

Pion pair production in e^+e^- annihilation

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(Received 23 March 2006; published 23 May 2006)

We present an analysis of the process $e^+e^- \rightarrow \gamma^*$ or $Z^* \rightarrow \pi\pi\gamma$, in the kinematical region where \sqrt{s} , the c.m. energy of the e^+e^- pair, is large but much below the Z -pole. The subprocess γ^* or $Z^* \rightarrow \pi\pi\gamma$ can be described by the convolution of the hard scattering coefficient γ^* or $Z^* \rightarrow q\bar{q}\gamma$ and the general distribution amplitude of two pions $q\bar{q} \rightarrow \pi\pi$. In the case of neutral pion, the production through γ^* is the dominant process, which can therefore be used to access the generalized distribution amplitudes (GDAs) of the pion, especially their C -even parts. The γZ interference term provides an alternative approach to extract the weak mixing angle $\sin\theta_W$ through measuring the helicity asymmetry in the process $e^+e^- \rightarrow \pi^0\pi^0\gamma$. In the case of charged pion pair production, the Bremsstrahlung process dominates and its interference with $e^+e^- \rightarrow \gamma^* \rightarrow \pi^+\pi^-\gamma$ can be applied to study the process $\gamma^* \rightarrow \pi\pi\gamma$ at the amplitude level.

DOI: [10.1103/PhysRevD.73.094021](https://doi.org/10.1103/PhysRevD.73.094021)

PACS numbers: 13.66.Bc, 13.88.+e, 14.40.Aq

I. INTRODUCTION

Generalized distribution amplitudes (GDAs) [1,2] are important nonperturbative objects for understanding how quarks and gluons form hadrons in hard exclusive processes. They describe the soft transition $q\bar{q} \rightarrow h\bar{h}$ or $gg \rightarrow h\bar{h}$, in the kinematical regime where the invariant mass of the hadron pair $h\bar{h}$ is small compared to the hard scale of the process. The typical processes to which the GDAs formalism can be applied are the two photon process $\gamma^*\gamma \rightarrow h\bar{h}$ [3–5] and hadron pair production $\gamma^*p \rightarrow p'h\bar{h}$ [6,7], for which factorization theorems have been proved [8]. In Ref. [9] we gave an analysis of the process $e^+e^- \rightarrow Z \rightarrow \pi\pi\gamma$ at the Z -pole energy, with small pion pair invariant mass. The subprocess $Z \rightarrow \pi\pi\gamma$ can be factorized into the convolution of a hard coefficient and the generalized distribution amplitude of two pions (2-pion GDA). In the case of charged pion pair production, the process is sensitive to the C -odd parts of 2-pion GDAs, and provide an opportunity to access them in e^+e^- annihilation. In this work we extend the analysis in Ref. [9] to a generalized process $e^+e^- \rightarrow \gamma^*$ or $Z^* \rightarrow \pi\pi\gamma$, in the kinematical region where \sqrt{s} is large but much below the Z -pole. Like the case of the production through Z boson analyzed in Ref. [9], a similar factorization holds for the case of the production through a virtual photon, which gives the dominant contribution for neutral pion pair production in the kinematical region we discuss here. Therefore, the investigation of the process $e^+e^- \rightarrow \gamma^* \rightarrow \pi^0\pi^0\gamma$ can provide detailed information about 2-pion GDAs, especially their C -even part. We also discuss the γZ interference term, which contains the weak mixing angle $\sin\theta_W$ and can be used as an alternative approach to extract its value. The Bremsstrahlung process $e^+e^- \rightarrow \gamma^*\gamma \rightarrow \pi^+\pi^-\gamma$ dominates in the case of charged pion

production, and its interference with $e^+e^- \rightarrow \gamma^* \rightarrow \pi^+\pi^-\gamma$ is a valuable tool for studying the process $\gamma^* \rightarrow \pi\pi\gamma$ at the amplitude level.

II. ANALYSIS OF THE PROCESS $\gamma^* \rightarrow \pi\pi\gamma$

We first study the subprocess of pion pair production

$$\gamma^*(q) \rightarrow \pi(p_1) + \pi(p_2) + \gamma(q') \quad (1)$$

in the framework of GDAs. It is convenient to investigate this subprocess in the pion pair center of mass frame [shown in Fig. 1(a)], where the kinematics is expressed as (choosing the z axis along the momentum of the real photon)

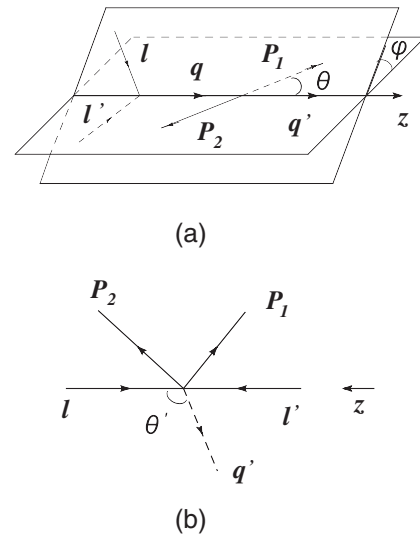


FIG. 1. The kinematics of pion pair production. (a) the c.m. frame of the pion pair, (b) the c.m. frame of e^+e^- .

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$$\begin{aligned}
q &= \frac{Q}{\sqrt{2}}v + \frac{Q}{\sqrt{2}}v', & q' &= \frac{Q^2 - W^2}{\sqrt{2}Q}v', \\
P &= \frac{Q}{\sqrt{2}}v + \frac{W^2}{\sqrt{2}Q}v', \\
p_1 &= \frac{\zeta Q}{\sqrt{2}}v + \frac{(1-\zeta)W^2}{\sqrt{2}Q}v' + \frac{\Delta_T}{2}, \\
p_2 &= \frac{(1-\zeta)Q}{\sqrt{2}}v + \frac{\zeta W^2}{\sqrt{2}Q}v' - \frac{\Delta_T}{2}
\end{aligned} \tag{2}$$

respectively. Here v and v' are two lightlike vectors which satisfy: $v^2 = v'^2 = 0$, $v \cdot v' = 1$, W is the invariant mass of the pion pair and $q^2 = Q^2 = s$, P is the total momentum of the pion pair. The skewness parameter ζ is the momentum fraction of plus momentum carried by $\pi(p_1)$ with respect to the pion pair:

$$\zeta = \frac{p_1^+}{P^+} = \frac{1 + \beta \cos\theta}{2}, \quad \beta = \sqrt{1 - \frac{4m_\pi^2}{W^2}}. \tag{3}$$

In the kinematical region $W^2 \ll s$, the process given in Eq. (1) can be analyzed in the framework of GDAs, which has been applied in Ref. [9] to the case where the virtual photon is replaced by the Z boson at $\sqrt{s} = M_Z$. From Fig. 2(a) and the one with the two photon vertices interchanged, the hadronic tensor of the process can be calculated as [1,9]

$$\begin{aligned}
T_{\gamma^* \rightarrow \pi\pi\gamma}^{\mu\nu} &= i \int d^4x e^{-iq \cdot x} \langle \pi(p_1) \pi(p_2) | T J_{em}^\mu(x) J_{em}^\nu(0) | 0 \rangle \\
&= -g_\perp^{\mu\nu} \sum_q \frac{e^2 e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z, \zeta, W^2),
\end{aligned} \tag{4}$$

which has the same form as that of the two photon process [1,3], and $\Phi_q(z, \zeta, W^2)$ is the GDA defined as [1]:

$$\Phi_q(z, \zeta, W^2) = \int \frac{dx^-}{2\pi} e^{iz(P^+ x^-)} \langle \pi(p_1) \pi(p_2) | \bar{\psi}(x^-) \gamma^+ \psi(0) | 0 \rangle, \tag{5}$$

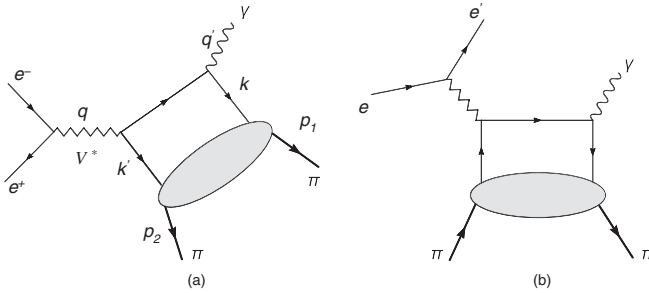


FIG. 2. (a) Diagram of the process $e^+e^- \rightarrow V^* \rightarrow \pi\pi\gamma$, where V can be a photon or Z boson. (b) Diagram of the DVCS process $e\pi \rightarrow e\pi\gamma$, the crossing related channel of the process depicted in (a).

where $z = k^+/P^+$ is the momentum fraction of the quark with respect to the hadronic system.

As discussed in Refs. [1,6], a useful constraint on the GDAs comes from charge conjugation invariance of the strong interactions:

$$\Phi_q^{\pi\pi}(z, \zeta, W^2) = -\Phi_q^{\pi\pi}(1-z, 1-\zeta, W^2). \tag{6}$$

Following this constraint one can define the C -even and C -odd parts (in [6] they are also called isoscalar and isovector parts, respectively) of 2-pion GDA as [3]

$$\Phi_q^\pm(z, \zeta, W^2) = \frac{1}{2} [\Phi_q^{\pi\pi}(z, \zeta, W^2) \pm \Phi_q^{\pi\pi}(z, 1-\zeta, W^2)], \tag{7}$$

where the superscript $\pi\pi$ represents $\pi^0\pi^0$ or $\pi^+\pi^-$. Then

$$\Phi_q^{\pi\pi}(z, \zeta, W^2) = \Phi_q^+(z, \zeta, W^2) + \Phi_q^-(z, \zeta, W^2). \tag{8}$$

The properties of $\Phi_q^\pm(z, \zeta, W^2)$ under the interchanges $z \rightarrow 1-z$ and $\zeta \rightarrow 1-\zeta$ can be easily derived from (8) [6]:

$$\Phi_q^+(z, \zeta, W^2) = -\Phi_q^+(1-z, \zeta, W^2) = \Phi_q^+(z, 1-\zeta, W^2), \tag{9}$$

$$\Phi_q^-(z, \zeta, W^2) = \Phi_q^-(1-z, \zeta, W^2) = -\Phi_q^-(z, 1-\zeta, W^2). \tag{10}$$

The $\zeta \rightarrow 1-\zeta$ exchange corresponds to the interchange of the two pions, thus in the case in which the final two pions are $\pi^0\pi^0$ we get

$$\Phi_q^{\pi^0\pi^0}(z, \zeta, W^2) = \Phi_q^{\pi^0\pi^0}(z, 1-\zeta, W^2). \tag{11}$$

Therefore there is only a C -even part for $\Phi_q^{\pi^0\pi^0}(z, \zeta, W^2)$. In the following we use $\Phi_q^\pm(z, \zeta, W^2)$ to represent the C -even/odd part of $\Phi_q^{\pi^+\pi^-}(z, \zeta, W^2)$ respectively. Isospin invariance implies [3]

$$\Phi_q^{\pi^0\pi^0}(z, \zeta, W^2) = \Phi_q^+(z, \zeta, W^2). \tag{12}$$

Thus according to (4), only $\Phi_q^+(z, \zeta, W^2)$ contributes to the process $\gamma^* \rightarrow \pi\pi\gamma$. Here we only consider the contributions coming from the u and d quark. Another consequence of isospin invariance is

$$\Phi_u^+ = \Phi_d^+, \quad \Phi_u^- = -\Phi_d^-. \tag{13}$$

Equation (4) shows that the process $e^+e^- \rightarrow \gamma^* \rightarrow \pi\pi\gamma$ can be also applied to probe Φ^+ , just like the two photon process $\gamma^*\gamma \rightarrow \pi\pi$, which has been proposed [3] to access Φ^+ . Interest on the process $e^+e^- \rightarrow \gamma^* \rightarrow \pi\pi\gamma$ relies also on the fact that this process is related to its crossed channel, the deeply virtual Compton scattering process (DVCS) $e\pi \rightarrow e\pi\gamma$ [shown in Fig. 2(b)], and is referred to as timelike DVCS. In the factorization of the DVCS process, the nonperturbative objects are the generalized parton distributions (GPDs) [10,11], which are important for understanding the parton angular momenta

inside the nucleon [12]. The simplest DVCS process is $e\pi \rightarrow e\pi\gamma$. However it is difficult to use the pion as the target, which makes the pion GPDs difficult to be experimentally accessed. It has been shown [13] that there is a crossing relation that connects GPDs and GDAs. Therefore to measure the process $e^+e^- \rightarrow \gamma^* \rightarrow \pi\pi\gamma$, the crossed channel of the DVCS process, can also give experimental indication about the GPDs of the pion. In the kinematical region $W^2 \ll s \ll M_Z^2$, the suitable facilities to investigate this process are B -factories. Finally, we remark that there is a suggestion [14] of using the transition distribution amplitude (TDA) [15] formalism to describe the process $e^+e^- \rightarrow \pi\pi\gamma$.

III. PHENOMENOLOGY ABOUT THE $e^+e^- \rightarrow \pi\pi\gamma$ PROCESS

A. Calculation of the cross-section

In this section we study the phenomenology associated with the process

$$e^+(l) + e^-(l') \rightarrow V^*(q) \rightarrow \pi(p_1) + \pi(p_2) + \gamma(q'), \quad (14)$$

where the intermediate vector boson V can be a photon or a Z boson, based on the analysis given in Sec. II and in Ref. [9]. We will consider the helicity of the electron in the analysis. It is useful to define two Lorentz invariant variables:

$$x = \frac{2q \cdot q'}{q^2}, \quad y = \frac{l \cdot q'}{q \cdot q'}. \quad (15)$$

The differential cross-section for the process $e^+e^- \rightarrow V^* \rightarrow \gamma\pi\pi$ is expressed as

$$d\sigma_{e^+e^- \rightarrow V^* \rightarrow \gamma\pi\pi} = \frac{\beta W(s - W^2)}{64(2\pi)^5 s^2} \times |\mathcal{A}_{e^+e^- \rightarrow V^* \rightarrow \pi\pi\gamma}|^2 dW d\Omega^* d\Omega', \quad (16)$$

in which $(|\mathbf{p}^*|, \Omega^*)$ is the momentum of pion 1 in the c.m. frame of the pion pair, and Ω' is the angle of the photon in the rest frame of the Z boson (that is, the c.m. frame of e^+e^-).

The amplitude of the process can be calculated from

$$\mathcal{A}_{e^+e^- \rightarrow V^* \rightarrow \gamma\pi\pi} = \sum_{i,j} \bar{v}(l') V^\mu u(l) \epsilon_\mu^{(i)*} \frac{1}{s - M_V^2} \mathcal{A}_{i,j}^{V^* \rightarrow \gamma\pi\pi}, \quad (17)$$

where $V^\mu = e\gamma^\mu(1 + \lambda_e\gamma_5)$ for lepton- γ vertex, and $V^\mu = e\gamma^\mu(c_V^l - \gamma_5 c_A^l)(1 + \lambda_e\gamma_5)/\sin 2\theta_W$ for lepton- Z boson vertex, with $\lambda_e = \pm 1$ the helicity of the electron. The amplitude of $V^* \rightarrow \pi\pi\gamma$ can be calculated from

$$\mathcal{A}_{i,j}^{V^* \rightarrow \gamma\pi\pi} = \epsilon_\alpha^{(i)} \epsilon_\beta^{(j)*} T_V^{\alpha\beta}(z, \zeta, W^2), \quad (18)$$

where ϵ and ϵ' are the polarization vectors of the inter-

mediate virtual boson (photon or Z) and the real photon, respectively. In our reference frame these vectors have the form:

$$\epsilon_\mu^{(\pm)} = \left(0, \frac{\mp 1}{\sqrt{2}}, \frac{-i}{\sqrt{2}}, 0\right), \quad \epsilon_\mu^0 = \left(\frac{|\mathbf{q}|}{\sqrt{s}}, 0, 0, \frac{q_0}{\sqrt{s}}\right), \quad (19)$$

$$\epsilon_\mu'^{(\pm)} = \left(0, \frac{\mp 1}{\sqrt{2}}, \frac{-i}{\sqrt{2}}, 0\right).$$

Using the notation of Ref. [16] the cross-section of the process can be written as:

$$\frac{d\sigma}{dW^2 d\cos\theta d\phi dy} = \frac{\alpha^3 \beta(s - W^2)}{32\pi s^4} \sum_{ab} L_{\mu\nu}^{ab} \mathcal{W}_{ab}^{\mu\nu} \chi_{ab}. \quad (20)$$

The indices a, b can be γ for the photon or Z for the Z boson. The relative propagator factors χ_{ab} are given by

$$\chi_{\gamma\gamma} = 1, \quad (21)$$

$$\chi_{\gamma Z} = \chi_{Z\gamma} = \frac{1}{\sin^2(2\theta_W)} \frac{s}{s - M_Z^2}, \quad (22)$$

$$\chi_{ZZ} = (\chi_{\gamma Z})^2. \quad (23)$$

which correspond to the contributions from γ , γZ interference and Z boson, respectively. The lepton tensors are:

$$L_{ab}^{\mu\nu} = -2s C_{ab} I_1(y) g_\perp^{\mu\nu} - i s D_{ab} I_2(y) \epsilon_\perp^{\mu\nu}. \quad (24)$$

The notation used in the above equation is

$$C_{\gamma\gamma} = 1, \quad (25)$$

$$C_{\gamma Z} = C_{Z\gamma} = (c_V^l - c_A^l \lambda_e), \quad (26)$$

$$C_{ZZ} = (c_V^{l2} + c_A^{l2}) - 2c_V^l c_A^l \lambda_e, \quad (27)$$

$$D_{\gamma\gamma} = \lambda_e, \quad (28)$$

$$D_{\gamma Z} = D_{Z\gamma} = (c_V^l \lambda_e - c_A^l), \quad (29)$$

$$D_{ZZ} = (c_V^{l2} + c_A^{l2}) \lambda_e - 2c_V^l c_A^l, \quad (30)$$

$$I_1(y) = \left(\frac{1}{2} - y + y^2\right), \quad (31)$$

$$I_2(y) = (1 - 2y). \quad (32)$$

The hadron tensors appearing in Eq. (20) are:

$$W_{\gamma\gamma}^{\mu\nu} = -g_\perp^{\mu\nu} \left| \sum_q e_q^2 V_q(\cos\theta, W^2) \right|^2, \quad (33)$$

$$\begin{aligned}
W_{\gamma Z}^{\mu\nu} = & -g_{\perp}^{\mu\nu} \operatorname{Re} \left\{ \left[\sum_q e_q c_V^q V_q^*(\cos\theta, W^2) \right] \right. \\
& \times \left[\sum_q e_q^2 V_q(\cos\theta, W^2) \right] \\
& + i\epsilon_{\perp}^{\mu\nu} \operatorname{Im} \left\{ \left[\sum_q e_q c_V^q V_q^*(\cos\theta, W^2) \right] \right. \\
& \times \left[\sum_q e_q^2 A_q(\cos\theta, W^2) \right] \left. \right\}, \quad (34)
\end{aligned}$$

$$\begin{aligned}
W_{ZZ}^{\mu\nu} = & -g_{\perp}^{\mu\nu} \left\{ \left| \sum_q e_q c_V^q V_q(\cos\theta, W^2) \right|^2 \right. \\
& + \left| \sum_q e_q c_A^q A_q(\cos\theta, W^2) \right|^2 \left. \right\} \\
& - 2i\epsilon_{\perp}^{\mu\nu} \operatorname{Im} \left\{ \left[\sum_q e_q c_V^q V_q^*(\cos\theta, W^2) \right] \right. \\
& \times \left[\sum_q e_q c_A^q A_q(\cos\theta, W^2) \right] \left. \right\}, \quad (35)
\end{aligned}$$

where

$$V_q(\cos\theta, W^2) = \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^+(z, \zeta, W^2), \quad (36)$$

$$A_q(\cos\theta, W^2) = \int_0^1 dz \frac{1}{z(1-z)} \Phi_q^-(z, \zeta, W^2). \quad (37)$$

The vector and axial-vector couplings to the Z boson appearing in the above equations are given by:

$$c_V^i = T_3^i - 2Q^i \sin^2\theta_W, \quad (38)$$

$$c_A^i = T_3^i, \quad (39)$$

where Q^i denotes the charge and T_3^i is the weak isospin of particle i (i.e., $T_3^i = +1/2$ for $i = u$, and $-1/2$ for $i = e^-, d, s$).

B. Results for the neutral case

In the previous subsection we have given the expression for the cross-section of the general process $e^+e^- \rightarrow \gamma^*$ or $Z^* \rightarrow \pi\pi\gamma$, in terms of 2-pion GDAs. There are interesting phenomenological implications coming from Eq. (16). As demonstrated in Ref. [9], measuring the charged pion pair production in the kinematical regime $\sqrt{s} = M_Z$, where the main contribution comes from Z boson, can provide an opportunity to access the C -odd part of 2-pion GDAs Φ_q^- . In this work, however, we consider a different kinematical region $W^2 \ll s \ll M_Z^2$. Consequently a different phenomenology emerges: the production through γ^* is dominant over the γZ interference and Z boson terms, which is a result coming from the factors given in Eqs. (21)–(23).

To avoid the competing contribution from the Bremsstrahlung process, in this subsection we limit our-

selves to the case of neutral pion pair production. Therefore the terms containing A_q in the hadronic tensor (24) do not contribute to the cross-section, and the calculation is simplified. Based on (16), we calculate the γ , γZ interference and Z contributions to the unpolarized differential cross-section of the process $e^+e^- \rightarrow \gamma^*$ or $Z^* \rightarrow \pi^0\pi^0\gamma$ as functions of \sqrt{s} . The results are shown in Fig. 3 by a solid line, dashed line and dotted line, respectively. For Φ_q^+ , which is needed in the calculation, we adopt the simple model given in [3]. The curves show that the contribution from the production through γ^* is about several orders of magnitude larger than those through γZ interference and Z boson, in the regime $W^2 \ll s \ll M_Z^2$. Therefore it would be suitable to investigate Φ_q^+ through the process $e^+e^- \rightarrow \gamma^* \rightarrow \pi^0\pi^0\gamma$, in the regime where \sqrt{s} is several GeV, where the cross-section is relatively large.

The γZ interference term is also interesting because it contains the weak mixing angle $\sin\theta_W$. To isolate this term one needs to go beyond the helicity-independent cross-section, that is, to consider the helicity dependent cross-section of the process. The helicity dependent differential cross-section is dominated by the γZ interference term:

$$\begin{aligned}
\frac{d\sigma(+\lambda_e) - d\sigma(-\lambda_e)}{2dW^2 d\cos\theta} = & -\frac{\alpha^3 \beta(s - W^2)}{6s^3} \\
& \times \chi_{\gamma Z} c_A^i \lambda_e (e_u c_V^u + e_d c_V^d) (e_u^2 + e_d^2) \\
& \times |V_u(\cos\theta, W^2)|^2, \quad (40)
\end{aligned}$$

since the Z boson term can be ignored compared to that of the γZ interference term.

As discussed previously, the unpolarized differential cross-section is dominated by the virtual photon produc-

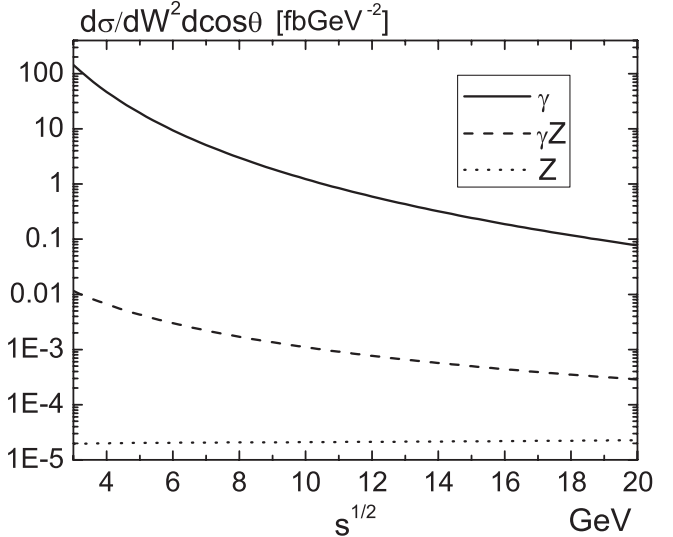


FIG. 3. The γ (solid line), γZ interference term (dashed line) and Z (dotted line) contributions to the helicity-independent differential cross-section of the process $e^+e^- \rightarrow V^* \rightarrow \pi^0\pi^0\gamma$ at $W = 0.7$ GeV, $\theta = 10^\circ$.

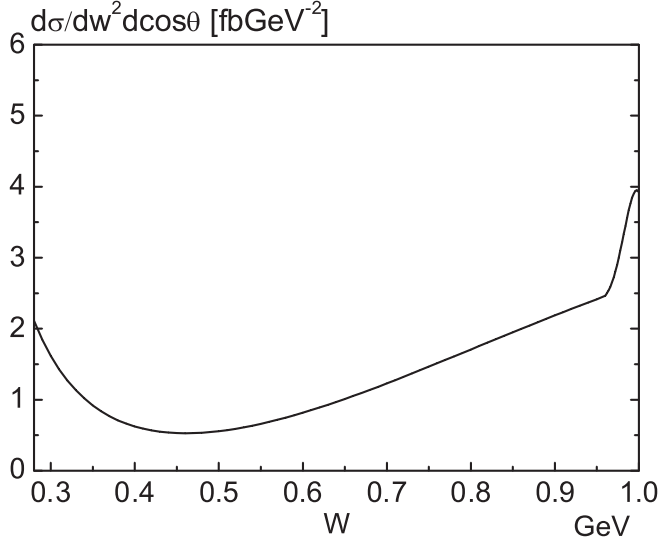


FIG. 4. The W -dependent differential cross-section of the process $e^+e^- \rightarrow \gamma^* \rightarrow \pi^0\pi^0\gamma$ at $\sqrt{s} = 10$ GeV and $\theta = 20^\circ$.

tion, which reads:

$$\frac{d\sigma(+\lambda_e) + d\sigma(-\lambda_e)}{2dW^2d(\cos\theta)} = \frac{\alpha^3\beta(s-W^2)}{12s^3} (e_u^2 + e_d^2)^2 \times |V_u(\cos\theta, W^2)|^2. \quad (41)$$

The W -dependence of this cross-section is shown in Fig. 4, for $\sqrt{s} = 10$ GeV and $\theta = 20^\circ$.

The ratio of the helicity dependent and helicity-independent cross-section, that is, the helicity asymmetry of the process is $(\lambda_e = +1)$

$$R = \frac{\sigma(+\lambda_e) - \sigma(-\lambda_e)}{\sigma(+\lambda_e) + \sigma(-\lambda_e)} = -2c_A^l \chi_{\gamma Z} \frac{e_u c_V^u + e_d c_V^d}{e_u^2 + e_d^2} = \frac{\frac{9}{10} - 2\sin^2\theta_W}{\sin^2(2\theta_W)} \frac{s}{s - M_Z^2}, \quad (42)$$

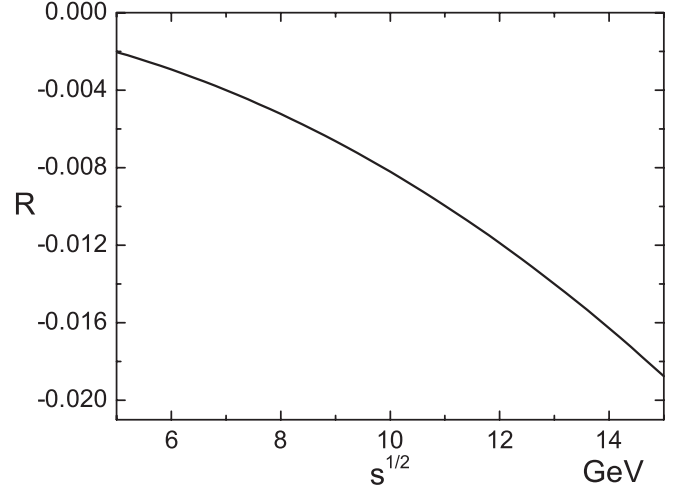


FIG. 5. The \sqrt{s} dependent ratio of the helicity dependent and helicity-independent cross-section of the process $e^+e^- \rightarrow V^* \rightarrow \pi^0\pi^0\gamma$.

which depends on s and $\sin\theta_W$. Therefore the measurement on this helicity asymmetry provides an alternative approach to extract the value of weak mixing angle $\sin\theta_W$. In Fig. 5 we show the \sqrt{s} dependent ratio R which is given in (42). For $\sqrt{s} = 10$ GeV, which is the typical c.m. energy of CLEO or B -factories, such as *BABAR* and *Belle*, this ratio is about 0.8%, which is a sizeable value.

C. Results for the charged case

In the case of charged pion pair production, besides the process $e^+e^- \rightarrow \gamma^* \rightarrow \pi^+\pi^-\gamma$, the Bremsstrahlung process $e^+e^- \rightarrow \gamma^*\gamma \rightarrow \pi^+\pi^-\gamma$ also needs to be considered. The cross-section of the former process is the same as that of neutral pion production, since only Φ_q^+ contributes. The differential cross-section of the Bremsstrahlung process has the form

$$\frac{d\sigma^B}{dW^2d(\cos\theta)d\phi dy} = \frac{\alpha^3\beta^3(s-W^2)}{16\pi s^2W^2} |F_\pi(W^2)|^2 \left((1-2x(1-x)) \frac{I_1(y)}{y(1-y)} \sin^2\theta + 4x(1-x)\cos^2\theta + \sqrt{x(1-x)}(1-2x) \frac{I_2(y)}{\sqrt{y(1-y)}} \sin 2\theta \cos\phi - x(1-x)2\sin^2\theta \cos 2\phi \right), \quad (43)$$

where $F_\pi(W^2)$ is the timelike pion form factor.

It is interesting to investigate the interference [9,17] between $e^+e^- \rightarrow \gamma^* \rightarrow \pi^+\pi^-\gamma$ and the Bremsstrahlung process, because it can be applied to study the subprocess $\gamma^* \rightarrow \pi^+\pi^-\gamma$ at the amplitude level, similar to the case $e\gamma \rightarrow e\pi\pi$ that has been discussed in Refs. [3,18]. Knowledge of the process $\gamma^* \rightarrow \pi^+\pi^-\gamma$ at the amplitude level can help to determine not only the magnitude of Φ_q^+ , but also its phase. The form of the interference term reads

$$\frac{d\sigma^I}{dW^2d(\cos\theta)d\phi dy} = \frac{\alpha^3\beta^2(s-W^2)}{16\pi s^2\sqrt{s}W} \left(\sqrt{x(1-x)} \operatorname{Re}\{F_\pi^*V\} \cos\theta + \left[\frac{xI_1(y)}{\sqrt{y(1-y)}} - (1-x)\sqrt{y(1-y)} \right] \operatorname{Re}\{F_\pi^*V\} \sin\theta \cos\phi + \lambda_e x \frac{I_2(y)}{2\sqrt{y(1-y)}} \operatorname{Im}\{F_\pi^*V\} \sin\theta \sin\phi \right), \quad (44)$$

where $V = \sum_q V_q$ for $q = u, d$. This interference term can be obtained from taking the difference of the cross-section in the exchange $(\theta, \phi) \rightarrow (\pi - \theta, \pi + \phi)$. The last term in Eq. (44) is the helicity dependent term, which can probe $\text{Im}\{F^*V\}$, and which together with the helicity-independent terms can be used to reconstruct the full complex amplitude of $\gamma^* \rightarrow \pi^+ \pi^- \gamma$, in the region where $F_\pi(W^2)$ is known. Conversely, these measurements could also be used in order to obtain information about the pion timelike form factor $F_\pi(W^2)$.

In Fig. 6 we give the curves of the differential cross-sections, coming from the Bremsstrahlung process and its interference with the process $e^+ e^- \rightarrow \gamma^* \rightarrow \pi^+ \pi^- \gamma$, respectively. For $F_\pi(W^2)$ we use the parametrization $N = 1$ given in Ref. [19]. We show the cross-sections for $\sqrt{s} = 5$ (solid line) and 10 GeV (dashed line) in Fig. 6. In the later case the terms containing $1 - x$ have sizeable contributions. In the lower panel of Fig. 6 we also show the contribution from the helicity dependent interference term (dotted line) for $\sqrt{s} = 5$ GeV.

Figure 6 shows that the contribution coming from the Bremsstrahlung process is 1 ~ 2 orders of magnitude larger than that from the interference term. According to the shape of F_π , the W -dependent curves of both the Bremsstrahlung and interference contributions have peaks in the region $W \sim m_\rho$. In the case of θ -dependence the curves of Bremsstrahlung and (unpolarized) interference contributions increase with θ from 0° to 90° , and then

decrease. The ϕ -dependent curve of the Bremsstrahlung contribution has a large value at $\phi = 180^\circ$ and a small value at $\phi = 0^\circ$, while that of the (unpolarized) interference contribution has a large (absolute) value at $\phi = 0^\circ$ and a small (absolute) value at $\phi = 180^\circ$. The y -dependence of the interference contribution shows that the curves have maximum magnitude in the region $y \rightarrow 0$ or 1 (due to the factor $\sqrt{y(1-y)}$ in the denominators of some terms in (44)), which is therefore the suitable region in order to study the interference term. A similar situation occurs in the case of the Bremsstrahlung process.

IV. SUMMARY

We have given a detailed analysis of the process $e^+ e^- \rightarrow \gamma^*$ or $Z^* \rightarrow \pi\pi\gamma$, in the kinematical region where \sqrt{s} is large compared to the invariant mass of the pion pair but much below the Z -pole. A factorization similar to which has been shown in Ref. [9] holds for the γ case: the amplitude of the process can be expressed as the convolution between the hard scattering $\gamma^* \rightarrow q\bar{q}\gamma$ and 2-pion GDAs which describe the soft transition $q\bar{q} \rightarrow \pi\pi$. Thus the investigation on the process $e^+ e^- \rightarrow \gamma^* \rightarrow \pi\pi\gamma$ can provide detailed information about 2-pion GDAs, especially their C -even part. The experimental measurement of this process can also provide experimental indication of pion GPDs, by virtue of the fact that this process is the crossed channel of the DVCS process $e\pi \rightarrow e\pi\gamma$, and

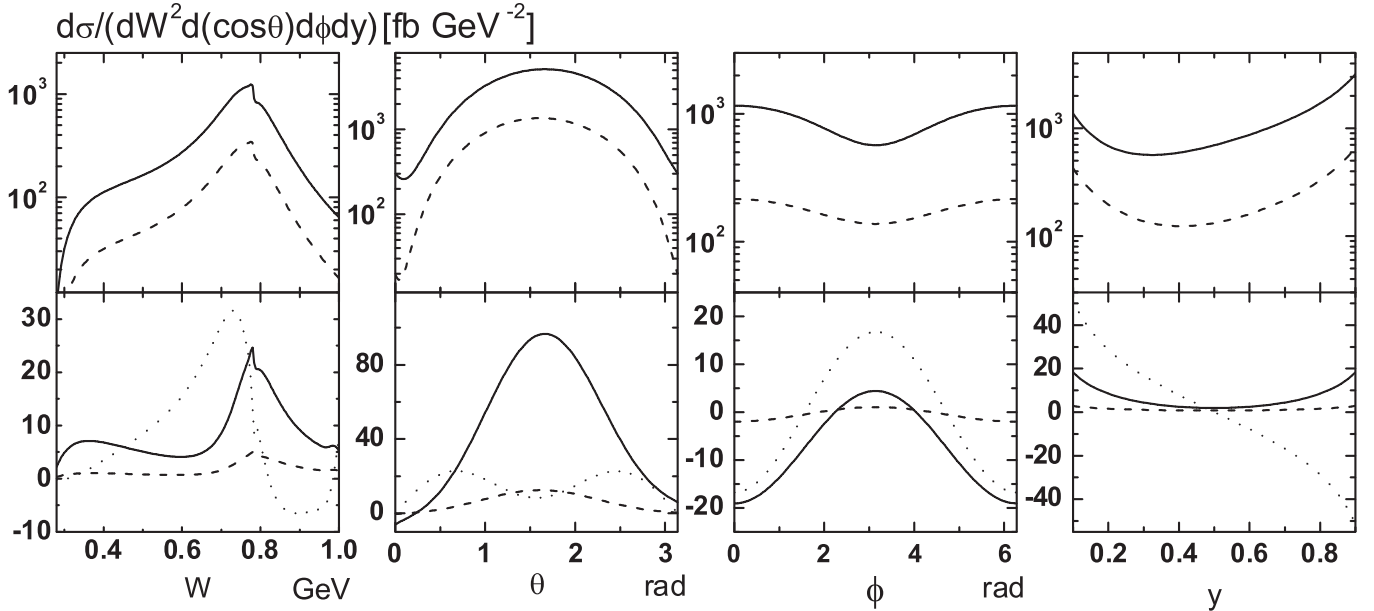


FIG. 6. Upper panel: the differential cross-sections of the Bremsstrahlung process, at $\sqrt{s} = 5$ (solid line) and 10 GeV (dashed line). Lower panel: contributions to the differential cross-sections coming from the unpolarized interference term between the Bremsstrahlung process and the process $e^+ e^- \rightarrow \gamma^* \rightarrow \pi^+ \pi^- \gamma$ at $\sqrt{s} = 5$ (solid line), 10 GeV (dashed line) and helicity dependent interference term (dotted line) at $\sqrt{s} = 5$ GeV. The first column: the W dependent differential cross-sections at $\theta = 20^\circ$, $\phi = 180^\circ$, $y = 0.3$; the second column: the θ dependent differential cross-sections at $W = 0.7$ GeV, $\phi = 180^\circ$, $y = 0.3$; the third column: the ϕ dependent differential cross-sections at $W = 0.7$ GeV, $\theta = 20^\circ$, $y = 0.3$; The fourth column: the y dependent differential cross-sections at $W = 0.7$ GeV, $\theta = 20^\circ$, $\phi = 180^\circ$.

there is a crossing relation between GPD and GDA. We studied the production of both neutral and charged pion. In the former case the γZ interference term can be applied to extract the value of the weak mixing angle $\sin\theta_W$ through the measurement of the helicity asymmetry in the process $e^+e^- \rightarrow \pi^0\pi^0\gamma$. In the last case the interference between the Bremsstrahlung process and the process $e^+e^- \rightarrow \gamma^* \rightarrow$

$\pi^+\pi^-\gamma$ is useful in order to study the subprocess $\gamma^* \rightarrow \pi^+\pi^-\gamma$ at the amplitude level.

ACKNOWLEDGMENTS

This work is supported by Fondecyt (Chile) under Projects No. 3050047 and No. 1030355.

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