# Perturbative renormalization factors for generic $\Delta S = 2$ four-quark operators in domain-wall QCD with improved gauge action

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We calculate one-loop renormalization factors of generic $\Delta S = 2$ four-quark operators for domain-wall QCD with the plaquette gauge action and the Iwasaki gauge action. The renormalization factors are presented in the modified minimal subtraction ($\overline{\rm MS}$) scheme with the naive dimensional regularization. As an important application we show how to construct the renormalization factors for the operators contributing to $K^0 - \bar{K}^0$ mixing in the supersymmetric models with the use of our results.

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## I. INTRODUCTION

Experimental studies of $K^0 - \bar{K}^0$ mixing provide us an opportunity to deduce the indirect $CP$ violation parameter $\epsilon_K$. In the standard model (SM), the low-energy effective Hamiltonian contains the dimension-six four-quark operator $\mathcal{O}_{LL} = \bar{s}\gamma_\mu(1-\gamma_5)d \cdot \bar{s}\gamma_\mu(1-\gamma_5)d$, and its hadronic matrix element $\langle \bar{K}^0|\mathcal{O}_{LL}|K^0\rangle$ is required to determine $\epsilon_K$ from the experimental results of $K^0 - \bar{K}^0$ mixing. On the other hand, the physics beyond the SM involves the four-quark operators with more general chiral structures. For example, the relevant operators in supersymmetric models are [1]

$$\mathcal{O}_1 = \bar{s}^a\gamma_\mu(1-\gamma_5)d^a\bar{s}^b\gamma_\mu(1-\gamma_5)d^b, \tag{1}$$

$$\mathcal{O}_2 = \bar{s}^a(1-\gamma_5)d^a\bar{s}^b(1-\gamma_5)d^b, \tag{2}$$

$$\mathcal{O}_3 = \bar{s}^a(1-\gamma_5)d^b\bar{s}^b(1-\gamma_5)d^a, \tag{3}$$

$$\mathcal{O}_4 = \bar{s}^a(1-\gamma_5)d^a\bar{s}^b(1+\gamma_5)d^b, \tag{4}$$

$$\mathcal{O}_5 = \bar{s}^a(1-\gamma_5)d^b\bar{s}^b(1+\gamma_5)d^a, \tag{5}$$

where $a$ and $b$ are color indices.

Lattice QCD should be an ideal tool to determine the above matrix elements from the first principles. In the past decades, much effort have been devoted to the calculation of $\langle \bar{K}^0|\mathcal{O}_{LL}|K^0\rangle$ by employing various quark and gauge actions [2–11]. On the contrary, the matrix elements for the operators of Eqs. (1)–(5) are less studied compared to the case of $\mathcal{O}_{LL}$ [12,13]. Our aim is at a detailed study of them with the use of the domain-wall quark formulation in lattice QCD, which is expected to realize full chiral symmetry at finite lattice spacing up to exponential fall-off of the explicit chiral symmetry breaking contributions [14,15] and have been successfully applied to the calculation of $\langle \bar{K}^0|\mathcal{O}_{LL}|K^0\rangle$ [6–8].

The matrix elements calculated on the lattice should be converted to those defined in some continuum regularization scheme [e.g., the modified minimal subtraction scheme ($\overline{\rm MS}$)]. This is achieved by the finite renormalization relating the lattice composite operators to the continuum counterparts defined in some regularization scheme. In this paper we present the perturbative results of the renormalization factors for the complete set of $\Delta S = 2$ four-quark operators consisting of physical quark fields in the domain-wall QCD (DWQCD). With the use of our results one can obtain the renormalization factor for an arbitrary $\Delta S = 2$ four-quark operator. This work is an extention of Refs. [16,17], where the renormalization factors for the four-quark operators relevant in the SM are evaluated.

This paper is organized as follows. In Sec. II we introduce the quark and gauge actions and the corresponding Feynman rules. Section III is devoted to explain our calculational procedure of the renormalization factors for the complete set of $\Delta S = 2$ four-quark operators. In Sec. IV, with the use of our results, we construct the renormalization factors for the SUSY operators in Eqs. (1)–(5). We briefly discuss the mean-field improvement in Sec. V. Our conclusions are summarized in Sec. VI.

The physical quantities are expressed in lattice units and the lattice spacing $a$ is suppressed unless necessary. We take SU($N$) gauge group with the gauge coupling $g$ and the second Casimir $C_F = (N^2-1)/(2N)$, while $N = 3$ is specified in the numerical calculations.

## II. ACTION AND FEYNMAN RULES

We take Shamir's domain-wall fermion action [14] given by

$$\begin{aligned} S_{\rm DW} = \sum_n \sum_{s=1}^{N_s}\Big[ & \frac{1}{2}\sum_\mu [\bar{\psi}(n)_s(-r+\gamma_\mu)U_\mu(n)\psi(n+\mu)_s \\ & + \bar{\psi}(n)_s(-r-\gamma_\mu)U_\mu^\dagger(n-\mu)\psi(n-\mu)_s] \\ & + \frac{1}{2}[\bar{\psi}(n)_s(1+\gamma_5)\psi(n)_{s+1} + \bar{\psi}(n)_s(1-\gamma_5)\psi(n)_{s-1}] \\ & + (M-1+4r)\bar{\psi}(n)_s\psi(n)_s\Big] \\ & + m\sum_n [\bar{\psi}(n)_{N_s}P_R\psi(n)_1 + \bar{\psi}(n)_1 P_L\psi(n)_{N_s}], \end{aligned} \tag{6}$$

 

where $n$ is a four-dimensional space-time coordinate and $s$ is a fifth-dimensional or "flavor" index bounded as $1 \leq s \leq N_s$. In this paper we conventionally take $N_s \to \infty$ limit to avoid complications arising from the finite $N_s$ such as mixing of operators with different chiralities. The domain-wall height $M$ is a parameter of the theory which we set $0 < M < 2$ to realize the massless fermion at tree level. $P_{R/L}$ is a projection operator $P_{R/L} = (1 \pm \gamma_5)/2$ and the Wilson parameter is set to $r = -1$. The "physical" quark field are defined by the boundary fermions in the fifth-dimensional space:

$$q(n) = P_R \psi(n)_1 + P_L \psi(n)_{N_s}, \tag{7}$$

$$\bar{q}(n) = \bar{\psi}(n)_{N_s} P_R + \bar{\psi}(n)_1 P_L, \tag{8}$$

whose mass is given by $m$. Our renormalization procedure is based on the Green functions consisting of only the physical quark fields.

For the gauge part of the action we employ the following form in four dimensions:

$$S_{\mathrm{gluon}} = \frac{1}{g^2}\Bigg[c_0 \sum_{\mathrm{plaquette}} \mathrm{Tr} U_{\mathrm{pl}} + c_1 \sum_{\mathrm{rectangle}} \mathrm{Tr} U_{\mathrm{rtg}} + c_2 \sum_{\mathrm{chair}} \mathrm{Tr} U_{\mathrm{chr}} + c_3 \sum_{\mathrm{parallelogram}} \mathrm{Tr} U_{\mathrm{plg}}\Bigg], \tag{9}$$

where the first term represents the standard plaquette action and the remaining terms are six-link loops formed by a $1 \times 2$ rectangle, a bent $1 \times 2$ rectangle (chair) and a 3-dimensional parallelogram. The coefficients $c_0, \ldots, c_3$ satisfy the normalization condition

$$c_0 + 8c_1 + 16c_2 + 8c_3 = 1. \tag{10}$$

The renormalization group (RG) improved gauge action is defined by setting the parameters to the value suggested by an approximate renormalization group analysis. In the following we adopt $c_1 = -0.331$, $c_2 = c_3 = 0$ (Iwasaki) [18] for the RG improved gauge action in addition to $c_1 = c_2 = c_3 = 0$ (plaquette). With these choices of parameters the RG improved gauge action is expected to realize smooth gauge field fluctuations approximating those in the continuum limit better than with the unimproved plaquette action.

Weak coupling perturbation theory is developed by expressing the link variable in terms of the gauge potential

$$U_{x,\mu} = \exp\left(igA_\mu\left(x + \tfrac{1}{2}\hat{\mu}\right)\right) \tag{11}$$

and expanding in terms of the gauge coupling. We adopt a covariant gauge fixing with a gauge parameter $\alpha$ defined by

$$S_{\mathrm{GF}} = \sum_x \frac{1}{2\alpha}\left(\nabla_\mu A^a_\mu\left(x + \frac{1}{2}\hat{\mu}\right)\right)^2, \tag{12}$$

where $\nabla_\mu f_n \equiv (f_{n+\hat{\mu}} - f_n)$.

The free part of the gluon action takes the form in momentum space

$$S_0 = \frac{1}{2}\int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4} \sum_{\mu,\nu} A^a_\mu(k)\left(G_{\mu\nu}(k) - \left(1 - \frac{1}{\alpha}\right)\hat{k}_\mu \hat{k}_\nu\right) A^a_\nu(-k), \tag{13}$$

where

$$G_{\mu\nu}(k) = \hat{k}_\mu \hat{k}_\nu + \sum_\rho (\hat{k}_\rho \delta_{\mu\nu} - \hat{k}_\mu \delta_{\rho\nu}) q_{\mu\rho} \hat{k}_\rho \tag{14}$$

with

$$\hat{k}_\mu = 2\sin\frac{k_\mu}{2} \tag{15}$$

and $q_{\mu\nu}$ is defined as

$$q_{\mu\nu} = (1 - \delta_{\mu\nu})(1 - (c_1 - c_2 - c_3)(\hat{k}^2_\mu + \hat{k}^2_\nu) - (c_2 + c_3)\hat{k}^2). \tag{16}$$

The gluon propagator can be written as

$$D_{\mu\nu}(k) = (\hat{k}^2)^{-2}\Big[\hat{k}_\mu \hat{k}_\nu + \sum_\sigma (\hat{k}_\sigma \delta_{\mu\nu} - \hat{k}_\nu \delta_{\mu\sigma}) \hat{k}_\sigma A_{\sigma\nu}\Big] - (1 - \alpha)\frac{\hat{k}_\mu \hat{k}_\nu}{(\hat{k}^2)^2} \tag{17}$$

$$= (\hat{k}^2)^{-2}\Big[(1 - A_{\mu\nu})\hat{k}_\mu \hat{k}_\nu + \delta_{\mu\nu}\sum_\sigma \hat{k}^2_\sigma A_{\nu\sigma}\Big] - (1 - \alpha)\frac{\hat{k}_\mu \hat{k}_\nu}{(\hat{k}^2)^2}, \tag{18}$$

where $A_{\mu\nu}$ is a function of $q_{\mu\nu}$ and $\hat{k}_\mu$, whose form we refer to the original literatures [18,19]. In this paper we will adopt the Feynman gauge ($\alpha = 1$) without loss of generality, since the renormalization factors for the composite operators do not depend on the choice of the gauge fixing condition.

Quark-gluon vertices are also identical to those in the $N_s$ flavor Wilson fermion. We need only one gluon vertex for our present calculation:

$$V^A_{1\mu}(k, p)_{st} = -igT^A\{\gamma_\mu \cos(-k_\mu/2 + p_\mu/2) - ir\sin(-k_\mu/2 + p_\mu/2)\}\delta_{st}, \tag{19}$$

where $k$ and $p$ represent incoming momentum into the vertex (see Fig. 1 of Ref. [20]) and $T^A (A = 1, \ldots, N^2 - 1)$ is a generator of color SU($N$).

The fermion propagator originally takes $N_s \times N_s$ matrix form in $s$-flavor space. In the present one-loop calculation, however, we do not need the whole matrix elements because we consider Green functions consisting of the physical quark fields. The relevant fermion propagators are restricted to the following three types:

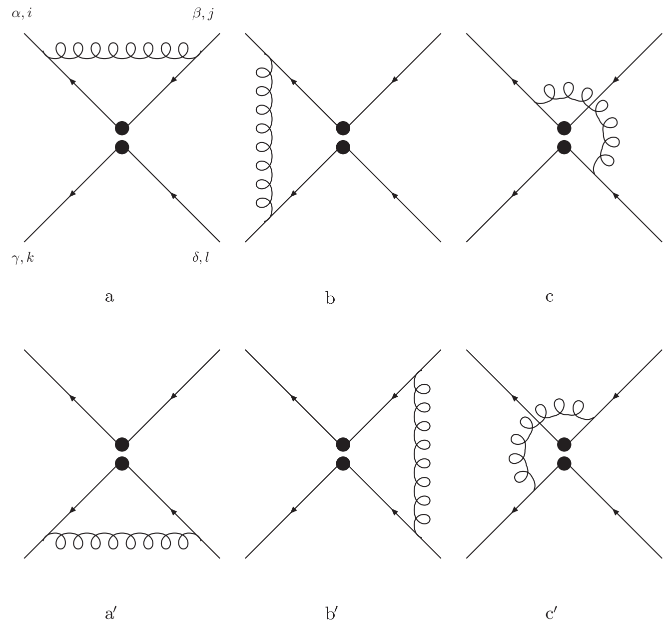


FIG. 1. One-loop vertex corrections for the four-quark operator. $\alpha$, $\beta$, $\gamma$, $\delta$ and $i$, $j$, $k$, $l$ label Dirac and color indices, respectively.

$$\langle q(-p)\bar{q}(p)\rangle = \frac{-i\gamma_\mu \sin p_\mu + (1 - We^{-\alpha})m}{-(1 - e^{\alpha}W) + m^2(1 - We^{-\alpha})} \equiv S_q(p), \tag{20}$$

$$\begin{aligned}\langle q(-p)\bar{\psi}(p,s)\rangle = {}& \frac{1}{F}[i\gamma_\mu \sin p_\mu - m(1 - We^{-\alpha})] \\ &\times (e^{-\alpha(N_s - s)}P_R + e^{-\alpha(s-1)}P_L) \\ &+ \frac{1}{F}[m[i\gamma_\mu \sin p_\mu - m(1 - We^{-\alpha})] - F] \\ &\times e^{-\alpha}(e^{-\alpha(s-1)}P_R + e^{-\alpha(N_s - s)}P_L),\end{aligned} \tag{21}$$

$$\begin{aligned}\langle \psi(-p,s)\bar{q}(p)\rangle = {}& \frac{1}{F}(e^{-\alpha(N_s - s)}P_L + e^{-\alpha(s-1)}P_R) \\ &\times [i\gamma_\mu \sin p_\mu - m(1 - We^{-\alpha})] \\ &+ \frac{1}{F}(e^{-\alpha(s-1)}P_L + e^{-\alpha(N_s - s)}P_R) \\ &\times e^{-\alpha}[m[i\gamma_\mu \sin p_\mu - m(1 - We^{-\alpha})] - F]\end{aligned} \tag{22}$$

with

$$W = 1 - M - r\sum_\mu (1 - \cos p_\mu), \tag{23}$$

$$\cosh(\alpha) = \frac{1 + W^2 + \sum_\mu \sin^2 p_\mu}{2|W|}, \tag{24}$$

$$F = 1 - e^{\alpha}W - m^2(1 - We^{-\alpha}), \tag{25}$$

where the argument $p$ in the factors $\alpha$ and $W$ is suppressed.

In the perturbative calculation of Green functions we assume that the external quark momenta and masses are much smaller than the lattice cutoff, so that the external quark propagators can be expanded in terms of them. We have the following expressions as leading term of the expansion:

$$\langle q\bar{q}\rangle_{\rm ext}(p) = \frac{1 - w_0^2}{i\not{p} + (1 - w_0^2)m}, \tag{26}$$

$$\langle q\bar{\psi}_s\rangle_{\rm ext}(p) = \langle q\bar{q}\rangle(p)(w_0^{s-1}P_L + w_0^{N_s - s}P_R), \tag{27}$$

$$\langle \psi_s\bar{q}\rangle_{\rm ext}(p) = (w_0^{s-1}P_R + w_0^{N_s - s}P_L)\langle q\bar{q}\rangle(p), \tag{28}$$

where $w_0 = 1 - M$.

## III. RENORMALIZATION FACTORS FOR GENERIC FOUR-QUARK OPERATORS

We consider the complete set of parity-conserving four-quark operators [4,21]:

$$Q_1^{\pm} \equiv \mathcal{O}_{VV+AA}^{\pm} \equiv \mathcal{O}_{VV}^{\pm} + \mathcal{O}_{AA}^{\pm}, \tag{29}$$

$$Q_2^{\pm} \equiv \mathcal{O}_{VV-AA}^{\pm} \equiv \mathcal{O}_{VV}^{\pm} - \mathcal{O}_{AA}^{\pm}, \tag{30}$$

$$Q_3^{\pm} \equiv \mathcal{O}_{SS-PP}^{\pm} \equiv \mathcal{O}_{SS}^{\pm} - \mathcal{O}_{PP}^{\pm}, \tag{31}$$

$$Q_4^{\pm} \equiv \mathcal{O}_{SS+PP}^{\pm} \equiv \mathcal{O}_{SS}^{\pm} + \mathcal{O}_{PP}^{\pm}, \tag{32}$$

$$Q_5^{\pm} \equiv \mathcal{O}_{TT}^{\pm}, \tag{33}$$

where

$$\mathcal{O}_{\Gamma\Gamma}^{\pm} = \tfrac{1}{2}[(\bar{q}_1^a\Gamma q_2^a)(\bar{q}_3^b\Gamma q_4^b) \pm (\bar{q}_1^a\Gamma q_4^a)(\bar{q}_3^b\Gamma q_2^b)] \tag{34}$$

with $\Gamma = \{\mathbf{1}, \gamma_\mu, \sigma_{\mu\nu}, \gamma_\mu\gamma_5, \gamma_5\} \equiv \{S, V, T, A, P\}$. $a$, $b$ denote color indices and summation over them is assumed. We should note that $q_i$ ($i = 1, 2, 3, 4$) are the physical quark fields defined by Eqs. (7) and (8). The parity-conserving parts of Eqs. (1)–(5) are expressed as linear combinations of the above operators. We find that Ref. [4] employs another choice of basis called Fierz eigenbasis, which is more convenient in considering the Fierz transformation of the four-quark operators.

We consider the following Green functions:

$$\langle Q_i^{\pm}\rangle_{\alpha\beta,\gamma\delta}^{ij,kl} \equiv \langle Q_i^{\pm}(q_1)_\alpha^i(\bar{q}_2)_\beta^j(q_3)_\gamma^k(\bar{q}_4)_\delta^l\rangle, \tag{35}$$

where $\alpha$, $\beta$, $\gamma$, $\delta$ are spinor indices and $i$, $j$, $k$, $l$ are color indices. It is convenient to decompose the above Green functions as

$$\begin{aligned}\langle \mathcal{O}^{\pm}_{\Gamma\Gamma}\rangle^{ij,kl}_{\alpha\beta,\gamma\delta} &\equiv \langle \mathcal{O}^{\pm}_{\Gamma\Gamma}(q_1)^i_\alpha (q_2)^j_\beta (q_3)^k_\gamma (q_4)^l_\delta\rangle \\ &= \tfrac{1}{2}\langle[(\bar{q}^a_1\Gamma q^a_2)(\bar{q}^b_3\Gamma q^b_4) \pm (\bar{q}^a_1\Gamma q^a_4)(\bar{q}^b_3\Gamma q^b_2)] \\ &\quad \times (q_1)^i_\alpha (q_2)^j_\beta (q_3)^k_\gamma (q_4)^l_\delta\rangle \\ &= \tfrac{1}{2}[\langle (q_1)^i_\alpha \bar{q}^a_1\Gamma q^a_2(\bar{q}_2)^j_\beta (q_3)^k_\gamma \bar{q}^b_3\Gamma q^b_4(\bar{q}_4)^l_\delta\rangle \\ &\quad \mp \langle (q_1)^i_\alpha \bar{q}^a_1\Gamma q^a_4(\bar{q}_4)^l_\delta (q_3)^k_\gamma \bar{q}^b_3\Gamma q^b_2(\bar{q}_2)^j_\beta\rangle]. \end{aligned} \tag{36}$$

After truncating the external quark propagators from $\langle Q^{\pm}_i\rangle$ with the multiplication of $i\not{p} + (1-w_0^2)m$, we obtain the vertex functions, which is written in the following form up to the one-loop level:

$$(1-w_0^2)^4(\Lambda^{\pm}_i)^{ij;kl}_{\alpha\beta;\gamma\delta} = (1-w_0^2)^4(\Lambda^{(0)\pm}_i + \Lambda^{(1)\pm}_i)^{ij;kl}_{\alpha\beta;\gamma\delta}, \tag{37}$$

where the superscript $(n)$ refers to the $n$th loop level and the subscript $i$ identifies the operators (29)–(33). The trivial factor $(1-w_0^2)^4$, which originates from the external quark propagators, is factored out for convenience. Since the renormalization factor does not depend on the external momenta $p_i$, we suppress them.

The tree-level vertex functions $\Lambda^{(0)\pm}_i$ are given by

$i=1$,

$$\tfrac{1}{2}[V\otimes V + A\otimes A \mp (V\odot V + A\odot A)]_{\alpha\beta;\gamma\delta}[1\tilde{\otimes}1]^{ij;kl}, \tag{38}$$

$i=2$,

$$\tfrac{1}{2}[V\otimes V - A\otimes A \mp (V\odot V - A\odot A)]_{\alpha\beta;\gamma\delta}[1\tilde{\otimes}1]^{ij;kl}, \tag{39}$$

$i=3$,

$$\tfrac{1}{2}[S\otimes S - P\otimes P \mp (S\odot S - P\odot P)]_{\alpha\beta;\gamma\delta}[1\tilde{\otimes}1]^{ij;kl}, \tag{40}$$

$i=4$,

$$\tfrac{1}{2}[S\otimes S + P\otimes P \mp (S\odot S + P\odot P)]_{\alpha\beta;\gamma\delta}[1\tilde{\otimes}1]^{ij;kl}, \tag{41}$$

$$i=5, \qquad \tfrac{1}{2}[T\otimes T \mp T\odot T]_{\alpha\beta;\gamma\delta}[1\tilde{\otimes}1]^{ij;kl}, \tag{42}$$

where $\otimes$, $\odot$ act on the Dirac spinor space as $[\Gamma\otimes\Gamma]_{\alpha\beta;\gamma\delta} \equiv (\Gamma)_{\alpha\beta}(\Gamma)_{\gamma\delta}$, $[\Gamma\odot\Gamma]_{\alpha\beta;\gamma\delta} \equiv (\Gamma)_{\alpha\delta}(\Gamma)_{\gamma\beta}$, and $\tilde{\otimes}$, $\tilde{\odot}$ on the color space as $[1\tilde{\otimes}1]^{ij;kl} \equiv \delta_{ij}\delta_{kl}$, $[1\tilde{\odot}1]^{ij;kl} \equiv \delta_{il}\delta_{kj}$.

Now let us consider the one-loop vertex corrections depicted by six diagrams in Fig. 1. Their total contribution yields the vertex function at one-loop level:

$i=1$,

$$\Lambda^{(1)\pm}_1 = \int_{-\pi}^{\pi}\frac{d^4k}{(2\pi)^4}[(I^a_{VV} + I^a_{AA}) +, \ldots, +(I^{c\prime}_{VV} + I^{c\prime}_{AA})], \tag{43}$$

$i=2$,

$$\Lambda^{(1)\pm}_2 = \int_{-\pi}^{\pi}\frac{d^4k}{(2\pi)^4}[(I^a_{VV} - I^a_{AA}) +, \ldots, +(I^{c\prime}_{VV} - I^{c\prime}_{AA})], \tag{44}$$

$i=3$,

$$\Lambda^{(1)\pm}_3 = \int_{-\pi}^{\pi}\frac{d^4k}{(2\pi)^4}[(I^a_{SS} - I^a_{PP}) +, \ldots, +(I^{c\prime}_{SS} - I^{c\prime}_{PP})], \tag{45}$$

$i=4$,

$$\Lambda^{(1)\pm}_4 = \int_{-\pi}^{\pi}\frac{d^4k}{(2\pi)^4}[(I^a_{SS} + I^a_{PP}) +, \ldots, +(I^{c\prime}_{SS} + I^{c\prime}_{PP})], \tag{46}$$

$$i=5, \qquad \Lambda^{(1)\pm}_5 = \int_{-\pi}^{\pi}\frac{d^4k}{(2\pi)^4}[I^a_{TT} +, \ldots, +I^{c\prime}_{TT}], \tag{47}$$

where

$$\begin{aligned} I^a_{\Gamma\Gamma} &= \frac{1}{2}g^2(T^AT^A\tilde{\otimes}1)K \\ &\quad\times\Bigg[\Bigg\{\gamma_\alpha\gamma_\beta\Gamma\gamma_\beta\gamma_\alpha\otimes\Gamma\Bigg(\frac{1}{4}A_{\alpha\beta}\sin^2k_\alpha\sin^2k_\beta \\ &\quad + \sum_\sigma A_{\alpha\sigma}\sin^2\frac{k_\sigma}{2}\cos^2\frac{k_\alpha}{2}\sin^2k_\beta\Bigg) \\ &\quad + \Gamma\otimes\Gamma\Bigg(4\Delta_3^2 - \frac{1}{4}\sum_\sigma\Delta^\alpha_{1,1} + \frac{1}{4}\sum_\alpha\sin^4k_\alpha + T\Bigg)\Bigg\} \\ &\quad \mp\{\otimes\leftrightarrow\odot\}\Bigg], \end{aligned} \tag{48}$$

$$\begin{aligned} I^b_{\Gamma\Gamma} &= \frac{1}{2}g^2(T^A\tilde{\otimes}T^A)K\Bigg[\Bigg\{\gamma_\alpha\gamma_\beta\Gamma\otimes\Gamma\gamma_\beta\gamma_\alpha\Bigg(\frac{1}{4}A_{\alpha\beta}\sin^2k_\alpha\sin^2k_\beta \\ &\quad + \sum_\sigma A_{\alpha\beta}\sin^2\frac{k_\sigma}{2}\cos^2\frac{k_\alpha}{2}\sin^2k_\beta\Bigg) \\ &\quad + \Gamma\otimes\Gamma\Bigg(4\Delta_3^2 - \frac{1}{4}\sum_\alpha\Delta^\alpha_{1,1} + \frac{1}{4}\sum_\sigma\sin^4k_\sigma + T\Bigg)\Bigg\} \\ &\quad \mp\{\otimes\leftrightarrow\odot\}\Bigg], \end{aligned} \tag{49}$$

$$I^c_{\Gamma\Gamma} = -\frac{1}{2}g^2(T^A\tilde{\otimes}T^A)K \times \Bigg[\Bigg\{\gamma_\alpha\gamma_\beta\Gamma\otimes\gamma_\alpha\gamma_\beta\Gamma\Big(\frac{1}{4}A_{\alpha\beta}\sin^2 k_\alpha \sin^2 k_\beta + \sum_\sigma A_{\alpha\sigma}\sin^2\frac{k_\sigma}{2}\cos^2\frac{k_\alpha}{2}\sin^2 k_\beta\Big) + \Gamma\otimes\Gamma\Big(4\Delta_3^2 - \frac{1}{4}\sum_\sigma \Delta^\sigma_{1,1} + \frac{1}{4}\sum_\sigma \sin^2 k_\sigma + T\Big)\Bigg\} \mp \{\otimes \leftrightarrow \odot\}\Bigg] \tag{50}$$

with

$$K \equiv \frac{4}{(4\sum_\mu \sin^2\frac{k_\mu}{2})^2}\frac{1}{\tilde{F}_0^2\tilde{F}^2}, \tag{51}$$

$$\tilde{F}_0 \equiv e^\alpha - w_0, \tag{52}$$

$$\tilde{F} \equiv e^{-\alpha} - W, \tag{53}$$

$$T \equiv r^2\Delta_1^2\tilde{F}^2 + 4r\Delta_3\Delta_1\tilde{F}, \tag{54}$$

$$\Delta_3 = \frac{1}{4}\sum_\mu \sin^2 k_\mu, \tag{55}$$

$$\Delta_1 = \sum_\mu \sin^2\frac{k_\mu}{2}, \tag{56}$$

$$\Delta^\mu_{1,1} = \sum_\nu(\delta_{\mu\nu} + A_{\mu\nu})\sin^2 k_\mu \sin^2 k_\nu. \tag{57}$$

Summation over repeated indices is assumed. The above expressions are obtained with the aid of useful formula presented in Refs. [16,17,20]. We should note that the other three contributions $I^{a',b',c'}_{\Gamma\Gamma}$ from Figs. 1(a), 1(b), and 1(c), are equal to $I^{a,b,c}_{\Gamma\Gamma}$ respectively. After all, the total contribution becomes

$$(I^a_{VV} + I^a_{AA}) +, \ldots, + (I^{c\prime}_{VV} + I^{c\prime}_{AA}) = g^2[V\otimes V + A\otimes A \mp (V\odot V + A\odot A)]_{\alpha\beta:\gamma\delta} \times [1\tilde{\otimes}1]^{ij:kl}K\Big[\Big(\frac{1}{N}\mp 1\Big)A_{SP} + \Big(2C_F - \frac{1}{N}\pm 1\Big)A_{VA} + 2C_F T\Big], \tag{58}$$

$$(I^a_{VV} - I^a_{AA}) +, \ldots, + (I^{c\prime}_{VV} - I^{c\prime}_{AA}) = g^2[V\otimes V - A\otimes A \mp (V\odot V - A\odot A)]_{\alpha\beta:\gamma\delta} \times [1\tilde{\otimes}1]^{ij:kl}K\Big[-\frac{1}{N}A_{SP} + \Big(2C_F + \frac{1}{N}\Big)A_{VA} + 2C_F T\Big] + g^2[S\otimes S - P\otimes P \mp (S\odot S - P\odot P)]_{\alpha\beta:\gamma\delta} \times [1\tilde{\otimes}1]^{ij:kl}K[\mp 2A_{SP} \pm 2A_{VA}], \tag{59}$$

$$(I^a_{SS} - I^a_{PP}) +, \ldots, + (I^{c\prime}_{SS} - I^{c\prime}_{PP}) = g^2[S\otimes S - P\otimes P \mp (S\odot S - P\odot P)]_{\alpha\beta:\gamma\delta} \times [1\tilde{\otimes}1]^{ij:kl}K[2C_F A_{SP} + 2C_F T], \tag{60}$$

$$(I^a_{SS} + I^a_{PP}) +, \ldots, + (I^{c\prime}_{SS} + I^{c\prime}_{PP}) = g^2[S\otimes S + P\otimes P \mp (S\odot S + P\odot P)]_{\alpha\beta:\gamma\delta} \times [1\tilde{\otimes}1]^{ij:kl}K[(2C_F \mp 1)A_{SP} \pm A_{VA} + 2C_F T] + g^2(1 - w_0^2)^2\Big([T\otimes T \mp T\otimes T]_{\alpha\beta:\gamma\delta} \times [1\tilde{\otimes}1]^{ij:kl}K\Big[\Big(\frac{2}{3N}\mp\frac{1}{3}\Big)A_{SP} + \Big(-\frac{2}{3N}\pm\frac{1}{3}\Big)A_{VA}\Big)\Big], \tag{61}$$

$$I^a_{TT} +, \ldots, + I^{c\prime}_{TT} = g^2[T\otimes T \mp T\odot T]_{\alpha\beta:\gamma\delta} \times [1\tilde{\otimes}1]^{ij:kl}K\Big[\Big(-\frac{2C_F}{3}\mp 1\Big)A_{SP} + \Big(\frac{8C_F}{3}\pm 1\Big)A_{VA} + 2C_F T\Big] + g^2[S\otimes S + P\otimes P \mp (S\odot S + P\odot P)]_{\alpha\beta:\gamma\delta} \times [1\tilde{\otimes}1]^{ij:kl}K\Big(\frac{2}{N}\pm 1\Big)[A_{SP} - A_{VA}], \tag{62}$$

where we use the Fierz rearrangements:

$$[S\otimes S][1\tilde{\odot}1] = \tfrac{1}{4}[S\odot S + V\odot V - T\odot T - A\odot A + P\odot P][1\tilde{\otimes}1], \tag{63}$$

$$[V\otimes V][1\tilde{\odot}1] = \tfrac{1}{4}[4S\odot S - 2V\odot V - 2A\odot A - 4P\odot P][1\tilde{\otimes}1], \tag{64}$$

$$[T\otimes T][1\tilde{\odot}1] = \tfrac{1}{4}[-6S\odot S - 2T\odot T - 6P\odot P][1\tilde{\otimes}1], \tag{65}$$

$$[A\otimes A][1\tilde{\odot}1] = \tfrac{1}{4}[-4S\odot S - 2V\odot V - 2A\odot A + 4P\odot P][1\tilde{\otimes}1], \tag{66}$$

$$[P \otimes P][1\tilde{\odot}1] = \frac{1}{4}[S \odot S - V \odot V - T \odot T + A \odot A + P \odot P][1\tilde{\otimes}1]. \quad (67)$$

Note that these formula do not include Fermi statistics.

Comparing the one-loop results to the tree-level ones we obtain

$$\Lambda_1^{\pm} = \Big[1 + g^2 \frac{N \mp 1}{N}\{\mp\langle A_{SP}\rangle + (N \pm 2)\langle A_{VA}\rangle + (N \pm 1)\langle T\rangle\}\Big]\Lambda_1^{(0)\pm}, \quad (68)$$

$$\Lambda_2^{\pm} = \Big[1 + g^2\Big\{-\frac{1}{N}\langle A_{SP}\rangle + \Big(2C_F + \frac{1}{N}\Big)\langle A_{VA}\rangle + 2C_F\langle T\rangle\Big\}\Big]\Lambda_2^{(0)\pm} + g^2\{\mp 2\langle A_{SP}\rangle \pm 2\langle A_{VA}\rangle\}\Lambda_3^{(0)\pm}, \quad (69)$$

$$\Lambda_3^{\pm} = [1 + g^2 2C_F\{\langle A_{SP}\rangle + \langle T\rangle\}]\Lambda_3^{(0)\pm}, \quad (70)$$

$$\Lambda_4^{\pm} = \Big[1 + g^2 \frac{1}{N}\{(N^2 \mp N - 1)\langle A_{SP}\rangle \pm N\langle A_{VA}\rangle + (N^2 - 1)\langle T\rangle\}\Big]\Lambda_4^{(0)\pm} + g^2\frac{2 \mp N}{3N}\{\langle A_{SP}\rangle - \langle A_{VA}\rangle\}\Lambda_5^{(0)\pm}, \quad (71)$$

$$\Lambda_5^{\pm} = \Big[1 + g^2\Big\{\Big(-\frac{2C_F}{3} \mp 1\Big)\langle A_{SP}\rangle + \Big(\frac{8C_F}{3} \pm 1\Big)\langle A_{VA}\rangle + 2C_F\langle T\rangle\Big\}\Big]\Lambda_5^{(0)\pm} + g^2\Big(\frac{2}{N} \pm 1\Big)\{\langle A_{SP}\rangle - \langle A_{VA}\rangle\}\Lambda_4^{(0)\pm} \quad (72)$$

with

$$\langle X\rangle = \int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4} K(k)X(k) \quad (73)$$

for $X = T$, $A_{VA}$, $A_{SP}$. We remark that $C_F\langle T + A_{VA}\rangle$ and $C_F\langle T + A_{SP}\rangle$ correspond to the one-loop vertex corrections to the (axial) vector and the (pseudo) scalar density. $A_{VA}$ and $A_{SP}$ are expressed as follows:

$$A_{VA} = 4\Delta_3^2 - \Delta_{1,1}^{\mu} + 4s_{\mu}^2\Delta_{1,0}^{\mu}, \quad (74)$$

$$A_{SP} = 16\Delta_3\Delta_{1,0}^{\mu} \quad (75)$$

with

$$\Delta_{1,0}^{\mu} = \sum_{\nu}(\delta_{\mu\nu} + A_{\mu\nu})\cos^2\frac{k_{\mu}}{2}\sin^2\frac{k_{\nu}}{2}. \quad (76)$$

The expressions of Eqs. (68) and (70) show important properties of $Q_1$ and $Q_3$ in DWQCD formalism: the one-loop vertex corrections are multiplicative. This is contrary to the Wilson case, in which the mixing operators with different chiralities appears at the one-loop level. The results for $Q_1^{\pm}$ in Eq. (68) are already obtained in Ref. [16].

Combining the contribution from the quark self-energy evaluated in Ref. [20] and the vertex corrections, we obtain the lattice renormalization factors:

$$Z_{ij}^{\pm \mathrm{lat}} = (1 - w_0^2)^2 Z_w^2 Z_2^2 V_{ij}^{\pm}, \quad (77)$$

where

$$Z_2 = 1 + \frac{g^2}{16\pi^2}C_F[\log(\lambda a)^2 + \Sigma_1], \quad (78)$$

$$Z_w = 1 - \frac{2w_0}{1 - w_0^2}\frac{g^2 C_F}{16\pi^2}\Sigma_3, \quad (79)$$

$$V_{ij}^{\pm} = \delta_{ij} + \frac{g^2}{16\pi^2}[\gamma_{ij}^{\pm}\log(\lambda a)^2 + v_{ij}^{\pm}]. \quad (80)$$

$\lambda$ is the fictitious gluon mass introduced to regularize the infrared divergences. $\Sigma_1$ and $\Sigma_3$ are finite parts of the renormalization factors of quark wave function and overall factor $(1 - w_0^2)$, whose numerical value is given in Ref. [17]. The matrix $v_{ij}^{\pm}$ is expressed as

$$v_{ij}^{\pm} = \begin{pmatrix} v_{11}^{\pm} & 0 & 0 & 0 & 0 \\ 0 & v_{22}^{\pm} & v_{23}^{\pm} & 0 & 0 \\ 0 & 0 & v_{33}^{\pm} & 0 & 0 \\ 0 & 0 & 0 & v_{44}^{\pm} & v_{45}^{\pm} \\ 0 & 0 & 0 & v_{54}^{\pm} & v_{55}^{\pm} \end{pmatrix}, \quad (81)$$

whose components are

$$v_{11}^{+} = \frac{16\pi^2(N-1)}{N}[-\langle\langle A_{SP}\rangle\rangle + (N+2)\langle\langle A_{VA}\rangle\rangle + (N+1)\langle T\rangle] + \gamma_{11}^{+}\log\pi^2, \quad (82)$$

$$v_{11}^{-} = \frac{16\pi^2(N+1)}{N}[\langle\langle A_{SP}\rangle\rangle + (N-2)\langle\langle A_{VA}\rangle\rangle + (N-1)\langle T\rangle] + \gamma_{11}^{-}\log\pi^2, \quad (83)$$

$$v_{22}^{\pm} = \frac{16\pi^2}{N}[-\langle\langle A_{SP}\rangle\rangle + N^2\langle\langle A_{VA}\rangle\rangle + (N^2-1)\langle T\rangle] + \gamma_{22}^{\pm}\log\pi^2, \quad (84)$$

$$v_{23}^{\pm} = \mp 16\pi^2[2\langle\langle A_{SP}\rangle\rangle - 2\langle\langle A_{VA}\rangle\rangle] + \gamma_{23}^{\pm}\log\pi^2, \quad (85)$$

$$v_{33}^{\pm} = \frac{16\pi^2(N+1)(N-1)}{N}[\langle\langle A_{SP}\rangle\rangle + \langle T\rangle] + \gamma_{33}^{\pm}\log\pi^2, \quad (86)$$

$$v_{44}^{\pm} = \frac{16\pi^2}{N}[(N^2 \mp N - 1)\langle\langle A_{SP}\rangle\rangle \pm N\langle\langle A_{VA}\rangle\rangle + (N^2-1)\langle T\rangle] + \gamma_{44}^{\pm}\log\pi^2, \quad (87)$$

$$v_{45}^{\pm} = \frac{16\pi^2(2 \mp N)}{3N}[\langle\langle A_{SP}\rangle\rangle - \langle\langle A_{VA}\rangle\rangle] + \gamma_{45}^{\pm}\log\pi^2, \tag{88}$$

$$v_{55}^{\pm} = \frac{16\pi^2}{N}\left[\frac{-N^2 \mp 3N + 1}{3}\langle\langle A_{SP}\rangle\rangle + \frac{4N^2 \pm 3N - 4}{3} \times \langle\langle A_{VA}\rangle\rangle + (N^2 - 1)\langle T\rangle\right] + \gamma_{55}^{\pm}\log\pi^2, \tag{89}$$

$$v_{54}^{\pm} = \frac{16\pi^2(2 \pm N)}{N}[\langle\langle A_{SP}\rangle\rangle - \langle\langle A_{VA}\rangle\rangle] + \gamma_{54}^{\pm}\log\pi^2 \tag{90}$$

with

$$\gamma_{ij}^{\pm} = \begin{pmatrix} \gamma_{11}^{\pm} & 0 & 0 & 0 & 0 \\ 0 & \gamma_{22}^{\pm} & \gamma_{23}^{\pm} & 0 & 0 \\ 0 & 0 & \gamma_{33}^{\pm} & 0 & 0 \\ 0 & 0 & 0 & \gamma_{44}^{\pm} & \gamma_{45}^{\pm} \\ 0 & 0 & 0 & \gamma_{54}^{\pm} & \gamma_{55}^{\pm} \end{pmatrix} = \begin{pmatrix} \frac{(N\mp1)(N\mp2)}{N} & 0 & 0 & 0 & 0 \\ 0 & \frac{(N+2)(N-2)}{N} & \mp6 & 0 & 0 \\ 0 & 0 & 4\frac{(N+1)(N-1)}{N} & 0 & 0 \\ 0 & 0 & 0 & \frac{4N^2\mp3N-4}{N} & \frac{2\mp N}{N} \\ 0 & 0 & 0 & \frac{6\pm3N}{N} & \mp3 \end{pmatrix}. \tag{91}$$

The infrared singularity in $\langle A_X\rangle$ is subtracted as

$$\langle\langle A_X\rangle\rangle = \int_{-\pi}^{\pi}\frac{d^4k}{(2\pi)^4}\left[K(k)A_X(k) - c_X\frac{\theta(\pi^2 - k^2)}{(k^2)^2}\right], \tag{92}$$

where $c_{SP} = 4$ and $c_{VA} = 1$.

The lattice operators and the continuum ones defined in the $\overline{\rm MS}$ scheme with the naive dimensional regularization (NDR) are related as

$$Q_i^{\pm\overline{\rm MS}}(\mu) = \frac{1}{(1 - \omega_0^2)^2 Z_\omega^2} Z_{ij}^{\pm}(\mu a) Q_j^{\pm{\rm lat}}(1/a) \tag{93}$$

with

$$Z_{ij}^{\pm}(\mu a) \equiv \frac{(Z_2^{\overline{\rm MS}})^2}{Z_2^2}\frac{V_{ij}^{\pm\overline{\rm MS}}}{V_{ij}^{\pm}} = \delta_{ij} + \frac{g^2}{16\pi^2}[(\gamma_{ij} - 2C_F\delta_{ij})\log(\mu a)^2 + z_{ij}^{\pm}] \tag{94}$$

$$z_{ij}^{\pm} = v_{ij}^{\pm\overline{\rm MS}} - v_{ij}^{\pm} + \delta_{ij}2C_F(\Sigma_1^{\overline{\rm MS}} - \Sigma_1) \tag{95}$$

$$\Sigma_1^{\overline{\rm MS}}({\rm NDR}) = \frac{1}{2}, \qquad v_{11}^{+\overline{\rm MS}}({\rm NDR}) = -5, \qquad v_{11}^{-\overline{\rm MS}}({\rm NDR}) = 6 \tag{96}$$

$$v_{ij}^{\pm\overline{\rm MS}}({\rm NDR}) = \begin{pmatrix} v_{22}^{\pm\overline{\rm MS}} & v_{23}^{\pm\overline{\rm MS}} & 0 & 0 \\ v_{32}^{\pm\overline{\rm MS}} & v_{33}^{\pm\overline{\rm MS}} & 0 & 0 \\ 0 & 0 & v_{44}^{\pm\overline{\rm MS}} & v_{45}^{\pm\overline{\rm MS}} \\ 0 & 0 & v_{54}^{\pm\overline{\rm MS}} & v_{55}^{\pm\overline{\rm MS}} \end{pmatrix} = \begin{pmatrix} -\frac{7}{6} & -1 & 0 & 0 \\ \pm\frac{3}{2} & \frac{19}{3} & 0 & 0 \\ 0 & 0 & \frac{17}{3}\mp1 & \frac{1}{3}\mp1 \\ 0 & 0 & 1 & \frac{1}{3}\mp2 \end{pmatrix} \tag{97}$$

where $v_{ij}^{\pm\overline{\rm MS}}$ are for the case of $N = 3$. The continuum counterparts of the wave function renormalization factor and the vertex corrections are evaluated by employing the same gauge fixing condition and the same infrared regulator as the lattice case. Here it should be noted that the one-loop vertex corrections require to specify the evanescent operators, which originates from the property that the Fierz transformation cannot be defined in the NDR scheme [22]. We employ the following evanescent operators:

$$E_{VV}^{\rm NDR} = \gamma_\alpha\gamma_\beta V\otimes V\gamma_\alpha\gamma_\beta - \{(D^2 - 2D + 2)V \otimes V + 2(1 - D)A\otimes A\}, \tag{98}$$

$$E_{AA}^{\rm NDR} = \gamma_\alpha\gamma_\beta A\otimes A\gamma_\alpha\gamma_\beta - \{2(1 - D)V \otimes V + (D^2 - 2D + 2)A\otimes A\}, \tag{99}$$

$$E_{SS}^{\rm NDR} = \gamma_\alpha\gamma_\beta S\otimes S\gamma_\alpha\gamma_\beta - \{(D^2 - 5D + 8)S \otimes S + (4 - D)P\otimes P + (2 - D)T\otimes T\}, \tag{100}$$

$$E_{PP}^{\rm NDR} = \gamma_\alpha\gamma_\beta P\otimes P\gamma_\alpha\gamma_\beta - \{(4 - D)S \otimes S + (D^2 - 5D + 8)P\otimes P + (2 - D)T\otimes T\}, \tag{101}$$

$$E_{TT}^{\rm NDR} = \gamma_\alpha\gamma_\beta T\otimes T\gamma_\alpha\gamma_\beta - \{6(2 - D)S \otimes S + 6(2 - D)P\otimes P + (D^2 - 2D + 4)T\otimes T\}, \tag{102}$$

where $D$ is the reduced space-time dimension.

Numerical values for $v_{ij}$ and $z_{ij}$ are evaluated by momentum integration, which is performed by a mode sum for a periodic box of a size $L^4$ after transforming the momentum variable through $k_\mu = q_\mu - \sin q_\mu$. We employ the size $L = 64$ for integrals, and numerical error is estimated by varying $L$ from 64 to 60. The results are presented in Tables I, II, III, IV, V, and VI for the plaquette gauge action and in Tables VII, VIII, IX, X, and XI for the Iwasaki

TABLE I. Numerical values for $v_{ij}$ $(i, j = 1, \ldots, 5)$ as a function of $M$ with plaquette gauge action.

| $M$ | $v_{11}^{+}$ | $v_{11}^{-}$ | $v_{22}^{\pm}$ | $v_{23}^{+}$ | $v_{23}^{-}$ |
|---|---|---|---|---|---|
| 0.1 | 13.5696(73) | 11.5369(73) | 13.2308(18) | 2.0327(66) | −2.0327(66) |
| 0.2 | 13.0548(73) | 12.5870(73) | 12.9768(18) | 0.4678(66) | −0.4678(66) |
| 0.3 | 12.6404(73) | 13.4380(73) | 12.7733(18) | −0.7976(66) | 0.7976(66) |
| 0.4 | 12.2775(72) | 14.1882(72) | 12.5960(18) | −1.9107(65) | 1.9107(65) |
| 0.5 | 11.9450(73) | 14.8801(73) | 12.4342(18) | −2.9352(66) | 2.9352(66) |
| 0.6 | 11.6307(72) | 15.5380(72) | 12.2819(18) | −3.9073(65) | 3.9073(65) |
| 0.7 | 11.3269(73) | 16.1781(73) | 12.1354(18) | −4.8512(66) | 4.8512(66) |
| 0.8 | 11.0275(72) | 16.8129(72) | 11.8484(18) | −5.7853(65) | 6.7247(66) |
| 0.9 | 10.7276(74) | 17.4523(74) | 11.7033(18) | −6.7247(66) | 7.6845(65) |
| 1.0 | 10.1073(73) | 18.7869(73) | 11.5539(18) | −7.6845(65) | 8.6796(66) |
| 1.1 | 9.7767(75) | 19.5039(75) | 11.3979(19) | −8.6796(66) | 9.7271(67) |
| 1.2 | 9.7767(75) | 19.5039(75) | 11.3979(19) | −9.7271(67) | 9.7271(67) |
| 1.3 | 9.4243(73) | 20.2719(73) | 11.2322(18) | −10.8475(66) | 10.8475(66) |
| 1.4 | 9.0419(75) | 21.1082(75) | 11.0529(19) | −12.0664(67) | 12.0664(67) |
| 1.5 | 8.6183(73) | 22.0373(73) | 10.8548(18) | −13.4191(65) | 13.4191(65) |
| 1.6 | 8.1375(74) | 23.0928(74) | 10.6301(18) | −14.9553(66) | 14.9553(66) |
| 1.7 | 7.5747(73) | 24.3277(73) | 10.3669(18) | −16.7531(66) | 24.3277(73) |
| 1.8 | 6.8864(73) | 25.8324(73) | 10.0440(18) | −18.9460(66) | 18.9460(66) |
| 1.9 | 5.9812(73) | 27.7944(73) | 9.6168(18) | −21.8132(66) | 21.8132(66) |

gauge action as a function of $M$. These values are evaluated with $N = 3$.

TABLE II. Numerical values for $v_{ij}$ $(i, j = 1, \ldots, 5)$ as a function of $M$ with plaquette gauge action.

| $M$ | $v_{33}^{\pm}$ | $v_{44}^{+}$ | $v_{44}^{-}$ | $v_{45}^{+}$ | $v_{45}^{-}$ |
|---|---|---|---|---|---|
| 0.1 | 10.182(12) | 11.1981(84) | 9.165(15) | 0.11293(37) | −0.5646(18) |
| 0.2 | 12.275(12) | 12.5090(84) | 12.041(15) | 0.02599(37) | −0.1299(18) |
| 0.3 | 13.970(12) | 13.5709(84) | 14.369(15) | −0.04431(37) | 0.2216(18) |
| 0.4 | 15.462(12) | 14.5067(83) | 16.417(15) | −0.10615(36) | 0.5307(18) |
| 0.5 | 16.837(12) | 15.3693(84) | 18.305(15) | −0.16307(36) | 0.8153(18) |
| 0.6 | 18.143(12) | 16.1893(83) | 20.097(15) | −0.21707(36) | 1.0854(18) |
| 0.7 | 19.412(12) | 16.9866(84) | 21.838(15) | −0.26951(37) | 1.3476(18) |
| 0.8 | 20.670(11) | 17.7771(83) | 23.562(15) | −0.32141(36) | 1.6070(18) |
| 0.9 | 21.935(12) | 18.5731(85) | 25.298(15) | −0.37359(37) | 1.8680(18) |
| 1.0 | 23.230(12) | 19.3878(84) | 27.072(15) | −0.42692(36) | 2.1346(18) |
| 1.1 | 24.573(12) | 20.2335(84) | 28.913(15) | −0.48220(37) | 2.4110(18) |
| 1.2 | 25.989(12) | 21.1250(86) | 30.852(15) | −0.54040(37) | 2.7020(19) |
| 1.3 | 27.504(12) | 22.0798(84) | 32.927(15) | −0.60264(37) | 3.0132(18) |
| 1.4 | 29.153(12) | 23.1193(86) | 35.186(15) | −0.67036(37) | 3.3518(19) |
| 1.5 | 30.983(12) | 24.2739(84) | 37.693(15) | −0.74550(36) | 3.7275(18) |
| 1.6 | 33.063(12) | 25.5853(85) | 40.541(15) | −0.83085(37) | 4.1542(18) |
| 1.7 | 35.496(12) | 27.1199(84) | 43.873(15) | −0.93073(37) | 4.6536(18) |
| 1.8 | 38.463(12) | 28.9900(84) | 47.936(15) | −1.05255(37) | 5.2628(18) |
| 1.9 | 42.337(12) | 31.4299(84) | 53.243(15) | −1.21184(37) | 6.0592(18) |

TABLE III. Numerical values for $v_{ij}$ $(i, j = 1, \ldots, 5)$ as a function of $M$ with plaquette gauge action.

| $M$ | $v_{55}^{+}$ | $v_{55}^{-}$ | $v_{54}^{+}$ | $v_{54}^{-}$ |
|---|---|---|---|---|
| 0.1 | 14.8119(33) | 12.7791(33) | −1.6939(55) | 0.3388(11) |
| 0.2 | 13.3406(33) | 12.8729(33) | −0.3898(55) | 0.0780(11) |
| 0.3 | 12.1530(33) | 12.9506(33) | 0.6647(55) | −0.1329(11) |
| 0.4 | 11.1099(32) | 13.0206(33) | 1.5922(54) | −0.3184(11) |
| 0.5 | 10.1512(33) | 13.0864(33) | 2.4460(55) | −0.4892(11) |
| 0.6 | 9.2429(33) | 13.1502(33) | 3.2561(54) | −0.6512(11) |
| 0.7 | 8.3622(33) | 13.2135(33) | 4.0427(55) | −0.8085(11) |
| 0.8 | 7.4920(32) | 13.2774(32) | 4.8211(54) | −0.9642(11) |
| 0.9 | 6.6181(33) | 13.3428(33) | 5.6039(55) | −1.1208(11) |
| 1.0 | 5.7264(33) | 13.4109(33) | 6.4038(55) | −1.2808(11) |
| 1.1 | 4.8031(33) | 13.4827(33) | 7.2330(55) | −1.4466(11) |
| 1.2 | 3.8324(34) | 13.5595(34) | 8.1059(56) | −1.6212(11) |
| 1.3 | 2.7953(33) | 13.6428(33) | 9.0396(55) | −1.8079(11) |
| 1.4 | 1.6679(34) | 13.7343(34) | 10.0553(56) | −2.0111(11) |
| 1.5 | 0.4177(33) | 13.8368(33) | 11.1826(55) | −2.2365(11) |
| 1.6 | −1.0018(33) | 13.9535(33) | 12.4627(55) | −2.4925(11) |
| 1.7 | −2.6633(33) | 14.0898(33) | 13.9609(55) | −2.7922(11) |
| 1.8 | −4.6917(33) | 14.2543(33) | 15.7883(55) | −3.1577(11) |
| 1.9 | −7.3491(33) | 14.4641(33) | 18.1777(55) | −3.6355(11) |

## IV. RENORMALIZATION FACTORS FOR SUSY OPERATORS

Let us consider the parity-conserving parts of the SUSY operators in Eqs. (1)–(5), which are relevant for $K^0 - \bar{K}^0$ mixing. They are related to the operators of Eqs. (29)–(33) as

$$\mathcal{O}_1 = Q_1^{+}, \tag{103}$$

$$\mathcal{O}_2 = Q_4^{+}, \tag{104}$$

$$\mathcal{O}_3 = -\tfrac{1}{2}(Q_4^{+} - Q_5^{+}), \tag{105}$$

$$\mathcal{O}_4 = Q_3^{+}, \tag{106}$$

$$\mathcal{O}_5 = -\tfrac{1}{2}Q_2^{+}, \tag{107}$$

where we take $q_1 = q_3 = s$ and $q_2 = q_4 = d$ in Eq. (34). With the use of these relations, the renormalization factors for $\mathcal{O}_i$ are expressed as

$$\mathcal{O}_1^{\overline{\rm MS}}(\mu) = \frac{1}{(1-\omega_0^2)^2 Z_\omega^2}[Z_{11}^{+}(\mu a)\mathcal{O}_1^{\rm lat}(1/a)], \tag{108}$$

$$\mathcal{O}_2^{\overline{\rm MS}}(\mu) = \frac{1}{(1-\omega_0^2)^2 Z_\omega^2}[(Z_{44}^{+}(\mu a) + Z_{45}^{+}(\mu a))\mathcal{O}_2^{\rm lat}(1/a) + 2Z_{45}^{+}(\mu a)\mathcal{O}_3^{\rm lat}(1/a)], \tag{109}$$

$$\mathcal{O}_3^{\overline{\rm MS}}(\mu) = \frac{1}{(1-\omega_0^2)^2 Z_\omega^2}\bigg[(Z_{55}^{+}(\mu a) - Z_{45}^{+}(\mu a))\mathcal{O}_3^{\rm lat}(1/a) - \frac{1}{2}(Z_{44}^{+}(\mu a) + Z_{45}^{+}(\mu a) - Z_{54}^{+}(\mu a) - Z_{55}^{+}(\mu a))\mathcal{O}_2^{\rm lat}(1/a)\bigg], \tag{110}$$

TABLE IV. Numerical values for $z_{ij}$ $(i, j = 1, \ldots, 5)$ as a function of $M$ with plaquette gauge action.

| $M$ | $z_{11}^{+}$ | $z_{11}^{-}$ | $z_{22}^{\pm}$ | $z_{23}^{+}$ | $z_{23}^{-}$ |
|---|---|---|---|---|---|
| 0.1 | −52.33012(24) | −39.2974(68) | −48.1580(13) | −3.0327(66) | 1.0327(66) |
| 0.2 | −51.4142(13) | −39.9464(53) | −47.50286(22) | −1.4678(66) | −0.5322(66) |
| 0.3 | −50.6609(11) | −40.4585(55) | −46.960537(34) | −0.2024(66) | −1.7976(66) |
| 0.4 | −50.0050(14) | −40.9157(51) | −46.49015(35) | 0.9107(65) | −2.9107(65) |
| 0.5 | −49.41704(64) | −41.3522(72) | −46.0729(17) | 1.9352(66) | −3.9352(66) |
| 0.6 | −48.8806(11) | −41.7879(54) | −45.698460(58) | 2.9073(65) | −4.9073(65) |
| 0.7 | −48.3854(18) | −42.2366(48) | −45.36057(67) | 3.8512(66) | −5.8512(66) |
| 0.8 | −47.92464(38) | −42.7100(61) | −45.05553(70) | 4.7853(65) | −6.7853(65) |
| 0.9 | −47.49355(14) | −43.2182(68) | −44.7810(12) | 5.7247(66) | −7.7247(66) |
| 1.0 | −47.0888(16) | −43.7734(50) | −44.53624(49) | 6.6845(65) | −8.6845(65) |
| 1.1 | −46.707975(79) | −44.3876(67) | −44.3212(12) | 7.6796(66) | −9.6796(66) |
| 1.2 | −46.3494(12) | −45.0765(55) | −44.137215(96) | 8.7271(67) | −10.7271(67) |
| 1.3 | −46.011799(83) | −45.8593(65) | −43.9864(10) | 9.8475(66) | −11.8475(66) |
| 1.4 | −45.6944(11) | −46.7608(56) | −43.872137(23) | 11.0664(67) | −13.0664(67) |
| 1.5 | −45.3964(14) | −47.8155(51) | −43.79961(34) | 12.4191(65) | −14.4191(65) |
| 1.6 | −45.11658(35) | −49.0719(70) | −43.7758(15) | 13.9553(66) | −15.9553(66) |
| 1.7 | −44.8518(17) | −50.6049(49) | −43.81064(62) | 15.7531(66) | −17.7531(66) |
| 1.8 | −44.59306(72) | −52.5390(73) | −43.9174(18) | 17.9460(66) | −19.9460(66) |
| 1.9 | −44.3086(13) | −55.1218(53) | −44.11077(18) | 20.8132(66) | −22.8132(66) |

TABLE V. Numerical values for $z_{ij}$ $(i, j = 1, \ldots, 5)$ as a function of $M$ with plaquette gauge action.

| $M$ | $z_{33}^{\pm}$ | $z_{44}^{+}$ | $z_{44}^{-}$ | $z_{45}^{+}$ | $z_{45}^{-}$ |
|---|---|---|---|---|---|
| 0.1 | −37.609(11) | −40.2919(79) | −36.259(14) | −0.77960(37) | 1.8980(18) |
| 0.2 | −39.3012(97) | −41.2018(64) | −38.734(13) | −0.69265(37) | 1.4633(18) |
| 0.3 | −40.6569(99) | −41.9248(66) | −40.722(13) | −0.62236(37) | 1.1118(18) |
| 0.4 | −41.8562(94) | −42.5675(61) | −42.478(13) | −0.56052(36) | 0.8026(18) |
| 0.5 | −42.976(12) | −43.1748(83) | −44.110(15) | −0.50360(36) | 0.5180(18) |
| 0.6 | −44.0595(97) | −43.7725(65) | −45.680(13) | −0.44959(36) | 0.2480(18) |
| 0.7 | −45.1374(92) | −44.3785(59) | −47.230(13) | −0.39715(37) | −0.0142(18) |
| 0.8 | −46.234(10) | −45.0075(72) | −48.793(14) | −0.34526(36) | −0.2737(18) |
| 0.9 | −47.368(11) | −45.6723(79) | −50.397(14) | −0.29307(37) | −0.5346(18) |
| 1.0 | −48.5631(93) | −46.3875(61) | −52.072(13) | −0.23975(36) | −0.8013(18) |
| 1.1 | −49.841(11) | −47.1675(78) | −53.847(14) | −0.18447(37) | −1.0777(18) |
| 1.2 | −51.228(10) | −48.0310(66) | −55.758(13) | −0.12627(37) | −1.3686(19) |
| 1.3 | −52.758(11) | −49.0006(76) | −57.848(14) | −0.06403(37) | −1.6799(18) |
| 1.4 | −54.472(10) | −50.1052(67) | −60.172(13) | 0.00369(37) | −2.0184(19) |
| 1.5 | −56.4282(95) | −51.3854(62) | −62.804(13) | 0.07884(36) | −2.3942(18) |
| 1.6 | −58.709(11) | −52.8977(81) | −65.853(15) | 0.16418(37) | −2.8209(18) |
| 1.7 | −61.4402(93) | −54.7304(60) | −69.483(13) | 0.26406(37) | −3.3203(18) |
| 1.8 | −64.836(12) | −57.0300(84) | −73.976(15) | 0.38589(37) | −3.9294(18) |
| 1.9 | −69.3305(97) | −60.0906(64) | −79.904(13) | 0.54518(37) | −4.7259(18) |

$$\mathcal{O}_4^{\overline{\rm MS}}(\mu) = \frac{1}{(1-\omega_0^2)^2 Z_\omega^2}[Z_{33}^{+}(\mu a)\mathcal{O}_4^{\rm lat}(1/a) - 2Z_{32}^{+}(\mu a)\mathcal{O}_5^{\rm lat}(1/a)], \tag{111}$$

$$\mathcal{O}_5^{\overline{\rm MS}}(\mu) = \frac{1}{(1-\omega_0^2)^2 Z_\omega^2}\left[Z_{22}^{+}(\mu a)\mathcal{O}_5^{\rm lat}(1/a) - \frac{1}{2}Z_{23}^{+}(\mu a)\mathcal{O}_4^{\rm lat}(1/a)\right], \tag{112}$$

where $Z_{ij}^{+}$ are given in Eq. (94).

TABLE VI. Numerical values for $z_{ij}$ $(i, j = 1, \ldots, 5)$ as a function of $M$ with plaquette gauge action.

| $M$ | $z_{55}^{+}$ | $z_{55}^{-}$ | $z_{54}^{+}$ | $z_{54}^{-}$ |
|---|---|---|---|---|
| 0.1 | −50.2390(38) | −44.2063(28) | 2.6939(55) | 0.6612(11) |
| 0.2 | −48.3667(54) | −43.8989(12) | 1.3898(55) | 0.9220(11) |
| 0.3 | −46.8402(52) | −43.6378(14) | 0.3353(55) | 1.1329(11) |
| 0.4 | −45.5041(54) | −43.4147(11) | −0.5922(54) | 1.3184(11) |
| 0.5 | −44.2900(34) | −43.2252(32) | −1.4460(55) | 1.4892(11) |
| 0.6 | −43.1594(51) | −43.0668(14) | −2.2561(54) | 1.6512(11) |
| 0.7 | −42.0874(58) | −42.93862(80) | −3.0427(55) | 1.8085(11) |
| 0.8 | −41.0558(43) | −42.8412(21) | −3.8211(54) | 1.9642(11) |
| 0.9 | −40.0507(39) | −42.7754(27) | −4.6039(55) | 2.1208(11) |
| 1.0 | −39.0594(56) | −42.74392(96) | −5.4038(55) | 2.2808(11) |
| 1.1 | −38.0704(40) | −42.7500(26) | −6.2330(55) | 2.4466(11) |
| 1.2 | −37.0717(53) | −42.7988(14) | −7.1059(56) | 2.6212(11) |
| 1.3 | −36.0494(41) | −42.8970(25) | −8.0396(55) | 2.8079(11) |
| 1.4 | −34.9872(52) | −43.0536(15) | −9.0553(56) | 3.0111(11) |
| 1.5 | −33.8626(54) | −43.2816(11) | −10.1826(55) | 3.2365(11) |
| 1.6 | −32.6439(37) | −43.5992(29) | −11.4627(55) | 3.4925(11) |
| 1.7 | −31.2805(57) | −44.03354(85) | −12.9609(55) | 3.7922(11) |
| 1.8 | −29.6816(33) | −44.6276(33) | −14.7883(55) | 4.1577(11) |
| 1.9 | −27.6450(53) | −45.4581(13) | −17.1777(55) | 4.6355(11) |

## V. MEAN FIELD IMPROVEMENT

Let us briefly explain the mean-field improvement on the renormalization factors of the four-quark operators in Eqs. (29)–(33). We follow the discussion in Sec. 6 of Ref. [17]. The renormalization factors with the mean field improvement for the four-quark operators are given by

$$Q_i^{\pm\overline{\rm MS}}(\mu) = \frac{u^2}{(1-\omega_0^2)^2 Z_\omega^2} Z_{ij}^{\pm}(\mu a) Q_j^{\pm{\rm lat}}(1/a), \tag{113}$$

$$Z_{ij}^{\pm} = \delta_{ij} + \frac{g^2}{16\pi^2}[(\gamma_{ij} - 2C_F\delta_{ij})\log(\mu a)^2 + z_{ij}^{\pm{\rm MF}}], \tag{114}$$

$$z_{ij}^{\pm{\rm MF}} = v_{ij}^{\pm\overline{\rm MS}} - v_{ij}^{\pm} + \delta_{ij}2C_F(\Sigma_1^{\overline{\rm MS}} - \Sigma_1) + \delta_{ij}16\pi^2 C_F T_{MF}, \tag{115}$$

where $T_{\rm MF}$ is the one-loop correction to the mean field factor defined by

$$u = 1 - g^2 C_F \frac{T_{\rm MF}}{2} + \cdots \tag{116}$$

and $w_0 = 1 - \tilde{M}$ with $\tilde{M} = M - 4(1-u)$. Note that the

TABLE VII. Numerical values for $v_{ij}$ $(i, j = 1, \ldots, 5)$ as a function of $M$ with Iwasaki gauge action.

| $M$ | $v_{11}^{+}$ | $v_{11}^{-}$ | $v_{22}^{\pm}$ | $v_{23}^{+}$ | $v_{23}^{-}$ |
|---|---|---|---|---|---|
| 0.1 | 14.1911(73) | 10.1069(73) | 13.5104(18) | 4.0842(66) | −4.0842(66) |
| 0.2 | 13.7193(73) | 11.0602(73) | 13.2761(18) | 2.6591(66) | −2.6591(66) |
| 0.3 | 13.3494(73) | 11.8105(73) | 13.0929(18) | 1.5389(66) | −1.5389(66) |
| 0.4 | 13.0327(72) | 12.4558(72) | 12.9365(18) | 0.5769(65) | −0.5769(65) |
| 0.5 | 12.7480(73) | 13.0378(73) | 12.7963(18) | −0.2899(66) | 0.2899(66) |
| 0.6 | 12.4836(72) | 13.5804(72) | 12.6664(18) | −1.0968(65) | 1.0968(65) |
| 0.7 | 12.2320(73) | 14.0990(73) | 12.5431(18) | −1.8670(66) | 1.8670(66) |
| 0.8 | 11.9874(72) | 14.6053(72) | 12.4237(18) | −2.6179(65) | 2.6179(65) |
| 0.9 | 11.7451(74) | 15.1082(74) | 12.3057(18) | −3.3631(66) | 3.3631(66) |
| 1.0 | 11.5011(73) | 15.6173(73) | 12.1871(18) | −4.1162(65) | 4.1162(65) |
| 1.1 | 11.2508(73) | 16.1409(73) | 12.0658(18) | −4.8901(66) | 4.8901(66) |
| 1.2 | 10.9896(75) | 16.6892(75) | 11.9395(19) | −5.6997(67) | 5.6997(67) |
| 1.3 | 10.7118(73) | 17.2742(73) | 11.8055(18) | −6.5624(66) | 6.5624(66) |
| 1.4 | 10.4101(75) | 17.9105(75) | 11.6602(19) | −7.5004(67) | 7.5004(67) |
| 1.5 | 10.0748(73) | 18.6191(73) | 11.4988(18) | −8.5444(65) | 8.5444(65) |
| 1.6 | 9.6912(74) | 19.4294(74) | 11.3142(18) | −9.7382(66) | 9.7382(66) |
| 1.7 | 9.2365(73) | 20.3887(73) | 11.0952(18) | −11.1523(66) | 11.1523(66) |
| 1.8 | 8.6700(73) | 21.5799(73) | 10.8217(18) | −12.9099(66) | 12.9099(66) |
| 1.9 | 7.9044(73) | 23.1801(73) | 10.4503(18) | −15.2758(66) | 15.2758(66) |

TABLE VIII. Numerical values for $v_{ij}$ $(i, j = 1, \ldots, 5)$ as a function of $M$ with Iwasaki gauge action.

| $M$ | $v_{33}^{\pm}$ | $v_{44}^{+}$ | $v_{44}^{-}$ | $v_{45}^{+}$ | $v_{45}^{-}$ |
|---|---|---|---|---|---|
| 0.1 | 7.384(12) | 9.4262(84) | 5.342(15) | 0.22690(37) | −1.1345(18) |
| 0.2 | 9.287(12) | 10.6170(84) | 7.958(15) | 0.14773(37) | −0.7386(18) |
| 0.3 | 10.785(12) | 11.5540(84) | 10.015(15) | 0.08549(37) | −0.4275(18) |
| 0.4 | 12.071(12) | 12.3596(83) | 11.783(15) | 0.03205(36) | −0.1603(18) |
| 0.5 | 13.231(12) | 13.0862(84) | 13.376(15) | −0.01610(36) | 0.0805(18) |
| 0.6 | 14.312(12) | 13.7632(83) | 14.860(15) | −0.06093(36) | 0.3047(18) |
| 0.7 | 15.344(12) | 14.4102(84) | 16.277(15) | −0.10372(37) | 0.5186(18) |
| 0.8 | 16.351(11) | 15.0416(83) | 17.660(15) | −0.14544(36) | 0.7272(18) |
| 0.9 | 17.350(12) | 15.6687(85) | 19.032(15) | −0.18684(37) | 0.9342(18) |
| 1.0 | 18.361(12) | 16.3033(84) | 20.420(15) | −0.22868(36) | 1.1434(18) |
| 1.1 | 19.401(12) | 16.9559(84) | 21.846(15) | −0.27167(37) | 1.3584(18) |
| 1.2 | 20.489(12) | 17.6392(86) | 23.339(15) | −0.31665(37) | 1.5832(19) |
| 1.3 | 21.649(12) | 18.3680(84) | 24.930(15) | −0.36458(37) | 1.8229(18) |
| 1.4 | 22.911(12) | 19.1606(86) | 26.661(15) | −0.41669(37) | 2.0834(19) |
| 1.5 | 24.315(12) | 20.0432(84) | 28.588(15) | −0.47469(36) | 2.3734(18) |
| 1.6 | 25.921(12) | 21.0524(85) | 30.791(15) | −0.54101(37) | 2.7051(18) |
| 1.7 | 27.824(12) | 22.2475(84) | 33.400(15) | −0.61957(37) | 3.0979(18) |
| 1.8 | 30.186(12) | 23.7315(84) | 36.641(15) | −0.71721(37) | 3.5861(18) |
| 1.9 | 33.364(12) | 25.7261(84) | 41.002(15) | −0.84865(37) | 4.2433(18) |

TABLE IX. Numerical values for $v_{ij}$ $(i, j = 1, \ldots, 5)$ as a function of $M$ with Iwasaki gauge action.

| $M$ | $v_{55}^+$ | $v_{55}^-$ | $v_{54}^+$ | $v_{54}^-$ |
|---|---|---|---|---|
| 0.1 | 16.6870(33) | 12.6028(33) | −3.4035(55) | 0.6807(11) |
| 0.2 | 15.3443(33) | 12.6852(33) | −2.2159(55) | 0.4432(11) |
| 0.3 | 14.2899(33) | 12.7510(33) | −1.2824(55) | 0.2565(11) |
| 0.4 | 13.3852(32) | 12.8083(33) | −0.4808(54) | 0.0962(11) |
| 0.5 | 12.5708(33) | 12.8607(33) | 0.2416(55) | −0.0483(11) |
| 0.6 | 11.8133(33) | 12.9102(33) | 0.9140(54) | −0.1828(11) |
| 0.7 | 11.0910(33) | 12.9580(33) | 1.5559(55) | −0.3112(11) |
| 0.8 | 10.3875(32) | 13.0055(32) | 2.1816(54) | −0.4363(11) |
| 0.9 | 9.6899(33) | 13.0530(33) | 2.8025(55) | −0.5605(11) |
| 1.0 | 8.9856(33) | 13.1018(33) | 3.4302(55) | −0.6860(11) |
| 1.1 | 8.2624(33) | 13.1525(33) | 4.0751(55) | −0.8150(11) |
| 1.2 | 7.5064(34) | 13.2061(34) | 4.7497(56) | −0.9499(11) |
| 1.3 | 6.7014(33) | 13.2638(33) | 5.4687(55) | −1.0937(11) |
| 1.4 | 5.8266(34) | 13.3270(34) | 6.2503(56) | −1.2501(11) |
| 1.5 | 4.8532(33) | 13.3976(33) | 7.1203(55) | −1.4241(11) |
| 1.6 | 3.7400(33) | 13.4782(33) | 8.1152(55) | −1.6230(11) |
| 1.7 | 2.4212(33) | 13.5735(33) | 9.2936(55) | −1.8587(11) |
| 1.8 | 0.7807(33) | 13.6905(33) | 10.7582(55) | −2.1516(11) |
| 1.9 | −1.4308(33) | 13.8449(33) | 12.7298(55) | −2.5460(11) |

TABLE X. Numerical values for $z_{ij}$ $(i, j = 1, \ldots, 5)$ as a function of $M$ with Iwasaki gauge action.

| $M$ | $z_{11}^+$ | $z_{11}^-$ | $z_{22}^\pm$ | $z_{23}^+$ | $z_{23}^-$ |
|---|---|---|---|---|---|
| 0.1 | −30.25980(58) | −15.1756(72) | −25.7458(17) | −5.0842(66) | 3.0842(66) |
| 0.2 | −29.43439(58) | −15.7753(72) | −25.1579(17) | −3.6591(66) | 1.6591(66) |
| 0.3 | −28.77133(70) | −16.2324(73) | −24.6815(18) | −2.5389(66) | 0.5389(66) |
| 0.4 | −28.20508(57) | −16.6282(71) | −24.2756(17) | −1.5769(65) | −0.4231(65) |
| 0.5 | −27.70614(68) | −16.9960(72) | −23.9211(18) | −0.7101(66) | −1.2899(66) |
| 0.6 | −27.25798(48) | −17.3548(70) | −23.6075(16) | 0.0968(65) | −2.0968(65) |
| 0.7 | −26.85022(60) | −17.7173(72) | −23.3281(17) | 0.8670(66) | −2.8670(66) |
| 0.8 | −26.47585(51) | −18.0938(70) | −23.0788(16) | 1.6179(65) | −3.6179(65) |
| 0.9 | −26.12985(57) | −18.4929(72) | −22.8570(17) | 2.3631(66) | −4.3631(66) |
| 1.0 | −25.80864(62) | −18.9249(72) | −22.6613(17) | 3.1162(65) | −5.1162(65) |
| 1.1 | −25.50947(72) | −19.3996(73) | −22.4912(18) | 3.8901(66) | −5.8901(66) |
| 1.2 | −25.23026(60) | −19.9299(73) | −22.3469(17) | 4.6997(67) | −6.6997(67) |
| 1.3 | −24.96938(47) | −20.5318(71) | −22.2298(16) | 5.5624(66) | −7.5624(66) |
| 1.4 | −24.72536(72) | −21.2258(74) | −22.1421(18) | 6.5004(67) | −8.5004(67) |
| 1.5 | −24.49673(63) | −22.0411(72) | −22.0875(17) | 7.5444(65) | −9.5444(65) |
| 1.6 | −24.28127(49) | −23.0195(71) | −22.0710(16) | 8.7382(66) | −10.7382(66) |
| 1.7 | −24.07481(51) | −24.2271(71) | −22.1002(16) | 10.1523(66) | −12.1523(66) |
| 1.8 | −23.86686(71) | −25.7767(73) | −22.1852(18) | 11.9099(66) | −13.9099(66) |
| 1.9 | −23.62377(52) | −27.8995(71) | −22.3364(16) | 14.2758(66) | −16.2758(66) |

TABLE XI. Numerical values for $z_{ij}$ $(i, j = 1, \ldots, 5)$ as a function of $M$ with Iwasaki gauge action.

| $M$ | $z_{33}^{\pm}$ | $z_{44}^{+}$ | $z_{44}^{-}$ | $z_{45}^{+}$ | $z_{45}^{-}$ |
|---|---|---|---|---|---|
| 0.1 | −12.119(12) | −15.8282(82) | −9.744(15) | −0.89357(37) | 2.4678(18) |
| 0.2 | −13.669(12) | −16.6654(83) | −12.006(15) | −0.81440(37) | 2.0720(18) |
| 0.3 | −14.873(12) | −17.3093(84) | −13.770(15) | −0.75216(37) | 1.7608(18) |
| 0.4 | −15.910(11) | −17.8654(82) | −15.288(15) | −0.69872(36) | 1.4936(18) |
| 0.5 | −16.856(12) | −18.3777(83) | −16.668(15) | −0.65056(36) | 1.2528(18) |
| 0.6 | −17.753(11) | −18.8709(81) | −17.968(15) | −0.60573(36) | 1.0287(18) |
| 0.7 | −18.629(12) | −19.3618(83) | −19.229(15) | −0.56294(37) | 0.8147(18) |
| 0.8 | −19.506(11) | −19.8634(81) | −20.481(15) | −0.52123(36) | 0.6061(18) |
| 0.9 | −20.402(12) | −20.3867(83) | −21.750(15) | −0.47983(37) | 0.3992(18) |
| 1.0 | −21.336(12) | −20.9442(83) | −23.060(15) | −0.43799(36) | 0.1899(18) |
| 1.1 | −22.326(12) | −21.5479(84) | −24.438(15) | −0.39499(37) | −0.0250(18) |
| 1.2 | −23.396(12) | −22.2132(84) | −25.913(15) | −0.35002(37) | −0.2499(19) |
| 1.3 | −24.573(11) | −22.9589(82) | −27.521(15) | −0.30209(37) | −0.4896(18) |
| 1.4 | −25.893(12) | −23.8092(85) | −29.310(15) | −0.24998(37) | −0.7501(19) |
| 1.5 | −27.404(12) | −24.7985(83) | −31.343(15) | −0.19198(36) | −1.0401(18) |
| 1.6 | −29.178(12) | −25.9758(82) | −33.714(15) | −0.12566(37) | −1.3717(18) |
| 1.7 | −31.329(11) | −27.4191(82) | −36.571(15) | −0.04710(37) | −1.7645(18) |
| 1.8 | −34.050(12) | −29.2617(84) | −40.172(15) | 0.05055(37) | −2.2527(18) |
| 1.9 | −37.750(11) | −31.7788(82) | −45.055(15) | 0.18199(37) | −2.9099(18) |

TABLE XII. Numerical values for $z_{ij}$ $(i, j = 1, \ldots, 5)$ as a function of $M$ with Iwasaki gauge action.

| $M$ | $z_{55}^{+}$ | $z_{55}^{-}$ | $z_{54}^{+}$ | $z_{54}^{-}$ |
|---|---|---|---|---|
| 0.1 | −29.4224(34) | −21.3382(31) | 4.4035(55) | 0.3193(11) |
| 0.2 | −27.7261(34) | −21.0670(31) | 3.2159(55) | 0.5568(11) |
| 0.3 | −26.3784(33) | −20.8395(33) | 2.2824(55) | 0.7435(11) |
| 0.4 | −25.2243(34) | −20.6474(31) | 1.4808(54) | 0.9038(11) |
| 0.5 | −24.1957(33) | −20.4855(32) | 0.7584(55) | 1.0483(11) |
| 0.6 | −23.2544(35) | −20.3512(30) | 0.0860(54) | 1.1828(11) |
| 0.7 | −22.3759(34) | −20.2430(32) | −0.5559(55) | 1.3112(11) |
| 0.8 | −21.5427(34) | −20.1606(30) | −1.1816(54) | 1.4363(11) |
| 0.9 | −20.7413(35) | −20.1044(31) | −1.8025(55) | 1.5605(11) |
| 1.0 | −19.9598(34) | −20.0761(32) | −2.4302(55) | 1.6860(11) |
| 1.1 | −19.1877(33) | −20.0778(33) | −3.0751(55) | 1.8150(11) |
| 1.2 | −18.4138(35) | −20.1135(32) | −3.7497(56) | 1.9499(11) |
| 1.3 | −17.6257(36) | −20.1881(30) | −4.4687(55) | 2.0937(11) |
| 1.4 | −16.8085(34) | −20.3088(33) | −5.2503(56) | 2.2501(11) |
| 1.5 | −15.9418(34) | −20.4862(32) | −6.1203(55) | 2.4241(11) |
| 1.6 | −14.9968(36) | −20.7350(31) | −7.1152(55) | 2.6230(11) |
| 1.7 | −13.9262(35) | −21.0785(31) | −8.2936(55) | 2.8587(11) |
| 1.8 | −12.6442(33) | −21.5540(33) | −9.7582(55) | 3.1516(11) |
| 1.9 | −10.9552(35) | −22.2310(31) | −11.7298(55) | 3.5460(11) |

TABLE XIII. Numerical values of $Z^{\pm}_{ij}$ $(i, j = 1, \ldots, 5)$ for plaquette gauge action (upper part) and Iwasaki gauge action (lower part).

| $\beta$ | $P$ | $g^2_{\overline{\rm MS}}(1/a)$ | $M$ | | |
|---|---|---|---|---|---|
| 6.0 | 0.59374 | 2.1793 | 1.80 | | |
| $\tilde{M}$ | $Z^+_{11}$ | $Z^-_{11}$ | $Z^{\pm}_{22}$ | $Z^+_{23}$ | $Z^-_{23}$ |
| 1.3112 | 0.7287 | 0.7289 | 0.7564 | 0.1378 | −0.1654 |
| | $Z^+_{33}$ | $Z^{\pm}_{32}$ | $Z^+_{44}$ | $Z^-_{44}$ | $Z^+_{45}$ |
| | 0.6325 | ±0.0207 | 0.6853 | 0.5613 | −0.0007790 |
| | $Z^-_{45}$ | $Z^+_{55}$ | $Z^-_{55}$ | $Z^+_{54}$ | $Z^-_{54}$ |
| | −0.02371 | 0.8674 | 0.7710 | −0.1125 | 0.03906 |

| $\beta$ | $P$ | $R$ | $g^2_{\overline{\rm MS}}(1/a)$ | $M$ | |
|---|---|---|---|---|---|
| 2.6 | 0.67063 | 0.45283 | 2.2479 | 1.80 | |
| $\tilde{M}$ | $Z^+_{11}$ | $Z^-_{11}$ | $Z^{\pm}_{22}$ | $Z^+_{23}$ | $Z^-_{23}$ |
| 1.4198 | 0.8062 | 0.8531 | 0.8425 | 0.09548 | −0.1239 |
| | $Z^+_{33}$ | $Z^{\pm}_{32}$ | $Z^+_{44}$ | $Z^-_{44}$ | $Z^+_{45}$ |
| | 0.7847 | ±0.02135 | 0.8158 | 0.7346 | −0.003395 |
| | $Z^-_{45}$ | $Z^+_{55}$ | $Z^-_{55}$ | $Z^+_{54}$ | $Z^-_{54}$ |
| | −0.01150 | 0.9207 | 0.8680 | −0.07719 | 0.03252 |

overall factor $(1-\omega_0^2)^2 Z_\omega^2$ is mean-field improved following the description in Ref. [17].

Now it is instructive to evaluate the magnitude of the renormalization factors in current representative simulations. We take Ref. [23] as an example, where a quenched simulation of domain-wall QCD was made at $\beta = 6.0$ for the plaquette gauge action with $M = 1.8$ and $\beta = 2.6$ for the Iwasaki gauge action with $M = 1.8$. The mean-field improved $\overline{\rm MS}$ coupling $g^2_{\overline{\rm MS}}(1/a)$ at the scale $\mu = 1/a$ is obtained from

$$\frac{1}{g^2_{\overline{\rm MS}}}(1/a) = P\frac{\beta}{6} + d_g + c_p \tag{117}$$

for the plaquette gauge action, while

$$\frac{1}{g^2_{\overline{\rm MS}}}(1/a) = (c_0 P + 8c_1 R)\frac{\beta}{6} + d_g + c_0 \cdot c_p + 8c_1 \cdot c_{R1} \tag{118}$$

for the Iwasaki gauge action, where the values of $d_g$ and $c_p$ are listed in Table XVI of Ref. [17]. $P$ denotes the expectation value of the plaquette and $R$ for the $1\times 2$ rectangular. Their values are taken from Ref. [7]. The domain-wall height $M = 1.8$ is replaced with $\tilde{M} = 1.3112$ (plaquette) and $\tilde{M} = 1.4198$ (Iwasaki), respectively, according to $\tilde{M} = M - 4(1-u)$ with $u = P^{1/4}$. The mean-field improved values for the renormalization factors $Z^{\pm}_{ij}$ at the scale $\mu = 1/a$ are given in Table XIII. We find reasonable magnitude of corrections to the tree-level results for all the renormalization factors.

## VI. CONCLUSION

In this paper we have evaluated the renormalization factors for the generic four-quark operators at the one-loop level in DWQCD with the plaquette and Iwasaki gauge actions. As an application we explain how to construct the renormalization factors for the SUSY operators by using our results. We also show that taking the parameters employed in Ref. [23] the numerical values for the renormalization factors result in reasonable magnitude with the mean-field improvement.

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[1] F. Gabbiani and A. Masiero, Nucl. Phys. **B322**, 235 (1989); J. S. Hagelin, S. Kelley, and T. Tanaka, Nucl. Phys. **B415**, 293 (1994); F. Gabbiani, E. Gabrielli, A. Masiero, and L. Silvestrini, Nucl. Phys. **B477**, 321 (1996).
[2] S. Aoki *et al.* (JLQCD Collaboration), Phys. Rev. Lett. **80**, 5271 (1998).
[3] W. Lee *et al.*, Phys. Rev. D **71**, 094501 (2005).
[4] S. Aoki *et al.* (JLQCD Collaboration), Phys. Rev. Lett. **81**, 1778 (1998); Phys. Rev. D **60**, 034511 (1999).
[5] D. Becirevic *et al.*, Eur. Phys. J. C **37**, 315 (2004).
[6] T. Blum and A. Soni, Phys. Rev. D **56**, 174 (1997); Phys. Rev. Lett. **79**, 3595 (1997).
[7] A. Ali Khan *et al.* (CP-PACS Collaboration), Phys. Rev. D **64**, 114506 (2001).
[8] Y. Aoki *et al.*, hep-lat/0508011.
[9] P. Dimopoulos *et al.* (ALPHA Collaboration), hep-ph/0601002.
[10] N. Garron *et al.*, Phys. Rev. Lett. **92**, 042001 (2004).
[11] T. DeGrand (MILC Collaboration), Phys. Rev. D **69**, 014504 (2004).
[12] C. R. Allton *et al.*, Phys. Lett. B **453**, 30 (1999).
[13] F. Berruto *et al.*, Nucl. Phys. B, Proc. Suppl. **140**, 365 (2005).
[14] Y. Shamir, Nucl. Phys. **B406**, 90 (1993).
[15] V. Furman and Y. Shamir, Nucl. Phys. **B439**, 54 (1995).
[16] S. Aoki, T. Izubuchi, Y. Kuramashi, and Y. Taniguchi, Phys. Rev. D **60**, 114504 (1999).
[17] S. Aoki, T. Izubuchi, Y. Kuramashi, and Y. Taniguchi, Phys. Rev. D **67**, 094502 (2003).
[18] Y. Iwasaki, Report No. UTHEP-118, 1983 (unpublished).

[19] P. Weisz, Nucl. Phys. **B212**, 1 (1983); P. Weisz and R. Wohlert, Nucl. Phys. **B236**, 397 (1984); **B247**, 544(E) (1984).

[20] S. Aoki, T. Izubuchi, Y. Kuramashi, and Y. Taniguchi, Phys. Rev. D **59**, 094505 (1999).

[21] A. Donini, V. Gimenez, G. Martinelli, M. Talevi, and A. Vladikas, Eur. Phys. J. C **10**, 121 (1999).

[22] A. J. Buras and P. H. Weisz, Nucl. Phys. **B333**, 66 (1990).

[23] J. Noaki *et al.* (CP-PACS Collaboration), Phys. Rev. D **68**, 014501 (2003).