Addendum to "Helicity conservation in gauge boson scattering at high energy"

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In a previous paper we have established that for any two-body process involving even numbers of transverse vector bosons and gauginos, the dominant asymptotic amplitudes obey helicity conservation (HC); i.e. the sum of the helicities of the two incoming particles equals to the sum of the helicities of the two outgoing ones. This HC has been proved to all orders in minimal supersymmetric standard mode (MSSM), provided $(s, |t|, |u|)$ are much larger than all masses in the model; but only to leading 1-loop logarithmic order in SM. In the present addendum, the validity of HC is extended to all two-body processes. Renormalizability is crucial for the HC validity, while all known anomalous couplings violate it.

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Asymptotic helicity conservation (HC) states that the dominant helicity amplitudes $F(a_{\lambda_1} b_{\lambda_2} \to c_{\lambda_3} d_{\lambda_4})$ for any two-body process

$$a_{\lambda_1} + b_{\lambda_2} \to c_{\lambda_3} + d_{\lambda_4}, \tag{1}$$

involving an even number of transverse gauge bosons, should satisfy

$$\lambda_1 + \lambda_2 = \lambda_3 + \lambda_4, \tag{2}$$

with λ_j denoting the ordinary helicities, and $(s, |t|, |u|)$ being much larger than all masses in either the standard model (SM) or the minimal supersymmetric standard model (MSSM) [1]. Particularly for MSSM, the validity of the HC theorem demanded also that the number of participating gauginos is again even.[1]

In case both initial (final) particles have spin $1/2$, while the final (initial) ones are bosons, the relation

$$\lambda_1 + \lambda_2 = \lambda_3 + \lambda_4 = 0, \tag{3}$$

should also be obeyed by the asymptotically dominant amplitudes [1]. It is obvious that (3) would had been impossible, if the number of transverse gauge bosons were not even.

This HC theorem has been proved in [1] to one-loop leading logarithmic order in SM, but to all orders in MSSM. For MSSM, in particular, the proof implied that all amplitudes violating (2), should vanish at energies much larger than all supersymmetric particle masses [1].

The very interesting property of HC is that it strongly reduces the number of independent amplitudes at high energy. As an example we quote the processes $e^- + e^+ \to W^- + W^+$ in SM or MSSM, where the validity of (2) and (3) forces the dominant amplitudes at high energies to have opposite helicities, for both, the $W^\mp$ and the $e^\mp$ pairs.

[1]Gaugino-Higgsino or sfermion mixings always disappear at high energies, where HC applies.

In the present addendum, the HC theorem is extended to processes involving an odd number of transverse gauge bosons, or an odd number of gauginos. Combining this with the results of [1], we obtain therefore that HC should be valid for all two-body processes. According to this, the dominant amplitudes at asymptotic energies, in either SM or MSSM, should always satisfy (2); while the additional validity of (3) only holds for processes involving an even number of transverse gauge bosons. As in [1], this extended HC theorem continues to be valid to one-loop leading logarithmic order in SM, while in MSSM it holds to all orders at high energies.

To make our point, we first look at various examples of processes not covered by [1], for which one-loop complete or leading logarithmic calculations exist in the literature [2].

The first concerns $g + b \to W^- + t$ which has been studied in either SM or MSSM, to one-loop leading logarithmic order [3]. For transverse W^-, it of course obeys the HC rule (2) [1]. But for longitudinal W^- also, it has been found in [3], that the asymptotically dominant amplitudes obey (2), to the above order. This is also true for $g + b \to H^- + t$ occurring in MSSM [3].

As a second example we consider the processes

$$q_{L(R)} + g \to \tilde{\chi}^0_i + \tilde{q}_{L(R)}, \qquad u_{L(R)} + g \to \tilde{\chi}^+_i + \tilde{d}_{L(R)}, \tag{4}$$

involving one transverse gauge boson. Both these processes have been found to respect[2] the HC rule (2), to the one-loop leading logarithmic order [4]. A similar property has also been observed by an explicit calculation for $q_{L(R)} + g \to \tilde{g} + \tilde{q}_{L(R)}$.

[2]The processes in (4) are of even order in the fermion-fermion-Higgs Yukawa couplings, and thus obey the U(1) symmetry discussed below.

Finally, as a third example we quote

$$\tilde{\chi}_i^0 + \tilde{\chi}_j^0 \to \gamma + Z, \tag{5}$$

which describes the annihilation of any pair of neutralinos to γZ. The full one-loop calculation of this process has been recently completed in MSSM, for any energy and scattering angle [5]. In performing this calculation, it has been observed that the dominant asymptotic amplitudes for a transverse Z respect both (2) and (3), as expected from [1]; while only (2) is obeyed for a longitudinal Z.

We next turn to the all-order proof of (2) in MSSM, following the same method as in [1], and working in the exact supersymmetric limit, with a vanishing Higgs-mixing parameter μ. All particles are massless in this limit, and a new global U(1) symmetry appears, whose charge we call ''formal helicity'' denoted by $\bar{\lambda}_i$ [1]. By definition, $\bar{\lambda}_i$ vanishes for the gauge fields, equals to the normal helicity for the fermion fields, to -1 for the L-sfermion and the Higgs fields, and to $+1$ for the R-sfermion fields [1]. All terms in this model are invariant under the above U(1), except the fermion-fermion-scalar interactions induced by the Yukawa terms in the superpotential, which violate it by 2 units.

Since for vanishing masses, crossing of a particle from an incoming to an outgoing state always changes its helicity, the only independent two-body processes we need to consider consist of the SUSY related pairs

$$\begin{aligned} &\tilde{V} + \tilde{f}_{L(R)} \to \tilde{H} + \tilde{f}'_{R(L)}, \\ &V + f_{L(R)} \to (H, G) + f'_{R(L)}, \end{aligned} \tag{6}$$

$$\begin{aligned} &\tilde{V} + f_{L(R)} \to \tilde{H} + f'_{R(L)}, \\ &V + \tilde{f}_{L(R)} \to (H, G) + \tilde{f}'_{R(L)}, \end{aligned} \tag{7}$$

involving an odd number of gauginos or an odd number of transverse gauge bosons, and the process

$$V + f_{L(R)} \to \tilde{V}' + \tilde{f}'_{L(R)}, \tag{8}$$

containing simultaneously one transverse gauge and one gaugino. In (6)–(8), f denotes fermions, $\tilde{f}$ sfermions, $\tilde{H}$ Higgsinos, (H, G) Higgs or Goldstone bosons, V transverse gauge bosons, and $\tilde{V}$ gauginos. The indicated chiralities of the fermions and sfermions, are the only ones allowed for the above amplitudes to be non vanishing in the considered model. Since we restrict to two-body processes, the aforementioned requirement that the number of participating transverse gauge or gauginos is odd, just means that this number can never be larger than 1.[3]

Turning now to the proof of (2), we first discuss the left-side process in (6). The diagrams contributing to it involve at most pairs of Hermitian conjugates of the above Yukawa terms, multiplied by a single such term, inducing a U(1) violation by two units of ''formal helicity'', i.e.

$$|\bar{\lambda}_{\tilde{V}} + \bar{\lambda}_{\tilde{f}} - \bar{\lambda}_{\tilde{H}} - \bar{\lambda}_{\tilde{f}'}| = 2. \tag{9}$$

Analyzing the possible discrete values of the ''formal'' and the corresponding ordinary helicities, we find that the asymptotically dominant amplitudes for the left process in (6) obey $|\bar{\lambda}_{\tilde{f}} - \bar{\lambda}_{\tilde{f}'}| = 2$, while the ordinary helicities satisfy

$$\lambda_{\tilde{V}} = \lambda_{\tilde{H}} = \pm\frac{1}{2} \quad \text{for} \quad \tilde{f}_{L(R)} \tag{10}$$

for the two $\tilde{f}$-chiralities, respectively. As a result, Eq. (2) is respected by the dominant amplitudes of the left process in (6).

Performing now a SUSY transformation to this process, under which the gaugino $\tilde{V}$ is transformed to a transverse gauge boson V carrying a helicity of the same sign [1], and $(\tilde{f}_{L(R)}, \tilde{f}'_{R(L)})$ are transformed to $(f_{L(R)}, f'_{R(L)})$, we end up with the right process in (6)

$$V + f_{L(R)} \to (H, G) + f'_{R(L)}, \tag{11}$$

where the ordinary helicities of the asymptotically dominant amplitudes satisfy $\lambda_V + \lambda_f = \lambda_{f'}$, in agreement with[4] (2).

Using then the equivalence theorem for the Goldstone case, we end up with the validity of (2) for the longitudinal W or Z-production process [6]

$$V + f_{L(R)} \to (W_{\text{long}}, Z_{\text{long}}) + f'_{R(L)}, \tag{12}$$

depending on the G charge. The validity of (2) for transverse W or Z, has already been established in [1].

As an illustration of the (6) case, we may quote

$$\begin{aligned} &\tilde{g} + \tilde{b}_{L(R)} \to \tilde{H}^- + \tilde{t}_{R(L)}, \\ &g + b_{L(R)} \to (G^-, H^-) + t_{R(L)}, \end{aligned} \tag{13}$$

where in the left process a gluino and a b-squark annihilate producing a Higgsino and a stop.

An identical treatment may be also done for the processes in (7).

An analogous study may also be made for the process (8) whose diagrams respect U(1), since they always involve pairs of Hermitian conjugates of the above Yukawa terms. The sum of the ''formal helicities'' in the initial and final states are therefore equal, for the asymptotically dominant amplitudes; which in turn again leads to the validity of (2)

[3]This is because the same treatment as the one presented in the paragraphs below, also leads to the conclusion that the two-body processes involving three (transverse) gauge and one scalar boson, or one gauge and three scalars, always vanish asymptotically, in agreement with HC.

[4]Note that the correspondence in the right part of (10) guarantees that the V-helicity in (11) must be of opposite sign to that of the f-helicity. Therefore the V and f' helicities carry the same sign in this process.

for the ordinary helicities. For reaching this conclusion, it is important to remember that the gauge and gaugino fields connected by SUSY transformations, always carry helicities of the same sign [1]. Thus, HC is valid also for amplitudes simultaneously involving one transverse gauge boson and one gaugino. As an illustration we quote the rather academic case

$$W^- + u_{L(R)} \to \tilde{W}^0 + \tilde{d}_{L(R)}. \tag{14}$$

The HC asymptotic rule (2) has therefore been established for all processes in (6)–(8). Combining this with the results of [1], we conclude that HC holds asymptotically for any two-body process. For this to be true though, we must always stay away from the infrared and collinear singularities of the two-body amplitudes; since these singularities are intimately related to the multibody processes, for which HC is known not to apply [7].

It is worth emphasizing that the renormalizability of the underlying theory is crucial for the validity of HC. All anomalous (nonrenormalizable) couplings we have looked at, violate HC [8]. An experimental program to test the high energy HC property, would therefore provide an important simple check of the nature of the basic interactions.

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