Static charged distributions in 2 + 1 gravity

Mauricio Cataldo*

Departamento de Física, Facultad de Ciencias, Universidad del Bío-Bío, Avenida Collao 1202, Casilla 5-C, Concepción, Chile

Norman Cruz†

Departamento de Física, Facultad de Ciencia, Universidad de Santiago, Casilla 307, Santiago, Chile

(Received 26 January 2006; published 22 May 2006)

Circularly symmetric charged perfect-fluid distributions are studied in three-dimensional gravity with a cosmological constant. We derive the Tolman-Oppenheimer-Volkoff equation of hydroelectrostatic equilibrium, and we discuss its applicability. A class of charged fluid distributions for $p = -\rho$ is considered. In this case, a particular model is obtained which represents a charged distribution whose mass is entirely of electromagnetic origin.

DOI: 10.1103/PhysRevD.73.104026 PACS numbers: 04.20.Jb, 04.40.Nr

I. INTRODUCTION

Properties of gravitation in (2 + 1) dimensions have received considerable attention in recent years. This three-dimensional Einstein gravity provides a model for a better understanding of the (3 + 1)-dimensional classical and quantum gravity. For many years, the study of classical solutions to (2 + 1)-dimensional gravity has received special attention. Of particular interest are static gravitational fields. A great number of works have been dedicated to the study of three-dimensional Bañados-Teitelboim-Zanelli (BTZ) black hole solutions [1,2]. The three-dimensional black hole is quite similar to its (3 + 1) counterpart—the Kerr solution. It has an ergosphere and an upper bound in angular momentum for any given mass [3]. Its geodesic structure was studied in detail in [4]. It was shown that a slight modification of this three-dimensional black hole solution yields an exact solution to string theory [5]. We can also list point mass solutions [6–8], static electrovacuum solutions [9,10], and static Einstein-Maxwell solutions with a cosmological constant (including charged black holes) [1,11–15]. Black hole solutions were also obtained in the framework of the nonlinear electrodynamics [16,17].

It is worthwhile to point out that the literature on interior solutions of (2 + 1) gravity is rather scarce. For example, perfect-fluid sources have been considered in Ref. [18], where the authors have shown that, in (2 + 1) gravity, one is able to determine all static circularly symmetric perfect-fluid solutions with the cosmological constant. On the contrary, there are no general static (3 + 1)-dimensional perfect-fluid solutions with a linear or a polytropic barotropic equation of state. In many cases, numerical calculations are required. Stationary circularly symmetric space-times have also been considered. In Ref. [19] the author derived interior solutions for a rigidly rotating perfect fluid in the presence of a cosmological constant, while all stationary space-times containing perfect-fluid sources with a constant energy density and a linear or a polytropic barotropic equation of state are found in Ref. [20].

Three-dimensional charged interior solutions have not been studied before in the literature as far as we know. In (3 + 1)-dimensional gravity a great number of works have been dedicated to this topic. The case of charged dust has received considerable attention. For example, Ref. [21] considered the general solution in which the fluid density equals the norm of the invariant charged density. Recently, new static charged dust solutions were found for the case of constant energy density [22]. The Schwarzschild interior solution of constant density has also been generalized to the case of a charged perfect fluid [23]. An interesting and new simple classification scheme of charged static spherically symmetric perfect-fluid solutions is given in Ref. [24].

For charged fluid spheres the gravitational field in the exterior region is described by Reissner-Nordström space-time, the unique asymptotically flat and spherically symmetric solution of the Einstein-Maxwell equations. Certain interior charged space-times which have the same form of the exterior Reissner-Nordström have been considered in Ref. [25]. This implies that the interior charged fluid distribution will have a natural matching on the boundary with the exterior Reissner-Nordström space-time.

In this paper we shall study circularly symmetric distributions of charged matter within the context of three-dimensional general relativity with a cosmological constant. For distributions with a negative cosmological constant, the exterior region is described by the electrovacuum BTZ solution.

We discuss the general conditions to obtain fluid charged distributions under hydrostatic equilibrium. In order to find exact solutions, we restrict ourselves to the simplified model where the fluid obeys the equation of state $p = -\rho$, which represents in some special cases electromagnetic mass models, previously discussed in four-dimensional gravity [25].

*Email address: mcataldo@ubiobio.cl
†Email address: ncruz@lauca.usach.cl

 © 2006 The American Physical Society

Specifically, we shall consider (2 + 1) space-times with metrics having the same form of the exterior three-dimensional electrovacuum solutions in order to have a natural matching on the boundary.

The organization of the paper is as follows: In Sec. II we present the field equations for circularly symmetric distributions of charged matter. The corresponding Tolman-Oppenheimer-Volkoff (TOV) equation is also derived. In Sec. III the hydroelectrostatic equilibrium and boundary conditions are discussed. In Sec. IV general results for three-dimensional space-time are discussed which have the same form of the circularly symmetric electrovacuum exterior solution, and a particular solution corresponding to an electromagnetic mass model is presented.

II. 2 + 1 FIELD EQUATIONS

We shall consider circularly symmetric static space-times only. This means that the metric can be written in the form

$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 d\phi^2, \tag{1}$$

where ν and λ are functions of the radial coordinate r only, $0 \le r \le \infty$, and $0 \le \phi \le 2\pi$.

We choose the (2 + 1)-dimensional comoving frame

$$\theta^{(0)} = e^{\nu/2}, \qquad \theta^{(1)} = e^{\lambda/2}, \qquad \theta^{(2)} = r. \tag{2}$$

The self-consistent Einstein-Maxwell equations with the cosmological constant Λ for a charged perfect-fluid distribution are given by

$$R_{(\alpha)(\beta)} - \tfrac{1}{2} R g_{(\alpha)(\beta)} + \Lambda g_{(\alpha)(\beta)} = -\kappa (T^{\mathrm{PF}}_{(\alpha)(\beta)} + T^{\mathrm{EM}}_{(\alpha)(\beta)}), \tag{3}$$

where

$$T^{\mathrm{PF}}_{(\alpha)(\beta)} = (p + \rho) u_{(\alpha)} u_{(\beta)} + p g_{(\alpha)(\beta)} \tag{4}$$

and

$$T^{\mathrm{EM}}_{(\mu)(\nu)} = -\frac{1}{4\pi}\left(F_{(\mu)}{}^{(\alpha)} F_{(\nu)(\alpha)} - \frac{1}{4} g_{(\mu)(\nu)} F_{(\alpha)(\beta)} F^{(\alpha)(\beta)} \right) \tag{5}$$

are the energy-momentum tensors for a perfect fluid and the electromagnetic field, respectively, and

$$F^{\mu\nu}{}_{;\nu} = -4\pi J^{\mu}, \tag{6}$$

where J^{μ} is the current three-vector defined by

$$J^{\mu} = \sigma(r) u^{\mu}, \tag{7}$$

$\sigma(r)$ being the proper charge density of the distribution and $u = u_{\alpha} dx^{\alpha} = u_{(\alpha)} \theta^{(\alpha)}$ the three-velocity of the fluid. Here ρ, p, and $F^{\mu\nu}$ are the energy density, the isotropic pressure, and the Maxwell tensor, respectively.

In our case the three-velocity is given by $u_{(\alpha)} = \delta^0_{\alpha}$ and the Maxwell tensor by

$$F = \tfrac{1}{2} F_{\mu\nu} dx^{\mu} \wedge dx^{\nu} = E_r dr \wedge dt = E_{(r)} \theta^{(1)} \wedge \theta^{(0)}, \tag{8}$$

where E_r and $E_{(r)}$ are the coordinate and the triad electric field, respectively, and they are functions of the radial coordinate. From (8) it is clear that the triadic component of the electric field $E_{(r)} = E_r e^{(\nu+\lambda)/2}$.

This implies that the Einstein-Maxwell equations take the form

$$-G_{(0)(0)} = -\frac{1}{2r}(e^{-\lambda})' = 8\pi\rho + E^2 + \Lambda, \tag{9}$$

$$-G_{(1)(1)} = \frac{1}{2r} \nu' e^{-\lambda} = 8\pi p - E^2 - \Lambda, \tag{10}$$

$$-G_{(2)(2)} = \frac{e^{-\lambda}}{2}\left(\nu'' + \frac{1}{2}[\nu'^2 - \nu'\lambda'] \right) = 8\pi p + E^2 - \Lambda, \tag{11}$$

and

$$\sigma(r) = \frac{e^{-\lambda/2}}{4\pi r}(rE)', \tag{12}$$

where $E \equiv E_{(r)}$ and the prime denotes d/dr.

From Eq. (12), for a given charge density $\sigma(r)$ one obtains that the electric field is given by

$$E(r) = \frac{4\pi}{r} \int_0^r x\sigma(x) e^{\lambda(x)/2} dx. \tag{13}$$

A self-consistent solution is found by solving Eqs. (9)–(12), but we shall solve the considered field equations by replacing Eq. (11) by the TOV equation. The TOV equation for a charged perfect fluid may be derived from the conservation equation $T^{\mu\nu}{}_{;\mu} = 0$, where $T_{\mu\nu} = T^{\mathrm{PF}}_{\mu\nu} + T^{\mathrm{EM}}_{\mu\nu}$. Then we have

$$\frac{dp}{dr} = -\frac{1}{2}(\rho + p)\nu' + \frac{1}{8\pi r^2}(r^2 E^2)'. \tag{14}$$

Since the exterior electric field is given by $E = q_e/r$, we shall write the interior electric field in the form

$$E(r) = \frac{q(r)}{r}, \tag{15}$$

where

$$q(r) = 4\pi \int_0^r x\sigma(x) e^{\lambda(x)/2} dx. \tag{16}$$

By using (15) and (16) the TOV equation can be written as

$$\frac{dp}{dr} = -\frac{1}{2}(\rho + p)\nu' + \frac{1}{8\pi r^2}(q^2(r))'. \tag{17}$$

From Eq. (9) we have

$$e^{\nu(r)} = e^{-\lambda(r)} = M(r) - \Lambda r^2, \tag{18}$$

where

$$M(r) = C - 16\pi \int_0^r x\rho(x)dx - 2\int_0^r xE^2(x)dx. \quad (19)$$

By demanding regularity of the metric coefficient $e^{\nu(r)}$ at $r = 0$, in order to include the origin to be part of the spacetime, we obtain that $C > 0$, since $M(r) - \Lambda r^2 > 0$ for $0 \leq r \leq a$, where a is the radius of the charged sphere.

Now, from Eq. (10) we have

$$\nu' = \frac{2(8\pi p - E^2 - \Lambda)r}{M(r) - \Lambda r^2}, \quad (20)$$

and, introducing it into Eq. (17), we finally obtain for the TOV equation

$$\frac{dp}{dr} = -\frac{(\rho + p)(8\pi p - E^2 - \Lambda)r}{M(r) - \Lambda r^2} + \frac{1}{8\pi r^2}(q^2(r))'. \quad (21)$$

Before studying particular interior solutions we analyze the behavior of charged perfect fluids in hydrostatic equilibrium using some general results derived from the TOV equation. Note that, in order to have a finite pressure gradient, the second term on the right-hand side of Eq. (21) must be finite in the range $0 \leq r \leq a$. In particular, if the charge within the fluid distribution varies with r obeying the constraint $q(r) = r^n$, then $n \geq 3/2$.

III. HYDROELECTROSTATIC EQUILIBRIUM AND BOUNDARY CONDITIONS

A. The general case for hydroelectrostatic equilibrium

In the following we find general conditions to have charged fluid distributions in hydrostatic equilibrium in 2 + 1 dimensions, which are valid irrespective of the equation of state $p = p(\rho)$ considered for the perfect fluid and of the form of the charge density $\sigma(r)$.

In the standard case, i.e. for a positive fluid pressure, the following physically reasonable conditions are generally assumed:

(1) The pressure p must vanish at the boundary of the fluid distribution $r = a$.

(2) The pressure p and the density ρ must be monotonically decreasing functions of r.

(3) The energy condition $p \leq \rho$ must be verified.

Evaluating Eq. (21) at the boundary of the fluid distribution, $r = a$, where the pressure p is zero, and considering that $M(r) - \Lambda r^2 > 0$, we obtain

$$\left.\frac{dp}{dr}\right|_{r=a} = \frac{\rho(Q^2/a^2 + \Lambda)a}{M(a) - \Lambda a^2} + \frac{1}{4\pi a^2} Q \left.\frac{dq(r)}{dr}\right|_{r=a}, \quad (22)$$

where Q is the total proper charge inside the fluid distribution, given by

$$Q = \pm 4\pi \int_0^a x^2 \sigma(x) e^{\lambda(x)/2} dx. \quad (23)$$

In order to have hydrostatic equilibrium, the pressure gradient must be negative throughout the distribution. Since the sign of the charge is the same throughout the fluid distribution, then $q^2(r)$ always increases as r increases, and the second term on the right-hand side of Eq. (22) is positive. Thus we have that $(Q^2/a^2 + \Lambda) < 0$ which implies the constraint

$$\Lambda < -Q^2/a^2. \quad (24)$$

So we conclude that Λ must be negative. In other words, hydrostatic equilibrium of charged perfect-fluid distributions with positive pressure exists only in anti–de-Sitter spaces.

This result is more restrictive than the noncharged case. As was demonstrated in [26], noncharged perfect fluids are in hydrostatic equilibrium even when $\Lambda = 0$ ($dp/dr|_{r=a} = 0$). Nevertheless, in that case, the central pressure is not related to the mass of the fluid distribution, which means that there is no possibility of gravitational collapse for any finite values of the mass and the radius [26,27]. This follows from the fact that gravity does not propagate in 2 + 1 dimensions, since the Weyl tensor is zero.

The presence of the electric charges in the fluid produces repulsive effects, which means that the hydrostatic equilibrium is possible only if an attractive force, generated by a negative cosmological constant, is included in order to balance the repulsive effects.

Now we shall briefly discuss some physical consequences of Eq. (21). The second term on the right-hand side represents the Coulomb repulsion of the charged distribution [28]. This term helps to stabilize the charged fluid sphere against collapse. In the Newtonian scheme, the Coulomb repulsion, due to the presence of electric fields, is always opposed to the gravitational force. Nevertheless, in general relativity all forms of energy are sources of gravitation. A consequence of this fact is visualized when the first term on the right-hand side of Eq. (21) is analyzed. The electric field tends to decrease the numerator but, on the other hand, also tends to decrease the denominator [see Eq. (19)], helping the gravitational attraction [29].

B. Linear barotropic equation of state

A widely used equation of state for fluid distribution is that corresponding to a linear barotropic fluid

$$p = \gamma\rho, \quad (25)$$

where the constant state parameter γ ranges over $-1 \leq \gamma \leq 1$ in order to hold the dominant energy conditions. Introducing Eq. (25) into Eq. (21) we obtain

$$\frac{dp}{dr} = -\frac{p(1/\gamma + 1)(8\pi p - E^2 - \Lambda)r}{M(r) - \Lambda r^2} + \frac{1}{8\pi r^2}(q^2(r))'. \quad (26)$$

At the boundary $r = a$, Eq. (26) becomes

$$\frac{dp}{dr}\bigg|_{r=a} = \frac{1}{4\pi a^2} Q \frac{dq(r)}{dr}\bigg|_{r=a}, \tag{27}$$

which means that the pressure gradient is always positive at the boundary. So, in order to have a fluid distribution in hydrostatic equilibrium, the pressure must be negative and the pressure gradient must be positive within the entire charged distribution. This implies that the pressure is an increasing function of r. The expression $p(1/\gamma + 1)$ is always positive for $-1 \leq \gamma \leq 1$, so the sign of the first term of Eq. (26) depends on the sign of the expression $-8\pi p + E^2 + \Lambda$. By demanding that the pressure be negative, i.e. $p = -\mid p \mid \leq 0$, we obtain that $-1 \leq \gamma < 0$, and the first term of Eq. (26) is positive if $8\pi \mid p \mid + E^2 + \Lambda \geq 0$. In this case we can have a positive or a negative cosmological constant.

The dust case, i.e. $p = 0$ ($\gamma = 0$), cannot be studied from Eq. (26), but this case is included in the considerations of the above subsection.

In the considered situation, the hydroelectrostatic equilibrium exists only for negative pressures since the equation of state (25) is a very strong condition on the energy density. Effectively, at the boundary $r = a$ the energy density vanishes as well as the pressure.

The fact that a simple equation of state, such as Eq. (25), does not allow us to have a charged fluid distribution in hydroelectrostatic equilibrium with physically reasonable conditions (such as monotonically decreasing pressure and energy density as functions of r) indicates that the interior solutions for $(2 + 1)$-charged stars must be described by nontrivial equations of state. As an example, let us consider a slight modification of the above studied constraint on the energy density and pressure. If we consider, for example, the equation of state of the form [24,30] $\rho = \rho_0 + p/\gamma$, with $\rho_0 \geq 0$ and $\gamma \neq 0$, then at the boundary $r = a$ the energy density does not vanish, and from Eq. (21) we obtain

$$\frac{dp}{dr}\bigg|_{r=a} = \frac{\rho_0(Q^2/a^2 + \Lambda)a}{M(a) - \Lambda a^2} + \frac{1}{4\pi a^2} Q \frac{dq(r)}{dr}\bigg|_{r=a}. \tag{28}$$

It is clear that in this case we can have charged fluid distributions in hydroelectrostatic equilibrium with either positive or negative pressures. Within the entire charged distribution the pressure gradient must be negative or positive, respectively.

The case $\gamma = -1$ will be studied in more detail.

C. A particular case: $p = -\rho$

We shall now consider the equation of state

$$p(r) = -\rho(r) \tag{29}$$

for the charged fluid distribution. In this case we have a negative pressure and the condition (2) is no longer valid. If $p = -\rho < 0$, Eq. (21) becomes

$$\frac{dp}{dr} = \frac{1}{8\pi r^2} \frac{dq^2(r)}{dr}, \tag{30}$$

for $0 \leq r \leq a$. Notice also that in this case our previous conclusion, that hydroelectrostatic equilibrium is only possible if a negative cosmological constant is included, is no longer valid. In this case the first term on the right-hand side of Eq. (21) becomes zero, which implies that the effects of the cosmological constant disappear.

On the other hand, since $p(r)$ is negative and it must vanish at $r = a$, the pressure must be an increasing function of r. This means that its derivative dp/dr is positive in the range $0 < r < a$ and, since $dq^2(r)/dr$ is also positive, hydroelectrostatic equilibrium is guaranteed in this case. Clearly in this case $d\rho/dr < 0$ and then the energy density decreases as r increases.

Notice that for $C = 1$ the total gravitational mass has exclusively an electromagnetic origin. The mass is derived completely from the charge of the electromagnetic fields. Effectively, from the TOV Eq. (30) we see that $p(r)$ is defined from the charge $q(r)$, and then the energy density ρ also. Such space-times are called relativistic electromagnetic mass models.

D. The matching conditions

As was proven above, charged spheres with positive pressure may exist in hydrostatic equilibrium only if a negative cosmological constant is included. This implies that, for anti–de-Sitter distributions of charged matter, the electrovacuum exterior solution corresponds to a charged BTZ black hole field which has the metric given by

$$ds^2 = (-m - \Lambda r^2 - q_e^2 \ln r)dt^2 - \frac{dr^2}{-m - \Lambda r^2 - q_e^2 \ln r} - r^2 d\phi^2, \tag{31}$$

where m and q_e are constants of integration, and $\Lambda = -1/l^2 < 0$. In this case, for $q_e = 0$, the constant m is the total gravitational mass of the space-time. For $q_e \neq 0$ the quasilocal mass at the spatial infinity is

$$M(\infty) = m + q_e^2 \ln r. \tag{32}$$

If the surface of the charged distribution is located at $r = a$, any interior solution given by the metric (1) and valid for $r \leq a$ must satisfy at the boundary the relations

$$e^{\nu(a)} = e^{-\lambda(a)} = -m - \Lambda a^2 - q_e^2 \ln a \tag{33}$$

for the metric functions

$$p(a) = 0 \tag{34}$$

for the pressure, and

$$q(a) = q_e \tag{35}$$

for the expression (16). Since the radius of the charged fluid distribution extends up to a, for $r > a$ we have the electrovacuum solution (33). At the boundary $r = a$, considering Eq. (18), we have

$$\begin{aligned} e^{-\lambda(a)} &= M(r)|_{r=a} - \Lambda r^2|_{r=a} \\ &= m - q_e^2 \ln r|_{r=a} - \Lambda r^2|_{r=a}. \end{aligned} \tag{36}$$

Then for the mass we get

$$m = M(r)|_{r=a} + q_e^2 \ln r|_{r=a}, \tag{37}$$

where $M(r)$ is given by Eq. (19).

IV. GENERAL RESULTS FOR THE CASE $e^{\nu} = e^{-\lambda}$

From the general form of a circularly symmetric gravitational field (1) we see that the electrovacuum solution (31) satisfies the condition

$$e^{\nu} = e^{-\lambda} \Longleftrightarrow g_{00} g_{11} = -1. \tag{38}$$

Tiwari *et al.* in Ref. [25] have considered spherically symmetric interior solutions which satisfy the condition (38), in order to have a natural matching with the Reissner-Nordström exterior solution.

We shall also assume this condition to be valid inside a charged perfect-fluid distribution of radius a, and then we will have a natural matching on the boundary $r = a$ with the electrovacuum $(2 + 1)$ solution (31).

From Eqs. (9) and (10) we obtain

$$\frac{e^{-\lambda}}{2r}(\nu' + \lambda') = 8\pi(\rho + p), \tag{39}$$

which implies that, for any solution of the form $e^{\nu} = e^{-\lambda}$, the condition (29) must be satisfied. It is interesting to note that Eq. (39) contains no charge term.

Since we are interested in finding solutions which have the form (38), we have that $G_{(0)(0)} = G_{(1)(1)}$ and the constraints (29) and (30) are valid for the pressure and its gradient. Therefore, we have a set of three differential equations (9) or (10), (12), and (30) containing four unknown functions. This means that we can solve the system when one more relation among the functions $\lambda(r)$, $\rho(r)$, $E(r)$, and $\sigma(r)$ is furnished.

In this case, the metric functions satisfy the condition $e^{\nu(r)} = e^{-\lambda(r)}$, where $e^{-\lambda(r)}$ is given by Eqs. (18) and (19).

A particular solution

As an example we introduce the charge density in the form

$$\sigma(r) = \sigma_0 e^{-\lambda/2}. \tag{40}$$

Then we have

$$q(r) = 2\pi\sigma_0 r^2, \tag{41}$$

$$E(r) = 2\pi\sigma_0 r, \tag{42}$$

$$\rho(r) = \pi\sigma_0^2(a^2 - r^2), \tag{43}$$

$$p(r) = \pi\sigma_0^2(r^2 - a^2), \tag{44}$$

$$e^{\nu(r)} = e^{-\lambda(r)} = C - 2\pi^2\sigma_0^2 r^2(4a^2 - r^2) + \frac{r^2}{l^2}, \tag{45}$$

where $\Lambda = -1/l^2$. In general, the total gravitational mass of this model has an electromagnetic origin only if the constant $C = 1$, i.e. for a charged distribution which generalizes the anti–de-Sitter space, for which we have

$$e^{\nu(r)} = e^{-\lambda(r)} = 1 + \frac{r^2}{l^2}.$$

When $C \neq 1$, we have a space-time with an angular lack or an angular excess. This implies that the space-time has a conic nature.

The matching at $r = a$ of the obtained solution implies that the exterior BTZ electrovacuum takes the form

$$\begin{aligned} ds^2 = {} & \left[-(-C + 6\pi^2\sigma_0^2 a^4 - 4\pi^2\sigma_0^2 a^4 \ln a) + \frac{r^2}{l^2} - 4\pi^2\sigma_0^2 a^4 \ln r\right] dt^2 \\ & - \frac{dr^2}{[-(-C + 6\pi^2\sigma_0^2 a^4 - 4\pi^2\sigma_0^2 a^4 \ln a) + \frac{r^2}{l^2} - 4\pi^2\sigma_0^2 a^4 \ln r]} - r^2 d\phi^2, \end{aligned} \tag{46}$$

where

$$m = -C + 6\pi^2\sigma_0^2 a^4 - 4\pi^2\sigma_0^2 a^4 \ln a \tag{47}$$

is the mass of the interior charged fluid distribution.

V. CONCLUDING REMARKS

We have studied circularly symmetric distributions of charged matter in $2 + 1$ dimensions. We have found that hydroelectrostatic equilibrium for a charged fluid with the equation of state $p = p(\rho)$ and $p > 0$ occurs only in anti-–de-Sitter spaces. In that case the exterior region is described by the electrovacuum BTZ solution. We call them "charged stars," since the possibility of collapse for certain finite values of the mass and radius is not ruled out.

Nevertheless, in the particular case $p = -\rho$ we have a distribution in hydroelectrostatic equilibrium, but there is no possibility of collapse. In this case we may have space-times corresponding to electromagnetic mass models. On the other hand, their simplicity allows us to obtain a particular solution when an appropriate form of the charge density is considered.

Lastly, although we have considered here static distributions, we want to note that the main motivation of this study is oriented to use the discovered similarities between $(2 + 1)$ and $(3 + 1)$ gravity in order to provide some insights into the study of $(3 + 1)$-stationary interior solutions. Since the discovery of the Kerr metric, many attempts have been made to find interior solutions for isolated rotating bodies. Since Schwarzschild, Kerr, and Kerr-Newman solutions are Petrov type D, most attempts were accomplished within Petrov type D stationary axisymmetric space-times. However, the search for a physically adequate interior Kerr (Kerr-Newman) solution is still a challenging problem in general relativity. The $(3 + 1)$ counterpart to the solution considered here is also a Petrov type D. So we are looking for the generalization of the $(2 + 1)$-obtained solution to the stationary case. This stationary version will be matched to the exterior rotating BTZ solution. We hope that these considerations in $(2 + 1)$ gravity will provide a procedure for constructing a stationary interior solution, with a Schwarzschild (Reissner-Nordström) static form, which will be adequately matched to the Kerr (and Kerr-Newman) exterior solution. This work is in progress.

ACKNOWLEDGMENTS

The authors acknowledge the hospitality of the Physics Departments of Universidad de Santiago (M. C.) and Universidad de Concepción (N. C.), and the Physics Department of Universidad Autónoma de Barcelona (M. C. and N. C.) where part of this work was done. This work was partially supported by CONICYT through FONDECYT Grant No. 1051086 (M. C.), No. 1040229 (N. C.); and MECESUP Grant No. USA0108 (M. C. and N. C.). It also was supported by the Dirección de Investigación de la Universidad del Bío-Bío (M. C.).

[1] M. Bañados, C. Teitelboim, and J. Zanelli, Phys. Rev. Lett. **69**, 1849 (1992).
[2] S. Carlip, Classical Quantum Gravity **12**, 2853 (1995).
[3] M. Bañados, M. Henneaux, C. Teitelboim, and J. Zanelli, Phys. Rev. D **48**, 1506 (1993).
[4] N. Cruz, C. Martínez, and L. Peña, Classical Quantum Gravity **11**, 2731 (1994).
[5] G. T. Horowitz and D. L. Welch, Phys. Rev. Lett. **71**, 328 (1993).
[6] J. R. Gott III and M. Alpert, Gen. Relativ. Gravit. **16**, 243 (1984).
[7] S. Deser, R. Jackiw, and G. 't Hooft, Ann. Phys. (N.Y.) **152**, 220 (1984).
[8] A. Staruszkiewicz, Acta Phys. Pol. **24**, 734 (1963).
[9] S. Deser and P. O. Mazur, Classical Quantum Gravity **2**, L51 (1985).
[10] J. R. Gott III, J. Z. Simon, and M. Alpert, Gen. Relativ. Gravit. **18**, 1019 (1986).
[11] P. Peldán, Nucl. Phys. **B395**, 239 (1993).
[12] E. Hirschmann and D. Welch, Phys. Rev. D **53**, 5579 (1996).
[13] M. Cataldo and P. Salgado, Phys. Rev. D **54**, 2971 (1996).
[14] M. Kamata and T. Koikawa, Phys. Lett. B **353**, 196 (1995).
[15] M. Cataldo and P. Salgado, Phys. Lett. B **448**, 20 (1999).
[16] M. Cataldo, N. Cruz, S. del Campo, and A. A. Garcia, Phys. Lett. B **484**, 154 (2000); Rev. Mex. Fis. **48**, Suppl. 3, 97 (2002).
[17] M. Cataldo and A. A. Garcia, Phys. Rev. D **61**, 084003 (2000).
[18] A. García and C. Campuzano, Phys. Rev. D **67**, 064014 (2003).
[19] A. García, Phys. Rev. D **69**, 124024 (2004).
[20] M. Cataldo, Phys. Rev. D **69**, 064015 (2004).
[21] W. B. Bonnor, Z. Phys. **160**, 59 (1960).
[22] M. Gürses, Phys. Rev. D **58**, 044001 (1998).
[23] F. de Felice, L. Siming, and Y. Yanqiang, Classical Quantum Gravity **16**, 2669 (1999).
[24] B. V. Ivanov, Phys. Rev. D **65**, 104001 (2002).
[25] R. N. Tiwari, J. R. Rao, and R. R. Kanakamedala, Phys. Rev. D **30**, 489 (1984).
[26] N. Cruz and J. Zanelli, Classical Quantum Gravity **12**, 975 (1995).
[27] S. Giddings, J. Abbott, and J. Santamarina, Gen. Relativ. Gravit. **16**, 751 (1984).
[28] J. D. Bekenstein, Phys. Rev. D **4**, 2185 (1971).
[29] P. Anninos and T. Rothmann, Phys. Rev. D **65**, 024003 (2002).
[30] B. V. Ivanov, gr-qc/0107032.