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Implementation of the multiple point principle in the two-Higgs doublet model of type II

Colin D. Froggatt,[1,*] Larisa Laperashvili,[2,†] Roman Nevzorov,[3,‡] Holger Bech Nielsen,[4,§] and Marc Sher[5,‖]

[1]*Department of Physics and Astronomy, Glasgow University, Glasgow, Scotland*
[2]*Institute of Mathematical Sciences, Chennai, India*
[3]*School of Physics and Astronomy, University of Southampton, Southampton, United Kingdom*
[4]*The Niels Bohr Institute, Copenhagen, Denmark*
[5]*Physics Department, College of William and Mary, Williamsburg, USA*

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The multiple point principle (MPP) is applied to the nonsupersymmetric two-Higgs doublet extension of the standard model (SM). The existence of a large set of degenerate vacua at some high energy scale caused by the MPP results in a few relations between Higgs self-coupling constants which can be examined at future colliders. The numerical analysis reveals that these MPP conditions constrain the mass of the SM-like Higgs boson to lie below 180 GeV for a wide set of MPP scales Λ and $\tan\beta$.

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I. INTRODUCTION

The success of the Standard Model (SM) strongly supports the concept of spontaneous $SU(2) \times U(1)$ symmetry breaking. The mechanism of electroweak symmetry breaking, in its minimal version, requires the introduction of a single doublet of scalar complex Higgs fields and leads to the existence of a neutral massive particle—the Higgs boson. Over the past two decades the upper [1] and lower [1–3] theoretical bounds on its mass have been established. Nevertheless there are good reasons to believe that the SM with the minimal Higgs content is not the ultimate theoretical structure responsible for electroweak symmetry breaking since it is unable to answer many fundamental questions. For example, if the SM is embedded in a more fundamental theory characterized by a much larger energy scale (e.g. the Planck scale $M_{Pl} \approx 10^{19}$ GeV) than the electroweak scale, then the Higgs mechanism suffers from a stability crisis. Indeed, due to the quadratically divergent radiative corrections, the Higgs boson tends to acquire a mass of order of the largest energy scale. Low-scale supersymmetry (SUSY) stabilizes the scale hierarchy, removing quadratic divergences. The unification of gauge coupling constants, which takes place in these models at high energies [4], is commonly considered as a manifestation of the ultimate underlying theory (e.g. superstring theory) accommodating gravity. However, the cosmological constant in SUSY models where supersymmetry is softly broken diverges quadratically, and enormous fine-tuning is required to keep its size around the observed value [5]. Theories with flat [6] and warped [7] extra spatial dimensions allow one to explain the hierarchy between the electroweak and Planck scales. They also provide new insights into gauge coupling unification [8] and the cosmological constant problem [9].

In this article we exploit the most economical approach addressing the hierarchy problem—the multiple point principle (MPP) [10], which does not require many new particles or extra dimensions to resolve this problem. MPP postulates the coexistence in Nature of many phases allowed by a given theory. It corresponds to a special (multiple) point on the phase diagram of the considered theory where these phases meet. At the multiple point the vacuum energy densities of the neighboring phases are degenerate.

The multiple point principle applied to the pure SM exhibits a remarkable agreement with the top-quark mass measurements. According to the MPP, the Higgs effective potential of the SM

$$V_{\rm eff}(\phi) = -m^2(\phi)\phi^2 + \frac{\lambda(\phi)}{2}\phi^4, \qquad (1)$$

which depends only on the norm of the Higgs field $\phi = (\chi^+, \chi^0)$, has two rings of minima in the Mexican hat with the same vacuum energy density [11]. The radius of the little ring equals the electroweak vacuum expectation value (VEV) of the Higgs field. The second vacuum was assumed to be near the Planck scale $\phi \approx M_{Pl}$.

The mass parameter in the effective potential (1) has to be of the order of electroweak scale ensuring the phenomenologically acceptable Higgs vacuum expectation value for the physical (first) vacuum. Since at high scales the ϕ^4 term in Eq. (1) strongly dominates the ϕ^2 term the derivative of $V_{\rm eff}(\phi)$ near the Planck scale takes the form:

$$\frac{dV_{\rm eff}(\phi)}{d\phi}\bigg|_{\phi=M_{Pl}} \approx \left(2\lambda(\phi) + \frac{1}{2}\beta_\lambda\right)\phi^3, \qquad (2)$$

where $\beta_\lambda(\lambda(\phi), g_t(\phi), g_i(\phi)) = \frac{d\lambda(\phi)}{d\log\phi}$ is the beta-function of $\lambda(\phi)$, which depends on $\lambda(\phi)$ itself, gauge $g_i(\phi)$ and top-quark Yukawa $g_t(\phi)$ couplings. Then the degeneracy of the vacua means that at the second vacuum the Higgs

*Electronic address: c.froggatt@physics.gla.ac.uk
†Electronic address: laper@imsc.res.in
‡On leave of absence from the Theory Department, ITEP, Moscow, Russia;
Electronic address: nevzorov@phys.soton.ac.uk
§Electronic address: hbech@alf.nbi.dk
‖Electronic address: sher@physics.wm.edu

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self-coupling and its derivative must be zero to very high accuracy.

When the Higgs self-coupling tends to zero at the Planck scale, the corresponding beta-function vanishes only for a unique value of the top-quark Yukawa coupling. Thus by virtue of MPP $\lambda(M_{Pl})$ and $g_t(M_{Pl})$ are determined. One can then compute quite precisely the top-quark (pole) and Higgs boson masses using the renormalization group flow (see [11]):

$$M_t = 173 \pm 4 \text{ GeV}, \qquad M_H = 135 \pm 9 \text{ GeV}. \tag{3}$$

Shifting the Higgs field VEV in the second vacuum down from the Planck scale by a few orders of magnitude decreases the values of the top-quark and Higgs masses, spoiling the agreement with the experimental data. The hierarchy between the electroweak and Planck scales might also be explained by MPP within the pure SM, if there exists a third degenerate vacuum [12].

The relationships between different couplings required by MPP could arise dynamically. For example a mild form of locality breaking in quantum gravity, due to baby universes say [13], may precisely fine-tune the couplings so that several phases with degenerate vacua coexist [14]. However a necessary ingredient of most models unifying gravity with other gauge interactions is supersymmetry. At the same time couplings in SUSY models are adjusted by the supersymmetry so that all global vacua are degenerate providing another possible origin for the MPP. In previous papers [15,16] the MPP assumption has been adapted to models based on $(N = 1)$ local supersymmetry-supergravity, that allowed an explanation for the small deviation of the cosmological constant from zero.

As the low-energy limit of an underlying SUSY theory the SM looks rather artificial. Indeed in order to give masses to all bosons and fermions in a manner consistent with supersymmetry at least two Higgs doublets must be introduced. It seems unnatural to assume that one of them remains light while another acquires a huge mass of the order of the cut-off scale Λ ($\Lambda \lesssim M_{Pl}$). Therefore in this article we study the nonsupersymmetric two Higgs doublet extension of the SM [2,17] supplemented by the MPP assumption, bearing in mind supersymmetry as a possible origin of the MPP. In the next section the SUSY inspired two Higgs doublet model of type II is outlined and the vacuum stability conditions in this model are specified. In Sec. III the MPP conditions are formulated and the ensuing relations between the Higgs self-couplings due to the MPP assumption are established. The Higgs spectrum in the MPP inspired two Higgs doublet model is discussed in Sec. IV. The restrictions on the Higgs self-couplings and the SM-like Higgs boson mass caused by the MPP are explored in Sec. V. Section VI contains our conclusions and outlook.

II. HIGGS BOSON POTENTIAL AND VACUUM STABILITY CONDITIONS

The most general renormalizable $SU(2) \times U(1)$ gauge invariant potential of the the two Higgs doublet model is given by

$$\begin{aligned} V_{\text{eff}}(H_1, H_2) = {}& m_1^2(\Phi)H_1^\dagger H_1 + m_2^2(\Phi)H_2^\dagger H_2 \\ & - [m_3^2(\Phi)H_1^\dagger H_2 + \text{h.c.}] + \frac{\lambda_1(\Phi)}{2}(H_1^\dagger H_1)^2 \\ & + \frac{\lambda_2(\Phi)}{2}(H_2^\dagger H_2)^2 + \lambda_3(\Phi)(H_1^\dagger H_1)(H_2^\dagger H_2) \\ & + \lambda_4(\Phi)|H_1^\dagger H_2|^2 + \bigg[\frac{\lambda_5(\Phi)}{2}(H_1^\dagger H_2)^2 \\ & + \lambda_6(\Phi)(H_1^\dagger H_1)(H_1^\dagger H_2) \\ & + \lambda_7(\Phi)(H_2^\dagger H_2)(H_1^\dagger H_2) + \text{h.c.}\bigg], \end{aligned} \tag{4}$$

where

$$H_n = \begin{pmatrix} \chi_n^+ \\ (H_n^0 + iA_n^0)/\sqrt{2} \end{pmatrix}.$$

It is easy to see that the number of couplings in the two Higgs doublet model (2HDM) compared with the SM grows from two to ten. Furthermore, four of them m_3^2, λ_5, λ_6 and λ_7 can be complex, inducing CP-violation. In what follows we suppose that mass parameters m_i^2 and Higgs self-couplings λ_i of the effective potential (4) depend only on the overall sum of the squared norms of the Higgs doublets, i.e.

$$\Phi^2 = \Phi_1^2 + \Phi_2^2,$$
$$\Phi_i^2 = H_i^\dagger H_i = \frac{1}{2}[(H_i^0)^2 + (A_i^0)^2] + |\chi_i^+|^2.$$

The running of these couplings is described by the 2HDM renormalization group equations (see [18,19]) where the renormalization scale is replaced by Φ.

At the physical minimum of the scalar potential (4) the Higgs fields develop vacuum expectation values

$$\langle\Phi_1\rangle = \frac{v_1}{\sqrt{2}}, \qquad \langle\Phi_2\rangle = \frac{v_2}{\sqrt{2}} \tag{5}$$

breaking the $SU(2) \times U(1)$ gauge symmetry and generating masses for the bosons and fermions. The overall Higgs norm $\langle\Phi\rangle = \sqrt{v_1^2 + v_2^2} = v = 246$ GeV is fixed by the Fermi constant. At the same time the ratio of the Higgs vacuum expectation values remains arbitrary. Hence it is convenient to introduce $\tan\beta = v_2/v_1$.

In general the interactions of the Higgs doublets H_1 and H_2 with quarks and leptons result in nondiagonal flavour transitions [20]. In particular these interactions contribute to the amplitude of $K^0 - \bar{K}^0$ oscillations and give rise to new channels of muon decay like $\mu \to e^- e^+ e^-$. The common way to suppress flavour changing processes is to impose a certain discrete Z_2 symmetry that forbids potentially dangerous couplings of the Higgs fields to quarks and leptons [20]. Phenomenologically viable two-Higgs doublet models obtained in such a way are classified according to the interactions of H_1 and H_2 with fermions. Our initial motivation encourages us to focus on the Higgs-fermion couplings inherited from the minimal supersymmetric standard model, which correspond to the Model II two Higgs doublet extension of the SM. The Lagrangian of the 2HDM of type II is invariant under the following symmetry transformations[1]:

$$H_1 \to -H_1, \qquad d_{Ri} \to -d_{Ri}, \qquad e_{Ri} \to -e_{Ri}, \qquad (6)$$

which forbid the couplings λ_6 and λ_7 in the Higgs boson potential (4). The discrete symmetry (6) also requires $m_3^2 = 0$. But usually a soft violation of the symmetry (6) by dimension-two terms is allowed, since it does not lead to Higgs-mediated tree-level flavor changing neutral currents. Henceforth we set $\lambda_6 = \lambda_7 = 0$ but retain a non-vanishing value for m_3^2.

The invariance under the symmetry transformations (6) ensures that only one Higgs doublet (H_1) interacts with the down-type quarks and leptons, whereas the second one couples only to up-type quarks [2,17]. As a result, the running masses of the t-quark (m_t), b-quark (m_b) and τ-lepton (m_τ) in the 2HDM of type II are given by

$$m_t(M_t) = \frac{h_t(M_t)v}{\sqrt{2}} \sin\beta, \qquad m_b(M_t) = \frac{h_b(M_t)v}{\sqrt{2}} \cos\beta,$$
$$m_\tau(M_t) = \frac{h_\tau(M_t)v}{\sqrt{2}} \cos\beta, \qquad (7)$$

where M_t is the top-quark pole mass. Since the running masses of the fermions of the third generation are known, Eq. (7) is used to derive the Yukawa couplings $h_t(M_t)$, $h_b(M_t)$ and $h_\tau(M_t)$, which play a crucial role in the 2HDM renormalization group flow.

Let us consider possible sets of global minima of the scalar potential of the 2HDM of type II with vanishing energy density at a high scale $\Phi \sim \Lambda$ and thereby degenerate with the electroweak scale vacuum. If we ignore the running of the Higgs self-couplings around this MPP scale Λ, then the most favorable situation occurs when

$$\lambda_1(\Lambda) = \lambda_2(\Lambda) = \lambda_3(\Lambda) = \lambda_4(\Lambda) = \lambda_5(\Lambda) = 0. \qquad (8)$$

In this case, for any vacuum configuration

$$\langle H_1 \rangle = \Phi_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad \langle H_2 \rangle = \Phi_2 \begin{pmatrix} \sin\theta \\ \cos\theta e^{i\omega} \end{pmatrix}, \qquad (9)$$

where $\Phi_1^2 + \Phi_2^2 = \Lambda^2$, the quartic part of the effective potential (4) goes to zero. Here, the gauge is fixed so that

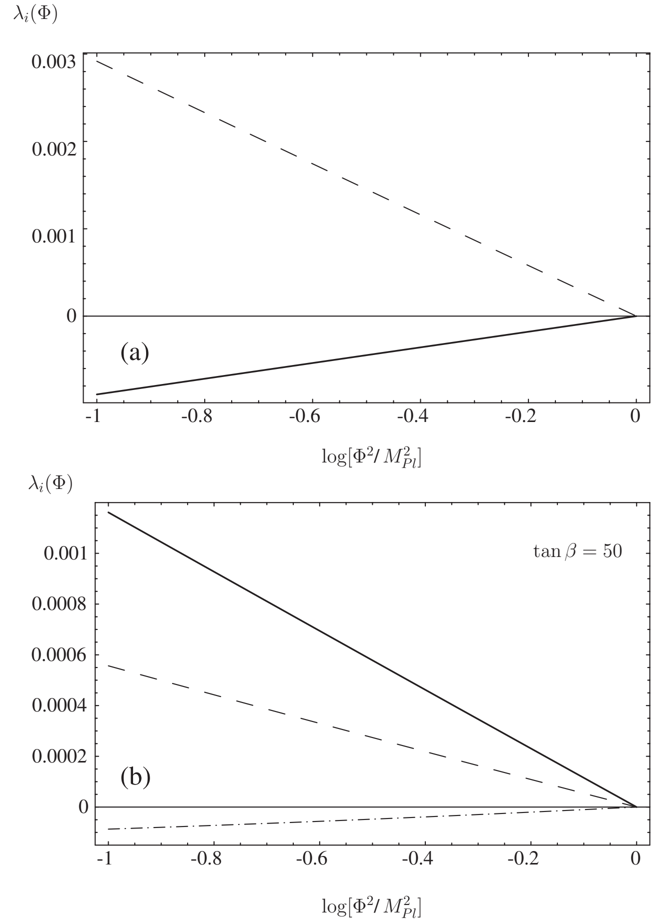

FIG. 1. The running of λ_1, λ_2 and $\tilde{\lambda}$ below M_{Pl} for $\lambda_i(M_{Pl}) = 0$, $M_t = 175$ GeV and $\alpha_3(M_Z) = 0.117$ for (a) $\tan\beta = 2$ and (b) $\tan\beta = 50$. The solid, dashed and dash-dotted lines correspond to λ_1, λ_2 and $\tilde{\lambda}$ respectively. The running of $\tilde{\lambda}$ is not shown for $\tan\beta = 2$ because $\tilde{\lambda}$ becomes complex when $\lambda_1 < 0$.

only the real part of the lower component of H_1 gets a vacuum expectation value.[2]

But the 2HDM renormalization group flow then leads to the instability of the vacua (9). In fact for moderate values of $\tan\beta$ the Higgs self-coupling λ_1 becomes negative just below the MPP scale [see Fig. 1(a)]. The renormalization group running of λ_2 exhibits the opposite behavior, because of the large and negative top-quark contribution to the corresponding beta-function. This means that, near the MPP scale, there is a minimum with a huge and negative energy density ($\sim -\Lambda^4$) where $\langle \Phi_2 \rangle = 0$ and $\langle \Phi_1 \rangle \lesssim \Lambda$.

The renormalization group flow of λ_1 only changes at very large $\tan\beta$ [see Fig. 1(b)]. The absolute value of the b-quark and τ-lepton contribution to β_{λ_1}, being negligible at the moderate values of $\tan\beta$, grows with increasing $\tan\beta$. At $\tan\beta \sim m_t(M_t)/m_b(M_t)$ their negative contribu-

[1]Here d_{Ri} and e_{Ri} denote the right-handed down-type quark and lepton fields.

[2]The $U(1)$ gauge invariance allows us to eliminate the imaginary part of the top component of H_2 as well.

tion to the beta-function of λ_1 prevails over the positive contributions coming from the loops containing Higgs and gauge bosons. The negative sign of β_{λ_1} results in $\lambda_1(\Phi) > 0$ if the overall Higgs norm Φ is less than Λ.

However the positive sign of λ_1 does not ensure the stability of the vacua (9). Substituting the vacuum configuration (9) into the quartic part of the 2HDM scalar potential and omitting all bilinear terms in the Higgs fields one finds for any Φ below the MPP scale:

$$\begin{aligned} V(H_1, H_2) \approx \frac{1}{2}(\sqrt{\lambda_1(\Phi)}\Phi_1^2 - \sqrt{\lambda_2(\Phi)}\Phi_2^2)^2 \\ + (\sqrt{\lambda_1(\Phi)\lambda_2(\Phi)} + \lambda_3(\Phi) \\ + \lambda_4(\Phi)\cos^2\theta)\Phi_1^2\Phi_2^2, \end{aligned} \tag{10}$$

Since the Higgs self-coupling λ_5 is taken to be zero at the scale Λ, it is not generated at any scale due to the form of the 2HDM renormalization group equations [18,19]. The Higgs scalar potential ((10)) attains its minimal value for $\cos\theta = 0$ if $\lambda_4 > 0$ or $\cos\theta = \pm 1$ when $\lambda_4 < 0$. Around the minimum the scalar potential can be written as

$$V(H_1, H_2) \approx \frac{1}{2}(\sqrt{\lambda_1(\Phi)}\Phi_1^2 - \sqrt{\lambda_2(\Phi)}\Phi_2^2)^2 + \tilde{\lambda}(\Phi)\Phi_1^2\Phi_2^2, \tag{11}$$

where

$$\tilde{\lambda}(\Phi) = \sqrt{\lambda_1(\Phi)\lambda_2(\Phi)} + \lambda_3(\Phi) + \min\{0, \lambda_4(\Phi)\}.$$

If at some intermediate scale the combination of the Higgs self-couplings $\tilde{\lambda}(\Phi)$ is less than zero, then there exists a minimum with negative energy density causing the instability of the vacua at the electroweak and MPP scales. Otherwise the Higgs effective potential is positive definite and the considered vacua are stable.

In Fig. 1(b) the Higgs self-couplings $\lambda_1(\Phi)$ and $\lambda_2(\Phi)$ as well as the combination $\tilde{\lambda}(\Phi)$ are plotted as a function of Φ for a large value of $\tan\beta$. It is clear that the vacuum stability conditions, i.e.

$$\lambda_1(\Phi) \gtrsim 0, \qquad \lambda_2(\Phi) \gtrsim 0, \qquad \tilde{\lambda}(\Phi) \gtrsim 0 \tag{12}$$

are not fulfilled simultaneously. The value of $\tilde{\lambda}(\Phi)$ tends to be negative for $\Phi < \Lambda$. So the above considerations demonstrate the failure of the original assumption (8), which therefore can not provide a self-consistent realization of the MPP in the 2HDM.

III. MPP CONDITIONS

At the next stage it is worth relaxing the conditions (8) and permitting $\lambda_1(\Lambda)$ and $\lambda_2(\Lambda)$ to take on nonzero values.[3] Again the Higgs self-coupling $\lambda_5(\Phi)$ remains zero at all scales. In order to avoid a huge and negative vacuum energy density in the global minimum of the 2HDM type II effective potential, the vacuum stability conditions (12) should be satisfied for any Φ in the interval: $v \lesssim \Phi \lesssim \Lambda$. In this case both terms in the quartic part of the scalar potential (11) are positive. In order to achieve the degeneracy of the vacua at the electroweak and MPP scales, they must go to zero separately at the scale Λ. For finite values of $\lambda_1(\Lambda)$ and $\lambda_2(\Lambda)$ the first term in the quartic part of the scalar potential (11) can be eliminated by the appropriate choice of Higgs vacuum expectation values

$$\Phi_1 = \Lambda\cos\gamma, \qquad \Phi_2 = \Lambda\sin\gamma, \qquad \tan\gamma = \left(\frac{\lambda_1}{\lambda_2}\right)^{1/4}, \tag{13}$$

at which $V(H_1, H_2)$ attains its minimal value if the vacuum stability conditions (12) are fulfilled. The vanishing of the second term in Eq. (11) requires a certain fine-tuning of the Higgs self-couplings at the MPP scale, namely $\tilde{\lambda}(\Lambda) = 0$. In order to get $\tilde{\lambda}(\Lambda) = 0$ at least one other Higgs self-coupling, $\lambda_3(\Lambda)$ or $\lambda_4(\Lambda)$, has to take on a nonzero value at the MPP scale. If the fine-tuning between the Higgs self-couplings mentioned above takes place, then the Higgs scalar potential (11) tends to zero at the MPP scale independently of the phase ω in the vacuum configuration (9).

At the first glance of Eq. (10), it even appears possible to get a set of degenerate vacua in which the energy density vanishes for any value of angle θ. This should correspond to

$$\lambda_3(\Lambda) = -\sqrt{\lambda_1(\Lambda)\lambda_2(\Lambda)}, \qquad \lambda_4(\Lambda) = 0. \tag{14}$$

Nevertheless the situation is not as promising as it first appears. The stability of the vacuum configuration (9) requires that the Higgs effective potential does not go to negative values in close vicinity to the MPP scale for $\Phi \gtrsim \Lambda$. In other words at the scale Λ there has to be a local minimum in which all partial derivatives of the 2HDM scalar potential go to zero. The degeneracy of the vacua at the MPP scale implies that they should vanish for any choice of θ and ω. Near the vacuum configuration parameterized by Eq. (9) and (13) the derivatives of $V(H_1, H_2)$ are proportional to

[3] This assumption does not look artificial if we take into account that the corresponding Higgs self-couplings differ from zero in any phenomenologically acceptable SUSY extension of the SM.

$$\frac{\partial V(H_1, H_2)}{\partial \Phi_i} \propto \frac{1}{2}\beta_{\lambda_1}\tan^{-2}\gamma + \frac{1}{2}\beta_{\lambda_2}\tan^2\gamma + \beta_{\lambda_3} + \beta_{\lambda_4}\cos^2\theta. \tag{15}$$

These partial derivatives tend to zero for any angles θ when $\beta_{\lambda_4} = 0$. However for $\lambda_4(\Lambda) = 0$ the beta-function of λ_4 at the scale Λ is given by

$$\beta_{\lambda_4} = \frac{1}{(4\pi)^2}[3g_2^2(\Lambda)g_1^2(\Lambda) + 12h_t^2(\Lambda)h_b^2(\Lambda)] \tag{16}$$

where g_2 and g_1 are the $SU(2)$ and $U(1)$ gauge coupling constants. It is always positive and thereby spoils the stability of the vacua given by Eq. (9) and (13). Thus our attempt to adapt the MPP idea to the 2HDM with $\lambda_4(\Lambda) = 0$ fails.

For nonzero values of $\lambda_1(\Lambda)$ and $\lambda_2(\Lambda)$ the self-consistent implementation of the MPP can only be obtained if $\lambda_3(\Lambda) \neq 0$ and $\lambda_4(\Lambda) \neq 0$. In order to ensure the degeneracy of the physical and the MPP scale vacua and to satisfy the vacuum stability constraints (12), the combination of the Higgs self-couplings $\tilde{\lambda}(\Lambda)$ and its derivative must vanish simultaneously at the scale Λ. Then the 2HDM effective potential possesses a set of local minima:

$$\langle H_1\rangle = \begin{pmatrix} 0 \\ \Phi_1 \end{pmatrix}, \qquad \langle H_2\rangle = \begin{pmatrix} \Phi_2 \\ 0 \end{pmatrix} \tag{17}$$

when $\lambda_4(\Lambda) > 0$ and

$$\langle H_1\rangle = \begin{pmatrix} 0 \\ \Phi_1 \end{pmatrix}, \qquad \langle H_2\rangle = \begin{pmatrix} 0 \\ \Phi_2 e^{i\omega} \end{pmatrix} \tag{18}$$

if $\lambda_4(\Lambda)$ is less than zero, in which the vacuum energy density tends to zero. The Higgs field norms Φ_1 and Φ_2 in the vacuum configurations ((17) and (18)) are determined by the equations for the extrema of the 2HDM scalar potential, whose solution is given by Eq. (13). We should notice here that the existence of the minimum (17) does not necessarily require the vanishing of $\lambda_5(\Lambda)$. Similar vacua with vanishing energy density can also be obtained for nonzero values of this Higgs self-coupling, if it satisfies the constraint: $|\lambda_5(\Lambda)| < \lambda_4(\Lambda)$. At the minimum (17) the $SU(2) \times U(1)$ symmetry is broken completely and the photon gains a mass of the order of Λ. Although this is not in conflict with phenomenology, since an MPP scale minimum is not presently realized in Nature, the scenario with $\lambda_4(\Lambda) < 0$ is more in compliance with the MPP philosophy, simply because it results in a larger set of degenerate vacua. In the minima (18) the photon remains massless and electric charge is conserved.

From the above considerations it becomes clear that the vacuum configurations (18) represent the largest possible set of local degenerate minima of the Higgs effective potential, which can be obtained in the 2HDM at the MPP high energy scale Λ for nonzero values of $\lambda_1(\Lambda)$ and $\lambda_2(\Lambda)$. The constraint on $\lambda_4(\Lambda)$ and the relationships between different Higgs self-couplings

$$\begin{cases} \lambda_5(\Lambda) = 0, & \lambda_4(\Lambda) < 0 \\ \tilde{\lambda}(\Lambda) = \left.\frac{d\tilde{\lambda}(\Phi)}{d\Phi}\right|_{\Phi=\Lambda} & = 0, \end{cases} \tag{19}$$

leading to the appearance of the degenerate vacua, should be identified with the MPP conditions. The conditions (19) have to be supplemented by the vacuum stability requirements (12), which must be valid everywhere from the electroweak scale to the MPP scale. Any failure of either the conditions (19) or the inequalities (12) prevents the consistent realization of the MPP in the 2HDM, when $\lambda_1(\Lambda) \neq 0$ and $\lambda_2(\Lambda) \neq 0$.

Differentiating $\tilde{\lambda}$ near the MPP scale, replacing the derivatives $\lambda_i'(\Lambda)$ by the corresponding one-loop beta-functions and setting $\tilde{\lambda}'(\Lambda)$ to zero, one obtains two relations between the gauge, Yukawa and Higgs self-couplings coupling constants at the MPP scale:

$$\lambda_3(\Lambda) = -\sqrt{\lambda_1(\Lambda)\lambda_2(\Lambda)} - \lambda_4(\Lambda), \tag{20}$$

$$\lambda_4^2(\Lambda) = \frac{6h_t^4(\Lambda)\lambda_1(\Lambda)}{(\sqrt{\lambda_1(\Lambda)} + \sqrt{\lambda_2(\Lambda)})^2} + \frac{(6h_b^4(\Lambda) + 2h_\tau^4(\Lambda))\lambda_2(\Lambda)}{(\sqrt{\lambda_1(\Lambda)} + \sqrt{\lambda_2(\Lambda)})^2} - 2\lambda_1(\Lambda)\lambda_2(\Lambda) - \frac{3}{8}(3g_2^4(\Lambda) + 2g_2^2(\Lambda)g_1^2(\Lambda) + g_1^4(\Lambda)). \tag{21}$$

The first of them follows from $\tilde{\lambda}(\Lambda) = 0$, whereas the second one comes from the vanishing of the derivative of $\tilde{\lambda}(\Phi)$ near the MPP scale. We note that, in the minimal SUSY model, the MPP conditions (19) are satisfied identically at any scale lying higher than the superparticle masses.

IV. HIGGS SPECTRUM

Keeping in mind that Eq. (20) and (21) relate different couplings at the scale Λ we can now explore the Higgs spectrum in the vicinity of the physical vacuum of the MPP inspired 2HDM of type II. The Higgs sector of the two Higgs doublet extension of the SM involves eight states. Two linear combinations of $\chi_1^\pm$ and $\chi_2^\pm$ are absorbed by the $W^\pm$ bosons after the spontaneous $SU(2) \times U(1)$ symmetry breaking at the electroweak scale. A linear combination of A_1 and A_2 become the longitudinal component of the Z boson. The others form two charged and three neutral scalar fields. One of the neutral Higgs bosons is CP-odd. The charged and CP-odd scalars do not interfere with each other and the CP-even states, because of the electric charge conservation and CP-invariance. They gain masses

$$m_{\chi^\pm}^2 = m_A^2 - \frac{\lambda_4}{2}v^2, \qquad m_A^2 = \frac{2m_3^2}{\sin 2\beta}, \tag{22}$$

where $m_{\chi^\pm}$ and m_A are the masses of the charged and pseudoscalar Higgs bosons. The direct searches for the rare B-meson decays ($B \to X_s \gamma$) place a lower limit on the charged Higgs scalar mass in the 2HDM of type II [21]:

$$m_{\chi^\pm} > 350 \text{ GeV}, \tag{23}$$

which is also valid in our case.

In the basis

$$h_1 = H_1^0 \cos\beta + H_2^0 \sin\beta, \qquad h_2 = -H_1^0 \sin\beta + H_2^0 \cos\beta \tag{24}$$

the mass matrix of the CP-even Higgs fields is expressed as (see [22])

$$M^2 = \begin{pmatrix} M_{11}^2 & M_{12}^2 \\ M_{21}^2 & M_{22}^2 \end{pmatrix} = \begin{pmatrix} \frac{\partial^2 V}{\partial v^2} & \frac{1}{v}\frac{\partial^2 V}{\partial v \partial\beta} \\ \frac{1}{v}\frac{\partial^2 V}{\partial v \partial\beta} & \frac{1}{v^2}\frac{\partial^2 V}{\partial \beta^2} \end{pmatrix},$$

$$M_{11}^2 = \left(\lambda_1 \cos^4\beta + \lambda_2 \sin^4\beta + \frac{\lambda}{2}\sin^2 2\beta\right) v^2,$$

$$M_{12}^2 = M_{21}^2 = \frac{v^2}{2}(-\lambda_1 \cos^2\beta + \lambda_2 \sin^2\beta + \lambda \cos 2\beta)\sin 2\beta, \tag{25}$$

$$M_{22}^2 = m_A^2 + \frac{v^2}{4}(\lambda_1 + \lambda_2 - 2\lambda)\sin^2 2\beta,$$

where $\lambda = \lambda_3 + \lambda_4$. Equations for the extrema of the Higgs boson effective potential are used to eliminate m_1^2 and m_2^2 from Eq. (22) and (25). The top-left entry of the CP-even mass matrix provides an upper bound on the lightest Higgs scalar mass-squared. The masses of the two CP-even eigenstates obtained by diagonalizing the matrix (25) are given by

$$m_{H,h}^2 = \frac{1}{2}\left(M_{11}^2 + M_{22}^2 \pm \sqrt{(M_{22}^2 - M_{11}^2)^2 + 4M_{12}^4}\right). \tag{26}$$

With increasing m_A the lightest Higgs boson mass grows and approaches its maximum value $\sqrt{M_{11}^2}$ for $m_A^2 \gg v^2$.

As follows from Eq. (22) and (25), the spectrum of the Higgs bosons in the 2HDM of type II supplemented by the MPP assumption is parameterized in terms of m_A, $\tan\beta$ and four Higgs self-couplings λ_1, λ_2, λ_3 and λ_4. Three other Higgs self-couplings λ_5, λ_6 and λ_7 vanish due to the MPP conditions (19). At the scale Λ the couplings $\lambda_3(\Lambda)$ and $\lambda_4(\Lambda)$ can be expressed in terms of $\lambda_1(\Lambda)$, $\lambda_2(\Lambda)$, $g_i(\Lambda)$ and $h_j(\Lambda)$. The gauge couplings at the MPP scale are fixed by $g_i(M_Z)$, which are extracted from the electroweak precision measurements. The running of the Yukawa couplings is mainly determined by $\tan\beta$. Thus, for a given scale Λ, the evolution of the Higgs couplings are governed by $\lambda_1(\Lambda)$, $\lambda_2(\Lambda)$ and $\tan\beta$. Therefore the Higgs masses and couplings depend on five variables

$$\lambda_1(\Lambda), \qquad \lambda_2(\Lambda) \qquad \Lambda \qquad \tan\beta, \qquad m_A. \tag{27}$$

It means that, owing to the MPP, the model suggested in this article has fewer free parameters compared to the 2HDM with an exact or softly broken Z_2 symmetry. Therefore it can be considered as the minimal nonsupersymmetric two Higgs doublet extension of the SM.

V. NUMERICAL ANALYSIS

The constraints on the Higgs masses in the 2HDM with an unbroken Z_2 symmetry (with $m_3^2 = 0$) have been examined in a number of publications [19,23,24]. An analysis performed assuming vacuum stability and the applicability of perturbation theory up to a high energy scale (e.g. the unification scale) revealed that all the Higgs boson masses lie below 200 GeV [24]. Stringent restrictions on the masses of the charged and pseudoscalar states were found. They do not exceed 150 GeV. This upper bound is considerably less than the lower experimental limit on $m_{\chi^\pm}$ (23) obtained in the 2HDM of type II. This shows that 2HDM with unbroken Z_2 symmetry is inconsistent with experimental data.

The aim of our study is to analyze the MPP scenario in the 2HDM of type II ($m_3^2 \neq 0$) and compare it with that in the SM. As part of the analysis sufficiently large values of $\tan\beta$ should be taken. The motivation for this is quite simple. The top-quark Yukawa coupling at the electroweak scale approaches its SM value for $\tan\beta \gg 1$. If simultaneously $\tan\beta$ is much less than $m_t(M_t)/m_b(M_t)$ the b-quark and τ-lepton Yukawa couplings remain small and can be disregarded. Since in the considered limit the beta-functions of h_t in the SM and 2HDM coincide, the renormalization group flows of the top-quark Yukawa coupling in these models are then identical and the main differences in the spectra are caused by the Higgs couplings.

For the numerical analysis we adopt the following procedure. At the first stage we fix the values of $\tan\beta$ ($\tan\beta = 10$) and the MPP scale ($\Lambda = M_{Pl}$). Then using the 2HDM renormalization group equations we calculate the gauge and Yukawa couplings at the scale Λ. For each given set of $\lambda_1(\Lambda)$ and $\lambda_2(\Lambda)$ we define $\lambda_3(\Lambda)$ and $\lambda_4(\Lambda)$ in accordance with Eq. (20) and (21), evolve the renormalization group equations down, determine the values of all Higgs self-couplings at the electroweak scale ($\mu = 175$ GeV) and study the Higgs spectrum as a function of the pseudoscalar mass. After that we investigate the dependence of the Higgs masses on $\lambda_1(\Lambda)$ and $\lambda_2(\Lambda)$. At the next stage we vary the value of $\tan\beta$ from 2 to 50 and MPP scale within the interval: 10 TeV $\lesssim \Lambda \lesssim M_{Pl}$. Finally the sensitivity of the Higgs masses to the choice of $\alpha_3(M_Z)$, $m_t(M_t)$ and renormalization scale μ is examined.

The results of our numerical study are summarized in Tables I and II and Figs. 2 and 3. The MPP assumption constrains $\lambda_1(\Lambda)$ and $\lambda_2(\Lambda)$ very strongly at moderate and large values of $\tan\beta$. In Fig. 2 different curves restrict the allowed range of the corresponding Higgs self−couplings

TABLE I. The lightest running Higgs mass for $m_A = 400$ GeV, $M_t = 175$ GeV and $\alpha_3(M_Z) = 0.117$ (m_h is given in GeV and calculated at the scale $\mu = 175$ GeV).

Λ	$\tan\beta$	$\lambda_1(\Lambda)$	$\lambda_2(\Lambda)$	m_h
$\Lambda = M_{Pl}$	$\tan\beta = 10$	1.0	0.005	137.8
		3.5	0.005	137.9
		0.25	0.005	138.5
		1.0	0.008	138.2
		1.0	0.001	136.8
	$\tan\beta = 2$	1.6	0.05	118.1
		3.2	0.05	128.3
		0.85	0.05	116.7
		1.6	0.08	127.4
		1.6	0.02	114.9
	$\tan\beta = 50$	1.0	0.01	140.9
		3.0	0.01	141.6
		0.1	0.01	141.8
		0.04	0.1	148.4
		0.01	4.0	170.7
$\Lambda = 10$ TeV	$\tan\beta = 10$	0.25	0.25	142.0
		0.45	0.45	166.6
		0.10	0.10	115.3
		0.25	0.45	168.2
		2.4	0.25	134.7
	$\tan\beta = 2$	0.3	0.3	103.2
		0.65	0.65	116.6
		0.16	0.16	95.6
		0.3	0.7	131.5
		4.0	0.3	72.4
	$\tan\beta = 50$	0.3	0.3	150.2
		0.64	0.64	188.9
		0.01	0.01	89.0
		0.1	4.0	321.9
		4.0	0.1	114.6

where $\lambda_4^2(\Lambda) \geq 0$. Outside the allowed range $\lambda_4(\Lambda)$ is complex and the Higgs effective potential (4) is nonhermitian. If the MPP scale is situated at very high energies (e.g. $\Lambda \simeq M_{Pl}$) then only extremely small values of $\lambda_2(\Lambda) \simeq 0.01$ are permitted for most of the large $\tan\beta$ region. The ratio of $\lambda_1(\Lambda)$ to $\lambda_2(\Lambda)$ is also limited so that $\lambda_1(\Lambda) \gg \lambda_2(\Lambda)$ since $\sin^2\gamma$ is bounded from below [see Fig. 2(a) and 2(b)]. These restrictions are substantially relaxed at $\tan\beta \sim 1$ and at very large values of $\tan\beta$ close to the upper limit coming from the validity of perturbation theory at high energies. When $\tan\beta$ is of the order of $m_t(M_t)/m_b(M_t)$ the Higgs self-coupling $\lambda_2(\Lambda)$ can be even much larger than $\lambda_1(\Lambda)$ as the low values of $\sin^2\gamma$ are not ruled out by the MPP in this case [see Fig. 3(a)].

TABLE II. The Higgs spectrum for $\Lambda = M_{Pl}$, $\tan\beta = 10$, $\lambda_1(M_{Pl}) = 1$ and $\lambda_2(M_{Pl}) = 0.005$ (all masses are given in GeV).

m_A	400	400	400	400	400	400	200	1000
m_t	165	165	165	165	165	170	165	165
μ	175	M_Z	400	175	175	180	175	175
$\alpha_3(M_Z)$	0.117	0.117	0.117	0.119	0.115	0.117	0.117	0.117
$m_h(\mu)$	137.8	143.3	131.4	137.0	138.6	146.6	136.7	137.9
$m_H(\mu)$	400.8	400.8	400.8	400.8	400.8	400.9	202.4	1000.3
$m_{\chi} \pm (\mu)$	406.7	406.7	406.7	405.2	407.8	411.0	213.1	1002.7

The restrictions on the Higgs self-couplings arising out of the MPP become weaker in the scenarios with a low MPP scale. The allowed regions of $\lambda_1(\Lambda)$ and $\lambda_2(\Lambda)$ enlarge because of the increase in the top-quark Yukawa coupling contribution to the right hand side of Eq. (21). As before, the admissible ranges of $\lambda_1(\Lambda)$ and $\lambda_2(\Lambda)$ expand at moderate and very large values of $\tan\beta$ [see Fig. 2(c), 2(d), and 3(b)].

The applicability of perturbation theory up to the scale Λ and the requirement of the stability of the degenerate vacua constrain the Higgs self-couplings further. While $\lambda_2(\Lambda)$ can vary from zero to its upper bound [see Fig. 2(a) and 2(c)], the Higgs self-coupling $\lambda_1(\Lambda)$ is limited from below and above for most of the $\tan\beta$ region. When the values of $\lambda_1(\Lambda)$ are too large they either violate perturbativity or make the term $\lambda_1(\Lambda) \cdot \lambda_2(\Lambda)$ in Eq. (21) so large that $\lambda_4^2(\Lambda)$ tends to be negative. Values of $\lambda_1(\Lambda)$ which are too small either reduce the top-quark Yukawa contribution to the right hand side of Eq. (21), so that $\lambda_4^2(\Lambda)$ turns out to be negative, or result in the changing of the sign of $\tilde{\lambda}(\Phi)$ during the renormalization group flow giving rise to vacuum instability. The allowed intervals of $\lambda_1(M_{Pl})$ are indicated in Table I for $\tan\beta = 10$ and $\lambda_2(M_{Pl}) = 0.005$, for $\tan\beta = 2$ and $\lambda_2(M_{Pl}) = 0.05$ and for $\tan\beta = 50$ and $\lambda_2(M_{Pl}) = 0.01$. Also Table I shows the admissible ranges of $\lambda_1(\Lambda) = \lambda_2(\Lambda)$ for $\Lambda = 10$ TeV and the three different values of $\tan\beta$. One can see that the lower bound on $\lambda_1(\Lambda)$ weakens at very large values of $\tan\beta \simeq m_t(M_t)/m_b(M_t)$ and in the scenarios when the MPP conditions (19) are fulfilled at low energies.

The restrictions on the Higgs self-couplings discussed above and the choice of m_3^2 needed to respect the lower limit on the charged scalar mass (23), deduced from the nonobservation of $B \to X_s\gamma$ decay, maintain a mass hierarchy in the Higgs sector of the MPP inspired 2HDM. Indeed the MPP, in conjunction with the requirements of vacuum stability and validity of perturbation theory, keep $\lambda_i v^2$ and the CP-even mass matrix elements M_{11}^2 and M_{12}^2 well below v^2 for a wide set of $\tan\beta$ ($\tan\beta \ll m_t(M_t)/m_b(M_t)$) and $\Lambda \gtrsim 10$ TeV. Then the pseudoscalar mass has to be substantially larger than v in order to suppress the branching ratio $B \to X_s\gamma$. As a consequence the masses of the heaviest scalar, pseudoscalar and charged Higgs bosons are confined around m_A while the lightest CP-even Higgs state has a mass of the order of the electroweak scale, i.e.:

$$m_h^2 \simeq M_{11}^2 - \frac{M_{12}^4}{m_A^2} + O\left(\frac{\lambda_i^2 v^4}{m_A^4}\right). \tag{28}$$

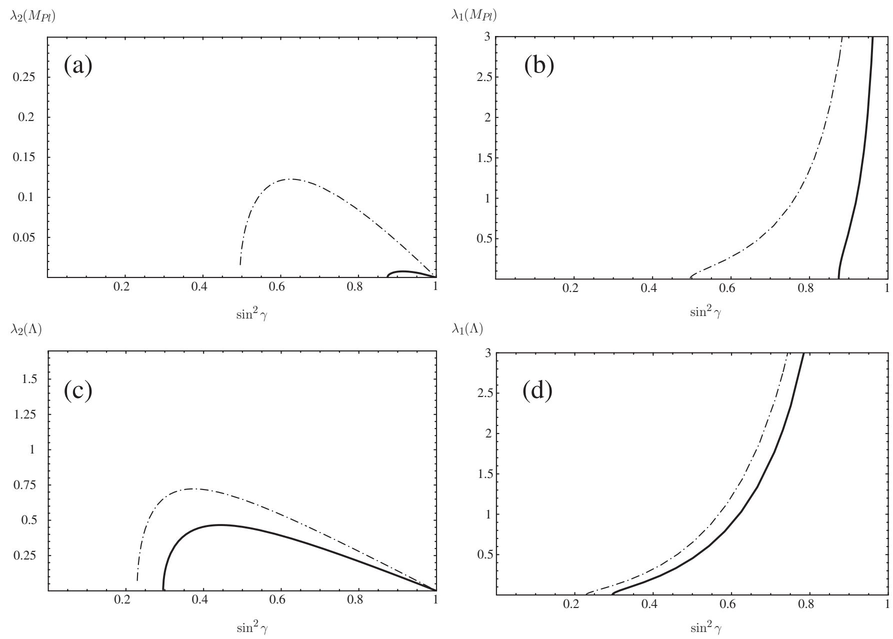

FIG. 2. The MPP bounds on (a) $\lambda_2(\Lambda)$ and (b) $\lambda_1(\Lambda)$, for $\Lambda = M_{Pl}$, as a function of $\sin^2\gamma$. The corresponding bounds on (c) $\lambda_2(\Lambda)$ and (d) $\lambda_1(\Lambda)$ for $\Lambda = 10$ TeV are also shown. The solid and dash-dotted curves represent the limits on the Higgs self-couplings for $\tan\beta = 10$ and $\tan\beta = 2$ respectively. The allowed range of the Higgs self-couplings lies below the curves.

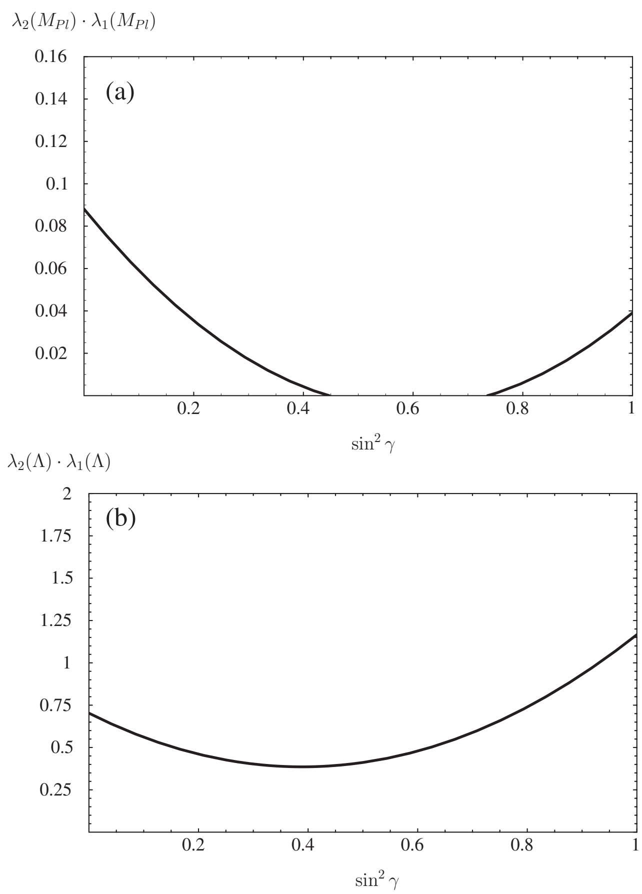

FIG. 3. The MPP restrictions on $\lambda_1(\Lambda) \cdot \lambda_2(\Lambda)$ for $\tan\beta = 50$ versus $\sin^2\gamma$ for (a) $\Lambda = M_{Pl}$ and (b) $\Lambda = 10$ TeV. The allowed part of the parameter space lies below the curves.

The results of the numerical analysis given in Table I indicate that, for a wide range of $\tan\beta$ ($2 \lesssim \tan\beta \ll 50$), the lightest Higgs boson mass is less than 180 GeV for any reasonable choice of the scale $\Lambda \gtrsim 10$ TeV. Furthermore, because of the large splitting among the Higgs boson masses, the lightest Higgs scalar has the same couplings to fermions, W and Z-bosons as the Higgs particle in the SM.

In order to illustrate how the MPP requires a value for m_h around the top-quark mass, let us assume that the MPP conditions (19) are fulfilled at the electroweak scale and $\lambda_1(\Lambda) = \lambda_2(\Lambda) = \lambda_0$. Then in the decoupling limit, when $m_A \gg \lambda_i v$, the lightest Higgs scalar mass is given by

$$m_h^2 \simeq \lambda_0 v^2 \cos^2 2\beta. \tag{29}$$

From Eq. (29) and the results of the numerical studies given in Table I, it becomes clear that the mass of the lightest Higgs particle grows with increasing λ_0 and $\tan\beta$. The allowed range of the Higgs self-couplings is constrained by the MPP assumption. In the interval $1 \ll \tan\beta \ll m_t(M_t)/m_b(M_t)$ the MPP constraint (21) implies that $\lambda_0^2 \leq \frac{3}{4}(h_t^4 - \frac{3}{4}g_2^4 - \frac{1}{2}g_2^2 g_1^2 - \frac{1}{4}g_1^4)$. As a result we get an upper limit on the mass of the lightest Higgs boson

$$m_h^2 \lesssim \sqrt{3(m_t^4 - (2\cos^4\theta_W + 1)M_Z^4)} \simeq \sqrt{3}m_t^2, \tag{30}$$

which is set by the top-quark mass. Here θ_W is the Weinberg angle.

A remarkable prediction for the mass of the SM-like Higgs boson appears if the MPP scale Λ is taken to a very high energy. For large $\tan\beta$, the admissible region of the

Higgs self-couplings shrinks drastically [see Fig. 2(a)] and only small enough values of $\lambda_2(\Lambda)$, $\lambda_3(\Lambda)$ and $\lambda_4(\Lambda)$ are allowed. Therefore the running of λ_2 resembles the renormalization group flow of λ in the SM with $\lambda(\Lambda) = 0$. Since, at large $\tan\beta$, the lightest Higgs boson mass is predominantly determined by the term proportional to $\lambda_2 v^2$ in the top-left entry of the CP-even Higgs mass matrix (25), $\lambda_1(\Lambda)$ affects m_h only marginally and the MPP prediction for the Higgs mass in the SM is almost reproduced when Λ is close to the Planck scale (see Table I and II). The main ambiguity in the calculation of the SM-like Higgs boson mass is related to the uncertainty of the top-quark mass measurements.[4] When the running top-quark mass changes from 165 GeV to 170 GeV m_h increases by 10 GeV (see Table II). The mass of the lightest Higgs scalar is less sensitive to the choice of the scale μ down to which the 2HDM renormalization group equations are assumed valid. It leads to only a 6 GeV uncertainty. Ultimately, for $\Lambda = M_{Pl}$ and relatively large values of $\tan\beta$, we find:

$$m_h = 137 \pm 12 \text{ GeV}. \tag{31}$$

The range of variation in the mass of the SM-like Higgs boson enlarges at moderate and very large values of $\tan\beta$. At $\tan\beta \simeq m_t(M_t)/m_b(M_t)$ the mass of the lightest Higgs scalar increases because the strict upper limit on $\lambda_2(\Lambda)$ is loosened, due to the large contribution to the beta-functions of the Higgs self-couplings coming from the loops containing the b-quark and the τ-lepton. Now the prediction (31) represents the lower bound on m_h for $\Lambda = M_{Pl}$, while the restriction on the lightest Higgs scalar mass from above tends to the upper bound on m_h in the SM—180 GeV. At moderate values of $\tan\beta$ the mass of the lightest Higgs particle decreases. As a result, at $\tan\beta = 2$, one can easily get $m_h = 114$ GeV without any modification of the MPP, such as that suggested for the SM in [25].

With a lowering of the MPP scale, the allowed range of m_h expands. At $\tan\beta = 50$ and $\Lambda = 10$ TeV the SM-like Higgs boson mass can be even as heavy as 300 GeV if $\lambda_2(\Lambda) \gg \lambda_1(\Lambda)$ (see Table I). However if $\tan\beta$ is not so large ($\tan\beta \ll m_t(M_t)/m_b(M_t)$), m_h remains lower than 180 GeV because of the stringent limit on $\lambda_2(\Lambda)$ [see Fig. 2(c)]. The upper bound on m_h is even stronger for moderate $\tan\beta$, where a considerable part of the parameter space is excluded by the unsuccesful Higgs searches at LEP.

VI. CONCLUSIONS

We have constructed a new minimal nonsupersymmetric two Higgs doublet extension of the SM by applying the MPP assumption to the SUSY inspired 2HDM. According to the MPP, λ_5 vanishes, preserving CP-invariance. Four other Higgs self-couplings obey two relationships (19) at some scale $\Lambda \gg v$ leading to the largest possible set of degenerate vacua in the SUSY inspired 2HDM. Usually the existence of a large set of degenerate vacua is associated with an enlarged global symmetry of the Lagrangian. The 2HDM is not an exception. When m_3^2, λ_5, λ_6 and λ_7 in the Higgs effective potential (4) vanish, the Lagrangian of the 2HDM is invariant under the transformations of an $SU(2) \times [U(1)]^2$ global symmetry. The additional $U(1)$ symmetry is nothing else than the Peccei-Quinn symmetry introduced to solve the strong CP problem in QCD [26]. The mixing term $m_3^2(H_1^\dagger H_2)$ in the effective potential (4), which is not forbidden by the MPP, breaks the extra $U(1)$ global symmetry softly so that a pseudo-Goldstone boson (the axion) does not appear in the particle spectrum. At high energies, where the contribution of the mixing term $m_3^2(H_1^\dagger H_2)$ can be safely ignored, the invariance under the Peccei-Quinn symmetry is restored giving rise to the set of degenerate vacua (18). The MPP predictions for λ_i (19)–(21) can be tested when the masses and couplings of the Higgs bosons are measured at the future colliders.

In the large $\tan\beta$ limit, when 2HDM approaches the SM, the allowed range of the Higgs self-couplings is severely constrained by the MPP conditions (19) and vacuum stability requirements (12). As a consequence, for most of the large $\tan\beta$ ($\tan\beta \gtrsim 2$) region the Higgs spectrum exhibits a hierarchical structure. While the heavy scalar, pseudoscalar and charged Higgs particles are nearly degenerate around m_A, and the latter should be greater than 350 GeV owing to the stringent limit on the $m_{\chi^\pm}$ (23) coming from the searches of the rare B-meson decays, the mass of the SM-like Higgs boson m_h does not exceed 180 GeV for any scale $\Lambda \gtrsim 10$ TeV. The theoretical bound on m_h obtained here is quite stringent. For comparison the lightest Higgs boson in the 2HDM with a softly broken Z_2 symmetry can be even heavier than 400 GeV [27] for $\Lambda \simeq 10$ TeV. So a fairly stringent constraint on m_h arises from the application of the MPP to the 2HDM of type II.

The bounds on m_h become even stronger if the MPP conditions are realized at high energies. In this case the MPP prediction for the Higgs mass obtained in the SM is reproduced. But, in contrast to the SM, lower values of $m_h \simeq 115$ GeV may be easily obtained in the MPP inspired 2HDM when $\tan\beta$ approaches 2. The restrictions on the Higgs self-couplings and the mass of the lightest Higgs particle are less stringent at very large values of $\tan\beta \simeq m_t(M_t)/m_b(M_t)$.

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[4]Unlike in the SM, the top-quark mass is not predicted by MPP in the 2HDM of type II

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