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Semileptonic decays of heavy baryons in the relativistic quark model

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Semileptonic decays of heavy baryons consisting of one heavy ($Q = b, c$) and two light ($q = u, d, s$) quarks are considered in the heavy-quark-light-diquark approximation. The relativistic quasipotential equation is used for obtaining masses and wave functions of both diquarks and baryons within the constituent quark model. The weak transition matrix elements are expressed through the overlap integrals of the baryon wave functions. The Isgur-Wise functions are determined in the whole accessible kinematic range. The exclusive semileptonic decay rates and different asymmetries are calculated with applying the heavy quark $1/m_Q$ expansion. The evaluated $\Lambda_b \to \Lambda_c l\nu$ decay rate agrees with its experimental value.

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I. INTRODUCTION

The description of heavy baryon properties represents a very interesting and important problem in quantum chromodynamics. Since the baryon is a three-body system, its theory is much more complicated compared to the two-body meson system. The quark-diquark picture of a baryon [1,2] is the popular approximation widely used to describe the baryon properties [1–5]. Such approximation allows to reduce the very complicated relativistic three-body problem to the two-body one. Recently, we evaluated the masses of the ground state heavy baryons in the framework of the relativistic quark model based on the quasipotential approach [6]. The heavy-quark-light-diquark picture of the heavy baryons was assumed. Both scalar and axial vector light diquarks were considered. The relatively large size of the light diquark was effectively taken into account by calculating the diquark-gluon interaction form factor through the overlap integral of the diquark wave functions. All the parameters of the quark model had fixed values which were determined from the previous studies of heavy and light meson properties [7–10]. The overall reasonable agreement (within a few MeV) of our model predictions for heavy baryon masses with the available experimental data supplies further support for the use of the heavy-quark-light-diquark approximation.

In this paper we continue the study of heavy baryon properties and apply our relativistic quark model for the calculation of their exclusive semileptonic decays. Such investigations are important, since they provide an additional source (complimentary to the heavy meson weak transitions) for determining the parameters of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, such as V_{cb}, from the comparison of the theoretical predictions with the experimental data. We limit our present consideration to the heavy-to-heavy ($b \to c$) transitions, where both the initial and final baryons contain heavy quarks. For such transitions the heavy quark effective theory (HQET), which is based on the $1/m_Q$ expansion of the QCD Lagrangian and the emerging heavy quark symmetry [11], provides the most effective constraints on theoretical models and significantly reduces the number of independent form factors in each order of the heavy quark expansion. In particular, transitions involving the Λ_Q ($Q = b, c$) baryons have the simplest structure, since the spectator light degrees of freedom (the scalar diquark) for these baryons have zero angular momentum.[1] In the heavy quark limit only one universal form factor, the so-called Isgur-Wise function, is required to describe the $\Lambda_Q \to \Lambda_{Q'}$ transition [12–14]. At the subleading order of the heavy quark expansion one mass parameter and one additional function emerge [15]. The consideration of the Ω_Q baryon decays is considerably more complicated, since light degrees of freedom (the axial vector diquark) now have spin 1. It is necessary to introduce two functions for parameterizing the $\Omega_Q \to \Omega_{Q'}^{(*)}$ transition in the heavy quark limit [12], and five additional functions and one mass parameter are needed at the subleading order in $1/m_Q$ [16]. Note that transition matrix elements between baryons with spectator diquarks having different spins (e.g., $\Lambda_b \to \Sigma_c$, which violate isospin symmetry) vanish in the heavy quark limit and can proceed only due to the subleading corrections [12]. The heavy quark symmetry alone does not allow the determination of the corresponding Isgur-Wise functions and mass parameters. Only the normalization of some of these functions is known at the point of zero recoil of the final heavy baryon. Thus, for the determination of these functions in the whole kinematic range the application of nonperturbative methods is necessary. Many different approaches were

[1]The structure of the decay matrix elements for the Λ_Q baryons is simpler than for heavy mesons, since in the latter case light degrees of freedom have spin 1/2.

previously used for the calculation of the Isgur-Wise functions of heavy baryons. However, most of them have important limitations. In some of these approaches the Isgur-Wise functions are calculated only at one kinematic point or in a limited region and then extrapolated to the whole kinematic range using an *ad hoc* ansatz, while other approaches assume some parameterization for the heavy baryon wave functions. The main aim of this paper is to determine the corresponding Isgur-Wise functions in the whole kinematic range through the overlap integrals of the heavy baryon wave functions in a consistent way within the relativistic quark model. These wave functions are known from the previous calculation of baryon masses [6]. On this basis exclusive semileptonic decay rates and different asymmetries can then be evaluated within the heavy quark expansion.

The paper is organized as follows. In Sec. II we describe our relativistic quark model and present predictions for the masses of ground-state light diquarks and heavy baryons in the heavy-quark-light diquark picture. In Sec. III we discuss the determination of the weak current matrix element between heavy baryon states. The relativistic transformation of the baryon wave function from the rest to the moving reference frame is presented. The general expressions for the weak matrix elements, decay rates and different asymmetries for heavy baryons with scalar and axial vector diquarks are given in Sec. IV. In Sec. V semileptonic decays of heavy baryons with the scalar diquark are considered using the heavy quark expansion. Explicit expressions for the leading and subleading Isgur-Wise functions are obtained as the overlap integrals of the baryon wave functions. The predictions for decay rates and the slope of the Isgur-Wise function are compared with the experimental data for the $\Lambda_b \to \Lambda_c e\nu$ decay. Semileptonic decay rates of heavy baryons with the axial vector diquark are studied in Sec. VI in the heavy quark limit. Finally, the comparison of our results for the heavy baryon semileptonic decay rates with previous theoretical predictions and our conclusions are given in Sec. VII.

II. RELATIVISTIC QUARK MODEL FOR HEAVY BARYONS

In the quasipotential approach and quark-diquark picture of heavy baryons [6] the interaction of two light quarks in a diquark and the heavy quark interaction with a light diquark in a baryon are described by the diquark wave function (Ψ_d) of the bound quark-quark state and by the baryon wave function (Ψ_B) of the bound quark-diquark state, respectively, which satisfy the quasipotential equation of the Schrödinger type

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_{d,B}(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M)\Psi_{d,B}(\mathbf{q}), \tag{1}$$

where the relativistic reduced mass is

$$\mu_R = \frac{E_1E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}, \tag{2}$$

and E_1, E_2 are given by

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \qquad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}. \tag{3}$$

Here $M = E_1 + E_2$ is the bound state mass (diquark or baryon), $m_{1,2}$ are the masses of light quarks (q_1 and q_2) which form the diquark or the masses of the light diquark (d) and heavy quark (Q) which form the heavy baryon (B), and $\mathbf{p}$ is their relative momentum. In the center of mass system the relative momentum squared on mass shell reads

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}. \tag{4}$$

The kernel $V(\mathbf{p}, \mathbf{q}; M)$ in Eq. (1) is the quasipotential operator of the quark-quark or quark-diquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive-energy states. In the following analysis we closely follow the similar construction of the quark-antiquark interaction in mesons which were extensively studied in our relativistic quark model [7]. For the quark-quark interaction in a diquark we use the relation $V_{qq} = V_{q\bar{q}}/2$ arising under the assumption about the octet structure of the interaction from the difference of the qq and $q\bar{q}$ color states. An important role in this construction is played by the Lorentz-structure of the confining interaction. In our analysis of mesons while constructing the quasipotential of the quark-antiquark interaction, we adopted that the effective interaction is the sum of the usual one-gluon exchange term with the mixture of long-range vector and scalar linear confining potentials, where the vector confining potential contains the Pauli terms. We use the same conventions for the construction of the quark-quark and quark-diquark interactions in the baryon. The quasipotential is then defined by [6,7,17]

(a) for the quark-quark (qq) interaction

$$\begin{aligned} V_{qq}(\mathbf{p}, \mathbf{q}; M) &= \bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}_{qq}(\mathbf{p}, \mathbf{q}; M) \\ &\quad \times u_1(q)u_2(-q), \\ \mathcal{V}_{qq}(\mathbf{p}, \mathbf{q}; M) &= \frac{1}{2}\bigg[\frac{4}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu \\ &\quad + V_{\rm conf}^V(\mathbf{k})\Gamma_1^\mu(\mathbf{k})\Gamma_{2;\mu}(-\mathbf{k}) \\ &\quad + V_{\rm conf}^S(\mathbf{k})\bigg], \end{aligned} \tag{5}$$

(b) for quark-diquark (Qd) interaction

$$V_{Qd}(\mathbf{p},\mathbf{q};M) = \frac{\langle d(P)|J^\mu|d(Q)\rangle}{2\sqrt{E_d(p)E_d(q)}}\bar{u}_Q(p) \times \frac{4}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma^\nu u_Q(q) + \psi_d^*(P)\bar{u}_Q(p)J_{d;\mu}\Gamma_Q^\mu(\mathbf{k}) \times V_{\rm conf}^V(\mathbf{k})u_Q(q)\psi_d(Q) + \psi_d^*(P)\bar{u}_Q(p)V_{\rm conf}^S(\mathbf{k})u_Q(q)\psi_d(Q), \tag{6}$$

where α_s is the QCD coupling constant; $\langle d(P)|J_\mu|d(Q)\rangle$ is the vertex of the diquark-gluon interaction which takes into account the diquark size [6] in terms of the diquark wave function overlap [$P = (E_d, -\mathbf{p})$ and $Q = (E_d, -\mathbf{q})$, $E_d = (M^2 - m_Q^2 + M_d^2)/(2M)$]. $D_{\mu\nu}(\mathbf{k})$ is the gluon propagator in the Coulomb gauge

$$D^{00}(\mathbf{k}) = -\frac{4\pi}{\mathbf{k}^2}, \qquad D^{ij}(\mathbf{k}) = -\frac{4\pi}{k^2}\left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right), \qquad D^{0i} = D^{i0} = 0, \tag{7}$$

and $\mathbf{k} = \mathbf{p} - \mathbf{q}$; γ_μ and $u(p)$ are the Dirac matrices and spinors

$$u^\lambda(p) = \sqrt{\frac{\epsilon(p)+m}{2\epsilon(p)}}\begin{pmatrix} 1 \\ \frac{\boldsymbol{\sigma}\mathbf{p}}{\epsilon(p)+m}\end{pmatrix}\chi^\lambda, \tag{8}$$

with $\epsilon(p) = \sqrt{\mathbf{p}^2 + m^2}$.

The diquark state in the confining part of the quark-diquark quasipotential (6) is described by the wave functions

$$\psi_d(p) = \begin{cases} 1 & \text{for scalar diquark} \\ \varepsilon_d(p) & \text{for axial vector diquark} \end{cases}, \tag{9}$$

where the four vector

$$\varepsilon_d(p) = \left(\frac{(\boldsymbol{\varepsilon}_d\mathbf{p})}{M_d}, \boldsymbol{\varepsilon}_d + \frac{(\boldsymbol{\varepsilon}_d\mathbf{p})\mathbf{p}}{M_d(E_d(p)+M_d)}\right) \tag{10}$$

is the polarization vector [$\varepsilon_d^\mu(p)p_\mu = 0$] of the axial vector diquark with momentum $\mathbf{p}$, $E_d(p) = \sqrt{\mathbf{p}^2 + M_d^2}$ and $\varepsilon_d(0) = (0, \boldsymbol{\varepsilon}_d)$ is the polarization vector in the diquark rest frame. The effective long-range vector vertex of the diquark can be presented in the form

$$J_{d;\mu} = \begin{cases} \frac{(P+Q)_\mu}{2\sqrt{E_d(p)E_d(q)}} & \text{for scalar diquark} \\ -\frac{(P+Q)_\mu}{2\sqrt{E_d(p)E_d(q)}} + \frac{i\mu_d}{2M_d}\Sigma_\mu^\nu \tilde{k}_\nu & \text{for axial vector diquark} \end{cases}, \tag{11}$$

where $\tilde{k} = (0, \mathbf{k})$. Here the antisymmetric tensor reads

$$(\Sigma_{\rho\sigma})_\mu^\nu = -i(g_{\mu\rho}\delta_\sigma^\nu - g_{\mu\sigma}\delta_\rho^\nu), \tag{12}$$

and the spin $\mathbf{S}_d$ of the axial vector diquark is given by $(S_{d;k})_{il} = -i\varepsilon_{kil}$. We choose the total chromomagnetic moment of the axial vector diquark $\mu_d = 0$ [18]. Such a choice appears to be natural, since the long-range chromomagnetic interaction of the diquark proportional to μ_d then vanishes in accord with the flux tube model.

The effective long-range vector vertex of the quark is defined by [7,19]

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}\tilde{k}^\nu, \qquad \tilde{k} = (0, \mathbf{k}), \tag{13}$$

where κ is the Pauli interaction constant characterizing the anomalous chromomagnetic moment of quarks. In the configuration space the vector and scalar confining potentials in the nonrelativistic limit reduce to

$$V_{\rm conf}^V(r) = (1-\varepsilon)V_{\rm conf}(r), \qquad V_{\rm conf}^S(r) = \varepsilon V_{\rm conf}(r), \tag{14}$$

with

$$V_{\rm conf}(r) = V_{\rm conf}^S(r) + V_{\rm conf}^V(r) = Ar + B, \tag{15}$$

where ε is the mixing coefficient.

The constituent quark masses $m_b = 4.88$ GeV, $m_c = 1.55$ GeV, $m_u = m_d = 0.33$ GeV, $m_s = 0.5$ GeV and the parameters of the linear potential $A = 0.18$ GeV^2 and $B = -0.3$ GeV have the usual values of quark models. The value of the mixing coefficient of vector and scalar confining potentials $\varepsilon = -1$ has been determined from the consideration of charmonium radiative decays [8] and the heavy quark expansion [9]. Finally, the universal Pauli interaction constant $\kappa = -1$ has been fixed from the analysis of the fine splitting of heavy quarkonia ^3P_J- states [8]. Note that the long-range chromomagnetic contribution to the potential in our model is proportional to $(1+\kappa)$ and thus vanishes for the chosen value of $\kappa = -1$.

The quasipotential (5) can be used for arbitrary quark masses. The substitution of the Dirac spinors into (5) results in an extremely nonlocal potential in the configuration space. Clearly, it is very hard to deal with such potentials without any additional transformations. In oder to simplify the relativistic qq potential, we make the following replacement in the Dirac spinors [6,7,10]:

$$\epsilon_{1,2}(p) = \sqrt{m_{1,2}^2 + \mathbf{p}^2} \to E_{1,2}. \tag{16}$$

This substitution makes the Fourier transformation of the potential (5) local, but the resulting relativistic potential becomes dependent on the diquark and baryon masses in a very complicated nonlinear way. We consider only the baryon ground states, which further simplifies our analysis, since all terms containing orbital momentum vanish. The detailed expressions for the relativistic quark potential can be found in Ref. [6]. The obtained masses of the light diquarks are given in Table I. The heavy baryon masses

TABLE I. Masses of light ground state diquarks (in MeV). S and A denote scalar and axial vector diquarks which are antisymmetric $[q, q']$ and symmetric $\{q, q'\}$ in flavour indices, respectively.

Quark content	Diquark type	Mass
$[u, d]$	S	710
$\{u, d\}$	A	909
$[u, s]$	S	948
$\{u, s\}$	A	1069
$\{s, s\}$	A	1203

TABLE II. Masses of the ground state heavy baryons (in MeV).

Baryon	$I(J^P)$	$M^{\rm theor}$ [6]	$M^{\rm exp}$ [20]
Λ_c	$0(\frac{1}{2}^+)$	2297	2284.9(6)
Σ_c	$1(\frac{1}{2}^+)$	2439	2451.3(7)
Σ_c^*	$1(\frac{3}{2}^+)$	2518	2515.9(2.4)
Ξ_c	$\frac{1}{2}(\frac{1}{2}^+)$	2481	2466.3(1.4)
Ξ_c'	$\frac{1}{2}(\frac{1}{2}^+)$	2578	2574.1(3.3)
Ξ_c^*	$\frac{1}{2}(\frac{3}{2}^+)$	2654	2647.4(2.0)
Ω_c	$0(\frac{1}{2}^+)$	2698	2697.5(2.6)
Ω_c^*	$0(\frac{3}{2}^+)$	2768	
Λ_b	$0(\frac{1}{2}^+)$	5622	5624(9)
Σ_b	$1(\frac{1}{2}^+)$	5805	
Σ_b^*	$1(\frac{3}{2}^+)$	5834	
Ξ_b	$\frac{1}{2}(\frac{1}{2}^+)$	5812	
Ξ_b'	$\frac{1}{2}(\frac{1}{2}^+)$	5937	
Ξ_b^*	$\frac{1}{2}(\frac{3}{2}^+)$	5963	
Ω_b	$0(\frac{1}{2}^+)$	6065	
Ω_b^*	$0(\frac{3}{2}^+)$	6088	

calculated in the heavy-quark-light diquark approximation are presented in Table II in comparison with the available experimental data [20]. There an overall good agreement of our predictions with experiment is found.

III. MATRIX ELEMENTS OF THE WEAK CURRENT FOR HEAVY BARYON DECAYS

In order to calculate the exclusive semileptonic decay rate of the heavy baryon, it is necessary to determine the corresponding matrix element of the weak current between baryon states. In the quasipotential approach, the matrix element of the weak current $J^W_\mu = \bar{Q}'\gamma_\mu(1-\gamma_5)Q$, associated with the heavy-to-heavy quark $Q \to Q'$ ($Q = b$ and $Q' = c$) transition, between baryon states with masses M_{B_Q}, $M_{B_{Q'}}$ and momenta p_{B_Q}, $p_{B_{Q'}}$ takes the form [21]

$$\langle B_{Q'}(p_{B_{Q'}})|J^W_\mu|B_Q(p_{B_Q})\rangle = \int \frac{d^3p d^3q}{(2\pi)^6} \bar{\Psi}_{B_{Q'}\mathbf{p}_{B_{Q'}}}(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q}) \times \Psi_{B_Q\mathbf{p}_{B_Q}}(\mathbf{q}), \tag{17}$$

where $\Gamma_\mu(\mathbf{p},\mathbf{q})$ is the two-particle vertex function and

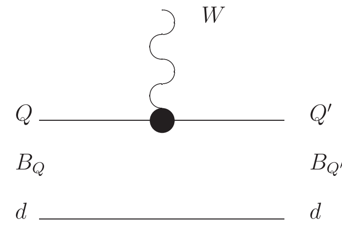

FIG. 1. Lowest order vertex function $\Gamma^{(1)}$ contributing to the current matrix element (17).

$\Psi_{B\mathbf{p}_B}$ are the baryon ($B = B_Q, B_{Q'}$) wave functions projected onto the positive-energy states of quarks and boosted to the moving reference frame with momentum $\mathbf{p}_B$.

The contributions to Γ come from Figs. 1 and 2. The contribution $\Gamma^{(2)}$ is the consequence of the projection onto the positive-energy states. Note that the form of the relativistic corrections resulting from the vertex function $\Gamma^{(2)}$ is explicitly dependent on the Lorentz structure of the quark-diquark interaction. In the heavy quark limit $m_Q \to \infty$ only $\Gamma^{(1)}$ contributes, while $\Gamma^{(2)}$ gives the subleading order contributions. The vertex functions are given by

$$\Gamma^{(1)}_\mu(\mathbf{p},\mathbf{q}) = \psi^*_d(p_d)\bar{u}_{Q'}(p_{Q'})\gamma_\mu(1-\gamma^5)u_Q(q_Q)\psi_d(q_d) \times (2\pi)^3\delta(\mathbf{p}_d - \mathbf{q}_d), \tag{18}$$

and

$$\begin{aligned}\Gamma^{(2)}_\mu(\mathbf{p},\mathbf{q}) = \psi^*_d(p_d)\bar{u}_{Q'}(p_{Q'})\Big\{&\gamma_\mu(1-\gamma^5) \\ &\times \frac{\Lambda^{(-)}_Q(k)}{\epsilon_Q(k)+\epsilon_Q(p_{Q'})}\gamma^0\mathcal{V}_{Qd}(\mathbf{p}_d-\mathbf{q}_d) \\ &+ \mathcal{V}_{Qd}(\mathbf{p}_d-\mathbf{q}_d)\frac{\Lambda^{(-)}_{Q'}(k')}{\epsilon_{Q'}(k')+\epsilon_{Q'}(q_Q)} \\ &\times \gamma^0\gamma_\mu(1-\gamma^5)\Big\}u_Q(q_Q)\psi_d(q_d),\end{aligned} \tag{19}$$

where the superscripts "(1)" and "(2)" correspond to Figs. 1 and 2, $\mathbf{k} = \mathbf{p}_{Q'} - \mathbf{\Delta}$; $\mathbf{k}' = \mathbf{q}_Q + \mathbf{\Delta}$; $\mathbf{\Delta} = M_{B_{Q'}}\mathbf{v}' - M_{B_Q}\mathbf{v}$; $\epsilon(p) = \sqrt{m^2 + \mathbf{p}^2}$;

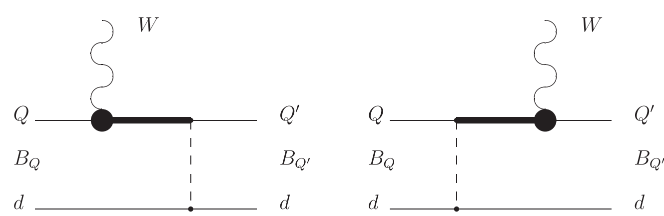

FIG. 2. Vertex function $\Gamma^{(2)}$ taking the quark interaction into account. Dashed lines correspond to the effective potential $\mathcal{V}_{Qd}$ in (6). Bold lines denote the negative-energy part of the quark propagator.

$$\Lambda^{(-)}(p) = \frac{\epsilon(p) - (m\gamma^0 + \gamma^0(\boldsymbol{\gamma}\mathbf{p}))}{2\epsilon(p)}.$$

Here [21]

$$p_{Q',d} = \epsilon_{Q',d}(p)v' \pm \sum_{i=1}^{3} n^{(i)}(v')p^i, \qquad v'^\mu = \frac{p^\mu_{B_{Q'}}}{M_{B_{Q'}}},$$
$$q_{Q,d} = \epsilon_{Q,d}(q)v \pm \sum_{i=1}^{3} n^{(i)}(v)q^i, \qquad v^\mu = \frac{p^\mu_{B_Q}}{M_{B_Q}}, \tag{20}$$

and $n^{(i)}$ are three four-vectors given by

$$n^{(i)\mu}(v) = \left\{v^i, \delta_{ij} + \frac{v^i v^j}{v^0 + 1}\right\}.$$

It is important to note that the wave functions entering the weak current matrix element (17) are not in the rest frame in general. For example, in the B_Q baryon rest frame ($\mathbf{v} = 0$), the final baryon is moving with the recoil momentum $\boldsymbol{\Delta}$. The wave function of the moving baryon $\Psi_{B_{Q'}\boldsymbol{\Delta}}$ is connected with the wave function in the rest frame $\Psi_{B_{Q'}\mathbf{0}} \equiv \Psi_{B_{Q'}}$ by the transformation [21]

$$\Psi_{B_{Q'}\boldsymbol{\Delta}}(\mathbf{p}) = D^{1/2}_{Q'}(R^W_{L_{\boldsymbol{\Delta}}})D^{\mathcal{I}}_d(R^W_{L_{\boldsymbol{\Delta}}})\Psi_{B_{Q'}\mathbf{0}}(\mathbf{p}), \qquad \mathcal{I} = 0, 1, \tag{21}$$

where R^W is the Wigner rotation, $L_{\boldsymbol{\Delta}}$ is the Lorentz boost from the baryon rest frame to a moving one, and the rotation matrix of the heavy quark spin $D^{1/2}(R)$ in spinor representation is given by

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} D^{1/2}_{Q'}(R^W_{L_{\boldsymbol{\Delta}}}) = S^{-1}(\mathbf{p}_{Q'})S(\boldsymbol{\Delta})S(\mathbf{p}), \tag{22}$$

where

$$S(\mathbf{p}) = \sqrt{\frac{\epsilon(p) + m}{2m}}\left(1 + \frac{\boldsymbol{\alpha}\mathbf{p}}{\epsilon(p) + m}\right)$$

is the usual Lorentz transformation matrix of the four-spinor. The rotation matrix $D^{\mathcal{I}}(R)$ of the diquark with spin $\mathcal{I}$ is equal to $D^0_d(R^W) = 1$ for the scalar diquark and $D^1_d(R^W) = R^W$ for the axial vector diquark.

IV. FORM FACTORS AND SEMILEPTONIC DECAY RATES

In this section we give the general parameterization of semileptonic decay matrix elements and the expressions for decay rates of heavy baryons with scalar and axial vector light diquarks.

A. Heavy baryons with the scalar diquark

The hadronic matrix elements for the semileptonic decay $\Lambda_Q \to \Lambda_{Q'}$ are parameterized in terms of six invariant form factors:

$$\langle\Lambda_{Q'}(v', s')|V^\mu|\Lambda_Q(v, s)\rangle = \bar{u}_{\Lambda_{Q'}}(v', s')[F_1(w)\gamma^\mu + F_2(w)v^\mu + F_3(w)v'^\mu]u_{\Lambda_Q}(v, s),$$
$$\langle\Lambda_{Q'}(v', s')|A^\mu|\Lambda_Q(v, s)\rangle = \bar{u}_{\Lambda_{Q'}}(v', s')[G_1(w)\gamma^\mu + G_2(w)v^\mu + G_3(w)v'^\mu]\gamma_5 u_{\Lambda_Q}(v, s), \tag{23}$$

where $u_{\Lambda_Q}(v, s)$ and $u_{\Lambda_{Q'}}(v', s')$ are Dirac spinors of the initial and final baryon with four-velocities v and v', respectively; $q = M_{\Lambda_{Q'}}v' - M_{\Lambda_Q}v$, and

$$w = v \cdot v' = \frac{M^2_{\Lambda_Q} + M^2_{\Lambda_{Q'}} - q^2}{2M_{\Lambda_Q}M_{\Lambda_{Q'}}}.$$

The helicity amplitudes are expressed in terms of these form factors [22] as

$$H^{V,A}_{1/2,0} = \frac{1}{\sqrt{q^2}}\sqrt{2M_{\Lambda_Q}M_{\Lambda_{Q'}}(w \mp 1)}[(M_{\Lambda_Q} \pm M_{\Lambda_{Q'}}) \times \mathcal{F}^{V,A}_1(w) \pm M_{\Lambda_{Q'}}(w \pm 1)\mathcal{F}^{V,A}_2(w) \pm M_{\Lambda_Q}(w \pm 1)\mathcal{F}^{V,A}_3(w)],$$
$$H^{V,A}_{1/2,1} = -2\sqrt{M_{\Lambda_Q}M_{\Lambda_{Q'}}(w \mp 1)}\mathcal{F}^{V,A}_1(w), \tag{24}$$

where the upper (lower) sign corresponds to $V(A)$ and $\mathcal{F}^V_i \equiv F_i$, $\mathcal{F}^A_i \equiv G_i$ $(i = 1, 2, 3)$. $H^{V,A}_{\lambda',\lambda_W}$ are the helicity amplitudes for weak transitions induced by vector (V) and axial vector (A) currents, where λ' and λ_W are the helicities of the final baryon and the virtual W-boson, respectively. The amplitudes for negative values of the helicities can be obtained using the relation

$$H^{V,A}_{-\lambda',-\lambda_W} = \pm H^{V,A}_{\lambda',\lambda_W}.$$

The total helicity amplitude for the $V - A$ current is then given by

$$H_{\lambda',\lambda_W} = H^V_{\lambda',\lambda_W} - H^A_{\lambda',\lambda_W}.$$

The total differential decay rate

$$\frac{d\Gamma}{dw} = \frac{d\Gamma_T}{dw} + \frac{d\Gamma_L}{dw} \tag{25}$$

is expressed in terms of the partial rates for transversely (T) and longitudinally (L) polarized W-bosons

$$\frac{d\Gamma_T}{dw} = \frac{G_F^2}{(2\pi)^3}|V_{QQ'}|^2\frac{q^2M^2_{\Lambda_{Q'}}\sqrt{w^2 - 1}}{12M_{\Lambda_Q}} \times [|H_{1/2,1}|^2 + |H_{-1/2,-1}|^2],$$
$$\frac{d\Gamma_L}{dw} = \frac{G_F^2}{(2\pi)^3}|V_{QQ'}|^2\frac{q^2M^2_{\Lambda_{Q'}}\sqrt{w^2 - 1}}{12M_{\Lambda_Q}} \times [|H_{1/2,0}|^2 + |H_{-1/2,0}|^2], \tag{26}$$

where G_F is the Fermi coupling constant and $V_{QQ'}$ is the relevant CKM matrix element.

The decay products in the semileptonic decay $\Lambda_Q \to \Lambda_{Q'}(\to \Lambda\pi) + W(\to l\nu)$ are highly polarized. The polarization of the decay products is usually expressed through different asymmetry parameters [22] defined as follows:

$$\begin{aligned}
a_T &= \frac{|H_{1/2,1}|^2 - |H_{-1/2,-1}|^2}{|H_{1/2,1}|^2 + |H_{-1/2,-1}|^2},\\
a_L &= \frac{|H_{1/2,0}|^2 - |H_{-1/2,0}|^2}{|H_{1/2,0}|^2 + |H_{-1/2,0}|^2},\\
\alpha' &= \frac{|H_{1/2,1}|^2 - |H_{-1/2,-1}|^2}{|H_{1/2,1}|^2 + |H_{-1/2,-1}|^2 + 2(|H_{1/2,0}|^2 + |H_{-1/2,0}|^2)},\\
\alpha'' &= \frac{|H_{1/2,1}|^2 - |H_{-1/2,-1}|^2 - 2(|H_{1/2,0}|^2 + |H_{-1/2,0}|^2)}{|H_{1/2,1}|^2 + |H_{-1/2,-1}|^2 + 2(|H_{1/2,0}|^2 + |H_{-1/2,0}|^2)},\\
\gamma &= \frac{2\,\mathrm{Re}(H_{-1/2,0}H^*_{1/2,1} + H_{1/2,0}H^*_{-1/2,-1})}{|H_{1/2,1}|^2 + |H_{-1/2,-1}|^2 + |H_{1/2,0}|^2 + |H_{-1/2,0}|^2}.
\end{aligned} \tag{27}$$

The average values of these asymmetry parameters ($\langle a_T\rangle$, $\langle a_L\rangle$, $\langle\alpha'\rangle$, $\langle\alpha''\rangle$, $\langle\gamma\rangle$) are calculated by separately integrating the numerators and denominators in (27) over w. The average longitudinal $\Lambda_{Q'}$ polarization $\langle P_L\rangle$ can be expressed in terms of $\langle a_T\rangle$, $\langle a_L\rangle$ as

$$\langle P_L\rangle = \frac{\langle a_T\rangle + R\langle a_L\rangle}{1+R}, \qquad R = \frac{\Gamma_L}{\Gamma_T}. \tag{28}$$

B. Heavy baryons with the axial vector diquark

The hadronic matrix elements for the semileptonic decay $\Omega_Q \to \Omega_{Q'}$ are parameterized in terms of six invariant form factors through expressions analogous to Eqs. (23). Then the helicity amplitudes and differential decay rates are given by Eqs. (24) and (26) with obvious mass replacements.

The invariant parameterization for the semileptonic decay $\Omega_Q \to \Omega^*_{Q'}$ reads:

$$\begin{aligned}
\langle\Omega^*_{Q'}(v',s')|V^\mu|\Omega_Q(v,s)\rangle &= \bar u_{\Omega^*_{Q'},\lambda}(v',s')[N_1(w)v^\lambda\gamma^\mu + N_2(w)v^\lambda v^\mu + N_3(w)v^\lambda v'^\mu + N_4(w)g^{\lambda\mu}]\gamma_5 u_{\Omega_Q}(v,s),\\
\langle\Omega^*_{Q'}(v',s')|A^\mu|\Omega_Q(v,s)\rangle &= \bar u_{\Omega^*_{Q'},\lambda}(v',s')[K_1(w)v^\lambda\gamma^\mu + K_2(w)v^\lambda v^\mu + K_3(w)v^\lambda v'^\mu + K_4(w)g^{\lambda\mu}]u_{\Omega_Q}(v,s),
\end{aligned} \tag{29}$$

where $u_{\Omega^*_Q,\mu}$ is the Rarita-Schwinger spinor for the Ω^*_Q, which obeys

$$\not v u_{\Omega^*_{Q'},\mu}(v,s) = u_{\Omega^*_{Q'},\mu}(v,s),$$
$$v^\mu u_{\Omega^*_{Q'},\mu}(v,s) = \gamma^\mu u_{\Omega^*_{Q'},\mu}(v,s) = 0.$$

The helicity amplitudes are given by [22]

$$\begin{aligned}
H^{V,A}_{1/2,0} &= \mp\frac{1}{\sqrt{q^2}}\frac{2}{\sqrt{3}}\sqrt{M_{\Omega_Q}M_{\Omega^*_{Q'}}(w\mp 1)}[(M_{\Omega_Q}w - M_{\Omega^*_{Q'}})\times\mathcal{N}^{V,A}_4(w) \mp (M_{\Omega_Q}\mp M_{\Omega^*_{Q'}})(w\pm 1)\mathcal{N}^{V,A}_1(w)\\
&\quad + M_{\Omega^*_{Q'}}(w^2-1)\mathcal{N}^{V,A}_2(w) + M_{\Omega_Q}(w^2-1)\mathcal{N}^{V,A}_3(w)],\\
H^{V,A}_{1/2,1} &= \sqrt{\frac{2}{3}}\sqrt{M_{\Omega_Q}M_{\Omega^*_{Q'}}(w\mp 1)}[\mathcal{N}^{V,A}_4(w) - 2(w\pm 1)\mathcal{N}^{V,A}_1(w)],\\
H^{V,A}_{3/2,1} &= \mp\sqrt{2M_{\Omega_Q}M_{\Omega^*_{Q'}}(w\mp 1)}\mathcal{N}^{V,A}_4(w),
\end{aligned} \tag{30}$$

where again the upper (lower) sign corresponds to $V(A)$ and $\mathcal{N}^V_i \equiv N_i$, $\mathcal{N}^A_i \equiv K_i$ ($i = 1, 2, 3$). The remaining helicity amplitudes can be obtained using the relation

$$H^{V,A}_{-\lambda',-\lambda_W} = \mp H^{V,A}_{\lambda',\lambda_W}.$$

Partial differential decay rates can be represented in the following form

$$\frac{d\Gamma_T}{dw} = \frac{G_F^2}{(2\pi)^3}|V_{QQ'}|^2\frac{q^2M^2_{\Omega^*_{Q'}}\sqrt{w^2-1}}{12M_{\Omega_Q}}[|H_{1/2,1}|^2 + |H_{-1/2,-1}|^2 + |H_{3/2,1}|^2 + |H_{-3/2,-1}|^2], \tag{31}$$

$$\frac{d\Gamma_L}{dw} = \frac{G_F^2}{(2\pi)^3}|V_{QQ'}|^2\frac{q^2M^2_{\Omega^*_{Q'}}\sqrt{w^2-1}}{12M_{\Omega_Q}}[|H_{1/2,0}|^2 + |H_{-1/2,0}|^2].$$

V. SEMILEPTONIC DECAYS OF HEAVY BARYONS WITH THE SCALAR DIQUARK

In the heavy quark limit $m_Q \to \infty$ ($Q = b, c$) the form factors (23) can be expressed through the single Isgur-Wise function $\zeta(w)$ [12]

$$\begin{aligned}
F_1(w) &= G_1(w) = \zeta(w),\\
F_2(w) &= F_3(w) = G_2(w) = G_3(w) = 0.
\end{aligned} \tag{32}$$

At subleading order of the heavy quark expansion two additional types of contributions arise [23]. The first one parameterizes $1/m_Q$ corrections to the HQET current and is proportional to the product of the parameter $\bar\Lambda = M_{\Lambda_Q} - m_Q$, which is the difference of the baryon and heavy quark masses in the infinitely heavy quark limit, and the leading order Isgur-Wise function $\zeta(w)$. The second one comes from the kinetic energy term in $1/m_Q$ correction to the HQET Lagrangian and introduces the additional function $\chi(w)$. Therefore the form factors are given by [23]

$$F_1(w) = \zeta(w) + \left(\frac{\bar\Lambda}{2m_Q} + \frac{\bar\Lambda}{2m_{Q'}}\right)[2\chi(w) + \zeta(w)],$$

$$G_1(w) = \zeta(w) + \left(\frac{\bar\Lambda}{2m_Q} + \frac{\bar\Lambda}{2m_{Q'}}\right)\left[2\chi(w) + \frac{w-1}{w+1}\zeta(w)\right],$$

$$F_2(w) = G_2(w) = -\frac{\bar\Lambda}{2m_{Q'}}\frac{2}{w+1}\zeta(w),$$

$$F_3(w) = -G_3(w) = -\frac{\bar\Lambda}{2m_Q}\frac{2}{w+1}\zeta(w). \qquad (33)$$

To calculate these semileptonic decay form factors in our model we substitute the vertex functions $\Gamma^{(1)}$ (18) and $\Gamma^{(2)}$ (19) in the weak current matrix element (17) between Λ_Q and $\Lambda_{Q'}$ baryons. It is important to take into account the relativistic transformation of the baryon wave functions (21) in this matrix element. The resulting structure of the decay matrix element is rather complicated, because it is necessary to integrate both over d^3p and d^3q. The δ function in expression (18) for $\Gamma^{(1)}$ permits us to perform one of these integrations and thus this contribution can be easily calculated. The calculation of the contribution of the vertex function $\Gamma^{(2)}$ (19) is more difficult, since here, instead of a δ function, we have a complicated structure, containing the heavy-quark-light-diquark interaction potential. Nevertheless, the application of the heavy quark $1/m_Q$ expansion considerably simplifies the calculation. We carry out this expansion up to the first order. Then we use the quasipotential equation to perform one of the integrations in the decay matrix element. The vertex function $\Gamma^{(1)}$ provides the leading order contribution, while $\Gamma^{(2)}$ contributes already at the subleading order. The resulting expressions for the semileptonic decay $\Lambda_Q \to \Lambda_{Q'}$ form factors up to subleading order in $1/m_Q$ are then given by

$$\begin{aligned}
F_1(w) &= \zeta(w) + \left(\frac{\bar\Lambda}{2m_Q} + \frac{\bar\Lambda}{2m_{Q'}}\right)[2\chi(w) + \zeta(w)] + 4(1-\varepsilon)(1+\kappa)\left[\frac{\bar\Lambda}{2m_{Q'}}\frac{1}{w-1} - \frac{\bar\Lambda}{2m_Q}(w+1)\right]\chi(w),\\
G_1(w) &= \zeta(w) + \left(\frac{\bar\Lambda}{2m_Q} + \frac{\bar\Lambda}{2m_{Q'}}\right)\left[2\chi(w) + \frac{w-1}{w+1}\zeta(w)\right] - 4(1-\varepsilon)(1+\kappa)\frac{\bar\Lambda}{2m_Q}w\chi(w),\\
F_2(w) &= -\frac{\bar\Lambda}{2m_{Q'}}\frac{2}{w+1}\zeta(w) - 4(1-\varepsilon)(1+\kappa)\left[\frac{\bar\Lambda}{2m_{Q'}}\frac{1}{w-1} + \frac{\bar\Lambda}{2m_Q}w\right]\chi(w),\\
G_2(w) &= -\frac{\bar\Lambda}{2m_{Q'}}\frac{2}{w+1}\zeta(w) - 4(1-\varepsilon)(1+\kappa)\frac{\bar\Lambda}{2m_{Q'}}\frac{1}{w-1}\chi(w),\\
F_3(w) &= -G_3(w) = -\frac{\bar\Lambda}{2m_Q}\frac{2}{w+1}\zeta(w) + 4(1-\varepsilon)(1+\kappa)\frac{\bar\Lambda}{2m_Q}\chi(w),
\end{aligned} \qquad (34)$$

where the leading order Isgur-Wise function of heavy baryons

$$\zeta(w) = \lim_{m_Q\to\infty}\int\frac{d^3p}{(2\pi)^3}\Psi_{\Lambda_{Q'}}\left(\mathbf{p} + 2\epsilon_d(p)\sqrt{\frac{w-1}{w+1}}\mathbf{e_\Delta}\right)\Psi_{\Lambda_Q}(\mathbf{p}), \qquad (35)$$

and the subleading function

$$\chi(w) = -\frac{w-1}{w+1}\lim_{m_Q\to\infty}\int\frac{d^3p}{(2\pi)^3}\Psi_{\Lambda_{Q'}}\left(\mathbf{p} + 2\epsilon_d(p)\sqrt{\frac{w-1}{w+1}}\mathbf{e_\Delta}\right)\frac{\bar\Lambda - \epsilon_d(p)}{2\bar\Lambda}\Psi_{\Lambda_Q}(\mathbf{p}), \qquad (36)$$

here $\mathbf{e_\Delta} = \mathbf{\Delta}/\sqrt{\mathbf{\Delta}^2}$ is the unit vector in the direction of $\mathbf{\Delta} = M_{\Lambda_{Q'}}\mathbf{v}' - M_{\Lambda_Q}\mathbf{v}$. It is important to note that in our model the expressions for the Isgur-Wise functions $\zeta(w)$ (35) and $\chi(w)$ (36) are determined in the whole kinematic range accessible in the semileptonic decays in terms of the overlap integrals of the heavy baryon wave functions, which are known from the baryon mass spectrum calculations. Therefore we do not need to make any assumptions about the baryon wave functions or/and extrapolate our form factors from the single kinematic point, as it was done in most of previous calculations.

For $(1-\varepsilon)(1+\kappa) = 0$ the HQET results (33) are reproduced. This can be achieved either setting $\varepsilon = 1$ (pure scalar confinement) or $\kappa = -1$. In our model we need a vector confining contribution (see Sec. II) and therefore use the latter option. This consideration gives us an additional justification, based on the HQET, for fixing one of the main parameters of the model κ.[2] In the heavy quark limit the wave functions of the initial Ψ_{Λ_Q} and final baryon

[2]It is important to note that the same value of κ is needed to get agreement with the HQET structure of the first order $1/m_Q$ corrections for $B \to D^{(*)}e\nu$ decays [9].

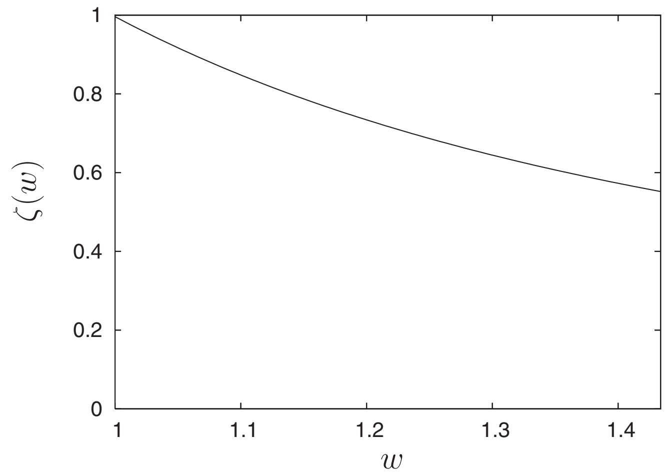

FIG. 3. The Isgur-Wise function $\zeta(w)$ for the $\Lambda_b \to \Lambda_c e\nu$ semileptonic decay.

$\Psi_{\Lambda_{Q'}}$ coincide, and thus the HQET normalization condition $\zeta(1) = 1$ is exactly reproduced. The subleading function $\chi(w)$ vanishes for $w = 1$. These functions, calculated with model wave functions for Λ_b and Λ_c baryons, are plotted in Figs. 3 and 4. The function $\chi(w)$ is very small in the whole accessible kinematic range, since it is roughly proportional to the ratio of the heavy baryon binding energy to the baryon mass.

Near the zero recoil point of the final baryon $w = 1$ these functions can be approximated by

$$\begin{aligned} \zeta(w) &= 1 - \rho_\zeta^2(w-1) + c_\zeta(w-1)^2 + \cdots, \\ \chi(w) &= \rho_\chi^2(w-1) + c_\chi(w-1)^2 + \cdots, \end{aligned} \tag{37}$$

where $\rho_\zeta^2 = -[d\zeta(w)/dw]_{w=1}$ is the slope and $2c_\zeta = [d^2\zeta(w)/d^2w]_{w=1}$ is the curvature of the Isgur-Wise functions, which are given in Table III. The values of $\zeta(1)$ for transitions between physical Λ_b (Ξ_b) and Λ_c (Ξ_c) baryons

TABLE III. Parameters of the Isgur-Wise functions for the $\Lambda_b \to \Lambda_c e\nu$ and $\Xi_b \to \Xi_c e\nu$ decays.

Decay	$\bar\Lambda$ (GeV)	ρ_ζ^2	c_ζ	ρ_χ^2	c_χ
$\Lambda_b \to \Lambda_c e\nu$	0.764	1.70	2.39	0.053	0.029
$\Xi_b \to \Xi_c e\nu$	0.970	2.27	3.87	0.045	0.036

are slightly different (by ~0.5%) from the heavy quark limit value 1 due to the distinction of the Λ_b (Ξ_b) and Λ_c (Ξ_c) baryon wave functions, calculated for finite values of the heavy quark masses.

Our model predictions for the form factors $F_i(w)$ and $G_i(w)$ ($i = 1, 2\ 3$) for the $\Lambda_b \to \Lambda_c e\nu$ semileptonic decay are plotted in Fig. 5. The corresponding differential decay distributions calculated both with inclusion of first order heavy quark corrections and in the heavy quark limit are plotted in Fig. 6.

The $\Lambda_b \to \Lambda_c$ differential decay rate near zero recoil [23]:

$$\lim_{w\to 1} \frac{1}{\sqrt{w^2-1}} \frac{d\Gamma(\Lambda_b \to \Lambda_c e\nu)}{dw} = \frac{G_F^2|V_{cb}|^2}{4\pi^3} M_{\Lambda_c}^3 (M_{\Lambda_b} - M_{\Lambda_c})^2 |G_1(1)|^2 \tag{38}$$

is governed by the square of the axial current form factor G_1, which near this point has the following expansion

$$G_1(w) = 1 - \hat\rho^2(w-1) + \hat c(w-1)^2 + \cdots, \tag{39}$$

where in our model with the inclusion of the first order heavy quark corrections (34)

$$\hat\rho^{\,2} = 1.51 \quad \text{and} \quad \hat c = 2.03.$$

This value of the slope parameter of the Λ_b-baryon decay form factor is in agreement with the recent experimental value obtained by the DELPHI Collaboration [24]

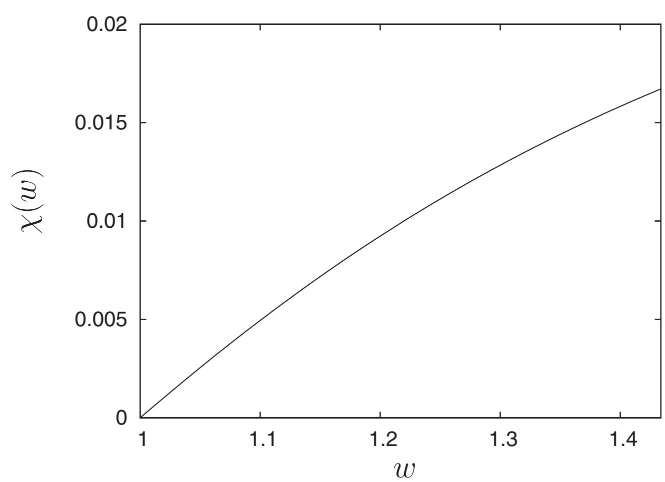

FIG. 4. The subleading function $\chi(w)$ for the $\Lambda_b \to \Lambda_c e\nu$ semileptonic decay.

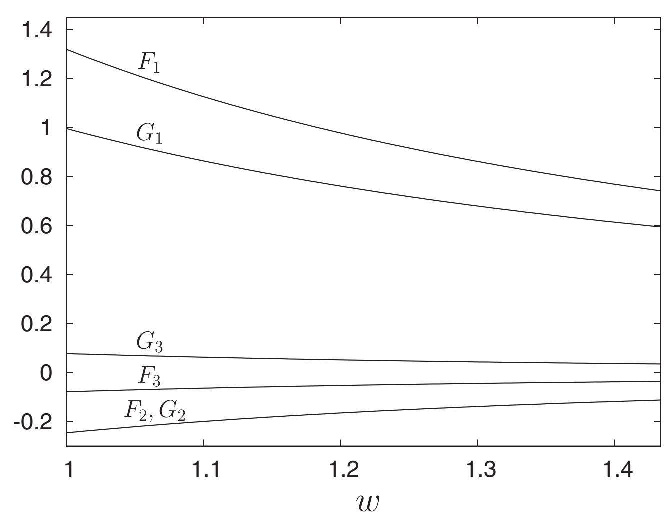

FIG. 5. Semileptonic decay form factors for $\Lambda_b \to \Lambda_c e\nu$.

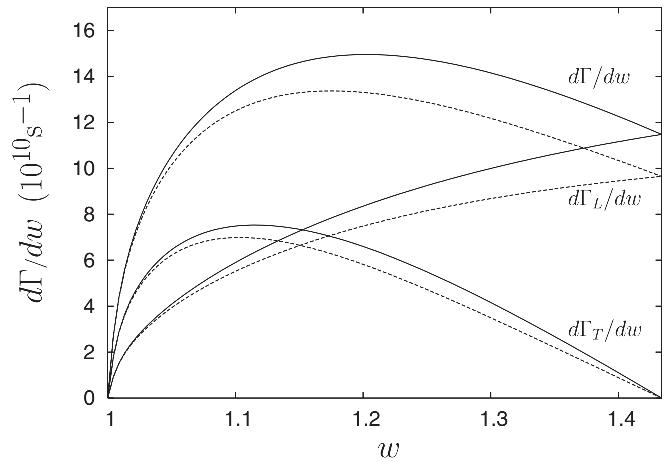

FIG. 6. Differential decay rates $d\Gamma/dw$ for the $\Lambda_b \to \Lambda_c e\nu$ semileptonic decay. Solid lines show decay rates including first order $1/m_Q$ corrections. Dashed lines correspond to decay rates in the heavy quark limit.

$$\hat{\rho}^2 = 2.03 \pm 0.46^{+0.72}_{-1.00}$$

and lattice QCD [25] estimate

$$\hat{\rho}^2 = 1.1 \pm 1.0.$$

Our prediction for the branching ratio of the semileptonic decay $\Lambda_b \to \Lambda_c e\nu$ for $|V_{cb}| = 0.041$ and $\tau_{\Lambda_b} = 1.23 \times 10^{-12}$ s [20]

$$\mathrm{Br}^{\mathrm{theor}}(\Lambda_b \to \Lambda_c l\nu) = 6.9\%$$

is in agreement with available experimental data [24,26]

$$\mathrm{Br}^{\exp}(\Lambda_b \to \Lambda_c l\nu) = \begin{cases} (5.0^{+1.1+1.6}_{-0.8-1.2})\% & \text{DELPHI [24]} \\ (8.1 \pm 1.2^{+1.1}_{-1.6} \pm 4.3)\% & \text{CDF [26]} \end{cases} \tag{40}$$

and the PDG branching ratio [20]

$$\mathrm{Br}^{\exp}(\Lambda_b \to \Lambda_c l\nu + \text{ anything}) = (9.1 \pm 2.1)\%. \tag{41}$$

Predictions of our model for the semileptonic decay rates (26) and averaged asymmetries (27) and (28) for $\Lambda_b \to \Lambda_c e\nu$ and $\Xi_b \to \Xi_c e\nu$ decays, both in the heavy quark limit and with inclusion of first order $1/m_Q$ corrections, are given in Table IV. In decay rate calculations we used for the Ξ_b mass the value from Table II and for other masses their experimental values [20]. Comparing results for the decay rates with and without first order $1/m_Q$ corrections we see that the inclusion of the subleading terms leads to a relatively small $\sim$14% increase of the total decay rates. Therefore, one can expect that higher order corrections should be small, and thus their account cannot substantially change the leading order predictions.

VI. SEMILEPTONIC DECAYS OF HEAVY BARYONS WITH THE AXIAL VECTOR DIQUARK

In the heavy quark limit $m_Q \to \infty$ the decay matrix element (29) is reduced to [12,16]

$$\langle \Omega^{(*)}_{Q'}(v', s')|\bar{h}^{(Q')}_{v'}\Gamma h^{(Q)}_v|\Omega_Q(v, s)\rangle = \bar{B}^{\Omega^{(*)}_{Q'}}_\mu(v', s')\Gamma B^{\Omega_Q}_\nu(v, s)[-g^{\mu\nu}\zeta_1(w) + v^\mu v'^\nu \zeta_2(w)], \tag{42}$$

where

$$B^{\Omega_Q}_\mu(v, s) = \frac{1}{\sqrt{3}}(\gamma_\mu + v_\mu)\gamma_5 u_{\Omega_Q}(v, s), \qquad B^{\Omega^*_Q}_\mu(v, s) = u_{\Omega^*_Q,\mu}(v, s). \tag{43}$$

The structure of the leading order in $1/m_Q$ corrections, which in the HQET can be parameterized in terms of five additional functions, can be found in Ref. [16].

In our model the corresponding semileptonic decay matrix element can be calculated using the same procedure as in the previous section. However such calculation is considerably more cumbersome (especially for the $\Gamma^{(2)}$ contribution), since now the spectator light diquark has spin equal to 1. Taking into account that in the Λ_Q baryon decays contributions of $1/m_Q$ corrections are rather small, we expect that in the case of Ω_Q baryon decays such corrections should be also relatively small. At present no bottom baryons with axial vector diquark have been observed yet, and, when observed, their semileptonic decays

TABLE IV. Theoretical predictions for semileptonic decay rates Γ (in 10^{10} s^{-1}) and averaged asymmetries for $\Lambda_b \to \Lambda_c e\nu$ and $\Xi_b \to \Xi_c e\nu$ for $|V_{cb}| = 0.041$. Branching ratios (Br) (in %) are calculated using experimental mean values [20] for the life times $\tau_{\Lambda_b} = 1.23 \times 10^{-12}$ s and $\tau_{\Xi_b} = 1.39 \times 10^{-12}$ s.

Decay	Γ	Br	Γ_L	Γ_T	R	$\langle a_T\rangle$	$\langle a_L\rangle$	$\langle P_L\rangle$	$\langle\alpha'\rangle$	$\langle\alpha''\rangle$	$\langle\gamma\rangle$
in $m_Q \to \infty$ limit											
$\Lambda_b \to \Lambda_c e\nu$	5.02	6.2	3.08	1.94	1.59	−0.483	−0.928	−0.756	−0.116	−0.521	0.562
$\Xi_b \to \Xi_c e\nu$	4.64	6.4	2.79	1.85	1.51	−0.455	−0.920	−0.735	−0.113	−0.503	0.587
with $1/m_Q$ corrections											
$\Lambda_b \to \Lambda_c e\nu$	5.64	6.9	3.48	2.16	1.61	−0.600	−0.940	−0.810	−0.142	−0.527	0.494
$\Xi_b \to \Xi_c e\nu$	5.29	7.4	3.21	2.08	1.54	−0.597	−0.935	−0.802	−0.146	−0.510	0.505

will be difficult to measure. Therefore it seems reasonable to limit our analysis here to the leading order of the heavy quark expansion. In the heavy quark limit only the lowest order vertex function $\Gamma^{(1)}$ (18) contributes to the decay matrix element (17). The resulting expressions for the weak decay matrix element exactly satisfy the HQET relation (42) and allow us to determine the Isgur-Wise functions $\zeta_1(w)$ and $\zeta_2(w)$ in the whole accessible kinematic range through the overlap integrals of the baryon wave functions. They are given by

$$\zeta_1(w) = \lim_{m_Q\to\infty} \int \frac{d^3p}{(2\pi)^3} \Psi_{\Omega_{Q'}}\left(\mathbf{p} + 2\epsilon_d(p) \times \sqrt{\frac{w-1}{w+1}}\mathbf{e}_\Delta\right)\Psi_{\Omega_Q}(\mathbf{p}), \tag{44}$$

$$\zeta_2(w) = \frac{1}{w+1}\zeta_1(w), \tag{45}$$

where $\mathbf{e}_\Delta = \mathbf{\Delta}/\sqrt{\mathbf{\Delta}^2}$ is the unit vector in the direction of $\mathbf{\Delta} = M_{\Omega_{Q'}}\mathbf{v}' - M_{\Omega_Q}\mathbf{v}$. The relation (45) follows from the relativistic spin transformation (21) of the spectator axial vector diquark. A similar relation was obtained also in Ref. [3]. The Isgur-Wise functions are plotted in Fig. 7.

Near the zero recoil point $w = 1$ the Isgur-Wise functions can again be approximated by

$$\zeta_i(w) = \zeta_i(1) - \rho^2_{\zeta_i}(w-1) + c_{\zeta_i}(w-1)^2 + \cdots, \tag{46}$$

where $\zeta_1(1) = 1$ and $\zeta_2(1) = 1/2$; $\rho^2_{\zeta_i} = -[d\zeta_i(w)/dw]_{w=1}$ is the slope and $2c_{\zeta_i} = [d^2\zeta_i/d^2w]_{w=1}$ is the curvature of the Isgur-Wise functions, which are given in Table V.

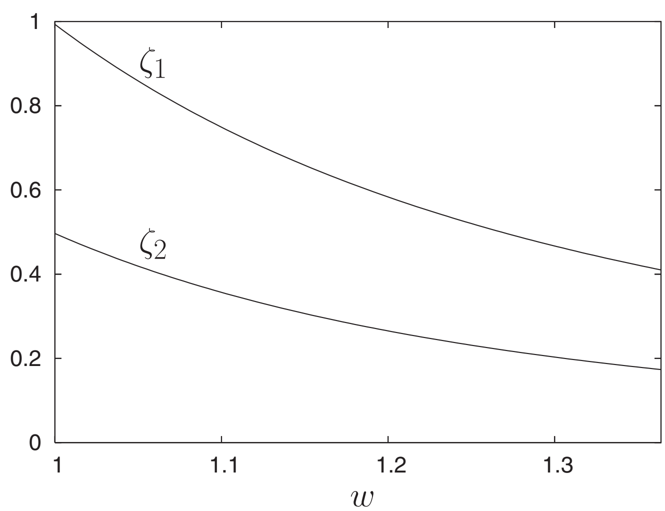

FIG. 7. The Isgur-Wise functions $\zeta_1(w)$ and $\zeta_2(w)$ for the $\Omega_b \to \Omega_c^{(*)} e\nu$ semileptonic decay.

TABLE V. Parameters of the Isgur-Wise functions for the $\Sigma_b \to \Sigma_c^{(*)} e\nu$, $\Xi'_b \to \Xi_c^{\prime(*)} e\nu$ and $\Omega_b \to \Omega_c^{(*)} e\nu$ decays.

Decay	$\bar\Lambda$ (GeV)	$\rho^2_{\zeta_1}$	c_{ζ_1}	$\rho^2_{\zeta_2}$	c_{ζ_2}
$\Sigma_b \to \Sigma_c^{(*)} e\nu$	0.942	2.17	3.62	1.34	2.44
$\Xi'_b \to \Xi_c^{\prime(*)} e\nu$	1.082	2.61	4.93	1.55	3.19
$\Omega_b \to \Omega_c^{(*)} e\nu$	1.208	2.99	6.21	1.74	3.91

The invariant form factors in the heavy quark limit can be expressed, using relation (45), in terms of the Isgur-Wise function $\zeta_1(w)$ as follows

$$F_1(w) = G_1(w) = -\frac{1}{3}\zeta_1(w),$$
$$F_2(w) = \frac{2}{3}\frac{2}{w+1}\zeta_1(w),$$
$$G_2(w) = G_3(w) = 0;$$
$$N_1(w) = -N_3(w) = K_3(w) = -\frac{1}{\sqrt{3}}\frac{2}{w+1}\zeta_1(w),$$

$$N_4(w) = -K_4(w) = -\frac{2}{\sqrt{3}}\zeta_1(w), \quad N_2(w) = K_1(w) = K_2(w) = 0. \tag{47}$$

The differential decay rates $d\Gamma/dw$ for $\Omega_b \to \Omega_c^{(*)} e\nu$ semileptonic decays are plotted in Figs. 8 and 9. The decay rates of bottom baryons with the axial vector diquark, calculated in the heavy quark limit using expressions (31), are given in Table VI. For masses of bottom baryons and the Ω_c^* we used the values from Table II and for other charmed baryons we used experimental mass values [20].

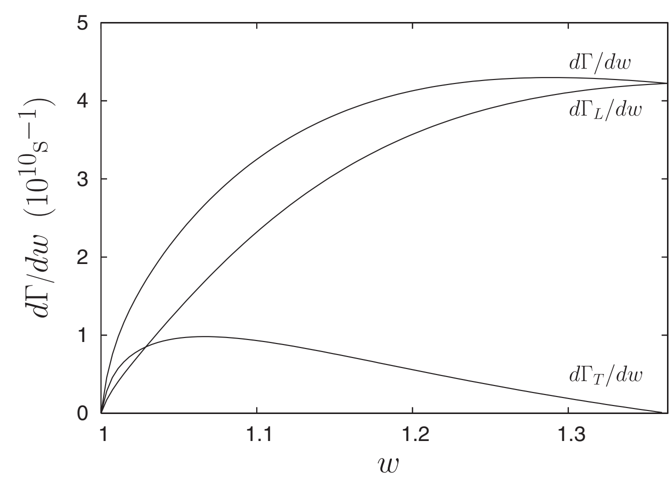

FIG. 8. Differential decay rates $d\Gamma/dw$ for the $\Omega_b \to \Omega_c e\nu$ semileptonic decay.

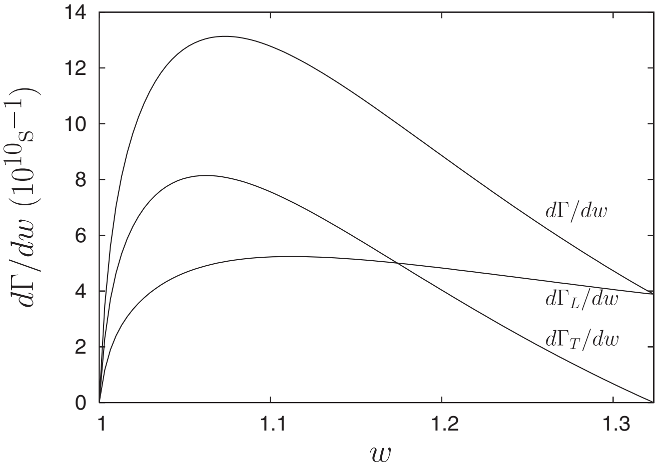

FIG. 9. Differential decay rates $d\Gamma/dw$ for the $\Omega_b \to \Omega_c^* e\nu$ semileptonic decay.

TABLE VI. Theoretical predictions for semileptonic decay rates Γ (in 10^{10} s^{-1}) of bottom baryons with the axial vector diquark in the heavy quark limit for $|V_{cb}| = 0.041$.

Decay	Γ	Γ_L	Γ_T	R
$\Sigma_b \to \Sigma_c e\nu$	1.44	1.23	0.21	5.87
$\Xi_b' \to \Xi_c' e\nu$	1.34	1.14	0.20	5.59
$\Omega_b \to \Omega_c e\nu$	1.29	1.09	0.20	5.31
$\Sigma_b \to \Sigma_c^* e\nu$	3.23	1.61	1.62	0.99
$\Xi_b' \to \Xi_c^* e\nu$	3.09	1.52	1.57	0.97
$\Omega_b \to \Omega_c^* e\nu$	3.03	1.48	1.55	0.95

VII. DISCUSSION AND CONCLUSIONS

The comparison of our model predictions with other theoretical calculations [3,27–33] is given in Table VII. In nonrelativistic quark models [3,27,28] form factors of the heavy baryon decays are evaluated at the single kinematic point of zero recoil and then different form factor parameterizations (pole, dipole) are used for decay rate calculations. The relativistic three-quark model [29], Bethe-Salpeter model [30] and light-front constituent quark model [31] assume Gaussian wave functions for heavy baryons. The authors of the recent nonrelativistic quark model [32] use for the form factor evaluations the set of variational wave functions, obtained from baryon spectra calculations without employing the quark-diquark approximation. Finally, Ref. [33] presents the recent QCD sum rule prediction. Calculations of Refs. [3,29,30] are done in the heavy quark limit only, while the rest include first order $1/m_Q$ corrections for the decays of Λ-type baryons. From Table VII we see that all theoretical models give close predictions for the semileptonic decays of heavy baryons with scalar diquark ($\Lambda_b \to \Lambda_c e\nu$ and $\Xi_b \to \Xi_c e\nu$), which are consistent with the available experimental data (40) and (41) for the $\Lambda_b \to \Lambda_c e\nu$ semileptonic decay. The results for averaged asymmetries of these decays (see Table IV) are also close in most of the considered approaches. Thus one can conclude that the precise measurement of the semileptonic $\Lambda_b \to \Lambda_c e\nu$ decay rate will allow an accurate determination of the CKM matrix element V_{cb} with small theoretical uncertainties.

All predictions for heavy baryon decays with the axial vector diquark listed in Table VII were obtained in the heavy quark limit. Here the differences between predictions are larger. The nonrelativistic quark model [27] gives for these decay rates values more than 2 times larger than other estimates. Our model values for these decay rates are the lowest ones. Among the relativistic quark models the closest to our predictions is given in [30]. Unfortunately, it will be difficult to measure such decays experimentally. Only Ω_b (which has not been observed yet) will decay predominantly weakly and thus has sizable semileptonic branching fractions, since a scalar ss diquark is forbidden by the Pauli principle. All other baryons with the axial vector diquark will decay predominantly strongly or electromagnetically and thus their weak branching ratios will be very small.

In summary, in this paper we calculated the semileptonic decay rates of heavy baryons in the framework of the relativistic quark model. Heavy baryons were considered in the heavy-quark-light-diquark approximation. The baryon wave functions were obtained previously in the process of the heavy baryon mass spectrum calculations. In our approach the spectator diquark is not treated as a

TABLE VII. Comparison of different theoretical predictions for semileptonic decay rates Γ (in 10^{10} s^{-1}) of bottom baryons.

Decay	this work	[27]	[28]	[3]	[29]	[30]	[31]	[32]	[33]
$\Lambda_b \to \Lambda_c e\nu$	5.64	5.9	5.1	5.14	5.39	6.09	5.08 ± 1.3	5.82	5.4 ± 0.4
$\Xi_b \to \Xi_c e\nu$	5.29	7.2	5.3	5.21	5.27	6.42	5.68 ± 1.5	4.98	
$\Sigma_b \to \Sigma_c e\nu$	1.44	4.3			2.23	1.65			
$\Xi_b' \to \Xi_c' e\nu$	1.34								
$\Omega_b \to \Omega_c e\nu$	1.29	5.4	2.3	1.52	1.87	1.81			
$\Sigma_b \to \Sigma_c^* e\nu$	3.23				4.56	3.75			
$\Xi_b' \to \Xi_c^* e\nu$	3.09								
$\Omega_b \to \Omega_c^* e\nu$	3.03			3.41	4.01	4.13			

pointlike object. The relatively large diquark size is taken into account by calculating the diquark-gluon form factor as the overlap integral of the diquark wave functions. The matrix element of the weak current between baryon states was considered using the quasipotential approach. The relativistic transformation of the baryon wave functions from the rest reference frame to the moving one as well as the negative-energy contributions to the decay matrix elements were explicitly taken into account. To simplify calculations and in order to compare with model-independent predictions of HQET the heavy quark expansion was applied up to subleading order for heavy baryon decays with a scalar light diquark. It was shown that all HQET relations in the leading and subleading order are exactly satisfied in our model if the long-range chromomagnetic interaction vanishes ($\kappa = -1$) in accord with our previous analysis of heavy meson decays. The leading and subleading Isgur-Wise functions were determined in terms of the overlap integrals of baryon wave functions. It was found by explicit calculation that the additional subleading function $\chi(w)$, arising from the kinetic energy term in the HQET Lagrangian, is negligibly small in the whole kinematic range. Decay rates as well as different averaged asymmetries both with and without $1/m_Q$ corrections were calculated. Moreover, it was shown that the subleading terms in the heavy quark expansion modify the results for decay rates by $\sim$14%. Thus one can expect that the influence of higher order corrections should be small.

The decays of heavy baryons with the axial vector diquark were considered in the heavy quark limit. All HQET relations are exactly satisfied in our model. The corresponding Isgur-Wise functions were determined in terms of the overlap integrals of the baryon wave functions. It was found that the relativistic transformation of the axial vector diquark spin leads to the relation (45) between baryon Isgur-Wise functions $\zeta_1(w)$ and $\zeta_2(w)$.

The calculated decay rates of heavy baryons were compared with the results of other theoretical approaches and available experimental data. One of the main advantages of our model is that it allows one to calculate consistently the heavy baryon wave functions from the consideration of the spectroscopy and then determine through the wave function overlap integrals the baryonic Isgur-Wise functions in the whole kinematic range accessible in semileptonic decays. Thus we do not need to make any assumptions about the form of the baryon wave functions or/and extrapolate the form factors from one point to the whole kinematic region using some *ad hoc* ansatz. No additional free parameters were introduced in our calculations. As it was pointed out above, we also consistently include relativistic effects. All this makes the presented results sufficiently accurate and reliable.

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