Baryon inhomogeneity generation in the quark-gluon plasma phase

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We discuss the possibility of generation of baryon inhomogeneities in a quark-gluon plasma phase due to moving $Z(3)$ interfaces. By modeling the dependence of effective mass of the quarks on the Polyakov loop order parameter, we study the reflection of quarks from collapsing $Z(3)$ interfaces and estimate resulting baryon inhomogeneities in the context of the early universe. We argue that in the context of certain low energy scale inflationary models, it is possible that large $Z(3)$ walls arise at the end of the reheating stage. Collapse of such walls could lead to baryon inhomogeneities which may be separated by large distances near the QCD scale. Importantly, the generation of these inhomogeneities is insensitive to the order, or even the existence, of the quark-hadron phase transition. We also briefly discuss the possibility of formation of quark nuggets in this model, as well as baryon inhomogeneity generation in relativistic heavy-ion collisions.

I. INTRODUCTION

Generation of baryon inhomogeneities in the early universe can have important implications for nucleosynthesis, and for the possibility of creating compact baryon rich objects [1]. Though, current observations do not support any strong deviation from the standard big-bang nucleosynthesis calculations. Calculations of inhomogeneous big-bang nucleosynthesis resulting from an inhomogeneous distribution of baryons in the universe, (such as those in Ref. [2,3]), therefore, can be used to constrain the baryon inhomogeneities present in the early universe.

There have been numerous investigations of the nature of baryon inhomogeneities generated during a first order quark-hadron phase transition [1,4]. In these investigations, baryon inhomogeneities arise due to moving bubble walls at the transition, with baryons getting concentrated in the remaining localized quark-gluon plasma (QGP) regions.

Main problems in implementing the scenario of Ref. [1] have been regarding the nature of the quark-hadron phase transition as well as the relevant length scales. Lattice calculations [5] tell us that for realistic values of quark masses, quark-hadron transition is at best a weak first order transition, and most likely it is a crossover. The scenario of Ref. [1] does not work in this case. Even if one allows for a possibility of strong first order transition, relevant length and time scales are such that the resulting baryon inhomogeneities are separated by very small distances. Typical separation between such baryonic lumps is of the order of separation between the nucleation sites of the hadronic

bubbles, which is at most of the order of few cm at the end of the quark-hadron transition for homogeneous nucleation [4,6]. In order that these baryonic lumps survive various dissipative processes, this separation needs to be at least of order of a meter at the transition stage [7]. There have been discussions of larger separations between baryon inhomogeneities invoking impurity induced inhomogeneous bubble nucleation [6], presence of density fluctuations [8,9] etc. However, all these scenarios still depend crucially on the assumption of a first order phase transition, and will not work if the quark-hadron transition was a crossover.

In this paper we propose a different scenario where baryon inhomogeneities are produced not due to moving quark-hadron phase boundaries, but due to moving $Z(N)$ interfaces. $Z(N)$ interfaces arise when one uses the expectation value of the Polyakov loop, $l(x)$, as the order parameter for the confinement-deconfinement phase transition of an $SU(N)$ gauge theory [10]. This order parameter transforms nontrivially under the center $Z(N)$ of the $SU(N)$ group and is nonzero above the critical temperature T_c. This breaks the global $Z(N)$ symmetry spontaneously above T_c, while the symmetry is restored below T_c in the confining phase where this order parameter vanishes. For QCD with $SU(3)$ color group, spontaneous breaking of the discrete $Z(3)$ symmetry in the QGP phase leads to the existence of domain walls (interfaces) across which $l(x)$ interpolates between different $Z(3)$ vacua. The properties and physical consequences of these $Z(3)$ interfaces have been discussed in the literature [11]. Though, we mention that it has also been suggested that these interfaces should not be taken as physical objects in the Minkowski space [12]. Similarly, it has also been subject of discussion whether it makes sense to talk about this $Z(3)$ symmetry in the presence of quarks [13]. The presence of quarks can be interpreted as leading to explicit breaking of

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$Z(3)$ symmetry, lifting the degeneracy of different $Z(3)$ vacua [14–17]. In this approach, with quarks, $Z(3)$ interfaces become unstable and move away from the region with the unique true vacuum. Thus, in the context of cosmology, if these walls were produced at some early stage (say after GUT scale inflation), it is likely that they will quickly disappear due to this pressure difference between different $Z(3)$ vacua. However, we will argue (in Sec. III) that in the context of certain low energy scale inflationary models it is possible that large $Z(3)$ domain walls may arise in the QGP phase near the quark-hadron transition stage and may lead to observational effects.

The basic idea of our model is that as $l(x)$ is the order parameter for the quark-hadron transition, physical properties such as effective mass of the quarks should be determined in terms of $l(x)$. This also looks natural from the expected correlation between the chiral condensate and the Polyakov loop. Thus, if there is spatial variation in the value of $l(x)$ in the QGP phase then effective mass of the quark traversing that region should also vary. For regions where $l(x) = 0$, quarks should acquire constituent mass as appropriate for the confining phase. As we will see below, $l(x)$ varies across a $Z(3)$ interface, acquiring small magnitude in the center of the wall. A quark passing through this interface, therefore, experiences a nonzero potential barrier leading to nonzero reflection coefficient for the quark. Because of this, as a closed domain wall collapses, quarks inside will stream through it. With a nonzero reflection coefficient, net baryon number density inside will grow, somewhat in the manner as in the conventional treatments of collapsing quark-hadron phase boundaries. This will lead to formation of baryonic lumps.

Important thing to realize is that all this happens in the QGP phase itself, with any possible quark-hadron transition being completely irrelevant to this discussion. The only relevance of the quark-hadron transition is that in the hadronic phase $l(x) = 0$ so all $Z(3)$ domain walls disappear. The final structure of the baryon inhomogeneities will therefore be decided by those $Z(3)$ interfaces which are last to collapse. As mentioned above, we will argue in Sec. III that it is possible that the size and separation of different collapsing domain walls may be of the order of a fraction of the horizon size just above the quark-hadron transition stage, i.e. of order of a km. If such large domain walls could form then the number of baryons trapped inside can be very large. Also, due to larger mass of the strange quark, reflection coefficient for them is larger than that for the u and d quarks. This leads naturally to strangeness rich quark nugget formation which, as we will show, can have baryon number as large as about 10^{44} within a size of order 1 m.

In a previous paper we have shown that at the intersection of the three different $Z(3)$ interfaces $l(x)$ vanish due to topological considerations, leading to a topological string whose core is in the confining phase [18]. Structure of this string is similar to the standard axionic string which forms at the junction of axionic domain walls [19]. With quarks contributing to explicit $Z(3)$ symmetry breaking, this will lead to decay of $Z(3)$ interfaces along with decay of the associated strings. As $l(x) = 0$ in the core of these strings, collapsing string loops will have larger reflection coefficients for quarks and will also contribute to formation of baryon inhomogeneities. However, unless this string network is very dense, large scale baryon inhomogeneities will mostly result from collapsing $Z(3)$ interfaces.

The mechanism discussed in this paper will also lead to generation of baryon fluctuations in the QGP formed in relativistic heavy-ion collision experiments, with the walls forming during the initial thermalization stage. The effects of explicit symmetry breaking due to quarks on the evolution of wall etc., as mentioned above, will not be much relevant there because of very short time scale available for the evolution of QGP. We plan to study this using detailed computer simulations in a future work.

The paper is organized in the following manner. In Sec. II we discuss structure of $Z(N)$ walls and give numerical results for the profile of $Z(3)$ walls for the case of QCD. Section III discusses how $Z(3)$ walls can form in the early universe. In Sec. IV baryon inhomogeneity generation due to quark reflection from collapsing $Z(3)$ walls is estimated. Numerical results and discussion are given in Sec. V.

II. STRUCTURE OF $Z(N)$ WALLS

We now start discussing the structure of $Z(N)$ interfaces. We will first focus on pure $SU(N)$ gauge theory and later discuss the case with quarks. In this case, an order parameter for the confinement-deconfinement phase transition is the Polyakov loop $l(x)$ which is defined as,

$$l(x) = \frac{1}{N} \, \text{tr}\left(P \exp\left(ig \int_0^\beta A_0(x, \tau) d\tau \right) \right). \quad (1)$$

Here P denotes path ordering, g is the gauge coupling, $\beta = 1/T$, with T being the temperature, $A_0(x, \tau)$ is the time component of the vector potential at spatial position x and Euclidean time τ. $l(x)$ is thus a complex scalar field. Under a global $Z(N)$ symmetry transformation, $l(x)$ transforms as,

$$l(x) \rightarrow \exp\left(\frac{2\pi i n}{N}\right) l(x), \qquad n = 0, 1, \ldots (N-1). \quad (2)$$

For temperatures above the critical temperature T_c, in the deconfining phase, the expectation value of the Polyakov loop $l_0 = \langle l(x) \rangle$ is nonzero corresponding to the finite free energy of isolated test quarks. This breaks the $Z(N)$ symmetry spontaneously. At temperatures below T_c, in the confining phase, l_0 vanishes, thereby restoring the $Z(N)$ symmetry [10].

For making estimates, we will use the effective potential proposed by Pisarski [15,16] (see, also Ref. [17]) for the

Polyakov loop $l(x)$ for the case of QCD with $N = 3$. The effective Lagrangian density is given by,

$$L = \frac{N}{g^2}|\partial_\mu l|^2 T^2 - V(l). \tag{3}$$

Here, $N = 3$ and $V(l)$ is the effective potential for the Polyakov loop given by,

$$V(l) = \left(-\frac{b_2}{2}|l|^2 - \frac{b_3}{6}(l^3 + (l^*)^3) + \frac{1}{4}(|l|^2)^2\right)b_4 T^4. \tag{4}$$

l_0 is then given by the absolute minimum of $V(l)$. Values of various parameters in Eqs. (3) and (4) are fixed in ref. [16,20] by making correspondence to lattice results [21]. Following [20], for three light quark flavors we take, $b_3 = 2.0$ and $b_4 = 0.6061 \times 47.5/16$, where the factor $47.5/16$ accounts for the extra degrees of freedom relative to the degrees of freedom of pure gauge theory. b_2 is taken as, $b_2(x) = (1 - 1.11/x)(1 + 0.265/x)^2(1 + 0.300/x)^3 - 0.487$, where $x = T/T_c$. With the coefficients chosen as above, l_0 approaches the value $y = b_3/2 + \frac{1}{2} \times \sqrt{b_3^2 + 4b_2(T = \infty)}$ for temperature $T \to \infty$. As in Ref. [16], the fields and the coefficients are rescaled as $l \to l/y$, $b_2(T) \to b_2(T)/y^2$, $b_3 \to b_3/y$ and $b_4 \to b_4 y^4$ to ensure proper normalization such that the expectation value of the order parameter l_0 goes to unity for temperature $T \to \infty$.

By writing $l = |l|e^{i\theta}$ we see that the b_3 term in Eq. (4) gives a $\cos(3\theta)$ term, leading to $Z(3)$ degenerate vacua for nonzero values of l, that is for $T > T_c$. The value of T_c is taken to be ~ 182 MeV [20]. The $Z(3)$ interface solution will correspond to a planar solution (say in the x-y plane) where l starts at one of the minimum of $V(l)$ at $z = -\infty$ and ends up at another minimum of $V(l)$ at $z = +\infty$.

In our earlier work we have given profile of this $Z(3)$ domain wall obtained by numerically minimizing the energy of a suitably chosen initial configuration, see Ref. [18] for details. Figure 1 gives the plot of $|l(z)|$ across the

FIG. 1. Profiles of the domain wall for T = 200 MeV (solid curve) and T = 300 MeV (dashed curve).

domain wall showing the profile for the domain wall solution for T = 200 and 300 MeV. Note that the value of $|l(z)|$ in the middle of the wall is smaller for T = 200 MeV than for T = 300 MeV. We thus expect that the effective quark mass will be larger for T = 200 MeV than for the case with T = 300 MeV inside the wall leading to larger reflection coefficient for T = 200 MeV. The surface tension of the wall for T = 200 and 300 MeV are found to be about 0.34 and 2.61 GeV/fm^2 respectively. In an earlier work [18] the surface tension was found to be about 7 GeV/fm^2 for T = 400 MeV. The values for T = 300 and 400 MeV are in reasonable agreement with the analytical estimates (for large temperatures) [22].

Let us now come back to the issue of quarks and the $Z(3)$ symmetry. The effect of quarks on this $Z(3)$ symmetry and $Z(3)$ interfaces etc. has been discussed in detail in the literature [13,14]. It has been suggested that in the presence of quarks, the $Z(3)$ symmetry becomes meaningless, and there is no sense in talking about $Z(3)$ interfaces etc. [13]. It has also been advocated in many papers, that one can take the effect of quarks in terms of explicit breaking of $Z(3)$ symmetry [14–16]. In such a case, the interfaces will survive, though they do not remain solutions of time independent equations of motion. It has been argued in Ref. [16] that the effects of quarks in terms of explicit symmetry breaking may be small, and the pure glue Polyakov model may be a good approximation. We will follow this interpretation, and assume that the effect of quarks is just to contribute explicit symmetry breaking terms which can make the interface and the string solution time dependent, but not invalid. With the explicit symmetry breaking, the interfaces will start moving away from the direction where true vacuum exists as in the conventional case of quark-hadron transition, and as mentioned above, for $Z(3)$ walls formed at some very early time in the universe (say near GUT scale), presumably all walls will disappear. This brings us to the issue of the formation and evolution of these $Z(3)$ walls and strings which we discuss in the next section.
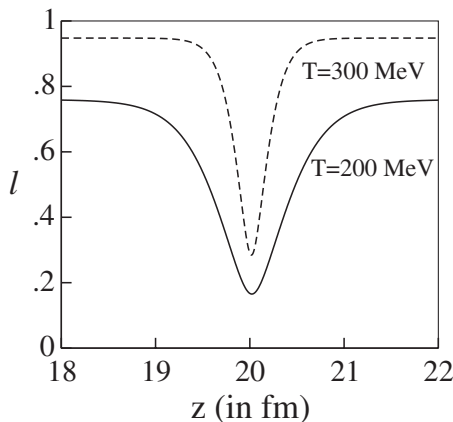
III. FORMATION OF $Z(3)$ WALLS IN THE EARLY UNIVERSE

The production of these $Z(3)$ walls and associated strings is, however, very different from the formation of conventional topological defects as here the symmetry is broken in the high temperature phase, it gets restored below T_c, the QCD transition temperature. To discuss the formation of these objects, one can use the standard Kibble mechanism [23] invoking causality argument at a very early stage of the universe. However, a concrete realization of the formation of $Z(3)$ walls can be achieved in the context of inflationary models, as we will discuss in this section.

We mention that a scenario for the formation of $Z(3)$ domain structure in the early universe has been discussed

in Ref. [24] where it was proposed that a novel phase transition may occur in the universe at a temperature of order 10 TeV. The basic idea in Ref. [24] is that if a large enough region was in a metastable $Z(3)$ vacuum of QCD initially then inflation can expand that region exponentially to superhorizon size. The tunneling rate for decay of this superhorizon size domain to the stable vacuum was estimated in Ref. [24] (see also, Ref. [25]) and it was concluded that bubble nucleation becomes effective when the universe temperature is around $10-20$ TeV, thereby leading to a new phase transition scale in the early universe. However, as will be clear from the discussion below, a crucial ingredient in the model of Ref. [24], namely, the assumption that such metastable domains survive the period of inflation, does not seem justified.

During inflation, the temperature of the universe is driven to almost zero value due to rapid expansion. This will lead to barriers between different $Z(3)$ phases disappear when energy density drops below the QCD scale due to expansion, either in equilibrium, or out of equilibrium. One expects then that $l(x)$ will roll down to the unique minimum of the effective potential if the inflation time scale is larger than the roll down time scale (which should not be much larger than 1 fm at the stage, when the energy density is of order of QCD scale, even for equations of motion in the expanding background). This will happen if the inflation energy scale is below about 10^9 GeV, as in the low scale inflation models discussed later in this paper. This will lead to restoration of $Z(3)$ symmetry during inflation. $Z(3)$ symmetry will be subsequently broken spontaneously as the universe reheats at the end of inflation to a temperature above T_c. $Z(3)$ domains and associated walls will then arise during this spontaneous symmetry breaking transition via the standard Kibble mechanism with typical sizes of the order of the correlation length at an appropriate stage during reheating (and therefore cannot have superhorizon sizes).

If the inflation time scale is much shorter than about $(10^9 \text{ GeV})^{-1}$, then the condensate will not have time to roll down to the minimum of the potential. It may be frozen (or might even decay during inflation), until reheating begins. (The nature of $l(x)$ in such nonequilibrium situations is not clear. One can think of certain gauge field configurations which in equilibrium lead to appropriate behavior of $l(x)$, but simply get redshifted during inflation.) It seems natural to assume that the potential energy of the condensate will be greatly reduced during inflation as the relevant spatial region becomes devoid of matter by rapid expansion. (With matter completely diluted away, the only relevant scales for this potential energy can be the QCD scale, or quark masses). When reheating begins, universe gets filled with high energy particles from the decay of inflaton. The net energy density of this matter may then be very low just at the beginning of reheating, but it would not mean that the universe is getting heated from almost zero temperature

upwards. Initially when the number of particles (from the decay of inflaton) is small, then mean free path of the particles will be larger than the Hubble size and the system will be completely out of equilibrium. As the density of these particles increases (and their energy decreases by multiple rescatterings), at some stage the mean free path will become shorter than the Hubble scale and system can be said to achieve (approximate) equilibrium. It seems clear that the energy density content of this matter will be far greater than the potential energy corresponding to any Polyakov loop condensate which could survive during the inflationary stage. Therefore, the nature of resulting thermal quark-gluon system will be completely dominated by the decay products of inflaton, with any background Polyakov loop condensate possibly surviving through the inflation making negligible contribution to it. In other words, the value of $l(x)$ (in equilibrium, or out of equilibrium) during reheating stage, and consequently, any resulting $Z(3)$ walls, should be entirely determined by the newly created matter and any memory of pre-existing Polyakov loop condensate will be lost. Therefore, in this case as well, one expects $Z(3)$ domain wall formation according to the Kibble mechanism, with typical sizes of the order of relevant correlation length at an appropriate stage during reheating. (Same conclusions will be reached in models of preheating with parametric resonance.)

There are models of inflation where preheating can lead to a short secondary stage of inflation [26] (see, also, [27]). However, the secondary inflation has short duration in these models, which seems inadequate to inflate the $Z(3)$ domains to superhorizon sizes. Formation of truly superhorizon $Z(3)$ domains (as envisaged in Ref. [24]) could be possible in the context of the so called *warm inflation* models where temperature does not become very low during inflation [28]. However the process of inflaton decay during inflation is very complex in these models [29]. For example, at any stage, the thermal system consists of particles which have been freshly generated, as any previously existing particles are diluted away by inflation. It is therefore not clear if one can think of this as a pre-existing $Z(3)$ domain in equilibrium, with temperature changing during inflation. Instead, the situation here appears to be closer to the case of high energy scale inflation, discussed above, where the matter is first diluted away, and then the space gets filled with completely new component of matter during reheating. Thus, even in warm inflation case, one may expect the behavior of $l(x)$, and hence $Z(3)$ domains, to be entirely determined by the matter-radiation which is created near the end of the inflation, leading to subhorizon $Z(3)$ domains.

We therefore conclude that with generic inflationary models, one expects formation of $Z(3)$ domains and associated walls (along with the strings [18]) to arise during the $Z(3)$ symmetry breaking transition at the reheating (or preheating) stage after the inflation via the standard

Kibble mechanism. For the evolution of this domain wall (and associated string [18]) network we note that the tension of the $Z(3)$ interface and this string is set by the QCD parameters and the temperature, hence their dynamics, as far as the tension forces are concerned, should be dominated by the background plasma (at least by its QGP component) for temperatures far above the QCD scale. However, the explicit symmetry breaking due to quarks leads to pressure difference between the metastable $Z(3)$ vacua and the true vacuum, and this should remain significant at high temperatures, again, because at high temperatures the only relevant scale is the temperature. As we mentioned above, estimate of this pressure difference for high temperatures are given in Ref. [24,25]. (There have also been discussions of CP violating effects associated with the metastable phases [30], such effects may be interesting in the context of our model). As mentioned above, due to this pressure difference one expects that regions of metastable phases will shrink quickly as walls enclosing the true vacuum expand. In this picture $Z(3)$ walls are unlikely to survive until late times, say until QCD scale, to play any significant role in the context of the universe.

Though one may still not completely rule out the possibility that the effects of explicit symmetry breaking due to quarks may not be dominant at high temperatures so that walls may survive until late times. In this context we note that the wall motion at high temperatures should be highly dissipative as quarks scattering from the walls will lead to friction. This is expected as the quark free energy depends on $l(x)$, hence there should be significant change in quark energy in crossing wall even at high temperatures (in a similar manner as discussed below), again, as T is the only relevant scale. For large friction the motion of wall in a local plasma rest frame will be strongly suppressed, with walls remaining almost frozen in the plasma. For example, it has been discussed in the literature that dynamics of light cosmic strings can be dominated by friction which strongly affects the coarsening of string network [31].

However, we will discuss below a scenario where in the context of low energy scale inflationary models it is possible that large $Z(3)$ walls, with sizes of order of a fraction of the horizon size at the QCD scale may arise. In such a scenario, with few large domain walls per horizon, the resulting inhomogeneities will be separated by large distances at the QCD transition scale (below which domain walls disappear as $l(x)$ becomes zero). With such large domain walls, number of baryons trapped inside can be very large. As the reflection coefficient for the s quark is larger than that for the u and d quarks, it may also lead to strangeness rich quark nugget formation [32] which can have baryon number as large as about 10^{44} when walls collapse down to the size of order 1 m. Even if walls are not of such large sizes, still resulting baryon inhomogeneities may have large enough magnitudes and distance scales to be able to survive until nucleosynthesis and affect abundances of elements. The model discussed in this paper can therefore be used to constrain various models of low scale inflation using calculations of inhomogeneous nucleosynthesis.

Recently inflationary models with low energy scale, near the electroweak scale, have been proposed which satisfy various requirements for inflation [33]. These models have very low reheating temperature T_{RH}, which is below the electroweak scale, and can be as low as 1 GeV. Let us consider, in some detail, formation of $Z(3)$ walls in the context of these models. At the end of inflation the universe is almost at zero temperature before reheating begins by the decay of the inflaton. As we discussed above, for inflation scales below about 10^9 GeV, this will lead to restoration of $Z(3)$ symmetry during inflation as $l(x)$ will have sufficient time to roll down to the unique minimum of the symmetry restored effective potential. Because of small coupling, decay of inflaton to other particles is very slow. However, due to very slow expansion rate of the universe near the electroweak scale, reheating still happens within one Hubble time in these models. (As opposed to high energy scale inflation where the universe undergoes significant expansion during reheating. Also, to keep our discussion simple, we are not discussing here the possibility of preheating due to parametric resonance.) We therefore have the situation where the universe is slowly (compared to the universe expansion scale) heated from a low temperature up to the reheat temperature T_{RH}. As the temperature becomes larger than the quark-hadron transition temperature T_c, $Z(3)$ symmetry will be spontaneously broken and $Z(3)$ domain walls will appear. (Note that the explicit symmetry breaking term can bias the formation of $Z(3)$ domains as the temperature rises above T_c. We will assume that thermal fluctuations, especially in view of continued heating by decay of inflaton, will dominate over any such bias.) Sizes of the resulting $Z(3)$ domains, and hence of $Z(3)$ walls initially should depend on the details of reheating mechanism. For conservative estimates one may assume that these domains may not be much bigger than the QCD scale at the formation stage. (Though reheating, starting from a low temperature, may allow much larger coherence lengths leading to larger domains initially.)

For low scale inflationary model we are considering, evolution of the dense network of $Z(3)$ walls depends crucially on relative importance of tension and pressure forces. The estimates of Ref. [24,25] for pressure difference between the metastable $Z(3)$ vacua and the true vacuum are valid for high temperatures and hence are inapplicable here. This is why, even the decay rate for the metastable vacua as calculated in Ref. [24,25] cannot be used here. We can use the effective potential in Eq. (4), though it does not have explicit $Z(3)$ symmetry breaking term. Still, one can check from Eq. (4) that at, and near, T_c,

the barrier between different $Z(3)$ vacua are much larger (by about a factor of 100) than the barrier between the broken and unbroken phase [18], and the surface tension of $Z(3)$ walls remains significant for temperatures near T_c.

On the other hand it seems reasonable to assume that the pressure difference between the metastable $Z(3)$ vacua and the true vacuum resulting from the explicit symmetry breaking term may become very small near T_c (see also, Ref. [16]). We will assume that this is the case. In such a case, the dynamics of $Z(3)$ walls near T_c will be controlled by the surface tension of the walls, with pressure difference remaining subdominant. This will also suppress decay of metastable phases by nucleation of true vacuum bubbles. The evolution of a network of such walls will then be like the standard domain walls which coarsens quickly and leads to few domain walls within the horizon volume. For example, if we take the reheat temperature to be 1 GeV, then one should get several large domain walls within the horizon while temperature approaches the quark-hadron transition temperature T_c. Important point here is that during reheating stage, the temperature should remain near T_c for large enough time so that the wall network can coarsen significantly with pressure difference remaining subdominant. At the end of the reheating stage, with temperature reaching few GeV, pressure term should become important and walls should evolve depending on expansion rate and wall velocity through the dissipative plasma. As mentioned above, in view of large friction due to quark scatterings, wall velocity may be very small and may help in retaining large sizes upto the stage of quark-hadron transition.

This scenario can lead to large $Z(3)$ domain walls at temperature near the QCD scale. If pressure term starts dominating early, then domain wall network may not be able to coarsen much and resulting walls will be smaller. Still resulting baryon inhomogeneities may have large enough scales to survive until nucleosynthesis and affect abundances of elements. In the optimistic scenario when temperature lasts near T_c for large enough time (depending on the details of reheating mechanism) so that pressure remains subdominant, one may get almost horizon size walls at the final reheat temperature of few GeV. Subsequent (dissipative) evolution of these walls, with expansion of the universe stretching such large walls, one can get walls which have sizes of order of a fraction of the horizon size at QCD scale. Also, as we mentioned above, there are models of inflation [26–28] in which larger domain walls can arise.

We will assume such an optimistic scenario, and work out the consequences of large $Z(3)$ domain walls near the QCD scale. As the walls evolve, there will be volume contribution of energy coming from the explicit symmetry breaking term. However, in the following calculations, we will neglect these effects. This is because, as explained below, such effects will require calculating reflection of

quarks from a potential barrier which depends on time (with temperature changing during wall collapse), which will require much more elaborate simulations.

IV. REFLECTION OF QUARKS FROM $Z(3)$ WALLS AND BARYON INHOMOGENEITY GENERATION

To model the dependence of effective quark mass on $l(x)$ we could use the color dielectric model of Ref. [34] identifying $l(x)$ with the color dielectric field χ in Ref. [34]. Effective mass of the quark was modeled in [34] to be inversely proportional to χ. This leads to divergent quark mass in the confining phase consistent with the notion of confinement. However, we know that the divergence of quark energy in the confining phase should be a volume divergence (effectively the length of string connecting the quark to the boundary of the volume). $1/l(x)$ dependence will not have this feature, hence we do not follow this choice. For the sake of simplicity, and for order of magnitude estimates at this stage, we will model the quark mass dependence on $l(x)$ in the following manner.

$$m(x) = m_q + m_0(l_0 - |l(x)|). \tag{5}$$

This is somewhat in the spirit of the expectation that a linear term in l should arise from explicit symmetry breaking due to quarks [14–16], though, as mentioned above, we are neglecting the effects of explicit symmetry breaking between different $Z(3)$ vacua. Hence we use $|l(x)|$ in Eq. (5). Here $l(x)$ represents the profile of the $Z(3)$ domain wall, and l_0 is the vacuum value of $|l(x)|$ appropriate for the temperature under consideration. m_q is the current quark mass of the quark as appropriate for the QGP phase with $|l(x)| = l_0$, with $m_u \simeq m_d = 10$ MeV and $m_s \simeq 140$ MeV. m_0 characterizes the constituent mass contribution for the quark. We will take $m_0 = 300$ MeV. Note that here $m(x)$ remains finite even in the confining phase with $l(x) = 0$. As mentioned above, this is reasonable since we are dealing with a situation where $l(x)$ differs from l_0 only in a region of thickness of order 1 fm (thickness of domain wall). For making conservative estimates, we will also give results for the choice $m_0 = m_q$. This will lead to small value for the potential barrier leading to small reflection coefficients. We will discuss resulting baryon inhomogeneities for all these cases. For very high temperatures (e.g. for calculating friction for wall motion), one should use appropriate thermal masses.

Another simplifying assumption we make is to model the potential barrier resulting from Eq. (5) as a rectangular barrier. Height of the barrier V_0 is taken to be equal to $m(x) - m_q$ given in Eq. (5) with the smallest value of $l(x)$ in the profile of the domain wall (Fig. 1). The width of the barrier d is taken to be equal to the width of the domain wall. Using Fig. 1, we take $d = 0.5$ fm and 1 fm for T = 300 MeV and 200 MeV, respectively. Transmission coefficient T for a quark of mass m_q with energy E for this potential barrier can be straightforwardly calculated from

the Dirac equation. We find:

$$T = \frac{4r^2}{4r^2 + (1 - r^2)^2 \sin^2(p_2 d)} \tag{6}$$

where, $r = \frac{p_2(E + m_q)}{p_1(E - V_0 + m_q)}$, $p_1^2 = E^2 - m_q^2$, and $p_2^2 = (E - V_0)^2 - m_q^2$. For $|E - V_0| < m_q$, p_2 is imaginary and $\sin^2(p_2 d)$ is replaced by $\sinh^2(p_2 d)$.

We now discuss the generation of baryon inhomogeneity. We will assume that there are on the average N_d domain walls per horizon volume and will present results for $N_d = 1$ and $N_d = 10$. As the walls collapse, there will be some reheating from decreasing surface area, and from explicit symmetry breaking due to quarks. However, we will neglect these effects in the present discussion, so that we can use a fixed potential barrier (corresponding to a fixed temperature) for calculating baryon transport across the wall. We will also assume that wall collapse is rapid, say with a velocity v_w equal to the velocity of sound $c/\sqrt{3}$ (it could be larger if wall tension completely dominates over the friction). In this case, walls should collapse away in a time smaller than the Hubble time. Thus for rough estimates, one can neglect the expansion of the universe while studying the collapse of a single domain wall (in contrast to Ref. [4,9]). Again, this has the simplification that one can use a fixed shape for the potential barrier, appropriate for a fixed temperature. As a fraction of quarks and antiquarks is reflected by the collapsing wall, thermal equilibrium should be maintained as in the conventional case [4]. This will lead to concentration of net baryon density inside such that we can use the transmission coefficient (Eq. (6)) for the net baryon number.

We mention here that in the context of heavy-ion collisions this assumption of rapid equilibration of reflected quarks and antiquarks may not hold true. In that case, the concentration of strange quarks as well as antiquarks may build up inside the collapsing walls which can lead to important effects such as enhancement of strange hadrons etc.

Let us denote by n_i and n_o the net baryon densities in quarks in the region inside the collapsing domain wall (with volume V_i), and the region outside of it (with volume $V_o = V_T - V_i$), respectively. V_T is the total, fixed, volume of the region neglecting the expansion of the universe as discussed above. Total baryon numbers are then given by $N_i = n_i V_i$ and $N_o = n_o V_o$ for inside and outside regions, respectively. The evolution equations for n_i and n_o can be written as follows (by straightforward modification of the approach used in [4,9]),

$$\dot{n}_i = \left[-\frac{2}{3} v_w T(v_w) n_i + \frac{n_o T(v_q^-) - n_i T(v_q^+)}{6} \right] \frac{S}{V_i} - n_i \frac{\dot{V}_i}{V_i}, \tag{7}$$

$$\dot{n}_o = \left[\frac{2}{3} v_w T(v_w) n_i - \frac{n_o T(v_q^-) - n_i T(v_q^+)}{6} \right] \frac{S}{V_o} + n_o \frac{\dot{V}_i}{V_o}. \tag{8}$$

Here dot denotes the time derivative and S is the surface area of the domain wall. $T(v_w)$ is the transmission coefficient for quarks which have thermal velocity parallel to the domain wall (with corresponding number density being $4n_i/6$). This is calculated by using Eq. (6) for the relative velocity v_w between the quark and the wall. $T(v_q^+)$ and $T(v_q^-)$ are transmission coefficients for quarks with thermal velocities towards the wall from inside and from outside, respectively, (corresponding densities being $n_i/6$ and $n_o/6$), calculated with appropriately Lorentz boosted energies. At these temperatures, the thermal velocities v_q of u,d,s quarks will be close to the speed of light c. (We mention here that the explicit symmetry breaking between different $Z(3)$ vacua will also lead to asymmetry in the transmission coefficients from the two sides of the wall. Though, for large enough potential barrier this difference may not be very significant, especially near T_c.)

The volume enclosed by the spherical collapsing wall is $V_i(t) = \frac{4\pi}{3} R(t)^3$ with the radius $R(t)$ given by

$$R(t) = \frac{r_H}{2N_d^{(1/3)}} - v_w(t - t_0) \tag{9}$$

where $r_H(= 2t)$ is the size of the horizon at the initial time $t_0 \simeq 30(\frac{150}{T(\text{MeV})})^2 \mu$ sec. We take fixed volume $V_T = r_H^3/N_d$ as appropriate for a single collapsing domain wall. With $R(t)$ given by Eq. (9), one has to solve Eqs. (7) and (8) simultaneously to get the detailed evolution of baryon density in the region enclosed by the collapsing domain wall. Baryon inhomogeneity will be produced as baryons are left behind the collapsing wall. We mention here that during the final stages of collapse of domain wall, baryon overdensities may be so large that the chemical potential becomes comparable to the temperature. This will have to be taken into account when calculating the reflection of quarks from the collapsing walls. However, we do not study the evolution of overdensities during those final stages, hence we can neglect the effects of the chemical potential.

For the profile of the baryon inhomogeneities, if $\rho(R)$ is the baryon density left behind at position R from the center of the collapsing spherical wall, then $N_i(R + dR) - N_i(R) = \rho(R) 4\pi R^2 dR$. With the time dependence of R given above, we get,

$$\rho(R) = \frac{dN_i}{dR} \frac{1}{4\pi R^2} = -\frac{\dot{N}_i}{4\pi v_w R^2}. \tag{10}$$

We mention here that the derivation of Eq. (10) assumes that baryons left behind by the collapsing interfaces do not diffuse away, while the derivation of equations for baryon

transport across the wall (Eqs. (7) and (8)) assumed that baryons in both regions homogenize, so that those equations could be written only in terms of two baryon densities, one for each region [4,9]. A more careful treatment should take proper account of baryon diffusion.

Equations (7) and (8) are numerically solved simultaneously to get the evolution of baryon densities n_i, and n_o. We have normalized the initial densities to the average baryon density of the universe n_{av} at that temperature. Initial values of n_i and n_o are thus equal to 1. We have checked that the total baryon number $N_i + N_o$ remains almost constant in time. We find that there are very small random fluctuations in the value of total baryon number, with no tendency of net increase or decrease over time. Numerical errors are therefore under control. Resulting profiles of baryon overdensity $\rho(R)$ is calculated using Eq. (9) and (10). We have used Mathematica routines for numerically solving these coupled differential equations.

Evolution of baryon inhomogeneities of varying amplitudes and length scales has been analyzed in detail in literature [35]. From Ref. [35] one can see that baryon inhomogeneities of initial magnitude $n_i/n_{av} \sim 1000$ near the QCD scale should survive relatively without any dissipation until the nucleosynthesis stage when temperature $T \sim 1$ MeV for all the values of length scales relevant for us, i.e. few tens of cm and above. (For example inhomogeneities with baryon to entropy ratio of about 10^{-5} almost do not change during their evolution. Inhomogeneities with larger amplitude eventually dissipate to this value. See, Ref. [35].) Though, the length scales in Ref. [35] are taken to be comoving at 100 MeV, the results there should apply for the order of magnitude estimates for the values of temperature we have considered $T \simeq 200$ MeV. Also, as these inhomogeneities in our model are produced above the quark-hadron transition, they may affect the quark-hadron transition dynamics [36]. As discussed in Ref. [36], modified dynamics of transition can lead to amplification of these already formed overdensities.

V. RESULTS AND DISCUSSION

In Fig. 2 we have given plots of n_i vs time (in microseconds) and of ρ vs R (in meters) for T = 200 MeV and for the choice of $m_0 = 300$ MeV in Eq. (5). (Again, initial values of n_i, n_o are normalized to the average baryon density of the universe n_{av}. To get absolute values of these densities, and of ρ, one should multiply by n_{av}.) We have taken the number of domain walls in a horizon volume N_d to be 10. We find that the size of the region inside which the baryon overdensity $\rho > 1000$ is about 10 m for u,d quarks while the size is about 60 m for the strange quark case. Baryon density sharply rises for small R. We see that for $R < 1m$, ρ rises to a value of about 20 000 for u,d quarks and to a value of about 6×10^5 for the strange quark. These overdensity magnitudes and sizes are large enough that they can survive until the time of nucleosynthesis and
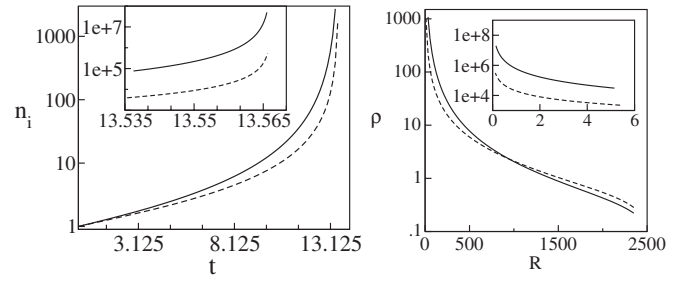

FIG. 2. Plots of n_i vs time t (in microseconds), and ρ vs R (in meters). The origin for t is chosen at the beginning of the wall collapse. Solid curves are for s quark and dashed curves for u,d quarks.

affect nuclear abundances. Typical separation between the inhomogeneities is the interdomain wall separation near the QCD scale (below which walls disappear), and hence can be very large in our model, of order of a km. (Of course, with the assumption that large size walls arise at the end of reheating stage in a low scale inflationary model, as discussed in Sec. III.) This corresponds to about 100–200 km length scale at the nucleosynthesis epoch, which is precisely the range of length scales which can have optimum effects on nucleosynthesis calculations in Ref. [3].

When we consider only one domain wall in the horizon ($N_d = 1$) then overdensities are larger. For example for the above cases, we find that within $R < 1$ m, ρ is larger by a factor of 2 to 4. Overdensities become much smaller for the u,d quark case if we take m_0 in Eq. (5) to be equal to m_q (instead of 300 MeV), as the potential barrier becomes much smaller than the typical quark energy leading to very small reflection coefficient. For example, for other parameters of Fig. 2, ρ is about 20 for $R < 1$ m for u,d quark. However, for the strange quark even with $m_0 = m_q$ the potential barrier is high enough with significant reflection of quarks and leads to $\rho = 120000$ for $R < 1$ m. For comparison we have also calculated overdensities occurring at T = 300 MeV. These are much smaller, first due to smaller domain wall width, and secondly due to larger value of l in the domain wall (see, Fig. 1), leading to smaller potential barrier (height as well as width). For example, with $m_0 = 300$ MeV, within $R < 1$ m we get $\rho = 5000$ for s quark, and $\rho = 400$ for u,d quarks.

With large overdensities occurring as in Fig. 2, there may be possibility of quark nugget formation [32]. Indeed we find that for certain cases, e.g. with the parameters of Fig. 2, total number of baryons can be very large, $\sim 10^{44}$ within $R = 1$ m. These regions will be dominated by strange quarks as is clear from Fig. 2. These seem to be favorable conditions for the formation of stable quark nuggets. If these survive cooling down through T_c, and survive until present then they may constitute dark matter, without affecting microwave background anisotropy or nucleosynthesis constraints.

We summarize main features of our model. We have discussed formation and evolution of $Z(3)$ domain walls in the early universe. We have argued that, in the context of low scale inflationary models with reheat temperature of order of few GeV, it is possible that large $Z(3)$ walls can arise near the QCD scale. (We also briefly mentioned other possibilities where large $Z(3)$ walls can arise in inflationary models based on thermal inflation, or warm inflation etc.) We study baryon inhomogeneities resulting from these walls. In our model, baryon inhomogeneities are produced not due to moving quark-hadron phase boundaries as in the conventional treatments, but due to moving $Z(3)$ domain walls. The variation in the value of the Polyakov loop order parameter across the wall leads to nonzero reflection coefficient for the quarks. As a closed domain wall collapses, a fraction of quarks inside it remains trapped leading to production of baryon inhomogeneities. Important thing is that all this happens in the QGP phase itself, with any possible quark-hadron transition being completely irrelevant. We have assumed that near T_c, the pressure difference between the metastable $Z(3)$ vacua and the true vacuum may be small so that surface tension may play a dominant role in the early evolution of domain walls, which form as the temperature of the universe crosses T_c during reheating stage at the end of inflation. The separation of the resulting inhomogeneities is then the separation between different collapsing domain walls, which may be of the order of a fraction of the horizon size near the quark-hadron transition stage. Resulting overdensities then have large enough magnitudes and sizes that they can survive until the stage of nucleosynthesis and affect the abundances of elements. We also find that if such large walls can form then strangeness rich quark nuggets of large baryon number (10^{44}) can form in our model. If the effects of pressure difference do not remain subdominant near T_c in the coarsening dynamics due to surface tension of walls, then resulting walls will not be as large. Still resulting baryon inhomogeneities may have large enough scales to survive until nucleosynthesis and affect abundances of elements. In view of tight constraints on models of inhomogeneous nucleosynthesis, our results can be used to constrain various models of low scale inflation (or other inflationary models, as discussed above).

The mechanism discussed in this paper will also lead to generation of baryon fluctuations in the QGP formed in relativistic heavy-ion collision experiments, with the walls forming during the initial thermalization stage. The effects of explicit symmetry breaking due to quarks, as discussed above, will not be much relevant there because of very short time scale available for the evolution of QGP. However, one cannot use simplifying assumptions about coarsening of $Z(3)$ walls for the heavy-ion case, as one can do for the case of the universe. Similarly, because of rapid cooling due to expansion, one will have to use time dependent potential barrier for estimating quark reflection from $Z(3)$ walls. We plan to study this using detailed computer simulations in a future work.

ACKNOWLEDGMENTS

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