Ultrahigh energy neutrino-nucleon scattering and parton distributions at small x

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The cross section for ultrahigh energy neutrino-nucleon scattering is very sensitive to the parton distributions at very small values of Bjorken x ($x \le 10^{-4}$). We numerically investigate the effects of modifying the behavior of the gluon distribution function at very small x in the DGLAP evolution equation. We then use the Color Glass Condensate formalism to calculate the neutrino-nucleon cross section at ultrahigh energies and compare the result with those based on modification of DGLAP evolution equation.

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I. INTRODUCTION

Neutrinos of ultrahigh energies $(E_{\nu} \ge 10^7 \text{ GeV})$ have been a puzzle for some time. One of the prime questions is where they come from, especially those above the GZK limit [1]. Possible sources include decays of super massive particles (dark matter?), acceleration in active galactic nuclei, and supernovae explosions [2]. Another question of interest is the cross section for the scattering of these ultrahigh energy neutrinos with nucleons. Here, part of the interest stems from the fact that if the cross section increases sufficiently rapidly, then the unitarity limit may be reached [3]. Another interest is what one can learn about the very small x parton distributions, since the energy dependence of the inclusive cross section is very sensitive to them.

The cross sections for scattering of neutrinos on nucleons at ultrahigh energies are dominated by the gluons in the nucleon while the contribution of sea quarks is suppressed by α_s since they come from gluon splitting via $g \rightarrow q\bar{q}$. For $x \le 10^{-2}$ the gluon distribution function of a nucleon is known to grow fast [4] with increasing Q^2 (virtuality of the gauge boson exchanged) and decreasing x as $(1/x)^{\beta}$, with beta less than 1. This implies that the structure functions, e.g., F_2 , in deep inelastic scattering will also increase, which would in turn mean a fast increase of the neutrino-nucleon total cross section. This fast growth of the total cross section can not continue indefinitely since it would violate unitarity (the Froissart bound). The parton (gluon) phase space density (number of partons per unit area and rapidity) is expected to be very high at very small Bjorken x which would lead to an overlap in transverse space and recombination of gluons which in turn could lead to saturation (a slow down of the growth of the structure functions) and the unitarization of the cross section.

At very small x, the nucleon is a very dense system of gluons and can be described via the Color Glass Condensate formalism [5] which resums large logs of energy as well as the large gluon density effects. It reduces to the BFKL formalism [6] in the limit that the gluon

density in a nucleon is small. The Color Glass Condensate is an all twist formalism and as such extends the domain of applicability of pQCD to high gluon density environments.

In this work, we consider different approaches to calculating the neutrino-nucleon total cross section at ultrahigh energies. First, we show the results from standard pQCD (DGLAP) [7] approach as well as the results from a unified DGLAP/BFKL approach, available in the literature [8]. We then consider the neutrino-nucleon cross section using the Color Glass Condensate formalism and gluon saturation based approaches. This involves modeling the quarkantiquark dipole cross section which is the basic ingredient in the structure functions. We compare the resulting neutrino-nucleon cross sections from different approaches and comment on the possibility of using future neutrino observatories to constrain the ultrahigh energy neutrinonucleon cross sections.

II. NEUTRINO-NUCLEON TOTAL CROSS SECTION

A. Leading twist pQCD

In perturbative QCD (pQCD), the cross section for the neutrino-nucleon cross section can be written as

$$\sigma_{\text{total}}^{\nu N}(s) = \int_0^1 dx \int_0^{xs} dQ^2 \frac{d^2 \sigma^{\nu N}}{dx dQ^2},$$
 (1)

where the differential cross section is given in terms of the quark and antiquark distribution functions

$$\frac{d^2 \sigma^{\nu N}}{dx dQ^2} = \frac{G_F^2}{\pi} \left(\frac{M_{W,Z}^2}{Q^2 + M_{W,Z}^2} \right)^2 \times [q(x, Q^2) + (1 - Q^2/xs)^2 \bar{q}(x, Q^2)].$$
(2)

Here G_F is the Fermi constant and $M_{W,Z}$ refer to the W or Z boson masses while s is the neutrino-nucleon center of mass energy. The total cross section is finite (unlike the photon exchange process) and is dominated by scales $Q \sim M_{W,Z}$. In what follows, we will restrict ourselves to charged

current exchanges, but the extension of work to the case of neutral current is trivial and we expect our results for the charged current exchange to hold equally well for the neutral current exchange.

In the standard leading twist (LT) pQCD approach, one parametrizes the x dependence of quark and antiquark distribution functions $q(x, Q^2)$, $\bar{q}(x, Q^2)$ at some initial scale Q_0 , typically taken to be of order of a GeV or so. The distribution functions are then given by DGLAP evolution equations at any other x and $Q > Q_0$. The parametrizations are fit to the available data on DIS, for example, at HERA. There are various parametrizations of parton distribution functions satisfying the DGLAP evolution equations, for example, CTEQ, MRST and GRV which differ in the choice of initial conditions and the degree of sophistication.

If the neutrino-nucleon center of mass energy is much higher than the exchanged momentum scale such that $\alpha_s \ln s/M_W^2 \sim 1$, it is more appropriate to use the BFKL formalism which resums these large logs rather than the DGLAP formalism. It is also possible to combine the two approaches in a phenomenological way such that both DGLAP and BFKL resummations are included [8]. In Fig. 1 we show the results of a DGLAP based calculation of the neutrino-nucleon total cross section, via charged current exchange due to Gandhi et al. [3], denoted GQRS, as well as a calculation due to Kutak *et al.*, denoted KK (unified), which uses a unified DGLAP and BFKL approach (shown here without gluon saturation effects). The cross section grows with the center of mass energy which can be parametrized in terms of the incident neutrino energy (in the range shown in Fig. 1) as $\sigma \sim$ $(E_{\nu}/1 \text{ GeV})^{0.402}$. It can be shown that this increase in the cross section is due to the growth of the parton distribution functions with decreasing Bjorken x [9]. While at the lowest energy the two results are identical, which shows small x effects resummed by BFKL are negligible, at



FIG. 1 (color online). The neutrino-nucleon cross section in leading twist pQCD via charged current exchange [3,8].

higher neutrino energy the two results can differ by a factor of 2 or larger. This signifies the fact that it is essential to include the contribution of small x partons properly at ultrahigh energies.

It is important to realize that the HERA data on DIS covers a limited kinematic region and that ultrahigh energy neutrino-nucleon cross sections are dominated by gluons at very small x and high Q^2 where there is no data. In the standard approach, one extrapolates the solution of the DGLAP evolution equations for parton distribution functions to smaller x, as needed. However, this requires making assumptions (or rather educated guesses) about the behavior of the distribution functions at small x. As we will show below, making rather plausible assumptions at small x, leads to large variations of the cross section at ultrahigh energies.

B. Gluon saturation

At very small Bjorken x, the gluon distribution function is expected to saturate, which would lead to a slow down of the growth of the neutrino-nucleon total cross section with energy. This is accomplished in the Color Glass Condensate (CGC) formalism which is an effective theory of QCD at high energy. The differential neutrino-nucleon cross section can be written in terms of the structure functions F_1 and F_2 (F_3 does not contribute at very small x) [3,5],

$$\frac{d^2\sigma}{dxdQ^2} = \frac{1}{2\pi} \frac{G_F^2}{(1+Q^2/M_W^2)^2} \times \left[(1-y)F_2(x,Q^2) + y^2 x F_1(x,Q^2) \right] \quad (3)$$

with

$$F_{2} = \frac{N_{c}Q^{2}}{4\pi^{3}} \int_{0}^{1} dz \int dr_{t}^{2}\sigma_{d}(x, r_{t}) \{4z^{2}(1-z)^{2}Q^{2}K_{0}^{2}(ar_{t}) + a^{2}[z^{2} + (1-z)^{2}]K_{1}^{2}(ar_{t})\}$$

$$F_{1} = \frac{1}{2x}\frac{N_{c}Q^{2}}{4\pi^{3}} \int_{0}^{1} dz \int dr_{t}^{2}\sigma_{d}(x, r_{t})a^{2}[z^{2} + (1-z)^{2}] \times K_{1}^{2}(ar_{t})$$
(4)

where $a^2 = z(1 - z)Q^2$ and K_0 and K_1 are modified Bessel functions, r_t is the size of the dipole and z is the fraction of the photon energy carried by the quark. The total cross section is the integral of (3) over x, from $x_{\min} = Q^2/s$ to 1 and over Q, where we choose Q_{\min} to be 10 GeV. The total cross section does not receive any appreciable contribution from scales below Q_{\min} . The essential ingredient in saturation based approaches is the dipole cross section which is the imaginary part of the forward scattering amplitude (hence the name dipole cross section) of a quark-antiquark dipole on the nucleon. The dipole cross section $\sigma_d(x, r_t)$ satisfies the JIMWLK evolution equation [10] which is the all twist generalization of the BFKL evolution equation. In

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practice since the JIMWLK evolution equation is a highly nonlinear equation, it is easier to parametrize the dipole cross section, in analogy to parametrizations of the standard parton distribution functions. The parametrizations of the dipole cross section are then used to calculate the structure functions in (4) and checked against available data in DIS [11–14]. The Color Glass Condensate formalism has also been successfully applied to particle production data in dA collisions at RHIC [15,16] (for a review see [17]). The dipole cross section depends sensitively on the value of the saturation scale Q_s and its energy dependence. While the overall magnitude of the saturation scale can not be determined from CGC itself, its energy (x) dependence is computed from CGC itself [18] and is in good agreement with the value extracted from HERA phenomenology which has been parametrized [11] as

$$Q_s^2(x) = (1 \text{ GeV}^2)(3 \times 10^{-4}/x)^{.28}.$$
 (5)

The value of the saturation scale Q_s compared to M_W determines whether one is in the saturation region ($Q_s \ge$ M_W), in the so called geometric scaling [19] region ($Q_s \leq$ $M_W \leq Q_s^2/\Lambda_{\rm OCD}$) or in the DGLAP region $(Q_s^2/\Lambda_{\rm OCD} \leq$ M_W). It is ideal to have a unified formalism which can address all three regions; however, such a formalism does not exist currently. One can either use the DGLAP evolution equation and modify it to include gluon saturation effects as in [8] or use the CGC formalism and add the contributions of the DGLAP region by using the standard pQCD expressions. We choose the later approach since we are mainly interested in the ultrahigh energy neutrino cross sections where the main contribution to the cross section comes from the very small x region. To do this, we introduce a cutoff x_0 below which we use the CGC expressions (4) while for $x > x_0$ we use (2) where the quark and antiquark distributions are taken from CTEQ parametrization.

One of the earlier parametrizations of the dipole cross section is due to Bartels *et al.* [12] which has been used to fit the HERA data. It is given by

$$\sigma_d(x, r_t) = \sigma_0 [1 - \exp(\pi^2 r_t^2 \alpha_s(\mu^2) x g(x, \mu^2) / (3\sigma_0))],$$
(6)

with $\mu^2 = .26/r_t^2 + 0.52$ and the gluon distribution function xg satisfies the DGLAP evolution equation. The overall constant σ_0 is the nucleon size and taken to be $\sigma_0 = 23$ mb. This parametrization includes higher twist effects but does not have the BFKL anomalous dimension. Another parametrization is due to Kharzeev *et al.* (KKT)[15] and has been used to fit the RHIC data on deuteron-nucleus collisions [15,16]. The dipole cross section in this parameterization is given by

$$\sigma_d(r_t, y) = \sigma_0 \left(\exp\left[-\frac{1}{4} [r_t^2 Q_s^2(y)]^{\gamma(y, r_t)} \right] - 1 \right)$$
(7)

where the saturation scale is given by $Q_s(y) =$

 $Q_0 \exp[\lambda(y - y_0)/2]$ with $y = \ln 1/x$ and $y_0 = 0.6$, $\lambda = 0.3$, $Q_0 = 0.62$ GeV. The anomalous dimension γ is

$$\gamma(y, r_t) = \frac{1}{2} \left(1 + \frac{\xi(y, r_t)}{\xi(y, r_t) + \sqrt{2\xi(y, r_t)} + 28\zeta(3)} \right)$$
(8)

where

$$\xi(y, r_t) = \frac{\log 1/r_t^2 Q_0^2}{(\lambda/2)(y - y_0)}.$$
(9)

This parameterization has the advantage that, unlike the one in (6), it has the BFKL anomalous dimension built in, which seems to be essential in describing the forward rapidity deuteron-gold data at RHIC. Using these two parametrizations of the dipole cross section, we calculate the neutrino-nucleon total cross section. We assume that quark (antiquark) distributions are known well for $x \ge x_0$ and use (2) to calculate the cross section for $x \ge x_0$. For $x \le x_0$, we use the saturation approach and calculate the cross section using (3) with the structure functions given by (4), using the two different parameterizations of the dipole cross section given in (6) and (7), denoted BGBK and KKT dipoles, respectively. We do not attempt to match the cross sections across x_0 which does introduce an additional uncertainty into our calculation, which we estimate to be less than 15% at the lower energies considered. To check the sensitivity of our results to the choice of x_0 , we try two different values of x_0 , first $x_0 = 10^{-4}$ and then $x_0 = 10^{-6}$. In case of BGBK dipoles, since gluon (as well as quark and antiquark) distribution function $xg(x, \mu^2)$ is not known well below $x \le 10^{-5}$, we consider three wildly different scenarios; (i) a continually growing distributions for $x \leq x$ 10^{-5} , (ii) a flat distribution for $x \le 10^{-5}$, and (iii) a distribution which falls by 1 order of magnitude for every decade of decreasing x for $x \le 10^{-5}$. A measurement of the neutrino-nucleon cross sections at ultrahigh energies would thus go a long way toward understanding the very small x parton distributions.

III. RESULTS AND DISCUSSION

In Fig. (2) we show our results for the neutrino-nucleon total cross section (via charged current exchange) for different neutrino energies for the case where $x_0 = 10^{-4}$ and BGBK denotes the Bartels *et al.* model of the dipole cross section given by (6) and KKT denotes the Kharzeev *et al.* parameterization given in (7). The subscript *I* refers to the case where the gluon (as well as quark and antiquark) distribution function $xg(x, \mu^2)$ in (6), taken from CTEQ6, keeps growing with *x* below $x = 10^{-5}$ while *II* refers to the case where the parton distribution functions below $x = 10^{-5}$ are flat and finally, case *III* corresponds to the case where the parton distribution functions below $x = 10^{-5}$ fall like a power. For neutrino energies less than 10^8 GeV, the cross section does not receive significant contributions from the region where $x < 10^{-5}$. This shows in Fig. 2 as



FIG. 2 (color online). The neutrino-nucleon cross section with $x_0 = 10^{-4}$, details are given in the text.

the three cases *I*, *II*, *III* (Bartels *et al.* dipole, denoted BGBK, with different behavior of parton distribution functions at small *x*) being almost identical for $E_{\nu} < 10^8$ GeV while the cross section calculated using the KKT parameterization of the dipole profile starts out below the other dipole models until neutrino energies of about 10^8-10^9 GeV after which it passes the BGBK *II*, *III* dipoles, due to the constancy or drop off of the BGBK gluon distribution function below $x = 10^{-5}$.

To see the sensitivity of our results to the choice of cutoff x_0 , we show the neutrino-nucleon cross section in Fig. 3 with the cutoff x_0 now taken to be 10^{-6} . Again, for $x > x_0$, we use the quark and antiquark distribution functions in (2) to calculate the cross section while for the region $x < x_0$ we use the saturation approach. While the BGBK *I* cross section changes by about 4% at the lowest and highest energies shown, it can change by 25% in the intermediate energy range, with the $x_0 = 10^{-6}$ case leading to a smaller cross section. This is due to the fact that the two ap-



FIG. 3 (color online). The same as in Fig. 2 but with $x_0 = 10^{-6}$.

proaches do not match at x_0 and contribution of the gluons to the BGBK cross section (3) in the $x = 10^{-4}-10^{-6}$ range is larger than the contribution of quarks and antiquarks to the standard cross section (2) in the same range. In this *x* range, saturation effects are negligible and the difference between the gluon distribution in (6) and the quark (antiquark) distribution in CTEQ is, at most, about 20% which, combined with the fact that we do not match the two distributions at x_0 , is responsible for the difference between the cross sections calculated using a different cutoff x_0 .

On the other hand, the case where we have the parton distribution functions falling off (BGBK *III*) is severely affected, by as much as a factor of 3 at the highest energy shown. On the contrary, the cross section using the KKT parametrization is rather robust, a change in x_0 from 10^{-4} to 10^{-6} changes the cross section by about 20% at $E_{\nu} = 10^{10}$ and 10% at $E_{\nu} = 10^{13}$. Depending on x_0 , the cross section given by Gandhi *et al.* [3] is about 1.65–2.0 times bigger than the KKT cross section at $E_{\nu} = 10^{12}$. It is clear that the assumptions made on the behavior of the gluon distribution function at very small x will determine the outcome of the calculated cross sections at high energy.

To compare our results to other saturation motivated studies, we show the ratio of our results for the neutrinonucleon cross section using the KKT parametrization, denoted KKT and the results of (screened) Kutak and Kwiecinski [8], denoted KK for the two choices of the parameter x_0 in Fig. 4. Since the numerical integrations involved are quite time consuming, we have taken rather large increments in the integration routines which leads to about 10% error on the KKT cross sections. This is the origin of the error bars shown in the figure. For neutrino energies more than 10^8-10^9 , the two approaches are in excellent agreement for $x_0 = 10^{-4}$ and within 10% (30%) at the highest (intermediate) energy for $x_0 = 10^{-6}$. This agreement is rather remarkable since the KK approach involves solving a phenomenologically unified DGLAP/



FIG. 4 (color online). Ratio of KKT and KK (screened) cross sections.

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BFKL equation with a nonlinear term motivated by the saturation physics while the results denoted KKT are based on a parameterization of the dipole profile which is motivated by the RHIC data on deuteron-gold collisions [17]. This is most likely due to the similar growth of the saturation scale in both cases since this growth is measured at HERA. It is also calculated very reliably in the Color Glass Condensate formalism [18] and is in excellent agreement with the measured value at HERA. The fact that the two rather different approaches give quite similar results for neutrino-nucleon cross section at high energies is very reassuring and gives us confidence that if and when the ultrahigh energy neutrino-nucleon cross sections are mea-

sured, one can have quite stringent constraints on saturation based calculations of the neutrino-nucleon cross section.

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