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Strings at future singularities

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We discuss the behavior of strings propagating in spacetimes which allow future singularities of either a sudden future or a Big-Rip type. We show that in general the invariant string size remains finite at sudden future singularities while it grows to infinity at a Big-Rip. This claim is based on the discussion of both the tensile and null strings. In conclusion, strings may survive a sudden future singularity, but not a Big-Rip where they are infinitely stretched.

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According to the claim of Ref. [1] sudden future singularities (a SFS) [2–5] are not strong singularities in the sense of Tipler and Królak [6]. In particular, geodesic equations do not feel anything special on the approach to a SFS (there is no geodesic incompleteness) and the only sign of such singularities may be experienced by the extended objects which may feel infinite tidal forces. On the other hand, Big-Rip (BR) [7,8] singularities are the strong ones and, according to Ref. [1], they are felt by geodesics and also lead to the destruction of structures.

In this context we discuss explicitly the behavior of extended objects such as fundamental strings [9] at the future singularities of either BR or a SFS type. We will discuss a possibility for a string to cross a SFS with its size conserved. We shall analyze a possibility to blow-up a string at BR and to collapse a string to a point at a SFS.

Firstly, it is interesting to remark that at the Schwarzschild horizon $r = r_s$ the metric is singular while the curvature invariants R, $R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ are finite. The a SFS is different from the respect that the metric is nonsingular at sudden future singularity for $t = t_s$, but all the curvature invariants R, $R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ have a blow-up due to the blow-up of the Riemann tensor as a consequence of the divergence of the second derivative of the scale factor. In fact, the geodesic deviation equation feel a SFS due to the divergence of the Riemann tensor.

According to string theory a free string which propagates in a curved spacetime sweeps out a world-sheet (a two-dimensional surface) in contrast to a point particle whose history is a worldline. The world-sheet action for such a string in the conformal gauge is given by [10]

$$S = -\frac{T}{2}\int d\tau d\sigma \eta^{ab} g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu, \qquad (1)$$

where $T = 1/2\pi\alpha'$ is the string tension, α' the Regge slope, τ and σ are the (spacelike and timelike, respectively) string coordinates, η^{ab} is a 2-dimensional world-sheet (flat) metric $(a, b = 0, 1)$, $X^\mu(\tau, \sigma)$ $(\mu, \nu = 0, 1, 2, 3)$ are the coordinates of the string world-sheet in a 4-dimensional spacetime with metric $g_{\mu\nu}$.

The equations of motion and the constraints for the action (1) are [10]

$$\ddot{X}^\mu + \Gamma^\mu_{\nu\rho}\dot{X}^\nu\dot{X}^\rho = \lambda(X''^\mu + \Gamma^\mu_{\nu\rho}X'^\nu X'^\rho), \qquad (2)$$

$$g_{\mu\nu}\dot{X}^\mu\dot{X}^\nu = -\lambda g_{\mu\nu}X'^\mu X'^\nu, \qquad (3)$$

$$g_{\mu\nu}\dot{X}^\mu X'^\nu = 0, \qquad (4)$$

where: $(\ldots)^\cdot \equiv \frac{\partial}{\partial\tau}$, $(\ldots)' \equiv \frac{\partial}{\partial\sigma}$, and $\lambda = 1$ for tensile strings, while $\lambda = 0$ for tensionless/null strings [11–13]. The meaning of the constraints (3) and (4) for a null string is as follows. The first one shows that the string moves with the speed of light and the second says that the velocity vector of an element of string is perpendicular to this element.

An important characteristic for strings is their invariant size for closed strings defined by [10]

$$S(\tau) = \int_0^{2\pi}\sqrt{g_{\mu\nu}X'^\mu X'^\nu}d\sigma. \qquad (5)$$

Expressing general spacetime coordinates as: $X^0 = t(\tau, \sigma)$, $X^1 = r(\tau, \sigma)$, $X^2 = \theta(\tau, \sigma)$, $X^3 = \varphi(\tau, \sigma)$ the equations of motion (2) for a string propagating in a Friedmann spacetime are:

$$\ddot{t} - \lambda t'' + aa_{,t}[f^2(r)(\dot{t}^2 - \lambda t'^2) + r^2(\dot{\theta}^2 - \lambda\theta'^2) + r^2\sin^2\theta(\dot{\varphi}^2 - \lambda\varphi'^2)] = 0, \qquad (6)$$

$$\ddot{r} - \lambda r'' + 2\frac{a_{,t}}{a}(\dot{t}\,\dot{r} - \lambda t'r') + krf^2(r)(\dot{r}^2 - \lambda r'^2) - \frac{r}{f^2(r)}(\dot{\theta}^2 - \lambda\theta'^2) - \frac{r\sin^2\theta}{f^2(r)}(\dot{\varphi}^2 - \lambda\varphi'^2) = 0, \qquad (7)$$

$$\ddot{\theta} - \lambda\theta'' + 2\frac{a_{,t}}{a}(\dot{t}\,\dot{\theta} - \lambda t'\theta') + \frac{2}{r}(\dot{r}\,\dot{\theta} - \lambda r'\theta') - \sin\theta\cos\theta(\dot{\varphi}^2 - \lambda\varphi'^2) = 0, \qquad (8)$$

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$$\ddot{\varphi} - \lambda\varphi'' + 2\frac{a_{,t}}{a}(\dot{t}\,\dot{\varphi} - \lambda t'\varphi') + \frac{2}{r}(\dot{r}\,\dot{\varphi} - \lambda r'\varphi') + 2\cot\theta(\dot{\theta}\,\dot{\varphi} - \varepsilon^2\theta'\varphi') = 0, \quad (9)$$

whereas the constraints (3) and (4) are given by

$$-\dot{t}^2 + \lambda t'^2 + a^2(t)f^2(r)(\dot{r}^2 - \lambda r'^2) + a^2(t)r^2(\dot{\theta}^2 - \lambda\theta'^2) + a^2(t)r^2\sin^2\theta(\dot{\varphi}^2 - \lambda\varphi'^2) = 0, \quad (10)$$

$$-\dot{t}t' - a^2(t)f^2(r)\dot{r}r' + a^2(t)r^2\dot{\theta}\theta' + a^2(t)r^2\sin^2\theta\dot{\varphi}\varphi' = 0. \quad (11)$$

Here $a(t)$ is the scale factor and $f^2(r) = 1/(1 - kr^2)$ with $k = 0, \pm1$—the curvature index. The invariant string size (5) is

$$S(\tau) = \int_0^{2\pi}\sqrt{-t'^2 + a^2f^2r'^2 + a^2r^2\theta'^2 + a^2r^2\sin^2\theta\varphi'^2}\,d\sigma. \quad (12)$$

One of the simplest string configurations is a circular string given by the ansatz [12,13]:

$$t = t(\tau), \qquad r = r(\tau), \qquad \theta = \theta(\tau), \qquad \varphi = \sigma, \quad (13)$$

It gives (6)–(9) as

$$\ddot{t} + aa_{,t}[f^2\dot{t}^2 + r^2\dot{\theta}^2 - \lambda r^2\sin^2\theta] = 0, \quad (14)$$

$$\ddot{r} + 2\frac{a_{,t}}{a}\dot{t}\,\dot{r} + krf^2\dot{r}^2 - \frac{r}{f^2}\dot{\theta}^2 + \lambda\frac{r\sin^2\theta}{f^2} = 0, \quad (15)$$

$$\ddot{\theta} + 2\frac{a_{,t}}{a}\dot{t}\,\dot{\theta} + \frac{\dot{r}}{r}\dot{\theta} + \lambda\sin\theta\cos\theta = 0, \quad (16)$$

$$\ddot{\varphi} = 0, \quad (17)$$

and the constraint (11) is fulfilled automatically, while (10) reads as

$$-\dot{t}^2 + a^2f^2\dot{r}^2 + a^2r^2\dot{\theta}^2 - \lambda a^2r^2\sin^2\theta = 0. \quad (18)$$

The invariant string size now reads as

$$S(\tau) = 2\pi a(t(\tau))r(\tau)\sin\theta(\tau). \quad (19)$$

Let us first discuss briefly the tensile strings and then the null strings. The flat Friedmann Universe admits a different circular ansatz (corresponding to the ansatz (13) for $\theta = 0$) given by Cartesian coordinates [14]

$$t = t(\tau), \qquad X = R(\tau)\cos\sigma, \qquad Y = R(\tau)\sin\sigma, \qquad Z = \text{const.} \quad (20)$$

After the application of the conformal time coordinate $\eta(t) = \int dt/a(t)$ the equations of motion and constraints (2)–(4) take the simple form

$$\ddot{\eta} + 2\frac{a_{,\eta}}{a}\dot{R}^2 = 0, \quad (21)$$

$$\ddot{R} + 2\frac{a_{,\eta}}{a}\dot{\eta}\,\dot{R} + \lambda R = 0, \quad (22)$$

$$\dot{\eta}^2 - \dot{R}^2 - \lambda R^2 = 0. \quad (23)$$

The scale factor in phantom cosmology which admits a Big-Rip in conformal time scales as [8]

$$a(\eta) = \eta^{-2/(3|\gamma|+2)}, \quad (24)$$

where the barotropic index $\gamma = -|\gamma| < 0$ for phantom models ($p = (\gamma - 1)\varrho$, p—the pressure, ϱ—the energy density). In terms of the conformal time coordinate the invariant string size is

$$S(\tau) = 2\pi a(\eta(\tau))R(\tau). \quad (25)$$

The simple solution of the system (21)–(23) is [14]

$$\eta(\tau) = \exp\left(\pm\sqrt{\frac{3|\gamma|+2}{2-3|\gamma|}}\tau\right), \qquad R(\tau) = \frac{\eta(\tau)}{2}\sqrt{3|\gamma|+2}, \quad (26)$$

so that the invariant string size reads as ($|\gamma| < 2/3$)

$$S(\tau) = \pi\sqrt{3|\gamma|+2}\times\exp\left(\pm\frac{3|\gamma|}{\sqrt{4-9\gamma^2}}\tau\right). \quad (27)$$

From (24) we see that a Big-Rip singularity $a(\eta) \to \infty$ appears for $\eta \to 0$. This singularity corresponds to the limit $\tau \to \infty$ and the "+" sign or $\tau \to -\infty$ and the "−" sign in (26). Then, it is clear from (27) that a string size is infinite in either of these limits. This means a string will be *infinitely stretched* at a Big-Rip singularity.

Now, let us discuss tensile strings at a sudden future singularity. Let us choose the following evolution of the scale factor of a SFS model, which presumably extends on both sides of sudden singularity, i.e.,

$$a(t) = 1 + \left(1 + \frac{t}{t_B}\right)^q(a_0 - 1) - \left(\frac{-t}{t_B}\right)^n, \qquad (t < 0) \quad (28)$$

$$\tilde{a}(t) = 1 + \left(1 - \frac{t}{t_C}\right)^q(a_0 - 1) - \left(\frac{t}{t_C}\right)^n, \qquad (t > 0) \quad (29)$$

Here the sudden singularity appears at $t = 0$, where $a_0 = a(0) = \text{const.}$, and $0 < q \leq 1$, $1 < n < 2$ [2–5]. The evolution begins with a Big-Bang singularity at $t = -t_B < 0$, faces a SFS at $t = 0$, and finally reaches a Big-Crunch singularity at $t = t_C > 0$. We will show that the invariant string size is finite at a sudden singularity. For simplicity, we apply asymptotic solutions around $t = 0$, i.e.,

$$a(t) \approx a_0 + \frac{q(a_0 - 1)}{t_B}t + \ldots, \qquad (t < 0), \quad (30)$$

$$a(t) \approx a_0 - \frac{q(a_0 - 1)}{t_C} t + \ldots, \qquad (t > 0). \tag{31}$$

Introducing the conformal time for (30) and (31) we have

$$\eta = \frac{1}{\beta} \ln| a_0 \pm \beta t |, \tag{32}$$

where $\beta = q(a_0 - 1)/t_{B,C}$, and the scale factor is

$$a(\eta) = e^{\beta\eta}. \tag{33}$$

Then, one can see that at a SFS, η is finite which means that the scale factor (33) is finite and so the invariant string size at a SFS is finite, too. This means a string may *cross smoothly* a SFS singularity and eventually approach a Big Crunch singularity where it may collapse to a zero size.

Let us now come to the discussion of the problem for the null strings ($\lambda = 0$ in (2)–(4)). We will briefly show that the null string can be considered as a collection of particles in which σ is assigned to a null particle in a collection, while τ is a parameter of a geodesic with a particular value of σ. Formally, $X^\mu(\tau, \sigma)$ is a geodesic with an index σ.

We note that the left-hand sides of the constraints (3) and (4) are the constants of motion for a collection of particles being the null string. The first claim is trivial, while the second requires the discussion of an absolute derivative of $g_{\mu\nu}\dot{X}^\mu X'^\nu$, i.e.,

$$\frac{d}{d\tau}(g_{\mu\nu}\dot{X}^\mu X'^\nu) = \dot{X}^\rho\nabla_\rho(g_{\mu\nu}\dot{X}^\mu X'^\nu) = \dot{X}_\nu\dot{X}^\rho\nabla_\rho X'^\nu = k_\nu((\mathcal{L}_{\vec{k}}\vec{\eta})^\nu + \eta^\rho\nabla_\rho k^\nu), \tag{34}$$

where $k^\mu = \dot{X}^\mu$ and $\eta^\mu = X'^\mu$. Since $\vec{\eta}(\tau, \sigma) = \phi^*\vec{\eta}(\tau_0, \sigma)$, where $p(\tau, \sigma) = \phi_\tau p_0(\sigma)$ is a geodesic attached to an index σ, with $p_0(\sigma) = \phi_{(\tau=\tau_0)}p_0(\sigma)$ (ϕ_τ is a map generated by a vector field $\vec{k}$), then according to the properties of the Lie derivative the first term vanishes. The second term vanishes due to the relation

$$k_\nu\eta^\rho\nabla_\rho k^\nu = \eta^\rho\nabla_\rho(k_\nu k^\nu) - \eta^\rho k^\nu\nabla_\rho k_\nu = -\eta^\rho k^\nu\nabla_\rho k_\nu = -k_\nu\eta^\rho\nabla_\rho k^\nu. \tag{35}$$

In conclusion, the evolution of the null string can *always be reduced* to the evolution of the geodesics.

Let us now consider a circular string (13) which is equivalent to a collection of particles with initial conditions $\varphi(\tau_0, \sigma) = \sigma$, $\dot{\varphi}(\tau_0, \sigma) = 0$. The first integrals of (14)–(17) for the null strings are

$$\dot{t}^2 = \frac{A}{a^2(t)}, \tag{36}$$

$$\dot{r} = \frac{B\sin\theta + P_3\cos\theta}{a^2(t)f(r)}, \tag{37}$$

$$\dot{\theta} = \frac{C}{a^2(t)r^2}, \tag{38}$$

$$\dot{\varphi} = 0, \tag{39}$$

where $A = P^2 + kL^2$, $B = P_1\cos\sigma + P_2\cos\sigma$, $C = L_1\cos\sigma + L_2\sin\sigma$ are constants independent of σ [1]. In terms of the cosmic time t we calculate the coordinate velocity components as

$$\frac{dr}{dt} = \frac{B'\sin\theta + P'_3\cos\theta}{a(t)f(r)}, \tag{40}$$

$$\frac{d\theta}{dt} = \frac{C'}{a(t)r^2}, \tag{41}$$

where $B' = B/|A|^{1/2}$, $C' = C/|A|^{1/2}$, $P'_3 = P_3/|A|^{1/2}$. Taking a horizontal plane $X^2 \equiv \theta = \theta_0 =$ const. we formally impose an additional initial condition as $\dot{\theta}(\tau_0, \sigma) = 0$, which requires $C = 0$. For this particular choice of θ we have for $k = 1, 0, -1$,

$$r(t) = \sin\left(D\left(\int_0^t \frac{dx}{a(x)}\right)\right), \tag{42}$$

$$r(t) = D\left(\int_0^t \frac{dx}{a(x)}\right), \tag{43}$$

$$r(t) = \sinh\left(D\left(\int_0^t \frac{dx}{a}\right)\right), \tag{44}$$

respectively, and $D = B'\sin\theta_0 + P'_3\cos\theta_0$.

Now let us come to the problem of the null strings at a SFS. Since for $t = t_s$ all the quantities in the expression for the invariant string size (19) are finite, then *the size of the string at a sudden future singularity is finite*. In all three cases $k = 0, \pm 1$ the string is spanned on the surface of either of the two cones of an angle θ_0 each. The cones are attached to each other with their apexes at $r = 0$. The cones have a common symmetry axis given by $\theta = 0$. For $k = +1$ the string oscillates around $r = 0$ and the frequency of its oscillations decreases while its invariant size (19) grows, though not to infinity. For $k = 0$ the string escapes from the point $r = 0$—its coordinate velocities (40) and (41) decrease while its size grows. For $k = -1$ the string escapes from $r = 0$, its coordinate velocities and size grow rapidly.

Now let us study the null strings at Big-Rip. We assume a simple model of evolution of the scale factor which admits such a singularity [7], i.e., $a(t) = a_R(t_m)[|\gamma| + 1 - (|\gamma| + 1)(\frac{t}{t_m})]^{-2/3|\gamma|}$, where $\gamma < 0$ and t_m is the time of a Big-Rip. The integral $\int dx/a(x)$ is convergent in an arbitrary small interval before a BR singularity. For $k = +1$ the frequency of oscillations of the string size is decreasing. Besides, the coordinate velocities (40) and (41) asymptotically tend to zero at a Big-Rip. For $k = 0$ the coordinate velocities (40) and (41) decrease rapidly in order to finally reach zero. Since $a(t) \to \infty$ at Big-Rip and all other quantities in the invariant string size (19) are finite, it means that *the string will be infinitely stretched at a Big-Rip*, i.e., its size $S \to \infty$. However, as concluded from the Eqs. (36)–(39), in the limit $a(t) \to \infty$ all the right-hand sides of these geodesic equations are zero which means that the four-velocity $dX^\mu/d\tau$ tends to zero at a Big-Rip. This is not the case for the acceleration vector $d^2X^\mu/d\tau^2$ which, as seen from the geodesic Eqs. (14)–(17), is not regular due to a blow-up of the scale factor $a(t)$ and its derivatives at a Big-Rip.

Finally, we briefly discuss strings at some other types of future singularities described in details in Refs. [15,16]. These singularities appear for the scale factor of the form $a(t) = a_s \exp\{h_0(t - t_s)^{1-\alpha}\}$ and $h_0 = \text{const.}$ If $0 < \alpha < 1$, then $a = a_s = \text{const.}$, $\varrho \to \infty$, $| p | \to \infty$ at $t = t_s$ (type III singularity of Ref. [15]). On the other hand, if $\alpha < -1$, then $a = a_s$, $\varrho \to 0$ (or finite), $| p | \to 0$ (or finite) and the higher order than two derivatives of the scale factor diverge at $t = t_s$ (type IV singularity of Ref. [16]). In the former case and for $k = +1$ the frequency of oscillations of a string approaching the singularity grows though remains finite at $t = t_s$. What is important from the point of view of the main task of our discussion is the fact that the invariant string size (19) remains finite at the singularity. For $k = 0, -1$ the coordinate velocities (40) and (41) near $t = t_s$ grow rapidly. The integral $\int dx/a(x)$ is convergent on an arbitrary interval of time before a singularity so that the coordinate velocities and the radial coordinate r are finite which means that the invariant string size is also finite at $t = t_s$. In the latter case for $k = +1$ one has almost homogeneous and finite frequency of oscillations of a string near t_s. The invariant size remains finite, too. For $k = 0$ the coordinate velocities remain almost constant near t_s and the invariant size is finite. For $k = -1$ one has a rapid growth of the coordinate velocities—since r remains finite, then the invariant size also remains finite.

Not restricting the value of the azimuthal coordinate $\theta = \theta(\tau)$ we have

$$\frac{1}{r^2} = [Z\cos(\theta + \zeta) - G]^2 + k, \tag{45}$$

where $\zeta = \arctan P_3'/B'$ and $Z = B'/(C'\sin\zeta)$. Since (45) restricts the value of r, then this $\theta = \theta(\tau)$ case does not change a general picture in which the strings are not infinitely stretched at these future singularities. Exact solutions for $\eta(\theta)$ can easily be given but since they do not contribute a new quality to the discussion we will not present them here.

In conclusion, we emphasize that bearing in mind the system of geodesic Eqs. (36)–(39) one can see that the geodesics may be extended through a sudden type, type III and type IV singularities discussed in this paper. This is a consequence of the finiteness of the right-hand sides of these equations since in all the cases we have studied the radial coordinate r was finite which implied the finiteness o the function $f(r)$. Then, strings may survive these singularities. On the other hand, at a Big-Rip, geodesic equations are singular (though the four-velocity is zero) and the strings will be infinitely stretched.

Finally, it is worth mentioning that the Big-Rip singularity leads to the growth of the energy scale up to the Planck scale so that the quantum effects seem to be inevitable. This means our classical string propagation analysis may be changed. For example, quantum effects may drive the universe to a future de Sitter phase instead of a Big-Rip [15], or these effects may prevent a future singularity [16,17]. According to our analysis and the earlier studies of the string propagation in simple cosmological backgrounds [9,14], it is clear that in the former case strings propagate smoothly towards the empty future, while in the latter case they do not show any singular behavior, too. Besides, in the case of an oscillating scale factor of Ref. [18], any time the scale factor is finite at future singularity one should not expect any blow-up of the string size there. In all of these cases the propagation of the quantum strings through future singularities may be different due to string excitation—the effect known as ''particle transmutation''. Actually, this effect was studied recently in Ref. [19] in the context of the propagation of (both classical and quantum) strings through a Big-Crunch/Big-Bang singularity in the ekpyrotic/cyclic scenario [20]. The classical part of the work given in Ref. [19] is complementary to our work about classical string propagation at Big-Rip and other future singularities. The quantum part of Ref. [19] may give some general insight into the quantum string propagation problem through Big-Rip, sudden future, type III and type IV singularities. The detailed study of this issue will be the matter of a separate paper.

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