

Factorization in $B \rightarrow K\pi\ell^+\ell^-$ decays

 Benjamín Grinstein¹ and Dan Pirjol²
¹*Department of Physics, University of California at San Diego, La Jolla, California 92093, USA*
²*Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*

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We derive factorization relations for the transverse helicity amplitudes in $B \rightarrow K\pi\ell^+\ell^-$ at leading order in Λ/m_b , in the kinematical region with an energetic kaon and a soft pion. We identify and compute a new contribution of leading order in Λ/m_b to the $B \rightarrow K\pi\ell^+\ell^-$ amplitude which is not present in the one-body decay $B \rightarrow K^*\ell^+\ell^-$. As an application we study the forward-backward asymmetry (FBA) of the lepton momentum angular distribution in $B \rightarrow K\pi\ell^+\ell^-$ decays on and off the K^* resonance. The FBA in these decays has a zero at $q_0^2 = q_0^2(M_{K\pi})$, which can be used, in principle, for determining the Wilson coefficients $C_{7,9}$ and testing the standard model. We point out that the slope of the $q_0^2(M_{K\pi}^2)$ curve contains the same information about the Wilson coefficients as the location of the zero, but is less sensitive to unknown nonperturbative dynamics. We estimate the location of the zero at leading order in factorization, and using a resonant model for the $B \rightarrow K\pi\ell^+\ell^-$ nonfactorizable amplitude.

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I. INTRODUCTION

The rare electroweak penguin decays $b \rightarrow s\gamma$ and $b \rightarrow s\ell^+\ell^-$ are sensitive probes of the flavor structure of the standard model (SM), and provide a promising testing ground for the study of new physics effects (see Ref. [1] for a recent review of the experimental situation). Several clean tests have been proposed in these decays, which are sensitive to the chiral structure of the quark couplings in the standard model. Examples of such tests involve measuring the photon polarization in $b \rightarrow s\gamma$ and the location of the zero of the forward-backward asymmetry in $b \rightarrow s\ell^+\ell^-$ [2].

Our understanding of these decays has advanced considerably over the past few years, through the derivation of factorization relations for exclusive $B \rightarrow K^{(*)}\ell^+\ell^-$ and $B \rightarrow K^*\gamma$ decays at large recoil. First derived at lowest order in perturbation theory [3,4], these factorization theorems were proved to all orders in α_s [5–11] using the soft-collinear effective theory [12,13].

In this paper we introduce a new factorization relation for the multibody rare decays $B \rightarrow K\pi\ell^+\ell^-$ in the kinematical region with a soft pion and an energetic kaon, at leading order in Λ/m_b . The schematic form of the factorization relation is given below in Eq. (40). This extends the application of factorization to multibody rare $B \rightarrow K\pi\ell^+\ell^-$ decays with small invariant mass.

A particularly clean test for new physics effects in these decays is based on the forward-backward asymmetry of the (charged) lepton momentum in $B \rightarrow K^*\ell^+\ell^-$ with respect to the decay axis $q = p_{\ell^+} + p_{\ell^-}$. This is defined as

$$A_{\text{FB}}(q^2) = \frac{1}{d\Gamma(q^2)/dq^2} \left[\int_0^1 d\cos\theta_+ \frac{d\Gamma(q^2, \theta_+)}{dq^2 d\cos\theta_+} - \int_{-1}^0 d\cos\theta_+ \frac{d\Gamma(q^2, \theta_+)}{dq^2 d\cos\theta_+} \right] \quad (1)$$

where θ_+ is the angle between \vec{p}_{ℓ^+} and \vec{q} in the rest frame of the lepton pair.

As pointed out in [2,14], due to certain form factor relations at large recoil [3,15], this asymmetry has a zero at q_0^2 which depends mostly on Wilson coefficients in the weak Hamiltonian with little hadronic uncertainty. The position of the zero was computed in [4,16,17], using the complete leading order factorization formula. The most updated result, including isospin violation effects, is [17]

$$q_0^2 = -2m_B m_b \frac{\text{Re}(C_7^{\text{eff}}(q_0^2))}{\text{Re}(C_9^{\text{eff}}(q_0^2))} (1 + \tau(\alpha_s)) + \delta_{\text{fact}} = \begin{cases} 4.15 \pm 0.27 \text{ GeV}^2 & (B^+ \rightarrow K^{*+}), \\ 4.36_{-0.31}^{+0.33} \text{ GeV}^2 & (B^0 \rightarrow K^{*0}). \end{cases} \quad (2)$$

Here $\tau \sim \alpha_s(m_b)$ is a radiative correction and δ_{fact} denotes factorizable corrections which break the form factor relations. Precise measurements of the position of the zero q_0^2 can give direct information about new physics effects through the values of the Wilson coefficient C_9^{eff} (with C_7^{eff} determined from $B \rightarrow X_s\gamma$ decays).

The branching ratios of the $B \rightarrow K^*\ell^+\ell^-$ exclusive modes have been measured [18] with the results

$$\mathcal{B}(B \rightarrow K^*\ell^+\ell^-) = \begin{cases} (7.8_{-1.7}^{+1.9} \pm 1.2) \times 10^{-7} & (\text{BABAR}), \\ (16.5 \pm 2.3 \pm 0.9 \pm 0.4) \times 10^{-7} & (\text{Belle}), \end{cases} \quad (3)$$

where the *BABAR* errors are statistical and systematic, and the Belle errors are statistical, systematic and from model dependence, respectively. Differential distributions of the q^2 spectrum are also available, as binned branching ratios. First measurements of the forward-backward asymmetry $A_{\text{FB}}(q^2)$ have been presented by the Belle Collaboration [18], but due to large errors the position and even existence of a zero are still inconclusive.

In practice, the K^* is always observed through its strong decay products $K^* \rightarrow K\pi$. We point out that this has several interesting implications. The multibody factorization relation proven here contains a new factorizable contribution to the decay amplitude of leading order in Λ/m_b , which is not present in the $B \rightarrow K^*\ell^+\ell^-$ factorization relation. This introduces a shift in the position of the zero of the forward-backward asymmetry (FBA) in this region. As an application of the new factorization relations, we compute the correction to the position of the zero arising from this effect.

Another novel effect is the existence of a zero of the FBA also for a nonresonant $K\pi$ pair, which occurs at a certain dilepton invariant mass $q_0^2(M_{K\pi})$ depending on the hadronic invariant mass $M_{K\pi}$. In principle, this extends the applicability of the SM test using the zero of the FB asymmetry also to nonresonant $B \rightarrow K\pi\ell^+\ell^-$ decays. In practice, however, the calculation of the position of the zero is complicated by the appearance of additional non-perturbative contributions to the amplitude. We estimate the unknown nonfactorizable amplitude in $B \rightarrow K\pi\ell^+\ell^-$ in terms of a K^* resonant model.

We propose an alternative test of the SM using the *slope* of the $q_0^2(M_{K\pi}^2)$ curve, which can be shown to contain the same information about the Wilson coefficients as the location of the zero itself. In contrast to the absolute position of the zero, which depends on less well-known hadronic parameters, the slope of the zero curve can be shown to be less sensitive to such effects.

In Sec. II we introduce the soft-collinear effective theory (SCET) formalism and write down the effective Lagrangian for the rare $B \rightarrow X_s e^+ e^-$ decay. Section III presents the factorization relations for the $B \rightarrow K\pi\ell^+\ell^-$ helicity amplitudes in the kinematical region with a soft pion and a hard kaon. Section IV lists the expressions for distributions in these decays, and gives a qualitative discussion of the zero of the FBA in the nonresonant region. Section V contains a numerical analysis of the asymmetry, and finally Sec. VI summarizes our results.

II. SCET FORMALISM

In the standard model the $\Delta S = 1$ rare $B \rightarrow X_s \ell^+ \ell^-$ decays are mediated by the weak Hamiltonian

$$H_W = -\frac{G_F}{\sqrt{2}} \lambda_t^{(s)} \sum_{i=1}^{10} C_i O_i(\mu) \quad (4)$$

with $\lambda_q^{(s)} = V_{tb} V_{ts}^*$. The dominant contributions come from the radiative penguin $O_7 = (em_b/4\pi^2) \bar{s} \sigma_{\mu\nu} P_R F^{\mu\nu} b$ and the two operators containing the lepton fields $\ell = e, \mu$,

$$\begin{aligned} O_9 &= \frac{\alpha}{\pi} (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \ell), \\ O_{10} &= \frac{\alpha}{\pi} (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \gamma_5 \ell). \end{aligned} \quad (5)$$

We use everywhere in this paper the operator basis for

O_{1-6} defined in Ref. [19]. Smaller contributions to the amplitude arise from T products of the operators in Eq. (4) with the electromagnetic current.

We choose the kinematics of the decay such that the total dilepton momentum $q_\mu = (p_{\ell^+} + p_{\ell^-})_\mu$ points along the $-\vec{e}_3$ direction, and has the components $q = (q^0, 0, 0, -|\vec{q}|)$, expressed in usual four-dimensional coordinates $a^\mu = (a^0, \vec{a})$. The hadronic system moves in the opposite direction $+\vec{e}_3$ in the B rest frame. We define the light-cone unit vectors $n^\mu = (1, 0, 0, 1)$, $\bar{n}^\mu = (1, 0, 0, -1)$. They can be used to project any vector a^μ onto light-cone directions, according to $a_+ = n_\mu a^\mu$ and $a_- = \bar{n}_\mu a^\mu$. Finally, we introduce a basis of orthogonal unit vectors $\varepsilon_\pm = \frac{1}{\sqrt{2}} \times (0, 1, \mp i, 0)$, $\varepsilon_0 = 1/\sqrt{q^2} (|\vec{q}|, 0, 0, q_0)$.

We will be interested in the kinematical region with $q^2 \ll m_b^2$, for which the hadronic system has a large light-cone momentum component along n . This defines the hard scale $Q \equiv \bar{n} \cdot p_X \sim m_b \gg \Lambda$, with $\Lambda \sim 500$ MeV the typical scale of the strong interactions.

The effective Hamiltonian in Eq. (4) is matched in the SCET₁ onto

$$H_W = -\frac{G_F}{\sqrt{2}} \lambda_t^{(s)} \frac{\alpha}{\pi} \{ (\bar{\ell} \gamma_\mu \ell) J_V^\mu + (\bar{\ell} \gamma_\mu \gamma_5 \ell) J_A^\mu \} \quad (6)$$

where the currents $J_{V,A}^\mu$ where each of the currents has the general form

$$\begin{aligned} J_i^\mu &= c_1^{(i)}(\omega) \bar{q}_{n,\omega} \gamma_\perp^\mu P_L b_\nu \\ &+ [c_2^{(i)}(\omega) v^\mu + c_3^{(i)}(\omega) n^\mu] \bar{q}_{n,\omega} P_R b_\nu \\ &+ b_{1L}^{(i)}(\omega_j) J^{(1L)\mu}(\omega_j) + b_{1R}^{(i)}(\omega_j) J^{(1R)\mu}(\omega_j) \\ &+ [b_{1v}^{(i)}(\omega_j) v^\mu + b_{1n}^{(i)}(\omega_j) n^\mu] J^{(10)}(\omega_j) \end{aligned} \quad (7)$$

with $i = V, A$, and integration over $\omega_j = (\omega_1, \omega_2)$ is implicit on the right-hand side. This expansion contains the most general operators up to order $O(\lambda)$, with $\lambda^2 = \Lambda/Q$. The subleading operators are defined as

$$\begin{aligned} J_\mu^{(1L,1R)}(\omega_1, \omega_2) &= \bar{q}_{n,\omega_1} \Gamma_{\mu\alpha}^{(1L,1R)} \left[\frac{1}{\bar{n} \cdot \mathcal{P}} ig \mathcal{B}_{\perp n}^\alpha \right]_{\omega_2} b_\nu, \\ J^{(10)}(\omega_1, \omega_2) &= \bar{q}_{n,\omega_1} \left[\frac{1}{\bar{n} \cdot \mathcal{P}} ig \mathcal{B}_n^\perp \right]_{\omega_2} P_L b_\nu, \end{aligned} \quad (8)$$

with $\{\Gamma_{\mu\alpha}^{(1L)}, \Gamma_{\mu\alpha}^{(1R)}\} = \{\gamma_\mu^\perp \gamma_\alpha^\perp P_R, \gamma_\alpha^\perp \gamma_\mu^\perp P_R\}$. The collinear gauge invariant fields are defined as $q_n = W^\dagger \xi_n$, $ig \mathcal{B}_\mu = W^\dagger [\bar{n} \cdot iD_c, iD_{c\mu}^\perp] W$, with ξ_n the collinear quark field and $W = \exp[-g(\bar{n} \cdot A_{n,q})/(\bar{n} \cdot q)]$ a Wilson line of the collinear gluon field. We use throughout the notations of Ref. [6] with $n \cdot v = \bar{n} \cdot v = 1$.

The Wilson coefficients of the leading order SCET operators appearing in the matching of $J_{V,A}^\mu$ are [12]

$$c_1^{(V)}(\omega, \mu) = \left(C_9^{\text{eff}} + 2m_b(\mu) \frac{n \cdot q}{q^2} C_7^{\text{eff}}(\mu)(1 + \tau(\omega, \mu)) \right) \times \left(1 - \frac{\alpha_s C_F}{4\pi} f_v(\omega, \mu) \right) + O(\alpha_s^2(Q)), \quad (9)$$

$$c_1^{(A)}(\omega, \mu) = C_{10} \left(1 - \frac{\alpha_s C_F}{4\pi} f_v(\omega, \mu) \right) + O(\alpha_s^2(Q)). \quad (10)$$

The effective Wilson coefficients $C_{7,9}^{\text{eff}}$ include the contributions of the operators O_{1-6} and O_8 . For convenience they are listed in the Appendix, together with the functions $f_v(\omega, \mu)$ containing the $O(\alpha_s(Q))$ contribution to the Wilson coefficient of the vector current in SCET, and $\tau(\omega, \mu)$ giving the additional contribution from the tensor current.

The Wilson coefficients of the $O(\lambda)$ SCET_I operators are given at leading order in $\alpha_s(Q)$ by

$$b_{1L}^{(V)}(\omega_1, \omega_2) = -\frac{2n \cdot q}{q^2} \left[C_7^{\text{eff}} + \frac{\bar{x}\omega}{8m_b} e_u \bar{C}_2 t_\perp(x, m_c) \right],$$

$$b_{1R}^{(V)}(\omega_1, \omega_2) = \left(2m_b(\mu) \frac{\bar{n} \cdot q}{q^2} C_7^{\text{eff}} + C_9^{\text{eff}} \right) \frac{1}{\omega} + \frac{2n \cdot q}{q^2} \left(\frac{\bar{x}\omega}{8m_b} e_u \bar{C}_2 t_\perp(x, m_c) \right), \quad (11)$$

$$b_{1L}^{(A)}(\omega_1, \omega_2) = 0, \quad (12)$$

$$b_{1R}^{(A)}(\omega_1, \omega_2) = C_{10} \frac{1}{\omega}, \quad (13)$$

with $\omega_1 = x\omega$, $\omega_2 = -\bar{x}\omega$, and $\bar{x} = 1 - x$. We neglect here smaller contributions from the operators O_{3-6} and the gluon penguin O_8 , which will be retained only in the leading order SCET Wilson coefficients $c_1^{(V,A)}(\omega, \mu)$. The complete expression can be extracted from Ref. [4]. The Wilson coefficients \bar{C}_i are defined in the Appendix. The function $t_\perp(x, m_c)$ appears in matching from graphs with both the photon and the transverse collinear gluon emitted from the charm loop [4] and is given in Eq. (A8) of the Appendix.

The coupling of the virtual photon $\gamma^* \rightarrow \ell^+\ell^-$ to the light quarks can also occur through diagrams with intermediate hard-collinear quarks propagating along the photon momentum [9]. (Such a description is appropriate only for a range of the dilepton invariant mass $q^2 \leq 1.5 \text{ GeV}^2$ which can be considered hard collinear; see below.) Such contributions are mediated by new terms in the SCET_I effective Lagrangian, which in the SM contains only one operator at leading order,

$$\mathcal{H}_{\text{sp}} = \frac{4G_F}{\sqrt{2}} \lambda_t^{(s)} \sum_{q=u,d,s} b_{\text{sp}}^{(q)}(\omega_j) (\bar{q}_{\bar{n},\omega_3} \not{P}_L b_\nu) \times \left(\bar{q}_{n,\omega_1} \frac{\not{q}}{2} P_L s_{n,\omega_2} \right). \quad (14)$$

The Wilson coefficient is given by

$$b_{\text{sp}}^{(q)}(z) = \frac{\lambda_u^{(s)}}{\lambda_t^{(s)}} \left(\bar{C}_2 + \frac{\bar{C}_1}{N_c} \right) \delta_{qu} - \left(\bar{C}_4 + \frac{\bar{C}_3}{N_c} \right) + O(\alpha_s(Q)) \quad (15)$$

where we neglect again smaller contributions proportional to C_8 .

For application to exclusive $B \rightarrow M$ form factors, with $M = \pi, \rho, \dots$ a light meson, the SCET_I effective Lagrangian in Eq. (6) has to be matched onto SCET_{II} operators. This requires taking into account the interaction with the soft spectator quark in the B meson [5,13]. Working to leading order in SCET_{II}, this matching contains two types of operators [3,5,6,8,9],

$$J_i^\mu \rightarrow c_1^{(i)}(\omega) O_{\text{nf}}^\mu + [c_2^{(i)}(\omega) v^\mu + c_3^{(i)}(\omega) n^\mu] O'_{\text{nf}} + J_{i,\text{f}}^\mu + \delta_{i,V} J_{\text{sp}}^\mu + \dots \quad (16)$$

The first type are the so-called ‘‘nonfactorizable’’ operators, denoted here as $O_{\text{nf}}^\mu, O'_{\text{nf}}$. They are defined such that they include the contributions of the leading operators in the SCET_I Lagrangian. The second type of operators are the so-called factorizable and spectator interaction operators, denoted as J_{f}^μ and J_{sp}^μ , respectively. Although their form is similar, they arise in matching from different operators in SCET_I, as follows. The operators J_{f}^μ are obtained from the $O(\lambda)$ operators in the SCET_I current. The spectator operators contribute only to J_{V}^μ in Eq. (6), and arise from the SCET_I weak nonleptonic effective Hamiltonian Eq. (14). The ellipses denote terms suppressed by powers of Λ/m_b .

The matrix elements of the nonfactorizable operators in Eq. (16) corresponding to a $B \rightarrow M_n$ transition are parametrized in terms of soft form factors. We define them as [3]

$$\langle M_n(p_M) | O'_{\text{nf}} | \bar{B} \rangle = 2E_M \zeta_0(E_M, \mu), \quad (17)$$

$$\langle M_n(p_M) | O_{\text{nf}}^\mu \varepsilon_{-\mu}^* | \bar{B} \rangle = 2E_M \zeta_\perp(E_M), \quad (18)$$

where in the first matrix element M_n is a pseudoscalar meson, and in the second M_n is a transversely polarized vector meson.

The factorizable operators in Eq. (16) are nonlocal soft-collinear four-quark operators. As mentioned above, they are of two types, denoted as factorizable (f) and spectator (sp) types. The $J_{i,\text{f}}^\mu$ operators have the generic form

$$J_{i,\text{f}}^\mu \sim \int dx dz dk_+ b^{(i)}(z) J(x, z, k_+) (\bar{q}_{k_+} \Gamma_S b_\nu) (\bar{q}_{n,\omega_1} \Gamma_C q_{n,\omega_2}) \quad (19)$$

where $b(z)$ are SCET_I Wilson coefficients, and J are jet functions. They are given in explicit form in Eq. (29) below. We use a momentum space notation for the nonlocal soft operator, defined by

$$(\bar{q}_{k_+}^i b_v^j)(0) = \int \frac{d\lambda}{4\pi} e^{-(i/2)\lambda k_+} \bar{q}\left(\lambda \frac{n}{2}\right) Y(\lambda, 0) b_v^j(0) \quad (20)$$

with $Y(\lambda, 0) = P \exp(ig \int d\alpha n_\mu A^\mu(\alpha n/2))$ a soft Wilson line along the direction n_μ .

There are two jet functions, defined as Wilson coefficients appearing in the matching of T products of the SCET_I currents in Eq. (8) with the ultrasoft-collinear subleading Lagrangian $\mathcal{L}_{q\xi}^{(1)}$, onto SCET_{II} [5,6,20]

$$\begin{aligned} T[\bar{q}_{n,\omega_1} i g \mathcal{B}_{n,\omega_2}^{\perp,\alpha}]^{ia}(0) [i g \mathcal{B}_n^{\perp} q_n]_{\omega=0}^{jb}(y) \\ = i \delta^{ab} \delta(y_+) \delta^{(2)}(y_\perp) \frac{1}{\omega} \int_0^1 dx \\ \times \int \frac{dk_+}{4\pi} e^{ik_+ y_- / 2} \left\{ J_{\parallel}(x, z, k_+) \left[(\not{n} \gamma_\perp^\alpha)^{ji} \left[\bar{q}_{n,x\omega} \frac{\not{n}}{2} q_{n,-\bar{x}\omega} \right] \right. \right. \\ \left. \left. + (\not{n} \gamma_5 \gamma_\perp^\alpha)^{ji} \left[\bar{q}_{n,x\omega} \frac{\not{n}}{2} \gamma_5 q_{n,-\bar{x}\omega} \right] \right] \right. \\ \left. + J_\perp(x, z, k_+) (\not{n} \gamma_\perp^\alpha \gamma_\perp^\beta)^{ji} \left[\bar{q}_{n,x\omega} \frac{\not{n}}{2} \gamma_\perp^\beta q_{n,-\bar{x}\omega} \right] \right\} \quad (21) \end{aligned}$$

with $\omega_1 = z\omega$ and $\omega = \omega_1 - \omega_2$. The jet functions are generated by physics at the hard-collinear scale $\mu_c^2 \sim Q\Lambda$, and have perturbative expansions in $\alpha_s(\mu_c)$. At lowest order in $\alpha_s(\mu_c)$ they are given by [5,6]

$$J_{\perp,\parallel}(x, z, k_+) = \frac{\pi \alpha_s C_F}{N_c} \delta(x - z) \frac{1}{\bar{x} k_+}. \quad (22)$$

Finally, the spectator-type operators have the form

$$\begin{aligned} J_{\text{sp}}^\mu = \int_0^1 dz b_{\text{sp}}(z) \int dk_- J_{\text{sp}}\left(k_- - \frac{q^2}{n \cdot q}\right) \\ \times (\bar{q}_{k_-} \gamma^\mu \not{n} \not{P}_L b_v) \left(\bar{s}_{n,z\omega} \frac{\not{n}}{2} P_L q_{n,-\bar{z}\omega} \right). \quad (23) \end{aligned}$$

The jet function $J_{\text{sp}}(k_-)$ is the same as the jet function appearing in the factorization relation for $B \rightarrow \gamma \ell \bar{\nu}$. It can be extracted from the results of Ref. [21] and is given at one-loop order by

$$J_{\text{sp}}(k_-) = \frac{1}{k_- + i\epsilon} \left(1 + \frac{\alpha_s C_F}{4\pi} \left(L^2 - 1 - \frac{\pi^2}{6} \right) \right) \quad (24)$$

with $L = \log[(-n \cdot q k_- - i\epsilon)/\mu^2]$.

The matrix elements of the factorizable and spectator operators in the $\bar{B} \rightarrow M_n$ transition at large recoil are computed as convolutions of the product of collinear and soft matrix elements. Adding also the nonfactorizable contribution, the generic form of the factorization relation for the hadronic matrix element for $\bar{B} \rightarrow M_n + \text{leptons}$ is

written as [5] (with $i = V, A$)

$$\begin{aligned} \langle M_n | J_i | \bar{B} \rangle = c_j^{(i)} 2 E_M \xi^{BM} + \int dx dz dk_+ b^{(i)}(z) J_j(x, z, k_+) \\ \times \langle 0 | \bar{q}_{k_+} \Gamma_S b_v | \bar{B}(v) \rangle \langle M_n | \bar{q}_{n,\omega_1} \Gamma_C q_{n,\omega_2} | 0 \rangle \\ + \int dx b_{\text{sp}}(x) \phi_M(x) \int dk_- J_{\text{sp}}(k_-) \langle 0 | \bar{q}_{k_-} \Gamma_S b_v | \bar{B} \rangle. \quad (25) \end{aligned}$$

The nonperturbative soft and collinear matrix elements appearing in this relation are given by the B -meson and light meson light-cone wave functions, respectively. We list here their expressions, adopting the following phase conventions for the meson states

$$\begin{aligned} (\pi^+, \pi^0, \pi^-) &= \left(u\bar{d}, \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), d\bar{u} \right), \\ (B^-, \bar{B}^0) &= (b\bar{u}, b\bar{d}). \end{aligned} \quad (26)$$

The light mesons' light-cone wave functions are given by

$$\begin{aligned} \langle \bar{K}_n(p_K) | \bar{s}_{n,\omega_1} \not{n} P_L q_{n,\omega_2} | 0 \rangle &= \frac{i}{2} f_K \bar{n} \cdot p_K \phi_K(x), \\ \langle \bar{K}_n^*(p_{K^*}, \eta) | \bar{s}_{n,\omega_1} \not{n} \gamma_\perp^\mu q_{n,\omega_2} | 0 \rangle &= \frac{1}{2} f_{K^*}^T \bar{n} \cdot p_{K^*} \eta_\perp^{*\mu} \phi_{K^*}^\perp(x), \end{aligned} \quad (27)$$

and the B -meson light-cone wave function is defined as [3]

$$\begin{aligned} \langle 0 | \bar{u}_{k_+}^i b_v^j | B^-(v) \rangle &= -\frac{i}{4} f_B m_B \\ &\times \left\{ \frac{1 + \not{v}}{2} [\not{n} \phi_+^B(k_+) + \not{n} \phi_-^B(k_+)] \gamma_5 \right\}_{ji}. \end{aligned} \quad (28)$$

The matrix element of the $\bar{q}_k b_v$ appearing in the last term of Eq. (25) can be obtained from this by the substitution $n_\mu \leftrightarrow \bar{n}_\mu$.

Factorization in multibody B decays

We consider here the application of the SCET formalism to rare $\bar{B} \rightarrow \bar{K}_n \pi \ell^+ \ell^-$ decays into final states containing

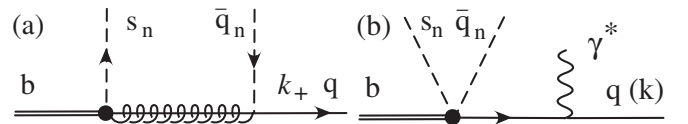


FIG. 1. Leading order SCET_I graphs contributing to the factorizable amplitude for $\bar{B} \rightarrow \bar{K}_n \pi + \text{leptons}$. (a) the T product $\{J_{V,A}^\mu, i\mathcal{L}_{q\xi_n}^{(1)}\}$, where the filled circle represents the SCET current $J_{V,A}^\mu$; (b) spectator-type contribution, with the virtual photon attaching to the light current quark; the intermediate quark propagator can be either hard collinear along the photon direction \bar{n}_μ , or hard, depending on the ratio $n \cdot q\Lambda/q^2$ being greater or less than 1, respectively. After matching onto SCET_{II} these graphs contribute to the operators $J_{i,f}$ and J_{sp} , respectively.

one energetic hadron \bar{K}_n and a soft hadron π . The heavy-light currents in Eq. (8) contribute to such processes again through T products with the ultrasoft-collinear subleading Lagrangian [13]. Typical diagrams in SCET_I contributing to these T products are shown in Fig. 1.

$$\begin{aligned}
 J_{i,\text{fact}}^\mu = & -\frac{1}{2\omega} \int dx dz dk_+ b_{1L}^{(i)}(z) J_\perp(x, z, k_+) (\bar{q}_{k_+} \not{x} \gamma_\mu^\perp \gamma_\perp^\lambda P_R b_v) \left(\bar{s}_{n,\omega_1} \frac{\not{x}}{2} \gamma_\perp^\lambda q_{n,\omega_2} \right) \\
 & -\frac{1}{2\omega} \int dx dz dk_+ b_{1R}^{(i)}(z) J_\parallel(x, z, k_+) (\bar{q}_{k_+} \not{x} \gamma_\mu^\perp P_R b_v) \left(\bar{s}_{n,\omega_1} \frac{\not{x}}{2} P_L q_{n,\omega_2} \right) \\
 & -\frac{1}{\omega} \int dx dz dk_+ [b_{1v}^{(i)}(z) v_\mu + b_{1n}^{(i)}(z) n_\mu] J_\parallel(x, z, k_+) (\bar{q}_{k_+} \not{x} P_L b_v) \left(\bar{s}_{n,\omega_1} \frac{\not{x}}{2} P_L q_{n,\omega_2} \right). \quad (29)
 \end{aligned}$$

The coefficients $b_j(z)$ are related to the Wilson coefficients $b_j(\omega_1, \omega_2)$ of Eqs. (11) as $b_j(z) \equiv b_j((1-z)\omega, z\omega)$; the labels of the collinear fields are parametrized as $\omega_1 = x\omega$, $\omega_2 = -\omega(1-x)$, $\omega = \omega_1 - \omega_2 = \bar{n} \cdot p_M$, with p_M the momentum of the collinear meson M_n produced by the collinear part of the operator.

Finally, the spectator-type factorizable operators arise from diagrams where the photon attaches to the light current quark; see Fig. 1(b). Although the spectator quark is not involved in these contributions, we will continue to use the same terminology as in the $B \rightarrow M_n$ case, due to the similarity of the corresponding operators.

The effective theory treatment of these contributions depends on the relative size of the virtuality of the photon q^2 and the typical hard-collinear scale $m_b \Lambda$. These two scales correspond to the two terms $k_- + q^2/n \cdot q + i\varepsilon$ in the propagator of the intermediate quark in Fig. 1(b).

Several approaches are used in the literature to deal with these contributions, which we briefly review in the following. One possible approach, used in [4,17] in QCD factorization, is to keep both terms in the propagator, and not expand in their ratio. From the point of view of the effective theory, this approach is equivalent to treating the photon as a hard-collinear mode moving along the photon \bar{n}_μ direction [9]. This approach is certainly appropriate for real photons, and for hard-collinear photons $q^2 \sim 1.5 \text{ GeV}^2$. It is not clear whether it can be also applied to photons with $q^2 \sim 4 \text{ GeV}^2$, as is the case here.

In this approach the spectator-type effective Hamiltonian in Eq. (14) contributes to exclusive decays through T -ordered products with the leading order SCET_I Lagrangian describing photon-quark couplings [9,21]. After matching onto SCET_{II}, these T products are matched onto one single operator, which can be written as an addition to J_V^μ , and is given by

$$\begin{aligned}
 J_{\text{sp}}^\mu = & \frac{8\pi^2}{q^2} \sum_{q=u,d,s} e_q \int_0^1 dz b_{\text{sp}}^{(q)}(z) \int dk_- J_{\text{sp}} \left(k_- - \frac{q^2}{n \cdot q} \right) \\
 & \times (\bar{q}_{k_-} \gamma_\mu \not{x} P_L b_v) \left(\bar{s}_{n,\omega_1} \frac{\not{x}}{2} P_L q_{n,\omega_2} \right). \quad (30)
 \end{aligned}$$

After integrating out the modes with virtuality $p_{\text{hc}}^2 \sim \Lambda Q$ connected with the hard-collinear degrees of freedom, these T products are matched onto SCET_{II} [5]. Performing a Fierz transformation of the four-quark operators, one finds the following result for the factorizable operators,

For consistency with the other factorizable operators included, we work to tree level in $\alpha_s(Q)$, but keep terms of $O(\alpha_s(\mu_c))$ in the matrix elements of the factorizable operators.

In the kinematical region we are interested in, a more appropriate treatment of these contributions makes use of an expansion in powers of $n \cdot q \Lambda / q^2 \sim 0.37$. This is similar to the approach adopted in Ref. [22] for weak annihilation contributions to $B \rightarrow \pi\ell^+\ell^-$. The SCET_{II} operators obtained in this way are similar to those in Eq. (30), except that the soft operator is local. Keeping terms to second order in $n \cdot q \Lambda / q^2$, the spectator operator in this approach reads

$$\begin{aligned}
 J_{\text{sp}}^\mu = & \frac{16\pi^2}{q^2} \sum_q e_q \left\{ \frac{q^\alpha}{q^2} (\bar{q} \gamma^\mu \gamma_\alpha \not{x} P_L b) \right. \\
 & \left. + \left(\frac{g^{\alpha\beta}}{q^2} - \frac{2q^\alpha q^\beta}{q^4} \right) (\bar{q} \gamma^\mu \gamma_\alpha (-i\overleftarrow{D}_\beta) \not{x} P_L b) \right\} \\
 & \times \int_0^1 dz b_{\text{sp}}^{(q)}(z) \left(\bar{s}_{n,\omega_1} \frac{\not{x}}{2} P_L q_{n,\omega_2} \right). \quad (31)
 \end{aligned}$$

In this paper we will adopt the latter approach to the treatment of the spectator amplitude, working at leading order in $n \cdot q \Lambda / q^2$. We quote our results in terms of the first approach, which has an additional convolution over k_- . However, it is straightforward to translate between the two approaches, simply by replacing

$$J_{\text{sp}}(k_-) \leftrightarrow \frac{n \cdot q}{q^2} \quad (32)$$

below [in, e.g., Eqs. (53) and (69)]. The advantage of this approach is that the predictions are independent on the details of the matrix elements of the nonlocal soft operator (the B -meson light-cone wave functions), but can be computed exactly in the soft pion limit to second order in the $n \cdot q \Lambda / q^2$ expansion.

In order to have a clean power counting of the transition amplitudes, we divide the phase space of the $B \rightarrow K\pi\ell^+\ell^-$ decay into several regions, shown in Fig. 2:

- (I) The region with one soft pion and one energetic kaon $B \rightarrow K_n \pi_S$, $E_\pi \sim \Lambda$, $E_K \sim Q$ and $M_{K\pi}^2 \sim \Lambda Q$. This region will be the main interest of our paper.

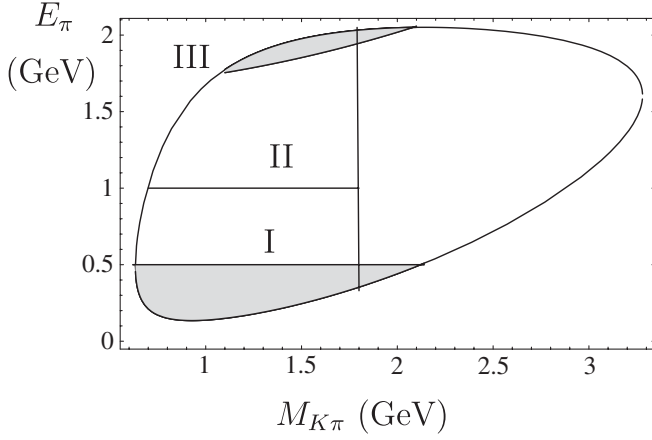


FIG. 2. The phase space of the decay $B \rightarrow K\pi\ell^+\ell^-$ at $q^2 = 4 \text{ GeV}^2$, in variables $(M_{K\pi}, E_\pi)$. The 3 regions shown correspond to (I) soft pion $E_\pi \sim \Lambda$; (II) collinear pion and kaon $E_\pi \sim Q$, $E_K > 1 \text{ GeV}$; (III) soft kaon $E_K < 1 \text{ GeV}$. The shaded region $E_\pi \leq 0.5 \text{ GeV}$ shows the region of applicability of chiral perturbation theory.

- (II) The region $B \rightarrow (K_n\pi_n)_{K^*}$ describing decays into an energetic $K\pi$ pair with a small invariant mass $M_{K\pi} \sim \Lambda$. This is dominated by one-body decays into a collinear meson $B \rightarrow K_n^*$, followed by $K_n^* \rightarrow K_n\pi_n$. This region will be treated essentially the same way as a one-body decay.
- (III) The region with a soft kaon and an energetic pion $E_K \sim \Lambda$, $E_\pi \sim Q$. The decay amplitude in this region is suppressed by Λ/Q relative to that in the other two regions I, II, and will be neglected in the rest of the paper.

We will use the SCET formalism described above to derive a factorization relation in region (I). The matrix elements of the nonfactorizable operators are parametrized in terms of soft nonperturbative matrix elements, in analogy with the $B \rightarrow M_n$ transition. We define them as complex functions of the momenta of the final state hadrons, with mass dimension zero,

$$\begin{aligned} \langle M_n M'_S | O_{\text{nf}}^\mu \varepsilon_{-\mu}^* | \bar{B} \rangle &= \zeta_\perp^{BMM'}(E_M, p_{M'}), \\ \langle M_n M'_S | O'_{\text{nf}} | \bar{B} \rangle &= \zeta_0^{BMM'}(E_M, p_{M'}). \end{aligned} \quad (33)$$

The matrix elements of the factorizable- and spectator-type operators are given again by convolutions as in Eq. (25), with a different soft matrix element,

$$\begin{aligned} &\langle M_n M'_S | J_i | \bar{B}(v) \rangle \\ &\sim \int dx dz dk_+ b^{(i)}(z) J_j(x, z, k_+) \langle M'_S | \bar{q}_{k_+} \Gamma_S b_v | \bar{B}(v) \rangle \\ &\quad \times \langle M_n | \bar{q}_{n, \omega_1} \Gamma_C q_{n, \omega_2} | 0 \rangle \\ &\quad + \int dx b_{\text{sp}}(x) \phi_M(x) \int dk_- J_{\text{sp}}(k_-) \langle 0 | \bar{q}_{k_-} \Gamma_S b_v | \bar{B} \rangle. \end{aligned} \quad (34)$$

These factorization relations contain several new hadronic nonperturbative matrix elements, which we define next. The new $\bar{B} \rightarrow \pi$ soft matrix element is defined in terms of the soft operator

$$O_\mu(k_+) = \int \frac{d\lambda}{4\pi} e^{-(i/2)\lambda k_+} \bar{u} \left(\lambda \frac{n}{2} \right) Y_n(\lambda, 0) \gamma_\mu^\perp \frac{\not{n}}{2} P_R b_v(0) \quad (35)$$

appearing in the term in Eq. (29) proportional to $b_{1R}(z)$, and in the spectator operator. In the latter, one has to take into account that the lightlike separation between the fields is along the direction \bar{n}_μ , rather than n_μ as in the factorizable operators $J_{i, \text{fact}}^\mu$.

The matrix element of the operator O_μ defines a soft function S as

$$\langle \pi^+(p_\pi) | O_\mu(k_+) | \bar{B}^0(v) \rangle = -(g_{\mu\nu}^\perp - i\varepsilon_{\mu\nu}^\perp) p_\pi^\nu S(k_+, t^2, p_\pi^+) \quad (36)$$

with $t = m_B v - p_\pi$. For simplicity of notation, we will drop the kinematical arguments of the soft function $S(k_+, t^2, \zeta)$ whenever no risk of confusion is possible, and show explicitly only its dependence on the integration variable k_+ . The matrix elements of the spectator operator in Eq. (30) are obtained from Eq. (36), with the replacements $n \leftrightarrow \bar{n}$ and $\varepsilon_{\mu\nu}^\perp \rightarrow -\varepsilon_{\mu\nu}^\perp$.

The function $S(k_+)$ is the B physics analog of a generalized parton distribution function (GPD), commonly encountered in nucleon physics [23]. The support of this function is the range $-n \cdot p_\pi \leq k_+ \leq \infty$, and its physical interpretation is different for positive and negative values of k_+ . For $k_+ > 0$ (the resonance region) the soft function gives the amplitude of finding a $u\bar{d}$ pair in the \bar{B}^0 meson, while for $k_+ < 0$ (the transition region), the soft function gives the amplitude for the $b \rightarrow u$ transition of the \bar{B}^0 meson into a π^+ meson. The soft function $S(k_+)$ is continuous at the transition point $k_+ = 0$ [23], which is important for ensuring the convergence of the k_+ convolutions in the factorization relation Eq. (40).

We recall here the main properties of the soft function $S(k_+)$, which were discussed in Refs. [24,25]. Time invariance of the strong interactions constrains it to be real. Its zeroth moment with respect to k_+ is given by

$$\int_{-p_\pi^+}^{\infty} dk_+ S(k_+, t^2, p_\pi^+) = -\frac{1}{4} n \cdot p f_T(t^2) \quad (37)$$

with $f_T(t^2)$ the $B \rightarrow \pi$ tensor form factor defined as

$$\langle \pi(p') | \bar{q} i \sigma_{\mu\nu} b | \bar{B}(p) \rangle = f_T(t^2) (p_\mu p'_\nu - p_\nu p'_\mu). \quad (38)$$

Its N th moments with respect to k_+ are related in a similar way to $\bar{B} \rightarrow \pi$ form factors of dimension $3 + N$ heavy-light currents of the form $\bar{q}(n \cdot iD)^N b_v$ [25].

In the soft pion region, chiral symmetry can be used to relate $S(k_+, t^2, \zeta)$ in the region $k_+ > 0$ to one of the B -meson light-cone wave functions $\phi_+^B(k_+)$ defined in

Eq. (28), according to [26]

$$S(k_+, t^2, \zeta) = \frac{gf_B m_B}{4f_\pi} \frac{1}{v \cdot p_\pi + \Delta} \phi_+^B(k_+). \quad (39)$$

Here g is the $BB^*\pi$ coupling appearing in the leading order heavy hadron chiral effective Lagrangian [27–30]. No such constraint is obtained using chiral symmetry for $S(k_+, t^2, \zeta)$ in the transition region ($k_+ < 0$).

Collecting all the contributions, the amplitude for $B \rightarrow M_n M'_S + \text{leptons}$ is given by a sum of factorizable and nonfactorizable terms, corresponding to the matrix elements of the SCET_{II} operators in Eq. (16). This leads to a factorization relation for such processes, which can be written schematically as

$$\begin{aligned} A(B \rightarrow [M_n M'_S] + \text{leptons}) &= c_i (\bar{n} \cdot p_M) \zeta^{BMM'} \\ &+ \int dz \int dx dk_+ b_i(z) J_j(x, z, k_+) \phi_M(x) S(k_+; p_{M'}) \\ &+ \int dx b_{\text{sp}}(x) \phi_M(x) \int dk_+ J_{\text{sp}}(k_+) S(k_+; p_{M'}). \end{aligned} \quad (40)$$

This factorization relation has several important properties [26]. First, the nonfactorizable contributions to the decay amplitudes of semileptonic and radiative decays satisfy symmetry relations following from the universality of the soft matrix element $\zeta_\perp^{BK\pi}$. They contribute only to the decays $\bar{B} \rightarrow [M_n M'_S]_{h=-1} \ell^+ \ell^-$ into final hadronic states with total helicity -1 . Second, the amplitude for $+1$ helicity is factorizable, and given by a convolution as seen in the second term of Eq. (40). Finally, the factorizable terms contain a new source of strong phases, arising from the region $k_+ \leq 0$ where the jet function develops a nonzero absorptive part. This represents a new, factorizable mechanism for generating final state rescattering.

Treating the spectator amplitudes in an expansion in powers of $n \cdot q\Lambda/q^2$ according to Eq. (31), the soft matrix elements are given by $B \rightarrow \pi$ form factors of dimension-3 and 4 local operators. The leading order term contains the form factors of the vector current

$$\langle \pi(p_\pi) | \bar{u} \gamma_\mu b | \bar{B}(p) \rangle = f_+(t^2) (p + p_\pi)_\mu + f_-(t^2) (p - p_\pi)_\mu. \quad (41)$$

The $f_\pm(t^2)$ form factors appear in the matrix element of Eq. (31) in the combination $f_+ - f_-$. In the hard photon approach, the last term of Eq. (40) has the form $A_{\text{sp}} \sim f_T(t^2) \int_0^1 dx b_{\text{sp}}(x) \phi_M(x)$, which follows from making the substitution of Eq. (32) in this relation, and using Eq. (37).

At subleading order in $n \cdot q\Lambda/q^2$, the form factors of dimension-4 currents $\bar{u} \gamma_\alpha i D_\beta b$ are also needed. They can be computed in the soft pion limit using chiral perturbation theory methods as discussed in Ref. [31].

In the following section we derive the detailed form of these factorization relations for the $\bar{B} \rightarrow K\pi\ell^+\ell^-$ decays.

III. FACTORIZATION RELATIONS FOR

$$\bar{B} \rightarrow K\pi\ell^+\ell^-$$

The decay amplitudes $\bar{B} \rightarrow \bar{K}_n \pi \ell^+ \ell^-$ into an energetic kaon and one soft pion can be parametrized in terms of 6 independent helicity amplitudes $H_\lambda^{(V,A)}(\bar{B} \rightarrow \bar{K}_n \pi)$ with $\lambda = \pm 1, 0$. They are defined as the matrix elements of the two hadronic currents in Eq. (6),

$$H_\lambda^{(V,A)}(\bar{B} \rightarrow \bar{K}_n \pi) = \varepsilon_\lambda^{\mu*} \langle \bar{K}_n \pi | J_{V,A}^\mu | \bar{B}(v) \rangle. \quad (42)$$

Working at leading order in $1/m_b$, the helicity amplitudes can be written as a sum of nonfactorizable and factorizable terms, arising from the corresponding SCET_{II} operators in Eq. (16),

$$H_\lambda^{(V,A)}(B \rightarrow K_n \pi) = \sum_{i=\text{nf}, \text{f}, \text{sp}} H_\lambda^{(V,A),i}(B \rightarrow K_n \pi). \quad (43)$$

The three contributions to each helicity amplitude are computed as described in Sec. II. The nonfactorizable terms are given in terms of the soft functions $\zeta_i^{BK\pi}$ defined in Eq. (33), and the factorizable and spectator contributions are given by factorization relations of the form shown in Eq. (40). In this section we present explicit results for the transverse helicity amplitudes.

We start by recalling the results for the one-body decays $B \rightarrow K_n^* \ell^+ \ell^-$. The factorization relations for this case are well known [4–7, 9, 10] and are given by (with $i = V, A$)

$$H_+^{(i)}(\bar{B} \rightarrow \bar{K}_n^*) = 0, \quad (44)$$

$$\begin{aligned} H_-^{(i)}(\bar{B} \rightarrow \bar{K}_n^*) &= c_1^{(i)} \bar{n} \cdot p_{K^*} \zeta_\perp^{BK^*} - m_B^2 \int_0^1 dz b_{1\perp}^{(i)}(z) \zeta_{J\perp}^{BK^*}(z). \end{aligned} \quad (45)$$

Note that the leading order spectator operator does not contribute to the decay with a transverse vector meson in the final state.

The function $\zeta_{J\perp}^{BK^*}(z)$ appearing in the factorizable term is defined as a convolution of the jet function with the light-cone wave functions of the K^* and B mesons,

$$\zeta_{J\perp}^{BK^*}(z) = \frac{f_B f_{K^*}^T}{m_B} \int dx dk_+ J_\perp(x, z, k_+) \phi_B^+(k_+) \phi_{K^*}^\perp(x). \quad (46)$$

Using the result for the jet function Eq. (22) at leading order in $\alpha_s(\mu_c)$, the integrals can be performed explicitly, and the function $\zeta_{J\perp}(z)$ is given by

$$\zeta_{J\perp}^{BK^*}(z) = \frac{\pi \alpha_s C_F}{N_c} \frac{f_B f_{K^*}^T}{m_B} \frac{1}{\lambda_{B+}} \frac{\phi_{K^*}^\perp(z)}{\bar{z}} \quad (47)$$

with the first inverse moment of the B wave function

$$\lambda_{B+}^{-1} = \int_0^\infty dk_+ \frac{\phi_+^B(k_+)}{k_+}. \quad (48)$$

The corresponding amplitudes for the charge conjugate mode $B \rightarrow K_n^* \ell^+ \ell^-$ are obtained from this by exchanging $H_+ \leftrightarrow H_-$. The vanishing of the right-handed helicity amplitude at leading order in Λ/m_b is a general result for the soft (nonfactorizable) component of the form factors in $B \rightarrow M_n$, combined with the absence of the factorizable contribution for this particular transition. This result is usually expressed as two exact symmetry relations among the tensor and vector $B \rightarrow V$ form factors at large recoil [3,15].

We proceed next to discuss the multibody decays $\bar{B} \rightarrow \bar{K} \pi \ell^+ \ell^-$, in the kinematical region with $q^2 \sim 4 \text{ GeV}^2$. According to the discussion in the subsection of Sec. II, the form of the factorization relation is different in the three regions of the Dalitz plot shown in Fig. 2. Our main interest is in region I, with one energetic kaon, and a soft pion. In this paper we prove a new factorization relation for the transverse helicity amplitudes in this region.

We consider for definiteness the mode $\bar{B}^0 \rightarrow K_n^- \pi^+ \ell^+ \ell^-$. Collecting the partial results in the subsection of Sec. II, we find the following results for the transverse helicity amplitudes in this mode (with $i = V, A$), valid in region I,

$$H_{\text{nf}}^{(i)}(\bar{B}^0 \rightarrow K_n^- \pi^+ \ell^+ \ell^-) = H_{\text{nf}}^{(i)} + \delta_{i,V} \frac{2}{3} H_{\text{sp}}^{(u)}, \quad (49)$$

$$H_{\text{f}}^{(i)}(\bar{B}^0 \rightarrow K_n^- \pi^+ \ell^+ \ell^-) = H_{\text{f}}^{(i)} \quad (50)$$

where the three terms correspond to the nonfactorizable, spectator, and factorizable terms in Eq. (43), respectively. They are given by

$$H_{\text{nf}}^{(i)} = c_1^{(i)}(\bar{n} \cdot p_K, \mu) \zeta_{\perp}^{BK\pi}, \quad (51)$$

$$H_{\text{f}}^{(i)} = -\frac{1}{2} f_K(\varepsilon_+^* \cdot p_\pi) \int_0^1 dz dx b_{\text{IR}}^{(i)}(z) \times \int_{-p_\pi^+}^{\infty} dk_+ J_{\parallel}(x, z, k_+) S(k_+) \phi_K(x), \quad (52)$$

$$H_{\text{sp}}^{(q)} = \frac{(4\pi)^2}{q^2} f_K(\bar{n} \cdot p_K)(\varepsilon_-^* \cdot p_\pi) \int_0^1 dx b_{\text{sp}}^{(q)}(x) \phi_K(x) \times \int_{-p_{\pi^-}}^{\infty} dk_- J_{\text{sp}}(k_-) S(k_-). \quad (53)$$

The nonfactorizable operators contribute only to the left-handed helicity amplitudes, and are given by the soft functions $\zeta_{\perp}^{BK\pi}$. They are the same for both $i = V$ and A amplitudes. Furthermore, the same soft functions would appear also in factorization relations for semileptonic decays into multibody states, such as $\bar{B} \rightarrow \pi_n \pi \ell \bar{\nu}$. This universality is the analog of the form factor relations for the nonfactorizable amplitudes [3,15], well known from one-body decays, to the multibody case.

The factorizable operators give nonvanishing contributions $H_{\text{f}}^{(i)}$ to the right-handed helicity amplitudes. The

appearance of these contributions is a new effect, specific to the multibody decays [10,26]. On the other hand, the spectator operator contributes only to the left-handed helicity amplitudes.

The helicity amplitudes for all other $\bar{B} \rightarrow K_n \pi \ell^+ \ell^-$ decays can be obtained in a similar way. The results are tabulated in Table I.

The structure of these results displays universality of hard-collinear effects. This is manifested as the fact that all factorizable terms $H_{\text{f}}^{(i)}$ depend on the same x, k_+ convolution,

$$I_{\text{f}}(z, p_\pi) \equiv \int_0^1 dx \int_{-p_\pi^+}^{\infty} dk_+ J_{\parallel}(x, z, k_+) S(k_+) \phi_K(x), \quad (54)$$

and all spectator contributions depend on the same k_- integral,

$$I_{\text{sp}}(n \cdot q) = \int_{-p_{\pi^-}}^{\infty} dk_- J_{\text{sp}}(k_-) S(k_-). \quad (55)$$

This universality is similar to that appearing in other factorization relations in exclusive decays. Examples are the relation between the rare leptonic decays $B_s \rightarrow \ell^+ \ell^- \gamma$ and the radiative leptonic decay $B \rightarrow \gamma \ell \bar{\nu}$ [21], and the relation among factorizable contributions in heavy-light form factors at large recoil, and the nonleptonic B decays into two light mesons [20]. The integral $I_{\text{sp}}(n \cdot q)$ appears also in the leading order factorization relation for exclusive semileptonic radiative decay $B \rightarrow \pi \gamma \ell \bar{\nu}$ [25], and could be determined from measurements of this decay.

Treating the spectator amplitude using the $n \cdot q \Lambda/Q^2$ expansion, the amplitude $H_{\text{sp}}^{(q)}$ can be obtained as explained from Eq. (53), at leading order in this expansion, by the substitution of Eq. (32),

$$H_{\text{sp}}^{(q)} = -\frac{4\pi^2}{q^2 \bar{n} \cdot q} f_K m_B \bar{n} \cdot p_K(\varepsilon_-^* \cdot p_\pi) f_T(t^2) \times \int_0^1 dz b_{\text{sp}}^{(q)}(z) \phi_K(z). \quad (56)$$

In the numerical estimates of this paper, we will use the leading order chiral perturbation theory result in Eq. (39)

TABLE I. The transverse helicity amplitudes $H_{\pm}^{(i)}$ with $i = V, A$, for the different charge states in $\bar{B} \rightarrow \bar{K}_n \pi \ell^+ \ell^-$ decays, at leading order in Λ/Q . The building blocks $H_{\text{nf}}^{(i)}$, $H_{\text{f}}^{(i)}$, $H_{\text{sp}}^{(q)}$ are given in Eqs. (51)–(53).

	$H^{(i)}$	$H_{\pm}^{(i)}$
$\bar{B}^0 \rightarrow K^- \pi^+ \ell^+ \ell^-$	$H_{\text{nf}}^{(i)} + \delta_{i,V} \frac{2}{3} H_{\text{sp}}^{(u)}$	$H_{\text{f}}^{(i)}$
$K_S \pi^0 \ell^+ \ell^-$	$\frac{1}{2} (H_{\text{nf}}^{(i)} - \delta_{i,V} \frac{1}{3} H_{\text{sp}}^{(d)})$	$\frac{1}{2} H_{\text{f}}^{(i)}$
$B^- \rightarrow K^- \pi^0 \ell^+ \ell^-$	$\frac{1}{\sqrt{2}} (H_{\text{nf}}^{(i)} + \delta_{i,V} \frac{2}{3} H_{\text{sp}}^{(u)})$	$\frac{1}{\sqrt{2}} H_{\text{f}}^{(i)}$
$K_S \pi^- \ell^+ \ell^-$	$\frac{1}{\sqrt{2}} (H_{\text{nf}}^{(i)} - \delta_{i,V} \frac{1}{3} H_{\text{sp}}^{(d)})$	$\frac{1}{\sqrt{2}} H_{\text{f}}^{(i)}$

for the soft function $S(k_+)$ in the resonance region $k_+ > 0$. The contribution to the k_+ convolutions in $H_f^{(i)}$ and $H_{\text{sp}}^{(q)}$ from the transition region $-n \cdot p_\pi \leq k_+ \leq 0$ will be neglected, which can be expected to be a good approximation for very soft pions (the region $E_\pi \leq 500$ MeV, corresponding to the lower shaded region in Fig. 2). We emphasize that, although the use of the chiral perturbation theory result for $S(k_+)$ is restricted to part of region (I), the factorization relations proved in this paper are valid over the entire region (I). Their unrestricted application requires a model for the soft function $S(k_+)$ whose validity extends beyond the limitations of chiral perturbation theory.

With these approximations, additional universality emerges, connecting the amplitudes in this problem to other B decays, to all orders in the perturbative expansion at the hard-collinear scale. The factorizable helicity amplitudes $H_f^{(i)}$ take a simpler form, and can be written as

$$H_f^{(i)} = \frac{1}{2} m_B^2 S_R(p_\pi) \int_0^1 dz b_{1R}^{(i)}(z) \zeta_J^{BK}(z) \quad (57)$$

where the nonperturbative dynamics is contained in the factorizable convolution defined as

$$\zeta_J^{BK}(z) = \frac{f_B f_K}{m_B} \int_0^1 dx \int_0^\infty dk_+ J_{\parallel}(x, z, k_+) \phi_B^+(k_+) \phi_K(x). \quad (58)$$

The same function appears also in the factorizable contribution to the $B \rightarrow K$ form factors at large recoil [5], and in the factorization relation for nonleptonic decays $B \rightarrow KM$ with $M = \pi, K, \dots$ a light meson [20]. The pion momentum dependence in Eq. (57) is contained in the function $S_R(p_\pi)$ given by [26]

$$S_R(p_\pi) = \frac{g}{f_\pi} \frac{\varepsilon_+^* \cdot p_\pi}{v \cdot p_\pi + \Delta - i\Gamma_{B^*}/2} \quad (59)$$

with $\Delta = m_{B^*} - m_B \simeq 50$ MeV.

A similar result is obtained for the spectator amplitude at leading order in chiral perturbation theory, for which we find (treating the q^2 as a hard-collinear scale)

$$H_{\text{sp}}^{(q)} = \frac{4\pi^2}{q^2} f_B f_K m_B (\bar{n} \cdot p_K) \left(\frac{g}{f_\pi} \frac{\varepsilon_-^* \cdot p_\pi}{v \cdot p_\pi + \Delta} \right) \times \int_0^1 dx b_{\text{sp}}^{(q)}(x) \phi_K(x) \int_0^\infty dk_- J_{\text{sp}}(k_-) \phi_+^B(k_-). \quad (60)$$

The k_- convolution in this relation is identical to that appearing in the leading order factorization relation for radiative semileptonic decays $B \rightarrow \gamma\ell\bar{\nu}$.

Adopting the approach of expanding in $n \cdot q\Lambda/q^2$, the result for $H_{\text{sp}}^{(q)}$ requires the $B \rightarrow \pi$ form factors, for which one finds at leading order in heavy hadron chiral perturbation theory [27–30]

$$m_B f_T(t^2) = f_+(t^2) - f_-(t^2) = -g \frac{f_B m_B}{f_\pi} \frac{1}{E_\pi + \Delta}. \quad (61)$$

We will use these expressions together with Eq. (56) in the numerical evaluations of Sec. V.

IV. DECAY RATES AND THE FB ASYMMETRY

The differential decay rate for $B \rightarrow K\pi\ell^+\ell^-$ is given by (see, e.g., [32])

$$\frac{1}{\Gamma_0} \frac{d^2\Gamma}{dq^2 d\cos\theta_+ dM_{K\pi}^2 dE_\pi} = \frac{q^2}{2(4\pi)^3 m_B^2 m_b^5} \{ 2\sin^2\theta_+ (|H_0^V|^2 + |H_0^A|^2) + (1 + \cos^2\theta_+) (|H_+^V|^2 + |H_+^A|^2 + |H_-^V|^2 + |H_-^A|^2) + 4\cos\theta_+ \text{Re}(H_-^V H_-^{A*} - H_+^V H_+^{A*}) \} \quad (62)$$

with $\Gamma_0 = G_F^2 \alpha^2 / (32\pi^4) |\lambda_t^{(s)}|^2 m_b^5$. We denoted θ_+ the angle between the direction of the positron momentum and the decay axis in the rest frame of the lepton pair, for a fixed configuration of the hadronic state $K\pi$ defined by $(M_{K\pi}, E_\pi)$.

Integrating over $\cos\theta_+$ one finds for the FBA defined as in Eq. (1)

$$A_{\text{FB}} \propto \text{Re}(H_-^V H_-^{A*} - H_+^V H_+^{A*}). \quad (63)$$

This defines a triply differential asymmetry depending on $(q^2, M_{K\pi}, E_\pi)$. Integrating also over E_π gives a doubly differential A_{FB} depending only on $(q^2, M_{K\pi})$. We denote them with the same symbol, and distinguish between them by their arguments.

The condition for a zero of the FBA can be written down straightforwardly using the expressions for the helicity

amplitudes in factorization given previously in Sec. III. The equation for the zero is different in the two regions (I) and (II), according to the different form of the factorization relations in each of them. It is convenient to write this equation in a common form in both regions, as

$$\text{Re}(c_1^{(V)}(M_{K\pi}, q^2) - a) = 0. \quad (64)$$

The quantity a stands for the contribution of the factorizable- and spectator-type amplitudes, and, in general, is a function of all kinematical variables $(M_{K\pi}, q^2, E_\pi)$.

In region (II) with a collinear kaon and pion, this correction is given by the (complex) quantity

$$a_{\text{II}} = -\frac{m_B^2}{E_{K^*} \bar{n} \cdot q} C_7^{\text{eff}} \frac{\zeta_{J\perp}^{BK^* \text{eff}}}{\zeta_{\perp}^{BK^*}} \quad (65)$$

where the factorizable coefficient, $\zeta_{J\perp}^{BK^* \text{eff}}$, which has implicit dependence on $(M_{K\pi}, q^2)$, is defined by

$$\int_0^1 dz b_{1L}^{(V)}(z) \zeta_{J\perp}^{BK^*}(z) = -\frac{2n \cdot q}{q^2} C_7^{\text{eff}} \zeta_{J\perp}^{BK^* \text{eff}}. \quad (66)$$

The zero of the FBA in region (II) was considered previously in Refs. [2,4,14,16,17], treating the problem as a one-body decay $B \rightarrow K^* \ell^+ \ell^-$.

In region (I) with a soft pion and a collinear kaon, this correction contains two terms, arising from the spectator and the factorizable contributions, respectively. We adopt everywhere in the following the leading order chiral perturbation theory results for the amplitudes given in Eqs. (57) and (60). Working at tree level in matching at the scale $\mu = Q$, but to all orders in the hard-collinear scale, one finds

$$a_1 = -e_q S_R^*(p_\pi) \frac{h_{\text{sp}}^{(q)}}{\zeta_{\perp}^{BK\pi}} + \left(\frac{m_B^2}{4E_K}\right)^2 |S_R(p_\pi)|^2 C_9^{\text{eff}} \frac{\zeta_J^{BK} \zeta_J^{BK \text{eff}}}{|\zeta_{\perp}^{BK\pi}|^2}. \quad (67)$$

The first term contains the contribution of the spectator amplitude, and depends on the charge states of the final and initial state through the superscript $q = u, d$, denoting the flavor of the quark attaching to the photon. The quantity $h_{\text{sp}}^{(q)}$, given by

$$h_{\text{sp}}^{(q)} = \frac{4\pi^2}{q^2} f_B f_K m_B (\bar{n} \cdot p_K) \int_0^1 dx b_{\text{sp}}^{(q)}(x) \phi_K(x) \times \int_0^\infty dk_- J_{\text{sp}}(k_-) \phi_+^B(k_-), \quad (68)$$

depends on kinematic variables only through the explicit factor of $\bar{n} \cdot p_K / q^2$, to the order in which we are working. For completeness, we quote also the expression for this amplitude at leading order in $n \cdot q \Lambda / q^2$, which is used in the actual numerical computation of Sec. V,

$$h_{\text{sp}}^{(q)} = \frac{4\pi^2}{q^2 \bar{n} \cdot q} f_B f_K m_B (\bar{n} \cdot p_K). \quad (69)$$

The first term in Eq. (67) contributes to $\text{Re}(a_1)$ with an undetermined sign, depending on the unknown $\zeta_{\perp}^{BK\pi}$.

The second term in Eq. (67) is due to the right-handed helicity amplitudes $H_+^{(i)}$. Its dependence on E_π is explicit in $S_R(p_\pi)$, and the remaining factors depend only on $(M_{K\pi}, q^2)$. The factorizable coefficients in the numerator are defined as

$$\int_0^1 dz b_{1R}^{(V)}(z) \zeta_J^{BK}(z) \equiv \frac{1}{\bar{n} \cdot p_K} C_9^{\text{eff}} \zeta_J^{BK \text{eff}}, \quad (70)$$

$$\int_0^1 dz b_{1R}^{(A)}(z) \zeta_J^{BK}(z) \equiv \frac{1}{\bar{n} \cdot p_K} C_{10} \zeta_J^{BK} \quad (71)$$

where we kept only the tree level matching result for

$b_{1R}^{(A)}(z)$. Numerical evaluation of these factorizable amplitudes in the next section shows that the contribution of this term to $\text{Re}(a_1)$ is positive.

To explore the implications of these results, let us assume as a starting point that the helicity amplitudes H_\pm in both regions are dominated by the nonfactorizable contributions, proportional to $\zeta_{\perp}^{BK\pi}$ [in region (I)], and $\zeta_{\perp}^{BK^*}$ [in region (II)]. This corresponds to taking $a = 0$ in both regions. In this approximation, the condition for the zero of the FBA reads simply $\text{Re}(c_1^{(V)}) = 0$, which can be solved exactly. For the $B \rightarrow M_n$ transition, this condition reproduces the well-known result following from the large energy form factor relations [2,4,14–16].

Since the zero of the FBA is related to the vanishing of the Wilson coefficient $\text{Re}(c_1^{(V)})$, such a zero must be present also for B decays into multibody states containing one energetic kaon [33]. In particular, the FBA in $B \rightarrow K_n \pi \ell^+ \ell^-$ must have a zero at a certain point $q_0^2 = q_0^2(M_{K\pi})$, which depends only on the invariant mass of the hadronic system. Adding the second term in Eq. (64) shifts the position of this zero, and introduces a dependence on the pion energy, $q_0^2 = q_0^2(M_{K\pi}, E_\pi)$. This extends the well-known result for the zero of the FBA in $B \rightarrow K^* \ell^+ \ell^-$ to multibody hadronic states.

It is interesting to comment on existing computations of the decay amplitudes and FB asymmetry in $B \rightarrow K \pi \ell^+ \ell^-$ [4,14,16,17], which keep only the K^* resonant amplitude. Of course, this is justified in region (II), where the pion and the kaon are collinear. However, in region (I) this contribution is in fact parametrically suppressed, since by leading order soft-collinear factorization, the $K_n^* K_n \pi_S$ vertex does not exist at leading order in $O(\Lambda/m_b)$.

Computing the factorizable corrections in region (I) parametrized by the term a_1 requires that we know the nonfactorizable soft function $\zeta_{\perp}^{BK\pi}$. There are several possible ways of determining $\zeta_{\perp}^{BK\pi}$ from data. For example, according to Eq. (49), the helicity amplitude $|H_-^A|$ receives no factorizable or spectator contributions. Assuming that it can be isolated, its measurement would give a clean determination of $|\zeta_{\perp}^{BK\pi}|$. Another method involves measuring $H_-^{(V)}$ in decays with a neutral kaon in the final state, for which the spectator contribution is small (see Table I). We assume that the factorizable coefficients can be computed in perturbation theory. We postpone a detailed numerical analysis for Sec. V, and discuss in the following general properties of the zero of the FBA which are independent of the details of the hadronic parameters.

Qualitative discussion of the zero of the FBA

Before proceeding with the details of the numerical study, we would like to discuss some of the qualitative properties of the zero of the FBA in the multibody decay $B \rightarrow K \pi \ell^+ \ell^-$. The general behavior of the solution can be seen by studying the solutions of the simplified equation

$$C_9 + 2m_b \frac{n \cdot q}{q^2} C_7 - a(M_{K\pi}) = 0 \quad (72)$$

where $a(M_{K\pi})$ denotes the factorizable correction. In this simplified version of Eq. (64) we have neglected additional dependence of a on q^2 , which is adequate if one is interested in the qualitative change in the zero in the FBA at fixed q^2 . We also have neglected here the radiative corrections. They introduce small logarithmic dependence on $M_{K\pi}$, and do not change the qualitative features of the solution.

It is convenient to write Eq. (72) in an equivalent form

$$\frac{n \cdot q}{q^2} = -\frac{1}{2m_b C_7} (C_9 - a(M_{K\pi})) \equiv \frac{F(M_{K\pi})}{m_B}. \quad (73)$$

The solution to this equation gives the location, q_0^2 , of the zero of the FBA:

$$q_0^2(M_{K\pi}) = \frac{m_B^2}{F} - \frac{M_{K\pi}^2}{F-1}. \quad (74)$$

The condition that the FBA zero lies in the physical region, $0 \leq q_0^2 \leq (m_B - M_{K\pi})^2$, imposes constraints on $F = F(M_{K\pi})$. The upper bound $q_0^2 \leq (m_B - M_{K\pi})^2$ implies that $F(M_{K\pi}) \geq 1$, while from $0 \leq q_0^2$ we learn that

$$F(M_{K\pi}) \geq 1 + \frac{M_{K\pi}^2}{m_B^2 - M_{K\pi}^2} \geq 1. \quad (75)$$

In terms of the correction a the condition for the existence of a FBA zero is therefore

$$a(M_{K\pi}) \leq C_9 + \frac{2m_b m_B}{m_B^2 - M_{K\pi}^2} C_7. \quad (76)$$

If the function $F = F(M_{K\pi})$ is roughly constant and satisfies the condition in (75), then Eq. (74) gives that the zero of the FBA decreases with increasing $M_{K\pi}$. These conditions hold if $a(M_{K\pi}) \ll C_9$. Since we expect the correction term to be small we also expect the FBA zero to decrease as $M_{K\pi}$ increases.

We can gain further insights into the solution by considering q_0^2 as a function of F for fixed $M_{K\pi}$ in Eq. (74). Figure 3 shows plots of q_0^2 vs F for several fixed values of $M_{K\pi}$. The sequence of curves moves down with increasing $M_{K\pi}$, which just restates the observation that at fixed F the zero decreases with increasing $M_{K\pi}$. The point $F = F_0$ corresponds to $a = 0$. The maxima of the curves are at $F_*(M_{K\pi}) = (1 - M_{K\pi}/m_B)^{-1}$, and, for physical values, they lie to the left of F_0 , that is, $F_* < F_0$. To see how q_0^2 depends on $M_{K\pi}$ when $F = F(M_{K\pi})$ is not constant, consider as a starting value a point on the top curve. An increase in $M_{K\pi}$ first moves the point down to a lower curve (as if F were constant), and then also along the lower curve to a different value of F . The region to the right of F_* is most interesting since we expect the physical function to lie in a region of F close to F_0 . In this region, if F increases with $M_{K\pi}$ then q_0^2 decreases with (increasing) $M_{K\pi}$. The

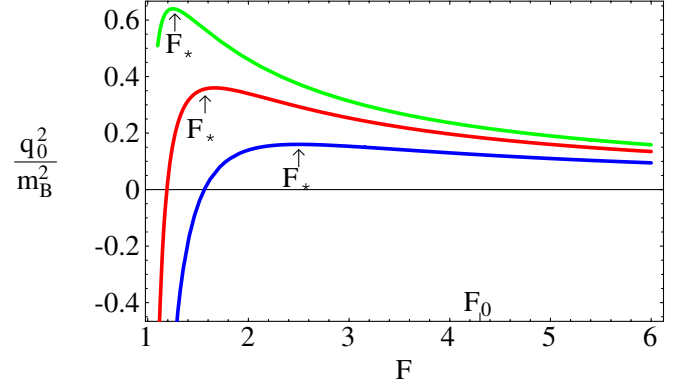


FIG. 3 (color online). Plot of the zero in the forward-backward asymmetry as a function of the parameter F of Eq. (73), for several values of $M_{K\pi}$. The top (green) curve has the lowest $M_{K\pi}$ and the bottom (blue) one has the highest $M_{K\pi}$. F_0 corresponds to the value of F in the absence of factorizable and spectator corrections. The maxima of the curves, labeled by F_* , lie to the left of F_0 for all relevant values of $M_{K\pi}$.

opposite is not necessarily true: whether q_0^2 increases with $M_{K\pi}$ or not depends on how steeply F decreases with $M_{K\pi}$.

Next, we would like to understand what the effect of changing $a \rightarrow a + \delta a$ is, corresponding to adding correction terms sequentially. A shift $\delta F > 0$ for fixed $M_{K\pi}$ corresponds to moving to the right along a fixed curve. In the region to the right of F_* this decreases q_0^2 . Note that $\delta a = (2m_b C_7 / m_B C_9) \delta F$ and $C_7 / C_9 < 0$. Therefore, an increase in a gives an increase in q_0^2 .

V. NUMERICAL STUDY

We investigate in this section the numerical effects of the new right-handed amplitude on the position of the zero of the FB asymmetry. As explained, we treat separately the decay amplitudes in the two regions (I) and (II), and ignore the contribution from region (III). For definiteness, we consider here the mode $\bar{B}^0 \rightarrow K^- \pi^+ \ell^+ \ell^-$ for which the soft pion detection efficiency is better than for the neutral pion modes.

The decay amplitudes in the collinear region (II) will be represented by a Breit-Wigner model, as

$$H_-^V(\bar{B}^0 \rightarrow K^- \pi^+ \ell^+ \ell^-) = h_-^V(\bar{B} \rightarrow \bar{K}^*) g_{K^* K \pi}(\varepsilon_-^* \cdot p_\pi) \times \text{BW}_{K^*}(M_{K\pi}), \quad (77)$$

$$H_-^A(\bar{B}^0 \rightarrow K^- \pi^+ \ell^+ \ell^-) = h_-^A(\bar{B} \rightarrow \bar{K}^*) g_{K^* K \pi}(\varepsilon_-^* \cdot p_\pi) \times \text{BW}_{K^*}(M_{K\pi}), \quad (78)$$

and $H_+^{V,A} = 0$, with $h_-^{V,A}$ the one-body helicity amplitudes for $B \rightarrow K^* \ell^+ \ell^-$ given above in Eq. (45). The Breit-Wigner function corresponding to a K^* resonance is defined as

$$\text{BW}_{K^*}(M) = \frac{1}{M^2 - M_{K^*}^2 + iM_{K^*}\Gamma_{K^*}}. \quad (79)$$

Finally, the $K^*0K^+\pi^-$ coupling with a charged pion can be determined from the total $K^* \rightarrow K\pi$ width, $\Gamma = g_{K^*K\pi}^2 P_\pi^3 / (16\pi m_{K^*}^2)$, with the result $g_{K^*K\pi} = 9.1$.

The factorization relations in region (I) require the non-factorizable amplitude $\zeta_{\perp}^{BK\pi}$. In the absence of experimental information about this quantity, we adopt a K^* resonance model for it, defined as

$$\zeta_{\perp}^{BK\pi}(M_{K\pi}, E_\pi) = \bar{n} \cdot p_{K^*} \zeta_{\perp}^{BK^*} g_{K^*K\pi}(\varepsilon_-^* \cdot p_\pi) \text{BW}_{K^*}(M_{K\pi}), \quad (80)$$

where the kinematical factor $\bar{n} \cdot p_{K^*} = 2E_{K^*} = \frac{1}{m_B}(m_B^2 - q^2)$ can be chosen corresponding to an on-shell K^* meson. For the kinematical dependence of the soft function $\zeta_{\perp}^{BK^*}(q^2)$ we adopt a modified pole shape [17,36]

$$\zeta_{\perp}^{BK^*}(q^2) = \frac{\zeta_{\perp}^{BK^*}(0)}{1 - 1.55 \frac{q^2}{m_B^2} + 0.575 \left(\frac{q^2}{m_B^2}\right)^2} \quad (81)$$

and quote results corresponding to the two values $\zeta_{\perp}^{BK^*}(0) = 0.3$ and 0.1 . These two choices should cover both cases of soft-dominated and hard-dominated tensor form factor.

We will use this model to define a FBA differential in $(q^2, M_{K\pi})$, integrated over the pion energy E_π . Separating the contributions from regions (I) and (II), this is given by

$$A_{\text{FB}}(q^2, M_{K\pi}) = \int_{(I)} dE_\pi \text{Re}[H^V H_-^{A*} - H_+^V H_+^{A*}] + \int_{(II)} dE_\pi \text{Re}[H^V H_-^{A*}]. \quad (82)$$

The integration over E_π can be simplified by approximating $\bar{n} \cdot p_K \approx m_B - \bar{n} \cdot q$ in region I. Then the result can be expressed in terms of three phase space integrals $I_{0,1,2}$, arising from region (I), and another integral \bar{I}_0 in region (II), defined as

$$I_j = \int_{E_\pi^{\min}}^{E_\pi^{\text{cut}}} dE_\pi \frac{|\varepsilon_+ \cdot p_\pi|^2}{(E_\pi + \Delta)^j}, \quad j = 0, 1, 2, \quad (83)$$

$$\bar{I}_0 = \int_{E_\pi^{\text{cut}}}^{E_\pi^{\max}} dE_\pi |\varepsilon_+ \cdot p_\pi|^2. \quad (84)$$

These integrals depend implicitly on $(M_{K\pi}, q^2)$. Numerically, for $E_\pi^{\text{cut}} = 500$ MeV we find $I_0 = 0.019$ GeV³, $I_1 = 0.047$ GeV², $I_2 = 0.12$ GeV, and $\bar{I}_0 = 0.12$ GeV³, at $M_{K\pi} = 1$ GeV and $q^2 = 4$ GeV².

The zero of the FBA is given by the solution of the equation

$$\text{Re}[c_1^{(V)} - a_{\text{sp}} - a_{\text{f}}] = 0, \quad (85)$$

where a_{sp} and a_{f} denote the factorizable contributions,

arising from the spectator amplitude, and from the factorizable (both in one-body and in the two-body amplitudes). The a_{sp} coefficient is given by

$$a_{\text{sp}} = -\frac{2}{3} \frac{g}{f_\pi} \frac{h_{\text{sp}}^{(u)}}{\bar{n} \cdot p_{K^*} g_{K^*} \zeta_{\perp}^{BK^*}} \frac{I_1}{I_0 + \bar{I}_0} \times [M_{K\pi}^2 - M_{K^*}^2 + iM_{K^*}\Gamma_{K^*}] \quad (86)$$

where $h_{\text{sp}}^{(q)}$ is defined in Eq. (69).

The factorizable term contains contributions from the one-body decay amplitude, and from the new right-handed amplitude appearing in the two-body mode,

$$a_{\text{f}} = -\frac{2m_B^2}{\bar{n} \cdot p_{K^*} \bar{n} \cdot q} C_7^{\text{eff}} \frac{\zeta_{J\perp}^{BK^* \text{eff}}}{\zeta_{\perp}^{BK^*}} \frac{\bar{I}_0}{I_0 + \bar{I}_0} + \frac{g^2 m_B^4}{4f_\pi^2 g_{K^*K\pi}^2 (\bar{n} \cdot p_K)^2 (\bar{n} \cdot p_{K^*})^2} C_9^{\text{eff}} \frac{\zeta_J^{BK} \zeta_J^{BK \text{eff}}}{(\zeta_{\perp}^{BK^*})^2} \times (M_{K\pi}^2 - M_{K^*}^2)^2 \frac{I_2}{I_0 + \bar{I}_0}. \quad (87)$$

We compute the factorizable matrix elements using the leading order jet functions from Eq. (22). This gives for the integrals of the factorizable functions $\zeta_J(z)$

$$\zeta_J^{BK} = \frac{\pi \alpha_s C_F}{N_c} \frac{f_B f_K}{m_B} \frac{1}{\lambda_{B^+}} \int_0^1 dz \frac{\phi_K(z)}{1-z}, \quad (88)$$

$$\zeta_{J\perp}^{BK^*} = \frac{\pi \alpha_s C_F}{N_c} \frac{f_B f_{K^*}^T}{m_B} \frac{1}{\lambda_{B^+}} \int_0^1 dz \frac{\phi_{K^*}^T(z)}{1-z}, \quad (89)$$

and for the effective ones

$$\zeta_J^{BK \text{eff}} = \zeta_J^{BK} \left[\left(1 + \frac{2m_b(\mu) \bar{n} \cdot q}{q^2} \frac{C_7^{\text{eff}}}{C_9^{\text{eff}}} \right) + \frac{e_u E_K^2}{m_b \bar{n} \cdot q} \frac{\bar{C}_2}{C_9^{\text{eff}}} I_J^{BK} \right], \quad (90)$$

$$\zeta_{J\perp}^{BK^* \text{eff}} = \zeta_{J\perp}^{BK^*} \left[1 + \frac{2e_u E_{K^*}}{8m_b} \frac{\bar{C}_2}{C_7^{\text{eff}}} I_{J\perp}^{BK^*} \right], \quad (91)$$

where

$$I_J^{BK} = \int_0^1 dz t_\perp(z, m_c) \phi_K(z) / \int_0^1 dz \frac{\phi_K(z)}{1-z}, \quad (92)$$

$$I_{J\perp}^{BK^*} = \int_0^1 dz t_\perp(z, m_c) \phi_{K^*}^T(z) / \int_0^1 dz \frac{\phi_{K^*}^T(z)}{1-z}. \quad (93)$$

For the computation of the integrals, we use the $K^{(*)}$ light-cone wave functions

$$\phi_K(x) = 6x\bar{x}(1 + 3a_{1K}(2x-1) + \frac{3}{2}a_{2K}[5(2x-1)^2 - 1]) \quad (94)$$

and analogous for $\phi_{K^*}^\perp(x)$, with coefficients a_{iK^*} . The values of the first two Gegenbauer moments are given in Table II. Also listed there are the remaining hadronic parameters used in the computation.

TABLE II. Input parameters used in the numerical computation.

m_b^{IS}	4.68 ± 0.03 GeV [37]	f_K	160 MeV
$\bar{m}_c(\bar{m}_c)$	1224 ± 57 MeV [38]	$f_{K^*}^T$	175 MeV
$\alpha_s(M_Z)$	0.119	a_{1K}	0.3
λ_{B^+}	350 MeV	a_{2K}	0.1
g	0.5 [39]	$a_{1K^*}^\perp$	0.2
f_B	200 MeV	$a_{2K^*}^\perp$	0.1
$\lambda_u^{(s)}/\lambda_t^{(s)}$	$-0.0106 + 0.0174i$ [40]	$g_{K^*K\pi}$	9.1

 TABLE III. Results for the effective Wilson coefficients and factorizable and spectator matrix elements. The values of the effective Wilson coefficients are at the scale $\mu = 4.8$ GeV and $q^2 = 4$ GeV². The factorizable matrix elements are computed at the scale $\mu_c = 1.5$ GeV.

$[C_9^{\text{eff}}]_{\text{NNLL}}$	$4.579 + 0.082i$	ζ_J^{BK}	0.036
$[C_7^{\text{eff}}]_{\text{NNLL}}$	$-0.388 - 0.020i$	$\zeta_{J\perp}^{BK^*}$	0.035
		$\frac{q^2 \bar{n} \cdot q}{\bar{n} \cdot p_K} h_{\text{sp}}^{(u)}$	$(0.135 + 0.124i)$ GeV ³

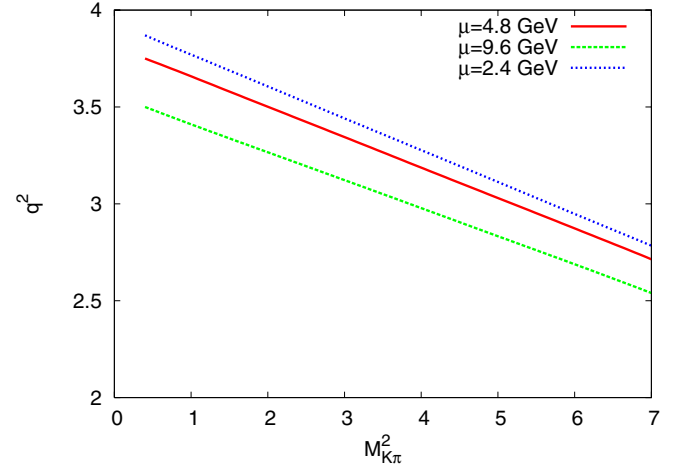
The resulting values of the factorizable matrix elements are tabulated in Table III, which also lists the effective Wilson coefficients $C_{7,9}^{\text{eff}}$ (computed at $\mu = 4.8$ GeV, $q^2 = 4$ GeV²). The effective matrix elements $\zeta_J^{BK^{\text{eff}}}$ and $\zeta_{J\perp}^{BK^{\text{eff}}}$ depend on $(M_{K\pi}, q^2)$, partly through implicit dependence of the integrals I_J^{BK} and $I_{J\perp}^{BK^*}$. To gain some understanding of the relative importance of the terms that contribute to the effective matrix elements, we quote these integrals at $M_{K\pi} = m_{K^*}$, $q^2 = 4.0$ GeV²: $I_J^{BK} = -0.704 - 2.564i$ and $I_{J\perp}^{BK^*} = -0.566 - 2.67i$.

It is straightforward to estimate the numerical importance of each term in the correction a appearing in Eq. (64) for the zero of the FBA. To this end, we evaluate at $M_{K\pi} = m_{K^*}$, $q^2 = 4.0$ GeV², and use $E_\pi^{\text{cut}} = 500$ MeV and $\zeta_{J\perp}^{BK^*}(0) = 0.3$ in the model of Eq. (81). We obtain (all in GeV units)

$$\begin{aligned}
 a_{\text{sp}} &\sim -(0.016 + 0.014i)(M_{K\pi}^2 - M_{K^*}^2 + iM_{K^*}\Gamma_{K^*}), \\
 a_f &\sim (0.533 + 0.321i) + (0.004 - 0.003i)(M_{K\pi}^2 - M_{K^*}^2)^2.
 \end{aligned} \tag{95}$$

We have kept explicit the rapidly varying dependence on the inverse Breit-Wigner function, so we may get some idea of the relative size of the coefficients. The first term in the factorizable amplitude can be regarded as a negative correction to $\text{Re}(C_9^{\text{eff}})$ in $c_1^{(V)}$ of about $\sim 10\%$. Thus it effectively shifts the zero of the FBA upwards by the same amount. The second terms in a_f and a_{sp} are negligible on resonance but may be important at large $M_{K\pi}$.

Using this model, we solve for the zeros of the FBA in $B \rightarrow K\pi\ell^+\ell^-$ decays, finding q_0^2 for given $(M_{K\pi}, E_\pi^{\text{cut}})$.


 FIG. 4 (color online). Plot of the position of the zero of the forward-backward asymmetry $q_0^2 = q_0^2(M_X)$ as a function of the invariant mass of the $K\pi$ system, obtained by neglecting the factorizable contributions to the helicity amplitudes, for different values of the renormalization point μ .

The results are shown in Figs. 4 and 5. In Fig. 4 we plot the result for $q_0^2(M_{K\pi})$ obtained by neglecting the factorizable and spectator terms [the solution to $\text{Re}(c_1^{(V)}) = 0$] for three values of the renormalization scale, $\mu = 2.4, 4.8, 9.6$ GeV. We used here next-to-next-to-leading logarithmic (NNLL) results for the Wilson coefficients and the 2-loop matrix elements of the operators $O_{1,2}$ obtained in Ref. [41]. The position of the zero at threshold is

$$q_0^2|_{M_{K\pi}=M_{K^*+M_\pi} = 3.75^{+0.12}_{-0.25} \text{ GeV}^2 \tag{96}$$

where the uncertainty includes only the scale dependence. This result depends only mildly on $M_{K\pi}$, as seen from Fig. 4.

In Fig. 5 we show also the effect of including the factorizable and spectator terms in Eq. (85), for three values of the cutoff on the pion energy $E_\pi^{\text{cut}} = 300$ MeV, 500 MeV, and 700 MeV separating regions I and II. The parameters used in evaluating this plot are listed in Tables II and III. For comparison, we present in Fig. 5 results with $\zeta_{J\perp}^{BK^*}(0) = 0.3$ (left panel) and $\zeta_{J\perp}^{BK^*}(0) = 0.1$ (right panel). The latter choice effectively amplifies the factorizable and spectator corrections, and can be taken as a conservative upper bound of these effects. The dependence of the results on E_π^{cut} is an artifact of the separation of regions discussed above and indicates the uncertainty in this procedure. For this reason, we have taken rather extreme values of E_π^{cut} . The overall trend of $q_0^2(M_{K\pi})$ decreasing towards the right of the plots is readily understood from the qualitative discussion in the subsection of Sec. IV: it follows from the correction term a being small. Similarly, that the inclusion of spectator and factorizable corrections tends to increase the value of q_0^2 for fixed $M_{K\pi}$ follows from positivity of a .

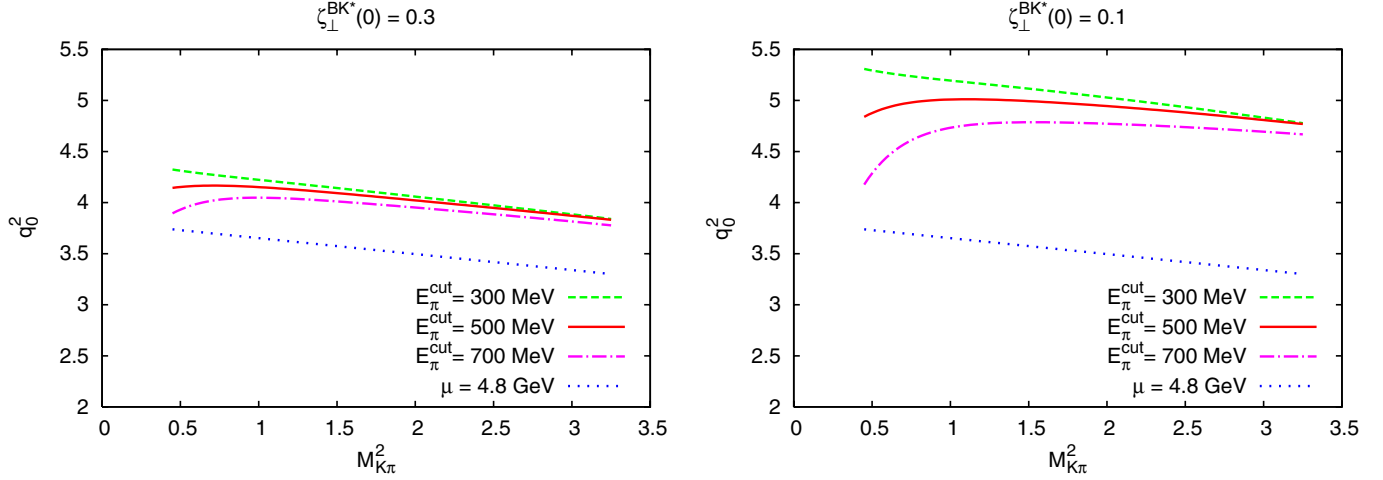


FIG. 5 (color online). Plot of the position of the zero of the forward-backward asymmetry $q_0^2 = q_0^2(M_{K\pi})$ as a function of the invariant mass of the $K\pi$ system. The plots show the change in the position of the zero due to the spectator and factorizable amplitudes, Eq. (64), for three values of the pion energy cutoff $E_\pi^{\text{cut}} = 300, 500, 700$ MeV separating regions I and II. The dotted (blue) line denotes the position of the zero in the absence of the factorizable and spectator contributions (for $\mu = 4.8$ GeV). The nonfactorizable matrix element is taken to be $\zeta_\perp^{\text{BK}^*}(0) = 0.3$ (left panel) and $\zeta_\perp^{\text{BK}^*}(0) = 0.1$ (right panel).

The results show a marked dependence (especially for small $M_{K\pi}$) of the zero position on the pion energy cutoff E_π^{cut} which separates regions (I) and (II). This is essentially due to the dominance of the factorizable contribution in region (II). A conservative way to use our results is to take the smaller value of $E_\pi^{\text{cut}} = 300$ MeV, for which the chiral perturbation theory result can be expected to be the most precise.

So far, our considerations were restricted to the case of $\bar{B}^0 \rightarrow K^- \pi^+ \ell^+ \ell^-$ decays. Going over to the CP conjugate mode $B^0 \rightarrow K^+ \pi^- \ell^+ \ell^-$, the position of the zero could change because of direct CP violation present in the spectator amplitude $H_{\text{sp}}^{(q)}$, which is furthermore enhanced by a $4\pi^2$ factor [see Eq. (53)]. Denoting the factorizable corrections analogous to a_{sp} and a_f for the CP conjugate mode with \bar{a}_i , we find

$$\bar{a}_{\text{sp}} \sim -(0.016 - 0.014i)(M_{K\pi}^2 - M_{K^*}^2 + iM_{K^*}\Gamma_{K^*}), \quad (97)$$

such that the CP asymmetry in the position of the zero is induced through the finite K^* width, and is small. Beyond tree level, such an effect will be introduced at order $\alpha_s(Q)$ through matching corrections to b_{sp} .

We consider next another observable, the slope of the curve for the zero of the FBA in Fig. 5. From Eq. (74), this is given by

$$\frac{dq_0^2(M_{K\pi}^2)}{dM_{K\pi}^2} = -\frac{1}{F-1} + \left(\frac{M_{K\pi}^2}{(F-1)^2} - \frac{m_B^2}{F^2} \right) \frac{dF(M_{K\pi}^2)}{dM_{K\pi}^2}. \quad (98)$$

$F(M_{K\pi}^2)$ is defined in Eq. (73) and depends on the Wilson coefficients $C_{7,9}$, also on the factorizable contributions $a(M_{K\pi})$. The last term contributes through the $M_{K\pi}$ dependence of $a(M_{K\pi})$, and is

$$\frac{dF(M_{K\pi}^2)}{dM_{K\pi}^2} = \frac{m_B}{2m_b C_7} \frac{da(M_{K\pi}^2)}{dM_{K\pi}^2} \simeq 0.02, \quad (99)$$

where we used the result in Eq. (95) for $a_{\text{sp}}(M_{K\pi})$ and neglected the tiny contribution from a_f . The contribution of this term to Eq. (98) is multiplied with a factor of order 1–2. Thus, even assigning this estimate a conservative error of $\sim 200\%$, its contribution to the slope in Eq. (98) for values $M_{K\pi} \sim 1$ GeV is negligible compared to the first term depending only on F (recall that $F \sim 4$). This is also seen in the curves in Fig. 5, whose slopes are essentially the same for all choices of the hadronic parameters considered. This shows that a measurement of the slope of the zero could provide a useful source of information about the Wilson coefficients $C_{7,9}$, but without the hadronic uncertainties associated with the absolute position of the zero.

VI. CONCLUSIONS

We studied in this paper the helicity structure of the exclusive rare $\bar{B} \rightarrow \bar{K} \pi \ell^+ \ell^-$ decays in the region of phase space with one energetic kaon and a soft pion. In this region the helicity amplitudes are given by new factorization relations, containing a universal soft matrix element, and a new nonperturbative matrix element for the $B \rightarrow \pi$ transition analogous to the off-forward parton distribution functions.

The most important difference with the $\bar{B} \rightarrow \bar{K}_n^* \ell^+ \ell^-$ decays at large recoil is the appearance in the multibody case of a nonvanishing right-handed helicity amplitude $\bar{B} \rightarrow [\bar{K}\pi]_{h=+1} \ell^+ \ell^-$ at leading order in Λ/m_b . This can be computed in factorization, in terms of the $B \rightarrow \pi$ off-forward matrix element of a nonlocal heavy-to-light operator. In the soft pion limit this nonperturbative matrix

element can be computed in chiral perturbation theory, and is related to the B -meson light-cone wave function [26].

We explored the implications of these results for the existence of a zero of the forward-backward asymmetry of the lepton momentum, pointing out two new results. First, the FBA has a zero also for nonresonant $B \rightarrow K\pi\ell^+\ell^-$ decays, occurring at a determined value of the dilepton invariant mass $q_0^2(M_{K\pi})$, depending on the hadronic invariant mass $M_{K\pi}$. Second, there are calculable corrections to the position of the zero, which can be computed in factorization. We use the factorization relations derived in this paper to compute these correction terms. We present explicit numerical results working at leading order in chiral perturbation theory [26], and show that the results for the zero of the FBA in $\bar{B} \rightarrow [\bar{K}\pi]_{K^*}\ell^+\ell^-$ hold to a good precision also in the nonresonant region.

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APPENDIX: EFFECTIVE WILSON COEFFICIENTS

We collect here for convenience the expressions for the effective Wilson coefficients used in the numerical study of Sec. IV. Working to NNLL order, they are given by

$$\begin{aligned}
 C_9^{\text{eff}} = & C_9 - \left(\bar{C}_1 + \frac{\bar{C}_2}{3} \right) \left(8G(m_c) + \frac{4}{3} \right) - \bar{C}_3 \left(8G(m_c) \right. \\
 & - \frac{4}{3}G(0) - \frac{16}{3}G(m_b) + \frac{2}{27} \left. \right) + \bar{C}_4 \left(4G(0) - \frac{8}{3}G(m_c) \right. \\
 & + \frac{16}{3}G(m_b) + \frac{14}{9} \left. \right) - \bar{C}_5 \left(8G(m_c) - 4G(m_b) - \frac{14}{27} \right) \\
 & - \bar{C}_6 \left(\frac{8}{3}G(m_c) - \frac{4}{3}G(m_b) + \frac{2}{9} \right) \\
 & - \frac{\alpha_s}{4\pi} \left[2\bar{C}_1 \left(F_1^{(9)}(q^2) + \frac{F_2^{(9)}(q^2)}{6} \right) \right. \\
 & \left. + \bar{C}_2 F_2^{(9)}(q^2) + C_8^{\text{eff}} F_8^{(9)} \right] \quad (\text{A1})
 \end{aligned}$$

and

$$\begin{aligned}
 C_7^{\text{eff}} = & C_7 - \frac{4}{9}\bar{C}_3 - \frac{4}{3}\bar{C}_4 + \frac{1}{9}\bar{C}_5 + \frac{1}{3}\bar{C}_6 \\
 & - \frac{\alpha_s}{4\pi} \left[\bar{C}_2 F_2^{(7)}(q^2) + C_8^{\text{eff}} F_8^{(7)}(q^2) \right]. \quad (\text{A2})
 \end{aligned}$$

They are expressed in terms of the modified Wilson coefficients \bar{C}_{1-6} , which are defined by expressing the operators O_{1-6} of Ref. [19] in terms of the basis of [42] using 4-dimensional Fierz identities. They are given by [4]

$$\begin{aligned}
 \bar{C}_1 &= \frac{1}{2}C_1, & \bar{C}_2 &= C_2 - \frac{1}{6}C_1, \\
 \bar{C}_3 &= C_3 - \frac{1}{6}C_4 + 16C_5 - \frac{8}{3}C_6, & \bar{C}_4 &= \frac{1}{4}C_4 + 8C_6, \\
 \bar{C}_5 &= C_3 - \frac{1}{6}C_4 + 4C_5 - \frac{2}{3}C_6, & \bar{C}_6 &= \frac{1}{2}C_4 + 2C_6,
 \end{aligned} \quad (\text{A3})$$

where C_i are the Wilson coefficients in the operator basis of Ref. [19]. Their complete NNLL expressions are given in Ref. [43]. The \bar{C}_i coefficients coincide with the Wilson coefficients in the basis of Ref. [42], but are different beyond leading log approximation. The relation between the two sets of coefficients can be found in Refs. [4,19]. The effective Wilson coefficient C_8^{eff} is given by

$$C_8^{\text{eff}} = C_8 + \frac{4}{3}\bar{C}_3 - \frac{1}{3}\bar{C}_5. \quad (\text{A4})$$

The one-loop function $G(m_q)$ is given by

$$G(m_q) = \int_0^1 dx x(1-x) \log \left(\frac{-q^2 x(1-x) + m_q^2 - i\epsilon}{\mu^2} \right).$$

The functions $F_{1,2}^{(9)}(q^2)$, $F_2^{(7)}(q^2)$ appearing in the 2-loop matching conditions are listed in Eqs. (54)–(56) of the second reference in Ref. [41]. The functions $F_8^{(7,9)}(q^2)$ are given in Eqs. (82), (83) of Ref. [4].

We list here the functions f_v and τ appearing in the expression of the Wilson coefficient $c_1^{(V)}$, Eq. (9),

$$\begin{aligned}
 f_v(\omega, \mu) = & \frac{1}{2} \log^2 \frac{m_b^2}{\mu^2} - \frac{5}{2} \log \frac{m_b^2}{\mu^2} + 2 \log \frac{m_b^2}{\mu^2} \log \frac{\omega}{m_b} \\
 & + 2 \log^2 \frac{\omega}{m_b} + 2 \text{Li}_2 \left(1 - \frac{\omega}{m_b} \right) \\
 & + \frac{3\omega - 2m_b}{m_b - \omega} \log \frac{\omega}{m_b} + \frac{\pi^2}{12} + 6, \quad (\text{A5})
 \end{aligned}$$

$$\tau(\omega, \mu) = \frac{\alpha_s C_F}{4\pi} \left[\frac{\omega}{m_b - \omega} \log \frac{\omega}{m_b} + \log \frac{m_b^2}{\mu^2} \right]. \quad (\text{A6})$$

In the numerical evaluation of the Wilson coefficient $c_1^{(V)}$ we replace the \overline{MS} mass $m_b(\mu)$ with the pole mass m_b^{pole} using the one-loop result

$$m_b(\mu) = m_b^{\text{pole}} \left(1 + \frac{\alpha_s C_F}{4\pi} \left(-6 \log \frac{\mu}{m_b} - 4 \right) \right) \quad (\text{A7})$$

and keeping only the term linear in α_s .

Finally, we give here the function $t_\perp(x, m_c)$ appearing in the Wilson coefficient of the subleading $O(\lambda)$ SCET_I operators. This is given in Eq. (27) of Ref. [4], which we reproduce here for completeness,

$$\begin{aligned}
 t_\perp(x, m_c) = & \frac{4m_B}{\bar{x}\omega} I_1(m_c) + \frac{4q^2}{\bar{x}^2\omega^2} [B_0(\bar{x}m_B^2 + xq^2, m_c) \\
 & - B_0(q^2, m_c)] \quad (\text{A8})
 \end{aligned}$$

with

$$B_0(s, m_c) = -2\sqrt{\frac{4m_c^2}{s} - 1} \arctan \frac{1}{\sqrt{\frac{4m_c^2}{s} - 1}},$$

$$I_1(m_c) = 1 + \frac{2m_c^2}{\bar{x}(m_B^2 - q^2)} [L_1(x_+) + L_1(x_-) - L_1(y_+) - L_1(y_-)].$$
(A9)

The function $L_1(x)$ and its arguments are defined as

$$L_1(x) = \log \frac{x-1}{x} \log(1-x) - \frac{\pi^2}{6} + \text{Li}_2\left(\frac{x}{x-1}\right),$$

$$x_{\pm} = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{m_c^2}{\bar{x}m_B^2 + xq^2}},$$
(A10)

$$y_{\pm} = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{m_c^2}{q^2}}.$$
(A11)

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