$$

$$\tau_{\mathrm{ABC}} = \mathcal{C}^2_{\mathrm{C(BA)}} - \mathcal{C}^2_{\mathrm{CB}} - \mathcal{C}^2_{\mathrm{CA}}. \tag{2.14}$$

Here

$$\begin{gathered}\mathcal{C}^2_{\mathrm{A(BC)}} = 4\det\rho_{\mathrm{A}}, \qquad \mathcal{C}^2_{\mathrm{B(AC)}} = 4\det\rho_{\mathrm{B}},\\ \mathcal{C}^2_{\mathrm{C(AB)}} = 4\det\rho_{\mathrm{C}},\end{gathered} \tag{2.15}$$

is a squared concurrence between A and the pair BC, B and the pair AC, C and the pair AB, respectively. One can also define the concurrence of AB inside ABC in terms of various combinations of ψ_{ijk}.

$$\mathcal{C}_{\mathrm{AB}} = (\det\rho_{\mathrm{C}} - \det\rho_{\mathrm{A}} - \det\rho_{\mathrm{B}} - \tfrac{1}{2}\tau_{\mathrm{ABC}})^{1/2}, \tag{2.16}$$

$$\mathcal{C}_{\mathrm{AC}} = (\det\rho_{\mathrm{B}} - \det\rho_{\mathrm{A}} - \det\rho_{\mathrm{C}} - \tfrac{1}{2}\tau_{\mathrm{ABC}})^{1/2}, \tag{2.17}$$

$$\mathcal{C}_{\mathrm{BC}} = (\det\rho_{\mathrm{A}} - \det\rho_{\mathrm{B}} - \det\rho_{\mathrm{C}} - \tfrac{1}{2}\tau_{\mathrm{ABC}})^{1/2}. \tag{2.18}$$

In Eqs. (2.11), (2.12), (2.13), (2.14), (2.15), (2.16), (2.17), and (2.18) each term scales under the rescaling of ψ_{ijk} homogeneously. Thus they are valid not only for the usual normalized vectors, satisfying the condition $\langle\Psi|\Psi\rangle = 1$, but also for vectors with arbitrary norm

$$|\Psi| \equiv \sqrt{\langle\Psi|\Psi\rangle} \neq 1. \tag{2.19}$$

One may try to interpret $\langle\Psi|\Psi\rangle$ for the states with $|\Psi^2| \neq 1$ as a number density rather than a probability density. One may also notice that

$$|\Psi|^2 \equiv \rho \equiv \mathrm{Tr}_{\mathrm{ABC}}|\Psi\rangle\langle\Psi| \tag{2.20}$$

and

$$\rho = \mathrm{Tr_A}\rho_{\mathrm{A}} = \mathrm{Tr_B}\rho_{\mathrm{B}} = \mathrm{Tr_C}\rho_{\mathrm{C}} \neq 1. \tag{2.21}$$

The difference between normalized and unnormalized vectors plays a significant role in our subsequent analysis because we are going to use the concepts of the 2-tangle and 3-tangle not for the calculation of probabilities in quantum mechanics, but for the calculation of black hole

[2] In this paper we will always write the usual determinant of a matrix ψ_{ij} as $\det\psi$, and the hyperdeterminant of a matrix ψ_{ijk} as $\mathrm{Det}\psi$.

entropy, which can be much greater than 1. In what follows we will discuss general states with norm $|\Psi| \neq 1$, and in the calculations of such objects as the 3-tangle or Cayley's hyperdeterminant we will use the states $|\Psi\rangle$ (2.7) without imposing any normalization constraints on ψ_{ijk}.

III. BLACK HOLES IN SUPERGRAVITY, STRING THEORY, AND ABC SYSTEM

A. Axion-dilaton extremal black holes and concurrence of a 2-qubit system

As a warm up to STU black holes 3-qubits relation we start with a simpler case of the so-called axion-dilaton black hole solutions with manifest $SL(2, Z)$-symmetry in [18] and display their relation to a 2-qubit system. In the case of $N = 2$ supergravity with one vector multiplet in a version without a prepotential the double-extremal axion-dilaton black holes were constructed in [18,27]. The double-extreme black holes solve the attractor equations [3] for the scalars and have everywhere constant scalars. The set of electric and magnetic charges is (p^0, p^1, q_0, q_1), and the entropy formula is given by the following $SL(2, \mathbb{Z})$-invariant expression

$$\frac{S}{\pi} = |p^0 q_1 - q_0 p^1|. \tag{3.1}$$

If we identify the charges with the components of a 2×2-matrix ψ_{ij}

$$\begin{pmatrix} p^0 \\ p^1 \\ q_1 \\ q_0 \end{pmatrix} = \begin{pmatrix} \psi_{00} \\ \psi_{01} \\ \psi_{10} \\ \psi_{11} \end{pmatrix}, \tag{3.2}$$

the entropy formula is proportional to the concurrence of a 2-qubit system:

$$S = \pi |\det \psi| = \frac{\pi}{2} \mathcal{C}, \qquad \psi = \begin{pmatrix} p^0 & p^1 \\ q^1 & q_0 \end{pmatrix} = \begin{pmatrix} \psi_{00} & \psi_{01} \\ \psi_{10} & \psi_{11} \end{pmatrix}. \tag{3.3}$$

Thus we have identified the features in the axion-dilaton black holes with some analogous features in a 2-qubit system AB in a pure state. In particular, the entropy formula for arbitrary integer charges is equal to the concurrence $\mathcal{C}$ of the 2-qubit system described by the unnormalized vector in Eq. (2.5).

B. STU black holes and 3-qubits

Consider type IIA string theory compactified on a Calabi-Yau space in the presence of D0, D2, D4, and D6 branes. The corresponding effective $N = 2$ supergravity is described by $N = 2$ gravitational multiplet and 3 vector multiplets. First we consider the simplest version of supergravity with the prepotential $F = \text{STU}$. The electric and magnetic charges of the graviphoton are denoted by (p^0, q_0), and the ones for the 3 vector multiplets are (p^1, q_1), (p^2, q_2), (p^3, q_3), respectively. These supergravity charges are known to originate from the number of D0, D2, D4, and D6 branes as follows: the number n_{D0} of D0 branes is q_0, the numbers k_{D2}, m_{D2}, l_{D2} of D2 branes wrapped on 3 2-cycles are q_1, q_2, q_3, respectively. The numbers k_{D4}, m_{D4}, l_{D4} of D4 branes wrapped on 3 4-cycles, dual to the relevant 2-cycles are p^1, p^2, p^3 and the number of D6 branes is p^0. Negative number of branes corresponds to a positive number of antibranes of the same kind.

Following Ref. [10], we can associate all magnetic charges with the presence of 1s in the ABC system according to a simple rule illustrated by Eq. (3.5). The state with the magnetic charge p^0 is the state $|000\rangle$ which has zero number of 1s. The state with charge p^1 corresponds to $|001\rangle$, which has 1 in the first position; p^2 corresponds to $|010\rangle$, which has 1 in the second position; p^3 corresponds to $|100\rangle$, which has 1 in the third position. (We count positions from the right to the left.) We associate electric charges with the presence of 0s in the ABC system. Thus the state q_0 corresponds to $|111\rangle$, which has no 0s; q_1, the state $|110\rangle$, has 0 in the first position; q_2, the state $|101\rangle$, has 0 in the second position; q_3, the state $|011\rangle$, has 0 in the third position. The signs are not explained by this rule, however, they have to be taken in a way so that the black hole entropy is an $[SL(2, \mathbb{Z})]^3$-invariant for integer charges and is defined by the properties of the ABC system. The explanation of signs is actually coming from the corresponding cube in p, q variables given in Fig. 2 which was presented in [12].

All S-, T-, and U-dualities in this basis are nonperturbative. However, one can switch to a different basis by performing an $Sp(8, \mathbb{Z})$ transformation which transforms both the symplectic section (X, F) as well as the charges (p, q), e.g.

$$\begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix}' = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix} \tag{3.4}$$

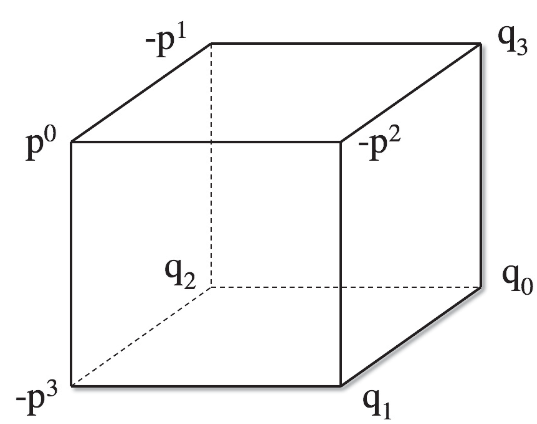

FIG. 2. The $2 \times 2 \times 2$ matrix corresponding to supergravity black holes [12].

with $A^T C - C^T A = B^T D - D^T B = 0$ and $A^T D - C^T B = 1$. In manifestly STU-symmetric version we have

$$\text{Supergravity}\quad\text{ABC}\quad\text{String Theory}\begin{pmatrix} p^0 \\ p^1 \\ p^2 \\ p^3 \\ q_0 \\ q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} \psi_{000} \\ -\psi_{001} \\ -\psi_{010} \\ -\psi_{100} \\ \psi_{111} \\ \psi_{110} \\ \psi_{101} \\ \psi_{011} \end{pmatrix} = \begin{pmatrix} n_{D6} \\ k_{D4} \\ m_{D4} \\ l_{D4} \\ n_{D0} \\ k_{D2} \\ m_{D2} \\ l_{D2} \end{pmatrix}. \tag{3.5}$$

It is important to stress here that the cubes, according to Fig. 1 as well as Fig. 2, have 3 magnetic and 1 electric charge in upper 4 corners, and 3 electric and 1 magnetic charge in lower 4 corners.

To associate these charges/numbers of branes with the elements of the ψ_{ijk} matrix one has to keep in mind that the entropy and the absolute value of the hyperdeterminant are invariant under the $[SL(2, \mathbb{Z})]^3$ subgroup of the symplectic $Sp(8, \mathbb{Z})$ transformations. We may go to an (S|TU), (T|US), or (U|ST) basis in which one of the duality transformations becomes perturbative and does not mix electric and magnetic charges. In this case either S, T, or U direction becomes different from the other two directions. Here are three possible options for $(p, q)'$ which one can get by returning to the symmetric STU basis:

(i) STU → (S|TU) → STU

$$\text{Supergravity}\quad\text{ABC}\quad\text{String Theory}\begin{pmatrix} dp^0 + cp^1 \\ bp^0 + ap^1 \\ dp^2 + cq_3 \\ dp^3 + cq_2 \\ aq_0 - bq_1 \\ -cq_0 + dq_1 \\ bp^3 + aq_2 \\ bp^2 + aq_3 \end{pmatrix} = \begin{pmatrix} \alpha_{000} \\ -\alpha_{001} \\ -\alpha_{010} \\ -\alpha_{100} \\ \alpha_{111} \\ \alpha_{110} \\ \alpha_{101} \\ \alpha_{011} \end{pmatrix} = \begin{pmatrix} dn_{D6} + ck_{D4} \\ bn_{D6} + ak_{D4} \\ dm_{D4} + cl_{D2} \\ dl_{D4} + cm_{D2} \\ an_{D0} - bk_{D2} \\ -cn_{D0} + dk_{D2} \\ bl_{D4} + am_{D2} \\ bm_{D4} + al_{D2} \end{pmatrix} \tag{3.6}$$

(ii) STU → (T|US) → STU

$$\text{Supergravity}\quad\text{ABC}\quad\text{String Theory}\begin{pmatrix} dp^0 + cp^2 \\ dp^1 + cq_3 \\ bp_0 + ap^2 \\ dp^3 + cq_1 \\ aq_0 - bq_2 \\ bp^3 + aq_1 \\ -cq_0 + dq_2 \\ bp^1 + aq_3 \end{pmatrix} = \begin{pmatrix} \beta_{000} \\ -\beta_{001} \\ -\beta_{010} \\ -\beta_{100} \\ \beta_{111} \\ \beta_{110} \\ \beta_{101} \\ \beta_{011} \end{pmatrix} = \begin{pmatrix} dn_{D6} + cm_{D4} \\ dk_{D4} + cl_{D2} \\ bn_{D6} + am_{D4} \\ dl_{D4} + ck_{D2} \\ an_{D0} - bm_{D2} \\ bl_{D4} + ak_{D2} \\ -cn_{D0} + dm_{D2} \\ bk_{D4} + al_{D2} \end{pmatrix} \tag{3.7}$$

(iii) STU → (U|ST) → STU

$$\text{Supergravity}\quad\text{ABC}\quad\text{String Theory}\begin{pmatrix} dp^0 + cp^3 \\ dp^1 + cq_2 \\ dp_2 + cq_1 \\ bp^0 + ap^3 \\ aq_0 - bq_3 \\ bp^2 + aq_1 \\ bp^1 + aq_2 \\ -cq_0 + dq_3 \end{pmatrix} = \begin{pmatrix} \gamma_{000} \\ -\gamma_{001} \\ -\gamma_{010} \\ -\gamma_{100} \\ \gamma_{111} \\ \gamma_{110} \\ \gamma_{101} \\ \gamma_{011} \end{pmatrix} = \begin{pmatrix} dn_{D6} + cl_{D4} \\ dk_{D4} + cm_{D2} \\ dm_{D4} + ck_{D2} \\ bn_{D6} + al_{D4} \\ an_{D0} - bl_{D2} \\ bm_{D4} + ak_{D2} \\ bk_{D4} + am_{D2} \\ -cn_{D0} + dl_{D2} \end{pmatrix} \tag{3.8}$$

According to [12], the black hole entropy of BPS black holes is given by

$$\frac{S}{\pi} = (W(p^\Lambda, q_\Lambda))^{1/2}, \tag{3.9}$$

where

$$W(p^\Lambda, q_\Lambda) = -(p \cdot q)^2 + 4((p^1 q_1)(p^2 q_2) + (p^1 q_1)(p^3 q_3) + (p^3 q_3)(p^2 q_2)) - 4p^0 q_1 q_2 q_3 + 4q_0 p^1 p^2 p^3 \tag{3.10}$$

and

$$p \cdot q = (p^0 q_0) + (p^1 q_1) + (p^2 q_2) + (p^3 q_3). \tag{3.11}$$

The function $W(p^\Lambda, q_\Lambda)$ is symmetric under transformations: $p^1 \leftrightarrow p^2 \leftrightarrow p^3$ and $q_1 \leftrightarrow q_2 \leftrightarrow q_3$ and we have to require that $W > 0$. In addition to these symmetries, one can also replace each p^Λ and q_Λ in the expression (3.10) for W by the combinations of p^Λ and q_Λ shown in the first column in Eqs. (3.6), (3.7), or (3.8).

As pointed out in [10], the classical expression for the entropy of the STU black holes $W(p^\Lambda, q_\Lambda)$ (3.10) can be represented in a very beautiful form:

$$S^{\mathrm{BPS}} = \pi\sqrt{W} = \frac{\pi}{2}\sqrt{-\det\psi}, \qquad \det\psi < 0, \tag{3.12}$$

where $\det\psi$ is the Cayley's hyperdeterminant of the unnormalized vector with components ψ_{ijk} related to p^Λ and q_Λ by Eq. (3.5). The BPS black hole entropy condition $W^{\mathrm{BPS}} > 0$ requires the related Cayley's hyperdeterminant to be negative.

Recently the entropy of some examples of extremal non-BPS STU black holes have been calculated in [28,29].[3] We will show in [31] that in the general case, the entropy of nonextremal black holes in STU model is equal to

$$S^{\text{non-BPS}} = \pi\sqrt{-W} = \pi\sqrt{\det\psi}, \qquad \det\psi > 0. \tag{3.13}$$

Thus we find that in all cases, including BPS and non-BPS, the classical supergravity entropy formula is

$$S = \pi\sqrt{|W(p,q)|} = \pi\sqrt{|\det\psi|} = \frac{\pi}{2}\sqrt{\tau_{\mathrm{ABC}}}. \tag{3.14}$$

Here $\tau_{\mathrm{ABC}} = 4|\det a|$ determines the threeway entanglement of the three qubits A, B, and C, and ψ_{ijk} defines an unnormalized vector with the coefficients depending on (p, q), see Eq. (3.5).

Note that because of the $[SL(2,\mathbb{Z})]^3$ invariance, the result of the calculation of the black hole entropy S^{BPS} does not change if instead of the hyperdeterminant of the matrix ψ_{ijk} defined in Eq. (3.5) one uses the hyperdeterminant of the matrix α_{ijk} defined in (3.6), the hyperdeterminant of the matrix β_{ijk} defined in (3.7), or the hyperdeterminant of the matrix γ_{ijk} defined in (3.8).

In string theory the microscopic entropy of the set of states with some number of branes was derived in [4] as

$$\ln d(p,q) = S_{\mathrm{micro}}(p,q). \tag{3.16}$$

Here $d(p,q)$ counts the total number of states for a given set of integers (p,q). In the limit of large (p,q)

$$S_{\mathrm{micro}}(p,q) \Rightarrow S_{\mathrm{macro}}(p,q). \tag{3.17}$$

For our STU model the specific calculation was performed in [20] in the context of M theory which by duality can be related to type IIB string theory with the relation between (p,q) and the numbers of D_0, D_2, D_4, D_6 branes shown in Eq. (3.5). Their expression for the square of the microscopic entropy in addition to the classical expression $\pi^2 W(p^\Lambda, q_\Lambda)$, which is quartic in charges, contained some extra terms quadratic in charges, which come from quantum corrections. We will come back to a more detailed discussion of these terms later.

The interest to the extremal black holes was enhanced during the last couple of years by the Ooguri, Strominger, and Vafa (OSV) conjecture [32] about the relation between extremal black holes and topological string theory, see for example [33] where these recent developments are presented. In these new developments it was important to differentiate between the so-called "large" and small black holes. The classical black hole entropy equal to $1/4$ of the area of the horizon, in the limit of very large charges when quantum corrections are small is important for defining two different kinds of extremal black holes which have analogies in definition of classes of states in ABC systems in quantum information theory.

(1) Large black holes, $S_{\mathrm{class}} \neq 0 \quad \rightarrow \quad$ entangled Greenberger, Horne, Zeilinger (GHZ) class of states, $|\det\psi| \neq 0$

(2) Small black holes, $S_{\mathrm{class}} = 0 \quad \rightarrow \quad$ nonentangled, bipartite, and W states, $|\det\psi = 0|$

We will present more details on GHZ canonical states and GHZ class of states with nonvanishing 3-tangle, as well as on nonentangled (completely separable), bipartite, and W states with vanishing 3-tangle in Sec. V. Here we only stress the fact that these two groups are differentiated by vanishing or nonvanishing 3-tangle which coincides with the vanishing or nonvanishing area of the horizon of the classical extremal black holes. We used here an expression S_{class} to emphasize that until now we were talking about black holes without taking into account stringy quantum corrections. With account of these corrections, the classical entropy formula changes, terms quadratic in charges have to be added to the quartic expression W [20]. Originally there was a discrepancy between the microscopic and macroscopic entropies. After R^2 quantum corrections were included into the supergravity action in [34],

[3]Examples of extremal nonsupersymmetric black holes were presented before in $N = 8$ theory in [30], where it was shown that the flip of the sign of one of the charges converts BPS solutions to non-BPS solutions.

the discrepancy with the microscopic entropy was removed. For large black holes the extra terms provide only a small correction. However, recently a new class of extremal black holes, small black holes, was identified, for which the quantum corrections play a crucial role. It was found in [23,24] that the small black holes with $S_{\rm class} = \frac{A_{\rm class}}{4} = 0$ actually acquire a nonvanishing entropy and a nonvanishing area of the horizon, $S_{\rm quant} = \frac{A_{\rm quant}}{2} \neq 0$. This phenomenon is known as a "stringy cloak for the classical singularity." This is a realization of the idea of a "stretched black hole horizon" proposed earlier by Susskind and Sen [35,36].

Completely separable states, including, e.g., the states with only one (electric or magnetic) charge, also have a classically vanishing entropy and area of the horizon. Recently it was found in the context of the Sen's new entropy function formalism [37] that the R^4 type quantum corrections may lead to a nonvanishing entropy and stretching of the horizon even for such states [25].

IV. BLACK HOLES, 3-QUBIT STATES, AND TWISTORS

The form of the STU black holes which we studied above is completely symmetric in STU variables. This model is described by the prepotential $F = \frac{X^1X^2X^3}{X_0}$. The symplectic section consists of four homogeneous coordinates X^Λ, depending on 3 special coordinates S, T, U, $X^\Lambda = \{X^0 = 1, X^1 = \mathrm{S}, X^2 = \mathrm{T}, X^3 = \mathrm{U}\}$ and four derivatives of the prepotential, $F_\Lambda \equiv \frac{\partial F}{\partial X^\Lambda} = \{F_0 = -\mathrm{STU}, F_1 = \mathrm{TU}, F_2 = \mathrm{SU}, F_3 = \mathrm{ST}\}$.

One can easily switch to the form in which one of the moduli is not on equal footing with others. In ABC system this would make one of the three friends, say Alice, not on equal footing with Bob and Charlie. In the black hole case we can use a symplectic transformation, a particular $Sp(8, \mathbb{Z})$ matrix, to transform into a new basis which has no prepotential [38]. In this new basis one of the moduli, say S, is removed from the set of new homogeneous coordinates, $\hat{X}^\Lambda$ and it shows up only in $\hat{F}_\Lambda$'s so that the total section is given by hatted coordinates $\hat{X}^\Lambda = \frac{1}{\sqrt{2}}\{1 - \mathrm{TU}, -(\mathrm{T} + \mathrm{U}), -(1 + \mathrm{TU}), (\mathrm{T} - \mathrm{U})\}$ and $\hat{F}_\Lambda = S\eta_{\Lambda\Sigma}\hat{X}^\Sigma$. Here $\eta_{\Lambda\Sigma} = (+ + - -)$. The (S|TU) coordinates now parametrize a coset space $\frac{SU(1,1)}{U(1)} \times \frac{SO(2,2)}{SO(2)\times SO(2)}$. The metric $\eta_{\Lambda\Sigma} = (+ + - -)$ reflects the manifest $SO(2, 2)$ symmetry.

In the relevant description of the ABC system one can say: Alice was promoted to the status of the $\hat{F}_\Lambda$ person whereas Bob and Charlie remain the $\hat{X}^\Lambda$-guys. Or, in an opposite mood one can say that Alice was excluded from the list of $\hat{X}^\Lambda$ persons and became an $\hat{F}_\Lambda$ person. Either way, she is not treated on equal footing with Bob and Charlie anymore. The corresponding transformation also produces the new hatted black hole charges ($\hat{p}^\Lambda$, $\hat{q}_\Lambda$). In terms of these hatted charges our lengthy expression for the entropy given by Eqs. (3.10) and (4.4) looks very simple [12]

$$\mathrm{Det}\ a = W(p(\hat{p}, \hat{q}), q(\hat{p}, \hat{q})) = \hat{p}^2\hat{q}^2 - (\hat{p}\cdot\hat{q})^2. \quad (4.1)$$

Here all contractions of the hatted 4-vectors are done with the metric $\eta_{\Lambda\Sigma} = (+ + - -)$, $\hat{p}^2 = \hat{p}^\Lambda\eta_{\Lambda\Sigma}\hat{p}^\Sigma = (\hat{p}^1)^2 + (\hat{p}^2)^2 - (\hat{p}^3)^2 - (\hat{p}^4)^2$, $\hat{p}\cdot\hat{q} \equiv \hat{p}^\Lambda\hat{q}_\Lambda$, etc. The duality invariant black hole entropy described by expression in Eq. (4.1) for STU black holes was discovered in the context of $N = 4$ string theory in [39].

The relevant 3-qubit entanglement in this basis is given by

$$\tau_{\rm ABC} = 4|\mathrm{Det}\psi| = 4|\hat{p}^2\hat{q}^2 - (\hat{p}\cdot\hat{q})^2| = 2|P^{\Lambda\Sigma}P_{\Lambda\Sigma}| = |(P - *P)\cdot(P + *P)|, \quad (4.2)$$

where the antisymmetric bivector $P^{\Lambda\Sigma}$ is defined as follows

$$P^{\Lambda\Sigma} \equiv \hat{p}^\Lambda\hat{q}^\Sigma - \hat{p}^\Sigma\hat{q}^\Lambda, \quad (4.3)$$

where $*P$ is a dual to P and $\hat{q}^\Lambda = \eta^{\Lambda\Sigma}\hat{q}_\Sigma$.

This construction may be easily compared with the description of the 3-qubit system in the context of twistor geometry [21]. Indeed, by some operation, closely related to the change of a basis in the black hole system which requires to put e.g. Alice on nonequal status with Bob and Charlie, the form of the 3-tangle is obtained in [21]:

$$\tau_{\rm ABC} = 4|\det\psi| = 4|(Z\cdot Z)(W\cdot W) - (Z\cdot W)^2| = 2|P^{\mu\nu}P_{\mu\nu}|, \quad (4.4)$$

where the bivector

$$P^{\mu\nu} \equiv Z^\mu W^\nu - Z^\nu W^\mu, \quad (4.5)$$

$Z\cdot Z = Z^\mu\eta_{\mu\nu}Z^\nu$, and $\eta_{\mu\nu} = (+ + + -)$, i.e. each vector Z^μ and W^μ is a complex vector in $SO(3.1)$ space.

Twistors associated with null vectors can be defined either in spaces with Minkowski signature $+ + + -$ or in spaces with $(+ + - -)$. The relation between the corresponding 2-component spinors is the following. For the case of null vectors, $Z^\mu E_{\mu\rm BC} = a_{0\rm BC}$, $W^\mu E_{\mu\rm BC} = a_{1\rm BC}$ of [21] one has to take the twistors $\lambda_{\rm B}^{\rm A}$ and $\tilde{\lambda}_{\rm C}^{\rm A}$ in $a_{\rm ABC} = \lambda_{\rm B}^{\rm A}\tilde{\lambda}_{\rm C}^{\rm A}$ (no summation in A) to be related via complex conjugation, $\tilde{\lambda} = \pm\bar{\lambda}$. In $(+ + - -)$ signature these two twistors $\lambda_{\rm B}$ and $\tilde{\lambda}_{\rm C}$ have to be completely independent real 2-component objects, since our $SO(2, 2)$ without any complexification is isomorphic to $SL(2, \mathbb{R}\times SL(2, \mathbb{R})$. This completes the translation from the black holes in the (S|TU) basis to the twistor form of the 3-qubit ABC system in the (A|BC) basis.

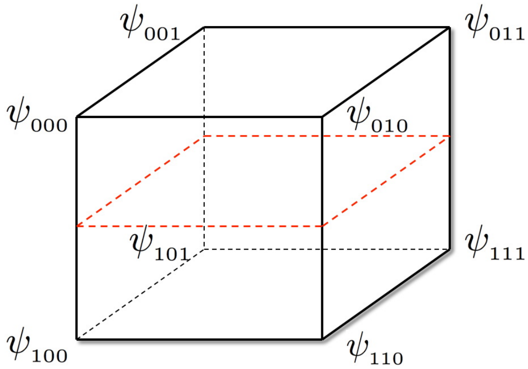

FIG. 3 (color online). The $2 \times 2 \times 2$ matrix corresponding to twistor picture of a 3-cubit in [21]. The combination of 4 upper corners forms a 4-vector Z. All lower corners are used to form a 4-vector W.

To make the relation between black holes and 3-qubit states in twistor form clear, let us look at the pictures. First, we can cut the 3-qubit cube in Fig. 1 by a horizontal surface so that all upper corners which have 0 in the first position are used for forming a 4-vector Z in [21]. All lower corners, which have 1 in the first position, are used to form a 4-vector W, see Fig. 3.

In order to see the relation between black holes and twistors we have to use a cube which appears after an $Sp(8, \mathbb{Z})$ duality transformation to the hatted basis, see Fig. 4.

In the twistor formulation of the 3-qubit system, the classification of the states proceeds in simple geometric terms related to properties of the Z^μ and W^μ vectors translated into the language of the twistor theory. Using our hatted vectors $\hat{q}$ and $\hat{p}$ we easily perform an analogous classification for black holes. Clearly, the cube in Fig. 4 with the vertical slice between front and back is related to the Fig. 3 after a rotation and renaming the corners.

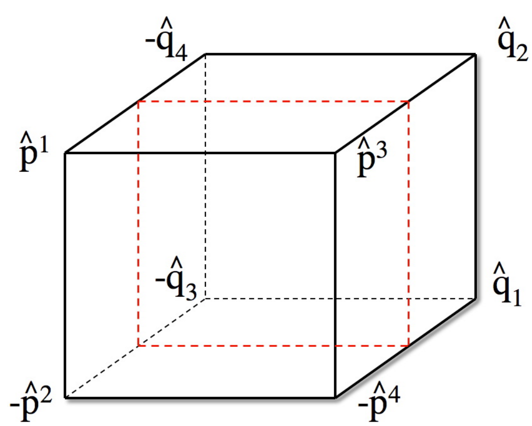

FIG. 4 (color online). The $2 \times 2 \times 2$ matrix corresponding to supergravity black holes [12] in the hatted basis, $\hat{p}$ and $\hat{q}$. One has to slice this cube vertically so that the back side is cut from the front side. In this way we will separate the 4 corners in the front forming a $\hat{p}$ vector and the 4 corners in the back forming a $\hat{q}$ vector.

V. CLASSIFICATION OF STATES OF EXTREMAL BLACK HOLES AND 3-QUBIT STATES

In ABC systems there are two groups of states, each with subdivisions, see Table I, where the values of 3-tangle and local entropies are given [22]. In group A one finds non-entangled product space (completely separable states) and bipartite entanglement (biseparable states). In group B of genuine entangled 3-qubit states there are two different classes: W class and GHZ class. In this classification only GHZ class of states [40] corresponds to large extremal black holes (i.e. to usual extreme black holes) since $\tau_{ABC} = (\frac{2S_{class}}{\pi})^2 \neq 0$.

All states except the GHZ state (i.e. completely separable, biseparable, and W-class states) have a vanishing 3-tangle/classical entropy $\tau_{ABC} = (\frac{2S_{class}}{\pi})^2 = 0$. All of these may describe the small black holes where small is defined by the vanishing area of the horizon of the classical black hole solution. We will find examples of all such black holes.

There are many ways to classify different states of the 3-qubit system. We found it most convenient to classify all possible states by discussing several ways to place charges to the corners of the cube shown in Figs. 1 and 2.

A. All states with vanishing 3-tangle and vanishing black hole entropy; small black holes

For all black holes with vanishing 3-tangle τ_{ABC}, i.e. with vanishing total black hole entropy, one has the following relations for the local entropies defined in (2.10):

$$S_A = \mathcal{C}_{AB}^2 + \mathcal{C}_{AC}^2, \tag{5.1}$$

$$S_B = \mathcal{C}_{AB}^2 + \mathcal{C}_{BC}^2, \tag{5.2}$$

$$S_C = \mathcal{C}_{CB}^2 + \mathcal{C}_{AC}^2. \tag{5.3}$$

1. Nonentangled product space, A-B-C state

An easy way to see the properties of a completely separable state is by looking at the cube which has just

TABLE I. Values of the local entropies S_A, S_B, S_C defined in (2.10) and the 3-tangle τ_{ABC} for the different classes.

Class	S_A	S_B	S_C	τ_{ABC}
A-B-C	0	0	0	0
A-BC	0	>0	>0	0
B-AC	>0	0	>0	0
C-AB	>0	>0	0	0
W	>0	>0	>0	0
GHZ	>0	>0	>0	>0

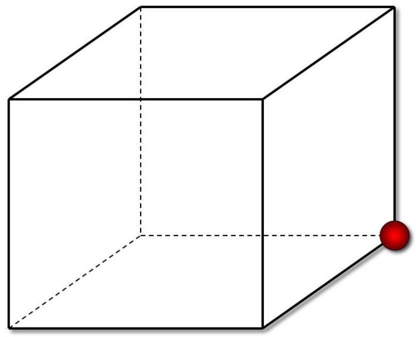

FIG. 5 (color online). The $2 \times 2 \times 2$ matrix with all entries vanishing except one, e.g. q_0. We show it by a corner with a circle. This corresponds to a nonentangled completely separable state describing a black hole with just one charge, q_0, with vanishing area of the horizon.

one corner with a nonvanishing entry. All concurrences are vanishing. As an example, we may consider a black hole with just one charge, e.g. q_0, with vanishing area of the horizon and null singularity, see Fig. 5. The corresponding quantum state is $|\Psi\rangle = q_0|111\rangle$, i.e. $\psi_{111} = q_0$ in the basis (3.5). For this state one has

$$S_A = S_B = S_C = 0, \qquad \mathcal{C}_{AB} = \mathcal{C}_{AC} = \mathcal{C}_{BC} = 0, \qquad \tau_{ABC} = 0. \tag{5.4}$$

Quantum corrections may stretch the horizon. As a result, this black hole may acquire a nonzero entropy proportional to $\sqrt{|q_0|} = \sqrt{|\Psi|}$ [25], see Sec. VI.

One could also consider a cube with two charges connected to each other by an edge, for example, q_0 and q_1, with $|\Psi\rangle = q_0|111\rangle + q_1|011\rangle$ see Fig. 6. This would also represent a completely separable state; all corresponding determinants would vanish.

It is instructive to see how the state $|\Psi\rangle = q_0|111\rangle$ looks in the S basis, in terms of the S-basis decomposition $|\Psi\rangle = \sum \alpha_{ijk}|ijk\rangle_\alpha$. From the dictionary Eq. (3.6) one finds that $|\Psi\rangle = q_0|111\rangle = aq_0|111\rangle_\alpha - cq_0|110\rangle_\alpha$. This state, up to numerical coefficients, coincides with the state shown in Fig. 6.

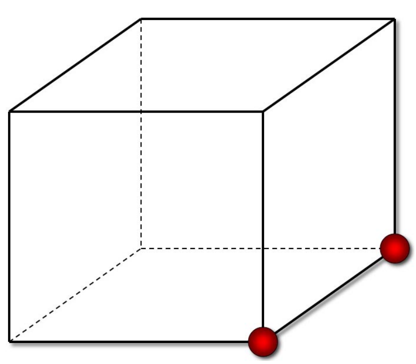

FIG. 6 (color online). The $2 \times 2 \times 2$ matrix with two charges connected to each other by an edge. This configuration also corresponds to a nonentangled completely separable state describing a black hole with vanishing area of the horizon, in the classical approximation.

Similarly, when we go to the T basis or U basis, we will get the states $|\Psi\rangle = aq_0|111\rangle_\beta - cq_0|101\rangle_\beta$ and $|\Psi\rangle = aq_0|111\rangle_\gamma - cq_0|011\rangle_\gamma$. In all of these cases we obtain states described by the cubes with the charge aq_0 in the same position as in Fig. 6 and with a second charge $-cq_i$ connected to it by an edge. All of these cases belong to the same class of completely separable states.

If one tries to add more charges, or place them differently (i.e. add charges p^Λ to an already existing charge q^Λ), one can only produce states that will not be completely separable. Therefore the simple cube with one entry, Fig. 5, represents the general class of all completely separable states.

2. Bipartite entanglement; A-BC state

In order to obtain a biseparable state one may consider a cube with two nonvanishing entries in the opposite corners of one side of the cube so that there is one nonvanishing 2-tangle, for example τ_{BC}. This state shown in Fig. 7 describes a black hole with charges q_0 and p^1, which corresponds to a quantum state $|\Psi\rangle = -p^1|001\rangle + q_0|111\rangle$ (the signs are due to the translation between the charges and ψ_{ijk}, Eq. (3.5)). In this case we have two nonvanishing entanglements between the 1-qubit and a 2-qubit system (or 2 local entropies).

$$S_A = 0, \tag{5.5}$$

$$S_B = \mathcal{C}_{BC}^2 = 4|q_0 p^1|^2 \neq 0, \tag{5.6}$$

$$S_C = \mathcal{C}_{BC}^2 = 4|q_0 p^1|^2 \neq 0. \tag{5.7}$$

This is the small black hole with just 2 charges and with classically vanishing entropy and the area of the horizon

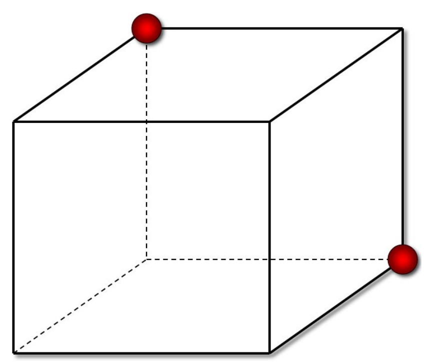

FIG. 7 (color online). The $2 \times 2 \times 2$ matrix with all entries vanishing except two entries on the same side but in opposite corners. They are shown by circles, one for $\psi_{111} = q_0$ and one for $\psi_{001} = -p^1$. This is the case of the small black hole with just 2 charges q_0 and p^1 and with classically vanishing area of the horizon.

[24]. When quantum corrections are included, which lead to quantum stretching of the horizon, the value of the new area is proportional to the only nonvanishing concurrence of the BC system inside the ABC system, $\mathcal{C}_{\rm BC} = 2|q_0 p^1|$, see Sec. VI. It is also a concurrence $\mathcal{C}_{\rm B(AC)} = 2|q_0 p^1|$ between Bob and the system of Alice-Charlie as well as a concurrence $\mathcal{C}_{\rm C(AB)} = 2|q_0 p^1|$ between Charlie and the system of Alice-Bob.

3. *W* class of states

Now let us consider 3 entries in the black hole case: q_0, p^1, and p^2 charges, as shown in Fig. 8. This is the state $|\Psi\rangle = -p^1|001\rangle - p^2|010\rangle + q_0|111\rangle$. None of the local entropies is vanishing, however we still have a vanishing 3-tangle and, at the classical level, vanishing entropy and the area of the horizon [24]. The corresponding black holes may be corrected and the area of the horizon with account of quantum corrections may depend on $q_0 p^1$ and $q_0 p^2$. Here again we will find that the stretched horizon depends on nonvanishing concurrences of the 2-qubit systems inside ABC, see Sec. VI.

$$\mathcal{C}_{\rm AB} = 2|q_0 p^1| \neq 0, \tag{5.8}$$

$$\mathcal{C}_{\rm BC} = 2|q_0 p^2| \neq 0, \tag{5.9}$$

$$\mathcal{C}_{\rm AC} = 2|p^1 p^2| \neq 0, \tag{5.10}$$

and

$$\mathrm{S}_{\rm A} = \mathcal{C}^2_{\rm AB} + \mathcal{C}^2_{\rm AC} = (p^1)^2((q_0)^2 + (p^2)^2), \tag{5.11}$$

$$\mathrm{S}_{\rm B} = \mathcal{C}^2_{\rm AB} + \mathcal{C}^2_{\rm BC} = (q_0)^2((p^1)^2 + (p^1)^2), \tag{5.12}$$

$$\mathrm{S}_{\rm C} = \mathcal{C}^2_{\rm CB} + \mathcal{C}^2_{\rm AC} = (p^2)^2((q_0)^2 + (p^1)^2). \tag{5.13}$$

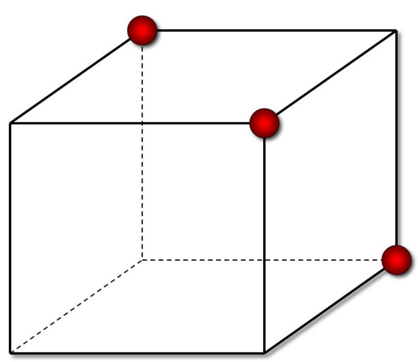

FIG. 8 (color online). The $2 \times 2 \times 2$ matrix with all entries vanishing except for $\psi_{111} = q_0$, $\psi_{001} = -p^1$, and $\psi_{010} = -p^2$, corresponding to the charges q_0, p^1, and p^2. The charges are always in opposite corners of each of these 3 sides. This state describes the small black hole [24] with just 3 charges and with classically vanishing area of the horizon.

B. Nonvanishing 3-tangle and entropy, GHZ states; large black holes

Here we have to satisfy the Eqs. (2.12), (2.13), and (2.14) with nonvanishing left-hand side. Using our cube pictures, we may immediately see that the configuration in Fig. 9 corresponds to a class of GHZ states, where we pick up some set of black hole charges in the expression for the nonvanishing entropy. For example, in the case of supersymmetric BPS black holes we may have nonvanishing charges q_0, p^1, p^2, p^3 with $q_0 p^1 p^2 p^3 > 0$. We place them as shown in Fig. 2. This is the cube in Fig. 9. The corresponding quantum state is $|\Psi\rangle = -p^1|001\rangle - p^2|010\rangle - p^3 100\rangle + q_0|111\rangle$. Now every side has two nonvanishing entries so that a concurrence associated with each side is nonvanishing. More importantly, the entropy and the 3-tangle also do not vanish,

$$\mathrm{S} = \pi\sqrt{|W(p,q)|} = \pi\sqrt{|\,\mathrm{det}\psi|} = \frac{\pi}{2}\sqrt{\tau_{\rm ABC}} \\ = 2\pi\sqrt{|q_0 p^1 p^2 p^3|}. \tag{5.14}$$

If however, $q_0 p^1 p^2 p^3 < 0$, this will be related to an extremal nonsupersymmetric non-BPS black hole with 4 charges, [28,29]. This is in general the case when $W < 0$ [30].

By using transformations preserving $\tau_{\rm ABC} = |\,\mathrm{det}\psi|$ (but not necessarily the sign of Detψ) one can always transform a state $|\Psi\rangle = -p^1|001\rangle - p^2|010\rangle - p^3 100\rangle + q_0|111\rangle$ to a canonical GHZ state describing only one electric and one magnetic charge in the same gauge group, say $\tilde{p}^0$ and $\tilde{q}_0$: $|\Psi\rangle = \tilde{p}^0|000\rangle + \tilde{q}_0|111\rangle$, see Fig. 10. The two charges corresponding to a canonical GHZ state are always at the opposite corners of the cube. One can easily check that for the canonical GHZ states, Fig. 10, the Cayley's hyperdeterminant detψ is always positive and

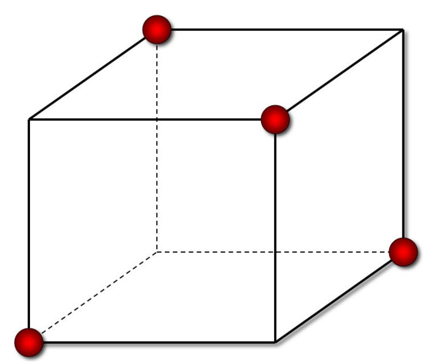

FIG. 9 (color online). The $2 \times 2 \times 2$ matrix with 4 nonvanishing charges, for example, q_0, p^1, p^2, p^3. This is a case of the large BPS and non-BPS black holes (depending on the sign of the product of these 4 charges) with just 4 charges and with classically nonvanishing area of the horizon. It belongs to the GHZ class of states, which may describe either BPS or non-BPS black holes.

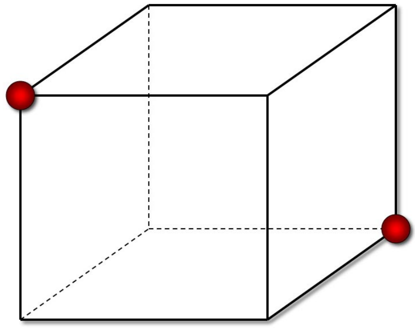

FIG. 10 (color online). The $2 \times 2 \times 2$ matrix with all entries vanishing, but two on the opposite diagonal of the cub. This is a case of the large non-BPS black hole with just 2 charges (in one gauge group, like p^0 and q_0) and with classically nonvanishing area of the horizon. It corresponds to the canonical GHZ state describing non-BPS black holes.

W is always negative, which corresponds to nonsupersymmetric non-BPS black holes.

Thus all extremal BPS and non-BPS black holes with nonvanishing entropy, i.e. all "usual," or large black holes, belong to the GHZ class of states of the ABC system, which is described by the lowest line in Table I. However, the theory of stringy black holes requires a more detailed classification than the standard 3-qubit classification provided by Table I. One encounters two inequivalent subclasses of GHZ states with respect to supersymmetry. The canonical GHZ states, Fig. 10 always correspond to nonsupersymmetric non-BPS black holes. Meanwhile the GHZ states described by Fig. 9 have the same 3-tangle (i.e. the same $|\det\psi|$), but the sign of $\det a$ may be either positive or negative. The states with $\det\psi > 0$ correspond to nonsupersymmetric non-BPS black holes, whereas the states with $\det\psi < 0$ correspond to supersymmetric BPS black holes.

VI. ENTROPY OF SMALL BLACK HOLES, THE NORM, AND 2-TANGLES IN ABC SYSTEMS

One of the goals of our paper was to obtain a better understanding of the intriguing relation between the entropy of the extreme BPS STU black holes and the 3-tangle discovered by Duff [10]. In this paper we extended his analysis for the axion-dilaton black holes and for the non-BPS STU black holes, and developed a new set of tools for investigation and classification of black holes, which have their counterparts in the theory of quantum information. Now we may apply our tools to the so-called extremal small black holes, which have a singular horizon with vanishing area and zero entropy at the classical level, but may acquire nonvanishing entropy and the area of horizon due to quantum corrections.

Let us first consider completely separable states such as a state with a single charge q_0 shown in Fig. 5. The corresponding small black holes with just one charge (eg. number of D0 branes) were studied in [25]. The value of the entropy due to R^4 corrections in the limit $q_0 \gg 1$ was found to be

$$S_{\mathrm{BH}} = \pi K\sqrt{\tfrac{2}{3}|q_0|}, \tag{6.1}$$

where K is some number. This entropy is also proportional to the area of the stretched horizon. As emphasized in [25], in order to verify this result one may need to check higher order corrections in R. If Eq. (6.1) is valid, one can represent it in the form that does not depend on the choice of a single charge q_Λ or p^Λ:

$$S_{\mathrm{BH}} = K\sqrt{\tfrac{2}{3}|\Psi|}, \tag{6.2}$$

where $|\Psi|$ is the norm of the state defined in Eq. (2.19). One can interpret this result as a consequence of the quantum stretching of the horizon conjectured by Susskind and Sen [35]. The classification in Table I does not attach any invariant concept to completely separable states, simply because in the quantum information theory all of these states are equally normalized: $|\Psi| = 1$. Meanwhile the entropy of the black holes is proportional to the square root of the wave function with a "stretched" norm $|\Psi| = |q_0| \neq 1$ (2.19). Thus we arrive at a simple intuitive interpretation: the stretching of the horizon of black holes with a single charge is related to the stretching of the norm of this state $|\Psi|$.

Now let us consider the bipartite case characterized by the charges q_0, p^1, numbers of D0 and D4 branes. In this case, the entropy with account of quantum corrections calculated in supergravity is given by

$$S_{\mathrm{quant}} = 4\pi\sqrt{|q_0 p^1|}. \tag{6.3}$$

This entropy was calculated in [23] by counting the number of microstates of string theory for $q_0 \gg p^1 \gg 1$. By comparing this answer with Fig. 7, one can see that the only parameter available in the classical cube is precisely the nonvanishing concurrence, $\mathcal{C} = 2|q_0 p^1|$, so we have the following interpretation of this result:

$$S_{\mathrm{quant}} = \pi\sqrt{\mathcal{C}}. \tag{6.4}$$

The radius of the stretched horizon r_h and the area of the horizon of the small black holes $A_{\mathrm{quant}} = 2S_{\mathrm{quant}}$ were calculated in [24]. Now we see that they have the following interpretation in terms of the concurrence of the 2-cubit state inside a 3-qubit state in quantum information theory:

$$A_{\mathrm{quant}} = 4\pi r_h^2 = 2\pi\sqrt{\mathcal{C}}. \tag{6.5}$$

It is amazing that the quantum corrected area of the horizon and entropy are related to the only nonvanishing concurrence for the case of the bipartite state q_0, p^1. One may wonder how quantum corrections in string theory could know about the concurrences in 3-qubit systems?

Is it just another coincidence or simply a consequence of the underlying symmetry of the theory?

Now let us make another step and discuss the entropy and the area of the horizon of the black holes in the bipartite or W state with nonvanishing q_0, p^1, p^2 charges. At the classical level, such black holes have a vanishing singular horizon with null singularity and zero entropy. Meanwhile quantum effects give the entropy [24]

$$\mathrm{S}_{\text{quant}} = 4\pi\sqrt{|q_0(p^1 + p^2)|} = \frac{\mathrm{A}_{\text{quant}}}{2}. \qquad (6.6)$$

These calculations, and the semiclassical approximation in general, require the condition that $q_0 \gg p^1,\ p^2 \gg 1$. In such case the term $p^1 p^2$, which is naturally expected from the cube picture, may be missing simply because it is supposed to be much smaller than the other two terms. In the limit $q_0 \gg p^1,\ p^2 \gg 1$ one can describe all results concerning the entropy of large and small black holes in the bipartite or W state by one simple equation preserving the symmetries of the system:

$$\mathrm{S}_{\text{total}} = \frac{\pi}{2}\sqrt{\tau_{ABC} + \frac{4c_2}{3}(\mathcal{C}_{\mathrm{AB}} + \mathcal{C}_{\mathrm{BC}} + \mathcal{C}_{\mathrm{CA}})}. \qquad (6.7)$$

Here c_2 is the second Chern class coefficient of the compactified manifold; in the example of K3 manifold $c_2 = 24$. Interestingly, $\mathcal{C}_{\mathrm{AB}} + \mathcal{C}_{\mathrm{BC}} + \mathcal{C}_{\mathrm{CA}}$ is equal to a half of the total area of a box with sides $|q_0|$, $|p^1|$, and $|p^2|$. The total entropy has two contributions, τ_{ABC}, which is quartic in charges, and $\frac{4c_2}{3}(\mathcal{C}_{\mathrm{AB}} + \mathcal{C}_{\mathrm{BC}} + \mathcal{C}_{\mathrm{CA}})$, which is quadratic in charges. Therefore for large black holes this expression in the leading approximation agrees with the result obtained in [12] and coincides with the result obtained by counting of states in string theory [20] and in supergravity with R^2 corrections [33] under the condition that $q_0 \gg p^1,\ p^2 \gg 1$. For small black holes the classical entropy vanishes, $\tau_{\mathrm{ABC}} = 0$, and the microscopic entropy calculated in string theory [23,24] is reproduced correctly by Eq. (6.7) in the approximation $q_0 \gg p^1,\ p^2 \gg 1$:

$$\mathrm{S}_{\text{small}} = \frac{\mathrm{A}_{\text{small}}}{2} = \pi\sqrt{\frac{c_2}{3}(\mathcal{C}_{\mathrm{AB}} + \mathcal{C}_{\mathrm{BC}} + \mathcal{C}_{\mathrm{CA}})}. \qquad (6.8)$$

One may go one step further and consider the small 1-charge black holes [25]. The modified entropy formula, under the conditions specified above, can be written as follows:

$$\mathrm{S}_{\text{total}} = \frac{\pi}{2}\sqrt{\tau_{\mathrm{ABC}} + \frac{4c_2}{3}(\mathcal{C}_{\mathrm{AB}} + \mathcal{C}_{\mathrm{BC}} + \mathcal{C}_{\mathrm{CA}}) + \frac{8K^2}{3}|\Psi|}. \qquad (6.9)$$

Here $|\Psi| = \sqrt{\langle\Psi|\Psi\rangle}$ is the norm of the wave function.

One can understand this equation as follows. For completely separable states with only one nonzero charge, this equation is reduced to Eqs. (6.1) and (6.2). For the bipartite and W states at large values of charges, the concurrences are much greater than $|\Psi|$, and the equation is reduced to (6.7), which is equivalent to Eq. (6.6) in the region of its applicability. Finally, for the GHZ states the 3-tangle is much greater than the concurrences, and we return to the equation $\mathrm{S} = \frac{\pi}{2}\sqrt{\tau_{\mathrm{ABC}}}$.

If Eq. (6.9) is correct beyond just representing various limiting cases, it may be a prediction for certain subleading corrections for the cases with $\tau_{\mathrm{ABC}} = 0$, or for the cases where $\tau_{\mathrm{ABC}} \neq 0$ but $\mathcal{C}_{\mathrm{AB}} + \mathcal{C}_{\mathrm{BC}} + \mathcal{C}_{\mathrm{CA}} \neq 0$. The relevant results on subleading corrections to the entropy of black holes with the classically finite horizon area were derived in [33]. One can also try to relate it to the black holes studied with the tools of topological string theory in [31].

VII. $E_{7(7)}$ QUARTIC INVARIANT AND CAYLEY'S HYPERDETERMINANT

In the previous investigation we mostly discussed axion-dilaton black holes and STU black holes. This covers a very broad class of extreme stringy black hole solutions. The STU black holes are described by 8 parameters and the classical entropy of these black holes is given by a square root of the absolute value of the Cayley's hyperdeterminant.

Now we are going to significantly generalize our results. The most general class of black holes in $N = 8$ supergravity/M theory is defined by 56 charges, and the entropy formula is given by the square root of the quartic Cartan-Cremmer-Julia $E_{7(7)}$ invariant [26,41–47],

$$\mathrm{S} = \pi\sqrt{|J_4|}, \qquad (7.1)$$

where the Cartan-Cremmer-Julia form of the invariant [42] depends on the central charge matrix Z,

$$J_4 = +\mathrm{Tr}(Z\bar{Z})^2 - \tfrac{1}{4}(\mathrm{Tr}Z\bar{Z})^2 + 4(\mathrm{Pf}Z + \mathrm{Pf}\bar{Z}), \qquad (7.2)$$

and the Cartan form [41] depends on the quantized charge matrix (x, y)

$$J_4 = -\mathrm{Tr}(xy)^2 + \tfrac{1}{4}(\mathrm{Tr}xy)^2 - 4(\mathrm{Pf}x + \mathrm{Pf}y). \qquad (7.3)$$

Here

$$Z_{\mathrm{AB}} = -\frac{1}{4\sqrt{2}}(x^{ab} + iy_{ab})(\Gamma^{ab})_{\mathrm{AB}} \qquad (7.4)$$

is the central charge matrix and

$$x^{ab} + iy_{ab} = -\frac{\sqrt{2}}{4}Z_{\mathrm{AB}}(\Gamma^{\mathrm{AB}})_{ab} \qquad (7.5)$$

is a matrix of the quantized charges related to some numbers of branes. The exact relation between the Cartan invariant in Eq. (7.3) and Cremmer-Julia invariant [42] in Eq. (7.2) has been established in [44].

The matrices of $SO(8)$ algebra are $(\Gamma^{ab})_{\mathrm{AB}}$ where (ab) are the 8 vector indices and (A, B) are the 8 spinor indices. The $(\Gamma^{ab})_{\mathrm{AB}}$ matrices can be considered also as $(\Gamma^{\mathrm{AB}})_{ab}$

matrices due to equivalence of the vector and spinor representations of the $SO(8)$ Lie algebra. The central charge matrix $Z_{\rm AB}$ can be brought to the canonical basis for the skew-symmetric matrix using an $SU(8)$ transformation. The eigenvalues z_i, $i = 1, 2, 3, 4$ are complex. In this way the content of a theory is reduced from 56 entries to 8.

Relation between the entropy of stringy black holes and the Cartan-Cremmer-Julia $E_{7(7)}$ invariant was established 10 years ago [26]. The stringy solutions in $N = 4$ theory characterized by 5 parameters were first found in [39]. Since that time many new black hole solutions have been found. In a systematic treatment in [45] in the context of the eigenvalues of the central charge matrix of $N = 8$ theory the meaning of these 5 parameters was clarified: $z_i = \rho_i e^{i\phi}$, from 4 complex values of $z_i = \rho_i e^{i\phi_i}$ one can remove 3 phases by an $SU(8)$ rotation, but the overall phase cannot be removed. Therefore a 5-parameter solution is called a generating solution for other black holes in $N = 8$ supergravity/M theory. Expression for their entropy is always given by $S = \pi\sqrt{|J_4|}$ for some subset of 5 of the 8 parameters mentioned above. Recently a new class of solutions was discovered, describing black rings. The maximal number of parameters for the known solutions is 7. The entropy of black ring solutions found so far was identified in [46] with the expression for $\pi\sqrt{|J_4|}$ for a subset of 7 out of 8 parameters mentioned above. That is why it would be most interesting to establish a possible relation between the general black hole/black ring entropy equation $S = \pi\sqrt{|J_4|}$ in $N = 8$ supergravity/M theory and some of the constructions of the theory of quantum information.

One could expect that this relation, if possible at all, may be quite involved and may require investigation of more complicated constructions, such as n-tangles for $n > 3$. However, we have found that this relation again involves only 3-tangles.

To find this relation, let us note that in x, y basis only $SO(8)$ symmetry is manifest, which means that every term in Eq. (7.3) is invariant only under $SO(8)$ symmetry. However, is was proved in [41,42] that the sum of all terms in Eq. (7.3) is invariant under the full $SU(8)$ symmetry, which acts as follows

$$\delta(x^{ab} \pm iy_{ab}) = (2\Lambda^{[a}_{[c}\delta^{b]}_{d]} \pm i\Sigma_{abcd})(x^{cd} \mp iy_{cd}). \quad (7.6)$$

The total number of parameters is 63, where 28 are from the manifest $SO(8)$ and 35 from the antisymmetric self-dual $\Sigma_{abcd} =^* \Sigma^{abcd}$. Thus one can use the $SU(8)$ transformation of the complex matrix $x^{ab} + iy_{ab}$ and bring it to the canonical form with some complex eigenvalues λ_I, $I = 1, 2, 3, 4$. The value of the quartic invariant (7.3) will not change.

$$(x^{ab} + iy_{ab})_{\rm can} = \begin{pmatrix} 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\lambda_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\lambda_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_4 & 0 \end{pmatrix}. \quad (7.7)$$

One can easily check that the Cartan-Cremmer-Julia quartic invariant J_4 depending on 4 complex eigenvalues λ_I can be represented as a Cayley's hyperdeterminant of a matrix ψ_{ijk}

$$J_4(\lambda) = -\det\psi, \quad (7.8)$$

where the relation between the complex coefficients λ_i, the parameters x_{ij} and y^{kl}, the matrix ψ_{ijk}, and the black hole charges p^i and q_k is given by the following dictionary:

$$\begin{aligned} \lambda_1 &= x_{12} + iy^{12} = a_{111} + ia_{000} = q_0 + ip^0, \\ \lambda_2 &= x_{34} + iy^{34} = a_{001} + ia_{110} = -p^1 + iq_1, \\ \lambda_3 &= x_{56} + iy^{56} = a_{010} + ia_{101} = -p^2 + iq_2, \\ \lambda_4 &= x_{78} + iy^{78} = a_{100} + ia_{011} = -p^3 + iq_3. \end{aligned} \quad (7.9)$$

The simplest way to prove it is to write the quartic $E_{7(7)}$ Cartan invariant in the canonical basis (x_{ij}, y^{ij}), $i, j = 1, \ldots, 8$:

$$\begin{aligned} J_4 = &-(x_{12}y^{12} + x_{34}y^{34} + x_{56}y^{56} + x_{78}y^{78})^2 \\ &- 4(x_{12}x_{34}x_{56}x_{78} + y^{12}y^{34}y^{56}y^{78}) \\ &+ 4(x_{12}x_{34}y^{12}y^{34} + x_{12}x_{56}y^{12}y^{56} + x_{34}x_{56}y^{34}y^{56} \\ &+ x_{12}x_{78}y^{12}y^{78} + x_{34}x_{78}y^{34}y^{78} + x_{56}x_{78}y^{56}y^{78}). \end{aligned} \quad (7.10)$$

Then one should compare it to the Cayley's hyperdeterminant (2.8) using the dictionary (7.9) given above, or an equivalent dictionary in the form similar to the one used in

Sec. III B, Eq. (3.5):

ABC STU Black Hole

$$\text{N} = 8 \text{ Black Hole} \begin{pmatrix} a_{000} \\ a_{001} \\ a_{010} \\ a_{100} \\ a_{111} \\ a_{110} \\ a_{101} \\ a_{011} \end{pmatrix} = \begin{pmatrix} p^0 \\ -p^1 \\ -p^2 \\ -p^3 \\ q_0 \\ q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} y^{12} \\ x_{34} \\ x_{56} \\ x_{78} \\ x_{12} \\ y^{34} \\ y^{56} \\ y^{78} \end{pmatrix}. \tag{7.11}$$

Our results imply that the entropy of the most general extremal BPS and non-BPS black hole and black ring solutions in $N = 8$ supergravity can be brought to a canonical basis where it depends only on 8 charges and can be represented by the same compact expression (3.14) as in the theory of STU black holes and as a 3-tangle in a 3-qubit system:

$$S_{(\text{BH,BR})} = \pi\sqrt{|J_4(\lambda)|} = \pi\sqrt{|\det\psi|} = \pi\sqrt{|W(p, q)|} = \frac{\pi}{2}\sqrt{\tau_{\text{ABC}}}. \tag{7.14}$$

The quartic invariant of the $E_{7(7)}$ J_4 is related to the octonionic Jordan algebra $J_3^{\mathbb{O}}$, see [47]. It is therefore natural, in view of our result (7.14), to expect that the 3-qubit system can be described by octonions, which was indeed shown in [48].

VII. CONCLUSIONS

Our work, following the recent work by Duff [10], demonstrated a lot of intriguing connections between extremal black holes and the ABC system in the quantum information theory. The new approach to the theory of stringy black holes may help us with the black hole and black ring classification and with interpretation of our results in terms of general quantum mechanical systems. It may also help us to represent our results in a different form, which may allow our intuition to grow in a previously unexpected way. In this paper we found that the entropy of the axion-dilaton extremal black hole is related to the concurrence of a 2-qubit state, whereas the entropy of the STU black holes, even if they are not BPS black holes, is related to the 3-tangle of a 3-qubit state. We identified usual black holes with the maximally entangled GHZ class of states, and small black holes with either separable, or bipartite entangled states or W class of states. We established a certain relation between 3-qubit states, twistors, and black holes. We found an expression for entropy and the area of the horizon of small black holes in terms of the concurrence of the 2-cubit states inside a 3-qubit state and its norm. Finally, we extended the previous results to the most general extremal BPS and non-BPS black hole and black ring solutions in $N = 8$ supergravity/M theory. To our own surprise, we have found that the expression for the entropy of these solutions in terms of the quartic $E_{7(7)}$ Cartan invariant [26] in Eq. (7.3) can be represented by the same compact expression in terms of the Cayley's hyperdeterminant (2.8) as a 3-tangle (2.9) and the entropy of STU black holes (3.14).

Our work was devoted to the implications of the quantum information theory to the theory of black holes. Even if some of these results eventually will be interpreted as coincidental, we may still appreciate the theory of quantum information for its heuristic potential, which allowed us to look at the theory of stringy black holes from a completely different perspective. However, we do not think that this is a one-way road. It is quite plausible that the enormous amount of highly nontrivial results obtained in the quantum theory of stringy black holes may lead to new insights in the theory of quantum information. We hope therefore that the parallel study of both sides of the story may be quite fruitful.

ACKNOWLEDGMENTS

It is a pleasure to thank T. Banks, M. Duff, S. Ferrara, A. Guth, C. Hull, S. Kachru, A. Maloney, J. Park, S. Shenker, M. Shmakova, A. Sinha, L. Susskind, and W. Taylor for useful conversations. This work was supported by NSF Grant No. PHY-0244728.

Note added.— Recently a paper by Levay [49] appeared where two important developments were made. First, it was shown that a pure 3-qubit state is real under certain conditions. Note that in general in quantum information theory the wave function is complex, and the system has a $[SL(2, \mathbb{C})]^3$ symmetry, whereas for black holes we have only $[SL(2, \mathbb{Z})]^3$ and the relevant ''wave function'' is real. Interestingly, two different conditions for reality found in [48] correspond to either BPS or non-BPS black holes. Second, it was established there that what in string theory is known as a stabilization of moduli near the black hole horizon, in quantum information theory is known as a procedure of finding the optimal local distillation protocol of a GHZ state from an arbitrary 3-qubit state. These statements provide additional links between the theory of extremal black holes and the quantum information theory.

[1] G. W. Gibbons and K. i. Maeda, Nucl. Phys. **B298**, 741 (1988); D. Garfinkle, G. T. Horowitz, and A. Strominger, Phys. Rev. D **43**, 3140 (1991); **45**, 3888 (1992).

[2] R. Kallosh, A. D. Linde, T. Ortin, A. W. Peet, and A. Van Proeyen, Phys. Rev. D **46**, 5278 (1992).

[3] S. Ferrara, R. Kallosh, and A. Strominger, Phys. Rev. D **52**, R5412 (1995); A. Strominger, Phys. Lett. B **383**, 39 (1996); S. Ferrara and R. Kallosh, Phys. Rev. D **54**, 1514 (1996); Phys. Rev. D **54**, 1525 (1996); K. Behrndt, G. Lopes Cardoso, B. de Wit, R. Kallosh, D. Lust, and T. Mohaupt, Nucl. Phys. **B488**, 236 (1997); S. Ferrara, G. W. Gibbons, and R. Kallosh, Nucl. Phys. **B500**, 75 (1997); G. W. Moore, hep-th/9807087; F. Denef, B. R. Greene, and M. Raugas, J. High Energy Phys. 05 (2001) 012; G. W. Moore, hep-th/0401049; R. Kallosh, J. High Energy Phys. 12 (2005) 022.

[4] A. Strominger and C. Vafa, Phys. Lett. B **379**, 99 (1996).

[5] C. M. Hull and P. K. Townsend, Nucl. Phys. **B438**, 109 (1995).

[6] E. Witten, Nucl. Phys. **B443**, 85 (1995).

[7] J. Polchinski, Phys. Rev. Lett. **75**, 4724 (1995).

[8] A. W. Peet, hep-th/0008241; C. V. Johnson, *D-branes* (Cambridge University Press, Cambridge, England, 2003); D. Youm, Phys. Rep. **316**, 1 (1999); T. Ortin, *Gravity and Strings* (Cambridge University Press, Cambridge, England, 2004); L. Susskind and J. Lindesay, *An Introduction to Black holes, Information and the String Theory Revolution; The holographic Universe* (World Scientific, Singapore, 2005).

[9] *The Physics of Quantum Information: Quantum Cryptography, Quantum Teleportation, Quantum Computation*, edited by D. Bouwmeester, A. K. Ekert, and A. Zeilinger (Springer, New York, 2000); M. A. Nielsen, I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, England, 2000).

[10] M. J. Duff, hep-th/0601134.

[11] M. Cvetic and D. Youm, Phys. Rev. D **53**, R584 (1996).

[12] K. Behrndt, R. Kallosh, J. Rahmfeld, M. Shmakova, and W. K. Wong, Phys. Rev. D **54**, 6293 (1996).

[13] A. Cayley, J. Math. Phys. (Cambridge, Mass.) **4**, 193 1945; I. M. Gelfand, M. M. Kapranov, and A. V. Zelevinsky, *Discriminants, Resultants, and Multi-dimensional Determinants* (Birkhauser, Boston, 1994).

[14] V. Coffman, J. Kundu, and W. Wootters, Phys. Rev. A **61**, 052306 (2000).

[15] A. Miyake and M. Wadati, Quantum Inf. Comput. **2**, 540 (2002).

[16] M. J. Duff, J. T. Liu, and J. Rahmfeld, Nucl. Phys. **B459**, 125 (1996).

[17] W. K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998).

[18] R. Kallosh and T. Ortin, Phys. Rev. D **48**, 742 (1993).

[19] I. R. Klebanov and A. A. Tseytlin, Nucl. Phys. **B475**, 179 (1996); V. Balasubramanian and F. Larsen, Nucl. Phys. **B478**, 199 (1996).

[20] J. M. Maldacena, A. Strominger, and E. Witten, J. High Energy Phys. 12 (1997) 002.

[21] P. Levay, Phys. Rev. A **71**, 012334 (2005).

[22] W. Dur, G. Vidal, and J. I. Cirac, Phys. Rev. A **62**, 062314 (2000).

[23] A. Dabholkar, Phys. Rev. Lett. **94**, 241301 (2005).

[24] A. Dabholkar, R. Kallosh, and A. Maloney, J. High Energy Phys. 12 (2004) 059.

[25] A. Sinha and N. V. Suryanarayana, hep-th/0601183.

[26] R. Kallosh and B. Kol, Phys. Rev. D **53**, R5344 (1996).

[27] R. Kallosh, Phys. Rev. D **54**, R4709 (1996); R. Kallosh, M. Shmakova, and W. K. Wong, Phys. Rev. D **54**, 6284 (1996).

[28] P. K. Tripathy and S. P. Trivedi, J. High Energy Phys. 03 (2006) 022.

[29] R. Kallosh, N. Sivanandam, and M. Soroush, J. High Energy Phys. 03 (2006) 060.

[30] T. Ortin, Phys. Lett. B **422**, 93 (1998).

[31] R. Kallosh, hep-th/0603003.

[32] H. Ooguri, A. Strominger, and C. Vafa, Phys. Rev. D **70**, 106007 (2004).

[33] A. Dabholkar, F. Denef, G. W. Moore, and B. Pioline, J. High Energy Phys. 10 (2005) 096; G. L. Cardoso, B. de Wit, J. Kappeli, and T. Mohaupt, J. High Energy Phys. 03 (2006) 074.

[34] G. Lopes Cardoso, B. de Wit, and T. Mohaupt, Phys. Lett. B **451**, 309 (1999); T. Mohaupt, Fortschr. Phys. **49**, 3 (2001).

[35] L. Susskind, Phys. Rev. D **49**, 6606 (1994); A. Sen, Nucl. Phys. **B440**, 421 (1995); A. Sen, Mod. Phys. Lett. A **10**, 2081 (1995).

[36] A. Sen, J. High Energy Phys. 07 (2005) 073.

[37] A. Sen, J. High Energy Phys. 03 (2006) 008; M. Alishahiha and H. Ebrahim, J. High Energy Phys. 03 (2006) 003.

[38] A. Ceresole, R. D'Auria, S. Ferrara, and A. Van Proeyen, Nucl. Phys. **B444**, 92 (1995).

[39] M. Cvetic and A. A. Tseytlin, Phys. Rev. D **53**, 5619 (1996) **55**, 3907 (1997)].

[40] D. M. Greenberger, M. Horne, and A. Zeilinger, in *Bell's Theorem, Quantum Theory and Conceptions of the Universe*, edited by M. Kafatos (Kluwer, Dordrecht, 1989).

[41] E. Cartan, *CEuvres complètes* (Editions du Centre National de la Recherche Scientifique, Paris, 1984).

[42] E. Cremmer and B. Julia, Nucl. Phys. **B159**, 141 (1979).

[43] M. Cvetic and C. M. Hull, Nucl. Phys. **B480**, 296 (1996); L. Andrianopoli, R. D'Auria, S. Ferrara, P. Fre, and M. Trigiante, Nucl. Phys. **B509**, 463 (1998); M. Bertolini and M. Trigiante, Nucl. Phys. **B582**, 393 (2000); N. R. Constable, C. V. Johnson, and R. C. Myers, J. High Energy Phys. 09 (2000) 039; M. Gunaydin, K. Koepsell, and H. Nicolai, Adv. Theor. Math. Phys. **5**, 923 (2002); M. Gunaydin, AIP Conf. Proc. No. 767 (AIP, New York, 2005), p. 268.

[44] V. Balasubramanian, F. Larsen, and R. G. Leigh, Phys. Rev. D **57**, 3509 (1998); M. Gunaydin, K. Koepsell, and H. Nicolai, Commun. Math. Phys. **221**, 57 (2001).

[45] S. Ferrara and J. M. Maldacena, Classical Quantum Gravity **15**, 749 (1998).

[46] I. Bena and P. Kraus, J. High Energy Phys. 12 (2004) 070; I. Bena, P. Kraus, and N. P. Warner, Phys. Rev. D **72**, 084019 (2005).

[47] S. Ferrara and M. Gunaydin, Int. J. Mod. Phys. A **13**, 2075 (1998).

[48] B. Bernevig and H.-D. Chen, J. Phys. A **36**, 8325 (2003).

[49] P. Levay, hep-th/0603136.