Post-Newtonian corrections to the motion of spinning bodies in nonrelativistic general relativity

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In this paper we include spin and multipole moment effects in the formalism used to describe the motion of extended objects recently introduced in hep-th/0409156. A suitable description for spinning bodies is developed and spin-orbit, spin-spin, and quadrupole-spin Hamiltonians are found at leading order. The existence of tidal as well as self-induced finite size effects is shown, and the contribution to the Hamiltonian is calculated in the latter. It is shown that tidal deformations start formally at $\mathcal{O}(v^6)$ and $\mathcal{O}(v^{10})$ for maximally rotating general and compact objects, respectively, whereas self-induced effects can show up at leading order. Agreement is found for the cases where the results are known.

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I. INTRODUCTION

In a recent paper an effective field theory (EFT) of gravity for nonspinning, spherically symmetric extended objects was introduced [1]. Within the post-Newtonian (PN) [2] framework this approach was coined NRGR (nonrelativistic general relativity) due to its similarities with EFT approaches to nonrelativistic bound states in QED and QCD [3]. However, the EFT formalism can be applied to a variety of scenarios, for instance the large small mass ratio case [4]. NRGR is relevant for understanding the gravitational power spectra emitted by binary systems, an important class of candidate signals for gravitational wave observatories such as LIGO or VIRGO [5,6]. The formalism allows for a clean separation of the long wavelength gravitational dynamics from the details of the internal structure. This separation enables us to calculate corrections to all orders in the point particle approximation. Furthermore, it was shown that the ambiguities [7,8] that plague the conventional PN calculations can be attributed to the presence of higher-dimensional world line terms in the action whose coefficients encode the short distance structure of the particles.

Building upon this idea, here we propose an extension of NRGR which allows for the inclusion of spin and multipole moments. Spin in general relativity (GR) has been considered previously in the literature from many different points of view (see for instance [9–12] and references therein), and is argued to play an important role in binary inspiral, particularly for black holes [13,14]. Within the PN approximation, spin effects for binary systems have been calculated using different techniques [13–17]. Dealing with spinning objects in the point particle approximation inevitably entails running into divergent integrals as one does in the nonspinning case. Regularization procedures, like Hadamard finite part [18] or considering contributions from different zones [19], were invoked when dealing with pointlike sources [17]. However, it has been argued, for instance for the proposal of [19], that the formalism is *"considerably complicated but it is inevitable that we have to adopt it to deal with divergences when we go to higher PN orders"* [16]. As it has been repeatedly emphasized in [1] that is not the case within an EFT approach. Here we will explicitly see how a systematic, and consistent to all orders approach, is translated into our case as well.

The outline of the paper is as follows. In the first section we review NRGR for nonspinning spherically symmetric objects highlighting the main results. Then, we will generalize the formalism to include internal angular as well as multipole moment degrees of freedom extending the work of Hanson and Regge in the realm of special relativity [20]. Afterwards we derive the power counting and Feynman rules of NRGR and calculate the leading spin-spin and spin-orbit potentials and show to reproduce known results [10,13,14]. A quadrupole-spin correction to the gravitational energy is obtained for the first time (to my knowledge) to leading order. The equivalence between different choices for the spin supplementary condition is also shown. Finally, we discuss the insertion of nonminimal terms in the world line action and its relevance to renormalization. The existence of two types of finite size effects encapsulated in a new set of coefficients, which can in principle be fixed by matching to the full theory, is predicted: those which have a renormalization group (RG) flow and naturally represent tidal spin effects induced by the companion, and those that do not have a RG flow and represent self-induced effects, such as the spin induced quadrupole moment due to the proper rotation of the objects [21,22]. By power counting it is shown that companion induced tidal effects start formally at 3PN, and 5PN for maximally rotating general and compact objects, respectively. Self-induced effects can show up at leading order. Details are relegated to appendices. We will study higher-order PN corrections, the radiative energy loss, matching, and new possible kinematic scenarios in future publications.

II. NRGR

In this section we will emphasize the main features of NRGR within the PN formalism; detailed calculation and further references can be found in the original proposal [1].

A. Basic philosophy

The traditional approach to the problem of motion was introduced by Fock [23] who split it into two subproblems. The *internal problem*, which consists of understanding the motion of each body around its center of mass, and the *external problem*, which determines the motion of the centers of mass of each body. Decomposing the problem this way allows us to naturally separate scales and henceforth calculate in a more systematic fashion. The price one pays is the necessity of a *matching* procedure which relies either in comparing with the full theory, if known, or extracting unknown parameters from experiment. This method is now called "effective theory," or EFT in the realm of quantum field theory (QFT), and has been used to great success in many different branches of physics [3]. While at first glance quantum field theoretical tools appear to introduce unnecessary machinery for classical calculations,[1] the power of the method will be shown to reside in two facts: It allows for the introduction of manifest power counting and naturally encapsulates divergences into text book renormalization procedures.[2] This means in addition that in the EFT it is straightforward to calculate the order at which a given term in the perturbative series first contributes to a given physical observable.

Here we are going to tackle the problem of motion by treating gravity coupled to point particle sources as the classical limit of an EFT, i.e. the "tree level approximation," within the PN formalism. Feynman diagrams will naturally show up as perturbative techniques to iteratively solve for the full Green functions of the theory. As it is known, GR coupled to distributional sources is not generically well defined due to its nonlinear character [25].[3] This can be seen as a formal obstacle to the PN expansion for point particle sources in GR. Within an EFT paradigm this problem does not even arise since one is not claiming to construct a full description to be applicable to all regimes, but an effective theory which will mimic GR coupled to extended objects within its realm of applicability. In addition, one can also argue that this EFT could be seen as the low energy regime of a quantum theory of gravity necessary to smear out pointlike sources.

The idea of describing low energy quantum gravity as an EFT is not new (for a review see [26]). What makes NRGR appealing is the uses of EFT to attack so-called *classical* problems. QFT has proven to be useful with classical calculations, as in electromagnetic radiation where we can think of photons (QED) to calculate a power spectrum. Here we will use the same idea introducing "gravitons" as the quantum of the metric field which will allow us to calculate the gravitational potential, from which the equations of motion (EOM) are derived, as well as gravitational radiation in a systematic fashion.

B. Effective theory of extended objects

The method of [1] is based in the explicit separation of the relevant scales of the problem: the size of the objects r_s (internal problem), the size of the orbit r (external problem), and the natural radiation wavelength r/v, where $v \ll c$ is the relative velocity in the PN frame. Finite size effects are treated by the inclusion of a tower of new terms in the world line action which are needed to regularize the theory.[4] For a nonspinning spherically symmetric particle the most general action consistent with the symmetries of GR is

$$S = \int (-m + c_R R + c_V v^\mu v^\nu R_{\mu\nu} + \cdots) d\tau, \qquad (1)$$

where $R_{\mu\nu}$ and R are the Ricci tensor and scalar, respectively. The series, involving higher-order Riemann-type insertions, must be truncated within the desired accuracy to have any predictive power. The coefficients of each of these new terms can be determined by comparison with the full theory. In this case the underlying theory is GR plus the internal equation of state of the objects. The beauty of this method is that, since these are 1-body properties, we can match using any relevant observable, for instance scattering processes, rather than solving the complete problem of motion explicitly.

As it was shown in [1], the terms proportional to c_R, c_V are generated by logarithmic divergences of the point particle approximation. However, it is possible to show that they are *unphysical* in the sense that they can be removed from the effective action by field redefinition (f.r.) and no trace is left in observable quantities [1,8]. Nevertheless, from here one concludes that not all divergences can be absorbed into the mass and new counterterms are necessary. Furthermore, at higher orders[5] it can be shown that (full Riemann dependent) finite size tidal effects are induced which cannot be removed from the theory. We will see here that allowing the objects to spin also introduces new terms in the world line action. For the sake of completeness, and given that the same idea will be used here later on, we will sketch the reasoning. One starts by calculating the effective action [28],

$$\Gamma[g_{\mu\nu}] = \frac{1}{m_p} \int \frac{d^4k}{(2\pi)^4} h_{\mu\nu}(-k) T^{\mu\nu}_{(1p)}(k) + \cdots, \qquad (2)$$

[1]QFT techniques have been recently used to calculate self-force effects in a curved space-time background [24].

[2]Also bear in mind that the classical solution is just the saddle point approximation to the path integral or what is known as the "tree level" approximation.

[3]I would like to thank Jorge Pullin for discussions on this point.

[4]A similar approach can be found in [27] within the realm of tensor-scalar theories. However, renormalization issues as well as spin effects were left undiscussed.

[5]Finite size effects first appear at order v^{10} for nonspinning particles.

where $g_{\mu\nu} = \eta_{\mu\nu} + \frac{h_{\mu\nu}}{m_p}$, and $h_{\mu\nu}$ the graviton field. Let us concentrate on the contributions to the one point function $T^{\mu\nu}_{(1p)}(k)$. As it was shown in [1] within dimensional regularization techniques (dim. reg.), the logarithmic divergences in $T^{\mu\nu}_{(1p)}(k)$ cannot be absorbed into the mass and a new counterterm of the form

$$T^{\mu\nu}_{ct}(k) = (2\pi)\delta(k\cdot v)[c_R(\eta^{\mu\nu}k^2 - k^\mu k^\nu) + \tfrac{1}{2}c_V k^2 v^\mu v^\nu] \tag{3}$$

is therefore needed. It is straightforward to conclude from here the necessity of including two new terms in the effective action as shown in (1). Within dim. reg. an arbitrary mass scale μ associated to the subtraction point at which the theory is renormalized is introduced. Given that the metric field does not pick any anomalous dimension at tree level we must have $\mu d\Gamma[g_{\mu\nu}]/d\mu = 0$. Thus the explicit dependence on the subtraction scale μ must be canceled by allowing the coefficients $c_{R,V}$ to vary with scale. The theory therefore exhibits nontrivial *classical* RG scaling. As we are going to show here spin dependent finite size effects are predicted by similar arguments.

C. The post-Newtonian expansion

Once the internal scale is taken into account by the introduction of a series of new terms in the 1-body world line action, the next scale we have to integrate out is the orbit scale. In order to do that we decompose the graviton field $h_{\mu\nu}$ into two pieces,

$$h_{\mu\nu}(x) = \bar{h}_{\mu\nu}(x) + H_{\mu\nu}(x), \tag{4}$$

where $H_{\mu\nu}$ represents the off-shell potential gravitons, with

$$\partial_i H_{\mu\nu} \sim \frac{1}{r}H_{\mu\nu} \qquad \partial_0 H_{\mu\nu} \sim \frac{v}{r}H_{\mu\nu}, \tag{5}$$

and $\bar{h}_{\mu\nu}$ describes an on-shell radiation field

$$\partial_\alpha \bar{h}_{\mu\nu} \sim \frac{v}{r}\bar{h}_{\mu\nu}. \tag{6}$$

We can now further decompose $H_{\mu\nu}$ by removing from it the large momentum fluctuations,

$$H_{\mu\nu}(x) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} H_{\mathbf{k}\mu\nu}(x^0). \tag{7}$$

The advantage of this redefinition is that now derivatives acting on any field in the EFT scale in the same way, $\partial_\mu \sim v/r$, so it is easy to count powers of v coming from derivative interactions.

The effective *radiation* NRGR Lagrangian, with the potential gravitons integrated out, can then be derived by computing the functional integral,

$$\exp[iS_{\rm NRGR}[x_a,\bar{h}]] = \int \mathcal{D}H_{\mu\nu}\exp[iS[\bar{h}+H,x_a] + iS_{GF}], \tag{8}$$

where S_{GF} is a suitable gauge fixing term. Equation (8) indicates that as far as the potential modes $H_{\mu\nu}$ are concerned $\bar{h}_{\mu\nu}$ is just a slowly varying background field. To preserve gauge invariance of the effective action, we choose S_{GF} to be invariant under general coordinate transformations of the background metric $\bar{g}_{\mu\nu}(x) = \eta_{\mu\nu} + \bar{h}_{\mu\nu}(x)$. This whole procedure is what is usually known as the "background field method," originally introduced by DeWitt [29] in canonical quantum gravity and used by t'Hooft and Veltman for the renormalization of gauge theories [30]. By expanding the Einstein-Hilbert action using (4), we can immediately read off Feynman rules [1]. For potential gravitons, which we are going to represent by a dashed line, the propagator is given by

$$\langle H_{\mathbf{k}\mu\nu}(x^0)H_{\mathbf{q}\alpha\beta}(0)\rangle = -(2\pi)^3\delta^3(\mathbf{k}+\mathbf{q})\frac{i}{\mathbf{k}^2}\delta(x_0)P_{\mu\nu;\alpha\beta}, \tag{9}$$

where $P_{\mu\nu;\alpha\beta} = \frac{1}{2}[\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta}]$. The radiation gravitons, which will be represented by a curly line, have the usual spin 2 massless propagator. A wavy line will be used for the full propagator. We also need to consider mass insertions which will just provide a vertex interaction [1],

$$\sum_a \frac{m_a}{m_p}\left[\frac{1}{2}h_{00} + h_{0i}\mathbf{v}_{ai} + \frac{1}{4}h_{00}\mathbf{v}_a^2 + \frac{1}{2}h_{ij}\mathbf{v}_{ai}\mathbf{v}_{ai}\right] + \cdots, \tag{10}$$

where h_{00}, h_{0i}, h_{ij} are evaluated on the point particle world line (the leading order graviton-mass vertex is shown in Fig. 1). Following standard power counting procedures, we arrive to the scaling laws for the NRGR fields shown in Table I [1,3]. In the last column we have introduced $m_p^2 = \frac{1}{32\pi G_N}$ the Planck mass and $L = mvr$ the angular momentum.

The effective action in (8) will be a function of the world line particles (treated as external sources) and the radiation field which allows us to calculate the energy loss due to gravitational radiation as well as the gravitational binding

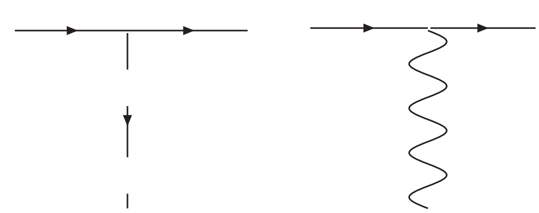

FIG. 1. Leading order mass vertex. The dashed line represents a potential graviton, whereas the wavy line stands for the full graviton propagator.

TABLE I. NRGR power counting rules.

$\mathbf{k}$	$H^{\mathbf{k}}_{\mu\nu}$	$\bar{h}_{\mu\nu}$	m/m_p
$1/r$	$r^2 v^{1/2}$	v/r	$\sqrt{Lv}$

potential from which the EOM are obtained. To get $S_{\rm eff}(x^a)$ we simply integrate out the radiation field $\bar{h}$. $S_{\rm eff}(x^a)$ has a real part which represents the effective potential for the 2-body system.[6] It also has an imaginary part that measures the total number of gravitons emitted by a given configuration $\{x^a_\mu\}$ over an arbitrarily large time $T \to \infty$,

$$\frac{1}{T}\,{\rm Im}S_{\rm eff}(x_a) = \frac{1}{2}\int dE d\Omega \frac{d^2\Gamma}{dEd\Omega}, \tag{11}$$

where $d\Gamma$ is the differential rate for graviton emission from the binary system from which the power spectrum is computed.

III. INTERNAL DEGREES OF FREEDOM I: SPINNING PARTICLE

Here we will follow closely the ideas developed in [20]. We will start formulating a Lagrangian formalism to deal with internal angular as well as multipole moment degrees of freedom which will enable us to describe a richer tensor structure. We will introduce the basic elements first, then we will construct the action and show how to reproduce Papapetrou equations for spinning particles in GR [32]. The issue of constraints, the correct number of degrees of freedom, and the angular-velocity/spin relationship will be discussed at the end of the section.

A. Basics

Given a space-time structure (g, M) we can always find at each point $x \in M$ a coordinate system where the metric looks locally flat at the point. Such a transformation can be expressed as

$$\eta_{IJ} = e^\mu_I e^\nu_J g_{\mu\nu}, \tag{12}$$

$$\eta^{IJ} e^\mu_I e^\nu_J = g^{\mu\nu}, \tag{13}$$

with $\eta_{IJ} \equiv (1, -1, -1, -1)$ the Minkowski metric and e^μ_I a set of $I = 0\ldots 3$ orthonormal basis vectors such that the tensor metric is diagonalized at the point. From now on, capital Latin letters will denote internal indexes [notice that they transform in $SO(3,1)$ due the residual Lorentz invariance]; the other conventions are as usual. Given a tetrad we can define its transport through the particle's world line using Fermi-Walker ideas as [33]

$$\dot{e}^{\,I}_{\ \mu} \equiv \frac{De^\mu_I}{d\lambda} = u^\alpha \nabla_\alpha e^\mu_I = -\Omega^{\mu\nu} e_{I\nu}, \tag{14}$$

where ∇_α is the covariant derivative compatible with g, namely $\nabla_\alpha g_{\mu\nu} \equiv g_{\mu\nu;\alpha} = 0$, and $\Omega^{\mu\nu}$ is an antisymmetric tensor which therefore preserves (12). One can invert the previous relation using (12),

$$\Omega_{\beta\alpha} = \eta^{IJ} e_{\beta I} \frac{De_{\alpha J}}{d\tau} = \left(\frac{de_{\alpha J}}{d\tau} - \Gamma^\sigma_{\alpha\gamma} e_{\sigma J} u^\gamma\right)\eta^{IJ} e_{\beta I}. \tag{15}$$

Notice that (15) implies the antisymmetry directly from $g_{\nu\mu;\mu} = 0$.

The introduction of e^μ_I is equivalent to adding an element of SO(3, 1) to the world line of the particle to describe rotations [20]. Following these ideas we will therefore construct an action in terms of the generalized coordinates and velocities $(x^\mu, u^\nu, e^I_\mu, \dot{e}^I_\mu)$. The number of degrees of freedom in e^μ_I is 3 more than we need to describe 3-rotations. We will see however that we can impose a set of kinematic constraints which will ensure the correct number.

B. Action principle and EOM

So far we have characterized the extra degrees of freedom we need in order to construct a Lagrangian for the spinning particle. In the process to construct the action we will demand, in addition to general covariance, internal Lorentz invariance as well as reparametrization invariance (RPI). This will naturally restrict ourselves to Lagrangians of the form $L(x^\mu, u^\nu, \Omega^{\mu\nu})$. It is however natural, instead of using $\Omega^{\mu\nu}$ as coordinates to treat them as velocities of angular degrees of freedom which will lead us to a natural interpretation of spin. It is easy to see there are four different scalar quantities (neglecting parity violating terms) we can consider (schematically)

$$a_1 = u^2 \tag{16}$$

$$a_2 = \Omega^2 \tag{17}$$

$$a_3 = u\Omega\Omega u \tag{18}$$

$$a_4 = \Omega\Omega\Omega\Omega, \tag{19}$$

where contractions are made with the space-time metric $g_{\mu\nu}$. Using these quantities our Lagrangian will be in principle a general expression of the form $L(a_1, a_2, a_3, a_4)$. We will neglect multipole moments throughout this section, the inclusion of which will be studied later on. The objects will be therefore considered symmetric with respect to their rotational axis.

In order to introduce the idea of spin we will define the antisymmetric tensor $S^{\mu\nu}$ and momentum p^μ by

$$\delta L = -p^\mu \delta u_\mu - \tfrac{1}{2} S^{\mu\nu}\delta\Omega_{\mu\nu}, \tag{20}$$

[6] Remember we are treating the world line of the particles as external sources, namely $x^a \equiv J$, therefore $S_{\rm eff}$ is just the partition function $e^{iS_{\rm eff}} \equiv \langle 0|0\rangle^J \sim e^{i\Gamma(J,\dot{J}\ldots)T}$, with $\Gamma(J)$ the effective action for the sources as $T \to \infty$ [31].

where the minus sign corresponds to the correct nonrelativistic limit [20]. From these definitions we will have

$$p^\alpha = -2u^\alpha \frac{\partial L}{\partial a_1} - 2\Omega^{\alpha\nu}\Omega_{\nu\rho}u^\rho \frac{\partial L}{\partial a_3} \tag{21}$$

$$S^{\mu\nu} = -4\Omega^{\mu\nu}\frac{\partial L}{\partial a_2} - 2(u^\mu\Omega^{\nu\lambda}u_\lambda - u^\nu\Omega^{\mu\lambda}u_\lambda)\frac{\partial L}{\partial a_3} - 8\Omega^{\nu\beta}\Omega_{\beta\alpha}\Omega^{\alpha\mu}\frac{\partial L}{\partial a_4}. \tag{22}$$

The variation of the action consists of two pieces. Let us concentrate first in the tetrad part. Using the definition of spin we will have to deal with

$$\begin{aligned}\delta S &= -\int d\tau S^{\alpha\beta}\delta\Omega_{\alpha\beta} \\ &= -\int d\tau\Big(-\frac{DS^{\alpha\beta}}{D\tau}e_{K\alpha} - \frac{De_{K\alpha}}{D\tau}S^{\alpha\beta} \\ &\quad + S^{\rho\nu}\frac{De_{J\nu}}{D\tau}e_{\rho K}e^{J\beta}\Big)\delta e^K_\beta.\end{aligned} \tag{23}$$

The equation of motion can be directly read from the above expression, multiplying by $e^{K\mu}$ we get [using (15)]

$$\frac{DS^{\mu\nu}}{D\tau} = S^{\mu\lambda}\Omega_\lambda{}^\nu - \Omega^\mu{}_\lambda S^{\lambda\nu} = p^\mu u^\nu - u^\mu p^\nu, \tag{24}$$

where the last equality follows from (21) and (22) [20]. Notice that we have not specified a Lagrangian up to this stage. It is easy to see from (24) that

$$\frac{DS^{IJ}}{D\tau} \equiv \frac{D(S^{\alpha\beta}e^I_\alpha e^J_\beta)}{D\tau} = 0, \tag{25}$$

which shows that spin projected with respect to the e^I_α frame remains constant. In addition, one can also show that the scalar $S^2 \equiv \frac{1}{2}S^{\mu\nu}S_{\mu\nu}$ is conserved. As a further property it is also instructive to notice that $S^{\mu\nu}S^*_{\mu\nu}$ is also a constant of the motion, where $S^*_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}S^{\alpha\beta}$.

In order the get the δx piece of δS, a shortcut can be taken by going to a locally flat coordinate system where the connection terms are zero at the point. Promoting the derivatives to covariant ones in this frame we will end up with

$$\frac{Dp_\gamma}{D\tau} = \frac{1}{2}S^{\alpha\beta}(\Gamma_{\alpha\beta\sigma,\gamma} - \Gamma_{\alpha\beta\gamma,\sigma})u^\sigma = -\frac{1}{2}R_{\gamma\sigma\alpha\beta}S^{\alpha\beta}u^\sigma, \tag{26}$$

where we have used the form of the Riemann tensor in a locally flat coordinate system. Written this way we can promote now the equation to all reference systems since it is covariant. We therefore recognize in (24) and (26) the well-known Papapetrou equations [32]. Remarkably, although in terms of the tetrad these equations depend on the choice of Lagrangian, as a function of spin and momentum the evolution equations are action independent as far as curvature terms are not inserted. Including curvature terms in the effective action will be relevant to introduce finite size effects and will modify these equations for extended objects.

C. Constraints and angular-velocity/spin relationship

In order to describe the correct number of degrees of freedom, we need to add a set of constraints to the EOM (24) and (26). A well-defined angular-velocity/spin relationship is also necessary to extend our power counting rules to the spinning case. We will show here that both features are related. Here we will closely follow [20], to which we refer the reader for details; other approaches may be found in [34]. This section relies on a basic knowledge of constrained systems (for further details see [35,36]).

It is natural to impose the following (covariant) constraints in the space of solutions:

$$V^\mu = S^{\mu\nu}p_\nu \approx 0, \tag{27}$$

where just three of the four components are independent, and "≈" stands for weakly vanishing [35]. This set of constraints are second class, namely, they have nonvanishing Poisson bracket among themselves, and therefore reduce the number of degrees of freedom from 6 to 3 SO(3) parameters as expected. It can be shown in addition there is a Lagrangian from which (27) kinematically follows [20].

We need to guarantee the constraints in (27) are preserved upon evolution. It is however possible to show from (24) and (26) that $\frac{DV^\mu}{d\tau} \approx 0$ will be satisfied provided

$$p^\alpha = mu^\alpha - \frac{1}{2m}R_{\beta\nu\rho\sigma}S^{\alpha\beta}S^{\rho\sigma}u^\nu, \tag{28}$$

with $m^2(S^2) \equiv p^2$ defined by (21) and (22). This also means that the difference between p^ν/m and u^ν is higher order in the PN expansion and we can consider $S^{\mu\nu}u_\nu = 0$ as well as $\frac{dS^{\mu\nu}}{d\tau} = 0$ to leading order.

It is possible to show that (27) implies $C_1 = S^{\mu\nu}S^*_{\mu\nu} \approx 0$, which is a first class constraint. There is therefore, in addition to RPI [$C_2 = p^2 - m(S^2) \approx 0$], a gauge freedom which can be attributed to the choice of the temporal vector of the tetrad e^0_μ, and a sensible choice of gauge is then $\psi^\mu = e^\mu_0 - p^\mu/m \approx 0$ [20]. This gauge, jointly with (27), also translates into a choice of center of mass of the object [10,14], and implies as well $\Omega^{\mu\nu}p_\nu = \frac{Dp^\mu}{D\tau}$, from which we get

$$\Omega_{\mu\nu} \sim S_{\mu\nu} - \frac{1}{2m}R_{\mu\nu\alpha\beta}S^{\alpha\beta} + RRSSS + \cdots, \tag{29}$$

where we have used (27) and (28). We can indeed obtain the angular-velocity/spin relationship by matching the evolution equation for the tetrad in a Minkowski background obtaining (see Appendix A)

$$S^{\mu\nu} = \frac{I}{(u^2)^{1/2}}\Big(\Omega^{\mu\nu} + \frac{I}{2m}R_{\mu\nu\alpha\beta}\Omega^{\alpha\beta} + \cdots\Big), \tag{30}$$

with I the moment of inertia, and a particular Lagrangian is chosen to ensure (27) [20]. The main results of this section are therefore Eqs. (28) and (30), from which we conclude that in a theory where (27) is kinematically imposed spin and angular velocity are naturally related and proportional in flat space. Within an EFT approach, these relationships are all we need to construct the NRGR extension for spinning bodies.

IV. NRGR FOR SPINNING BODIES

A. Power counting

The power counting rules in NRGR have been developed in [1]. Here we are going to extend them to include spin degrees of freedom. As we shall show, the only necessary change is the inclusion of spin insertions at the vertices.

First of all notice that, from the constraints at leading order,

$$S^{\mu\nu}u_\nu = 0 \rightarrow S^{j0} = S^{jk}u^k, \tag{31}$$

which implies a suppression of the temporal components with respect to the spatial ones. In other words, spin is represented by a 3-vector, $S^k = \frac{1}{2}\epsilon^{kij}S_{ij}$, in the rest frame of the particle as expected. We will concentrate here in compact objects like neutron stars or black holes where the natural length scale can be taken to be their Schwarzschild radius $r_s \sim Gm$ (for general objects see Appendix B), and hence a momentum of inertia scaling as $I \sim m^3/m_p^4$. For the spin angular momentum we will have

$$S = I\omega = I\frac{v_{\rm rot}}{r_s} \sim m v_{\rm rot} r_s < m r_s \sim L v. \tag{32}$$

We therefore see that spin gets suppressed with respect to the orbital angular momentum, even for the maximally rotating case ($v_{\rm rot} = 1$). We can also assume a different kinematical configuration with the particles corotating, namely $\frac{v_{\rm rot}}{r_s} = \frac{v}{r}$. In the former $S \sim (mvr_s^2)/r = L(r_s^2/r^2) \sim Lv^4$. We will generally power count spin as

$$S \sim Lv^s, \tag{33}$$

with $s = 1$ and $s = 4$ in the maximally rotating and corotating scenario, respectively.

To obtain the scaling laws we have assumed the usual proportionality at leading order between spin and angular velocity which was obtained from (30). As usual, subleading scalings will be naturally taken into account by the insertion of higher-order terms in the world line action. What we have learned here is that spin effects are in any case subleading in the PN expansion, and the scaling laws developed in [1] still hold and spin contributions can be treated as a perturbation.

B. Feynman rules: Spin-graviton vertex

To construct the effective theory for gravitons we need to expand the metric around a Minkowski background, namely $g_{\alpha\beta} = \eta_{\alpha\beta} + \frac{h_{\alpha\beta}}{m_p}$. There is however a subtle point in doing this given that (13) leads to

$$\begin{aligned} \eta_{IJ}e^I_\mu e^J_\nu &= \eta_{\mu\nu} + \frac{h_{\mu\nu}}{m_p}, \\ e^I_\mu &= \Lambda^I_\mu + \delta e^I_\mu \rightarrow \delta e^I_\mu = \frac{1}{2m_p}h_{\mu\nu}\Lambda^{\nu I} + \cdots, \end{aligned} \tag{34}$$

where $\Lambda^{I\alpha}$ is an element of the Lorentz group. We will also need $\delta e^{\mu I}$ which is defined through the inverse metric [37],

$$\begin{aligned} g^{\mu\nu} &= \eta^{\mu\nu} + h^{\mu\nu}, \\ h^{\mu\nu} &= \eta^{\mu\mu'}\eta^{\nu\nu'}(-h_{\mu'\nu'} + h^\alpha_{\mu'}h_{\alpha\nu'} - \cdots), \end{aligned} \tag{35}$$

where indices are raised with $\eta^{\mu\nu}$. We will therefore have

$$\delta e^{I\mu} = -\frac{1}{2m_p}\eta^{\mu\nu}h_{\nu\alpha}\Lambda^{\alpha I} + \cdots, \tag{36}$$

One can immediately see how to proceed by comparison with what has been done in [20] within flat space where the angular velocity was defined as

$$\Omega^{\mu\nu}_M \equiv \Lambda^{\mu\alpha}\frac{d\Lambda^\nu_\alpha}{d\tau}, \tag{37}$$

with $\Lambda^{\mu\nu}$ describing the *rotation* of the particle, and M stands for Minkowski. Expanding the action using (34) and (36), we will thus obtain the spin-graviton interaction to all orders in a flat space background as a function of the graviton field and (37) (see Appendix C for details). To leading order in the weak field expansion (see Fig. 2),

$$L_0 = \frac{1}{2m_p}h_{\alpha\gamma,\beta}S^{\alpha\beta}_M u^\gamma, \tag{38}$$

where $S^{\mu\nu}_M = -\frac{\partial L}{\partial\Omega_{\mu\nu}}$ at $g_{\mu\nu} = \eta_{\mu\nu}$ (we will drop the M from now on). The expression in (38) is remarkably action independent if written in terms of spin and the graviton field. The Lagrangian dependence enters in the so far unknown function $S(\Omega)$. This conclusion however just applies to the leading term and a choice of action is necessary to obtain the Feynman rules to all orders. At next to leading order in the weak gravity limit we will have

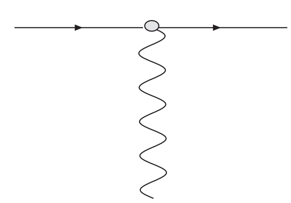

FIG. 2. Leading order spin-graviton vertex interaction. The blow represents a spin insertion.

$$L_1 = \frac{1}{4m_p^2} S^{\beta\gamma} u^\mu h_\gamma^\lambda \Big(\frac{1}{2} h_{\beta\lambda,\mu} + h_{\mu\lambda,\beta} - h_{\mu\beta,\lambda}\Big), \quad (39)$$

where a particular action has been chosen to ensure (27). A different choice of Lagrangian will imply different Feynman rules. However, bear in mind that different actions will differ in the spin/angular-velocity relationships and might not lead kinematically to (27). The physics will be invariant once these differences are taken into account.

To calculate in the EFT we need to match (38) and (39) into NRGR using the power counting rules developed in [1] plus the spin insertions. Up to 2PN, for maximally rotating compact objects we will get for the spinning part of the NRGR Lagrangian

$$L_{\rm 1PN}^{\rm NRGR} = -\frac{1}{2m_p} H_{i0,k} S^{ki}, \quad (40)$$

$$L_{\rm 1.5PN}^{\rm NRGR} = -\frac{1}{2m_p}(H_{ij,k} S^{ki} u^j + H_{00,k} S^{k0}), \quad (41)$$

$$L_{\rm 2PN}^{\rm NRGR} = -\frac{1}{2m_p}(H_{0j,k} S^{k0} u^j + H_{i0,0} S^{0i}) + \frac{1}{4m_p^2} S^{ij}(H_j^\lambda H_{0\lambda,i} - H_j^k H_{0i,k}). \quad (42)$$

The procedure follows systematically as shown in Appendix C.

It is an useful exercise to check the gauge invariance of (38), or in other words to obtain the leading stress energy tensor. It is straightforward to calculate $T^{\mu\nu}_{(1)} = -\frac{\partial L}{\partial g_{\mu\nu}}$ at $g_{\mu\nu} = \eta_{\mu\nu}$ getting

$$T^{\mu\nu}_{(1)} = -\tfrac{1}{2}\partial_\beta (S^{\beta\mu} u^\nu + S^{\beta\nu} u^\mu), \quad (43)$$

which agrees at zero order with the original proposal of Dixon [38] and also Bailey and Israel [39] (see Appendix D).

The Ward identity,

$$\partial_\mu T^{\mu\nu}_{(1)} \sim \partial_\beta \frac{dS^{\beta\nu}}{d\tau} = 0, \quad (44)$$

is therefore obeyed since spin is constant to leading order in the PN expansion.

C. Leading order graviton exchange

Our goal from now on is to calculate the leading order piece (one graviton exchange) of the potential energy due to spin-orbit (Fig. 3) and spin-spin (Fig. 4) couplings coming from (40) and (41).[7] The leading order spin-orbit

[7]Self-energy terms are not considered since they yield scaleless integrals.

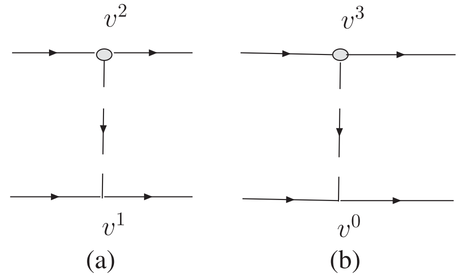

FIG. 3. Leading order spin-orbit interaction. Diagram (a) takes into account a v^1 mass insertion. Diagram (b) corresponds to the v^3 spin-graviton vertex.

contribution is the sum of two pieces,

$$\text{Fig. 3} = \frac{-im_2}{4m_p^2}\int dt dt' \frac{d^3p}{(2\pi)^3}\Big(\frac{e^{-i\vec{p}(\vec{x}(t)-\vec{y}(t'))}}{\vec{p}^2} \times \delta(t-t') P_{0\epsilon;\alpha\gamma}\Big) S_1^{\beta\alpha} u_1^\gamma(t) u_2^\epsilon (2-\delta_0^\epsilon). \quad (45)$$

We need to distinguish two different cases: the temporal and spatial derivative. The temporal derivative will just hit $\delta(t-t')$ and can be integrated by part bringing down a velocity factor. To leading order we would need to consider $\epsilon = \gamma = 0$. However, $P_{00;\alpha 0} = 0$ unless $\alpha = 0$, which leads to a term proportional to $S^{00} = 0$. There is then no contribution from the temporal derivative and we just need to concentrate in the spatial part and the terms,

$$-im_2 G_N \partial_j \int dt \frac{1}{|\vec{x}(t) - \vec{y}(t)|} P_{0\epsilon;\alpha\gamma} S_1^{j\alpha} u_1^\gamma(t) u_2^\epsilon(t)(2-\delta_0^\epsilon). \quad (46)$$

Notice that we have three possible contributions, one coming from $\epsilon = 0$, $\alpha = l$, $\gamma = k$, another where $\epsilon = l$, $\alpha = k$, $\gamma = 0$, and finally $\epsilon = \alpha = \gamma = 0$. The latter looks at first as a v^0 piece, however this is misleading since our spin choice implies $S^{j0} = S^{jl} u^l$. Adding all the terms one gets

$$\text{Fig. 3} = i\int dt \frac{-2G_N m_2}{|\vec{x}(t) - \vec{y}(t)|^2}((\vec{n}\times\vec{u}_1)\cdot\vec{S}_1 - (\vec{n}\times\vec{u}_2)\cdot\vec{S}_1), \quad (47)$$

where $\vec{n}$ is the unit vector in the $(\vec{x} - \vec{y})$ direction and we have used $S_{kj} = \epsilon_{kji} S^i$. Joining the mirror image we will

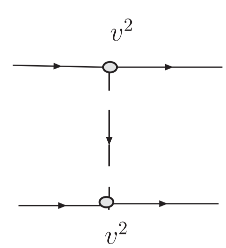

FIG. 4. Leading order spin-spin interaction.

end up with

$$V_{SO} = \frac{2G_N}{r^2}\mu(\vec{n}\times\vec{v})\cdot\left(\left(1+\frac{m_1}{m_2}\right)\vec{S}_2 + \left(1+\frac{m_2}{m_1}\right)\vec{S}_1\right), \tag{48}$$

for the spin-orbit potential, where μ is the reduced mass, $r = |\vec{x}(t) - \vec{y}(t)|$ and $\vec{v} \equiv \vec{u}_1 - \vec{u}_2$.

Let us now consider the spin-spin interaction. The leading order contribution is (see Fig. 4)

$$\frac{i}{4m_p^2}\partial_{y_k}\partial_{x_{k'}}\int dt \frac{d^3p}{(2\pi)^3} e^{i\vec{p}(\vec{x}(t)-\vec{y}(t))}\frac{1}{\vec{p}^2}P_{j'0;j0}S_1^{jk}S_2^{j'k'}. \tag{49}$$

Using $P_{j'0;j0} = -\frac{1}{2}\delta_{jj'}$ it is straightforward to show

$$V_{SS} = -\frac{G_N}{r^3}(\vec{S}_1\cdot\vec{S}_2 - 3\vec{S}_1\cdot\vec{n}\vec{S}_2\cdot\vec{n}), \tag{50}$$

for the spin-spin binding potential. It is easy to see by power counting that $V_{SO}dt \sim Lv^3$ and $V_{SS}dt \sim Lv^4$, effectively 1.5PN and 2PN for maximally rotating compact objects.

By comparison with the results in [10], it is immediate to notice there is a mismatch in the spin-orbit contribution (48), which can be traced back to the choice of spin supplementary condition in (27). As it was noticed in [10,12,13,15], this discrepancy is associated to the choice of center of mass of each body. It can be shown there is a coordinate transformation that relates the center of mass choice which follows from (27) and the so-called baryonic coordinates (implicitly used in [10]), where the center of mass is defined through the baryonic density, and one has $S^{i0} = \frac{1}{2}S^{ij}u^j$. Had we calculated the spin-orbit term within the baryonic condition we would obtain

$$\bar{V}_{SO} = \frac{2G_N}{r^2}\mu(\vec{n}\times\vec{v})\cdot\left(\left(1+\frac{3m_1}{4m_2}\right)\vec{S}_2 + \left(1+\frac{3m_2}{4m_1}\right)\vec{S}_1\right), \tag{51}$$

in complete agreement with the result in [10]. The leading order spin-spin interaction does not get affected by this new choice.

D. EOM

Even though (51) is in total agreement with the spin-orbit potential in baryonic coordinates, the calculation in the covariant approach does not reproduce the well-known fact that the generalized Lagrangian from which the EOM are derived turns out to be acceleration dependent [9,10,12,13,15]. Indeed, (48) reproduces the gravitational potential in [13] up to this acceleration dependent piece which does not follow from a graviton exchange. As we shall show in Appendix E, the solution to this puzzle lies on the fact that a noncanonical algebra develops which naturally reconciles both approaches. Instead of following that path here, it is instructive to remark there is a coordinate transformation which leads to a canonical structure. Not surprisingly this map transforms the covariant choice into the baryonic one, where it has been explicitly shown there is no need for an acceleration dependent piece in the action. We can therefore proceed from the potentials in (50) and (51) and the standard Euler-Lagrange formalism to obtain the EOM within the baryonic supplementary condition. It can be easily shown that they are given by (in relative coordinates)

$$\vec{a} \equiv \vec{a}_1 - \vec{a}_2 = -\frac{G_N M}{r^2}\vec{n} + \vec{a}_{SO} + \vec{a}_{SS} \tag{52}$$

$$\vec{a}_{SO} = \frac{G_N}{r^3}(3\vec{n}(\vec{n}\times\vec{v})\cdot\vec{\chi} + 2\vec{v}\times\vec{\chi} + 3\vec{n}\cdot\vec{v}(\vec{n}\times\vec{\chi})) \tag{53}$$

$$\vec{a}_{SS} = -\frac{3G_N}{\mu r^4}(\vec{n}(\vec{S}_1\cdot\vec{S}_2 - 5\vec{S}_2\cdot\vec{n}\vec{S}_2\cdot\vec{n}) + \vec{S}_2(\vec{n}\cdot\vec{S}_1) + \vec{S}_1(\vec{n}\cdot\vec{S}_2)) \tag{54}$$

$$\frac{d\vec{S}_1}{dt} = \frac{G_N}{r^3}\left(\vec{L}\times\vec{S}_1\left(2+\frac{3m_2}{2m_1}\right) + \vec{S}_1\times\vec{S}_2 + 3(\vec{n}\cdot\vec{S}_2)\vec{n}\times\vec{S}_1\right);$$
$$\frac{d\vec{S}_2}{dt} = 1 \leftrightarrow 2, \tag{55}$$

where we have introduced $M = m_1 + m_2$, $\vec{\chi} = (2 + \frac{3m_2}{2m_1})\vec{S}_1 + (2 + \frac{3m_1}{2m_2})\vec{S}_2$, and $\vec{L} = \mu r\vec{n}\times\vec{v}$.

From the symmetries of the action we can directly construct the conserved quantities, in particular, the energy. First of all notice that the spin-orbit *force* does not do any work, namely $\vec{a}_{SO}\cdot\vec{v} = 0$, from which we conclude that the conserved energy is nothing but

$$E = \frac{1}{2}\mu\vec{v}^2 - \frac{G_N M\mu}{r} + V_{SS}. \tag{56}$$

In order to compare with the results in [13] within the covariant supplementary condition, we restrict ourselves now to the case of nearly circular orbits to express (56) in terms of the orbital angular frequency ω and spin. Taking an angular average for all quantities we obtain from (52),

$$r(\omega, S) = \frac{M^{1/3}}{\omega^{2/3}}\left(1 - \frac{1}{3}\frac{\omega}{M}\vec{l}\cdot\vec{\chi} - \frac{1}{2}\frac{\omega^{4/3}}{\mu M^{5/3}}(\vec{S}_1\cdot\vec{S}_2 - 3\vec{l}\cdot\vec{S}_1\vec{l}\cdot\vec{S}_2)\right) \tag{57}$$

$$E(\omega, S) = -\frac{1}{2}(M\omega)^{2/3}\left(1 + \frac{4}{3}\frac{\omega}{M}\vec{l}\cdot\vec{\chi} + \frac{\omega^{4/3}}{\mu M^{5/3}}(\vec{S}_1\cdot\vec{S}_2 - 3\vec{l}\cdot\vec{S}_1\vec{l}\cdot\vec{S}_2)\right), \tag{58}$$

with $\vec{l}$ the unit vector in the $\vec{L}$-direction. We therefore conclude that energy as a function of spin and angular frequency in (58) matches that of [13,15] independently of the spin supplementary condition. As a final comment, let us remark that our results also agree with those of Buonanno *et al.* [40], which appeared after we had completed our work, where a similar procedure is advocated within baryonic coordinates. However, the covariant spin supplementary condition and finite size effects are left undiscussed in [40].

V. INTERNAL DEGREES OF FREEDOM II: PERMANENT MULTIPOLE MOMENTS

By now it should be easy to visualize how are we going to include multipole moments in terms of the e^I_μ fields. Let us assume for instance the particle has an intrinsic, permanent quadrupole moment Q^{IJ}. In order to couple it to the gravitational field, the following term can be introduced:

$$\int d\tau R_{\mu\alpha\beta\gamma} e^\mu_I e^\beta_J Q^{IJ} u^\alpha u^\gamma. \tag{59}$$

Notice in fact this is just a generalization of what has been done in [1]. In fact, it can be shown (see Appendix C) this quadrupole term is naturally obtained if we include the nonspherical contribution of the tensor of inertia in the spin part of the Lagrangian.

From the gauge fixing condition $e^\mu_0 \approx p^\mu/m$, we immediately see that the Q^{0I} components do not contribute at all given that replacing e^μ_0 in (59) gives rise to vanishing terms. We can therefore, as expected, concentrate just in the spatial components.

It is easy to see that (59) will naturally reproduce the quadrupole gravitational energy piece, $(Q_{ij}x^i x^j)/r^5$, in the potential. In order to obtain a nontrivial contribution, we calculate a correction to the binding energy due to a quadrupole-spin interaction to leading order. After matching (59) into NRGR and using the expression for the Riemann tensor in the weak gravity approximation, the one potential graviton exchange in Fig. 5 gives

$$V_{QS} = \frac{3G_N}{r^4}[(Q^i_{1i} - 5Q^{ik}_1 n_i n_k)\vec{n}\cdot(\vec{u}_2 \times \vec{S}_2) + Q^{jk}_1 n_k((2\vec{u}_2 + \vec{u}_1)\times\vec{S}_2)_j + Q^{ij}_1(u_{1j} - 5(\vec{u}_1\cdot\vec{n})n_j)(\vec{n}\times\vec{S}_2)_i], \tag{60}$$

for the quadrupole-spin potential within the covariant spin condition, where we have used $Q^{ij} = Q^{ji}$ and the Euclidian metric (δ^{ij}) to raise and lower indexes. A similar expression is obtained from the mirror image $1 \leftrightarrow 2$. It is easy to show it corresponds to a 3.5PN contribution for maximally rotating neutron stars or black holes. Notice that for a spherically symmetric object ($Q^{ij} \sim \delta^{ij}$) $E_{QS} \to 0$ as one would have guessed.[8] Therefore, the coupling is effectively to the traceless piece of the quadrupole. Higher-order multipole moments are easily handled by similar procedures.

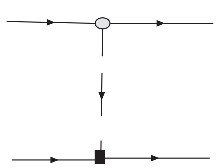

FIG. 5. Leading order quadrupole-spin one graviton exchange. The black square represents a quadrupole insertion.

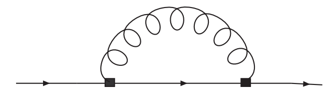

FIG. 6. Leading order diagram whose imaginary part gives rise to the quadrupole radiation power spectrum. The black boxes are quadrupole insertions and the curly propagator a radiation graviton.

A. Quadrupole radiation

It is instructive to notice that (59) will directly lead to the well-known quadrupole radiation formula. The leading order piece will be of the form (for on-shell gravitons)

$$\frac{1}{2m_p} R_{i0j0} Q^{ij}_{TF}, \tag{61}$$

where TF stands for the traceless piece. By calculating the imaginary part of Fig. 6 we can immediately obtain (we skip details which can be found for an identical calculation in [1])

$$\text{Im Fig. 6} = -\frac{1}{80m^2_{Pl}}\int\frac{d^3\mathbf{k}}{(2\pi)^3 2|\mathbf{k}|}\mathbf{k}^4|Q^{ij}_{TF}(|\mathbf{k}|)|^2, \tag{62}$$

from which the power radiated follows:

$$P = \frac{G_N}{5\pi T}\int_0^\infty d\omega\,\omega^6|Q^{ij}_{TF}(\omega)|^2 = \frac{G_N}{5}\langle\dddot{Q}^{ij}_{TF}\dddot{Q}^{ij}_{TF}\rangle, \tag{63}$$

with dots as time derivatives and the bracket representing time averaging. This is the celebrated quadrupole radiation formula.

VI. DIVERGENCES, NONMINIMAL INSERTIONS, AND FINITE SIZE EFFECTS FOR SPINNING BODIES

We are going to discuss here the appearance of divergences and their consequent renormalization. This will lead us to the study of higher-order terms in the world line action and their Wilson coefficients, which will encode the information about the internal structure of the body. In addition to terms coming from logarithmic UV divergen-

[8] This is reminiscent of the vanishing of the c_V contributions in [1].

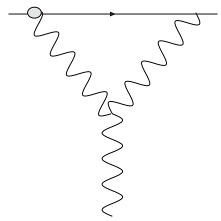

FIG. 7. Two graviton contributions to the one point function in effective action with a single spin insertion.

ces, we will encounter power law divergences whose associated Wilson coefficients do not have scale dependence. This distinction will turn out to be connected with tidal deformations vs self-induced effects as we shall see.

A. A cursory first look

Let us study the one point function in the effective action with spin insertions. Let us start with the diagram shown in Fig. 7. The spin-graviton Feynman rules derived from (38) differ from mass insertions in two main points: its tensor structure and its dependence on the graviton momentum. Figure 7 then contributes for potential gravitons terms proportional to

$$\int \frac{d^3\mathbf{q}}{(2\pi)^3}\left(\frac{\mathbf{q}\cdot\mathbf{k}, \mathbf{q}^2, \mathbf{k}^2}{\mathbf{q}^2(\mathbf{q}+\mathbf{k})^2}\right), \tag{64}$$

where $\mathbf{k}$ is the external graviton momentum. None of these integrals are logarithmically divergent and therefore can be absorbed as pure counterterms in the original Lagrangian.

Let us concentrate now on the divergent piece coming from diagrams like in Fig. 8. It can be shown that this diagram contain terms such as (in d-dimensions)

$$I(\mathbf{k}) = \int \frac{d^{d-1}\mathbf{p}}{(2\pi)^{d-1}} \frac{d^{d-1}\mathbf{q}}{(2\pi)^{d-1}} \frac{(\mathbf{q}\cdot\mathbf{k})(\mathbf{p}\cdot\mathbf{k})}{\mathbf{q}^2\mathbf{p}^2(\mathbf{q}+\mathbf{p}+\mathbf{k})^2}, \tag{65}$$

as well as integrals with $\mathbf{q}\cdot\mathbf{p}$ in the numerator. These integrals contain power as well as logarithmic UV divergences. It is clear that these divergences cannot be absorbed into the original Lagrangian since they involve in principle higher-order derivatives of the metric. By general covariance and parity conservation, there is a limited set of nonzero terms one can build up with the right structure to cancel the previous divergences. In what follows we will

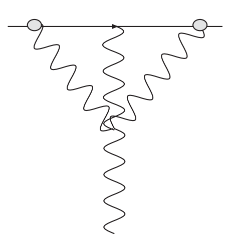

FIG. 8. Three graviton contribution to the one point function in the effective action with two spin insertions.

study a possible set of new insertions in the world line action which are generated by renormalization.

B. Nonminimal insertions I: Self-induced effects

We will consider here terms which are not total derivatives and cannot be removed by f.r. Let us proceed systematically. Let us start with terms linear in Riemann and no further derivatives acting on external fields. The first nonzero terms we can construct are

$$\begin{aligned}
\mathcal{O}^1_{RS^2} &\equiv \frac{C^1_{RS^2}}{m_p} R_{\alpha\beta\mu\nu} S^{\alpha\beta} S^{\mu\nu} \\
\mathcal{O}^2_{RS^2} &\equiv \frac{C^2_{RS^2}}{m_p} R_{\alpha\beta\mu\nu} S^{\alpha\mu} S^{\beta\nu} \\
\mathcal{O}^3_{RS^2} &\equiv \frac{C^3_{RS^2}}{m_p} R_{\alpha\beta\mu\nu} S^{\alpha\gamma} S^{\mu}_{\gamma} u^\beta u^\nu.
\end{aligned} \tag{66}$$

It is possible to show that they are physically equivalent, namely, they are proportional up to f.r. removable terms. By simple inspection it is easy to see they are similar to the quadrupole moment insertion in (59). It is therefore natural to expect these terms to describe self-induced quadrupole effects rather than tidal deformations. It is straightforward to show, after matching into NRGR for potential gravitons, that $\mathcal{O}^i_{RS^2} \sim \sqrt{L} v^{2s+2}$ with $C^i_{RS^2} \sim 1/m$. This immediately tells us $\mathcal{O}^i_{RS^2}$ cannot be generated from a logarithmic UV divergence. It is possible to show nonetheless that these terms can be generated from power law divergences (see Appendix G). At 2(5)PN these terms generate a gravitational potential for a maximally rotating (corotating) neutron star or black hole A coupled to a nonspinning one B of the form

$$V_{S^2O} = C^{\rm tot}_{RS^2(A)} \frac{G_N m_B}{2r^3} (3(\vec{S}_A\cdot\vec{n})^2 - \vec{S}_A\cdot\vec{S}_A), \tag{67}$$

with $C^{\rm tot}_{RS^2(A)} = (-C^1_{RS^2} - \frac{1}{2}C^2_{RS^2} + \frac{1}{4}C^3_{RS^2})_A$. A spinning particle will tend to deform and therefore generate multipole (mass) moments which will thus produce a binding energy term equivalent, as in this case, to a quadrupole interaction [21].[9] It is well known that rotating black holes, or neutron stars, have a quadrupole moment given by $Q_{bh} = -aS^2/m$ $(G = c = 1)$, with m, S the mass and spin, respectively. For a black hole $a = 1$ [41], for neutron stars a ranges between 4 and 8 depending on the equation of state of the neutron star matter [22]. This will give us a straightforward matching for $C^{\rm tot}_{RS^2(A)}$ in (67) which is consistent by dimensional analysis with what we expect from naturalness arguments. Furthermore, these coefficients contribute to the one point function, and thus will show

[9]It will in principle vary in time and henceforth radiate. This effect will be naturally taken into account similarly as we did for the quadrupole.

up in the metric solution for a rotating neutron star or black hole. For the case of a black hole, the Kerr-Newman spacetime does not have any logarithmic dependence and therefore, every coefficient associated to a non-f.r. removable term[10], which contributes to the one point function, must be scale independent. Similarly for neutron stars. This provides a natural characterization for self-induced effects. Tidal effects in the other hand will be then associated to coefficients which do not contribute to the one point function, and moreover are scale dependent. Tidally induced effects will be therefore naturally generated by logarithmic UV divergences.

Once self-induced spin-multipole moments are included it is no longer necessary to introduce a nondynamical permanent multipole. Adding more spin insertions without derivatives will have the same type of behavior we encountered above, namely, the Wilson coefficients will scale with negative powers of the mass and therefore they cannot be generated from logarithmic UV divergences. It is indeed possible to show that we can in principle hook together n spin tensors leading to terms scaling as $\sqrt{L}v^{4+n(s-1)}$ after matching into NRGR. Given that n could be any number of spin insertions, it appears as if we will have no predictive power for the case $s = 1$. However, it can be shown that terms with a large number of spin insertions can be rewritten in terms of interactions with no more than four spin insertions [20]. The case of three and four spin insertions does not modify the leading order expression in (67).

C. Nonminimal insertions II: Tidal deformations

Another type of world line insertions we could in principle generate are those having derivatives of the Riemann tensor and more spin insertions. We will need to introduce terms like

$$\begin{aligned} &D_\epsilon R_{\alpha\beta\mu\nu} S^{\epsilon\mu} S^{\nu\beta} u^\alpha; \\ &D^2 R_{\mu\nu\alpha\beta} S^{\mu\sigma} S^\alpha_\sigma u^\nu u^\beta; \\ &D_\rho D_\sigma R_{\mu\nu\alpha\beta} S^{\sigma\mu} S^{\rho\alpha} u^\nu u^\beta; \\ &D_\sigma D^2 R_{\beta\rho\mu\epsilon} S^{\rho\sigma} S^\epsilon_\gamma S^{\beta\gamma} u^\mu \ldots. \end{aligned} \tag{68}$$

We will concentrate here in tidal effects and therefore in those terms generated by logarithmic UV divergences. To lowest order it can be shown we will have $D^2\mathcal{O}^i_{RS^2}$ (with $i = 1, 2, 3$) coming from diagrams like Fig. 8. We will generically denote their Wilson coefficients by C_{D^2}. Given that these expressions possess different tensorial structure, the RG flow will naturally decouple. By dimensional analysis it is easy to conclude that

$$\mu \frac{dC_{D^2}}{d\mu} \sim \frac{m}{m_p^4}. \tag{69}$$

As it was pointed out before, these new insertions will not contribute to the one point function.[11] However, they will in principle be observable for more complicated ambient metric, such as the field produced by a binary companion.

Let us power count this effect. After matching into NRGR for potential gravitons we will get, to leading order,

$$C_{D^2}(\partial^4 H_{\mathbf{k}} d^3\mathbf{k}) S^2 d\tau \sim \frac{m}{m_p^4}\frac{1}{r^4}\frac{v^2}{\sqrt{L}} L^2 v^{2s}\frac{r}{v} \sim \sqrt{L}v^{6+2s}, \tag{70}$$

which would make it a 4PN contribution for maximally rotating compact objects. A careful inspection shows however that the leading piece from these terms goes as derivatives of $\delta(x_1 - x_2)$, which is a contact interaction (the $\mathbf{k}^2$ piece cancels the propagator). As a consequence, the first long range interaction coming from C_{D^2}, and therefore the lowest companion induced tidal effect, will scale as Lv^{8+2s}, a 5PN contribution for maximally rotating compact objects (formally at 3PN as shown in Appendix B). At this order new terms will also start to contribute [for instance the third and fourth expressions in (68); see Appendix G for details]. The reasoning in previous sections can be easily extended to the n-point function and higher Riemann insertions.

VII. CONCLUSIONS

In this paper we have extended the formalism initially proposed in [1] to include internal degrees of freedom like spin as well as multipole moments. As a first step, we have developed in a suitable fashion the description of spinning bodies in GR to include a richer tensor structure extending the previous work done in the realm of special relativity by Hanson and Regge [20]. We have shown that a self-consistent action principle can be implemented and Papapetrou equations [32] recovered. Permanent multipole moments are naturally introduced by adding new degrees of freedom in the world line action. Using this formalism we have extended NRGR, its power counting, and Feynman rules with which we have reproduced the well-known spin-spin and spin-orbit effects at leading order [13]. A quadrupole-spin correction to the binding energy was obtained for the first time (to my knowledge) as well as the quadrupole radiation formula recovered. The equivalence between different choices for the spin supplementary condition was explicitly shown. We have shown afterwards the appearance of divergences at higher orders in the PN expansion and its consequent regularization. The type of

[10]As it was noticed in [1], f.r. removable terms can in principle show up in the one point function. However, they can be washed away by a coordinate transformation. Here we will concentrate in terms which are not f.r. removable.

[11]For instance $D^2\mathcal{O}^i_{RS^2} = 0$ on shell.

divergences are twofold: logarithmic and power law UV divergences. This distinction was shown to be associated to tidal vs self-induced effects. Renormalization through the insertion of nonminimal terms in the effective action was implemented and the RG flow obtained. A finite size cutoff was invoked in the case of power law divergences and its respective Wilson coefficients set by naturalness. In the EFT spirit it is likely that all terms which are consistent with the symmetries will contribute to the effective action. In fact, self-induced spin effects are naturally expected and the lack of scale dependence just responds to the fact that it is a 1-body effect on itself due to its proper rotation which does not get renormalized.[12] A partial matching into the full theory was accomplished by comparison with known results [21,22]. Self-induced effects could in principle appear at leading orders in the PN expansion in the case of maximally rotating compact bodies, for which tidal deformations were shown to first appear at 5PN, although formally at 3PN for general objects. Within the power of the EFT, most of the conclusions are based on dimensional grounds without detailed calculations.

Several aspects remain still to be worked out. In addition to the matching calculation and the issue of finite size effects, higher-order corrections are yet to be obtained even though the formalism is already set and just computational work is needed. Including dynamical properties for multipole moments as well as backreaction effects is also to be worked out. Moreover, new kinematical scenarios, like a 3-body system and the large small mass ratio case, are currently under study. All these issues, including the radiative energy loss due to spin, will be covered in forthcoming publications.

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APPENDIX A: ANGULAR-VELOCITY/SPIN RELATIONSHIP

By RPI we know the theory has vanishing Hamiltonian and dynamics is generated by the constraints C_1, C_2. It can be also shown that the Lagrange multiplier associated to C_1 has been set to zero by the condition $\psi \approx 0$ [20]. Using the Hamiltonian equations for the tetrad and position we have in the realm of special relativity,

$$\begin{aligned} \frac{dx^\mu}{d\lambda} &= [x^\mu, \xi C_2]_{pb} = 2\xi p^\mu \to \xi = \frac{(u^2)^{1/2}}{2m} \\ \frac{de^I_\mu}{d\lambda} &= [e^I_\mu, \xi C_2]_{pb} = 2\xi f'(S^2) S^{\nu\mu} e^I_\nu \to \Omega^{\mu\nu} \\ &= \frac{(u^2)^{1/2}}{m} f'(S^2) S^{\mu\nu}, \end{aligned} \tag{A1}$$

where ξ is a Lagrange multiplier, $[,]_{pb}$ stands for the Poisson bracket and $f(S^2) \equiv m^2(S^2)$. By comparison with (29) we conclude, by matching to the zero curvature case,

$$\Omega_{\mu\nu} = (u^2)^{1/2} \frac{f'(S^2)}{m} \left(S_{\mu\nu} - \frac{1}{2m} R_{\mu\nu\alpha\beta} S^{\alpha\beta} + \cdots \right). \tag{A2}$$

The Lagrangian dependence of this expression is encoded in the function $f(S^2)$ defined by (21) and (22). It is possible now to construct a Lagrangian ($\bar{L}$) using all the freedom we showed previously, that will ensure (27) kinematically [20]. Such a procedure is therefore preferred given that the unphysical degrees of freedom are cut off kinematically rather than cut by hand. As it has been shown in [20] $\bar{L}$ is however not unique. There is still a remnant freedom of the form $f'(S^2) \sim A$, with A a constant.[13] We can henceforth set A in order to recover the well-known relationship between angular velocity and spin in flat space, namely $S \sim I\Omega$, with I the moment of inertia. One then solves for $S^{\mu\nu}$ in (A2) order by order to get (30).

One could still argue that the angular-velocity/spin relation should be obtained directly from $S_M = (\frac{\partial \bar{L}}{\partial \Omega})_M$ rather than using the EOM as we did. One should however bear in mind that dynamics naturally help us to power count within an EFT approach. Higher-order corrections are taken into account by insertions in the world line action [1].

APPENDIX B: FORMAL POWER COUNTING

Here we will comment on the power counting from a formal point of view without assuming any specific properties of the objects. This will introduce new parameters which should be adjusted depending on the constituents. For an object of characteristic length R, the spin magnitude for a rotating velocity $v_{\rm rot}$ is $S \sim L \frac{R}{r} \frac{v_{\rm rot}}{v}$, formally nonperturbative effects. For maximally rotating, and corotating, bodies one gets $S \sim L \frac{R}{rv}$ and $S \sim L \frac{R^2}{r^2}$ respectively. Nevertheless, it is naturally expected that $\epsilon \equiv R/r \ll 1$ and a new perturbative parameter is introduced ($\epsilon \sim v^2$ for neutron stars or black holes). It is easy to show that the leading spin-orbit effect for maximally rotating objects

[12] Formally speaking, self-induced effects do not get renormalized as a consequence of the fact that they are derived from the coupling to the conserved stress energy and the metric field does not get renormalized classically.

[13] Regge trajectories are of constant slope.

scales as $v\epsilon L$, a subleading contribution formally at 0.5PN. The first long range contribution from C_{D^2} to the effective action will now scale as $Lv^6\epsilon^2$ for $v_{\rm rot} = 1$, a 3PN effect, effectively at 4PN for $\epsilon \sim v$. This brings hope to potentially observe these effects in the future.

APPENDIX C: GOING TO ALL ORDERS

As we pointed out the *unphysical* states can be washed away kinematically by (27) if a suitable Lagrangian ($\bar{L}$) is chosen [20]. Moreover, the leading order spin-graviton vertex was shown to be Lagrangian independent to leading order. This however cannot be translated to higher orders. Here we will show how to proceed to obtain the Feynman rules to all orders.

By local Lorentz invariance and general covariance, we know $\bar{L}$ is a function of the metric and angular velocity. We can rewrite $\bar{L} \equiv \bar{L}(\Omega^{IJ}, \eta^{IJ})$ which shrinks to $\Omega^{IJ} \equiv e^\mu_I e^\nu_J \Omega_{\mu\nu}$ all the metric dependence. Within an EFT framework, the explicit form of the Lagrangian given in [20] is not necessary, since we can always obtain its NRGR counterpart by expanding $\bar{L}$ around a Minkowski background,

$$\bar{L} = \bar{L}(\Omega^{IJ}_M) + \left(\frac{\partial \bar{L}}{\partial \Omega^{IJ}}\right)_M \delta\Omega^{IJ} + \cdots \frac{1}{n!}\left(\frac{\partial^n \bar{L}}{\partial \Omega^n}\right)_M \delta^n\Omega \ldots, \tag{C1}$$

where $\delta\Omega^{IJ}(\delta e, h) \equiv \Omega^{IJ} - \Omega^{IJ}_M$, and Ω^{IJ}_M is defined by (37). Using that $S_M = I\Omega_M$ on shell, the NRGR Lagrangian turns out to be (schematically)

$$\bar{L} = \bar{L}(\Omega_M) - \frac{1}{2}S_M\delta\Omega - \frac{I}{2}\delta\Omega\delta\Omega. \tag{C2}$$

By expanding $\delta\Omega(\delta e, h)$ in (C2), we will therefore generate the spin-graviton vertices to all orders in the weak field limit.[14] The next step to construct the EFT is to match into NRGR using the power counting rules thus far developed [1]. That is an straightforward task. The terms in the NRGR Lagrangian for maximally rotating compact objects are shown in (40)–(42) up to 2PN.

Notice also that the second piece in (C2) generates contributions which are not explicitly spin dependent, although the coupling is proportional to the moment of inertia. For spherically symmetric objects, we will have for instance a term of the form

$$\frac{I}{2}\Gamma^\mu_{\alpha\nu}\Gamma_{\mu\beta}{}^\nu u^\alpha u^\beta, \tag{C3}$$

which can be easily shown to be proportional to $R_{\mu\nu}u^\mu u^\nu$ and henceforth f.r. removable. For nonspherical bodies we will get

$$R_{\mu\nu\alpha\beta}e^\beta_K e^\nu_J I^{KJ} u^\mu u^\alpha, \tag{C4}$$

with I^{KJ} the inertia tensor defined by $I^{KJ} = \sum_p m_p(\vec{x}^2_p\delta^{KJ} - x^J_p x^K_p)$ with p labeling the internal structure of the body. Given that the symmetric piece leads to a Ricci tensor, the only physically observable contribution will come from the quadrupole piece $Q^{KJ} = \sum_p m_p x^J_p x^K_p$ as expected. Therefore, by adding the nonspherical structure into its rotational part we will automatically account for its internal quadrupole moment structure. Higher-order multipoles do not follow this procedure and they should be added depending on the physical situation.

[14]We will show in Appendix F that the spin part of the action can indeed be rewritten in terms of the Ricci rotation coefficients in a more compelling fashion.

APPENDIX D: STRESS ENERGY TENSOR FOR SPINNING OBJECTS

As it was shown by Dixon, a spinning particle can be described by the following stress energy tensor [38,42]:

$$T^{\alpha\beta}_D = \sum_A \int d\tau p^\alpha u^\beta \frac{\delta^4(x^\mu - x^\mu_A(\tau))}{\sqrt{-g}} - \frac{1}{2}\nabla_\mu\left[(S^{\mu\alpha}u^\beta + S^{\mu\beta}u^\alpha)\frac{\delta^4(x^\mu - x^\mu_A(\tau))}{\sqrt{-g}}\right]. \tag{D1}$$

The Papapetrou equations can be recovered as a consequence of Einstein equations, namely $T_{\alpha\beta;\beta} = 0$ [42].[15]

It can be shown also that (D1) is obtained from our formalism. By definition the stress energy tensor is defined such that

$$\delta S = -\frac{1}{2}\int d^4x\sqrt{-g}T^{\alpha\beta}\delta g_{\alpha\beta}. \tag{D2}$$

The variation of the action is in principle tricky due to the presence of the constraint $e^I_\mu e_{I\nu} = g_{\mu\nu}$. Using

$$\delta e^I_\mu = \tfrac{1}{2}e^I_\nu g^{\nu\beta}\delta g_{\beta\mu} \tag{D3}$$

$$\delta g^{\mu\nu} = -g^{\alpha\nu}g^{\beta\mu}\delta g_{\alpha\beta}, \tag{D4}$$

we will therefore get

$$\frac{-1}{2}\int d\tau S^{IJ}\delta\Omega_{IJ} = \frac{-1}{2}\int d^4x\sqrt{-g}T^{\alpha\beta}_{D({\rm spin})}\delta g_{\alpha\beta} \tag{D5}$$

as expected. It is important to remark that, even though at first sight (D1) looks action independent, it utterly depends on the relationship between spin and angular velocity which by itself depends on the particular Lagrangian.

APPENDIX E: THE EOM IN THE COVARIANT GAUGE: NONCOMMUTATIVE ALGEBRA

As it is known relativistic N-body EOM cannot be derived from an ordinary Lagrangian beyond the 1PN level, provided Lorentz invariance is preserved [43].

[15]However, they do not decouple using (D1). They can be separately recovered by using the stress energy tensor proposed by Bailey and Israel [39], plus imposing the symmetry condition $T^{\mu\nu} = T^{\nu\mu}$ by hand [16].

However, the latter theorem does not follow if the position coordinates are not canonical variables, and this is exactly what happens once the second class constraints in (27), and the gauge fixing condition $\psi^\mu = 0$, are imposed strongly in the phase space [35]. As shown in [20], the Poisson brackets are modified by the Dirac algebra [35,36]. The interesting commutation relations are those of x_a^i and spin, to leading order we have [20]

$$[x_a^i, x_a^j]_{db} = \frac{S_a^{ji}}{m_a^2} \qquad [x_a^k, S_a^{ij}]_{db} = \frac{1}{m_a}(S_a^{ki} v_a^j - S_a^{kj} v_a^i), \tag{E1}$$

with $a = 1, 2$, and db stands for Dirac brackets.

It can be easily noticed that our expressions in (48) and (50) for the potentials within the covariant condition coincides with that of [9,12,13] up to the acceleration piece. Therefore, all we have to do in order to prove the equivalence is to explicitly show that the new term derived from the acceleration part agrees with the extra factor generated by the noncanonical brackets. Let us start with the position dynamics. It is easy to show from (E1) that the following new term,

$$\frac{d}{dt}\left(\left[\vec{x}_1, -\frac{G_N m_2}{r}\right]_{db}\right) = G_N \frac{m_2}{m_1}\frac{d}{dt}\left(\frac{\vec{n}\times\vec{S}_1}{r^2}\right), \tag{E2}$$

will appear into the acceleration of body 1. Adding the piece coming from the second body, it is straightforward to show that the extra factor is equivalently obtained by adding a term (in relative coordinates),

$$\frac{\mu}{2M}\vec{v}\cdot\left(\vec{a}\times\left(\frac{m_1}{m_2}\vec{S}_2 + \frac{m_2}{m_1}\vec{S}_1\right)\right) \tag{E3}$$

into the Lagrangian, where it is understood that wherever the acceleration appears in higher-order terms one substitutes the leading order EOM. This agrees with [12,13] and the equivalence is thus proven. In addition, it is easy to show there is a coordinate transformation that leads to a canonical algebra, to leading order [20],

$$\vec{x}_a \to \vec{x}_a - \frac{1}{2m_a}\vec{S}_a \times \vec{v}_a. \tag{E4}$$

Still missing is the precession of spin. Using (E1) one can show that the EOM for spin ends up being

$$\frac{d\vec{S}_1}{dt} = 2\left(1+\frac{m_2}{m_1}\right)\frac{\mu G_N}{r^2}(\vec{n}\times\vec{v})\times\vec{S}_1 + \frac{m_2 G_N}{r^2}(\vec{n}\cdot\vec{v}_1)\vec{S}_1. \tag{E5}$$

It is easy to show now that the following PN shift,

$$\vec{S}_a \to (1 - \tfrac{1}{2}\vec{v}_a^2)\vec{S}_a + \tfrac{1}{2}\vec{v}_a(\vec{v}_a\cdot\vec{S}_a), \tag{E6}$$

which jointly with (E4) leads to a canonical algebra, reproduces the well-known spin precession [see (52)]. The map in (E4) and (E6) connects the covariant choice with the baryonic one. The baryonic condition does not preserve Lorentz invariance, and an acceleration independent Lagrangian exists.[16] As a consequence, (51) will not be invariant under the usual linear realization of the Poincaré group [12,43]. Given that (E4) and (E6) are PN shifts it is also immediate to conclude that the power counting rules developed in this paper do not get affected by the new choice.

APPENDIX F: SPIN-GRAVITON COUPLING REVISED

By RPI we know that the Lagrangian must be of the form

$$L = -p^\mu u_\mu - \tfrac{1}{2}S^{\mu\nu}\Omega_{\mu\nu}. \tag{F1}$$

Here we shall show that the spin part of the action can be rewritten as

$$S_{\rm spin} \sim \frac{1}{2}\int d\tau\, \omega_\mu^{IJ} S_{IJ} u^\mu, \tag{F2}$$

with ω_μ^{IJ} the Ricci rotation coefficients. Notice this coupling is generally covariant and RPI by construction. We could have chosen to add spin this way into the NRGR Lagrangian given that S^{IJ} can be treated as a constant external source [see Eq. (25)]. In fact, the momentum dynamics [Eq. (26)] is also recovered following similar steps as we did before. It is therefore natural to expect that both formalisms agree which indeed follows almost straightforwardly by definition. In terms of a tetrad and the Levi-Cività connection, the Ricci rotation coefficients can be written as [37]

$$\omega_\mu^{IJ} = \Gamma^J{}_\mu{}^I + e_\alpha^J \partial_\mu e^{\alpha I}. \tag{F3}$$

Given that (F2) is defined on the world line, we can use the tetrad field transported by the particle. We will thus get for the spin part of the action [using (15)]

$$u^\mu \omega_{\mu IJ} S^{IJ} = S^{IJ}\left(-\Gamma_{JI\mu}u^\mu + \frac{de_{\alpha I}}{d\tau}e_J^\alpha\right) = S^{IJ}\Omega_{IJ}, \tag{F4}$$

as advertised. Written this way it is clear how spin and gravity couple to each other, with spin playing the role of a ‘‘gravitational charge‘‘ coupled to a connection of a spin 2 field.

APPENDIX G: NAIVE POWER COUNTING

Here we will schematically show the type of new insertions which are generated by divergences in the one point function. Let us start by calculating the effective action with n_s spin and n_m mass leading order insertions as shown in Fig. 9. This diagram will scale as (after matching into NRGR for potential gravitons)

[16] As shown by Schafer [44], substituting the leading order EOM in the acceleration dependent Lagrangian of [12,13] is also equivalent to the map to baryonic coordinates.

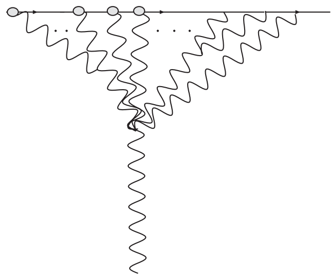

FIG. 9. A typical contribution to the one point function in the effective action coming from leading order mass and spin insertions.

$$\frac{1}{m_p^{n_s+n_m-1}}\left(\frac{m}{m_p}\right)^{n_m} L^{n_s} \frac{v^{sn_s}}{m_p^{n_s}} \frac{v^2 m_p}{\sqrt{L}} \frac{r}{r^d v}, \qquad (G1)$$

where r^d is introduced for dimensional reasons. By using NRGR power counting [1], each diagram should scale as $\sqrt{L}$ and therefore,

$$d = 2n_s + n_m - 1 \rightarrow \text{Fig. } 9 \sim \sqrt{L} v^{2d}. \qquad (G2)$$

To consider the type of terms that can be generated by renormalization in the one point function, and will contribute to physical observables, we need as a necessary condition $\tilde{d} \equiv d - 2 \geq 0$. This however is not sufficient given that using Bianchi identities it can be shown that contraction of covariant derivatives with the full Riemann tensor are equivalent to derivatives of the Ricci tensor and henceforth f.r. removable terms. We can nonetheless enumerate some cases. Let us concentrate in logarithmic divergences first. For $\tilde{d} = 0$ we have $n_s = n_m = 1$, and it is easy to see there are not any new terms generated by Fig. 7. The case $\tilde{d} = 1$ has either $n_s = 2$, $n_m = 0$ or $n_s = 1$, $n_m = 2$. None of these diagrams have logarithmic divergences (for potential gravitons) which could generate an observable term. In fact, the only observable term which can be written down with $\tilde{d} = 1$, and either $n_s = 1, 2$ is the first term in (68), which can be shown to be a subleading self-induced effect. For $\tilde{d} = 2$ we have either $n_s = 2$, $n_m = 1$ and Fig. 8 which leads to the finite size effects we discussed in the paper, or $n_s = 1$, $n_m = 3$ which can be shown does not generate observable terms. For $\tilde{d} = 3$ we can have $n_s = 1, 2, 3$ plus mass insertions. Some of these diagrams will contribute observable terms. After matching into NRGR such terms start out at $\mathcal{O}(v^{10})$ for maximally rotating compact objects. The procedure follows systematically with higher-order terms.

In addition to logarithmic divergences, it is easy to see that diagrams like Fig. 9 will also have power law divergences. For instance, the term $\mathcal{O}^3_{RS^2}$ can be generated by diagram Fig. 8 and its coefficient scale as $(m\Lambda^2)/m_p^4$. Assuming a cutoff of order $\Lambda \sim 1/r_s$, we will have $C^3_{RS^2} \sim m/(r_s^2 m_p^4) \sim 1/m$ as expected.

It is therefore straightforward to conclude from all we have seen thus far that companion induced finite size effects due to spin start out at 5PN (formally at 3PN) for maximally rotating compact objects. For the sake of completeness here are the terms which will contribute to the potential energy,

$$\begin{aligned} &D^2\mathcal{O}^i_{RS^2}, \quad D_\rho D_\sigma R_{\mu\nu\alpha\beta} S^{\sigma\mu} S^{\rho\alpha} u^\nu u^\beta, \\ &D_\sigma D_\epsilon D_\gamma R_{\beta\rho\mu\nu} S^{\beta\sigma} S^{\epsilon\rho} S^{\mu\gamma} u^\nu, \quad D^2 D_\sigma R_{\beta\rho\mu\nu} S^{\beta\sigma} S^{\epsilon\rho} S^{\mu}_{\epsilon} u^\nu, \\ &D^2 D_\sigma R_{\beta\rho\mu\nu} S^{\rho\sigma} S^{\epsilon\mu} S^{\nu}_{\epsilon} u^\beta, \quad D^2 D_\gamma R_{\mu\nu\alpha\beta} S^{\gamma\sigma} S^{\sigma\nu} S^{\mu\beta} u^\alpha, \\ &D^2 D_\gamma R_{\mu\nu\alpha\beta} S^{\gamma\sigma} S^{\sigma\beta} S^{\mu\nu} u^\alpha. \end{aligned} \qquad (G3)$$

Other possible terms, like $D^2 D_\gamma R_{\sigma\mu\nu\alpha} S^{\gamma\sigma} S^{\nu\alpha} u^\mu$, can be shown to be subleading. The reasoning shown here can be easily extended to the case of higher-order Riemann insertions with similar conclusions.

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