Baryon spectrum in large N_c chiral soliton and in quark models

Vladimir B. Kopeliovich and Andrei M. Shunderuk

Institute for Nuclear Research of RAS, Moscow 117312, Russia

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Strangeness contents of baryons are calculated within the rigid rotator model for arbitrary number of colors N_c. The problem of extrapolation to realistic value $N_c = 3$ is noted, based on explicit calculations and comparison of the rigid rotator and rigid oscillator variants of the model. Some features of exotic baryon spectra ($\{\overline{10}\}$, $\{27\}$, and $\{35\}$-plets of baryons) obtained in the chiral soliton approach can be understood in terms of simplified quark ($4q\bar{q}$) wave functions. The effective mass of strange antiquark in different $SU(3)$ multiplets of pentaquarks should depend on the particular multiplet, to link the predictions of soliton and quark models. The estimate of the 6_F and $\bar{3}_F$ diquarks mass difference can be made from comparison with chiral soliton model results for masses of exotic baryons from different $SU(3)$ multiplets. The masses of baryons partners with different values of spin J are also estimated.

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I. INTRODUCTION

Description of hadrons structure in terms of their quark constituents is generally accepted, but the alternative description within e.g. topological soliton (Skyrme) model [1,2] and its modifications also is useful and has certain advantages in comparison with traditional approaches. The chiral (topological) soliton approach is based on general principles and few ingredients incorporated in the effective chiral Lagrangian, this is the reason for apparent simplifications in comparison, for example, with attempts to solve relativistic many-body problems. To simplify the latter, some additional objects like diquarks and triquarks have been phenomenologically introduced and discussed especially intensively after recent observations of the so-called pentaquarks [3,4].[1] The concept of diquarks "as an organizing principle for hadron spectroscopy" is considered in details in [12], see also [13]. The concepts of diquarks, triquarks, or other correlated quark clusters are certainly of useful heuristic value, although their properties have not been deduced rigorously from basic QCD Lagrangian. It should be noted that diquarks present in different physical states, baryons, or mesons, can have different properties like the effective mass and size, even for the same quantum numbers.[2]

In the present paper we perform explicit calculation of the strangeness contents of exotic and nonexotic baryon states at an arbitrary number of colors N_c, and discuss connection of the chiral soliton approach (CSA) and simple quark (pentaquark) model for exotic baryon states, in the realistic $N_c = 3$ case. Although there was intensive discussion of connections of the rigid rotator model (RRM) and the bound state model (BSM) in the literature, mainly in the large N_c limit [15–22], explicit analytical calculations of observable quantities at arbitrary N_c were lacking still, except several cases [8,21,22]. The rotation-vibration approach (RVA) described in [22,23] and references in these papers, includes both rotational (zero modes) and vibrational degrees of freedom of solitons and is a generalization of both RRM and BSM, which appear therefore as particular variants of RVA when certain degrees of freedom are frozen (see also discussion in Sec. III).[3] As our studies have shown, there is an essential difference between results of RR calculation and BS model (in its commonly accepted version) in the next-to-leading term contributions of the $1/N_c$ expansion for the mass splittings inside $SU(3)$ multiplets of baryons. Since the expansion parameter is large, there is a problem of extrapolation from the large N_c limit to the real $N_c = 3$ world. This problem of extrapolation to realistic value of N_c we note in the BSM, persists in RVA as well.

Some features of exotic baryon spectra obtained previously within the topological soliton model [7,26–29] can be understood in the framework of pentaquark model, independently of its particular variant (see, e.g. [30]). The Gell-Mann–Okubo relations which are valid in any model where the $SU(3)$ symmetry breaking is introduced in a definite way, mimic the mass splittings of simple quark models, where they are mainly due to the mass difference between strange and nonstrange quarks.

Comparison of the results of calculation within the soliton model—if we believe that CSA provides the cor-

[1] A contradictive present situation with experimental observation of possible pentaquark states is discussed, e.g. in [5,6]. Consideration of baryon states in the present paper is relevant independently on particular values of masses, widths, and other properties of exotic baryon states measured experimentally. A detailed discussion of theoretical predictions of these states can be found in [7–11]

[2] Some analogy with nuclei can be noted: two, three, etc. nucleon clusters play an important role in the structure of heavy nuclei, however, it is not possible to evaluate their properties from those of deuteron, helium, etc., only. See, e.g. [14] for discussion of the role of femtometer toroidal structures in nuclei.

[3] The approach of [22] was criticized in [24], and response to this criticism was given in [25].

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rect description, of course—with the naive quark-diquark model allows us to get information about properties of constituents (quarks, antiquarks, diquarks), e.g. mass differences of diquarks with different quantum numbers, in qualitative agreement with other estimates. Quantitatively, however, the mass difference between "bad" and "good" diquarks obtained in this way contains considerable uncertainties. Another result of interest is relatively strong dependence of the mass of strange antiquark on the $SU(3)$ baryon multiplet under discussion. It is shown as well that the partners of baryon resonances with different J^P predicted within quark models are present also in the CSA, although they have usually higher energy. Some of these questions have been addressed in talks [8], and here we add more rigour to this consideration.

In the next section strangeness contents of nonexotic and exotic states are calculated at an arbitrary number of colors N_c, in Sec. III these results are compared with that of the bound state model, its rigid oscillator (RO) variant, for a small enough value of the flavor symmetry breaking mass. In Sec. IV comparison with the simple pentaquark model is performed, the partners of baryon states with different spin are discussed in Sec. V, and the final section contains some conclusions.

II. STRANGENESS CONTENTS OF BARYONS FOR AN ARBITRARY NUMBER OF COLORS

We begin our consideration with scalar strangeness contents (C_S in what follows) of baryons, nonexotic and exotic, which defines the mass splittings within $SU(3)$ multiplets of baryons in the chiral soliton approach, by the following reasons. First, strangeness content of baryons or baryon resonances is important and a physically transparent characteristic of these states, not calculated yet analytically for an arbitrary number of colors N_c.[4] Second, the behavior of this quantity as a function of N_c allows to make some conclusions (mostly pessimistic) about the possibility of extrapolation from large N_c to a realistic world with $N_c = 3$. A comparison of different variants of the model at arbitrary N_c and at $N_c \to 3$ also allows to make conclusions about the reliability of the whole CSA.

The spectrum of observed baryon states is obtained within chiral (topological) soliton models by means of quantization of the motion of starting classical field configuration (usually it is $SU(2)$ configuration, although it may be some other configuration as well) in $SU(3)$ collective coordinates space. In the rigid rotator approximation the mass formula for the quantized states is [26,31–34]

$$M(p,q,Y,I,J) = M_{cl} + \left[C_2(SU_3) - J(J+1) - \frac{N_c^2}{12}\right]\frac{1}{2\Theta_K} + \frac{J(J+1)}{2\Theta_\pi} + \Delta M, \tag{1}$$

The second order Casimir operator $C_2(SU_3) = (p^2 + q^2 + pq)/3 + p + q$ for the (p, q)-multiplet, Y, I, J are the hypercharge, isospin, and spin of baryon, Θ_π and Θ_K are the moments of inertia, of the order of (5–6) GeV^{-1} and (2–3) GeV^{-1}. The mass splittings within multiplets of baryons are defined by the following relation [31], see also [26,32–34] where details of evaluation and expressions for the moments of inertia can be found

$$\Delta M = \left[\Gamma\left(\frac{F_K^2}{F_\pi^2}\mu_K^2 - \mu_\pi^2\right) + (F_K^2 - F_\pi^2)\tilde{\Gamma}\right]C_S, \tag{2}$$

if the configuration mixing is not included.

$$\Gamma = \frac{F_\pi^2}{2}\int (1 - c_f)d^3r \tag{3}$$

is the so-called σ term, one of the characteristics of the classical configuration, and

$$\tilde{\Gamma} = \frac{1}{4}\int c_f\left(f'^2 + \frac{2s_f^2}{r^2}\right)d^3r, \tag{4}$$

TABLE I. Strangeness contents of the octet, decuplet, and antidecuplet of baryons at arbitrary $N = N_c$, for unmixed states. $Y' = S + 1$, states which appear only if $N > 3$ are marked by $*$.

$[p, q]$	$C_S(N)$	$C_S(N = 3)$
$[1, (N-1)/2]$		
$Y' = 1, I = 1/2$	$2(N+4)/[(N+3)(N+7)]$	7/30
$Y' = 0, I = 0$	$3/(N+7)$	9/30
$Y' = 0, I = 1$	$(3N+13)/[(N+3)(N+7)]$	11/30
$Y' = -1, I = 1/2$	$4/(N+7)$	12/30
$*Y' = -1, I = 3/2$	$(4N+18)/[(N+3)(N+7)]$	⋯
$[3, (N-3)/2]$		
$Y' = 1, I = 3/2$	$2(N+4)/[(N+1)(N+9)]$	7/24
$Y' = 0, I = 1$	$(3N+7)/[(N+1)(N+9)]$	8/24
$*Y' = 0, I = 2$	$(3N+15)/[(N+1)(N+9)]$	⋯
$Y' = -1, I = 1/2$	$(4N+6)/[(N+1)(N+9)]$	9/24
$*Y' = -1, I = 3/2$	$4(N+3)/[(N+1)(N+9)]$	⋯
$*Y' = -1, I = 5/2$	$(4N+22)/[(N+1)(N+9)]$	⋯
$Y' = -2, I = 0$	$5/(N+9)$	10/24
$*Y' = -2, I = 1$	$(5N+9)/[(N+1)(N+9)]$	⋯
$*Y' = -2, I = 2$	$(5N+17)/[(N+1)(N+9)]$	⋯
$*Y' = -2, I = 3$	$(5N+29)/[(N+1)(N+9)]$	⋯
$[0, (N+3)/2]$		
$Y' = 2, I = 0$	$3/(N+9)$	6/24
$Y' = 1, I = 1/2$	$(4N+9)/[(N+3)(N+9)]$	7/24
$Y' = 0, I = 1$	$(5N+9)/[(N+3)(N+9)]$	8/24
$Y' = -1, I = 3/2$	$(6N+9)/[(N+3)(N+9)]$	9/24
$*Y' = -2, I = 2$	$(7N+9)/[(N+3)(N+9)]$	⋯

[4] Numerical calculations for the "octet" and "decuplet" of baryons have been performed recently, however (Herbert Weigel, private communication, see also [22]).

f is the profile function of the Skyrmion, the values of physical masses μ_K, μ_π and decay constants F_K, F_π are taken from the experiment. C_S is the so-called strangeness content of the quantized baryon state. Within the RR model [31] rotation of incident $SU(2)$ configuration is described with the help of matrix of collective coordinates $A(t) \in SU(3)$, which is usually parametrized as $A = A_1(SU_2) \times \exp(i\nu\lambda_4)A_2(SU_2)\exp(i\rho\lambda_8/\sqrt{3})$, where $SU(2)$ rotation matrices depend each of 3 variables, λ_4, λ_8 are Gell-Mann matrices, see e.g. [35].

The wave functions of baryons in $SU(3)$ space, $\Psi(p, q; Y, I, I_3)$ are just $SU(3)$ Wigner functions depending on 8 parameters incorporated in matrix A: the integers (p, q) define the $SU(3)$ multiplet under consideration, Y, I, I_3 are hypercharge, isospin, and its 3d projection of particular baryon state. For arbitrary (odd) number of colors the hypercharge is connected with strangeness by relation $Y = S + NB/3$, see e.g. [2,8,15] (we omit the index c in N_c in most of the formulas and in Tables I and II). It is more convenient therefore to use for the $B = 1$ case the quantity $Y' = S + 1$, as we do in the tables and in Fig. 1. Within this parametrization the only flavor changing parameter is ν, which defines the deviation in "strange direction," and strangeness content

$$C_S = \tfrac{1}{2}\langle\Psi_B(\nu)|\sin^2\nu|\Psi_B(\nu)\rangle, \tag{5}$$

see the appendix, where explicit examples of ν-dependent wave functions of some baryon states are given. The main contribution to the baryon mass operator, depending on flavor symmetry breaking (FSB) mass m_K equals to [31]

TABLE II. Strangeness contents for unmixed states of the {27}-plet (spin $J = 3/2$) and {35}-plet ($J = 5/2$) of baryons, for arbitrary N and numerically for $N = 3$. States which exist only for $N > 3$ are marked with $*$.

$[2, (N+1)/2]$	$C_S(N)$	$C_S(N=3)$
$Y' = 2, I = 1$	$(3N+23)/[(N+5)(N+11)]$	32/112
$Y' = 1, I = 3/2$	$(4N^2 + 65N/2 - 3/2)/[(N+1)(N+5)(N+11)]$	33/112
$Y' = 1, I = 1/2$	$(4N+24)/[(N+5)(N+11)]$	36/112
$Y' = 0, I = 2$	$(5N^2 + 39N - 26)/[(N+1)(N+5)(N+11)]$	34/112
$Y' = 0, I = 1$	$(5N^2 + 33N + 8)/[(N+1)(N+5)(N+11)]$	38/112
$Y' = 0, I = 0$	$5/(N+11)$	5/14
$*Y' = -1, I = 5/2$	$(6N^2 + \frac{91}{2}N - \frac{101}{2})/[(N+1)(N+5)(N+11)]$	⋯
$Y' = -1, I = 3/2$	$(6N^2 + 38N - 8)/[(N+1)(N+5)(N+11)]$	40/112
$Y' = -1, I = 1/2$	$(6N + 7/2)/[(N+1)(N+11)]$	43/112
$*Y' = -2, I = 3$	$(7N^2 + 52N - 75)/[(N+1)(N+5)(N+11)]$	⋯
$*Y' = -2, I = 2$	$(7N^2 + 43N - 24)/[(N+1)(N+5)(N+11)]$	⋯
$Y' = -2, I = 1$	$(7N+2)/[(N+1)(N+11)]$	46/112
$[4, (N-1)/2]$		
$Y' = 2, I = 2$	$(3N+25)/[(N+3)(N+13)]$	34/96
$Y' = 1, I = 5/2$	$(4N^2 + 85N/3 - 79)/[(N-1)(N+3)(N+13)]$	21/96
$Y' = 1, I = 3/2$	$(4N+24)/[(N+3)(N+13)]$	36/96
$*Y' = 0, I = 3$	$(5N^2 + \frac{104}{3}N - 133)/[(N-1)(N+3)(N+13)]$	⋯
$Y' = 0, I = 2$	$(5N^2 + \frac{74}{3}N - 67)/[(N-1)(N+3)(N+13)]$	26/96
$Y' = 0, I = 1$	$(5N+23)/[(N+3)(N+13)]$	38/96
$*Y' = -1, I = 7/2$	$(6N^2 + 41N - 187)/[(N-1)(N+3)(N+13)]$	⋯
$*Y' = -1, I = 5/2$	$(6N^2 + \frac{88}{3}N - 110)/[(N-1)(N+3)(N+13)]$	⋯
$Y' = -1, I = 3/2$	$(6N^2 + 21N - 55)/[(N-1)(N+3)(N+13)]$	31/96
$Y' = -1, I = 1/2$	$(6N+22)/[(N+3)(N+13)]$	40/96
$*Y' = -2, I = 4$	$(7N^2 + \frac{142}{3}N - 241)/[(N-1)(N+3)(N+13)]$	⋯
$*Y' = -2, I = 3$	$(7N^2 + 34N - 153)/[(N-1)(N+3)(N+13)]$	⋯
$*Y' = -2, I = 2$	$(7N^2 + 24N - 87)/[(N-1)(N+3)(N+13)]$	⋯
$Y' = -2, I = 1$	$(7N^2 + 52N/3 - 43)/[(N-1)(N+3)(N+13)]$	36/96
$Y' = -2, I = 0$	$7/(N+13)$	42/96
$*Y' = -3, I = 9/2$	$(8N^2 + \frac{161}{3}N - 295)/[(N-1)(N+3)(N+13)]$	⋯
$*Y' = -3, I = 7/2$	$(8N^2 + \frac{116}{3}N - 196)/[(N-1)(N+3)(N+13)]$	⋯
$*Y' = -3, I = 5/2$	$(8N^2 + 27N - 119)/[(N-1)(N+3)(N+13)]$	⋯
$*Y' = -3, I = 3/2$	$(8N^2 + 56N/3 - 64)/[(N-1)(N+3)(N+13)]$	⋯
$Y' = -3, I = 1/2$	$(8N - 31/3)/[(N-1)(N+13)]$	41/96

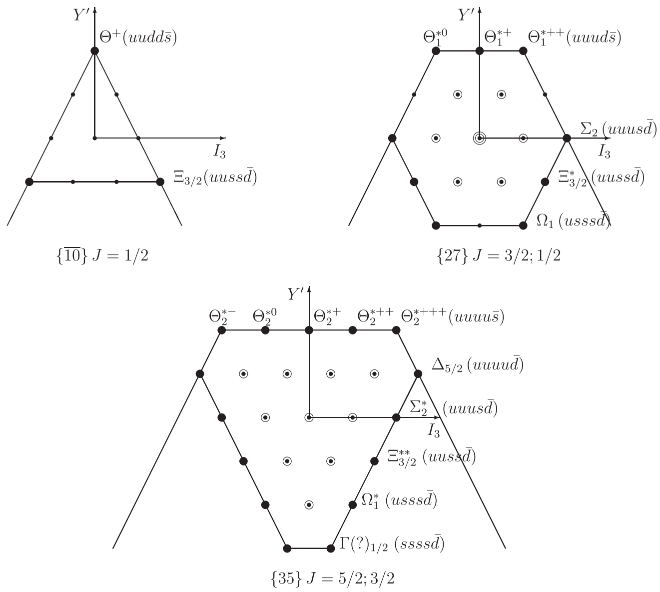

FIG. 1. The $I_3 - Y'$ diagrams ($Y' = S + 1$) for multiplets of pentaquark baryons, antidecuplet, $\{27\}$ and $\{35\}$-plets. For $N > 3$ these diagrams should be extended within long lines, as shown in the picture. Quark contents are given for manifestly exotic states, when $N = 3$.

$$\Delta M = m_K^2 \Gamma \frac{\langle 1 - D_{88}(\nu)\rangle}{3} = \frac{1}{2} m_K^2 \Gamma \langle \sin^2 \nu \rangle, \qquad (6)$$

since $D_{88} = Tr(A^\dagger \lambda_8 A \lambda_8)/2 = 1 - 3(\sin^2\nu)/2$, $m_K^2 = F_K^2 \mu_K^2 / F_\pi^2 - \mu_\pi^2$.

The second term in (2), proportional to $\tilde{\Gamma}$ gives relatively small contribution in comparison with the first term; it is however not negligible for realistic values of masses and parameters. When $m_K \to 0$, then the ν rotation becomes zero mode. More details can be found e.g. in [8,26,34].

The quantity $\langle D_{88}\rangle$ can be calculated using Clebsh-Gordan coefficients for an arbitrary number of colors N which have been presented previously for few cases in [21,36] (however, the strangeness contents have not been calculated). Another method of calculations which we prefer here is to use the baryons wave functions in the $SU(3)$-configuration space, as it was described, e.g. in [32]. For large N generalization of the octet, the decuplet of baryons, and for the Θ^+ baryon this method has been used recently in [8] to calculate strangeness contents of these baryons. For exotic baryon multiplets, "antidecuplet," "$\{27\}$," and "$\{35\}$"-plets (shown in Fig. 1) we present here strangeness contents and wave functions for the first time (see the appendix). In Tables I and II strangeness contents are given for an arbitrary number of colors, and also numerically for $N = 3$.

It can be seen easily from Tables I and II that for the fixed value of strangeness, C_S decreases as $1/N$ with increasing N—in agreement with the fact that a fixed number of quarks are strange, whereas the total number of constituent quarks is N, or $N + 2$ for "pentaquarks." The difference of strangeness contents of states from different $SU(3)$ multiplets, but with the same value of strangeness, decreases as $1/N^2$ or faster. E.g., the difference of C_S for the nucleon with $I = 1/2$ and delta ($I = 3/2$) decreases like $1/N^3$ [8].

Any chain of states within definite $SU(3)$ multiplet, satisfying the relation $I = \pm Y/2 + C$, i.e. which belong to such straight lines in $(I - Y')$-plane, has equidistant behavior due to Gell-Mann–Okubo relations.[5] According to these, the mass splitting and strangeness contents within the $SU(3)$ multiplets can be presented in the form

$$C_S(p, q, Y', I) = a(p, q)Y' + b(p, q)[Y'^2/4 - I(I + 1)] + c(p, q), \qquad (7)$$

where $a(p, q)$, $b(p, q)$, being constants within any $SU(3)$ multiplet, are different for different multiplets (p, q). Linear behavior of masses of any chain of states with $I = \pm Y'/2 + C$ follows then immediately. Since $Y' = S + 1$, (7) can be easily rewritten in terms of strangeness S and isospin I.

From Table I we easily obtain

$$a(\text{"}\{8\}\text{"}) = -\frac{N + 2}{(N + 3)(N + 7)}, \quad b(\text{"}\{8\}\text{"}) = -\frac{2}{(N + 3)(N + 7)}, \quad c(\text{"}\{8\}\text{"}) = \frac{3}{(N + 7)}, \qquad (8)$$

and for decuplet:

$$a(\text{"}\{10\}\text{"}) = -\frac{N + 2}{(N + 1)(N + 9)}, \quad b(\text{"}\{10\}\text{"}) = -\frac{2}{(N + 1)(N + 9)}, \quad c(\text{"}\{10\}\text{"}) = \frac{3}{(N + 9)}. \qquad (9)$$

For antidecuplet $I = 1 - Y'/2$, relation (7) takes the form

$$C_S = (a + 3b/2)Y' - 2b + c,$$

and we obtain from Table I two relations:

$$a(\text{"}\{\overline{10}\}\text{"}) + \frac{3}{2} b(\text{"}\{\overline{10}\}\text{"}) = -\frac{N}{(N + 3)(N + 9)}, \quad -2b(\text{"}\{\overline{10}\}\text{"}) + c(\text{"}\{\overline{10}\}\text{"}) = \frac{5N + 9}{(N + 3)(N + 9)}. \qquad (10)$$

For "$\{27\}$"-plet we have from Table II:

[5] The validity of Gell-Mann–Okubo relations for the octet and decuplet of baryons at an arbitrary number of colors has been noted long ago in the paper [37] where the $1/N$ expansion and induced representation methods were developed for describing baryon properties.

$$a(\text{"}\{27\}\text{"}) = \frac{-(N^2 + 11N/4 - 13/4)}{(N+1)(N+5)(N+11)},$$
$$b(\text{"}\{27\}\text{"}) = \frac{-(3N - 17)}{2(N+1)(N+5)(N+11)}, \qquad (11)$$
$$c(\text{"}\{27\}\text{"}) = \frac{5}{(N+11)},$$

and for "{35}"-plet:

$$a(\text{"}\{35\}\text{"}) = \frac{-(N^2 + N/2 - 31/2)}{(N-1)(N+3)(N+13)},$$
$$b(\text{"}\{35\}\text{"}) = \frac{-(5N/3 - 11)}{(N-1)(N+3)(N+13)}, \qquad (12)$$
$$c(\text{"}\{35\}\text{"}) = \frac{5N^2 + 44N/3 - 1}{(N-1)(N+3)(N+13)}.$$

In all cases at large N, $a(p,q) \sim c(p,q) \sim 1/N$, and $b(p,q) \sim 1/N^2$. A feature of interest is that the step in C_S per unit strangeness for decuplet, $\delta_{10} = (N-1)/[(N+1)(N+9)]$, is greater than that for antidecuplet, $\delta_{\overline{10}} = N/[(N+3)(N+9)]$, although they coincide for $N = 3$, and we do not consider the case of $N = 1$.[6]

It can be seen also from Tables I and II that the parameter for expansion $C_S = (\alpha/N)[1 + \beta/N + \ldots]$ is $\sim 7/N, 9/N, 11/N, 13/N, \ldots$, for the octet, decuplet, {27}, and {35}-plets, so, it increases with increasing values of (p, q) defining the multiplet [8]. E.g., for multiplets $[p, q] = [0, (N+3m)/2]$ the expansion parameter is $(3m+6)/N$. The authors of [22] came to similar conclusions considering the decay matrix element for Θ^+-baryon: "Any approach…that employs $1/N$ expansion methods for exotic baryon matrix elements seems questionable" (Subsection VI B of [22]). As we show here, for nonexotic baryons such an expansion method is questionable also, for the bound state model as well as for the RVA.

III. COMPARISON OF RIGID ROTATOR AND OSCILLATOR MODELS AT LARGE N

When flavor symmetry breaking mass m_K is small enough, it is possible to compare directly the results of the rigid rotator and oscillator models at arbitrary N. In the RR model any baryon state is ascribed, at first, to definite $SU(3)$-multiplet (p, q) with some value of spin J which depends on the multiplet, and as a next step the mass splitting within each multiplet can be calculated in the first order in FSB mass m_K, precisely for an arbitrary number of colors N (previous section). In the bound state model [36,38,39] expansion in $1/N$ is made from the beginning, the states are labeled by their strangeness (flavor in general case), spin, and isospin. The J, I-dependent energy is calculated as the hyperfine splitting correction of the order $\sim 1/N$, and each state can be ascribed to definite $SU(3)$-multiplet, according to its quantum numbers S, I, and J. When $m_K \to 0$, there is no need to consider the full bound state model, because it reduces in this limit to the simplified rigid oscillator version [39,40].

A. Nonexotic baryon states

In this subsection we follow mainly to the discussion in [41]. For the rigid rotator model we shall use the above expressions (2)–(6), i.e.

$$\Delta M = m_K^2 \Gamma C_S, \qquad (13)$$

which corresponds to first order in flavor symmetry breaking mass squared m_K^2. This approximation becomes more precise as $m_K^2 \to 0$. In this limit the RR model and soft, or slow rotator model provide the same results.[7]

From Table I we obtain for the components of the octet, providing expansion in parameter $1/N$:

$$\delta M_N = \frac{2(N+4)}{(N+3)(N+7)} m_K^2 \Gamma = \left(\frac{2}{N} - \frac{12}{N^2} + O(N^{-3})\right) m_K^2 \Gamma, \qquad (14)$$

$$\delta M_\Lambda = \frac{3}{(N+7)} m_K^2 \Gamma = \left(\frac{3}{N} - \frac{21}{N^2} + O(N^{-3})\right) m_K^2 \Gamma, \qquad (15)$$

$$\delta M_\Sigma = \frac{3N+13}{(N+3)(N+7)} m_K^2 \Gamma = \left(\frac{3}{N} - \frac{17}{N^2} + O(N^{-3})\right) m_K^2 \Gamma, \qquad (16)$$

$$\delta M_\Xi = \frac{4}{(N+7)} m_K^2 \Gamma = \left(\frac{4}{N} - \frac{28}{N^2} + O(N^{-3})\right) m_K^2 \Gamma. \qquad (17)$$

For arbitrary nonexotic $SU(3)$ multiplets it is a matter of

[6] It should be mentioned that it is a convention to identify the multiplet $[p, q] = [3, (N-3)/2]$ with the decuplet. In this case the difference $Y^{\max} - Y^{\min} = p + q = (N+3)/2$ coincides with that of antidecuplet $[0, (N+3)/2]$. It is usually assumed for the generalization of any $SU(3)$ multiplet that spin and isospin of baryon state is fixed when the number of colors N_c increases. Another logical possibility for generalization of the decuplet, based on symmetry principle, is the multiplet $[N, 0]$, see e.g. discussion in [8].

[7] The opposite to the rigid rotator is the assumption that during the rotation it is sufficient time for changing the Skyrmion profiles under the influence of FSB terms in the Lagrangian (so-called soft, or slow rotator approximation, see [42] where static properties of baryons have been calculated within this approximation). Evidently, both rigid and soft rotator approximations converge when $m_K \to 0$, and estimates show also that for $B = 1$ the RR model is more justified in the realistic case, whereas for large baryon numbers the soft rotator model can be better [8].

simple algebra to show, using the ν-dependent wave functions of baryons, that for not large values of S the strangeness content of baryon equals

$$C_S \simeq \frac{2+|S|}{N}, \tag{18}$$

so, minimal strangeness content exists and decreases like $1/N$.

Let us compare this with the results of the RO approach. The bound state soliton model is in fact the particular case of the more general rotation-vibration approach (RVA) described in detail in [22], see also references in this paper. In the rigid oscillator model parametrization of the matrix $A(t)$ is used, somewhat different from that described above: $A(t) = A_{SU(2)}(t)S(t)$, matrix $S(t) = \exp(i\mathcal{D})$ describes strangeness changing movement of soliton in $SU(3)$ space [36,38]:

$$\mathcal{D} = \sum_{a=4}^{7} d_a \lambda_a, \tag{19}$$

so, deviation into "strange" direction is defined by two-component spinor $D = (d_4 - id_5, d_6 - id_7)^T/\sqrt{2}$. Comparison with the RR parametrization above allows to conclude that $D^\dagger D \simeq \nu^2/2$. The Hamiltonian of the RO model is of the oscillator type and can be quantized appropriately [36,39]. The average deviation $|D|$ into strange direction for arbitrary negative S can be estimated easily as

$$|D|_S \sim \frac{2|S|+1}{[16m_K^2\Gamma\Theta_K + N^2]^{1/4}}, \tag{20}$$

for $S < 0$. At fixed $|S|$ it decreases with increasing N and FSB mass m_K. However, (20) does not hold for positive S. The quantity Θ_K, similar to Γ, is defined by incident $SU(2)$ chiral field configuration [36,39], and can be called the moment of inertia of Skyrmion relative to the motion into strange direction. It is assumed again that during the motion in the oscillator potential the classical configuration does not change its form, that is the reason why the model is called the rigid oscillator one.

The order N^0 contributions to the nonexotic baryon masses are

$$\Delta M_0(RO) = \omega_- + \omega_+ + \omega_-|S|, \tag{21}$$

where

$$\omega_\pm = \frac{N}{8\Theta_K}(\mu \pm 1), \tag{22}$$

$$\mu = \sqrt{1 + (m_K/M_0)^2}, \qquad M_0 = \frac{N}{4\sqrt{\Gamma\Theta_K}}. \tag{23}$$

In lowest order in m_K we obtain easily:

$$\omega_- \simeq m_K^2 \frac{\Gamma}{N}, \qquad \omega_+ \simeq \frac{N}{4\Theta_K} + m_K^2\frac{\Gamma}{N}. \tag{24}$$

The first two terms in (21) come from the zero-point energy. To order m_K^2 this gives

$$\Delta M_0(RO) \simeq \frac{N}{4\Theta_K} + \frac{m_K^2\Gamma}{N}(2+|S|). \tag{25}$$

The term $N/(4\Theta_K)$ is well known to appear in the RR approach [7,8,26], and we also see that the term linear in m_K^2 agrees with the RR approach, in the order $N^0 \sim 1$.

The $O(1/N)$ contributions were studied in [36,39], and the result was expressed in terms of the hyperfine splitting (HFS) constants

$$c = 1 - \frac{\Theta_\pi}{2\mu\Theta_K}(\mu - 1) = 1 - \frac{4\Theta_\pi\Gamma m_K^2}{N^2} + O(m_K^4), \tag{26}$$

$$\bar{c} = 1 - \frac{\Theta_\pi}{\mu^2\Theta_K}(\mu - 1) = 1 - \frac{8\Theta_\pi\Gamma m_K^2}{N^2} + O(m_K^4). \tag{27}$$

The $O(1/N)$ term as stated in [39] and obtained also in [33,34], is

$$\Delta E_{HFS} = \frac{J(J+1)}{2\Theta_\pi} + \frac{1}{2\Theta_\pi}\{(c-1)[J(J+1) - I(I+1)] + (\bar{c} - c)I_S(I_S+1))\} \tag{28}$$

with $I_S = |S|/2$-isospin carried by kaon field.[8] At $m_K = 0$ (flavor symmetric case) $c = \bar{c} = 1$, and the hyperfine splitting correction reduces to the well-known quantum rotational correction $J(J+1)/2\Theta_\pi$. The relations take place in the linear in m_K^2 approximation:

$$\bar{c} \simeq 2c - 1, \tag{29}$$

which ensures validity of the Gell-Mann–Okubo relations, and

$$\bar{c} \simeq c^2, \tag{30}$$

which is used sometimes in literature. However, relation (30) does not hold for antiflavor (positive strangeness), see the next subsection. In the expression (28), the term linear in m_K^2 is found to be

$$\delta M_{1/N}(RO) = 2\frac{\Gamma m_K^2}{N^2}[I(I+1) - J(J+1) - I_S(I_S+1)], \tag{31}$$

and for $J = 1/2$ we can compare this with the RR results for the octet, (14)–(17). Collecting the terms $\sim m_K^2\Gamma$ from (25) and (35) we obtain

[8]In [36,39] the last term in the bracket of (28) was given as $(\bar{c} - c)Y^2/4$, the correct formula was given first in [40] and for the general case in [33]. Details of the evaluation can be found also in [34].

$$\delta M_N(RO) \simeq \frac{2}{N} m_K^2 \Gamma;$$
$$\delta M_\Lambda(RO) \simeq \left(\frac{3}{N} - \frac{3}{N^2}\right) m_K^2 \Gamma;$$
$$\delta M_\Sigma(RO) \simeq \left(\frac{3}{N} + \frac{1}{N^2}\right) m_K^2 \Gamma;$$
$$\delta M_\Xi(RO) \simeq \left(\frac{4}{N} - \frac{4}{N^2}\right) m_K^2 \Gamma. \tag{32}$$

Obviously, there is no agreement between (14)–(17) and (32) for all 4 components of the octet.

Now, let us consider the decuplet of baryons, i.e. the $(3, (N-3)/2)$ multiplet of $SU(3)$, $J = 3/2$. The terms linear in m_K^2 as it follows from Table I, are

$$\delta M_\Delta = \frac{2(N+4)}{(N+1)(N+9)} m_K^2 \Gamma = \left(\frac{2}{N} - \frac{12}{N^2} + O(N^{-3})\right) m_K^2 \Gamma, \tag{33}$$

$$\delta M_{\Sigma^*} = \frac{3N+7}{(N+1)(N+9)} m_K^2 \Gamma = \left(\frac{3}{N} - \frac{23}{N^2} + O(N^{-3})\right) m_K^2 \Gamma, \tag{34}$$

$$\delta M_{\Xi^*} = \frac{2(2N+3)}{(N+1)(N+9)} m_K^2 \Gamma = \left(\frac{4}{N} - \frac{34}{N^2} + O(N^{-3})\right) m_K^2 \Gamma. \tag{35}$$

$$\delta M_\Omega = \frac{5}{(N+9)} m_K^2 \Gamma = \left(\frac{5}{N} - \frac{45}{N^2} + O(N^{-3})\right) m_K^2 \Gamma. \tag{36}$$

They satisfy the usual equal splitting rule for decuplet, with the splitting

$$\frac{N-1}{(N+1)(N+9)} m_K^2 \Gamma = \left(\frac{1}{N} - \frac{11}{N^2} + O(N^{-3})\right) m_K^2 \Gamma. \tag{37}$$

Within the RO variant we should use (31) with $J = 3/2$ and $I = J - I_S$, and obtain in this way for the components of the decuplet quite different results.

A possible way to remove disagreement between the RR model and RO variant of the bound state model is the following [41]. The RO calculation involves normal-ordering ambiguities in quartic terms, which can correct the overall shift of masses and the term linear in strangeness that already appeared in the leading order in $1/N$. Let us assume that the normal-ordering corrections change the $O(1/N)$ mass formula by an extra additive term [41]

$$\Delta M(\text{norm.ord.}) = -6 \frac{\Gamma m_K^2}{N^2} (2 + |S|), \tag{38}$$

which is proportional to the order 1 contribution, but is down by a power of N.

Then, the $O(1/N)$ term in the mass formula becomes [41]

$$\delta M(\text{RO, norm.ord.}) = \frac{\Gamma m_K^2}{N^2} \left[-12 + 2I(I+1) - 2J(J+1) - \frac{S^2}{2} - 7|S| \right], \tag{39}$$

and the $O(m_K^2 \Gamma / N^2)$ terms of the RO approach agree with the RR calculations for all the octet and decuplet masses.

These results show that there should be a specific normal-ordering prescription that brings the two approaches in complete agreement [41]. As it is well known [16,22], in the large N limit both RR and RO approaches coincide. But the next-to-leading order corrections in the $1/N$-expansion are large, including the normal-ordering correction, so the problem of extrapolation to the real world with $N = 3$ cannot be solved by means of $1/N$ expansion. It should be noted also that besides the $1/N$ corrections we discussed here there can be also corrections of other types, e.g. corrections of dynamical nature to static characteristics of Skyrmions. By this reason, even if the proper way to remove the difference between RR and RO models is found, it may not mean that the whole problem of extrapolation to real $N = 3$ world is resolved.

B. Positive strangeness states

To calculate the HFS correction in this case, the substitution $\mu \to -\mu$ should be made in the above expressions for the HFS constants c and $\bar{c}$, and we have in this case:

$$c_{\bar{S}} = 1 - \frac{\Theta_\pi}{2\mu\Theta_K}(\mu + 1) = 1 - \frac{\Theta_\pi}{\Theta_K} + \frac{8\Theta_K \Gamma m_K^2}{N^2} + O(m_K^4) \tag{40}$$

and

$$\bar{c}_{\bar{S}} = 1 + \frac{\Theta_\pi}{\mu^2 \Theta_K}(\mu + 1) = 1 + \frac{2\Theta_\pi}{\Theta_K} - \frac{24\Theta_K \Gamma m_K^2}{N^2} + O(m_K^4). \tag{41}$$

In the difference from the negative strangeness case, for positive strangeness (antiflavor in general case) the constants $c \neq 1$ at $m_K = 0$, and approximate equality $\bar{c} \simeq c^2$ is strongly violated now. For the energy of states with antiflavor we have from (28)

$$\Delta E_{\rm HFS+FSB} = \frac{J(J+1)}{2\Theta_\pi} + \frac{1}{2\Theta_K}[-J(J+1) + I(I+1) + 3I_S(I_S+1)] + \frac{m_K^2\Gamma}{N^2}\{3N - 2[-J(J+1) + I(I+1) + 7I_S(I_S+1)]\}. \quad (42)$$

The case of exotic $S = +1$ states is especially interesting. In this case $I_S = 1/2$, $J = I + 1/2$, and within the RO model we obtain, using the expressions for $c_{\bar{S}}$ and $\bar{c}_{\bar{S}}$:

$$M_{\Theta_0,J=1/2} = \frac{2N+3}{4\Theta_K} + \frac{3}{8\Theta_\pi} + m_K^2\Gamma\left(\frac{3}{N} - \frac{9}{N^2}\right) + M_{cl}, \quad (43)$$

$$M_{\Theta_1,J=3/2} = \frac{2N+1}{4\Theta_K} + \frac{15}{8\Theta_\pi} + m_K^2\Gamma\left(\frac{3}{N} - \frac{7}{N^2}\right) + M_{cl}, \quad (44)$$

$$M_{\Theta_2,J=5/2} = \frac{2N-1}{4\Theta_K} + \frac{35}{8\Theta_\pi} + m_K^2\Gamma\left(\frac{3}{N} - \frac{5}{N^2}\right) + M_{cl}. \quad (45)$$

The terms of zero's order in m_K coincide exactly with those given above by RR mass formula (1) applied to exotic multiplets $\{\overline{10}\}$, $J = 1/2$, $\{27\}$, $J = 3/2$, and $\{35\}$, $J = 5/2$. As it was expected, there is additional contribution $N/(4\Theta_K)$ to the energy of exotic states compared with nonexotic states, in agreement with the RR model result.[9] Let us compare this with the mass splitting correction $\sim m_K^2$, obtained within the RR model, see Tables I and II:

$$\delta M_{\Theta_0} \simeq m_K^2\Gamma\left(\frac{3}{N} - \frac{27}{N^2}\right), \quad (46)$$

$$\delta M_{\Theta_1} \simeq m_K^2\Gamma\left(\frac{3}{N} - \frac{25}{N^2}\right), \quad (47)$$

$$\delta M_{\Theta_2} \simeq m_K^2\Gamma\left(\frac{3}{N} - \frac{23}{N^2}\right). \quad (48)$$

There is considerable difference between RR and RO models in FSB terms, proportional to m_K^2. This difference can be eliminated if the contribution given by (39)

$$\Delta M(\text{norm.ord.}, S = 1) = -18 m_K^2 \frac{\Gamma}{N^2} \quad (49)$$

is added to the RO result, similar to the case of the octet and decuplet of baryons considered in [41] and in the previous subsection. Evidently, the difference between RR and RO models should be kept in mind, when comparison of predictions of both variants is made. However, in the literature discussing relevance of the pentaquarks predictions within CSA this difference was not taken into account.

Other states with values of strangeness different from $S = 1$ which could be ascribed to exotic multiplets can be considered similarly, but it is technically a more complicated problem.

C. Comparison of the total mass splittings

Also, it is more difficult to calculate the total mass splittings, especially for exotic $SU(3)$ multiplets in the RO model. An important restriction for the whole mass splitting of any $SU(3)$ multiplet follows from expression (2), since $s_\nu^2 \leq 1$:

$$\Delta M \leq \frac{1}{2}\left(\frac{F_K^2}{F_\pi^2}\mu_K^2 - \mu_\pi^2\right)\Gamma. \quad (50)$$

This restriction is useful for the comparison of different quantization schemes.

Within the RR model it is convenient to use the Gell-Mann–Okubo formulas (7), substituting in this formula $Y^{\max} = (p + 2q)/3$, $I(Y^{\max}) = p/2$, and $Y^{\min} = -(q + 2p)/3$, $I(Y^{\min}) = q/2$ (recall that $Y = N/3 + S$ for an arbitrary number of colors).

For decuplet $[p, q] = [3, (N-3)/2]$ from (9) we obtain

$$\Delta_{\rm RR}^{\rm tot}(10) = m_K^2\Gamma\frac{N^2 + 4N - 15}{2(N+1)(N+9)} \simeq \frac{m_K^2\Gamma}{2}\left(1 - \frac{6}{N} + \frac{36}{N^2}\right). \quad (51)$$

Within the RO model, for any multiplet (p, q) the total mass splitting in the leading in $1/N$ approximation is given by

$$\Delta^{\rm tot}M(p, q) = \Delta Y\omega_- \simeq m_K^2\frac{\Gamma}{N}(p + q). \quad (52)$$

It turned out that in this approximation for $N = 3$ the total mass splitting within the decuplet is 8 times greater than within the rigid rotator approximation (51), for the octet the difference is 4 times, as noted already in [8].

The hyperfine splitting correction can be calculated with the help of formula (39), where for the decuplet we should take $J = 3/2$, $I = 3/2$ for $S = 0$, and $I = (N-3)/4$ for $S = -(N+3)/2$. Then we obtain

$$\Delta_{\rm RO}^{\rm tot}(\{10\}) = \frac{m_K^2\Gamma}{2}\left(1 - \frac{6}{N} + \ldots\right) \quad (53)$$

in agreement with the first two terms in the $1/N$ expansion of the above formula (51). Note, that it would be no agreement without the addition of special normal-ordering contribution (38) [41]. However, there is no agreement in

[9] It was shown explicitly in [34] (formula (52) in the appendix) that within the RR model the energy difference between exotic and nonexotic baryon states (25) due to the difference of corresponding Casimir operators equals $\Delta E = (NB + 3)/(4\Theta_K)$ for arbitrary odd B. Note that if the expression for $\Delta E_{\rm HFS}$ (28) is used with the term $(c^2 - c)I_S(I_S + 1)$ instead of $(\bar{c} - c)I_S(I_S + 1)$, as sometimes in the literature, then the results of the RR model cannot be reproduced correctly within BSM.

the next order terms in the $1/N$ expansion. Of course, one should not expect such agreement because the RO model we are using here does not take into account such contributions. A similar agreement between RR and RO results takes place for the total mass splitting of the octet $[p, q] = [1, (N-1)/2]$.

Let us consider as the next example the antidecuplet $[p, q] = [0, (N+3)/2]$ multiplet which is a generalization of (0, 3) antidecuplet for an arbitrary N. In this case there is equidistant position of the components with different hypercharge, in view of Gell-Mann–Okubo relations, and $Y^{\max} = (N+3)/3$, $Y^{\min} = -(N+3)/6$, $\Delta Y = p + q = (N+3)/2$, and the mass splitting of this multiplet is

$$\Delta^{\rm tot}_{\rm RR} M(\text{"}\overline{10}\text{"}) = m_K^2 \Gamma \frac{N\Delta_Y}{(N+3)(N+9)} = m_K^2 \Gamma \frac{N}{2(N+9)}. \tag{54}$$

Within the BSM and its RO variant we have, without hyperfine splitting correction,

$$\Delta^{\rm tot}_{\rm RO} M(\text{"}\overline{10}\text{"}) \simeq \Delta Y \omega_- \simeq \frac{N(N+3)}{16\Theta_K}(\mu - 1) \simeq \frac{N+3}{2N} m_K^2 \Gamma. \tag{55}$$

We cannot, however, calculate the HFS correction in this case, because expression (42) is not sufficient for this purpose. To calculate the hyperfine correction for states with strangeness $S < 1$ we should, in terms of the quark model, make a summation of spins of nonstrange quarks, strange antiquark, and several strange quarks, in correspondence with strangeness S. This is a more complicated problem to be solved starting from incident Lagrangian.

To conclude this subsection, we obtained agreement between the RR and modified RO models in the total mass splitting of nonexotic baryon multiplets in two leading orders of $1/N$ expansion, and for exotic multiplets—only in first leading order. The next order contributions in the RO model are not calculated yet. Anyway, since the expansion parameter is large, like $6/N$, the knowledge of several terms of such expansion may be not so useful for extrapolation to the real $N = 3$ world.

IV. QUARK WAVE FUNCTIONS OF PENTAQUARKS

The connection between chiral soliton models and the quark models of exotic states has been discussed intensively, and different opinions have been revealed, from that both models are dual [10,43], or complementary to each other, to that they are essentially different, and predict different states; in particular, in [44] the states were predicted which are absent in the simplest quantization scheme of the chiral soliton models—the partners of states with different spin, but same flavor quantum numbers, including isospin. Here we show that some features of exotic baryons spectra obtained within the chiral soliton approach can be illustrated in terms of the quark model, as it was shown at first [45] for the case of the antidecuplet. Any model with $SU(3)$ flavor symmetry and its violation in a special way mimics the quark model in view of Gell-Mann–Okubo type relations (Sec. II). There are, however, some distinctions, mainly in the quantitative estimates of mass differences of different diquarks and partners of exotic baryon states.

Under the *simple quark model* of baryons we mean the model where mass splittings within $SU(3)$ multiplets are defined mainly by the difference between strange and nonstrange quark masses. It is a common feature of phenomenological models discussed recently in connection with the observation of pentaquarks [3,4]. Here we shall reserve a possibility that strange quark mass can be different in different $SU(3)$ multiplets, as well as strange antiquark mass is different from the mass of strange quark. There is nothing special in this assumption; even the effective masses of electrons are slightly different in different atoms due to different binding energies. Strong interactions of strange quarks and antiquarks with (u, d) quarks are different, which can lead to considerable difference of effective masses.

Under the *simplistic*, or *oversimplified quark model* we mean the model where strange quark and antiquark masses are equal, as well as they are equal in different $SU(3)$ multiplets. The striking property of exotic spectra within CSA is that the mass splitting within the antidecuplet in the RR model, in the first order of perturbation theory for $N = 3$ equals exactly that of the decuplet, as it follows from values of C_S presented in Table I, therefore the simplistic quark model contradicts the results of CSA for $N = 3$.

As it follows from the formulas of the preceding section, the RO variant of the bound state model in the leading in the $1/N$ approximation corresponds to the simple quark model, with the strange quark mass

$$m_s \simeq m_K^2 \frac{\Gamma}{N}, \tag{56}$$

which is of the order $N^0 \sim 1$ (as it follows from the above results, the relation is rather $m_s \simeq m_K^2\Gamma/(N+9)$, considerably smaller numerically for $N = 3$). The antiflavor excitation energy ω_+ is greater than ω_-, so, one could decide that the effective mass of the strange antiquark is greater than the mass of the strange quark. Within the RR variant of the CSA the difference $\omega_+ - \omega_-$ is reproduced by the difference of rotational energies of different $SU(3)$ multiplets, due to the difference of Casimir operators of exotic and nonexotic multiplets, and can be ascribed to the contribution of the effective mass of the additional quark-antiquark pair, $m_{q\bar{q}} \sim 1/\Theta_K$ (see, e.g. the appendix of [8,34]). Within the bound state model and its RO variant calculations of spectra of exotic multiplets (not only positive strangeness components) are absent still, as mentioned above.

Relation (56) is in agreement with the known relation $m_s|\langle\bar{q}q\rangle| \simeq F_K^2\mu_K^2/8$ [46], with the proper relation between quark condensate $\langle\bar{q}q\rangle$ and F_π^2/Γ. Sometimes in the literature the relation is used to obtain Γ or other quantities for arbitrary N from the value at $N=3$: $\Gamma(N) = \Gamma(N=3)\times(N/3)$. We want to note here that this is really an arbitrary and not justified prescription, since any relation of the type $\Gamma(N) = \Gamma(N=3)[(N+a)/(3+a)]$ with any real (positive) constant a gives the correct value for $N=3$, but different at large N.

A. Quark contents of exotic baryons in pentaquark approximation

We call q the lightest quarks, u, d, and s denotes as usually the strange quark, (c, b—the charmed or beauty quark). We consider here the case of strangeness, the charmed or beautiful states can be obtained by simple substitution $s\to c$, etc.

Quark contents of antidecuplet. First we recall that the minimal quark content of the components of $\{\overline{10}\}$-plet is, for $N=3$ [45]:

$$\begin{aligned}
\Theta^+&: |\overline{10},2,0,0\rangle = |uudd\bar{s}\rangle;\\
N^*&: |\overline{10},1,1/2,-1/2\rangle = |udd(Q\bar{Q})_{N^{*0}}\rangle, |\overline{10},1,1/2,1/2\rangle = |uud(Q\bar{Q})_{N^{*+}}\rangle;\\
\Sigma^*&: |\overline{10},0,1,-1\rangle = |sdd(Q\bar{Q})_{\Sigma^{*-}}\rangle,\ldots,|\overline{10},0,1,1\rangle = |suu(Q\bar{Q})_{\Sigma^{*+}}\rangle;\\
\Xi^*_{3/2}&: |\overline{10},-1,3/2,-3/2\rangle = |ssdd\bar{u}\rangle,\ldots,|\overline{10},-1,3/2,3/2\rangle = |ssuu\bar{d}\rangle.
\end{aligned} \tag{57}$$

Here we use the notation $|N(p,q),Y,I,I_3\rangle$ for the components of the multiplet $N(p,q) = (p+1)(q+1)(p+q+2)/2$ with hypercharge Y, isospin I, and its third projection I_3. The minimal quark content (i.e. the number of u, d, s quarks or antiquarks) of manifestly exotic states Θ^+ and $\Xi^*_{3/2}$ is unique within pentaquark approximation, the condition for this is $I=(5+S)/2$ for $S\le 0$, since the number of nonstrange quarks and antiquarks equals $5+S$ and each of them has isospin 1/2. This uniqueness of the quark contents allows to obtain the mass splitting within simple quark model and to compare with results of the chiral soliton (rigid rotator version) model described above.

In the model with $\bar{3}_F$ diquarks [4,45] the flavor part of the wave function of Θ^+ is made of two isoscalar diquarks:

$$\Psi_{\Theta^+} = \tfrac{1}{2}[u_1d_2 - u_2d_1][u_3d_4 - u_4d_3]\bar{s} \tag{58}$$

which corresponds exactly to isospin $I=0$. Other components of the antidecuplet can be obtained by action of U-spin, or V-spin and isospin operators ($Ud=s$, $U\bar{s}=-\bar{d}$, etc., see e.g. [45]).

The quark contents and the wave function of cryptoexotic states N^* and Σ^* depend on the particular model: $(Q\bar{Q})_B = \alpha_B s\bar{s} + \beta_B u\bar{u} + \gamma_B d\bar{d}$ with coefficients α, β, γ depending not only on the particular baryon under consideration but also on the variant of the model and on the mixing between different $SU(3)$ multiplets. Within the diquark model [4,45] one obtains

$$\Psi_{N^{*+}} = \frac{1}{\sqrt{3}}([us]_{12}[ud]_{34}\bar{s} + [ud]_{12}[us]_{34}\bar{s} - [ud]_{12}[ud]_{34}\bar{d}), \tag{59}$$

with $[us]_{12} = (u_1s_2 - u_2s_1)/\sqrt{2}$, and similarly for other cryptoexotic components of the antidecuplet, see Table III.

The wave function of the Ξ -quartet does not contain $(s\bar{s})$ pair as a consequence of isotopic invariance: we can obtain components $\Xi^{*-}_{3/2}$, $\Xi^{*0}_{3/2}$; $\Xi^{*+}_{3/2}$ from $\Xi^{*--}_{3/2}$ by the acting operator I^+, and the $(s\bar{s})$ pair does not appear.

The upper component of the antidecuplet Θ^+ (see Fig. 1) contains one antiquark with the mass $m_{\bar{s}}$, the lower component $\Xi_{3/2}$ contains two strange quarks with the mass $2m_s$, therefore, the whole splitting due to the mass of the strange quark is $1m_s$, within the simplistic model [45], and within pentaquark approximation, of course. This should be compared with the total splitting $3m_s$ for the decuplet, where minimal content varies from (qqq) for Δ-isobar to (sss) for Ω-hyperon. The particular weight of $(s\bar{s})$ pair in intermediate components (with strangeness 0 and -1) depends on the assumption concerning the structure of their wave function. It can be different in different models, e.g. diquark-diquark or diquark-triquark models and even for different variants of the diquark model. In the model [4] the equidistant behavior was obtained for the antidecuplet [45]. But such behavior of antidecuplet spectrum does not follow in general from the above consideration.[10]

Quark contents of $\{27\}$*-plet*. The $\{27\}$-plet has the upper $S=+1$, $I=1$ component with content $qqqq\bar{s}$ of mixed symmetry and manifestly exotic components with $S=-1, I=2$, $S=-2, I=3/2$, and $S=-3, I=1$, the

[10]In the paper [4] the mixing between the pentaquark octet and antidecuplet was studied, but their mixing with lowest baryon octet was neglected. Strong interactions do not conserve the number of additional quark-antiquark pairs, therefore, this mixing takes place inevitably and will push the states considered towards higher energies. The nonexotic octet and decuplet of baryons should be included into consideration for self-consistency of any model. The paper [20] contains a similar remark.

TABLE III. Masses of components of $\{\overline{10}\}$, and components with maximal isospin for $\{27\}$, $J = 3/2$ and $\{35\}$, $J = 5/2$-plets of exotic baryons (in MeV, the nucleon mass is input, $N = 3$). The first line after notations of the components shows the contribution of the strange quarks/antiquark masses within the simple model, $m_{s\bar{s}}$ is the mass of the $s\bar{s}$ pair taken usually to the sum of masses of s and $\bar{s}$ quarks. The next line is the result of calculation without configuration mixing, the second line of numbers—configuration mixing included according to [26]. Calculations correspond to case A of paper [26]: $\Theta_K = 2.84\ \mathrm{GeV}^{-1}$, $\Theta_\pi = 5.61\ \mathrm{GeV}^{-1}$, $\Gamma = 1.45$ GeV, which allowed to obtain the mass of Θ^+ hyperon close to the observed value 1.54 GeV.

$\vert\overline{10}, 2, 0\rangle$	$\vert\overline{10}, 1, 1/2\rangle$	$\vert\overline{10}, 0, 1\rangle$	$\vert\overline{10}, -1, 3/2\rangle$		
$m_{\bar{s}}$	$2m_{s\bar{s}}/3$	$m_s + m_{s\bar{s}}/3$	$2m_s$		
1503	1594	1684	1775		
1539	1661	1764	1786		
$\vert 27, 2, 1\rangle$	$\vert 27, 1, 3/2\rangle$	$\vert 27, 0, 2\rangle$	$\vert 27, -1, 3/2\rangle$	$\vert 27, -2, 1\rangle$	
$m_{\bar{s}}$	$m_{s\bar{s}}/2$	m_s	$2m_s$	$3m_s$	
1672	1692	1711	1828	1944	
1688	1826	1718	1850	1987	
$\vert 35, 2, 2\rangle$	$\vert 35, 1, 5/2\rangle$	$\vert 35, 0, 2\rangle$	$\vert 35, -1, 3/2\rangle$	$\vert 35, -2, 1\rangle$	$\vert 35, -3, 1/2\rangle$
$m_{\bar{s}}$	0	m_s	$2m_s$	$3m_s$	$4m_s$
2091	1796	1910	2023	2136	2250
2061	1792	1918	2046	2175	2306

components with $S = 0$, $I = 3/2$, or $I = 1/2$ are cryptoexotic:

$$\begin{aligned}
\Theta_1&: |27, 2, 1, -1\rangle = |dddu\bar{s}\rangle, \ldots, |27, 2, 1, 1\rangle = |uuud\bar{s}\rangle;\\
\Delta^*&: |27, 1, 3/2, -3/2\rangle = |ddd(Q\bar{Q})_{\Delta^{*-}}\rangle, \ldots, |27, 1, 3/2, 3/2\rangle = |uuu(Q\bar{Q})_{\Delta^{*++}}\rangle;\\
\Sigma_2&: |27, 0, 2, -2\rangle = |sddd\bar{u}\rangle, \ldots, |27, 0, 2, 2\rangle = |suuu\bar{d}\rangle;\\
\Xi^*_{3/2}&: |27, -1, 3/2, -3/2\rangle = |ssdd\bar{u}\rangle, \ldots, |27, -1, 3/2, 3/2\rangle = |ssuu\bar{d}\rangle;\\
\Omega_1&: |27, -2, 1, -1\rangle = |sssd\bar{u}\rangle, \ldots, |27, -2, 1, 1\rangle = |sssu\bar{d}\rangle,
\end{aligned} \tag{60}$$

so, the energy gap is $2m_s$ for 4 units of strangeness, $m_s/2$ in average. Evidently, the upper $S = +1$, $I = 1$ component of the $\{27\}$-plet, as well as $S = +1$ component of the $\{35\}$-plet cannot be obtained in the flavor antisymmetric diquark model [4]. The flavor symmetric diquarks of the type 6_F (isovectors in the $S = 0$ case) must be invoked for this purpose.

Indeed, if the diquark is $\bar{3}_F$, then we have according to the well-known group-theoretical relation:

$$\bar{3} \otimes \bar{3} \otimes \bar{3} = \overline{10} \oplus 8 \oplus 8 \oplus 1, \tag{61}$$

and there appears only an antidecuplet from the known pentaquark states (Fig. 1), and two octets of baryons. If one diquark is $\bar{3}$, and the other is 6_F, we obtain

$$6 \otimes \bar{3} \otimes \bar{3} = (15 \oplus 3) \otimes \bar{3} = 27 \oplus 10 \oplus 8 \oplus 8 \oplus 1. \tag{62}$$

If both diquarks are 6_F, then

$$\begin{aligned}
6 \otimes 6 \otimes \bar{3} &= (15 \oplus 15 \oplus \bar{6}) \otimes \bar{3}\\
&= 35 \oplus 10 \oplus 27 \oplus 10 \oplus 8 \oplus \overline{10} \oplus 8.
\end{aligned} \tag{63}$$

So, in the latter case all known pentaquark states can be obtained.[11]

Let us denote (q_1q_2) the flavor symmetric diquark, 6_F in flavor, with spin $J = 1$ ($\bar{3}_C$ in color). Then realization of the wave function of $\{27\}$-plet of pentaquarks via diquarks is:

$$|27, 2, 1, 1\rangle = (u_1u_2)[u_3d_4]\bar{s}, \tag{64}$$

other components can be obtained with the help of U-spin and isospin $I^\pm$ operators:

[11]For example, the $S = +1$ component of the antidecuplet made of two isovector diquarks is $\Psi_{\Theta^+} = [u_1u_2d_3d_4 + d_1d_2u_3u_4 - \frac{1}{2}(u_1d_2 + u_2d_1)(u_3d_4 + d_3u_4)]\bar{s}$. In the diquark-triquark model [3] the diquark within the triquark is color-symmetric (6_c) and antitriplet in flavor, so, this model should be modified to provide $\{27\}$ and $\{35\}$-plets of pentaquarks

$$|27, 1, 3/2, 3/2\rangle = (u_1 u_2)[[u_3 s_4]\bar{s} - [u_3 d_4]\bar{d}]/\sqrt{2},$$
$$|27, 0, 2, 2\rangle = -(u_1 u_2)[u_3 s_4]\bar{d},$$
$$|27, -1, 3/2, 3/2\rangle = -(u_1 s_2)[u_3 s_4]\bar{d},$$
$$|27, -2, 1, 1\rangle = (s_1 s_2)[u_3 s_4]\bar{d}. \tag{65}$$

It follows that the weight of the $s\bar{s}$ pair within $S = 0$ component is $1/2$, therefore, the contribution of the strange quark mass equals m_s in this case, similar to $|27, 2, 1\rangle$ state. The $S = -1$, $I = 2$ components have content $sqqq\bar{q}$, from $sddd\bar{u}$ to $suuu\bar{d}$, and it does not contain the $s\bar{s}$-pair. Therefore, its mass contains $1m_s$, similar to $S = +1$, $I = 1$ component see Table III). Remarkably, that chiral soliton calculation provides very close results for masses of $S = +1$ and $S = -1$, $I = 2$ components of $\{27\}$-plet, (Table 2 and Fig. 4 of [26]): the difference of masses equals 0.03 GeV, see Table III which is the modification of Table 5 in [8].

The effect of configuration mixing is especially important for cryptoexotic components of the antidecuplet ($Y = 1$ and 0) which mix with similar components of the lowest baryon octet. As it is known from quantum mechanics, in this case mixing makes the splitting between the octet and antidecuplet greater and pushes the upper state to higher energy. The mixing of the manifestly exotic state $\Xi_{3/2} \in \{\overline{10}\}$ with the corresponding component of $\{27\}$-plet pushes it down, as a result the total mass splitting within $\overline{10}$ becomes smaller due to mixing.

For the cryptoexotic component of $\{27\}$-plet the mixing effect is especially large: $\sim$20% admixture of Δ-isobar from decuplet pushes this component by an additional 130 MeV above nucleon and makes it even higher in energy than the nearest strange Σ_2 state.

Quark contents of 35-plet. The wave function of the $\{35\}$-plet, the largest multiplet of pentaquarks, is symmetric in flavor indices of 4 quarks. The $I = 2$ upper components of this multiplet has quark content $qqqq\bar{s}$, from $dddd\bar{s}$ to $uuuu\bar{s}$:

$$\Theta_2^{**}: |35, 2, 2, -2\rangle = |dddd\bar{s}\rangle; \ldots, |35, 2, 2, 2\rangle = |uuuu\bar{s}\rangle. \tag{66}$$

The intermediate components can be obtained easily by applying the isospin operators I^+ or I^-. Evidently, it has the largest possible isospin for the $S = +1$ pentaquark. The strange antiquark contribution into the mass equals $m_{\bar{s}}$, obviously (and $m_{\bar{s}} = m_s$ in simplistic model). The $S = 0$ components of $\{35\}$-plet with isospin $I = 5/2$ has minimal content $qqqq\bar{q}$ (evidently, $I = 5/2$ is the maximal possible value of isospin of any pentaquark):

$$\Delta_{5/2}: |35, 1, 5/2, -5/2\rangle = |dddd\bar{u}\rangle, \ldots, |35, 1, 5/2, 5/2\rangle = |uuuu\bar{d}\rangle,$$

and do not contain strange quarks at all. By this reason, the $I = 5/2$, $S = 0$ component is the lightest component of the $\{35\}$-plet, and has smaller strangeness content than nucleon and Δ, again in agreement with calculation within CSA [26]. The components with $S = -1$, $S = -2$, etc. should contain strange quarks in the wave function:

$$\Sigma_2: |35, 0, 2, -2\rangle = |sddd\bar{u}\rangle, \ldots, |35, 0, 2, 2\rangle = |suuu\bar{d}\rangle;$$
$$\Xi_{3/2}^{**}: |35, -1, 3/2, -3/2\rangle = |ssdd\bar{u}\rangle, \ldots, |35, -1, 3/2, 3/2\rangle = |ssuu\bar{d}\rangle;$$
$$\Omega_1^{*}: |35, -2, 1, -1\rangle = |sssd\bar{u}\rangle, \ldots, |35, -2, 1, 1\rangle = |sssu\bar{d}\rangle;$$
$$\Gamma?: |35, -3, 1/2, -1/2\rangle = |ssss\bar{u}\rangle, |35, -3, 1/2, 1/2\rangle = |ssss\bar{d}\rangle, \tag{67}$$

and there is no place for the $s\bar{s}$ pair.[12] The 4-quark part of the wave function of the $\{35\}$-plet is symmetric in flavors and can be easily made of two flavor symmetric 6_F diquarks, e.g. $\{ddds\} = (dd)(ds) + (ds)(dd)$, $\{ddss\} = (dd)(ss) + (ss)(dd) + (ds)(ds)$, etc.

The lowest $S = -4$, $I = 1/2$ isodoublet has $4m_s$ contribution in the mass. As a result, we have the mass gap $4m_s$ between $S = 0$, $I = 5/2$ state and $S = -4$, $I = 1/2$ state: $1m_s$ for the unit of strangeness. But the gap between $S = +1$ and $S = -4$ components is only $3m_s$ for 5 units of strangeness, $3m_s/5$ for one unit in average. The result of chiral soliton model calculation [26] is in rough agreement with the mass splitting given by the above wave function with $m_s \simeq$ 130–140 MeV. All exotic components of $\{35\}$-plet mix with components of higher irreducible representations ($\{64\}$-plet, etc.) and slightly move down in energy after mixing. Positions of states obtained within CSA are shown in Fig. 2 with $\times$. Predictions of the simplistic quark model with $m_s = m_{\bar{s}} = 130$ MeV are shown with circles. For $\{27\}$-plet the location of state with $S = -1$ is identified with that of CSA, the same for the $S = 0$ component of the $\{35\}$-plet.

Summing up, within the simplistic quark model we have the following hierarchy of the energy gaps per unit strangeness (in average) between highest and lowest components of the $SU(3)$ multiplets: $m_s/3$; $m_s/2$; $3m_s/5$ for $\{\overline{10}\}$, $\{27\}$, and $\{35\}$-plets, but the individual splittings, in general, do not follow such simple law and are model dependent. Obviously, this is in contradiction with CSA approach

[12] The notation Γ for the $S = -4$, $I = 1/2$ component of $\{35\}$-plet is not generally accepted, still.

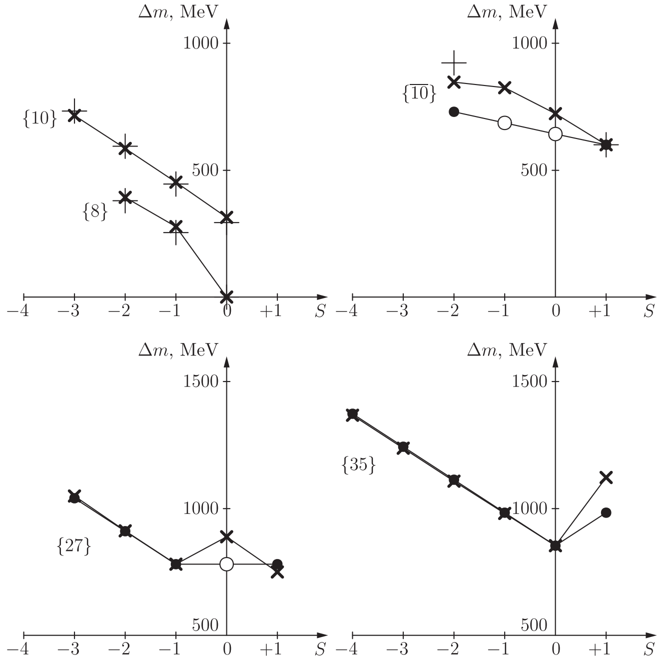

FIG. 2. Schematic picture of the mass splittings within chiral soliton model ($N_c = 3$). The upper left figure corresponds to the nonexotic octet and decuplet, the upper right one—to exotic antidecuplet, the lower—to {27}-plet with spin $J = 3/2$ and to {35}-plet ($J = 5/2$) of exotic baryons. Experimental data are shown by direct crosses +, position of states obtained within CSA with configuration mixing is marked by ×. The circles show position of states within the simplistic quark model with $m_s = m_{\bar{s}} \simeq 130$ MeV; full circles show manifestly exotic states with unique quark contents and empty circles—cryptoexotic states. For the antidecuplet the fit is made for the state with $S = 1$, see also discussion in the text.

results, and we should allow the masses of strange quarks to be different within different $SU(3)$ multiplets. Then the following relations take place, according to the results presented in Table III (configuration mixing included):

$$[2m_s - m_{\bar{s}}]_{\overline{10}} \simeq 250 \text{ MeV}; \qquad [m_s - m_{\bar{s}}]_{27} \simeq 30 \text{ MeV}, \qquad [m_s]_{27} \simeq 135 \text{ MeV}; \qquad [m_s]_{35} \simeq 130 \text{ MeV}; \qquad [m_{\bar{s}}]_{35} \simeq 270 \text{ MeV}. \tag{68}$$

Only one relation takes place for the antidecuplet, and if we assume that the mass of strange quark within the antidecuplet is close to that within higher multiplets, i.e. about 130–135 MeV, then the strange antiquark within $\overline{10}$ should be very light, ~10–20 MeV only. The strange antiquark is heavier within {27}-plet, about 100 MeV, and much heavier within {35}-plet. Recall that now the observed mass splitting within antidecuplet is about 320 MeV, if the observed Ξ^{--} state [47] belongs to the antidecuplet, and not to the higher multiplet. To fit the simplistic quark model, the splitting of the antidecuplet should be smaller, about 130–150 MeV, but this will be in disagreement with CSA. Some decrease of the strange (or kaonic) inertia Θ_K in comparison with the value used to obtain the numbers in Table III [8,26] would increase all masses of exotic states, but would not make much influence on the mass splittings inside of $SU(3)$ multiplets. Experimental studies of exotic spectra could help in solving this problem, the present situation with searches of baryons from higher $SU(3)$ multiplets has been discussed recently in [48].

B. Diquarks mass difference estimate

Comparison with the results of chiral soliton approach allows to estimate the difference of the diquarks masses as well.

In the rigid or soft rotator approximation there is contribution to the mass difference of the different $SU(3)$ multiplets due to different rotation energy (second order Casimir operators) of these multiplets. For {27}- and $\{\overline{10}\}$-plets this difference is

$$M^{\rm rot}_{27,J=3/2} - M^{\rm rot}_{\overline{10}} = \frac{3}{2\Theta_\pi} - \frac{1}{2\Theta_K}. \tag{69}$$

This difference can be naturally ascribed to the difference of effective masses of 6_F and $\bar{3}_F$ diquarks (see (61) and (62) above). This quantity is about 100 MeV, more precisely, 91 MeV if we take the same values of moments of inertia, as in Table III. The difference of rotational energies of {35}-plet which contains two 6_F diquarks (see (63)) and {27}-plet is

$$M^{\rm rot}_{35,J=5/2} - M^{\rm rot}_{27,J=3/2} = \frac{5}{2\Theta_\pi} - \frac{1}{2\Theta_K}. \tag{70}$$

Numerically this is considerably greater than in the former case, about 270 MeV. The real picture may be considerably more complicated; besides effective masses of diquarks the interaction energy between different diquarks can be substantially different. This means that there is no simple additivity of the diquark masses within topological soliton approach. Roughly, we can conclude however that the mass difference between 6_F and $\bar{3}_F$ diquarks is between 100 and 270 MeV, the latter value is close to the estimate given, e.g. in [12].

Consideration of charmed or beautiful states can be made in close analogy with that for strangeness. One could consider $SU(4)(u, d, c, s)$ or even $SU(5)(u, d, c, s, b)$ symmetry, but since this symmetry is badly violated, it has not much significance for practical use. Instead, the (u, d, c) and (u, d, b) $SU(3)$ symmetry groups are often considered. The {35}-plet is again remarkable: within $SU(4)$ it should belong to the most symmetric {120}-plet which can be described by spinor T_r^{iklm}, $(i, k \ldots r = u, d, s, c)$, corresponding Young tableau is (4, 0, 1); within $SU(5) \times (u, d, s, c, b)$ it belongs to 315-plet with Young tableau (4, 0, 0, 1). The $S = 0$, or $c = 0$, or $b = 0$ components of {35}-plet which do not contain $s\bar{s}$ or $c\bar{c}$, or $b\bar{b}$ in the wave function is a common component of the $\{35\}$-plets in each of the $SU(3)$ groups, which is a remarkable property of this $I = 5/2$, $S = c = b = 0$ state consisting of light u, d quarks only.

V. PARTNERS OF EXOTIC STATES WITH DIFFERENT VALUES OF SPIN

Within quark models there are partners of states with same flavor quantum numbers (strangeness and isospin), but with different values of spin [44]. Existence of partners of exotic baryons has been demonstrated and discussed also in [49] in large N_c QCD. At the same time, within CSA the value of spin equals to the value of "right" isospin, as a result of the hedgehog nature of the basic classical configuration. A natural question is: where are such partners within CSA, if they exist at all? The answer is that they are present as well, although belong to different $SU(3)$ multiplets. Here we give one simple example: the $J^P = 3/2^+$ partner of the antidecuplet with spin $J = 1/2$ found its place within $\overline{35}$-plet $(p, q) = (1, 4)$ (septuquark or heptaquark), as shown in Fig. 3. The mass of this state is considerably greater due to a large difference of the Casimir operators $C_2(SU_3)$:

$$\Delta M^{\rm rot}_{\overline{35}-\overline{10}} = M(\overline{35}, J = 3/2) - M(\overline{10}) = \frac{3}{2\Theta_K} + \frac{3}{2\Theta_\pi} \tag{71}$$

which is about 750–800 MeV, greater than several tens of MeV obtained in [44]. The spectrum of these states for some reasonable values of model parameters is given in Table 6 of [8], and we shall not reproduce it here.

There are also partners of nonexotic baryon states. For example, the $J^P = 5/2^+$ partners of the decuplet ($J^P = 3/2^+$) are contained within $\{35\}$-plet (4, 1), the difference of rotational energies is

$$\Delta M^{\rm rot}_{35-10} = M(35, J = 5/2) - M(10, J = 3/2) = \frac{1}{2\Theta_K} + \frac{5}{2\Theta_\pi} \tag{72}$$

which is about 600–700 MeV. The analog with $J^P = 3/2^+$ of the baryon octet ($J^P = 1/2^+$) is contained within $\{27\}$-plet and has energy by 0.7–0.85 GeV greater than masses of lowest octet. Moreover, for any baryon multiplet one can find partners with a greater value of spin within some $SU(3)$ multiplet with other (greater, as a rule) values of (p, q). So, all partners noted are present in the CSA as well, but have considerably greater energy. It was assumed in [4] that the $J = 3/2$ partners of exotic baryon states have considerably greater energy than the $J = 1/2$ ground states, and estimates made here can be considered as justification of this assumption within chiral soliton model.

Another kind of partners are states with the same value of spin (and parity), but another value of isospin. Such partners are absent within multiplets of nonexotic baryons (octet and decuplet) and for the antidecuplet, but exist for complicated multiplets, $\{27\}$ and $\{35\}$-plets. The mass difference between such partners is due to FSB contributions in (1), since rotational energy is the same, and is usually within few tens of MeV.

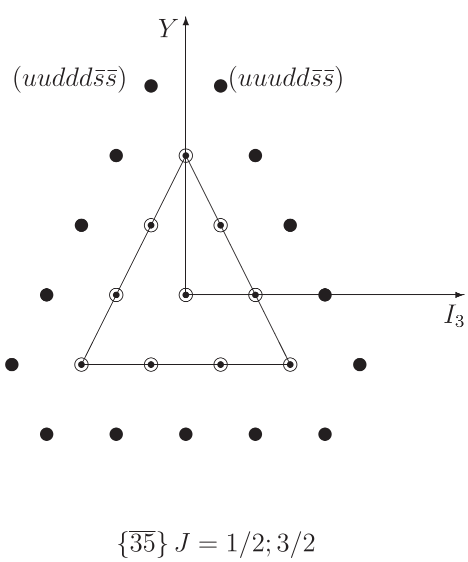

FIG. 3. Partners of the components of the exotic antidecuplet located within $\{\overline{35}\}$-plet.

VI. CONCLUSIONS

Calculations of the strangeness contents of exotic baryons, performed in the present paper at arbitrary N for the first time, have shown that the expansion parameter for this quantity is large and increases for exotic states in comparison with nonexotic [8,41]. There is common agreement that the rigid rotator model and the bound state approach provide the same results in the limit $N_c \to \infty$, but there is crucial difference in the following in $1/N_c$-expansion terms for different variants of the model—rigid rotator variant and bound state model. There is a way to reach coincidence in the next-to-leading in $1/N_c$-expansion terms by means of appropriate resolution of some ambiguities in the BSM [41], but it is valid for large enough N_c, only. This makes questionable the possibility of extrapolation from the large N_c to real $N_c = 3$ world, and provides grounds for scepticism that conclusions made in the limit $N_c \to \infty$—e.g. concerning existence or nonexistence of exotic baryon resonances—are valid in the realistic case $N_c = 3$ [8]. This problem has been noted recently also for the quantities different from spectra of baryons, e.g. for widths of exotic resonances [22,50]. The existence of pentaquark states by itself seems to be without any doubt within CSA [8,22], although the prediction of their particular properties like mass and width contains considerable uncertainties, and some kind of phenomenological extrapolation should be and has been made for this purpose, as e.g. in [26,28,29].

We have considered also some general properties of the pentaquark wave functions, mainly their quark contents for the realistic $N_c = 3$ case. The peculiarity of manifestly exotic states is that their quark contents are model independent (within the pentaquark approximation), whereas the contents as well as wave functions of cryptoexotic states depend on the particular variant of the model.

The mass splittings within multiplets of pentaquarks (negative strangeness) expected within the simple quark model are reproduced in the chiral soliton model (its rigid rotator variant), due to Gell-Mann–Okubo relations. In particular, the lightest component of $\{35\}$-plet, the $\Delta_{5/2}$, which does not contain strange quarks or antiquarks within pentaquark approximation, is the lightest one in chiral soliton model as well. For positive strangeness components of pentaquarks multiplets the link between CSM and QM requires strong dependence of effective strange antiquark mass on the $SU(3)$ multiplet to which the pentaquark belongs. Configuration mixing pushes the spectra towards the simplistic model—nice property which reasons are not clear yet.

The bound state model (its RO variant), in the leading in $1/N_c$ order, corresponds to the *simplistic* variant of the quark model with the unique value of the strange quark (antiquark) mass, $m_s \simeq m_K^2\Gamma/N$. The next-to-leading order corrections for spectrum of exotic baryons with $S < 1$ and correspondence with the simple quark model still remain to be investigated.

The partners of baryons multiplets with different J, discussed in the literature [44,49], for example, the $J^P = 3/2^+$ partner of the $1/2^+$ antidecuplet [44], exist within chiral soliton models as well [8]. They are the parts of higher multiplets and have considerably greater energy than the states with the lowest value of spin.

In view of considerable theoretical uncertainties connected, in particular, with the problem of extrapolation to realistic value of N_c, experimental searches for pentaquark states could be decisive. Even if the existence of narrow pentaquarks is not confirmed, they can exist as broader resonances of higher mass, and their studies will be useful for checking and development of theoretical ideas.[13]

ACKNOWLEDGMENTS

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APPENDIX: WAVE FUNCTIONS OF BARYONS IN THE $SU(3)$ CONFIGURATION SPACE FOR AN ARBITRARY NUMBER OF COLORS

In the rigid rotator quantization scheme the wave functions of baryon states are some combinations of the $SU(3)$ Wigner D-functions. Such functions are quite well known for the case of $N_c = 3$ and for octet and decuplet of baryons [32,35]. Here we present these functions for an arbitrary number of colors and for exotic baryon multiplets, since they are still absent in the literature. As in [32,35], we have:

$$\Psi(Y, I, I_3; Y_R, J, J_3) = \sum_{M_L} D^{I*}_{I_3,M_L}(\alpha, \beta, \gamma) f^{Y,I;Y_R,J}_{M_L,M_R}(\nu) \times D^{J*}_{M_R,-J_3}(\alpha', \beta', \gamma') \exp(iY_R\rho), \tag{A1}$$

where $D^I_{M_1,M_2}$ are the well-known $SU(2)$ Wigner functions, right hypercharge $Y_R = N/3$ and $Y'_R = 1$ for the case of baryons we consider here, right isospin $I_R = J$, spin of the baryon state, due to the hedgehog structure of the classical $B = 1$ configuration, $M_R = M_L + (Y_R - Y)/2$ due to the properties of λ_4 rotations. There are obvious restrictions $-I \leq M_L \leq I$, and $-J \leq M_R \leq J$, and this leaves in the sum (A1) few allowed terms. When the isospin of the state equals $I = 0$, only one term is present in (A1). Nontrivial ν dependence is contained in the function $f^{Y,I;Y_R,J}_{M_L,M_R}(\nu)$ only, which we present here. For the sake of brevity we label it further as f_{M_L}, since other labels can be obtained easily, and we use notation $Q_{ikl\ldots} = \sqrt{(N+i)(N+k)(N+l)\ldots}$ for arbitrary integers $i, k, l \ldots$, some of them can be negative.

Antidecuplet: $[p, q] = [0, (N_c + 3)/2]$

$$\Theta^+ \colon f_0 = f^{2,0;1,1/2}_{0,-1/2} = \frac{Q_{3,5,7}}{4} s_\nu c_\nu^{(N+1)/2}, \tag{A2}$$

$$Q_{3,5,7} = \sqrt{(N+3)(N+5)(N+7)};$$

$$N^* \colon f_{-1/2} = f^{1,1/2;1,1/2}_{-1/2,-1/2} = \frac{Q_{5,7}}{8}(2 - (N+3)s_\nu^2)c_\nu^{(N-1)/2},$$

$$f_{1/2} = \frac{Q_{5,7}}{4} c_\nu^{(N+1)/2}, \tag{A3}$$

[13]The situation with observation of the Θ^+ pentaquark state does not become less dramatic: recently CLAS Collaboration disavowed their previous result on Θ^+ photoproduction on deuterons [51], whereas DIANA Collaboration reinforced their result on Θ^+ production by kaons in Xe chamber [52].

$$\Sigma^{**}\colon f_{-1} = \frac{Q_{1,5,7}}{8\sqrt{6}} s_\nu (4 - (N+3)s_\nu^2) c_\nu^{(N-3)/2}, \quad f_0 = \frac{Q_{1,5,7}}{4\sqrt{3}} s_\nu c_\nu^{(N-1)/2}, \tag{A4}$$

$$\Xi_{3/2}\colon f_{-3/2} = \frac{Q_{-1,1,5,7}}{32\sqrt{3}} s_\nu^2 (6 - (N+3)s_\nu^2) c_\nu^{(N-5)/2}, \quad f_{-1/2} = \frac{Q_{-1,1,5,7}}{16} s_\nu^2 c_\nu^{(N-3)/2}. \tag{A5}$$

For each baryon state functions $f(\nu)$ are normalized according to [35]

$$\int \left(\sum_m f_m^2\right) s_\nu^2 ds_\nu^2 = 1. \tag{A6}$$

The orthogonality conditions of wave functions of states with the same spin, strangeness, and isospin, but from different $SU(3)$ multiplets, take the form, in view of orthogonality of different $SU(2)$ D-functions:

$$\int \left(\sum_m f_m g_m\right) s_\nu^2 ds_\nu^2 = 0, \tag{A7}$$

which can be easily verified using wave functions given here.

27-plet: $[p, q] = [2, (N_c + 1)/2]$

$$\Theta_1\colon f_{-1} = \frac{Q_{1,3,9}}{4\sqrt{2}} s_\nu c_\nu^{(N-1)/2}, \quad f_0 = \frac{Q_{1,3,9}}{4\sqrt{3}} s_\nu c_\nu^{(N+1)/2}, \quad f_1 = \frac{Q_{1,3,9}}{4\sqrt{6}} s_\nu c_\nu^{(N+3)/2}, \tag{A8}$$

$$\Delta^*\colon f_{-3/2} = \frac{\sqrt{3}Q_{3,9}}{48}(6 - 3(N+1)s_\nu^2) c_\nu^{(N-3)/2}, \quad f_{-1/2} = \frac{\sqrt{3}Q_{3,9}}{48}(6 - 2(N+1)s_\nu^2) c_\nu^{(N-1)/2}, \quad f_{1/2} = \frac{\sqrt{3}Q_{3,9}}{48}(6 - (N+1)s_\nu^2) c_\nu^{(N+1)/2}, \quad f_{3/2} = \frac{\sqrt{3}Q_{3,9}}{8} c_\nu^{(N+3)/2}, \tag{A9}$$

$$\Sigma_2\colon f_{-2} = \frac{Q_{-1,3,9}}{16\sqrt{15}} s_\nu (12 - 3(N+1)s_\nu^2) c_\nu^{(N-5)/2}, \quad f_{-1} = \frac{Q_{-1,3,9}}{32\sqrt{5}} s_\nu (12 - 2(N+1)s_\nu^2) c_\nu^{(N-3)/2}, \quad f_0 = \frac{Q_{-1,3,9}}{16\sqrt{30}} s_\nu (12 - (N+1)s_\nu^2) c_\nu^{(N-1)/2}, \quad f_1 = \frac{3Q_{-1,3,9}}{8\sqrt{15}} s_\nu c_\nu^{(N+1)/2}, \tag{A10}$$

$$\Xi^*_{3/2}\colon f_{-3/2} = \frac{Q_{-1,3,7,9}}{64\sqrt{5}} s_\nu^2 (8 - 2(N+1)s_\nu^2) c_\nu^{(N-5)/2}, \quad f_{-1/2} = \frac{Q_{-1,3,7,9}}{32\sqrt{15}} s_\nu^2 (8 - (N+1)s_\nu^2) c_\nu^{(N-3)/2}, \quad f_{1/2} = \frac{Q_{-1,3,7,9}}{8\sqrt{5}} s_\nu^2 c_\nu^{(N-1)/2}, \tag{A11}$$

$$\Omega_1\colon f_{-1} = \frac{Q_{-1,3,5,7,9}}{64\sqrt{15}} s_\nu^3 (4 - (N+1)s_\nu^2) c_\nu^{(N-5)/2}, \quad f_0 = \frac{Q_{-1,3,5,7,9}}{16\sqrt{10}} s_\nu^3 c_\nu^{(N-3)/2}, \tag{A12}$$

35-plet: $[p, q] = [4, (N_c - 1)/2]$

$$\Theta_2\colon f_{-2} = \frac{Q_{-1,1,11}}{4\sqrt{3}} s_\nu c_\nu^{(N-3)/2}, \quad f_{-1} = \frac{Q_{-1,1,11}}{2\sqrt{15}} s_\nu c_\nu^{(N-1)/2}, \quad f_0 = \frac{Q_{-1,1,11}}{4\sqrt{5}} s_\nu c_\nu^{(N+1)/2}, \quad f_1 = \frac{Q_{-1,1,11}}{2\sqrt{30}} s_\nu c_\nu^{(N+3)/2}, \quad f_2 = \frac{Q_{-1,1,11}}{4\sqrt{15}} s_\nu c_\nu^{(N+5)/2}, \tag{A13}$$

$$\Delta_{5/2}\colon f_{-5/2} = \frac{\sqrt{5}Q_{1,11}}{120}(10 - 5(N-1)s_\nu^2) c_\nu^{(N-5)/2}, \quad f_{-3/2} = \frac{\sqrt{5}Q_{1,11}}{120}(10 - 4(N-1)s_\nu^2) c_\nu^{(N-3)/2}, \quad f_{-1/2} = \frac{\sqrt{5}Q_{1,11}}{120}(10 - 3(N-1)s_\nu^2) c_\nu^{(N-1)/2}, \quad f_{1/2} = \frac{\sqrt{5}Q_{1,11}}{120}(10 - 2(N-1)s_\nu^2) c_\nu^{(N+1)/2}, \tag{A14}$$

$$f_{3/2} = \frac{\sqrt{5}Q_{1,11}}{120}(10 - (N-1)s_\nu^2) c_\nu^{(N+3)/2}, \quad f_{5/2} = \frac{\sqrt{5}Q_{1,11}}{12} c_\nu^{(N+5)/2},$$

$$\Sigma_2^*\colon f_{-2} = \frac{Q_{1,9,11}}{48\sqrt{10}} s_\nu (8 - 4(N-1)s_\nu^2) c_\nu^{(N-5)/2}, \quad f_{-1} = \frac{Q_{1,9,11}}{48\sqrt{5}} s_\nu (8 - 3(N-1)s_\nu^2) c_\nu^{(N-3)/2}, \quad f_0 = \frac{Q_{1,9,11}}{16\sqrt{30}} s_\nu (8 - 2(N-1)s_\nu^2) c_\nu^{(N-1)/2}, \quad f_1 = \frac{Q_{1,9,11}}{24\sqrt{10}} s_\nu (8 - (N-1)s_\nu^2) c_\nu^{(N+1)/2}, \tag{A15}$$

$$f_2 = \frac{Q_{1,9,11}}{6\sqrt{2}} s_\nu c_\nu^{(N+3)/2},$$

$$\Xi^{**}_{3/2}\colon f_{-3/2} = \frac{Q_{1,7,9,11}}{96\sqrt{15}} s_\nu^2 (6 - 3(N-1)s_\nu^2) c_\nu^{(N-5)/2},$$

$$f_{-1/2} = \frac{Q_{1,7,9,11}}{96\sqrt{5}} s_\nu^2 (6 - 2(N-1)s_\nu^2) c_\nu^{(N-3)/2},$$

$$f_{1/2} = \frac{Q_{1,7,9,11}}{48\sqrt{10}} s_\nu^2 (6 - (N-1)s_\nu^2) c_\nu^{(N-1)/2},$$

$$f_{3/2} = \frac{Q_{1,7,9,11}}{8\sqrt{6}} s_\nu^2 c_\nu^{(N+1)/2}, \quad \text{(A16)}$$

$$\Omega^*_1\colon f_{-1} = \frac{Q_{1,5,7,9,11}}{192\sqrt{15}} s_\nu^3 (4 - 2(N-1)s_\nu^2) c_\nu^{(N-5)/2},$$

$$f_0 = \frac{Q_{1,5,7,9,11}}{96\sqrt{15}} s_\nu^3 (4 - (N-1)s_\nu^2) c_\nu^{(N-3)/2}, \quad \text{(A17)}$$

$$f_1 = \frac{Q_{1,5,7,9,11}}{24\sqrt{6}} s_\nu^3 c_\nu^{(N-1)/2},$$

$$\Gamma_{1/2}\colon f_{-1/2} = \frac{Q_{1,3,5,7,9,11}}{192\sqrt{30}} s_\nu^4 (2 - (N-1)s_\nu^2) c_\nu^{(N-5)/2},$$

$$f_{1/2} = \frac{Q_{1,3,5,7,9,11}}{96\sqrt{6}} s_\nu^4 c_\nu^{(N-3)/2}. \quad \text{(A18)}$$

Wave functions of other states presented in Tables I and II and also states with another possible value of spin have been obtained as well, but we shall not give them here for the sake of brevity.

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