# Enhancement of the $\bar{\nu}_e$ flux from astrophysical sources by two-photon annihilation interactions

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The ratio of antielectron to total neutrino flux,  $\Phi_{\bar{\nu}_e}: \Phi_{\nu}$ , expected from  $p\gamma$  interactions in astrophysical sources is  $\leq 1:15$ . We point out that this ratio is enhanced by the decay of  $\mu^+\mu^-$  pairs, created by the annihilation of secondary high energy photons from the decay of the neutral pions produced in  $p\gamma$  interactions. We show that, under certain conditions, the  $\Phi_{\bar{\nu}_e}:\Phi_{\nu}$  ratio may be significantly enhanced in gamma-ray burst (GRB) fireballs, and that detection at the Glashow resonance of  $\bar{\nu}_e$  in kilometer scale neutrino detectors may constrain GRB fireball model parameters, such as the magnetic field and energy dissipation radius.

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# I. INTRODUCTION

High energy neutrinos are expected to be produced in astrophysical sources mainly by  $p\gamma$  interactions, leading to the production and subsequent decay of charged pions:  $\pi^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \nu_\mu$  (see, e.g., [1] for recent reviews). Neutrino oscillations lead in this case to an observed ratio of  $\bar{\nu}_{\rho}$  flux to the total  $\nu$  flux of  $\simeq 1.15$  [2] (or lower, in case muons suffer significant electromagnetic energy loss prior to decay [3]). For neutrinos produced in inelastic pp(pn)nuclear collisions, where both  $\pi^+$ 's and  $\pi^-$ 's are produced, the ratio is  $\simeq 1.6$ , and it was suggested that measurements of the  $\nu_e$  to  $\bar{\nu}_e$  flux ratio at the W-resonance may allow one to probe the physics of the sources by discriminating between the two primary modes of pion production,  $p\gamma$  and pp collisions [4]. This test for discriminating between the two mechanisms is complicated by the fact that the ratio of  $\Phi_{\bar{\nu}_e}$  to  $\Phi_{\nu}$  produced in  $p\gamma$  interactions can be enhanced to a value similar to that due to inelastic nuclear collisions in sources where the optical depth to  $p\gamma$  interactions is large (e.g. [3]). In this case, neutrons produced in  $p\gamma \rightarrow n\pi^+$  interactions are likely to interact with photons and produce  $\pi^-$  before escaping the source, leading to production of roughly equal numbers of  $\pi^+$ 's and  $\pi^{-}$ 's.

In this paper, we point out that the  $\Phi_{\bar{\nu}_e}: \Phi_{\nu}$  ratio from  $p\gamma$  interactions may be enhanced above 1:15 also in sources with small  $p\gamma$  optical depth. Neutral pions, which are created at roughly the same rate as charged pions in  $p\gamma$  interactions, decay to produce high energy  $\gamma$ -rays. These  $\gamma$ -rays typically carry  $\sim 10\%$  of the initial proton energy, and may therefore interact with the low energy photons (with which the protons interact to produce pions) to produce  $\mu^+\mu^-$  pairs. The decay of muons yields (after vacuum oscillations)  $\Phi_{\bar{\nu}_e}: \Phi_{\nu} \simeq 1:5$ , thus enhancing the  $\bar{\nu}_e$  fraction.

We discuss below a specific example, the widely considered fireball model of gamma-ray bursts (GRBs). In this model, the observed  $\gamma$ -rays are produced by synchrotron radiation of shock-accelerated electrons in the magnetic field which is assumed to be a fraction of the total energy (see [5] e.g. for reviews). The protons are expected to coaccelerate with electrons to ultrahigh energy [6], and produce high energy neutrinos by  $p\gamma$  interactions [7]. We calculate below the additional neutrino flux, due to the decay of muons produced by secondary photon annihilation, for a typical long duration GRB, and show that the enhanced  $\bar{\nu}_e$  flux may be detectable at the Glashow resonance ( $\bar{\nu}_e e \rightarrow W^- \rightarrow$  anything [8]) in kilometer scale neutrino detectors such as IceCube [9].

The enhancement of  $\Phi_{\bar{\nu}_e}: \Phi_{\nu}$  due to  $\gamma\gamma$  interactions in  $p\gamma$  sources makes the discrimination between  $p\gamma$  and pp neutrino sources more difficult. On the other hand, it may provide a new handle on the physics of the source. We show below that for GRBs the enhancement of  $\bar{\nu}_e$  flux depends on model parameters which are poorly constrained by observations, namely, the magnetic field strength and the energy dissipation radius. Detection of  $\bar{\nu}_e$ 's at the Glashow resonance, in conjunction with  $\gamma$ -ray detection, may therefore constrain these parameters.

# **II. FIREBALL MODEL AND PHOTON SPECTRUM**

The minimum observed GRB fireball radius r may be estimated by requiring that it is optically thin to Thomson scatterings:  $\tau'_{Th} = \sigma_{Th} n' r' \leq 1$  (denoting the comoving and local lab. frame variables with and without a prime, respectively). Here n' is the density of scatterers in the fireball,  $r' = r/\Gamma$  is the size of the interaction region and  $\Gamma$ is the bulk Lorentz factor. The radius at which  $\tau'_{Th} \approx 1$  is the photospheric radius  $r_{ph}$ . For a kinetic luminosity  $L_k$  of the fireball, mostly carried by the baryons, the number density of the baryons, and of the leptons which are coupled to the baryons, is  $n'_b \approx L_k/(4\pi r^2 \Gamma^2 m_p c^3)$ . The observed isotropic equivalent  $\gamma$ -ray luminosity of a long duration GRB is  $L_{52} = L_{\gamma}/10^{52} \text{ erg/s} \sim 1$ . Assuming  $L_{\gamma} = \varepsilon_e L_k$  with  $\varepsilon_e \sim 0.05\varepsilon_{e,-1.3}$  (a parametrization which is motivated below), the photospheric radius is SOEBUR RAZZAQUE, PETER MÉSZÁROS, AND ELI WAXMAN

$$r_{\rm ph} = \frac{\sigma_{\rm Th} L_{\gamma} / \varepsilon_e}{4\pi \Gamma^3 m_p c^3} \approx 7.4 \times 10^{12} \frac{L_{52}}{\varepsilon_{e,-1.3} \Gamma_{2.5}^3} \,\,{\rm cm},\qquad(1)$$

for  $\Gamma_{2.5} = \Gamma/316 \sim 1$ . The radius at which the bulk kinetic energy dissipation occurs, e.g. by internal shocks, is in general  $r \gtrsim r_{\rm ph}$ .

The  $\gamma$ -ray spectrum of a GRB fireball at a dissipation radius  $r = 10^{14} r_{14}$  cm peaks at a typical energy

$$\epsilon_{\gamma,\text{pk}} = \hbar c \Gamma^2 (3\gamma_{e,\min}^{\prime 2} qB') / (2m_e c^2)$$
  
~ 500(\varepsilon\_{e,-1,3}^3 \varepsilon\_{B,-1} L\_{52} \Gamma\_{2,5}^2 / r\_{14}^2)^{1/2} keV, (2)

due to synchrotron radiation by electrons with a Lorentz factor  $\gamma'_{e,\min} \approx \varepsilon_e(m_p/m_e)$ . Here,  $\gamma'_{e,\min}$  is at the lower end of a  $\propto 1/\gamma'_e^{p}$  distribution of electron Lorentz factor, with  $p \geq 2$ , created by Fermi acceleration in the shock. The magnetic field is assumed to be  $B'^2/8\pi \approx$  $\varepsilon_B L_k/(4\pi r^2 \Gamma^2 c)$ , where  $\varepsilon_B \sim 0.1 \varepsilon_{B,-1}$  is the equipartion value, currently unconstrained in the GRB prompt phase. Note that  $\epsilon_{\gamma,\text{pk}} \propto 1/r$  will be larger than the above value for  $r \approx r_{\text{ph}}$ , with other parameters fixed.

For a GRB at a luminosity distance  $d_L$  the observed  $\gamma$ -ray spectrum is generally approximated with a broken power-law Band fit [10],

$$\frac{dN_{\gamma}}{d\epsilon_{\gamma}} \approx \frac{L_{\gamma}}{4\pi d_L^2 \epsilon_{\gamma, pk}^2} \begin{cases} (\epsilon_{\gamma}/\epsilon_{\gamma, pk})^{-1}; & \epsilon_{\gamma} < \epsilon_{\gamma, pk} \\ (\epsilon_{\gamma}/\epsilon_{\gamma, pk})^{-2}; & \epsilon_{\gamma} > \epsilon_{\gamma, pk}. \end{cases}$$
(3)

The spectrum deviates from this at low energy, becoming  $dN_{\gamma}/d\epsilon_{\gamma} \propto \epsilon_{\gamma}^{3/2}$  for  $\epsilon_{\gamma} \lesssim \epsilon_{\gamma,\text{sa}}$ , the energy below which synchrotron self-absorption becomes dominant. Theoretical modeling indicates a value [11]

$$\epsilon_{\gamma,\text{sa}} \approx 2.4 (\Gamma^2 \gamma'_{e,\min} n'_e r[q\hbar c]^4 B'^2 / [m_e^3 c^6])^{1/3}$$
  
$$\sim 8 (\epsilon_{B,-1} L_{52}^2 / [\epsilon_{e,-1.3} \Gamma_{2.5}^2 r_{14}^3])^{1/3} \text{ keV}, \quad (4)$$

for p = 2. The differential number density of photons is

$$dN_{\gamma}^{\prime}/d\epsilon_{\gamma}^{\prime} \approx L_{\gamma}/(4\pi r^{2}c\epsilon_{\gamma,\mathrm{pk}}^{2}) \begin{cases} (\epsilon_{\gamma,\mathrm{sa}}^{\prime}/\epsilon_{\gamma,\mathrm{pk}}^{\prime})^{-1}(\epsilon_{\gamma}^{\prime}/\epsilon_{\gamma,\mathrm{sa}}^{\prime})^{3/2}; & \epsilon_{\gamma}^{\prime} < \epsilon_{\gamma,\mathrm{sa}}^{\prime} \\ (\epsilon_{\gamma}^{\prime}/\epsilon_{\gamma,\mathrm{pk}}^{\prime})^{-1}; & \epsilon_{\gamma,\mathrm{pk}}^{\prime} > \epsilon_{\gamma}^{\prime} > \epsilon_{\gamma,\mathrm{sa}}^{\prime} \\ (\epsilon_{\gamma}^{\prime}/\epsilon_{\gamma,\mathrm{pk}}^{\prime})^{-2}; & \epsilon_{\gamma}^{\prime} > \epsilon_{\gamma,\mathrm{pk}}^{\prime}. \end{cases}$$
(5)

Electron synchrotron radiation produces a power law  $\gamma$ -ray spectrum at energies above  $\epsilon_{\gamma,pk}$  [see Eq. (2)] which depends on the maximum Lorentz factor. Other mechanisms can contribute to an extension of the  $\gamma$ -ray spectrum in Eq. (3) to high energies. High energy electrons can inverse Compton scatter synchrotron photons up to an energy similar to the maximum shock-accelerated electron energy (which we derive shortly) in the Klein-Nishina limit in one mechanism. Here we consider ultrahigh energy  $\gamma$ -rays from  $\pi^0$  decays which are produced by  $p\gamma$  interactions of shock-accelerated protons with synchrotron photons as  $p\gamma \rightarrow \Delta^+ \rightarrow p\pi^0 \rightarrow p\gamma\gamma$ .

The maximum proton energy is calculated by equating its acceleration time  $t'_{\rm acc} \approx \epsilon'_p/(qcB')$  to the shorter of the dynamic time  $t'_{\rm dyn} \approx r/(2\Gamma c)$  and the synchrotron cooling time  $t'_{\rm syn} \approx 6\pi m_p^4 c^3/(\sigma_{\rm Th} m_e^2 \epsilon'_p B'^2)$  as

$$\epsilon_{p,\max} = (6\pi m_p^4 c^4 q \Gamma^2 / [\sigma_{\text{Th}} m_e^2 B'])^{1/2}$$
  

$$\approx 3.3 \times 10^{11} (\varepsilon_{e,-1.3} \Gamma_{2.5}^6 r_{14}^2 / [\varepsilon_{B,-1} L_{52}])^{1/4} \text{ GeV}$$
  

$$\epsilon_{p,\max} = \frac{qB'r}{2} \approx 5.5 \times 10^{11} \left(\frac{\varepsilon_{B,-1} L_{52}}{\varepsilon_{e,-1} \Gamma_{2.5}^2}\right)^{1/2} \text{ GeV}, \quad (6)$$

respectively for  $t'_{\rm acc} = t'_{\rm syn}$  and  $t'_{\rm acc} = t'_{\rm dyn}$ . For electrons,  $t'_{\rm dyn} \gg t'_{\rm syn}$  typically and the maximum electron energy, using Eq. (6) for electrons, is  $\epsilon_{e,\max} \approx$  $10^6 \varepsilon_{e,-1.3}^{1/4} \varepsilon_{B,-1}^{-1/4} \Gamma_{52}^{3/2} \Gamma_{14}^{1/2}$  GeV.

At a given incident proton energy  $\epsilon_p$ , the threshold photon energy leading to a  $\Delta^+$  resonance interaction is

$$\epsilon_{\gamma,\Delta^+} \simeq \frac{0.3\Gamma^2}{(\epsilon_p/\text{GeV})} \text{ GeV.}$$
 (7)

The optical depth for this interaction may be calculated using a delta-function approximation with a cross-section  $\sigma_{p\gamma} \approx 10^{-28} \text{ cm}^2$  as

$$\tau'_{p\gamma}(\epsilon'_p) = \sigma_{p\gamma} \frac{r}{\Gamma} \left( \frac{dN'_{\gamma,\Delta^+}}{d\epsilon'_{\gamma,\Delta^+}} \right) d\epsilon'_{\gamma,\Delta^+}$$
(8)

using Eq. (5). Note that the target photon spectrum, within parenthesis, is now evaluated at the  $\Delta^+$  resonance energy [see Eq. (7)] for an incident proton energy  $\epsilon'_p$ . In particular, Eq. (7) may be used to replace  $\epsilon'_{\gamma,\Delta^+}$  by  $\epsilon'_p$  in Eq. (5). As a result, the optical depth in Eq. (8) is expressed as a function of  $\epsilon'_p$ . The spectral shape of the optical depth is then  $\propto (\epsilon'_p)^{q-1}$ , where q is the spectral index,  $dN'_{\gamma}/d\epsilon'_{\gamma} \propto$  $(\epsilon'_{\gamma})^{-q}$ , in Eq. (5). Also the order is reversed, i.e. q = 2, 1, -3/2 in  $\tau'_{p\gamma} \propto (\epsilon'_p)^{q-1}$  instead of q = -3/2, 1, 2 in  $dN'_{\gamma}/d\epsilon'_{\gamma} \propto (\epsilon'_{\gamma})^{-q}$ , according to the condition in Eq. (7).

Protons lose  $\approx 20\%$  of their energy by  $p\gamma$  interactions to  $\pi^0$  and  $\epsilon_{\gamma} \approx 0.1\epsilon_p$  for each secondary photon. With an equal probability to produce  $\pi^0$  and  $\pi^+$  in each  $p\gamma$  interaction, the resulting  $\pi^0$  decay photon flux is

$$\frac{dN_{\gamma}}{d\epsilon_{\gamma}} = \min[1, \tau'_{p\gamma}] \frac{0.2}{4} \frac{(\xi_p/\varepsilon_e)L_{\gamma}}{4\pi d_L^2 \epsilon_{\gamma}^2}.$$
(9)

Here  $\xi_p$  is the proton fraction undergoing shock accelera-

tion. For  $\xi_p = 1$  and  $\varepsilon_e = 0.05$  we have  $\tau'_{p\gamma} \approx 1$  which leads to the observed flux level in Eq. (3). Secondary pions from  $\Delta^+$  decay, and subsequent decay photons and neutrinos follow the  $dN_p/d\epsilon_p \propto \epsilon_p^{-p}$  spectral shape of the protons for a constant optical depth. For an optical depth of spectral shape  $\propto \epsilon_p^{q-1}$ , the resulting pion, photon and neutrino spectra would be  $dN/d\epsilon \propto \epsilon^{q-1-p}$ . The  $\pi^0$  decay photon spectrum in Eq. (9) is then  $dN_\gamma/d\epsilon_\gamma \propto \epsilon_\gamma^{-2}$  between  $\epsilon_\gamma = 0.03\Gamma^2/\epsilon_{\gamma,pk}$  GeV<sup>2</sup> and  $0.03\Gamma^2/\epsilon_{\gamma,sa}$  GeV<sup>2</sup>,  $\propto \epsilon_\gamma^{-1}$  below  $\epsilon_\gamma = 0.03\Gamma^2/\epsilon_{\gamma,pk}$  GeV<sup>2</sup> and  $\propto \epsilon_\gamma^{-9/2}$  above  $\epsilon_\gamma = 0.03\Gamma^2/\epsilon_{\gamma,sa}$  GeV<sup>2</sup> due to self-absorption following Eqs. (5) and (8).

Note that, the luminosity of shock-accelerated protons is  $1/\varepsilon_e = 20$  times the shock-accelerated electron luminosity. In the fast cooling scenario, valid in the GRB internal shocks, the electrons synchrotron radiate all their energy into observed  $\gamma$ -rays. Thus  $L_p \approx L_\gamma/\varepsilon_e$ . In a single  $p\gamma$  interaction the secondary pion (charged or neutral) luminosity is  $L_\pi \approx 0.2L_p \approx 4L_\gamma/\varepsilon_{e,-1.3}$ . The neutrino luminosity of all flavors from charged pion decay, assuming equal energy for all 4 final leptons, is  $L_\nu \approx (1/2) \times (3/4)L_\pi \approx 1.5L_\gamma/\varepsilon_{e,-1.3}$ . The 1/2 factor arises from the equal probability of  $\pi^0$  and  $\pi^+$  production. The neutrinos carry away energy from the fireball, and the rest of the pion decay ( $e^+$  from  $\pi^+$  and  $\gamma\gamma$  from  $\pi^0$ ) energy is electromagnetic (e.m.), with a luminosity  $L_{e.m.} \approx L_\pi - L_\nu \approx 2.5L_\gamma/\varepsilon_{e,-1.3}$ . A significant fraction of the  $\pi^0$ -decay  $L_\gamma$ 

[Eq. (9)] would be converted to muon pairs and subsequently to neutrinos, as we discuss next. A substantial (small) fraction of the rest of  $L_{e.m.}$  would be emitted at  $r > r_{ph}$  ( $r \approx r_{ph}$ ) as low energy photons with a luminosity not significantly above the observed  $\gamma$ -ray luminosity. These, however, do not affect substantially the neutrino flux calculated from very high energy  $\gamma$ -rays interacting with soft photons.

# **III. TWO-PHOTON PAIR PRODUCTION**

High energy  $\gamma$ -rays can produce lepton pairs,  $l^+l^-$  ( $l = e, \mu$ ), with other photons which are above a threshold energy  $\omega_{\text{th}} = m_l c^2$  in the center of mass (c.m.) frame of interaction. For an incident (target) photon of energy  $\epsilon'_{\gamma,i}$  ( $\epsilon'_{\gamma,t}$ ) in the comoving GRB fireball frame,  $\omega = (2\epsilon'_{\gamma,i}\epsilon'_{\gamma,t})^{1/2}$ , and the cross-section for  $l^+l^-$  pair production may be written, ignoring the logarithmic rise factor at high energy, as  $\sigma_{\gamma\gamma \rightarrow l^+l^-} \approx \pi r_e^2 (m_l c^2 / \omega)^2$ , where  $r_e$  is the classical electron radius. The corresponding optical depth is

$$\tau_{\gamma\gamma}(\epsilon_{\gamma,i}') = \frac{r}{\Gamma} \int \sigma_{\gamma\gamma}(\epsilon_{\gamma,i}';\epsilon_{\gamma,t}') \frac{dN_{\gamma,t}'}{d\epsilon_{\gamma,t}'} d\epsilon_{\gamma,t}'.$$
 (10)

Given the power-law dependence of the photon distribution in Eq. (5), we may calculate the  $l^+l^-$  pair production opacities by integrating Eq. (10) piecewise as

 $m_{\tau}^2 c^4 \Gamma^2$ 

$$I_{\tau}^{+} I_{\tau}^{-}(\boldsymbol{\epsilon}_{\gamma}) = r_{e}^{2} m_{e}^{2} c^{3} L_{\gamma} / (8r\boldsymbol{\epsilon}_{\gamma,\mathrm{pk}}^{2} \boldsymbol{\epsilon}_{\gamma}) \begin{cases} \frac{1}{2} \left[ \left(\frac{\epsilon_{\gamma,\mathrm{pk}}}{\epsilon_{l,\mathrm{th}}}\right)^{2} - \left(\frac{\epsilon_{\gamma,\mathrm{pk}}}{\epsilon_{\gamma,\mathrm{pk}}}\right)^{2} \right]; \boldsymbol{\epsilon}_{\gamma} < \frac{1}{2} \epsilon_{\gamma,\mathrm{pk}} \\ \left(\frac{\epsilon_{\gamma,\mathrm{pk}}}{\epsilon_{l,\mathrm{th}}} - 1\right) + \frac{1}{2} \left[ 1 - \left(\frac{\epsilon_{\gamma,\mathrm{pk}}}{\epsilon_{\gamma}}\right)^{2} \right] \\ \frac{m_{l}^{2} c^{4} \Gamma^{2}}{2\epsilon_{\gamma,\mathrm{sa}}} > \boldsymbol{\epsilon}_{\gamma} > \frac{m_{l}^{2} c^{4} \Gamma^{2}}{2\epsilon_{\gamma,\mathrm{pk}}} \\ \frac{2}{3} \left(\frac{\epsilon_{\gamma,\mathrm{pk}}}{\epsilon_{\gamma,\mathrm{sa}}}\right) \left(1 - \frac{\epsilon_{l,\mathrm{th}}}{\epsilon_{\gamma,\mathrm{sa}}}\right)^{3/2} + \left(\frac{\epsilon_{\gamma,\mathrm{pk}}}{\epsilon_{\gamma,\mathrm{sa}}} - 1\right) \\ + \frac{1}{2} \left[ 1 - \left(\frac{\epsilon_{\gamma,\mathrm{pk}}}{\epsilon_{\gamma}}\right)^{2} \right]; \boldsymbol{\epsilon}_{\gamma} > \frac{m_{l}^{2} c^{4} \Gamma^{2}}{2\epsilon_{\gamma,\mathrm{sa}}}. \end{cases}$$

 $(1 \Gamma (\epsilon_{x, pk})) = (\epsilon_{x, pk}) 2 =$ 

Here we defined the threshold energy for lepton pair production as  $\epsilon_{l,\text{th}} = m_l^2 c^4 \Gamma^2 / 2\epsilon_{\gamma}$ . Note that the high energy photons produce  $e^+e^-$  pairs dominantly at lower energy. The ratio of the two opacities  $\kappa = \tau_{\gamma\gamma \to \mu^+\mu^-} / \tau_{\gamma\gamma \to e^+e^-}$ becomes unity for higher energy photons, since the crosssection is the same for  $\mu^+\mu^-$  and  $e^+e^-$  pair productions above the muon pair production threshold energy.

 $\tau_{\gamma\gamma-}$ 

#### A. Muon decay neutrino flux

Muon pairs decay to neutrinos as  $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$  and  $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$  shortly after they are created in the c.m. frame of the  $\gamma\gamma$  collision. In the observer's frame  $\epsilon_\mu \approx \epsilon_{\gamma}/2$ , and the particle pairs move radially along the incident photon's direction. For simplicity we assume that the  $\nu_e$  and  $\nu_\mu$  created from  $\mu$ -decay carry 1/3 of the muon energy each. The observed neutrino energies are then  $\epsilon_{\gamma}/6$  for each flavor. The neutrino source flux, which is the same

for  $\nu_e$ ,  $\nu_\mu$ ,  $\bar{\nu}_e$  and  $\bar{\nu}_\mu$  previous to any flavor oscillation in vacuum, is

$$\epsilon_{\nu}\Phi^{s}_{\nu,\gamma\gamma} \equiv \epsilon_{\nu}\frac{dN_{\nu}}{d\epsilon_{\nu}} = \min[1,\tau_{\gamma\gamma\to\mu^{+}\mu^{-}}]\kappa\epsilon_{\gamma}\frac{dN_{\gamma}}{d\epsilon_{\gamma}}.$$
 (12)

The high energy muons produced from  $\gamma\gamma$  interactions may lose a significant fraction of their energy by synchrotron radiation before they decay into neutrinos (with a decay time  $t_{dec}$ ), if their energy is above a break energy

$$\epsilon_{\mu,\text{sb}} = (6\pi m_{\mu}^5 c^5 \Gamma^2 / [t_{\text{dec}} \sigma_{\text{Th}} m_e^2 B'])^{1/2}$$
  
\$\approx 5 \times 10^7 (\varepsilon\_{e,-1.3} \Gamma\_{2.5}^4 r\_{14}^2 / [\varepsilon\_{B,-1} L\_{52}])^{1/2} \text{ GeV. (13)}

The corresponding neutrino break energy from muon decay is  $\epsilon_{\nu,sb} = \epsilon_{\mu,sb}/6$ . For  $\epsilon_{\nu} \ge \epsilon_{\nu,sb}$ , the neutrino flux index would steepen by a factor 2 [12].



FIG. 1 (color online). Source flux of  $\nu_e$  (same for  $\nu_{\mu}$ ,  $\bar{\nu}_e$  and  $\bar{\nu}_{\mu}$ ) from  $\gamma\gamma \rightarrow \mu^+\mu^-$  interaction and subsequent  $\mu$ -decays, compared to the canonical  $\nu_e$  flux (same for  $\bar{\nu}_{\mu}$ ) from single  $p\gamma \rightarrow n\pi^+$  interactions and subsequent  $\pi^+$ ,  $\mu^+$  decays. This is for a long duration GRB of  $L_{\gamma} = 10^{52}$  erg/s at redshift  $z \sim 0.1$ . The solid, dashed, dot-dashed and dotted lines are for magnetic field parameters  $\varepsilon_B = 10^{-1}$ ,  $10^{-2}$ ,  $10^{-3}$  and  $10^{-4}$  respectively, for different  $\Gamma$  and r combinations.

We have plotted in Fig. 1 the  $\nu_e$  flux at the source,  $\epsilon_{\nu}^2 \Phi_{\nu}^s$ (same for  $\nu_e$ ,  $\nu_{\mu}$ ,  $\bar{\nu}_e$  and  $\bar{\nu}_{\mu}$ ), previous to any vacuum oscillation, arising from  $\gamma \gamma \rightarrow \mu^+ \mu^-$  interactions and the associated muon decays, for a GRB of isotropic equivalent luminosity  $L_{\gamma} = 10^{52}$  erg/s, which is average for a long GRB [5]. We assume a redshift of  $z \sim 0.1$ , which is near the low end of observed redshifts; there have been a few spectroscopic redshifts observed in the 0.1–0.2 in the past 8 years, and indirect redshift measures, such as time lags, indicate many more bursts in this range among the BATSE sample (see, e.g., Ref. [5] and references therein). Also plotted are the  $\nu_e$  source flux:  $\epsilon_{\nu}^2 \Phi_{\nu}^s = \min[1, \tau'_{p\gamma}] \times$  $(0.2/8)L_{\gamma}/(4\pi d_L^2 \varepsilon_e)$ , from  $p\gamma \rightarrow \Delta^+ \rightarrow n\pi^+$  interactions and subsequent  $\pi^+$  and  $\mu^+$  decays. Different panels are for different bulk Lorentz factor  $\Gamma$  and dissipation radii r.

#### B. Neutrino flavor oscillation and flux on earth

While neutrinos are created via weak interactions as flavor eigenstates, their propagation is determined by the mass eigenstates. The flavor eigenstates  $\nu_{\alpha}$  and the mass eigenstates  $\nu_j$  are mixed through a unitary matrix defined as  $\nu_{\alpha} = \sum_j U_{\alpha j}^* \nu_j$ , where  $\alpha = e$ ,  $\mu$ ,  $\tau$  and j = 1, 2, 3 for three known flavors. The probability for flavor change by vacuum oscillation is given by  $\mathcal{P}_{\nu_{\alpha} \to \nu_{\beta}} = \sum_j |U_{\beta j}|^2 \cdot$  $|U_{\alpha j}|^2$ , for the neutrino propagation from their sources to Earth over astrophysical distances.

We use the standard expression for  $U_{\alpha,j}$  with solar mixing angle  $\theta_{\odot} \equiv \theta_{12} = 32.5^{\circ}$  and atmospheric mixing angle  $\theta_{\text{atm}} \equiv \theta_{23} = 45^{\circ}$  [13]. The unknown mixing angle  $\theta_{13}$  and the *CP* violating phase may be assumed to be zero given the current upper bounds from reactor experiments. Using these values for  $U_{\alpha,j}$  and  $\mathcal{P}_{\nu_{\alpha} \to \nu_{\beta}}$  results in a relationship between the source neutrino fluxes  $\Phi_{\nu}^{s}$  and the expected neutrino fluxes on Earth  $\Phi_{\nu}$  which is given by

$$\begin{bmatrix} \Phi_{\nu_e} \\ \Phi_{\nu_{\mu}} \\ \Phi_{\nu_{\tau}} \end{bmatrix} \approx \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.2 & 0.4 & 0.4 \\ 0.2 & 0.4 & 0.4 \end{bmatrix} \begin{bmatrix} \Phi_{\nu_e}^{s} \\ \Phi_{\nu_{\mu}}^{s} \\ \Phi_{\nu_{\tau}}^{s} \end{bmatrix}.$$
(14)

For antineutrinos  $\mathcal{P}_{\bar{\nu}_{\alpha} \to \bar{\nu}_{\beta}}$  is the same as above.

Different production mechanisms produce  $\nu$  and  $\bar{\nu}$  fluxes at the source with different flavor proportions. Their production ratios may be expressed as normalized vectors, shown in the left-hand side of Eqs. (15) and (16). The corresponding flux ratios at Earth, using Eq. (14), are shown in the right-hand side of Eqs. (15) and (16) below

$$p\gamma \rightarrow n\pi^{+} \begin{bmatrix} 1\\1\\0 \end{bmatrix} \begin{bmatrix} 0\\1\\0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.8\\0.6\\0.6 \end{bmatrix} \begin{bmatrix} 0.2\\0.4\\0.4 \end{bmatrix}, \quad (15)$$

$$\phi\gamma \rightarrow \mu^{+}\mu^{-} \begin{bmatrix} 1\\1\\0 \end{bmatrix} \begin{bmatrix} 1\\1\\0 \end{bmatrix} \begin{bmatrix} 1\\1\\0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.8\\0.6\\0.6 \end{bmatrix} \begin{bmatrix} 0.8\\0.6\\0.6 \end{bmatrix} \begin{bmatrix} 0.8\\0.6\\0.6 \end{bmatrix}. \quad (16)$$

The source  $\nu$ -fluxes plotted in Fig. 1 will be modified accordingly. Note that  $\Phi_{\bar{\nu}_e}(\gamma\gamma)/\Phi_{\bar{\nu}_e}(p\gamma) \approx 4$  for the same initial  $\gamma\gamma$  and  $p\gamma$  flux levels. The  $\bar{\nu}_e$ -flux component is 1/5 (1/15) of the total  $\nu$ -flux from  $\gamma\gamma$  ( $p\gamma$ ). The



FIG. 2. Observed ratio of electron antineutrino flux to the total neutrino flux  $\Phi_{\nu_e}/\Phi_{\nu}$  for the  $p\gamma$  and  $\gamma\gamma$  source fluxes plotted in Fig. 1. This is for a long duration GRB of  $L_{\gamma} = 10^{52}$  erg/s at redshift  $z \sim 0.1$ , the solid, dashed, dot-dashed and dotted lines indicate magnetic field parameters  $\varepsilon_B = 10^{-1}$ ,  $10^{-2}$ ,  $10^{-3}$  and  $10^{-4}$  respectively, for different  $\Gamma$  and r combinations. Note that the 1/15 ratio from single  $p\gamma \rightarrow n\pi^+$  interactions is enhanced by  $\gamma\gamma \rightarrow \mu^+\mu^-$  interactions in certain energy ranges which depend upon the GRB model parameters.

observed ratio of  $\bar{\nu}_e$  to total  $\nu$  fluxes from both the single  $p\gamma$  interactions and  $\gamma\gamma$  interactions can be calculated using Eqs. (15) and (16) from the source fluxes as

$$\frac{\Phi_{\bar{\nu}_e}}{\Phi_{\nu}} = \frac{0.2\Phi^{s}_{\bar{\nu}_e,p\gamma} + 0.8\Phi^{s}_{\bar{\nu}_e,\gamma\gamma}}{3\Phi^{s}_{\bar{\nu}_e,p\gamma} + 4\Phi^{s}_{\bar{\nu}_e,\gamma\gamma}}.$$
(17)

We have plotted this ratio in Fig. 2 for different GRB model parameters. The ratio is enhanced from the canonical  $(p\gamma)$  1/15 value in the energy range where  $\gamma\gamma$  interactions contribute significantly (see Fig. 1). Interestingly, the enhancement takes place over a small energy range which may be explored to learn about the GRB model parameters as we discuss next.

## **IV. NEUTRINO DETECTION**

We consider here the antielectron neutrino detection channel at the Glashow resonance energy  $\epsilon_{\nu,\text{res}} = m_W^2 c^2 / 2m_e \approx 6.4 \text{ PeV}$  [8]. The number of electrons in the 2 km<sup>3</sup> effective IceCube volume is  $N_{e,\text{eff}} \approx 6 \times 10^{38}$ and the corresponding number of downgoing  $\bar{\nu}_e$  events from a point source of flux  $\Phi_{\bar{\nu}_e}$  is [4]

$$N_{\bar{\nu}_e} \approx \Delta t N_{e,\text{eff}} \frac{\pi g^2 (\hbar c)^2}{4m_e c^2} \Phi_{\bar{\nu}_e}(\boldsymbol{\epsilon}_{\nu,\text{res}}).$$
(18)

Here  $g^2 \simeq 0.43$  from the standard model of electroweak theory, and  $\Delta t$  is the duration of the emission.

We have plotted in Fig. 3 the expected number of  $\bar{\nu}_e$ events at  $\epsilon_{\nu, \text{res}}$  from a GRB fireball for various value of r,  $\varepsilon_B$  and  $\Gamma$ . We have assumed here that shock-accelerated protons interact once with synchrotron photons (two top panels), losing  $\sim 20\%$  of their energy. For very high  $p\gamma$ opacity the protons can lose most of their energy through the  $p\gamma$  and  $n\gamma$  interaction chains. This could lead to the  $\nu_e$ -fluxes from both  $p\gamma$  and  $\gamma\gamma$  plotted in Fig. 1 to roughly increase by a factor five. The  $p\gamma$  source  $\nu$ -fluxes may reach ratios  $\Phi_{\nu}^{s} = \Phi_{\bar{\nu}}^{s} = [1, 2, 0]$ , from  $\pi^{+}$  and  $\pi^{-}$  decays, which at Earth would be  $\Phi_{\nu} = \Phi_{\bar{\nu}} = [1, 1, 1]$  for  $\epsilon_{\nu} \leq$  $\epsilon_{\mu,\rm sb}/2$ . The resonant  $\bar{\nu}_e$ -events in such case, from both  $\gamma\gamma$ and  $p\gamma$  fluxes, are plotted in Fig. 3 (two bottom panels). For simplicity, we assumed  $\Phi_{\nu} \propto \Phi_{\gamma} \propto \min[1, 0.2\tau'_{p\gamma}]$  for the  $p\gamma \nu$ -flux and  $\gamma$ -flux in Eq. (12). Also  $\Phi_{\bar{\nu}} = [1, 1, 1]$ from  $p/n\gamma$  interactions for  $\tau'_{p\gamma} \ge 2$ .

The background for astrophysical  $\bar{\nu}_e$  detection is mostly due to atmospheric prompt neutrinos from cosmic-ray generated charm meson decays. A parametrization for the  $\nu_e$  and  $\bar{\nu}_e$  atmospheric flux is given by [14]

$$\Phi_{\nu_e+\bar{\nu}_e}^{\text{atm}} = \begin{cases} \frac{1.5 \times 10^{-5} \epsilon_{\bar{\nu}}^{-2.77}}{1+3 \times 10^{-8} \epsilon_{\nu}}; & \epsilon_{\nu} < 1.2 \times 10^{6} \text{ GeV} \\ \frac{4.9 \times 10^{-4} \epsilon_{-3.02}}{1+3 \times 10^{-8} \epsilon_{\nu}}; & \epsilon_{\nu} > 1.2 \times 10^{6} \text{ GeV} \end{cases} \text{GeV}^{-1} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}.$$
(19)

The corresponding background  $\bar{\nu}_e$ -events at the Glashow resonance energy is  $\leq 10^{-7}$  for a GRB within a  $\sim 100$  s time window, allowing the full directional uncertainty ( $2\pi$  sr), given the poor current knowledge of the  $\nu_e$  or  $\bar{\nu}_e$  signal reconstruction in neutrino Cherenkov detectors, from Eq. (18).

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### V. DISCUSSION

The results of Fig. 3 and Eqs. (15) and (16) show that  $\gamma\gamma$ interactions in astrophysical sources can enhance the observed  $\Phi_{\bar{\nu}_{a}}:\Phi_{\nu}$  flux ratio. A different source of enhancement of the  $\bar{\nu}_{e}$  flux may be pp interactions [3,4]. In GRBs, however, the optical depth to pp is low [7], except in buried jets leading to  $\nu$  precursors [15], where pp interactions are expected to lead to  $\nu$ 's at energies  $\sim$ TeV. The number of resonant  $\bar{\nu}_{e}$  events arising from  $p\gamma$  interactions is essentially independent of  $\varepsilon_B$  (for  $\varepsilon_B \leq 10^{-2}$ ) for any  $\Gamma$ . On the other hand, the number of resonant  $\bar{\nu}_{e}$  events arising from  $\gamma \gamma$  interactions varies significantly with  $\Gamma$  and r. It may become as large as the  $p\gamma$  contribution for  $10^{-2} \leq$  $\varepsilon_B \lesssim 10^{-3}, 100 \lesssim \Gamma \lesssim 300$  and  $r_{\rm ph} \lesssim r \lesssim 3r_{\rm ph}$ . For long bursts of average isotropic equivalent luminosity at a redshift  $\sim 0.1$ , which from past experience are electromagnetically detected every few years, IceCube could probe the  $\bar{\nu}_e$ 



FIG. 3 (color online). Resonant  $\bar{\nu}_e$  events in IceCube from  $\gamma\gamma$  and  $p\gamma$  interactions in a GRB, as a function of the magnetization  $\varepsilon_B$  for Lorentz factor  $\Gamma$  of 100 (solid lines), 178 (dashed lines), 316 (dot-dashed lines), 500 (dotted lines) and fireball radii equal to the photospheric radius (*two left panels*) and 10 times the photospheric radii (*two right panels*). We used single  $p\gamma$  interactions (*two top panels*) and multiple  $p\gamma$  and subsequent  $n\gamma$  interactions proportional to  $\tau_{p\gamma}$  (*two bottom panels*) for comparison. Other GRB parameters are z = 0.1,  $\Delta t = 30$  s,  $L_{\gamma} = 10^{52}$  erg/s and  $\varepsilon_e = 0.05$ .

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enhancement, and thus the value of the magnetization parameter and dissipation radius, by measuring the  $\Phi_{\bar{\nu}_e}: \Phi_{\nu}$  flux ratio. Finally, we note that a moderate excess of  $\nu_e$  events compared to  $\nu_{\mu}$  and  $\nu_{\tau}$  events may also be an indication for the presence of a  $\gamma\gamma$  component.

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