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Structure of the $D_{s0}(2317)$ and the strong coupling constant $g_{D_{s0}DK}$ with the light-cone QCD sum rules

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In this article, we take the point of view that the charmed scalar meson $D_{s0}(2317)$ is the conventional $c\bar{s}$ meson and calculate the strong coupling constant $g_{D_{s0}DK}$ within the framework of the light-cone QCD sum rules approach. The numerical values for the large scalar-DK coupling constant $g_{D_{s0}DK}$ support the hadronic dressing mechanism. Just like the scalar mesons $f_0(980)$ and $a_0(980)$, the $D_{s0}(2317)$ may have small scalar $c\bar{s}$ kernel of the typical $c\bar{s}$ meson size. The strong coupling to the hadronic channels (or the virtual mesons loops) may result in smaller mass than the conventional scalar $c\bar{s}$ meson in the constituent quark models, and enrich the pure $c\bar{s}$ state with other components. The $D_{s0}(2317)$ may spend part (or most part) of its lifetime as virtual DK state.

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I. INTRODUCTION

The constituent quark model provides a rather successful description of the spectrum of the mesons in terms of quark-antiquark bound states, which fit into the suitable multiplets reasonably well. However, the scalar mesons below 2 GeV present a remarkable exception as the structures of those mesons are not unambiguously determined yet [1]. The light scalar mesons are the subject of an intense and continuous controversy in clarifying the hadron spectroscopy; the more elusive things are the constituent structures of the $f_0(980)$ and $a_0(980)$ mesons with almost the degenerate masses. Furthermore, the discovery of the two strange-charmed mesons $D_{s0}(2317)$ and $D_{s1}(2460)$ with spin-parity 0^+ and 1^+ respectively has triggered hot debate on their nature, understructures, and whether it is necessary to introduce the exotic states [2]. The mass of the $D_{s0}(2317)$ is significantly lower than the values of the 0^+ state mass from the quark models and lattice simulations [3]. The difficulties to identify the $D_{s0}(2317)$ and $D_{s1}(2460)$ states with the conventional $c\bar{s}$ mesons are rather similar to those appearing in the light scalar mesons below 1 GeV. Those two states $D_{s0}(2317)$ and $D_{s1}(2460)$ lie just below the DK and D^*K threshold, respectively, which are analogous to the situation that the scalar mesons $a_0(980)$ and $f_0(980)$ lie just below the $K\bar{K}$ threshold and couple strongly to the nearby channels. The mechanism responsible for the low-mass charmed scalar meson may be the same as the light scalar nonet mesons, the $f_0(600)$, $f_0(980)$, $a_0(980)$ and $K_0^*(800)$ [4–6]. There have been a lot of explanations for their nature, for example, conventional $c\bar{s}$ states [7–9], two-meson molecular state [10], four-quark states [11], etc. If we take the scalar mesons $a_0(980)$ and $f_0(980)$ as four-quark states with the constituents of scalar diquark-antidiquark substructures, the masses of the scalar nonet mesons below 1 GeV can be naturally explained [5,6].

There are other possibilities beside the four-quark state explanations, for example, the scalar mesons $a_0(980)$, $f_0(980)$ and $D_{s0}(2317)$ may have bare $q\bar{q}$ and $c\bar{s}$ kernels in the P-wave states with strong coupling to the nearby threshold, respectively, the S-wave virtual intermediate hadronic states (or the virtual mesons loops) play a crucial role in the composition of those bound states (or resonances due to the masses below or above the thresholds). The hadronic dressing mechanism (or unitarized quark models) takes the point of view that the $f_0(980)$, $a_0(980)$ and $D_{s0}(2317)$ mesons have small $q\bar{q}$ and $c\bar{s}$ kernels of the typical $q\bar{q}$ and $c\bar{s}$ mesons size, respectively. The strong couplings to the virtual intermediate hadronic states (or the virtual mesons loops) may result in smaller masses than the conventional scalar $q\bar{q}$ and $c\bar{s}$ mesons in the constituent quark models, enrich the pure $q\bar{q}$ and $c\bar{s}$ states with other components [12,13]. Those mesons may spend part (or most part) of their lifetime as virtual $K\bar{K}$ and DK states [12,13]. Despite what constituents they may have, we have the fact that they lie just a little below the $K\bar{K}$ and DK threshold, respectively, the strong interactions with the $K\bar{K}$ and DK thresholds will significantly influence their dynamics, although the decay $D_{s0}(2317) \to DK$ is kinematically suppressed. It is interesting to investigate the possibility of the hadronic dressing mechanism.

In this article, we take the point of view that the scalar mesons $f_0(980)$, $a_0(980)$ and $D_{s0}(2317)$ are the conventional $q\bar{q}$ and $c\bar{s}$ state, respectively, and calculate the values of the strong coupling constant $g_{D_{s0}DK}$ within the framework of the light-cone QCD sum rules approach. The light-cone QCD sum rules approach carries out the operator product expansion near the light-cone $x^2 \approx 0$ instead of the short distance $x \approx 0$ while the nonperturbative matrix elements are parametrized by the light-cone distribution

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amplitudes which classified according to their twists instead of the vacuum condensates [14,15].

The article is arranged as follows: in Sec. II, we derive the strong coupling constant $g_{D_{s0}DK}$ within the framework of the light-cone QCD sum rules approach; in Sec. III, we discuss the numerical results; and in Sec. IV, we conclude.

II. STRONG COUPLING CONSTANT $g_{D_{s0}DK}$ WITH LIGHT-CONE QCD SUM RULES

In the following, we write down the definition for the strong coupling constant $g_{D_{s0}DK}$,

$$\langle K(q)D(p)|D_{s0}(p+q)\rangle = g_{D_{s0}DK}. \tag{1}$$

We study the strong coupling constant $g_{D_{s0}DK}$ with the scalar interpolating current $J_{D_{s0}}(x)$ and choose the two-point correlation function $T_\mu(p, q)$,

$$T_\mu(p,q) = i\int d^4x e^{ip\cdot x}\langle K(q)|T\{J_\mu^D(x)J_{D_{s0}}(0)\}|0\rangle, \tag{2}$$

$$J_{D_{s0}}(x) = \bar{c}(x)s(x), \tag{3}$$

$$J_\mu^D(x) = \bar{u}(x)\gamma_\mu\gamma_5 c(x). \tag{4}$$

Here the axial-vector current $J_\mu^D(x)$ interpolates the pseudoscalar D meson, and the external K state has four momentum q with $q^2 = M_K^2$. The correlation function $T_\mu(p, q)$ can be decomposed as

$$T_\mu(p,q) = T_p(p^2,(p+q)^2)p_\mu + T_q(p^2,(p+q)^2)q_\mu, \tag{5}$$

due to the tensor analysis.

According to the basic assumption of current-hadron duality in the QCD sum rules approach [16], we can insert a complete series of intermediate states with the same quantum numbers as the current operators $J_{D_{s0}}(x)$ and $J_\mu^D(x)$ into the correlation function $T_\mu(p, q)$ to obtain the hadronic representation. After isolating the ground state contributions from the pole terms of the $D_{s0}(2317)$ and D mesons, we get the following result,

$$\begin{aligned} T_p(p^2,(p+q)^2)p_\mu &= \frac{\langle 0 \mid J_\mu^D \mid D(p)\rangle\langle DK \mid D_{s0}\rangle\langle D_{s0}(p+q)|J_{D_{s0}} \mid 0\rangle}{(M_D^2-p^2)(M_{D_{s0}}^2-(p+q)^2)} + \cdots \\ &= \frac{ig_{D_{s0}DK}f_Df_{D_{s0}}M_{D_{s0}}p_\mu}{(M_D^2-p^2)(M_{D_{s0}}^2-(p+q)^2)} + \cdots, \end{aligned} \tag{6}$$

where the following definitions have been used,

$$\begin{aligned} \langle D_{s0}(p+q) \mid J_{D_{s0}}(0) \mid 0\rangle &= f_{D_{s0}}M_{D_{s0}}, \\ \langle 0 \mid J_\mu^D(0) \mid D(p)\rangle &= if_Dp_\mu. \end{aligned} \tag{7}$$

Here we have not shown the contributions from the high resonances and continuum states explicitly as they are suppressed due to the double Borel transformation. In the ground state approximation, the tensor structure $T_q(p^2, (p+q)^2)q_\mu$ has no contributions and neglected.

In the following, we briefly outline the operator product expansion for the correlation function $T_\mu(p, q)$ in perturbative QCD theory. The calculations are performed at the large spacelike momentum regions $(p+q)^2 \ll 0$ and $p^2 \ll 0$, which correspond to the small light-cone distance $x^2 \approx 0$ required by the validity of the operator product expansion approach. We write down the propagator of a massive quark in the external gluon field in the Fock-Schwinger gauge first [17],

$$\begin{aligned} \langle 0|T\{q_i(x_1)\bar{q}_j(x_2)\}|0\rangle = i\int \frac{d^4k}{(2\pi)^4}e^{-ik(x_1-x_2)}\Big\{\frac{\not k+m}{k^2-m^2}\delta_{ij} \\ - \int_0^1 dv g_s G_a^{\mu\nu}(vx_1+(1-v)x_2) \\ \times\Big(\frac{\lambda^a}{2}\Big)_{ij}\Big[\frac{1}{2}\frac{\not k+m}{(k^2-m^2)^2}\sigma_{\mu\nu} \\ - \frac{1}{k^2-m^2}v(x_1-x_2)_\mu\gamma_\nu\Big]\Big\}. \end{aligned} \tag{8}$$

Here $G_a^{\mu\nu}$ is the gluonic field strength, and g_s denotes the strong coupling constant. Substituting the above c quark propagator and the corresponding K meson light-cone distribution amplitudes into the correlation function $T_\mu(p, q)$ in Eq. (2) and completing the integrals over the variables x and k, finally we obtain the result,

$$\begin{aligned} T_p(p^2,(p+q)^2) &= if_K\int_0^1 du\Big\{\frac{M_K^2}{m_s}\varphi_p(u)\frac{1}{m_c^2-(p+uq)^2} - 2\Big[m_cg_2(u) + \frac{M_K^2}{6m_s}\varphi_\sigma(u)(p\cdot q+uM_K^2)\Big]\frac{1}{[m_c^2-(p+uq)^2]^2}\Big\} \\ &\quad + if_{3K}M_K^2\int_0^1 dv(2v-3)\int \mathcal{D}\alpha_i\varphi_{3K}(\alpha_i)\frac{1}{\{m_c^2-[p+q(\alpha_1+v\alpha_3)]^2\}^2} \\ &\quad - 4if_Km_cM_K^2\Big\{\int_0^1 dv(v-1)\int_0^1 d\alpha_3\int_0^{\alpha_3}d\beta\int_0^{1-\beta}d\alpha\frac{\Phi(\alpha,1-\alpha-\beta,\beta)}{\{[p+(1-\alpha_3+v\alpha_3)q]^2-m_c^2\}^3} \\ &\quad + \int_0^1 dv\int_0^1 d\alpha_3\int_0^{1-\alpha_3}d\alpha_1\int_0^{\alpha_1}d\alpha\frac{\Phi(\alpha,1-\alpha-\alpha_3,\alpha_3)}{\{[p+(\alpha_1+v\alpha_3)q]^2-m_c^2\}^3}\Big\}. \end{aligned} \tag{9}$$

In calculation, the following two-particle and three-particle K meson light-cone distribution amplitudes are useful [14,15,17–19],

$$\begin{aligned}
\langle K(q)|\bar{u}(x)\gamma_\mu\gamma_5 s(0)|0\rangle &= -if_K q_\mu \int_0^1 du\, e^{iuq\cdot x}[\varphi_K(u) + x^2 g_1(u)] + f_K\Big(x_\mu - \frac{q_\mu x^2}{q\cdot x}\Big)\int_0^1 du\, e^{iuq\cdot x} g_2(u),\\
\langle K(q)|\bar{u}(x)i\gamma_5 s(0)|0\rangle &= \frac{f_K M_K^2}{m_s}\int_0^1 du\, e^{iuq\cdot x}\varphi_p(u),\\
\langle K(q)|\bar{u}(x)\sigma_{\mu\nu}\gamma_5 s(0)|0\rangle &= i(q_\mu x_\nu - q_\nu x_\mu)\frac{f_K M_K^2}{6m_s}\int_0^1 du e^{iuq\cdot x}\varphi_\sigma(u),\\
\langle K(q)|\bar{u}(x)\sigma_{\alpha\beta}\gamma_5 g_s G_{\mu\nu}(vx)s(0)|0\rangle &= if_{3K}[(q_\mu q_\alpha g_{\nu\beta} - q_\nu q_\alpha g_{\mu\beta}) - (q_\mu q_\beta g_{\nu\alpha} - q_\nu q_\beta g_{\mu\alpha})]\\
&\quad\times\int \mathcal{D}\alpha_i \varphi_{3K}(\alpha_i) e^{iq\cdot x(\alpha_1 + v\alpha_3)},\\
\langle K(q)|\bar{u}(x)\gamma_\mu\gamma_5 g_s G_{\alpha\beta}(vx)s(0)|0\rangle &= f_K\Big[q_\beta\Big(g_{\alpha\mu} - \frac{x_\alpha q_\mu}{q\cdot x}\Big) - q_\alpha\Big(g_{\beta\mu} - \frac{x_\beta q_\mu}{q\cdot x}\Big)\Big]\int \mathcal{D}\alpha_i \varphi_\perp(\alpha_i) e^{iq\cdot x(\alpha_1 + v\alpha_3)}\\
&\quad + f_K\frac{q_\mu}{q\cdot x}(q_\alpha x_\beta - q_\beta x_\alpha)\int \mathcal{D}\alpha_i \varphi_\parallel(\alpha_i) e^{iq\cdot x(\alpha_1 + v\alpha_3)},\\
\langle K(q)|\bar{u}(x)\gamma_\mu g_s \tilde{G}_{\alpha\beta}(vx)s(0)|0\rangle &= if_K\Big[q_\beta\Big(g_{\alpha\mu} - \frac{x_\alpha q_\mu}{q\cdot x}\Big) - q_\alpha\Big(g_{\beta\mu} - \frac{x_\beta q_\mu}{q\cdot x}\Big)\Big]\int \mathcal{D}\alpha_i \tilde{\varphi}_\perp(\alpha_i) e^{iq\cdot x(\alpha_1 + v\alpha_3)}\\
&\quad + if_K\frac{q_\mu}{q\cdot x}(q_\alpha x_\beta - q_\beta x_\alpha)\int \mathcal{D}\alpha_i \tilde{\varphi}_\parallel(\alpha_i) e^{iq\cdot x(\alpha_1 + v\alpha_3)}.
\end{aligned}\tag{10}$$

Here the operator $\tilde{G}_{\alpha\beta}$ is the dual of the $G_{\alpha\beta}$, $\tilde{G}_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\delta\rho}G^{\delta\rho}$, $\mathcal{D}\alpha_i$ is defined as $\mathcal{D}\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3\delta(1-\alpha_1-\alpha_2-\alpha_3)$ and $\Phi(\alpha_1,\alpha_2,\alpha_3) = \varphi_\perp + \varphi_\parallel - \tilde{\varphi}_\perp - \tilde{\varphi}_\parallel$. The twist-3 and twist-4 light-cone distribution amplitudes can be parametrized as

$$\begin{aligned}
\varphi_p(u,\mu) &= 1 + \Big(30\eta_3 - \frac{5}{2}\rho^2\Big)C_2^{1/2}(2u-1) + \Big(-3\eta_3\omega_3 - \frac{27}{20}\rho^2 - \frac{81}{10}\rho^2\tilde{a}_2\Big)C_4^{1/2}(2u-1),\\
\varphi_\sigma(u,\mu) &= 6u(1-u)\Big(1 + \Big(5\eta_3 - \frac{1}{2}\eta_3\omega_3 - \frac{7}{20}\rho^2 - \frac{3}{5}\rho^2\tilde{a}_2\Big)C_2^{3/2}(2u-1)\Big),\\
\phi_{3K}(\alpha_i,\mu) &= 360\alpha_1\alpha_2\alpha_3^2\Big(1 + \lambda_3(\alpha_1-\alpha_2) + \omega_3\frac{1}{2}(7\alpha_3 - 3)\Big),\\
\phi_\perp(\alpha_i,\mu) &= 30\delta^2(\mu)(\alpha_1-\alpha_2)\alpha_3^2\Big(\frac{1}{3} + 2\epsilon(\mu)(1-2\alpha_3)\Big),\\
\phi_\parallel(\alpha_i,\mu) &= 120\delta^2(\mu)\epsilon(\mu)(\alpha_1-\alpha_2)\alpha_1\alpha_2\alpha_3,\\
\tilde{\phi}_\perp(\alpha_i,\mu) &= 30\delta^2(\mu)\alpha_3^2(1-\alpha_3)\Big(\frac{1}{3} + 2\epsilon(\mu)(1-2\alpha_3)\Big),\\
\tilde{\phi}_\parallel(\alpha_i,\mu) &= -120\delta^2(\mu)\alpha_1\alpha_2\alpha_3\Big(\frac{1}{3} + \epsilon(\mu)(1-3\alpha_3)\Big),\\
g_2(u,\mu) &= \frac{10}{3}\delta^2(\mu)u(1-u)(2u-1),
\end{aligned}\tag{11}$$

where $C_2^{1/2}$, $C_4^{1/2}$ and $C_2^{3/2}$ are Gegenbauer polynomials, $\epsilon = \frac{21}{8}\omega_4$, $\eta_3 = (f_{3K}/f_K)(m_q + m_s/M_K^2)$ and $\rho^2 = m_s^2/M_K^2$ [14,15,17–19]. The parameters in the light-cone distribution amplitudes can be estimated from the QCD sum rules approach [14,15,17–19]. In this article, the energy scale μ is chosen to be $\mu = 1$ GeV.

Now we perform the double Borel transformation with respect to the variables $Q_1^2 = -p^2$ and $Q_2^2 = -(p+q)^2$ for the correlation function $T_p(p^2,(p+q)^2)$ in Eq. (6), and obtain the analytical expression for the invariant function in the hadronic representation,

$$B_{M_2^2}B_{M_1^2}T_p(Q_1^2,Q_2^2) = ig_{D_{s0}DK}f_D f_{D_{s0}}M_{D_{s0}} \times e^{-M_D^2/M_1^2}e^{-M_{D_{s0}}^2/M_2^2} + \cdots. \tag{12}$$

Here we have not shown the contributions from the high resonances and continuum states explicitly for simplicity. In order to match the duality regions below the thresholds s_0 and s_0' for the interpolating currents $J_\mu^D(x)$ and $J_{D_{s0}}(x)$ respectively, we can express the correlation function T_p at the level of quark-gluon degrees of freedom into the following form,

$$T_p(p^2,(p+q)^2) = i\int ds ds' \frac{\rho(s,s')}{(s-p^2)[s'-(p+q)^2]}, \tag{13}$$

then we perform the double Borel transformation with respect to the variables $Q_1 = -p^2$ and $Q_2^2 = -(p+q)^2$ directly. However, the analytical expressions for the spectral density $\rho(s,s')$ is hard to obtain, we have to resort to some approximations. As the contributions from the higher twist terms are suppressed by more powers of $1/-p^2$ or $1/-(p+q)^2$, the continuum subtractions will not affect the results remarkably, here we will use the expressions in Eq. (9) for the three-particle (quark-antiquark-gluon) twist-3 and twist-4 terms. In fact, their contributions are of minor importance; the dominating contributions come from the two-particle twist-3 terms involving the $\varphi_p(u)$ and $\varphi_\sigma(u)$. We perform the same trick as Refs. [17,20] and expand the amplitudes $\varphi_p(u)$ and $\varphi_\sigma(u)$ in terms of polynomials of $1-u$,

$$\varphi_p(u) + \frac{d\varphi_\sigma(u)}{6du} = \sum_{k=0}^{N} b_k(1-u)^k = b_k\left(\frac{s-m_c^2}{s-p^2}\right)^k, \tag{14}$$

then introduce the variable s' and the spectral density is obtained. After straightforward but cumbersome calculations, we can obtain the final expression for the double Borel transformed correlation function $T_p(M_1^2, M_2^2)$ at the level of quark-gluon degrees of freedom. The masses of the charmed mesons are $M_{D_{s0}} = 2.317$ GeV and $M_D = 1.865$ GeV, $M_D/(M_D + M_{D_{s0}}) \approx 0.45$, there exists an overlapping working window for the two Borel parameters M_1^2 and M_2^2. It is convenient to take the value $M_1^2 = M_2^2$, $u_0 = M_1^2/(M_1^2+M_2^2) = \frac{1}{2}$, $M^2 = (M_1^2 M_2^2)/(M_1^2+M_2^2)$, furthermore, the K meson light-cone distribution amplitudes are known quite well at the value $u_0 = \frac{1}{2}$. We can introduce the threshold parameter s_0 and make the simple replacement,

$$e^{-(m_c^2+u_0(1-u_0)M_K^2)/M^2} \to e^{-(m_c^2+u_0(1-u_0)M_K^2)/M^2} - e^{-s_0/M^2}$$

to subtract the contributions from the high resonances and continuum states [17],

$$\begin{aligned} B_{M_2^2}B_{M_1^2}T_p = i\Bigg\{&\frac{f_K M^2 M_K^2}{m_s}\left(e^{-(m_c^2+u_0(1-u_0)M_K^2)/M^2} - e^{-s_0/M^2}\right)\left(\varphi_p(u_0) + \frac{d\varphi_\sigma(u_0)}{6du_0}\right) + e^{-(m_c^2+u_0(1-u_0)M_K^2)/M^2}\Bigg[-2f_K m_c g_2(u_0) \\ &+ f_{3K}M_K^2\int_0^{u_0} d\alpha_1 \int_{u_0-\alpha_1}^{1-\alpha_1}\frac{d\alpha_3}{\alpha_3}\varphi_{3K}(\alpha_1, 1-\alpha_1-\alpha_3, \alpha_3)\left(2\frac{u_0-\alpha_1}{\alpha_3} - 3\right) \\ &- \frac{2f_K m_c M_K^2}{M^2}(1-u_0)\int_{1-u_0}^1 \frac{d\alpha_3}{\alpha_3^2}\int_0^{\alpha_3} d\beta \int_0^{1-\beta} d\alpha \Phi(\alpha, 1-\alpha-\beta, \beta) \\ &+ \frac{2f_K m_c M_K^2}{M^2}\left(\int_0^{1-u_0}\frac{d\alpha_3}{\alpha_3}\int_{u_0-\alpha_3}^{u_0} d\alpha_1 \int_0^{\alpha_1} d\alpha + \int_{1-u_0}^1 \frac{d\alpha_3}{\alpha_3}\int_{u_0-\alpha_3}^{1-\alpha_3} d\alpha_1 \int_0^{\alpha_1} d\alpha\right)\Phi(\alpha, 1-\alpha-\alpha_3, \alpha_3)\Bigg]\Bigg\}. \end{aligned} \tag{15}$$

A slight different manipulation (with the techniques taken in the Ref. [4]) for the dominating contributions comes from the terms involving the two-particle twist-3 light-cone distribution amplitudes $\varphi_p(u)$ and $\varphi_\sigma(u)$ leads to the following result,

$$\begin{aligned} B_{M_2^2}B_{M_1^2}T_p = ie^{-(m_c^2+u_0(1-u_0)M_K^2)/M^2}\Bigg\{&\frac{f_K M^2 M_K^2}{m_s}\sum_{k=0}^{N} b_k\left(\frac{M^2}{M_1^2}\right)^k\left(1 - e^{-(s_0-m_c^2)/M^2}\sum_{i=0}^{k}\frac{(\frac{s_0-m_c^2}{M^2})^i}{i!}\right) - 2f_K m_c g_2(u_0) \\ &+ f_{3K}M_K^2\int_0^{u_0} d\alpha_1\int_{u_0-\alpha_1}^{1-\alpha_1}\frac{d\alpha_3}{\alpha_3}\varphi_{3K}(\alpha_1, 1-\alpha_1-\alpha_3, \alpha_3)\left(2\frac{u_0-\alpha_1}{\alpha_3} - 3\right) - \frac{2f_K m_c M_K^2}{M^2}(1-u_0) \\ &\times \int_{1-u_0}^1 \frac{d\alpha_3}{\alpha_3^2}\int_0^{\alpha_3} d\beta\int_0^{1-\beta} d\alpha\Phi(\alpha, 1-\alpha-\beta, \beta) + \frac{2f_K m_c M_K^2}{M^2}\left(\int_0^{1-u_0}\frac{d\alpha_3}{\alpha_3}\int_{u_0-\alpha_3}^{u_0} d\alpha_1\int_0^{\alpha_1} d\alpha \right. \\ &\left. + \int_{1-u_0}^1\frac{d\alpha_3}{\alpha_3}\int_{u_0-\alpha_3}^{1-\alpha_3} d\alpha_1 \int_0^{\alpha_1} d\alpha\right)\Phi(\alpha, 1-\alpha-\alpha_3, \alpha_3)\Bigg\}. \end{aligned} \tag{16}$$

In deriving the above expressions for $\varphi_p(u) + [d\varphi_\sigma(u)]/(6du)$, we have neglected the terms $\sim M_K^4$, here $u_0 = M_1^2/(M_1^2 + M_2^2)$ and $M^2 = (M_1^2 M_2^2)/(M_1^2 + M_2^2)$.

Matching the Eq. (12) with the Eqs. (15) and (16) below the threshold s_0, we obtain two sum rules for the strong coupling constant $g_{D_{s0}DK}$,

$$g_{D_{s0}DK}=\frac{1}{f_D f_{D_{s0}} M_{D_{s0}}} e^{(M_{D_{s0}}^2/M_2^2)+(M_D^2/M_1^2)}\left\{\frac{f_K M^2 M_K^2}{m_s}\left(e^{-m_c^2+u_0(1-u_0)M_K^2/M^2}-e^{-s_0/M^2}\right)\left(\varphi_p(u)+\frac{d\varphi_\sigma(u)}{6du}\right)\right.$$
$$+e^{-(m_c^2+u_0(1-u_0)M_K^2)/M^2}\left[-2f_K m_c g_2(u_0)+f_{3K}M_K^2\int_0^{u_0}d\alpha_1\int_{u_0-\alpha_1}^{1-\alpha_1}\frac{d\alpha_3}{\alpha_3}\varphi_{3K}(\alpha_1,1-\alpha_1-\alpha_3,\alpha_3)\right.$$
$$\times\left(2\frac{u_0-\alpha_1}{\alpha_3}-3\right)-\frac{2f_K m_c M_K^2}{M^2}(1-u_0)\int_{1-u_0}^1\frac{d\alpha_3}{\alpha_3^2}\int_0^{\alpha_3}d\beta\int_0^{1-\beta}d\alpha\Phi(\alpha,1-\alpha-\beta,\beta)$$
$$\left.\left.+\frac{2f_K m_c M_K^2}{M^2}\left(\int_0^{1-u_0}\frac{d\alpha_3}{\alpha_3}\int_{u_0-\alpha_3}^{u_0}d\alpha_1\int_0^{\alpha_1}d\alpha+\int_{1-u_0}^1\frac{d\alpha_3}{\alpha_3}\int_{u_0-\alpha_3}^{1-\alpha_3}d\alpha_1\int_0^{\alpha_1}d\alpha\right)\Phi(\alpha,1-\alpha-\alpha_3,\alpha_3)\right]\right\}; \quad (17)$$

$$g_{D_{s0}DK}=\frac{1}{f_D f_{D_{s0}} M_{D_{s0}}} e^{(M_{D_{s0}}^2/M_2^2)+(M_D^2/M_1^2)-(m_c^2+u_0(1-u_0)M_K^2)/M^2}\left\{\frac{f_K M^2 M_K^2}{m_s}\sum_{k=0}^N b_k\left(\frac{M^2}{M_1^2}\right)^k\left(1-e^{-(s_0-m_c^2)/M^2}\sum_{i=0}^k\frac{(\frac{s_0-m_c^2}{M^2})^i}{i!}\right)\right.$$
$$-2f_K m_c g_2(u_0)+f_{3K}M_K^2\int_0^{u_0}d\alpha_1\int_{u_0-\alpha_1}^{1-\alpha_1}\frac{d\alpha_3}{\alpha_3}\varphi_{3K}(\alpha_1,1-\alpha_1-\alpha_3,\alpha_3)\left(2\frac{u_0-\alpha_1}{\alpha_3}-3\right)$$
$$-\frac{2f_K m_c M_K^2}{M^2}(1-u_0)\int_{1-u_0}^1\frac{d\alpha_3}{\alpha_3^2}\int_0^{\alpha_3}d\beta\int_0^{1-\beta}d\alpha\Phi(\alpha,1-\alpha-\beta,\beta)$$
$$\left.+\frac{2f_K m_c M_K^2}{M^2}\left(\int_0^{1-u_0}\frac{d\alpha_3}{\alpha_3}\int_{u_0-\alpha_3}^{u_0}d\alpha_1\int_0^{\alpha_1}d\alpha+\int_{1-u_0}^1\frac{d\alpha_3}{\alpha_3}\int_{u_0-\alpha_3}^{1-\alpha_3}d\alpha_1\int_0^{\alpha_1}d\alpha\right)\Phi(\alpha,1-\alpha-\alpha_3,\alpha_3)\right\} \quad (18)$$

corresponding to the Eq. (15) and (16) respectively.

III. NUMERICAL RESULTS AND DISCUSSIONS

The parameters are taken as $m_s=(140\pm10)$ MeV, $m_c=(1.25\pm0.10)$ GeV, $\lambda_3=1.6\pm0.4$, $f_{3K}=(0.45\pm0.15)\times10^{-2}$ GeV^2, $\omega_3=-1.2\pm0.7$, $\delta^2=(0.20\pm0.06)$ GeV^2, $\omega_4=0.2\pm0.1$, $\tilde{a}_2=0.25\pm0.15$ [14,15,17–19], $f_K=0.160$ GeV, $M_K=498$ MeV, $M_{D_{s0}}(2317)=2.317$ GeV, $M_D=1.865$ GeV, $f_D=(0.23\pm0.02)$ GeV [21], and $f_{D_{s0}}=(0.225\pm0.025)$ GeV [8]. The duality thresholds s_0 in Eqs. (17) and (18) are taken as $s_0=(6.1-6.5)$ GeV to avoid possible contaminations from the high resonances and continuum states, from the Fig. 1, we can see that the numerical results are not sensitive to the threshold parameter s_0 in this region. The Borel parameters are chosen as 10 GeV$^2\le M_1^2=M_2^2\le20$ GeV^2 and 5 GeV$^2\le M^2\le10$ GeV^2, in those regions, the values of the strong coupling constant $g_{D_{s0}DK}$ are rather stable from the sum rule in Eq. (17) with the simple subtraction, which are shown in the Figs. 1–7. However, the values from the sum rule in Eq. (18) with the more sophisticated subtraction are not stable according to the variations of the Borel parameter M^2.

The uncertainties of the five parameters δ^2, ω_4, ω_3, λ_3 and $\tilde{a}_2$ can not result in large uncertainties for the numerical values. The main uncertainties come from the five parameters f_{3K}, m_s, m_c, f_D and $f_{D_{s0}}$, small variations of those parameters can lead to relatively large changes for the numerical values, which are shown in the Figs. 2–6, respectively. Taking into account all the uncertainties, finally we obtain the numerical results for the strong coupling constant,

$$g_{D_{s0}DK}=(9.3^{+2.7}_{-2.1})\ \text{GeV}. \quad (19)$$

The strong coupling constant $g_{D_{s0}DK}$ can be related to the parameter h in the heavy-light Chiral perturbation theory [22,23],

$$g_{SP\pi}=\sqrt{M_S M_P}\frac{M_S^2-M_P^2}{M_S}\frac{|h|}{f_\pi}. \quad (20)$$

Here the S are the scalar heavy mesons with 0^+, the P are the heavy pseudoscalar mesons with 0^-, and the π stand for the light pseudoscalar mesons. The parameter h has been estimated with the light-cone QCD sum rules [23], quark models [24], Adler-Weisberger type sum rule [25], and extracted from the experimental data [9], the values are

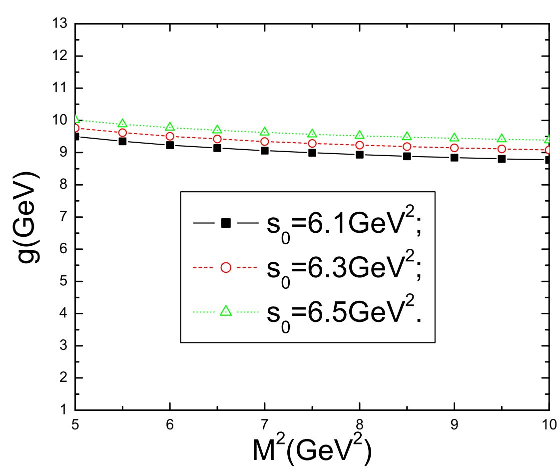

FIG. 1 (color online). The $g_{D_{s0}DK}$ with the parameters M^2 and s_0.

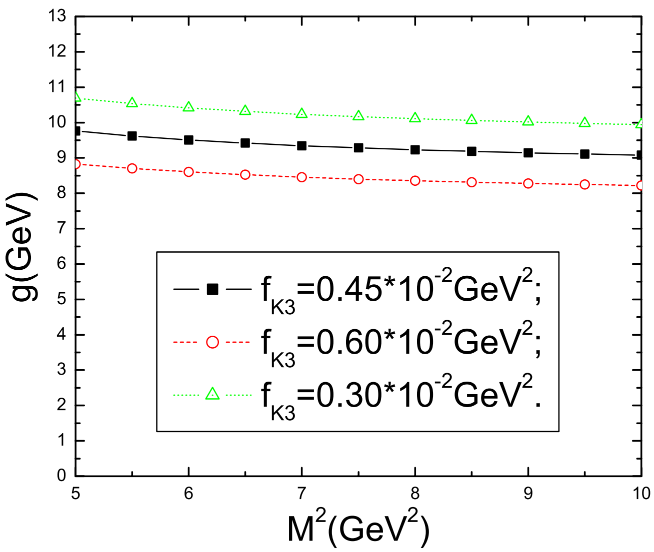

FIG. 2 (color online). The $g_{D_{s0}DK}$ with the parameters M^2 and f_{K3}.

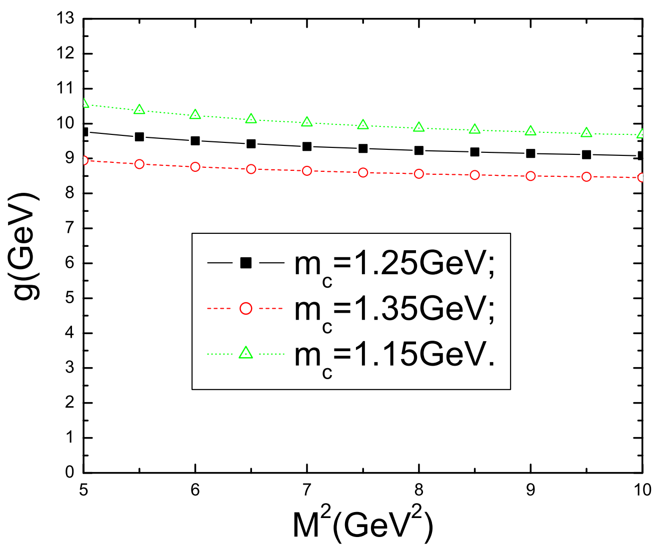

FIG. 4 (color online). The $g_{D_{s0}DK}$ with the parameters M^2 and m_c.

listed in Table I, from those values we can estimate the values of the corresponding strong coupling constant $g_{D_{s0}DK}$ in the $SU(3)$ limit for the light pseudoscalar mesons. The value of the dimensionless effective coupling constant $\Gamma/k = 0.46(9)$ from Lattice QCD [27] is somewhat smaller than the values extracted from the experimental data $\Gamma/k = 0.73^{+28}_{-24}$, here the Γ is the decay width and the k is the decay momentum. Our numerical values $g_{D_{s0}DK} = (9.3^{+2.7}_{-2.1})$ GeV are somewhat larger compared to the existing estimations in Refs. [9,23–25] and about 4 times as large as the energy scale $M_{D_{s0}} = 2.317$ GeV, and favor the hadronic dressing mechanism.

Here we will take a short digression to discuss the hadronic dressing mechanism [12,13]; one can consult the original literature for the details. In the conventional constituent quark models, the mesons are taken as quark-antiquark bound states. The spectrum can be obtained by solving the corresponding Schrödinger's or Dirac's equations with the phenomenological potential which trying to incorporate the observed properties of the strong interactions, such as the asymptotic freedom and confinement. The solutions can be referred as confinement bound states or bare quark-antiquark states (or kernels). If we switch on the hadronic interactions between the confinement bound states and the free ordinary two-meson states, the situation

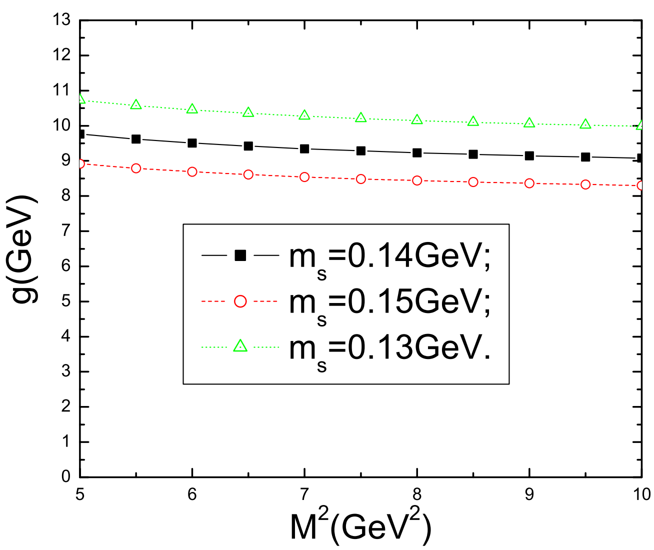

FIG. 3 (color online). The $g_{D_{s0}DK}$ with the parameters M^2 and m_s.

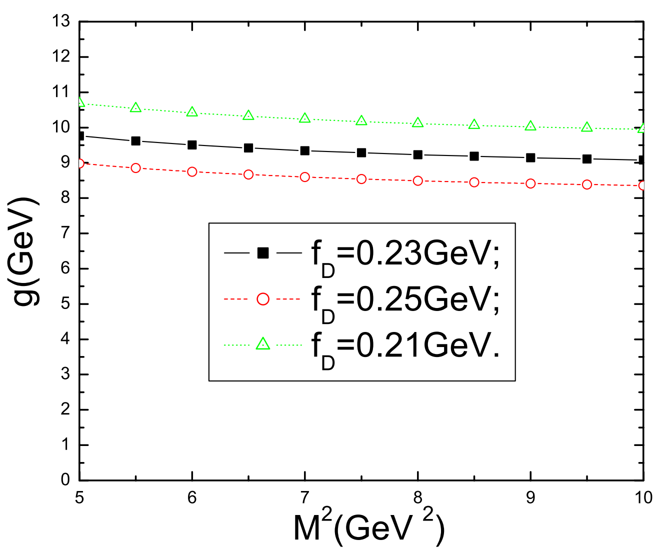

FIG. 5 (color online). The $g_{D_{s0}DK}$ with the parameter M^2 and f_D.

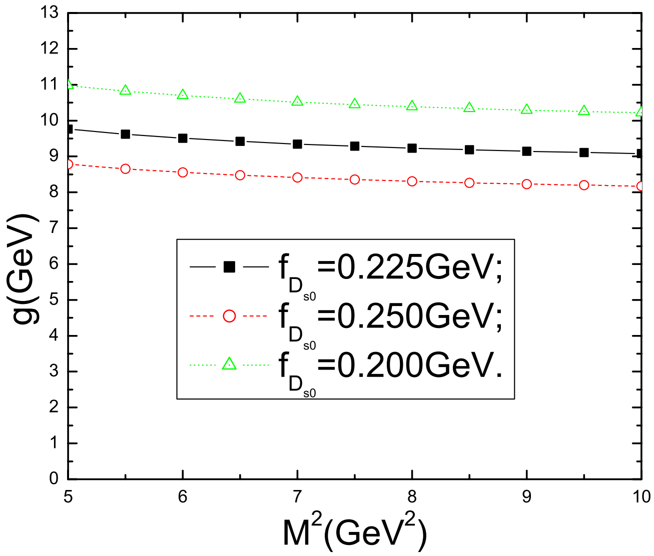

FIG. 6 (color online). The $g_{D_{s0}DK}$ with the parameters M^2 and $f_{D_{s0}}$.

becomes more complex. With the increasing hadronic coupling constants, the contributions from the hadronic loops of the intermediate mesons become larger and the bare quark-antiquark states can be distorted greatly. There may be double poles or several poles in the scattering amplitudes with the same quantum number as the bare quark-antiquark kernels; some stem from the bare quark-antiquark kernels while the others originate from the continuum states. The strong coupling may enrich the bare quark-antiquark states with other components, for example, virtual mesons pairs and spend part (or most part) of their lifetime as virtual mesons pairs.

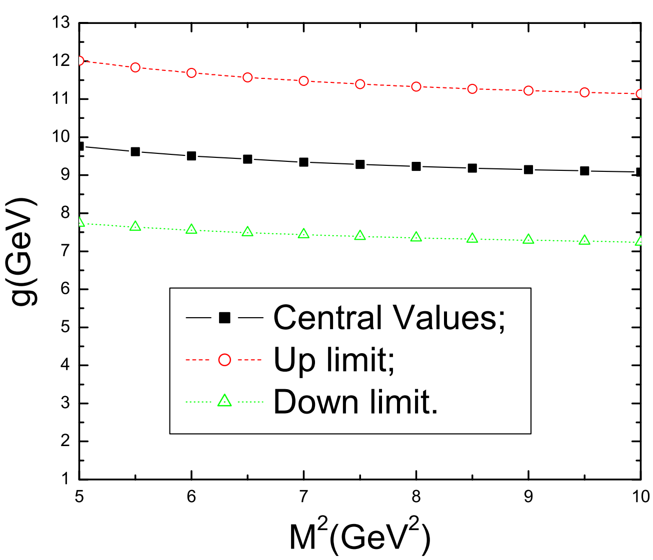

FIG. 7 (color online). The $g_{D_{s0}DK}$ with the parameter M^2.

TABLE I. Numerical values for the parameter h, and the corresponding values for the strong coupling constant $g_{D_{s0}DK}$ in the $SU(3)$ limit.

$\|h\|$	$g_{D_{s0}DK}$ (GeV)	Reference
0.52 ± 0.17	5.5 ± 1.8	[23]
0.536	5.68	[24]
<0.93	<9.86	[25]
$0.57 - 0.74$	6.0–7.8	[9]
	10.203	[26]
$0.88^{+0.26}_{-0.20}$	$9.3^{+2.7}_{-2.1}$	This work

The large values for the strong coupling constant $g_{D_{s0}DK}$ obviously support the hadronic dressing mechanism, the $D_{s0}(2317)$ (just like the scalar mesons $f_0(980)$ and $a_0(980)$, see Ref. [4]) can be taken as having small scalar $c\bar{s}$ kernel of typical meson size with large virtual S-wave DK cloud. In Ref. [26], the authors analyze the unitarized two-meson scattering amplitudes from the heavy-light Chiral Lagrangian, and observe that the scalar meson $D_{s0}(2317)$ appears as the bound state pole with the strong coupling constant $g_{D_{s0}DK} = 10.203$ GeV. Our numerical results $g_{D_{s0}DK} = (9.3^{+2.7}_{-2.1})$ GeV are certainly reasonable and can make robust predictions. However, we take the point of view that the scalar meson $D_{s0}(2317)$ be bound state in the sense that it appears below the DK threshold; its constituents may be the bare $c\bar{s}$ state, the virtual DK pair and their mixing, rather than the DK bound state.

IV. CONCLUSIONS

In this article, we take the point of view that the charmed scalar meson $D_{s0}(2317)$ is the conventional $c\bar{s}$ meson and calculate the strong coupling constant $g_{D_{s0}DK}$ within the framework of the light-cone QCD sum rules approach. The numerical values for the scalar-DK coupling constant $g_{D_{s0}DK}$ are compatible with the existing estimations although somewhat larger, the large values support the hadronic dressing mechanism. Just like the scalar mesons $f_0(980)$ and $a_0(980)$, the scalar meson $D_{s0}(2317)$ may have small $c\bar{s}$ kernel of typical $c\bar{s}$ meson size. The strong coupling to virtual intermediate hadronic states (or the virtual mesons loops) can result in smaller mass than the conventional scalar $c\bar{s}$ meson in the constituent quark models, enrich the pure $c\bar{s}$ state with other components. The $D_{s0}(2317)$ may spend part (or most part) of its lifetime as virtual DK state.

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