

**Virtual meson cloud of the nucleon and generalized parton distributions**

B. Pasquini and S. Boffi

*Dipartimento di Fisica Nucleare e Teorica, Università degli Studi di Pavia and INFN, Sezione di Pavia, Pavia, Italy*

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We present the general formalism required to derive generalized parton distributions within a convolution model where the bare nucleon is dressed by its virtual meson cloud. In the one-meson approximation the Fock states of the physical nucleon are expanded in a series involving a bare nucleon and two-particle, meson-baryon, states. The baryon is assumed here to be either a nucleon or a  $\Delta$  described within the constituent-quark model in terms of three-valence quarks; correspondingly, the meson, assumed to be a pion, is described as a quark-antiquark pair. Explicit expressions for the unpolarized generalized parton distributions are obtained and evaluated in different kinematics.

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**I. INTRODUCTION**

The fundamental role of a nonperturbative pion cloud surrounding the nucleon is well explained in quantum chromodynamics (QCD) as a consequence of the spontaneously-broken chiral symmetry. The pion cloud associated with chiral-symmetry breaking was first discussed in the context of deep-inelastic scattering (DIS) by Feynman [1] and Sullivan [2]. As realized by Thomas [3], it can give an explanation of the flavor-symmetry violation in the sea-quark distributions of the nucleon thus naturally accounting for the excess of  $\bar{d}$  (anti)quarks over  $\bar{u}$  (anti)quarks as observed through the violation of the Gottfried sum rule [4–7]. Although the nucleon's nonperturbative antiquark sea cannot be ascribed entirely to its virtual meson cloud [8], the role of mesons in understanding these data has been extensively discussed in connection with parton distributions (for reviews, see Refs. [9–11]).

A similar important role is expected to be played by the meson cloud in the case of generalized parton distributions (GPDs) that have recently been introduced and discussed in connection with deeply virtual Compton scattering (DVCS) and hard exclusive meson production (for reviews, see Refs. [12–15]). Because of their intrinsic nonperturbative nature GPDs cannot be calculated directly within QCD. Up to now, only first results of their Mellin moments have been obtained by lattice calculations (see, e.g., Ref. [16]). Therefore, in order to guide the planning of possible experiments one has still to rely on models.

The first calculation performed in the MIT bag model [17] did not consider the pion cloud explicitly. Further calculations have been performed in the chiral quark-soliton model [18–21]. The model is based on an effective relativistic quantum field theory where the instanton fluctuations of the gluon field are simulated by a pion field binding the constituent quarks inside the nucleon. The model is theoretically justified in the limit of the large number of colors  $N_c$ , and in the leading order in the  $1/N_c$  expansion it is not possible to obtain results for separate flavors, only special flavor combinations being

nonzero. GPDs have also been calculated within the Nambu-Jona-Lasinio model [22]. A first attempt to explicitly include the meson cloud in a model calculation of GPDs has been discussed in Ref. [23] in terms of double distributions [24].

A complete and exact overlap representation of GPDs has recently been worked out within the framework of light-cone quantization [25,26]. A Fock-state decomposition of the hadronic state is performed in terms of  $N$ -parton Fock states with coefficients representing the momentum light-cone wave function (LCWF) of the  $N$  partons. The same approach has been followed in Refs. [27–29], where GPDs both in the chiral-even and chiral-odd sector were derived assuming that at the low-energy scale valence quarks can be interpreted as the constituent quarks treated in constituent-quark models (CQMs). This assumption is based on the idea that there exists a scale  $Q_0^2$  where the short range (perturbative) part of the interaction is negligible and, therefore, the glue and sea are suppressed, while a long range (confining) part of the interaction produces a proton composed by (three) valence quarks, mainly [30]. Jaffe and Ross [31] proposed to ascribe the quark model calculations of matrix elements to that hadronic scale  $Q_0^2$ . In this way, quark models summarizing a great deal of hadronic properties may substitute for low-energy parametrizations, while evolution to larger momentum  $Q^2$  is dictated by perturbative QCD.

In this paper, we will study the possibility of integrating meson-cloud effects into the valence-quark contribution to GPDs. Along the lines originally proposed in Refs. [32,33], a meson-baryon Fock-state expansion is used to construct the state  $|\tilde{N}\rangle$  of the physical nucleon. In the one-meson approximation the state  $|\tilde{N}\rangle$  is pictured as being part of the time a bare nucleon,  $|N\rangle$ , and part of the time a baryon-meson system,  $|BM\rangle$ . In this framework, it will be shown how to apply the convolution approach used for the standard parton distributions in deep inelastic scattering [34] to the case of GPDs. The main idea of the convolution approach is that there are no interactions among the particles in a multiparticle Fock state during the interaction with the

hard photon. Therefore the external probe can scatter either on the bare nucleon,  $|N\rangle$ , or on one of the constituents of the higher Fock states,  $|BM\rangle$ . In the so-called DGLAP region, with one (anti)quark emitted and reabsorbed by the physical nucleon, the GPDs are obtained by folding the quark GPDs within bare constituents (nucleons, pions, deltas, etc.) with the probability amplitudes describing the distribution of these constituents in the dressed initial and final nucleon. In contrast, in the so-called ERBL region where a  $q\bar{q}$  pair is emitted from the initial nucleon, the GPDs are obtained from the overlap between wave functions of Fock states with different parton number, which corresponds, in the meson-cloud model, to the contribution of the interference terms between the  $|BM\rangle$  component in the initial state and the bare nucleon in the final state.

The model is revisited in Sec. II where all the necessary ingredients that one can find scattered in the literature are consistently rederived to construct the LCWF in the meson-cloud model. The definition of the unpolarized GPDs are introduced in Sec. III, and the convolution formulas for the GPDs in the meson-cloud model are explicitly derived in Sec. IV. In Sec. V we describe the model calculation considering the case of a pure pion cloud, and assuming a constituent-quark model to construct the LCWFs of the bare nucleon and the constituents of the baryon-meson Fock state. Numerical results are discussed in Sec. VI and some conclusions are drawn in the final section. Derivation of auxiliary quantities is detailed in three appendices.

## II. THE MESON-CLOUD MODEL FOR THE NUCLEON

The basic assumption throughout this paper is that the physical nucleon  $\tilde{N}$  is made of a bare nucleon  $N$  dressed by the surrounding meson cloud so that the state of the physical nucleon is decomposed according to the meson-baryon Fock-state expansion as a superposition of a bare-nucleon state and states containing virtual mesons associated with recoiling baryons. This state, with four-momentum  $p_N^\mu = (p_N^-, p_N^+, \mathbf{p}_{N\perp}) \equiv (p_N^-, \tilde{p}_N)$  and helicity  $\lambda$ , is an eigenstate of the light-cone Hamiltonian

$$H_{\text{LC}} = \sum_{B,M} [H_0^B(q) + H_0^M(q) + H_I(N, BM)], \quad (1)$$

i.e.

$$H_{\text{LC}}|\tilde{p}_N, \lambda; \tilde{N}\rangle = \frac{\mathbf{p}_{N\perp}^2 + M_N^2}{p_N^+}|\tilde{p}_N, \lambda; \tilde{N}\rangle. \quad (2)$$

Here  $H_0^B(q)$  stands for the effective-QCD Hamiltonian which governs the constituent-quark dynamics, and leads to the confinement of three quarks in a baryon state; analogously,  $H_0^M(q)$  describes the quark interaction in a meson state. Thus we assume that the three- and two-quark states with the quantum numbers of a baryon and a meson are the eigenstates of  $H_0^B(q)$  and  $H_0^M(q)$ , e.g.

$$H_0^B|\tilde{p}_B, \lambda; B\rangle = \frac{\mathbf{p}_{B\perp}^2 + M_B^2}{p_B^+}|\tilde{p}_B, \lambda; B\rangle, \quad (3)$$

$$H_0^M|\tilde{p}_M, \lambda; M\rangle = \frac{\mathbf{p}_{M\perp}^2 + M_M^2}{p_M^+}|\tilde{p}_M, \lambda; M\rangle. \quad (4)$$

In Eq. (1),  $H_I(N, BM)$  is the nucleon-baryon-meson interaction, and the sum is over all the possible baryon and meson configurations in which the nucleon can virtually fluctuate. Using perturbation theory, we can expand the nucleon wave function in terms of the eigenstates of the bare Hamiltonian  $H_0 \equiv H_0^B(q) + H_0^M(q)$ , i.e.

$$\begin{aligned} |\tilde{p}_N, \lambda; \tilde{N}\rangle = \sqrt{Z} & \left( |\tilde{p}_N, \lambda; N\rangle + \sum_{n_1}' \frac{|n_1\rangle\langle n_1|H_I|\tilde{p}_N, \lambda; N\rangle}{E_N - E_{n_1} + i\epsilon} \right. \\ & + \sum_{n_1, n_2}' \frac{|n_2\rangle\langle n_2|H_I|n_1\rangle\langle n_1|H_I|\tilde{p}_N, \lambda; N\rangle}{(E_N - E_{n_2} + i\epsilon)(E_N - E_{n_1} + i\epsilon)} \\ & \left. + \dots \right), \quad (5) \end{aligned}$$

where  $\sum'$  indicates the summation over  $BM$  intermediate states, and  $Z$  is the wave function renormalization constant. In the one-meson approximation, we truncate the series expansion of Eq. (5) to the first order in  $H_I$ , and as a result we obtain

$$\begin{aligned} |\tilde{p}_N, \lambda; \tilde{N}\rangle &= \sqrt{Z}|\tilde{p}_N, \lambda; N\rangle + \sum_{B,M} |\tilde{p}_N, \lambda; N(BM)\rangle \\ &= \sqrt{Z}|\tilde{p}_N, \lambda; N\rangle + \sum_{B,M} \int \frac{d^2p_B^+ d^2\mathbf{p}_{B\perp}}{2(2\pi)^3 p_B^+} \\ &\quad \times \int \frac{d^2p_M^+ d^2\mathbf{p}_{M\perp}}{2(2\pi)^3 p_M^+} \sum_{\lambda', \lambda''} \frac{\langle B(\tilde{p}_B, \lambda')M(\tilde{p}_M, \lambda'')|H_I|N(\tilde{p}_N, \lambda)\rangle}{E_N - E_B - E_M} |\tilde{p}_B, \lambda'; B\rangle |\tilde{p}_M, \lambda''; M\rangle. \quad (6) \end{aligned}$$

In Eq. (6), the normalization factor  $\sqrt{Z}$  only affects the bare core  $|N\rangle$ , and not the meson-baryon component. As discussed in details in Refs. [8,35], this prescription is consistent with assuming that the nucleon-baryon-meson coupling constant in  $H_I$  is taken equal to the renormalized value  $g_{NBM}$ , related to lowest order to the bare coupling  $g_{NBM}^0$  via  $g_{NBM} = \sqrt{Z}g_{NBM}^0$ .

Finally, the hadron states in Eq. (6) are normalized as

$$\begin{aligned} \langle p'^+, \mathbf{p}'_\perp, \lambda'; H | p^+, \mathbf{p}_\perp, \lambda; H \rangle \\ = 2(2\pi)^3 p^+ \delta(p'^+ - p^+) \delta^{(2)}(\mathbf{p}'_\perp - \mathbf{p}_\perp) \delta_{\lambda\lambda'}. \end{aligned} \quad (7)$$

### A. The nucleon wave function

In this Section starting from Eq. (6) we derive the explicit general expression of the nucleon wave function on the basis of bare-nucleon and baryon-meson Fock states.

We first evaluate the energy denominator in Eq. (6) using the following expression for the energy of the particles in terms of light-front variables

$$E = \frac{1}{2} \left( p^+ + \frac{\mathbf{p}_\perp^2 + M^2}{p^+} \right). \quad (8)$$

If we write the momenta of the baryon,  $\tilde{p}_B$ , and the meson,  $\tilde{p}_M$ , in terms of the intrinsic (nucleon rest-frame) variables, i.e.

$$\begin{aligned} p_B^+ &= y p_N^+, & p_M^+ &= (1-y) p_N^+, \\ \mathbf{p}_{B\perp} &= \mathbf{k}_\perp + y \mathbf{p}_{N\perp}, & \mathbf{p}_{M\perp} &= -\mathbf{k}_\perp + (1-y) \mathbf{p}_{N\perp}, \end{aligned} \quad (9)$$

$$\begin{aligned} |\tilde{p}_N, \lambda; \tilde{N}\rangle &= \sqrt{Z} |\tilde{p}_N, \lambda; N\rangle + \sum_{B,M} \int \frac{dy d^2 \mathbf{k}_\perp}{2(2\pi)^3} \frac{1}{\sqrt{y(1-y)}} \\ &\times \sum_{\lambda', \lambda''} \phi_{\lambda' \lambda''}^{\lambda(N, BM)}(y, \mathbf{k}_\perp) |y p_N^+, \mathbf{k}_\perp + y \mathbf{p}_{N\perp}, \lambda'; B\rangle | (1-y) p_N^+, -\mathbf{k}_\perp + (1-y) \mathbf{p}_{N\perp}, \lambda''; M\rangle, \end{aligned} \quad (14)$$

where we introduced the function  $\phi_{\lambda' \lambda''}^{\lambda(N, BM)}(y, \mathbf{k}_\perp)$  to define the probability amplitude for a nucleon with helicity  $\lambda$  to fluctuate into a virtual  $BM$  system with the baryon having helicity  $\lambda'$ , longitudinal momentum fraction  $y$  and transverse momentum  $\mathbf{k}_\perp$ , and the meson having helicity  $\lambda''$ , longitudinal momentum fraction  $1-y$  and transverse momentum  $-\mathbf{k}_\perp$ , i.e.

$$\phi_{\lambda' \lambda''}^{\lambda(N, BM)}(y, \mathbf{k}_\perp) = \frac{1}{\sqrt{y(1-y)}} \frac{V_{\lambda', \lambda''}^\lambda(N, BM)}{M_N^2 - M_{BM}^2(y, \mathbf{k}_\perp)}. \quad (15)$$

We note that Eq. (14) is equivalent to the expression of the nucleon wave function obtained in the framework of ‘‘old-fashioned’’ time-ordered perturbation theory in the infinite momentum frame (see Ref. [32]).

we have

$$\begin{aligned} (E_N - E_B - E_M) &= \frac{1}{2p_N^+} \left( M_N^2 - \frac{M_B^2 + \mathbf{k}_\perp^2}{y} - \frac{M_M^2 + \mathbf{k}_\perp^2}{1-y} \right) \\ &\equiv \frac{1}{2p_N^+} (M_N^2 - M_{BM}^2(y, \mathbf{k}_\perp)), \end{aligned} \quad (10)$$

where

$$M_{BM}^2(y, \mathbf{k}_\perp) \equiv \frac{M_B^2 + \mathbf{k}_\perp^2}{y} + \frac{M_M^2 + \mathbf{k}_\perp^2}{1-y}, \quad (11)$$

is the invariant mass of the baryon-meson fluctuation.

Furthermore, the transition amplitude  $\langle B(\tilde{p}_B, \lambda') M(\tilde{p}_M, \lambda'') | H_I | N(\tilde{p}_N, \lambda) \rangle$  in Eq. (6) can be rewritten as

$$\begin{aligned} \langle B(\tilde{p}_B, \lambda') M(\tilde{p}_M, \lambda'') | H_I | N(\tilde{p}_N, \lambda) \rangle \\ = (2\pi)^3 \delta(p_B^+ + p_M^+ - p_N^+) \delta^{(2)}(\mathbf{p}_{B\perp} + \mathbf{p}_{M\perp} - \mathbf{p}_{N\perp}) \\ \times V_{\lambda', \lambda''}^\lambda(N, BM), \end{aligned} \quad (12)$$

where the vertex function  $V_{\lambda', \lambda''}^\lambda(N, BM)$  has the following general expression

$$V_{\lambda', \lambda''}^\lambda(N, BM) = \bar{u}_{N\alpha}(\tilde{p}_N, \lambda) v^{\alpha\beta\gamma} \chi_\beta(\tilde{p}_M, \lambda'') \psi_\gamma(\tilde{p}_B, \lambda'). \quad (13)$$

Here  $u_N$  is the nucleon spinor,  $\chi$  and  $\psi$  are the field operators of the intermediate meson and baryon, respectively, and  $\alpha, \beta, \gamma$  are bi-spinor and/or vector indices depending on the representation used for particles of given type.

Using the results of Eqs. (10) and (12), we find

By imposing the normalization of the nucleon state as in Eq. (7), from Eq. (14) we obtain the following condition on the normalization factor  $Z$

$$1 = Z + P_{BM/N}, \quad (16)$$

with

$$\begin{aligned} P_{BM/N} &= \sum_{B,M} \int \frac{dy d^2 \mathbf{k}_\perp}{2(2\pi)^3} \\ &\times \frac{1}{y(1-y)} \sum_{\lambda', \lambda''} \frac{|V_{\lambda', \lambda''}^{1/2}(N, BM)|^2}{[M_N^2 - M_{BM}^2(y, \mathbf{k}_\perp)]^2}. \end{aligned} \quad (17)$$

Here  $P_{BM/N}$  is the probability of fluctuation of the nucleon in a baryon-meson state, and, accordingly,  $Z$  gives the probability to find the bare nucleon in the physical nucleon.

### B. Partonic content of the nucleon wave function

In this Section we derive the expression of the nucleon wave function (14) in terms of the constituent partons of the nucleon core and of the meson-baryon components.

The light-front state of the bare nucleon is given by

$$|\tilde{p}_N, \lambda; N\rangle = \sum_{\tau_i, \lambda_i} \int \left[ \frac{dx}{\sqrt{x}} \right]_3 [d^2\mathbf{k}_\perp]_3 \times \Psi_\lambda^{N,[f]}(\{x_i, \mathbf{k}_{i\perp}; \lambda_i, \tau_i\}_{i=1,2,3}) \times \prod_{i=1}^3 |x_i p_N^+, \mathbf{p}_{i\perp}, \lambda_i, \tau_i; q\rangle, \quad (18)$$

where  $\Psi_\lambda^{N,[f]}(\{x_i, \mathbf{k}_{i\perp}; \lambda_i, \tau_i\}_{i=1,2,3})$  is the momentum light-cone wave function which gives the probability amplitude for finding in the nucleon three quarks with momenta  $(x_i p_N^+, \mathbf{p}_{i\perp} = \mathbf{k}_{i\perp} + x_i \mathbf{p}_{N\perp})$ , and spin and isospin variables  $\lambda_i$  and  $\tau_i$ , respectively. In Eq. (18) and in the following formulas, the integration measures are defined by

$$\left[ \frac{dx}{\sqrt{x}} \right]_N = \left( \prod_{i=1}^N \frac{dx_i}{\sqrt{x_i}} \right) \delta\left(1 - \sum_{i=1}^N x_i\right), \quad (19)$$

$$[d^2\mathbf{k}_\perp]_N = \left( \prod_{i=1}^N \frac{d^2\mathbf{k}_{i\perp}}{2(2\pi)^3} \right) 2(2\pi)^3 \delta\left(\sum_{i=1}^N \mathbf{k}_{i\perp}\right). \quad (20)$$

Next we consider the component of the meson-baryon Fock state in Eq. (14).

The light-front state of the baryon is given by

$$|\tilde{p}_B, \lambda'; B\rangle = \sum_{\tau_i, \lambda_i} \int \left[ \frac{dx}{\sqrt{x}} \right]_3 [d^2\mathbf{k}_\perp]_3 \times \Psi_{\lambda'}^{B,[f]}(\{x_i, \mathbf{k}_{i\perp}; \lambda_i, \tau_i\}_{i=1,2,3}) \times \prod_{i=1}^3 |x_i p_B^+, \mathbf{p}_{i\perp}, \lambda_i, \tau_i; q\rangle, \quad (21)$$

where now the intrinsic variables of the quarks  $x_i$  and  $k_i^+$  refer to the baryon rest frame, i.e.  $x_i = p_i^+ / p_B^+$  and  $\mathbf{p}_{i\perp} = \mathbf{k}_{i\perp} + x_i \mathbf{p}_{B\perp}$  ( $i = 1, 2, 3$ ).

An analogous expression holds for the light-front state of the meson, i.e.

$$|\tilde{p}_M, \lambda''; M\rangle = \sum_{\tau_i, \lambda_i} \int \frac{dx_4 dx_5}{\sqrt{x_4 x_5}} \frac{d\mathbf{k}_{4\perp} d\mathbf{k}_{5\perp}}{16\pi^3} \delta(1 - x_4 - x_5) \times \delta^{(2)}(\mathbf{k}_{4\perp} + \mathbf{k}_{5\perp}) \times \Psi_{\lambda''}^{M,[f]}(\{x_i, \mathbf{k}_{i\perp}; \lambda_i, \tau_i\}_{i=4,5}) \times \prod_{i=4}^5 |x_i p_M^+, \mathbf{p}_{i\perp}, \lambda_i, \tau_i; q\rangle, \quad (22)$$

with  $x_i = p_i^+ / p_M^+$ ,  $\mathbf{p}_{i\perp} = x_i \mathbf{p}_{M\perp} + \mathbf{k}_{i\perp}$  ( $i = 4, 5$ ).

When we insert the expressions of the baryon and meson states in Eq. (14), it is convenient to rewrite the kinematical variables of the partons as follows.

For  $i = 1, 2, 3$ :

$$x_i = \frac{p_i^+}{p_B^+} = \frac{p_i^+}{p_N^+} \frac{p_N^+}{p_B^+} = \frac{\xi_i}{y},$$

$$\mathbf{p}_{i\perp} = x_i \mathbf{p}_{B\perp} + \mathbf{k}_{i\perp} = x_i (\mathbf{k}_\perp + y \mathbf{p}_{N\perp}) + \mathbf{k}_{i\perp} = \xi_i \mathbf{p}_{N\perp} + \mathbf{k}_{i\perp} + x_i \mathbf{k}_\perp \equiv \xi_i \mathbf{p}_{N\perp} + \mathbf{k}'_{i\perp},$$

where  $\xi_i = p_i^+ / p_N^+$  is the fraction of the longitudinal momentum of the nucleon carried by the quarks in the baryon, and  $\mathbf{k}'_{i\perp}$  is the intrinsic transverse momentum of the quarks with respect to the nucleon rest frame.

For  $i = 4, 5$ :

$$x_i = \frac{p_i^+}{p_M^+} = \frac{p_i^+}{p_N^+} \frac{p_N^+}{p_M^+} = \frac{\xi_i}{1-y},$$

$$\mathbf{p}_{i\perp} = x_i \mathbf{p}_{M\perp} + \mathbf{k}_{i\perp} = x_i (-\mathbf{k}_\perp + (1-y) \mathbf{p}_{N\perp}) + \mathbf{k}_{i\perp} = \xi_i \mathbf{p}_{N\perp} + \mathbf{k}_{i\perp} - x_i \mathbf{k}_\perp \equiv \xi_i \mathbf{p}_{N\perp} + \mathbf{k}'_{i\perp},$$

with  $\xi_i = p_i^+ / p_N^+$  and  $\mathbf{k}'_{i\perp}$  the intrinsic variables of the quarks in the meson with respect to the nucleon rest frame. Accordingly we transform the variables of integration as follows.

For  $i = 1, 2, 3$ :

$$x_i \rightarrow \xi_i = y x_i, \quad \mathbf{k}_{i\perp} \rightarrow \mathbf{k}'_{i\perp} = \mathbf{k}_{i\perp} + x_i \mathbf{k}_\perp.$$

For  $i = 4, 5$ :

$$x_i \rightarrow \xi_i = (1-y) x_i, \quad \mathbf{k}_{i\perp} \rightarrow \mathbf{k}'_{i\perp} = \mathbf{k}_{i\perp} - x_i \mathbf{k}_\perp.$$

The meson-baryon component of the nucleon wave function in Eq. (14) can then be written as

$$|\tilde{p}_N, \lambda; N(BM)\rangle = \int dy d^2\mathbf{k}_\perp \int_0^y \prod_{i=1}^3 \frac{d\xi_i}{\sqrt{\xi_i}} \int_0^{1-y} \prod_{i=4}^5 \frac{d\xi_i}{\sqrt{\xi_i}} \int \prod_{i=1}^5 \frac{d\mathbf{k}'_{i\perp}}{[2(2\pi)^3]^4} \delta\left(y - \sum_{i=1}^3 \xi_i\right) \delta^{(2)}\left(\mathbf{k}_\perp - \sum_{i=1}^3 \mathbf{k}'_{i\perp}\right) \delta\left(1 - \sum_{i=1}^5 \xi_i\right) \times \delta^{(2)}\left(\sum_{i=1}^5 \mathbf{k}'_{i\perp}\right) \sum_{\lambda', \lambda''} \sum_{\lambda_i, \tau_i} \frac{V_{\lambda', \lambda''}^\lambda(N, BM)}{M_N^2 - M_{BM}^2(y, \mathbf{k}_\perp)} \tilde{\Psi}_{\lambda'}^{B,[f]}(\{\xi_i, \mathbf{k}'_{i\perp}; \lambda_i, \tau_i\}_{i=1,2,3}) \tilde{\Psi}_{\lambda''}^{M,[f]}(\{\xi_i, \mathbf{k}'_{i\perp}; \lambda_i, \tau_i\}_{i=4,5}) \times \prod_{i=1}^5 |\xi_i p_N^+, \mathbf{k}'_{i\perp} + \xi_i \mathbf{p}_{N\perp}, \lambda_i, \tau_i; q\rangle, \quad (23)$$

where the wave functions  $\tilde{\Psi}_{\lambda'}^{B,[f]}$  and  $\tilde{\Psi}_{\lambda''}^{M,[f]}$  incorporate the Jacobian  $\mathcal{J}$  of the transformation  $x_i \rightarrow \xi_i$ , i.e.

$$\tilde{\Psi}_{\lambda'}^{B,[f]}(\{\xi_i, \mathbf{k}'_{i\perp}; \lambda_i, \tau_i\}_{i=1,2,3}) = \sqrt{\mathcal{J}(\xi_1, \xi_2, \xi_3)} \tilde{\Psi}_{\lambda'}^{B,[f]}(\{x_i, \mathbf{k}_{i\perp}; \lambda_i, \tau_i\}_{i=1,2,3}) = \frac{1}{y^{3/2}} \tilde{\Psi}_{\lambda'}^{B,[f]}(\{x_i, \mathbf{k}_{i\perp}; \lambda_i, \tau_i\}_{i=1,2,3}), \quad (24)$$

$$\tilde{\Psi}_{\lambda''}^{M,[f]}(\{\xi_i, \mathbf{k}'_{i\perp}; \lambda_i, \tau_i\}_{i=4,5}) = \sqrt{\mathcal{J}(\xi_4, \xi_5)} \tilde{\Psi}_{\lambda''}^{M,[f]}(\{x_i, \mathbf{k}_{i\perp}; \lambda_i, \tau_i\}_{i=4,5}) = \frac{1}{(1-y)} \tilde{\Psi}_{\lambda''}^{M,[f]}(\{x_i, \mathbf{k}_{i\perp}; \lambda_i, \tau_i\}_{i=4,5}). \quad (25)$$

Finally, by introducing the following definition

$$\tilde{\Psi}_{\lambda}^{5q,[f]}(y, \mathbf{k}_{\perp}; \{\xi_i, \mathbf{k}'_{i\perp}; \lambda_i, \tau_i\}_{i=1,\dots,5}) \equiv \sum_{\lambda', \lambda''} \frac{V_{\lambda', \lambda''}^{\lambda}(N, BM)}{M_N^2 - M_{BM}^2(y, \mathbf{k}_{\perp})} \tilde{\Psi}_{\lambda'}^{B,[f]}(\{\xi_i, \mathbf{k}'_{i\perp}, \lambda_i, \tau_i\}_{i=1,2,3}) \tilde{\Psi}_{\lambda''}^{M,[f]}(\{\xi_i, \mathbf{k}'_{i\perp}, \lambda_i, \tau_i\}_{i=4,5}), \quad (26)$$

Eq. (23) can be simplified to the following expression:

$$\begin{aligned} |\tilde{p}_N, \lambda; N(BM)\rangle &= \int dy d^2\mathbf{k}_{\perp} \int_0^y \prod_{i=1}^3 \frac{d\xi_i}{\sqrt{\xi_i}} \int_0^{1-y} \prod_{i=4}^5 \frac{d\xi_i}{\sqrt{\xi_i}} \int \prod_{i=1}^5 \frac{d\mathbf{k}'_{i\perp}}{[2(2\pi)^3]^4} \delta\left(y - \sum_{i=1}^3 \xi_i\right) \delta^{(2)}\left(\mathbf{k}_{\perp} - \sum_{i=1}^3 \mathbf{k}'_{i\perp}\right) \delta\left(1 - \sum_{i=1}^5 \xi_i\right) \\ &\times \delta^{(2)}\left(\sum_{i=1}^5 \mathbf{k}'_{i\perp}\right) \sum_{\lambda_i, \tau_i} \tilde{\Psi}_{\lambda}^{5q,[f]}(y, \mathbf{k}_{\perp}; \{\xi_i, \mathbf{k}'_{i\perp}; \lambda_i, \tau_i\}_{i=1,\dots,5}) \prod_{i=1}^5 |\xi_i p_N^+, \mathbf{k}'_{i\perp} + \xi_i \mathbf{p}_{N\perp}, \lambda_i, \tau_i; q\rangle, \end{aligned} \quad (27)$$

where  $\tilde{\Psi}_{\lambda}^{5q,[f]}$  can be interpreted as the probability amplitude for finding in the nucleon a configuration of five partons composed by two clusters of three and two quarks, with total momentum  $(y p_N^+, \mathbf{p}_{B\perp})$  and  $((1-y)p_N^+, \mathbf{p}_{M\perp})$ , respectively.

### III. THE UNPOLARIZED GENERALIZED PARTON DISTRIBUTIONS

In the definition of GPDs it is useful to choose a symmetric frame of reference where the virtual photon momentum  $q^\mu$  and the average nucleon momentum  $\bar{p}_N^\mu = \frac{1}{2}(p_N^\mu + p_N'^\mu)$  are collinear along the  $z$  axis and in opposite directions, i.e.

$$\begin{aligned} p_N &= \left[ \frac{m^2 + \Delta_{\perp}^2/4}{2(1+\xi)\bar{p}_N^+}, (1+\xi)\bar{p}_N^+, -\frac{\Delta_{\perp}}{2} \right] \equiv \left[ \frac{m^2 + \Delta_{\perp}^2/4}{2(1+\xi)\bar{p}_N^+}, \bar{p}_N \right], \\ p_N' &= \left[ \frac{m^2 + \Delta_{\perp}^2/4}{2(1-\xi)\bar{p}_N^+}, (1-\xi)\bar{p}_N^+, +\frac{\Delta_{\perp}}{2} \right] \equiv \left[ \frac{m^2 + \Delta_{\perp}^2/4}{2(1-\xi)\bar{p}_N^+}, \bar{p}_N' \right]. \end{aligned} \quad (28)$$

Furthermore,  $Q^2 = -q^\mu q_\mu$  is the spacelike virtuality that defines the scale of the process,  $t = \Delta^2 = (p_N'^\mu - p_N^\mu)^2$  is the invariant transferred momentum square, and the skewness  $\xi$  describes the longitudinal change of the nucleon momentum,  $2\xi = -\Delta^+/\bar{p}_N^+$ .

According to Ref. [12], for each flavor  $q$  the soft amplitude corresponding to unpolarized GPDs reads

$$F_{\lambda'_N \lambda_N}^q(\bar{x}, \xi, \Delta_{\perp}) = \frac{1}{2\sqrt{1-\xi^2}} \int \frac{dz^-}{2\pi} e^{i\bar{x}\bar{p}_N^+ z^-} \langle p_N', \lambda'_N | \bar{\psi}\left(-\frac{1}{2}z\right) \gamma^+ \psi\left(\frac{1}{2}z\right) | p_N, \lambda_N \rangle \Big|_{z^+ = z_{\perp} = 0}, \quad (29)$$

where  $\bar{x}$  defines the fraction of the quark light-cone momentum ( $\bar{k}^+ = \bar{x}\bar{p}_N^+$ ),  $\lambda_N$  ( $\lambda'_N$ ) is the helicity of the initial (final) nucleon, and the quark-quark correlation function is integrated along the light-cone distance  $z^-$  at equal light-cone time ( $y^+ = 0$ ) and zero transverse separation ( $\mathbf{z}_{\perp} = 0$ ) between the quarks. The leading-twist (twist-two) part of this amplitude can be parametrized as

$$\begin{aligned} F_{\lambda'_N \lambda_N}^q(\bar{x}, \xi, \Delta_{\perp}) &= \frac{1}{2\bar{p}_N^+ \sqrt{1-\xi^2}} \bar{u}(\bar{p}'_N, \lambda'_N) \gamma^+ u(\bar{p}_N, \lambda_N) H^q(\bar{x}, \xi, \Delta_{\perp}) + \frac{1}{2\bar{p}_N^+ \sqrt{1-\xi^2}} \bar{u}(\bar{p}'_N, \lambda'_N) \\ &\times \frac{i\sigma^{+\nu} \Delta_{\nu}}{2M_N} u(\bar{p}_N, \lambda_N) E^q(\bar{x}, \xi, \Delta_{\perp}), \end{aligned} \quad (30)$$

where  $H^q(\bar{x}, \xi, \mathbf{\Delta}_\perp)$  and  $E^q(\bar{x}, \xi, \mathbf{\Delta}_\perp)$  are the chiral-even helicity conserving and helicity flipping GPDs for partons of flavor  $q$ , respectively. Taking different nucleon-helicity combinations, we have

$$\begin{aligned} F_{++}^q(\bar{x}, \xi, \mathbf{\Delta}_\perp) &= F_{--}^q(\bar{x}, \xi, \mathbf{\Delta}_\perp) \\ &= H^q(\bar{x}, \xi, \mathbf{\Delta}_\perp) - \frac{\xi^2}{1 - \xi^2} E^q(\bar{x}, \xi, \mathbf{\Delta}_\perp), \end{aligned} \quad (31)$$

$$\begin{aligned} F_{-+}^q(\bar{x}, \xi, \mathbf{\Delta}_\perp) &= -(F_{+-}^q(\bar{x}, \xi, \mathbf{\Delta}_\perp))^* \\ &= \eta \frac{\sqrt{t_0 - t}}{2M} \frac{1}{\sqrt{1 - \xi^2}} E^q(\bar{x}, \xi, \mathbf{\Delta}_\perp), \end{aligned} \quad (32)$$

where

$$\eta = \frac{\Delta^1 + i\Delta^2}{|\mathbf{\Delta}_\perp|}, \quad (33)$$

and

$$-t_0 = \frac{4\xi^2 M_N^2}{1 - \xi^2} \quad (34)$$

is the minimal value for  $-t$  at given  $\xi$ .

#### IV. THE CONVOLUTION MODEL FOR THE UNPOLARIZED GPDS

Before deriving the convolution formulas for the GPDs in the meson-cloud model, we need to specify our conventions for the kinematical variables. The Fock expansion of the nucleon wave function in Eq. (18) as well as the LCWF for the hadron states in Eqs. (21), (22), and (27) do not depend on the momentum of the hadron, but only on the momentum coordinates of the constituents relative to the hadron momentum. This fact reflects the well known result in light-front dynamics that the center of mass motion can be separated from the relative motion of the constituents. On the other hand, the arguments of the LCWF can most easily be identified in reference frames where the hadron has zero transverse momentum. We call such frames ‘‘hadron frames,’’ and more specifically we introduce the names ‘‘hadron-in’’ and ‘‘hadron-out’’ for the frames where the incoming and outgoing hadron has zero transverse momentum, respectively [25]. In general, we denote the momenta of constituents belonging to the incoming hadrons with unprimed, and the momenta of constituents belonging to the outgoing hadron with primed variables. Furthermore, we label quantities in the hadron-in (hadron-out) frame with an additional tilde (hat). The relations between the momenta of the constituents in the ‘‘average frame’’ defined in Eq. (28) and the variables in the hadron frames are obtained via a transverse boost, i.e. a transformation that leaves the plus component of any four-vector  $z$  unchanged. It reads

$$[z^+, z^-, \mathbf{z}_\perp] \rightarrow \left[ z^+, z^- - \frac{\mathbf{z}_\perp \cdot \mathbf{b}_\perp}{b^+} + \frac{z^+ \mathbf{b}_\perp^2}{2(b^+)^2}, \mathbf{z}_\perp - \frac{z^+}{b^+} \mathbf{b}_\perp \right], \quad (35)$$

where the two parameters  $b^+$  and  $\mathbf{b}_\perp$  are given by  $b^+ = (1 + \xi)\bar{p}_N^+$  and  $\mathbf{b}_\perp = -\mathbf{\Delta}_\perp/2$  for the transformation from the average frame to the hadron-in frame. Likewise, a transverse boost with parameters  $b^+ = (1 - \xi)\bar{p}_N^+$  and  $\mathbf{b}_\perp = +\mathbf{\Delta}_\perp/2$  leads from the average frame to the hadron-out frame.

In the following we will derive the convolution formulas for the GPDs in the three different regions corresponding to  $\xi \leq \bar{x} \leq 1$ ,  $-\xi \leq \bar{x} \leq \xi$ , and  $-1 \leq \bar{x} \leq -\xi$ .

#### A. The region $\xi \leq \bar{x} \leq 1$

In this region the GPDs describe the emission of a quark from the nucleon with momentum fraction  $\bar{x} + \xi$  and its reabsorption with momentum fraction  $\bar{x} - \xi$ . In the meson-cloud model, the virtual photon can hit either the bare nucleon  $N$  or one of the higher Fock states. As a consequence, the DVCS amplitude can be written as the sum of two contributions

$$\begin{aligned} F_{\lambda'_N \lambda_N}^q(\bar{x}, \xi, \mathbf{\Delta}_\perp) &= Z F_{\lambda'_N \lambda_N}^{q, \text{bare}}(\bar{x}, \xi, \mathbf{\Delta}_\perp) \\ &\quad + \delta F_{\lambda'_N \lambda_N}^q(\bar{x}, \xi, \mathbf{\Delta}_\perp), \end{aligned} \quad (36)$$

where  $F^{q, \text{bare}}$  is the contribution from the bare nucleon, described in terms of Fock states with three-valence quarks, and  $\delta F^q$  is the contribution from the  $BM$  Fock components of the nucleon state, corresponding to five-parton configurations. This last term can further be split into two contributions, with the active quark belonging either to the baryon ( $\delta F^{q/BM}$ ) or to the meson ( $\delta F^{q/MB}$ ), i.e.

$$\begin{aligned} \delta F_{\lambda'_N \lambda_N}^q(\bar{x}, \xi, \mathbf{\Delta}_\perp) &= \sum_{B, M} [\delta F_{\lambda'_N \lambda_N}^{q/BM}(\bar{x}, \xi, \mathbf{\Delta}_\perp) \\ &\quad + \delta F_{\lambda'_N \lambda_N}^{q/MB}(\bar{x}, \xi, \mathbf{\Delta}_\perp)]. \end{aligned} \quad (37)$$

The valence-quark contribution corresponding to the diagram of Fig. 1 can be calculated in terms of the light-front overlap representation derived in Ref. [25], and applied here to the case of  $N = 3$  valence quarks. It reads

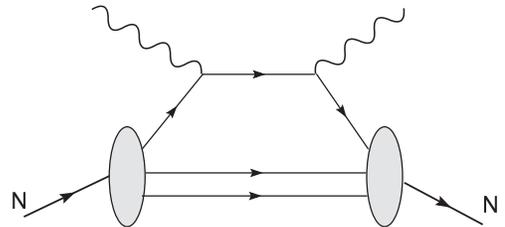


FIG. 1. Deeply virtual Compton scattering from the bare nucleon.

$$\begin{aligned}
 F_{\lambda'_N \lambda_N}^{q, \text{bare}}(\bar{x}, \xi, \Delta_\perp) &= \frac{1}{(1-\xi^2)} \sum_{\lambda_i \tau_i} \sum_{j=1}^3 \delta_{s_j q} \int [d\bar{x}]_3 [d^2\bar{\mathbf{k}}_\perp]_3 \\
 &\times \delta(\bar{x} - \bar{x}_j) \Psi_{\lambda'_N}^{N, [f]*}(\{\hat{x}'_i, \hat{\mathbf{k}}'_i; \lambda_i, \tau_i\}) \\
 &\times \Psi_{\lambda_N}^{N, [f]}(\{\bar{x}_i, \bar{\mathbf{k}}_i; \lambda_i, \tau_i\}), \quad (38)
 \end{aligned}$$

where the LCWF  $\Psi_{\lambda}^{N, [f]}(\{\bar{x}_i, \bar{\mathbf{k}}_{i\perp}; \lambda_i, \tau_i\})$  is the bare-nucleon LCWF of Eq. (18), and  $s_j$  labels the quantum numbers of the  $j$ th active parton. The integration in Eq. (38) is over the average quark transverse momenta  $\bar{\mathbf{k}}_{i\perp}$  and the average quark longitudinal momentum fractions  $\bar{x}_i = \bar{k}_i^+ / \bar{p}_N^+$ . The kinematical variables appearing as arguments in the LCWFs in Eq. (38) have been defined in the ‘‘hadron-in’’ and ‘‘hadron-out’’ frames following the conventions of Ref. [25]. In particular, the momenta of the partons belonging to the incoming hadron are given by

$$\bar{x}_i = \frac{\bar{x}_i}{1+\xi}, \quad \bar{\mathbf{k}}_{i\perp} = \bar{\mathbf{k}}_{i\perp} + \frac{\bar{x}_i}{1+\xi} \frac{\Delta_\perp}{2}, \quad (39)$$

for the spectator quarks ( $i \neq j$ );

$$\bar{x}_j = \frac{\bar{x}_j + \xi}{1+\xi}, \quad \bar{\mathbf{k}}_{j\perp} = \bar{\mathbf{k}}_{j\perp} - \frac{1-\bar{x}_j}{1+\xi} \frac{\Delta_\perp}{2}, \quad (40)$$

for the active quark. Likewise, the LCWF arguments for the outgoing hadron are explicitly given by

$$\hat{x}'_i = \frac{\bar{x}_i}{1-\xi}, \quad \hat{\mathbf{k}}'_{i\perp} = \bar{\mathbf{k}}_{i\perp} - \frac{\bar{x}_i}{1-\xi} \frac{\Delta_\perp}{2}, \quad (41)$$

for the spectator quarks ( $i \neq j$ );

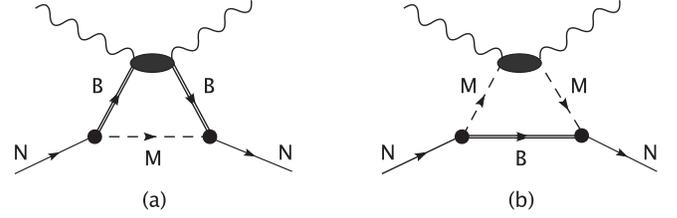


FIG. 2. Deeply virtual Compton scattering from the virtual (a) baryon and (b) meson components of a dressed nucleon.

$$\hat{x}'_j = \frac{\bar{x}_j - \xi}{1-\xi}, \quad \hat{\mathbf{k}}'_{j\perp} = \bar{\mathbf{k}}_{j\perp} + \frac{1-\bar{x}_j}{1-\xi} \frac{\Delta_\perp}{2}, \quad (42)$$

for the active quark.

We now consider the contribution from the  $BM$  component of the nucleon wave function, starting from the  $\delta F_{\lambda'_N \lambda_N}^{q/BM}$  term in Eq. (37) represented in Fig. 2(a). In this case, the baryon is taken out from the initial nucleon with a fraction  $\bar{y}_B + \xi$  of the average plus-momentum  $\bar{p}_N^+$ , and after the interaction with the initial and final photons is reinserted back into the final nucleon with a fraction  $\bar{y}_B - \xi$  of the average plus-momentum  $\bar{p}_N^+$ . The transverse momentum of the baryon is  $\bar{\mathbf{p}}_{B\perp} - \Delta_\perp/2$  before, and  $\bar{\mathbf{p}}_{B\perp} + \Delta_\perp/2$  after the scattering process. The meson substate is a spectator during the whole scattering process, with momentum in the initial and final state equal to  $(\bar{p}_M^+, -\bar{\mathbf{p}}_{B\perp})$ .

The baryon contribution to the  $\delta F_{\lambda'_N \lambda_N}^q$  scattering amplitude can easily be evaluated by calculating the matrix element in Eq. (29) between the  $BM$  components of the initial and final nucleon expressed in the hadron-in and hadron-out frame, respectively. For the initial state, they are given by

$$\begin{aligned}
 &\sum_{B, M} \int \frac{d\tilde{y}_B}{\sqrt{\tilde{y}_B}} \frac{d\tilde{y}_M}{\sqrt{\tilde{y}_M}} \delta(1 - \tilde{y}_B - \tilde{y}_M) \int \frac{d^2\tilde{\mathbf{k}}_{B\perp} d^2\tilde{\mathbf{k}}_{M\perp}}{2(2\pi)^3} \delta^2(\tilde{\mathbf{k}}_{B\perp} + \tilde{\mathbf{k}}_{M\perp}) \\
 &\times \sum_{\lambda', \lambda''} \phi_{\lambda' \lambda''}^{\lambda_N(N, BM)}(\tilde{y}_B, \tilde{\mathbf{k}}_{B\perp}) |(\tilde{y}_B + \xi) \bar{p}_N^+, \bar{\mathbf{p}}_{B\perp} - \frac{\Delta_\perp}{2}, \lambda') | \bar{p}_M^+, -\bar{\mathbf{p}}_{B\perp}, \lambda''). \quad (43)
 \end{aligned}$$

The coordinates of the baryon and meson states in the average frame are related to the variables in the hadron-in frame through the transverse boost in Eq. (35) as explained before. They are explicitly given by

$$\tilde{y}_B = \frac{\bar{y}_B + \xi}{1+\xi}, \quad \tilde{\mathbf{k}}_{B\perp} = \bar{\mathbf{p}}_{B\perp} - \frac{1-\bar{y}_B}{1+\xi} \frac{\Delta_\perp}{2}, \quad (44)$$

$$\tilde{y}_M = \frac{\bar{y}_M}{1+\xi}, \quad \tilde{\mathbf{k}}_{M\perp} = \bar{\mathbf{p}}_{M\perp} + \frac{\bar{y}_M}{1+\xi} \frac{\Delta_\perp}{2}. \quad (45)$$

Analogously, the  $BM$  components of the final nucleon state are given by

$$\sum_{B,M} \int \frac{d\hat{y}'_B}{\sqrt{\hat{y}'_B}} \frac{d\hat{y}'_M}{\sqrt{\hat{y}'_M}} \delta(1 - \hat{y}'_B - \hat{y}'_M) \int \frac{d^2\hat{\mathbf{k}}'_{B\perp} d^2\hat{\mathbf{k}}'_{M\perp}}{2(2\pi)^3} \delta^2(\hat{\mathbf{k}}'_{B\perp} + \hat{\mathbf{k}}'_{M\perp}) \\ \times \sum_{\lambda', \lambda''} \phi_{\lambda' \lambda''}^{\lambda'_N(N, BM)}(\hat{y}'_B, \hat{\mathbf{k}}'_{B\perp}) |(\bar{y}_B - \xi) \bar{p}_N^+, \bar{\mathbf{p}}_{B\perp} + \frac{\Delta_\perp}{2}, \lambda'\rangle | \bar{p}_M^+, -\bar{\mathbf{p}}_{B\perp}, \lambda''\rangle, \quad (46)$$

where the coordinates in the hadron-out frame and in the average frame are related by

$$\hat{y}'_B = \frac{\bar{y}_B - \xi}{1 - \xi}, \quad \hat{\mathbf{k}}'_{B\perp} = \bar{\mathbf{p}}_{B\perp} + \frac{1 - \bar{y}_B}{1 - \xi} \frac{\Delta_\perp}{2}, \quad (47)$$

$$\hat{y}'_M = \frac{\bar{y}_M}{1 - \xi}, \quad \hat{\mathbf{k}}'_{M\perp} = \bar{\mathbf{p}}_{M\perp} - \frac{\bar{y}_M}{1 - \xi} \frac{\Delta_\perp}{2}. \quad (48)$$

The final result for the baryon contribution to the  $\delta F_{\lambda'_N \lambda_N}^q$  scattering amplitude is

$$\delta F_{\lambda'_N \lambda_N}^{q/BM}(\bar{x}, \xi, \Delta_\perp) = \frac{1}{\sqrt{1 - \xi^2}} \sum_M \sum_{\lambda, \lambda', \lambda''} \int_{\bar{x}}^1 \frac{d\bar{y}_B}{\bar{y}_B} \int \frac{d^2\bar{\mathbf{p}}_{B\perp}}{2(2\pi)^3} F_{\lambda' \lambda}^{q/B} \left( \frac{\bar{x}}{\bar{y}_B}, \frac{\xi}{\bar{y}_B}, \Delta_\perp \right) \phi_{\lambda \lambda''}^{\lambda'_N(N, BM)}(\bar{y}_B, \tilde{\mathbf{k}}_{B\perp}) [\phi_{\lambda' \lambda''}^{\lambda'_N(N, BM)}(\hat{y}'_B, \hat{\mathbf{k}}_{B\perp})]^*, \quad (49)$$

where

$$F_{\lambda' \lambda}^{q/B} \left( \frac{\bar{x}}{\bar{y}_B}, \frac{\xi}{\bar{y}_B}, \Delta_\perp \right) = \frac{1}{2\sqrt{1 - \xi^2/\bar{y}_B^2}} \int \frac{dz^-}{2\pi} e^{i\bar{p}_B^+ z^- \bar{x}/\bar{y}_B} \langle p_B^+, \mathbf{p}'_{B\perp}, \lambda' | \bar{\psi} \left( -\frac{z}{2} \right) \gamma^+ \psi \left( \frac{z}{2} \right) | p_B^+, \mathbf{p}_{B\perp}, \lambda \rangle \Big|_{z^+ = z_\perp = 0} \quad (50)$$

is the scattering amplitude from the active baryon in the  $BM$  component of the nucleon. The overlap representation of  $F_{\lambda \lambda'}^{q/B}$  in terms of LCWFs is given explicitly in Appendix A.

Analogously, we can derive the meson contribution to the scattering amplitude, corresponding to the case when the meson takes part to the interaction process while the baryon remains as a spectator [see Fig. 2(b)]. In such a case the role of the meson and baryon substates is interchanged with respect to the situation described before. The meson is taken out from the initial nucleon with a fraction  $\bar{y}_M + \xi$  of the average plus-momentum  $\bar{p}_N^+$ , and after the interaction with the initial and final photons is reinserted back into the final nucleon with a fraction  $\bar{y}_M - \xi$  of the average plus-momentum  $\bar{p}_N^+$ . The transverse momentum of the meson is  $\bar{\mathbf{p}}_{M\perp} - \Delta_\perp/2$  before, and  $\bar{\mathbf{p}}_{M\perp} + \Delta_\perp/2$  after the scattering process. Vice versa, the baryon substate is inert during the whole scattering process, with the same momentum  $(\bar{p}_B^+, -\bar{\mathbf{p}}_{M\perp})$  in the initial and final state. Therefore the meson contribution to the  $\delta F_{\lambda'_N \lambda_N}^q$  scattering amplitude is given by

$$\delta F_{\lambda'_N \lambda_N}^{q/MB}(\bar{x}, \xi, \Delta_\perp) = \frac{1}{\sqrt{1 - \xi^2}} \sum_B \sum_{\lambda, \lambda', \lambda''} \int_{\bar{x}}^1 \frac{d\bar{y}_M}{\bar{y}_M} \int \frac{d^2\bar{\mathbf{p}}_{M\perp}}{2(2\pi)^3} F_{\lambda' \lambda}^{q/M} \left( \frac{\bar{x}}{\bar{y}_M}, \frac{\xi}{\bar{y}_M}, \Delta_\perp \right) \phi_{\lambda \lambda''}^{\lambda'_N(N, BM)}(1 - \bar{y}_M, -\tilde{\mathbf{k}}_{M\perp}) \\ \times [\phi_{\lambda' \lambda'}^{\lambda'_N(N, BM)}(1 - \hat{y}'_M, -\hat{\mathbf{k}}_{M\perp})]^*, \quad (51)$$

where

$$F_{\lambda' \lambda}^{q/M} \left( \frac{\bar{x}}{\bar{y}_M}, \frac{\xi}{\bar{y}_M}, \Delta_\perp \right) = \frac{1}{2\sqrt{1 - \xi^2/\bar{y}_M^2}} \int \frac{dz^-}{2\pi} e^{i\bar{p}_M^+ z^- \bar{x}/\bar{y}_M} \langle p_M^+, \mathbf{p}'_{M\perp}, \lambda' | \bar{\psi} \left( -\frac{z}{2} \right) \gamma^+ \psi \left( \frac{z}{2} \right) | p_M^+, \mathbf{p}_{M\perp}, \lambda \rangle \Big|_{z^+ = z_\perp = 0} \quad (52)$$

is the scattering amplitude from the active meson in the  $BM$  component of the nucleon. In Eq. (51), the meson coordinates in the hadron-in and hadron-out frames are related to the variables in the average frame by

$$\tilde{y}_M = \frac{\bar{y}_M + \xi}{1 + \xi}, \quad \tilde{\mathbf{k}}_{M\perp} = \bar{\mathbf{p}}_{M\perp} - \frac{1 - \bar{y}_M}{1 + \xi} \frac{\Delta_\perp}{2}, \quad (53)$$

$$\hat{y}'_M = \frac{\bar{y}_M - \xi}{1 - \xi}, \quad \hat{\mathbf{k}}'_{M\perp} = \bar{\mathbf{p}}_{M\perp} + \frac{1 - \bar{y}_M}{1 - \xi} \frac{\Delta_\perp}{2}. \quad (54)$$

The explicit expression of  $F_{\lambda \lambda'}^{q/M}$  in terms of LCWFs is given in Appendix B.

### 1. Forward limit

In the limit  $\Delta^\mu \rightarrow 0$ , where  $\bar{x}$  goes over to the parton momentum fraction  $x$ , and  $\xi = \Delta_\perp = 0$ , the scattering amplitude without nucleon helicity flip reduces to the ordinary parton distribution, i.e.

$$F_{++}^q(x, 0, 0) = F_{--}^q(x, 0, 0) = q(x) = Zq^{\text{bare}}(x) + \delta q(x). \quad (55)$$

In Ref. [27], we derived the forward limit of the valence-quark contribution, i.e.  $F_{++}^{q,\text{bare}}(x, 0, 0) = q^{\text{bare}}(x)$ . Here we show that the forward limit of Eqs. (49) and (51) gives the  $\delta q(x)$  contribution to the parton distribution considered in Ref. [9] within the meson-cloud model.

In the case of the active baryon, we have  $\tilde{y}_B = \hat{y}'_B = y_B$  and  $\tilde{\mathbf{k}}_{B\perp} = \hat{\mathbf{k}}'_{B\perp} = \mathbf{k}_{B\perp}$ , and Eq. (49) reduces to the following expression

$$\delta F_{++}^{q/BM}(x, 0, 0) = \sum_M \int_x^1 dy_B \int \frac{d^2 \mathbf{k}_{B\perp}}{(2\pi)^3} \frac{1}{y_B} \sum_{\lambda, \lambda''} |\phi_{\lambda\lambda''}^{+(N,BM)}(y_B, \mathbf{k}_{B\perp})|^2 F_{++}^{q/B}\left(\frac{x}{y_B}, 0, 0\right) = \sum_M \int_x^1 \frac{dy_B}{y_B} q_B\left(\frac{x}{y_B}\right) f_{BM,N}(y_B), \quad (56)$$

where the splitting function

$$f_{BM,N}(y_B) = \int \frac{d^2 \mathbf{k}_{B\perp}}{(2\pi)^3} \sum_{\lambda, \lambda''} |\phi_{\lambda\lambda''}^{+(N,BM)}(y_B, \mathbf{k}_{B\perp})|^2 \quad (57)$$

coincides with the definition given in Ref. [9].

Analogously, in the case of the active meson we have  $\tilde{y}_M = \hat{y}'_M = y_M$  and  $\tilde{\mathbf{k}}_{M\perp} = \hat{\mathbf{k}}'_{M\perp} = \mathbf{k}_{M\perp}$  and Eq. (51) reduces to the following expression

$$\begin{aligned} \delta F_{++}^{q/MB}(x, 0, 0) &= \sum_B \sum_{\lambda, \lambda''} \int_x^1 \frac{dy_M}{y_M} \int \frac{d^2 \mathbf{k}_{M\perp}}{(2\pi)^3} |\phi_{\lambda\lambda''}^{+(N,BM)}(1 - y_M, -\mathbf{k}_{M\perp})|^2 F_{++}^{q/MB}\left(\frac{x}{y_M}, 0, 0\right) \\ &= \sum_B \int_x^1 \frac{dy_M}{y_M} f_{MB,N}(y_M) q_M\left(\frac{x}{y_M}\right), \end{aligned} \quad (58)$$

where we use the definition

$$f_{MB,N}(y_M) = \int \frac{d^2 \mathbf{k}_{M\perp}}{(2\pi)^3} \sum_{\lambda, \lambda''} |\phi_{\lambda\lambda''}^{+(N,BM)}(1 - y_M, -\mathbf{k}_{M\perp})|^2. \quad (59)$$

Since the probability for the dressed nucleon to consist of a bare baryon and meson is independent of which one interacts with the probe, we assumed in Eq. (59) the following condition

$$f_{MB,N}(y_M) = f_{BM,N}(1 - y_M). \quad (60)$$

This property cannot be derived *a priori* from the definition of the convolution model, but it is an additional physical input which may be used to restrict the vertex functions of the model as we will discuss in Sec. V. Moreover, the relation (60) automatically ensures both momentum and baryon number sum rules.

As a final result, the contribution of the higher Fock states to the parton distribution is given by

$$\begin{aligned} \delta q(x) &= \sum_{MB} \left[ \int_x^1 \frac{dy}{y} f_{MB/N}(y) q_M\left(\frac{x}{y}\right) \right. \\ &\quad \left. + \int_x^1 \frac{dy}{y} f_{BM/N}(y) q_B\left(\frac{x}{y}\right) \right], \end{aligned} \quad (61)$$

which coincides with the formulation of the meson-cloud model in deep-inelastic process (see, for example, Ref. [9] and references therein).

### B. The region $-1 \leq \bar{x} \leq -\xi$

In this region, the scattering amplitude describes the emission of an antiquark from the nucleon with momentum fraction  $-(\bar{x} + \xi)$  and its reabsorption with momentum fraction  $-(\bar{x} - \xi)$ . As a consequence, the only nonvanishing contribution can come from the active antiquark in the meson substate of the  $BM$  Fock component of the nucleon

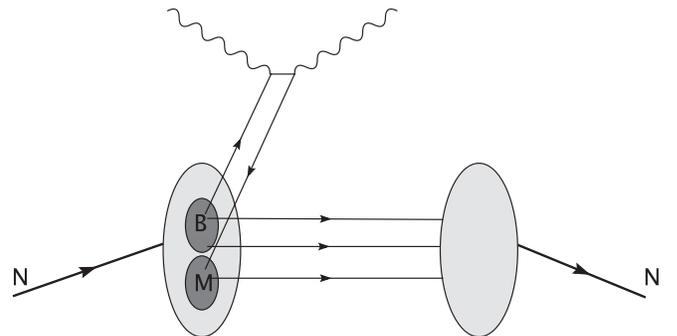


FIG. 3. Deeply virtual Compton scattering in the ERBL region.

wave function, i.e.

$$F_{\lambda'_N \lambda_N}^q(\bar{x}, \xi, \mathbf{\Delta}_\perp) = \delta F_{\lambda'_N \lambda_N}^{q/MB}(\bar{x}, \xi, \mathbf{\Delta}_\perp), \quad (62)$$

where  $\delta F_{\lambda'_N \lambda_N}^{q/MB}(\bar{x}, \xi, \mathbf{\Delta}_\perp)$  corresponds to the meson scattering amplitude illustrated in Fig. 2(b), and is given by

$$\begin{aligned} \delta F_{\lambda'_N \lambda_N}^{q/MB}(\bar{x}, \xi, \mathbf{\Delta}_\perp) &= \frac{1}{\sqrt{1-\xi^2}} \sum_B \sum_{\lambda, \lambda', \lambda''} \int_{-\bar{x}}^1 \frac{d\bar{y}_M}{\bar{y}_M} \int \frac{d^2 \bar{\mathbf{p}}_{M\perp}}{2(2\pi)^3} F_{\lambda'\lambda}^{q/M} \left( \frac{\bar{x}}{\bar{y}_M}, \frac{\xi}{\bar{y}_M}, \mathbf{\Delta}_\perp \right) \phi_{\lambda''\lambda}^{\lambda_N(N, BM)}(1 - \bar{y}_M, -\tilde{\mathbf{k}}_{M\perp}) \\ &\times [\phi_{\lambda''\lambda'}^{\lambda'_N(N, BM)}(1 - \hat{y}'_M, -\hat{\mathbf{k}}_{M\perp})]^*. \end{aligned} \quad (63)$$

Here the relations between the coordinates in the hadron frames and in the average frame are the same as in Eqs. (53) and (54). In addition, we note that Eq. (63) corresponds to the same convolution formula (51), with the integration range over  $\bar{y}_M$  between  $-\bar{x}$  and 1, and with the explicit LCWF overlap representation of  $F_{\lambda'\lambda}^{q/M}$  in the range  $-1 \leq \bar{x} \leq -\xi$  given in Appendix B.

### 1. Forward limit

The scattering amplitude from the meson substate has the following forward limit:

$$F_{++}^{q/M} \left( \frac{x}{y_M}, 0, 0 \right) = q \left( \frac{x}{y_M} \right) = -\bar{q} \left( -\frac{x}{y_M} \right). \quad (64)$$

As a consequence, in the forward limit Eq. (63) reduces to

$$\begin{aligned} \delta F_{++}^{q/MB}(x, 0, 0) &= -\sum_B \int_{-x}^1 \frac{dy_M}{y_M} \bar{q}_M \left( -\frac{x}{y_M} \right) f_{MB, N}(y_M) \\ &= -\delta \bar{q}(-x). \end{aligned} \quad (65)$$

### C. The region $-\xi \leq \bar{x} \leq \xi$

In this region, the scattering amplitude describes the emission of a quark-antiquark pair from the initial nucleon. As discussed in Ref. [25], in the Fock-state decomposition of the initial- and final-state partons we have to consider only terms where the initial state has the same parton content as the final state plus an additional quark-antiquark pair. In the present meson-cloud model, the initial state is

given by the five-parton component in Eq. (27), while the final state is described by the three-valence-quark configuration given in Eq. (18), multiplied by the normalization factor  $\sqrt{Z}$ . To label the coordinates of the initial and final partons we follow the conventions explained in Ref. [25]. This means that we use the same numbering for the spectator partons in the LCWFs of the initial and final-state partons. Thus the three partons in the outgoing nucleon are numbered not as  $i = 1, 2, 3$ , but as  $i = 1, \dots, 5$  with the labels of the active quark ( $j$ ) and the active antiquark ( $j'$ ) omitted. In principle, we can distinguish between two cases: i) the active quark belongs to the baryon substate ( $j = 1, 2, 3$ ), and the active antiquark is in the parton configuration of the meson substate of the initial nucleon; ii) both the active quark and antiquark belong to the meson substate of the initial nucleon ( $j = 4$  and  $j' = 5$ ) and the baryon is a spectator during the scattering process. However, this last contribution is vanishing because it involves the overlap of two orthogonal states, i.e. the wave functions of the baryon in the initial state and of the bare nucleon in the final state. As a consequence, the only nonvanishing contribution corresponds to the case i) which is pictured in Fig. 3.

The final result for the scattering amplitude in the ERBL region is derived using the overlap representation of Ref. [25] with the initial wave function describing the initial 5-parton configuration given in Eq. (27) and the final state described by the valence-quark wave function in Eq. (18). It reads

$$\begin{aligned} F_{\lambda'_N \lambda_N}^q &= \frac{\sqrt{Z}}{(1-\xi)(1+\xi)^2} \sum_{B, M} \sum_{\lambda_i, \tau_i} \sum_{j=1}^3 \sum_{j'} \delta_{s_j, s_q} \delta_{\lambda_j, -\lambda_{j'}} K_c \int d\bar{y}_B \int d\bar{y}_M \delta(1 - \bar{y}_B - \bar{y}_M) \int d^2 \tilde{\mathbf{k}}_{B\perp} \int d^2 \tilde{\mathbf{k}}_{M\perp} \delta(\tilde{\mathbf{k}}_{B\perp} + \tilde{\mathbf{k}}_{M\perp}) \\ &\times \int d\bar{x}_j \int \prod_{\substack{i=1 \\ i \neq j, j'}} d\bar{x}_i \int d^2 \bar{\mathbf{k}}_{j\perp} \int \prod_{\substack{i=1 \\ i \neq j, j'}} d^2 \bar{\mathbf{k}}_{i\perp} \frac{1}{[2(2\pi)^3]^3} \delta(\bar{x}_j - \bar{x}) \delta \left( \bar{y}_B - \sum_{\substack{i=1 \\ i \neq j}}^3 \frac{\bar{x}_i}{1+\xi} - \frac{\bar{x}_j + \xi}{1+\xi} \right) \delta \left( 1 - \xi - \sum_{\substack{i=1 \\ i \neq j, j'}}^5 \bar{x}_i \right) \\ &\times \delta^2 \left( \tilde{\mathbf{k}}_{B\perp} - \sum_{i=1}^3 \bar{\mathbf{k}}_{i\perp} + \frac{\mathbf{\Delta}_\perp}{2} (1 - \bar{y}_B) \right) \delta^2 \left( \frac{\mathbf{\Delta}_\perp}{2} - 5 \sum_{\substack{i=1 \\ i \neq j, j'}}^5 \bar{\mathbf{k}}_{i\perp} \right) \tilde{\Psi}_{\lambda_N}^{5q, [f]}(\bar{y}_B, \tilde{\mathbf{k}}_{B\perp}; \{\bar{x}_i, \tilde{\mathbf{k}}_{i\perp}, \lambda_i, \tau_i\}_{i=1, \dots, 5}) \\ &\times [\tilde{\Psi}_{\lambda'_N}^{3q, [f]}(\{\hat{x}'_i, \hat{\mathbf{k}}'_{i\perp}, \lambda_i, \tau_i\}_{i=1, 2, 3})]^*, \end{aligned} \quad (66)$$

where  $K_c$  is a color factor which comes from the color component of the initial- and final-state wave functions. By taking

the color component equal to  $\sum_{ijk} 1/\sqrt{6}\epsilon_{ijk}|q^i q^j q^k\rangle$  and  $\sum_{ij} 1/\sqrt{3}\delta_{ij}|q^i q^j\rangle$  for the baryon and meson state, respectively, we have  $K_c = 1/\sqrt{3}$ .

In Eq. (66) the coordinates of the initial partons in the hadron-in frame are related to the parton momenta in the average frame by

$$\begin{aligned} \tilde{x}_i &= \frac{\bar{x}_i}{1+\xi}, & \tilde{\mathbf{k}}_{i\perp} &= \bar{\mathbf{k}}_{i\perp} + \frac{\bar{x}_i}{1+\xi} \frac{\Delta_\perp}{2}, & \text{for } i \neq j, j', \\ \tilde{x}_j &= \frac{\bar{x}_j + \xi}{1+\xi}, & \tilde{\mathbf{k}}_{j\perp} &= \bar{\mathbf{k}}_{j\perp} - \frac{1-\bar{x}_j}{1+\xi} \frac{\Delta_\perp}{2}, & \tilde{x}_{j'} &= -\frac{\bar{x}_j - \xi}{1+\xi}, & \tilde{\mathbf{k}}_{j'\perp} &= -\bar{\mathbf{k}}_{j\perp} - \frac{1+\bar{x}_j}{1+\xi} \frac{\Delta_\perp}{2}, \end{aligned} \quad (67)$$

where for the average momentum of the spectators partons we used the standard definition  $\bar{k}_i = \frac{1}{2}(k_i + k'_i)$ , while for the active quark we defined  $\bar{k}_j = \frac{1}{2}(k_j - k'_j)$ , i.e. half the relative momentum between the active quark and antiquark. The corresponding relations for the coordinates of the final partons in the hadron-out and average frames are

$$\hat{x}'_i = \frac{\bar{x}_i}{1-\xi}, \quad \hat{\mathbf{k}}'_{i\perp} = \bar{\mathbf{k}}_{i\perp} - \frac{\bar{x}_i}{1-\xi} \frac{\Delta_\perp}{2}, \quad \text{for } i \neq j, j'. \quad (68)$$

Finally, the five-parton component of the initial-state wave function  $\tilde{\Psi}_{\lambda_N}^{5q,[f]}$  in Eq. (66) is given by  $\tilde{\mathbf{k}}_{i\perp}$   $\tilde{\kappa}_{i\perp}$

$$\begin{aligned} \tilde{\Psi}_{\lambda_N}^{5q,[f]}(\tilde{y}_B, \tilde{\mathbf{k}}_{B\perp}; \{\tilde{x}_i, \tilde{\kappa}_{i\perp}, \lambda_i, \tau_i\}_{i=1,\dots,5}) &= \frac{1}{\tilde{y}_B^{3/2}} \frac{1}{\tilde{y}_M} \sum_{\lambda', \lambda''} \frac{V_{\lambda', \lambda''}^{\lambda_N}(N, BM)}{M_N^2 - M_{BM}^2(\tilde{y}_B, \tilde{\mathbf{k}}_{B\perp})} \tilde{\Psi}_{\lambda'}^{B,[f]}(\{\tilde{\zeta}_i, \tilde{\mathbf{k}}_{i\perp}, \lambda_i, \tau_i\}_{i=1,2,3}) \\ &\times \tilde{\Psi}_{\lambda''}^{M,[f]}(\{\tilde{\zeta}_i, \tilde{\mathbf{k}}_{i\perp}, \lambda_i, \tau_i\}_{i=4,5}) \end{aligned} \quad (69)$$

with the coordinates in the baryon-in ( $\zeta_i$ , with  $i = 1, 2, 3$ ) and meson-in ( $\zeta_i$ , with  $i = 4, 5$ ) frames defined as

$$\begin{aligned} \tilde{\zeta}_i &= \frac{\tilde{x}_i}{\tilde{y}_B}, & \tilde{\mathbf{k}}_{i\perp} &= \tilde{\mathbf{k}}_{i\perp} - \tilde{\zeta}_i \tilde{\mathbf{k}}_{B\perp}, & i &= 1, 2, 3, \\ \tilde{\zeta}_i &= \frac{\tilde{x}_i}{\tilde{y}_M}, & \tilde{\mathbf{k}}_{i\perp} &= \tilde{\mathbf{k}}_{i\perp} - \tilde{\zeta}_i \tilde{\mathbf{k}}_{M\perp}, & i &= 4, 5. \end{aligned} \quad (70)$$

## V. MODEL FOR THE PION CLOUD OF THE PROTON

In this section we specify the ingredients for the model calculation of the unpolarized GPDs of the proton. We restrict ourselves to consider only the pion-cloud contribution disregarding the contributions from mesons of higher masses which are suppressed. As a consequence, the accompanying baryon in the  $|B\pi\rangle$  component of the dressed proton is a nucleon or a  $\Delta$ .

### A. Vertex functions

The vertex functions for the transition  $p \rightarrow B\pi$  are given by

$$\begin{aligned} V_{\lambda', 0}^\lambda(p, N\pi) &= i g_{pN\pi} \bar{u}_{\lambda'}(\tilde{p}'_N) \gamma_5 u_\lambda(\tilde{p}_p), \\ V_{\lambda', 0}^\lambda(p, \Delta\pi) &= i g_{p\Delta\pi} \bar{u}_{\lambda'}^\mu(\tilde{p}'_\Delta) (p_p - p'_\Delta)_\mu u_\lambda(\tilde{p}_p) \end{aligned} \quad (71)$$

where  $u_\lambda(\tilde{p}_N)$  and  $u_{\lambda'}^\mu(\tilde{p}_\Delta)$  are the nucleon spinor and the Rarita-Schwinger spinor, respectively, defined in Appendix C.

Because of the extended nature of the vertices one has to replace the coupling constants in Eq. (71) with phenomenological vertex form factors,  $G_{NBM}(y, k_\perp^2)$ , which parametrize the unknown dynamics at the vertices. We use the following parametrization [36]

$$G_{NBM}(y, k_\perp^2) = g_{NBM} \left( \frac{\Lambda_{BM}^2 + M_N^2}{\Lambda_{BM}^2 + M_{BM}^2(y, k_\perp^2)} \right)^2, \quad (72)$$

where  $\Lambda_{BM}$  is the cutoff parameter and the invariant mass of the baryon-meson fluctuation  $M_{BM}^2$  is given in Eq. (11). We note that the dipole parametrization of the vertex form factors satisfies the condition  $G_{NBM}(y, k_\perp^2) = G_{NBM}(1-y, k_\perp^2)$  which automatically guarantees the property (60) for the splitting functions.

For the coupling constants we use the numerical values given in Refs. [36,37], i.e.  $g_{NN\pi}^2/4\pi = 13.6$  and  $g_{N\Delta\pi}^2/4\pi = 11.08 \text{ GeV}^{-2}$ , with  $g_{NN\pi} = g_{pp\pi^0}$  and  $g_{N\Delta\pi} = g_{p\Delta^+\pi^-}$ . The coupling of a given type of transition with different isospin components is obtained in terms of isospin Clebsch-Gordan coefficients, i.e.  $g_{pn\pi^+} = -\sqrt{2}g_{pp\pi^0}$ ,  $g_{p\Delta^0\pi^+} = -g_{p\Delta^+\pi^0}/\sqrt{2} = g_{p\Delta^+\pi^-}/\sqrt{3}$ .

The violation of the Gottfried sum rule and flavor symmetry puts constraints on the magnitude of the cutoff parameters. We use the values  $\Lambda_{BM} = 1.0 \text{ GeV}$  and  $\Lambda_{BM} = 1.3 \text{ GeV}$  for the  $\pi N$  and  $\pi\Delta$  components, respectively, because they give contributions to the  $\bar{u}$  and  $\bar{d}$  which are consistent with the requirement that the meson-cloud component of the sea-quark contribution cannot be larger than the measured sea quark and also with flavor-symmetry violation [36].

With the specified parameters, in the case of the  $p \rightarrow B\pi$  transition one has [37],

$$P_{N\pi/p} = P_{p\pi^0/p} + P_{n\pi^+/p} = 3P_{p\pi^0/p} = 13\%,$$

$$P_{\Delta\pi/p} = P_{\Delta^{++}\pi^-/p} + P_{\Delta^+\pi^0/p} + P_{\Delta^0\pi^+/p} = 2P_{\Delta^{++}\pi^-/p} = 11\%.$$

$$\Psi_{\lambda}^{[f]}(\{x_i, \mathbf{k}_{i\perp}; \lambda_i, \tau_i\}_{i=1,\dots,N}) = 2(2\pi)^3 \frac{1}{\sqrt{M_0}} \prod_{i=1}^N \left(\frac{\omega_i}{x_i}\right)^{1/2} \sum_{\mu_1,\dots,\mu_N} \Psi_{\lambda}^{[c]}(\{\mathbf{k}_i; \mu_i, \tau_i, \mu_i\}_{i=1,\dots,N}) \prod_{i=1}^N D_{\mu_i\lambda_i}^{1/2*}(R_{cf}(\vec{k}_i)), \quad (73)$$

where  $\Psi_{\lambda}^{[c]}$  is the canonical wave function, and  $D_{\mu_i\lambda_i}^{1/2*}(R_{cf}(\vec{k}_i))$  are the Melosh rotations defined in Ref. [27]. In Eq. (73),  $\omega_i = \sqrt{m_i^2 + \mathbf{k}_i^2}$  is the energy of the  $i$ -th quark, and  $M_0 = \sum_i \omega_i$  is the free mass of the system of  $N$  noninteracting quarks.

In the case of the nucleon, we adopt the canonical wave function of the relativistic CQM model of Ref. [38], given by a product of a space and a spin-isospin term which is SU(6) symmetric. For a more detailed discussion of the model see Ref. [38].

The  $\Delta$  is described as a state of isospin  $T = 3/2$  obtained as a pure spin-flip excitation of the nucleon, with the corresponding spatial part of the wave function equal to that of the nucleon.

Finally, the canonical wave function of the pion is taken from Ref. [39] and reads

$$\Psi_{\pi}^{[c]}(\vec{k}_1, \vec{k}_2; \mu_1, \mu_2) = \frac{i}{\pi^{3/4} \beta^{3/2}} \left(\frac{1}{2} \mu_1 \frac{1}{2} \mu_2 |00\right) \times \exp(-k^2/(2\beta^2)), \quad (74)$$

with  $\vec{k} = \vec{k}_1 = -\vec{k}_2$ , and  $\beta = 0.3659$  GeV. The choice of the model from Ref. [39] is consistent with the CQM we adopt for the nucleon and the  $\Delta$ , since the central potential between the two constituent quarks in the pion is described as a linear confining term plus Coulomb-like interaction. As explained in more details in Ref. [39], the canonical expression (74) represents a variational solution to the mass equation. The phase of the pion wave function (74) is consistent with that of the antiquark spinors of Ref. [40].

## VI. RESULTS

The general formalism developed in the preceding sections has been applied to calculate the unpolarized GPDs  $H^q$  and  $E^q$  for the proton dressed by a pion cloud. The different contributions coming from the  $p \rightarrow BM$  fluctuations are derived from the basic  $p \rightarrow p\pi^0$  and  $p \rightarrow \Delta^{++}\pi^-$  transitions. Because of the isospin relations between the different charged channels and the SU(6) symmetry of the spin-isospin part of the proton wave function, in the region  $\xi \leq \bar{x} \leq 1$  the other contributions are ob-

## B. LCWFs in the constituent-quark model

As explained in details in Ref. [27], the LCWFs of the baryons and pion in Eqs. (21) and (22), respectively, can be expressed in terms of the canonical wave functions, solutions of the instant-form Hamiltonian for the  $N$ -valence quarks of the hadron, through the following relation

tained making use of the following relations:

$$\begin{aligned} \delta F^{u/n\pi^+} &= 2\delta F^{d/p\pi^0}, & \delta F^{d/n\pi^+} &= 2\delta F^{u/p\pi^0}, \\ \delta F^{u/\Delta^0\pi^+} &= \frac{1}{4}\delta F^{u/\Delta^+\pi^0}, & \delta F^{d/\Delta^0\pi^+} &= \delta F^{d/\Delta^+\pi^0}, \\ \delta F^{u/\Delta^{++}\pi^-} &= \frac{2}{4}\delta F^{u/\Delta^+\pi^0}, & \delta F^{u/\pi^0p} &= \delta F^{d/\pi^0p}, \\ \delta F^{u/\pi^+n} &= 4\delta F^{u/\pi^0p}, & \delta F^{u/\pi^+\Delta^0} &= \delta F^{u/\pi^0\Delta^+}, \\ \delta F^{d/\pi^-\Delta^{++}} &= 3\delta F^{d/\pi^0\Delta^+}. \end{aligned} \quad (75)$$

Similarly, in the  $-1 \leq \bar{x} \leq -\xi$  region we have the following relations:

$$\begin{aligned} \delta F^{d/\pi^+n} &= 4\delta F^{d/\pi^0p}, & \delta F^{d/\pi^+\Delta^0} &= \delta F^{d/\pi^0\Delta^+}, \\ \delta F^{u/\pi^-\Delta^{++}} &= 3\delta F^{u/\pi^0\Delta^+}, & \delta F^{d/\pi^0p} &= \delta F^{u/\pi^0p}. \end{aligned} \quad (76)$$

In addition, for  $\xi \leq \bar{x} \leq 1$ , we have

$$\begin{aligned} \delta F^{u/\pi^0p}(\bar{x}, \xi, t) &= -\delta F^{d/\pi^0p}(-\bar{x}, \xi, t), \\ \delta F^{d/\pi^0p}(\bar{x}, \xi, t) &= -\delta F^{u/\pi^0p}(-\bar{x}, \xi, t), \\ \delta F^{d/\pi^-\Delta^{++}}(\bar{x}, \xi, t) &= -\delta F^{u/\pi^-\Delta^{++}}(-\bar{x}, \xi, t). \end{aligned} \quad (77)$$

The multidimensional integration required for the numerical computation of the different contributions to  $F_{\lambda'_N\lambda_N}^q$  was implemented in a parallel computation using the parallelized version of the VEGAS routine of Ref. [41]. In this way one makes easier an otherwise time-consuming computation. The results presented in this Section have been obtained for some combinations of  $t$  and  $\xi$  as an example of the effects introduced by the sea.

First let us study the forward limit,  $\xi = 0$ ,  $t = 0$ .

In Fig. 4 the spin-averaged  $H^q$  and the helicity-flip  $E^q$  GPDs are plotted together with the separated contributions from the bare proton (dashed-dotted line), the baryon (dashed lines) and the meson (dotted lines) in the baryon-pion fluctuation. All these contributions add up incoherently to give the total result (full curves). The bare-proton contribution is the same as that already calculated in Ref. [27], rescaled by the wave function renormalization constant  $Z$ . It is always positive within its support ( $0 \leq \bar{x} \leq 1$ ) with the exception of  $E^d$  for which it is

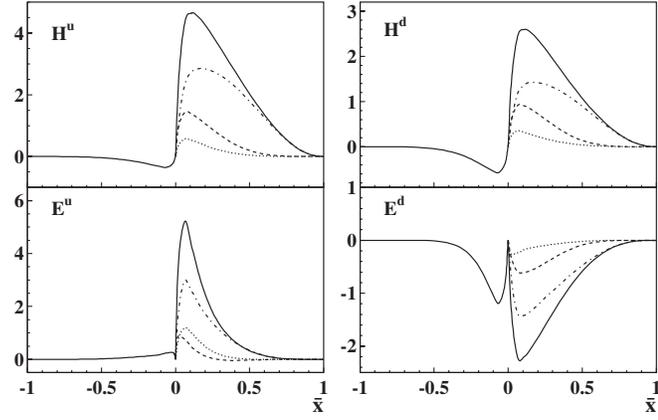


FIG. 4. The different contributions to the spin-averaged ( $H^q$ , upper panels) and helicity-flip ( $E^q$ , lower panels) generalized parton distributions calculated in the meson-cloud model for flavors  $u$  (left panels) and  $d$  (right panels), at  $\xi = 0$  and  $t = 0$ . Dashed lines: baryon contribution from the  $|BM\rangle$  component. Dotted lines: meson contribution from the  $|BM\rangle$  component. Dashed-dotted lines: contribution from the bare nucleon. Full curves: total result as a sum of the different contributions.

negative. The same behavior characterizes the baryon contribution from the baryon-pion fluctuation that is also limited to the range  $0 \leq \bar{x} \leq 1$ , consistently with the assumption that the only active degrees of freedom for such a baryon are the valence quarks. Both contributions vanish at the end points of their support. The sea-quark contribution, extending all over the full range  $-1 \leq \bar{x} \leq 1$ , is determined by the antiquark residing in the meson of the baryon-pion fluctuation. The resulting effect of the pion cloud is thus to add a contribution for negative  $\bar{x}$  and to increase the magnitude of the GPDs for positive  $\bar{x}$  with respect to the case of the bare proton. In particular, for positive and small  $\bar{x}$  the pion-cloud contribution as a whole is comparable to that of the bare proton, confirming the important role of the sea at small  $\bar{x}$  found within the chiral quark-soliton model [18,20,21]. Since for  $\bar{x} > 0$  the contribution of the baryon-meson fluctuation is effective only at low  $\bar{x}$  values, the same faster fall off of  $E^q$  with respect to  $H^q$  for  $\bar{x} \rightarrow 1$  is obtained as in Ref. [27]. This is a consequence of the decreasing role of the Melosh transform to generate angular momentum in  $E^q$  with increasing quark momentum.

In all cases at  $\bar{x} = 0$  the GPDs have a zero. This is due to fact that in the overlap integrals the various terms of the proton wave function are taken at one of their end points. This peculiar feature was discussed in Ref. [42] and explained as an artifact due to the truncated Fock-state expansion. The singular behavior of the usual parton distributions for  $\bar{x} \rightarrow 0$  cannot be obtained from any finite number of Fock-state contributions, all of which vanish at  $\bar{x} = 0$ .

In any case, the forward limit of the first moment sum rules for the spin-averaged GPDs,

$$\int_{-1}^1 dx H^u(x, 0, 0) = 2, \quad \int_{-1}^1 dx H^d(x, 0, 0) = 1, \quad (78)$$

is correctly fulfilled. For the helicity-flip GPDs the first moment sum rule reads

$$\int_{-1}^1 dx E^q(x, 0, 0) = \kappa^q, \quad (79)$$

where  $\kappa^u$  ( $\kappa^d$ ) is the anomalous magnetic moment of the  $u$  ( $d$ ) quarks, with  $\kappa^u + \kappa^d = 3(\kappa^p + \kappa^n)$  and  $\kappa^u - \kappa^d = \kappa^p - \kappa^n$ ,  $\kappa^p$  and  $\kappa^n$  being the proton and neutron anomalous magnetic moments, respectively.

Experimentally, we have  $\kappa^p = 1.793$  and  $\kappa^n = -1.913$ , thus giving  $\kappa^u = 1.673$  and  $\kappa^d = -2.033$ . Without the pion cloud the values of the nucleon anomalous magnetic moments were found to be rather far from the experimental ones, i.e.  $\kappa^p = 0.91$  and  $\kappa^n = -0.82$ , corresponding to  $\kappa^u = 1.0$  and  $\kappa^d = -0.74$  [27]. This result is, however, in agreement with analogous light-front calculations with pointlike quarks [43–45] and the values derived in the forward limit of GPDs derived in the Nambu-Jona-Lasinio model [22]. Including the pion cloud we have  $\kappa^u = 1.14$  and  $\kappa^d = -1.03$ , corresponding to  $\kappa^p = 1.10$  and  $\kappa^n = -1.07$ , closer to the phenomenological values. Similar effects are also expected for the pion-cloud contribution to the form factors at finite value of  $t$ , improving the agreement between the CQM predictions and the experimental results [46].

In Fig. 5 the same results plotted in Fig. 4 are reorganized to show the isoscalar  $u + d$  and isovector  $u - d$  combinations. In the large- $N_c$  limit  $H^u + H^d$  and  $E^u - E^d$  appear in leading order in  $N_c$ , while  $H^u - H^d$  and  $E^u + E^d$  appear in subleading order [14]. Also in our model  $H^u + H^d$  and  $E^u - E^d$  are found to be larger than  $H^u - H^d$  and  $E^u + E^d$ . The behavior of the isoscalar combinations is also very similar to that provided by the chiral quark-soliton model [18,20]. The sea-quark contribution is

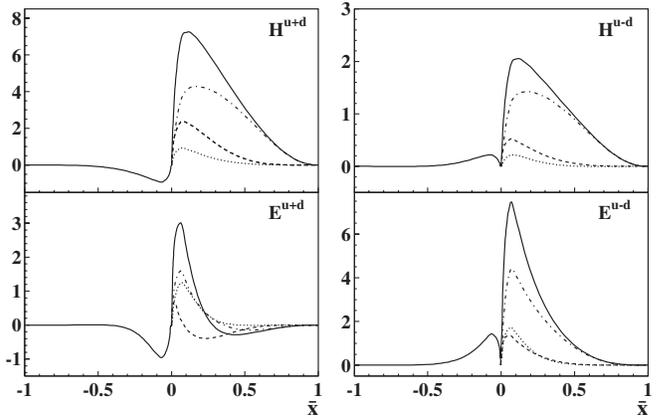


FIG. 5. Isoscalar ( $u + d$ , left panels) and isovector ( $u - d$ , left panels) combinations of the spin-averaged (upper panels) and helicity-flip (lower panels) generalized parton distributions calculated in the meson-cloud model, at  $\xi = 0$  and  $t = 0$ .

an odd (even) function of  $\bar{x}$  for the isoscalar (isovector) combination. As discussed above, the isovector combinations vanish at  $\bar{x} = 0$  due to the truncated Fock-state expansion of the proton wave function. Would the dip at  $\bar{x} = 0$  be filled by including the whole series expansion, the behavior of the isovector combinations would resemble that derived in the chiral quark-soliton model [21], where GPDs in the neighborhood of  $\bar{x} = 0$  are mostly determined by the Dirac sea. In our model the sea contribution comes only from the antiquark present in the one-pion component of the cloud.

Anyway, since there are no gluons in the model the total momentum of the proton is carried by quarks and antiquarks only. Therefore the second moment sum rule,

$$\int_{-1}^1 dx x (H^u + H^d)(x, 0, 0) = M^Q = 1, \quad (80)$$

and the spin sum rule,

$$\int_{-1}^1 dx x (H^u + H^d + E^u + E^d)(x, 0, 0) = 2J^Q = 1 \quad (81)$$

are consistently fulfilled in the model. In addition,

$$\int_{-1}^1 dx x (E^u + E^d)(x, 0, 0) = 2J^Q - M^Q = 0. \quad (82)$$

Going beyond the forward limit we first discuss the  $t$  dependence of the GPDs at  $\xi = 0$  by comparing results in Fig. 4 at  $t = 0$  with those in Fig. 6 at  $t = -0.2 \text{ GeV}^2$ . The relative contribution of the different components is not modified by switching on the momentum transfer  $t$ , only the overall magnitude is decreased. This is in agreement with the common believe that the main part of the  $t$  dependence of the GPDs is exhibited by their first moments, i.e. by the quark Dirac and Pauli form factors.

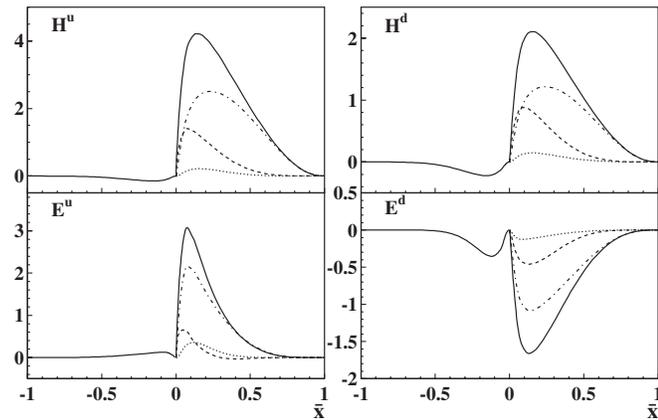


FIG. 6. The different contributions to the spin-averaged ( $H^q$ , upper panels) and helicity-flip ( $E^q$ , lower panels) generalized parton distributions calculated in the meson-cloud model for flavors  $u$  (left panels) and  $d$  (right panels), at  $\xi = 0$  and  $t = -0.2 \text{ GeV}^2$ . Line style as in Fig. 4.

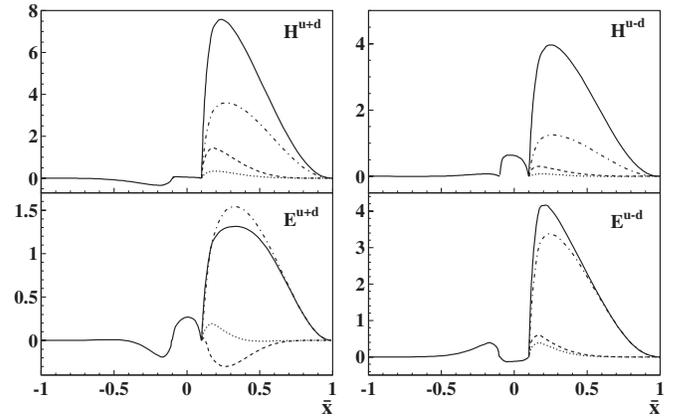


FIG. 7. Isoscalar ( $u + d$ , left panels) and isovector ( $u - d$ , left panels) combinations of the spin-averaged (upper panels) and helicity-flip (lower panels) generalized parton distributions calculated in the meson-cloud model, at  $\xi = 0.1$  and  $t = -0.2 \text{ GeV}^2$ . Line style as in Fig. 4.

With a nonvanishing  $\xi$  one can explore the ERBL region with  $|\bar{x}| \leq \xi$  where in our model only transition amplitudes between the bare-proton and baryon-meson components are contributing. The combined dependence of the isoscalar and isovector combinations of GPDs on  $\xi$  and  $t$  is shown in Figs. 7–9. The  $t$  dependence at constant  $\xi = 0.1$  can be extracted from Figs. 7 and 8, while the  $\xi$  dependence at constant  $t = -0.5 \text{ GeV}^2$  is deduced from Figs. 8 and 9.

From the isoscalar and isovector combinations of GPDs plotted in Fig. 7 at  $\xi = 0.1$  and  $t = -0.2 \text{ GeV}^2$  we see that GPDs in the ERBL region are rather regular functions over the whole range, with zeros at the endpoints  $\bar{x} = \pm \xi$ . This result is quite different from the oscillatory behavior predicted by the chiral quark-soliton model [18] where the valence contribution of the discrete level is a smooth

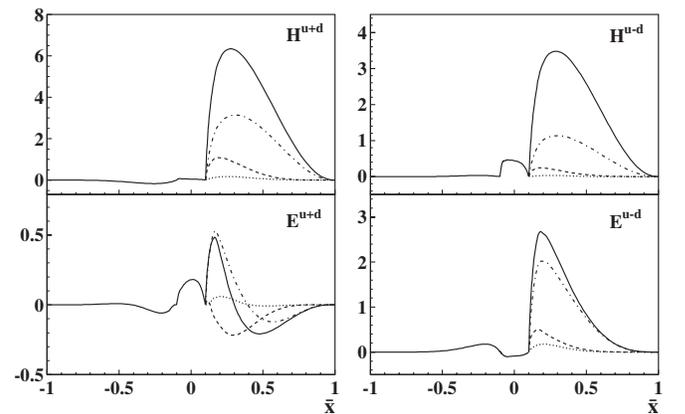


FIG. 8. Isoscalar ( $u + d$ , left panels) and isovector ( $u - d$ , left panels) combinations of the spin-averaged (upper panels) and helicity-flip (lower panels) generalized parton distributions calculated in the meson-cloud model, at  $\xi = 0.1$  and  $t = -0.5 \text{ GeV}^2$ . Line style as in Fig. 4.

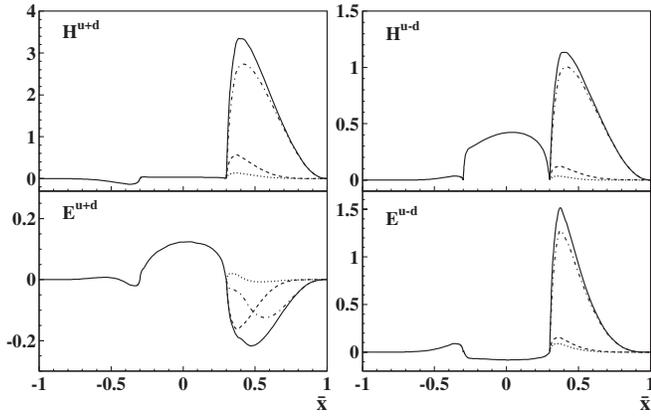


FIG. 9. Isoscalar ( $u + d$ , left panels) and isovector ( $u - d$ , left panels) combinations of the spin-averaged (upper panels) and helicity-flip (lower panels) generalized parton distributions calculated in the meson-cloud model, at  $\xi = 0.3$  and  $t = -0.5 \text{ GeV}^2$ .

function extending into the ERBL region and adding to the sea contribution. Here this is forbidden because the support of the valence contribution is limited to the DGLAP region. In addition, the transition amplitude between the bare-proton and the baryon-meson components vanishes at the boundary of the ERBL region. As discussed in Ref. [15], the points  $\bar{x} = \pm\xi$  correspond to very peculiar parton configurations involving one parton with vanishing momentum in the initial or final-state hadron. As a consequence, when approaching e.g.  $\xi$  from below, one probes a quark-antiquark pair with one momentum fraction finite and the other going to zero, a configuration similar to the one of a meson distribution amplitude at its endpoints. On the other hand leading-twist GPDs must be continuous at  $\bar{x} = \pm\xi$  to avoid logarithmically divergent scattering amplitudes in DVCS. In fact, this condition is fulfilled here because also approaching  $\bar{x} = \pm\xi$  from the DGLAP region in our model GPDs are constrained to go to zero. This generates a discontinuity of the first derivative of GPDs at  $\bar{x} = \pm\xi$  which, however, is not in contradiction with general principles. Including higher Fock-states of the type suggested in Ref. [47] and/or considering evolution to a higher scale will fill up the zero at  $\bar{x} = \pm\xi$ .

In the DGLAP region both for  $\bar{x} \geq \xi$  and  $\bar{x} \leq -\xi$  no striking difference arises in Fig. 7 for the spin-averaged GPDs  $H^{u\pm d}$  with respect to the results in the forward limit shown in Fig. 5, while for the helicity-flip GPDs the (negative)  $d$  contribution coming from the baryon in the  $|BM\rangle$  component is responsible for a broader shape at  $\bar{x} \geq \xi$ . The  $t$  dependence of the different and opposite contributions to  $E^u$  and  $E^d$  is also responsible for the small size and the oscillatory behavior of  $E^{u+d}$  at  $t = -0.5 \text{ GeV}^2$  (Fig. 8). Increasing  $\xi$ , therefore compressing the support for the valence contribution, this effect is even more visible producing a negative  $E^{u+d}$  for  $\bar{x} \geq \xi$  as in Fig. 9, while the behavior in the ERBL region remains the same.

## VII. CONCLUSIONS

The convolution model for the physical nucleon, where the bare nucleon is dressed by its virtual meson cloud, has a long and successful history in explaining properties such as form factors and parton distributions. In this paper it has been revisited and applied for the first time to study GPDs. A light-front wave function overlap representation is obtained in the one-meson approximation by inserting a Fock-state expansion involving a bare nucleon and meson-baryon states. The model fulfills the support condition and general sum rules such as the number, momentum and angular momentum sum rules. Explicit expressions for the unpolarized GPDs have been derived and applied to the case of the meson being a pion and the baryon being either a nucleon or a  $\Delta$ .

This meson-cloud model gives the possibility to link GPDs calculated in the light-front formalism to the nucleon description in terms of constituent quarks including a sea contribution already at a low-energy scale and providing a suitable input for the evolution to higher scales. The results presented in this paper for different kinematics show an important contribution of the meson cloud at low  $\bar{x}$  and a smooth contribution of the sea in the ERBL region and for negative  $\bar{x}$ . As an effect of the truncated Fock-state expansion, characteristic nodes occur at the endpoints of the DGLAP and ERBL regions, i.e. at  $\bar{x} = 0$  in the forward case and at  $\bar{x} = \pm\xi$  in the off-forward case, where the wave function has to vanish. However, this artifact will disappear under evolution of GPDs to higher scales. In addition, since the contribution in the ERBL region is vanishing in the forward limit, it can not be easily inferred from parametrizations in terms of parton distributions. Therefore, the present calculation gives new insights to model the off-forward features of the GPDs, and can be further used as input at the hadronic scale to study the behavior under evolution at higher scales.

## ACKNOWLEDGMENTS

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## APPENDIX A: BARYON SCATTERING AMPLITUDE

In this section we give the LCWF overlap representation of the baryon scattering amplitude given in Eq. (50), and evaluated in the nucleon average frame where the plus and transverse components of the momentum of the initial and final baryon are given by

$$\begin{aligned}
p_B^+ &= \bar{p}_B^+ \left(1 + \frac{\xi}{\bar{y}_B}\right), & \mathbf{p}_{B\perp} &= \bar{\mathbf{p}}_{B\perp} - \frac{\Delta}{2}, \\
p_B'^+ &= \bar{p}_B'^+ \left(1 - \frac{\xi}{\bar{y}_B}\right), & \mathbf{p}'_{B\perp} &= \bar{\mathbf{p}}_{B\perp} + \frac{\Delta}{2}.
\end{aligned} \tag{A1}$$

This scattering amplitude gives a nonvanishing contribution only in the region  $\xi \leq \bar{x} \leq 1$ , where it describes the emission of a quark from the baryon with momentum fraction  $\bar{x} + \xi/\bar{y}_B$  of the average plus momentum  $\bar{p}_B^+$ , and its reabsorption with  $\bar{x} - \xi/\bar{y}_B$ . By analogy with the nucleon case, we introduce the names ‘‘baryon-in’’ and ‘‘baryon-out’’ for frames where the incoming and outgoing baryon has zero transverse momentum. The fraction of plus momentum and the transverse components of the momentum of partons before the scattering process in the

baryon-in frame will be denoted with  $\bar{\zeta}_i$ , and  $\bar{\mathbf{k}}_{i\perp}$ , respectively. The corresponding quantities for the final partons in the baryon-out frames are defined by  $\hat{\zeta}_i'$  and  $\hat{\mathbf{k}}_{i\perp}'$ . Furthermore, defining the average momentum of the partons as  $\bar{k}_i^+ = (k_i^+ + k_i'^+)/2$ , we introduce the fraction  $\bar{\zeta}_i = \bar{k}_i^+/\bar{p}_B^+ = \bar{x}_i/\bar{y}_B$ . When the active parton is taken out from the baryon it carries a fraction  $\bar{\zeta}_j + \xi/\bar{y}_B$  of the average plus momentum  $\bar{p}_B^+$ , and its transverse momentum is  $\bar{\mathbf{k}}_{j\perp} - \Delta_{\perp}/2$ . The arguments of the initial-state LCWF in the baryon-in frame are obtained from the variables of the partons in the average frame by means of the transverse boost in Eq. (35) with parameters  $\mathbf{b}_{\perp} = \bar{\mathbf{p}}_{B\perp} - \Delta_{\perp}/2$  and  $b^+ = \bar{p}_B^+(1 + \xi/\bar{y}_B)$ . Furthermore, by using the spectator condition,  $k_i' = \bar{k}_i = k_i$ , one obtains

$$\begin{aligned}
\bar{\zeta}_i &= \frac{\bar{\zeta}_i}{1 + \xi/\bar{y}_B}, & \bar{\mathbf{k}}_{i\perp} &= \bar{\mathbf{k}}_{i\perp} - \frac{\bar{\zeta}_i}{1 + \xi/\bar{y}_B} \left( \bar{\mathbf{p}}_{B\perp} - \frac{\Delta_{\perp}}{2} \right), & \text{for } i \neq j, \\
\bar{\zeta}_j &= \frac{\bar{\zeta}_j + \xi/\bar{y}_B}{1 + \xi/\bar{y}_B}, & \bar{\mathbf{k}}_{j\perp} &= \bar{\mathbf{k}}_{j\perp} - \bar{\mathbf{p}}_{B\perp} + \frac{1 - \bar{\zeta}_j}{1 + \xi/\bar{y}_B} \left( \bar{\mathbf{p}}_{B\perp} - \frac{\Delta_{\perp}}{2} \right).
\end{aligned} \tag{A2}$$

In the final state, the active parton has a fraction of the average plus momentum  $\bar{p}_B^+$  of the baryon equal to  $\bar{\zeta}_j - \xi/\bar{y}_B$ , and a transverse momentum  $\bar{\mathbf{k}}_{j\perp} + \Delta_{\perp}/2$ . The arguments of the final-state LCWF in the baryon-out frame are obtained from the variables of the partons in the average frame by means of the transverse boost in Eq. (35) with parameters  $\mathbf{b}_{\perp} = \bar{\mathbf{p}}_{B\perp} + \Delta_{\perp}/2$  and  $b^+ = \bar{p}_B^+(1 - \xi/\bar{y}_B)$  and are given by

$$\begin{aligned}
\hat{\zeta}_i' &= \frac{\bar{\zeta}_i}{1 - \xi/\bar{y}_B}, & \hat{\mathbf{k}}_{i\perp}' &= \bar{\mathbf{k}}_{i\perp} - \frac{\bar{\zeta}_i}{1 - \xi/\bar{y}_B} \left( \bar{\mathbf{p}}_{B\perp} + \frac{\Delta_{\perp}}{2} \right), & \text{for } i \neq j, \\
\hat{\zeta}_j' &= \frac{\bar{\zeta}_j - \xi/\bar{y}_B}{1 - \xi/\bar{y}_B}, & \hat{\mathbf{k}}_{j\perp}' &= \bar{\mathbf{k}}_{j\perp} - \bar{\mathbf{p}}_{B\perp} + \frac{1 - \bar{\zeta}_j}{1 - \xi/\bar{y}_B} \left( \bar{\mathbf{p}}_{B\perp} + \frac{\Delta_{\perp}}{2} \right).
\end{aligned} \tag{A3}$$

Using a similar procedure as in the calculation of the valence-quark contribution to the scattering amplitude in the region  $\xi \leq \bar{x} \leq 1$ , one finds that the LCWF overlap representation of  $F_{\lambda\lambda'}^{q/B}$  is given by

$$F_{\lambda\lambda'}^{q/B} \left( \frac{\bar{x}}{\bar{y}_B}, \frac{\xi}{\bar{y}_B}, \Delta_{\perp} \right) = \frac{1}{(1 - \xi^2/\bar{y}_B^2)} \sum_{j=1}^3 \sum_{\lambda_i, \tau_i} \delta_{s_j q} \int [d\bar{\zeta}]_3 [d^2\bar{\mathbf{k}}_{\perp}]_3 \delta \left( \frac{\bar{x}}{\bar{y}_B} - \bar{\zeta}_j \right) \Psi_{\lambda'}^{B,[f]*} (\{\bar{\zeta}_i, \bar{\mathbf{k}}_{i\perp}; \lambda_i, \tau_i\}) \Psi_{\lambda}^{B,[f]} (\{\hat{\zeta}_i', \hat{\mathbf{k}}_{i\perp}'; \lambda_i, \tau_i\}). \tag{A4}$$

## APPENDIX B: MESON GENERALIZED PARTON DISTRIBUTIONS

In this section we derive the LCWF overlap representation of the meson scattering amplitude in Eq. (52), with the momenta of the incoming and outgoing meson in the average frame given by

$$p_M^+ = \bar{p}_M^+ \left(1 + \frac{\xi}{\bar{y}_M}\right), \quad \mathbf{p}_{M\perp} = \bar{\mathbf{p}}_{M\perp} - \frac{\Delta}{2}, \quad p_M^+ = \bar{p}_M'^+ \left(1 - \frac{\xi}{\bar{y}_M}\right), \quad \mathbf{p}'_{M\perp} = \bar{\mathbf{p}}_{M\perp} + \frac{\Delta}{2}. \tag{B1}$$

In the following we separately discuss the contributions in the DGLAP region for positive and negative  $\bar{x}$ .

### 1. Region $\xi \leq \bar{x} \leq 1$

The derivation of  $F^{q/M}$  in terms of meson LCWFs goes along the same lines as in the case of the baryon discussed in Appendix A. The final result is given by

$$F_{\lambda'\lambda}^{q/M}\left(\frac{\bar{x}}{\bar{y}_M}, \frac{\xi}{\bar{y}_M}, \Delta_\perp\right) = \frac{1}{\sqrt{1 - \xi^2/\bar{y}_M^2}} \sum_{j=1}^2 \sum_{\lambda_i, \tau_i} \delta_{s_j q} \int [d\bar{\xi}]_2 [d^2\bar{\mathbf{k}}_\perp]_2 \delta\left(\frac{\bar{x}}{\bar{y}_M} - \bar{\xi}_j\right) \Psi_{\lambda'}^{M,[f]*}(\{\hat{\xi}'_i, \hat{\mathbf{k}}'_{i\perp}; \lambda_i, \tau_i\}) \Psi_\lambda^{M,[f]}(\{\bar{\xi}_i, \bar{\mathbf{k}}_{i\perp}; \lambda_i, \tau_i\}), \quad (\text{B2})$$

where  $\{\bar{\xi}_i\}, \{\bar{\mathbf{k}}_{i\perp}\}$  are the coordinates of the initial partons in the meson-in frame. They are related to the coordinates in the average frame by the same transformation as in Eq. (A2), with the substitution  $\bar{y}_B \rightarrow \bar{y}_M$ , and  $\mathbf{p}_{B\perp} \rightarrow \mathbf{p}_{M\perp}$ . Likewise, in Eq. (B2)  $\{\hat{\xi}'_i\}, \{\hat{\mathbf{k}}'_{i\perp}\}$  are the coordinates of the final partons in the meson-out frame and satisfy the same relations as in Eq. (A3) with  $\bar{y}_B$  and  $\mathbf{p}_{B\perp}$  replaced by  $\bar{y}_M$  and  $\mathbf{p}_{M\perp}$ , respectively.

## 2. Region $-1 \leq \bar{x} \leq -\xi$

In this region, the meson contribution to the scattering amplitude is given by

$$F_{\lambda'\lambda}^{q/M}\left(\frac{\bar{x}}{\bar{y}_M}, \frac{\xi}{\bar{y}_M}, \Delta_\perp\right) = -\frac{1}{\sqrt{1 - \xi^2/\bar{y}_M^2}} \sum_{j=1}^2 \sum_{\lambda_i, \tau_i} \delta_{s_j \bar{q}} \int [d\bar{\xi}]_2 [d^2\bar{\mathbf{k}}_\perp]_2 \delta\left(\frac{\bar{x}}{\bar{y}_M} + \bar{\xi}_j\right) \Psi_{\lambda'}^{M,[f]*}(\{\hat{\xi}'_i, \hat{\mathbf{k}}'_{i\perp}; \lambda_i, \tau_i\}) \times \Psi_\lambda^{M,[f]}(\{\bar{\xi}_i, \bar{\mathbf{k}}_{i\perp}; \lambda_i, \tau_i\}), \quad (\text{B3})$$

where the arguments of the LCWFs are given by the same expression as in the region  $\xi \leq \bar{x} \leq 1$ .

## APPENDIX C: VERTEX FUNCTIONS

In this Appendix we work out the case of the  $N \rightarrow N\pi$  and  $N \rightarrow \Delta\pi$  transitions in the light-front formalism. Vertex functions for such transitions can be found in several places (see, e.g., Refs. [9,48,49]), but it is convenient to show their derivation.

The light-front vectors are defined as

$$A^\mu = (A^-, A^+, \mathbf{A}_\perp), \quad (\text{C1})$$

with

$$A^\pm = A^0 \pm A^3, \quad \mathbf{A}_\perp = (A^1, A^2). \quad (\text{C2})$$

We also use the notations  $A_{R,L} = A^1 \pm iA^2$  and  $\tilde{A} = (A^+, \mathbf{A}_\perp)$ .

The light-front nucleon spinors  $u_\lambda(\tilde{p})$  are given by

$$u_{1/2}(\tilde{p}) = \frac{1}{\sqrt{2p^+}} \begin{pmatrix} p^+ + m \\ p_R \\ p^+ - m \\ p_R \end{pmatrix}, \quad (\text{C3})$$

$$u_{-1/2}(\tilde{p}) = \frac{1}{\sqrt{2p^+}} \begin{pmatrix} -p_L \\ p^+ + m \\ p_L \\ m - p^+ \end{pmatrix}.$$

The gamma matrices are defined as in Ref. [50].

A similar expansion for the  $\Delta$  field involves the Rarita-Schwinger spinors given by

$$u_{3/2}^\mu(\tilde{p}) = \epsilon_{+1}^\mu(\tilde{p}) u_{1/2}(\tilde{p}),$$

$$u_{1/2}^\mu(\tilde{p}) = \sqrt{\frac{2}{3}} \epsilon_0^\mu(\tilde{p}) u_{1/2}(\tilde{p}) + \sqrt{\frac{1}{3}} \epsilon_{+1}^\mu(\tilde{p}) u_{-1/2}(\tilde{p}), \quad (\text{C4})$$

$$u_{-1/2}^\mu(\tilde{p}) = \sqrt{\frac{2}{3}} \epsilon_0^\mu(\tilde{p}) u_{-1/2}(\tilde{p}) + \sqrt{\frac{1}{3}} \epsilon_{-1}^\mu(\tilde{p}) u_{1/2}(\tilde{p}),$$

$$u_{-3/2}^\mu(\tilde{p}) = \epsilon_{-1}^\mu(\tilde{p}) u_{-1/2}(\tilde{p}),$$

where the polarization vectors are given by

$$\epsilon_{+1}^\mu(\tilde{p}) = \left(-\sqrt{2} \frac{p_R}{p^+}, 0, \left(-\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}\right)\right),$$

$$\epsilon_0^\mu(\tilde{p}) = \frac{1}{m} \left(\frac{\mathbf{p}_\perp^2 - m^2}{p^+}, p^+, \mathbf{p}_\perp\right), \quad (\text{C5})$$

$$\epsilon_{-1}^\mu(\tilde{p}) = \left(\sqrt{2} \frac{p_L}{p^+}, 0, \left(\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}\right)\right).$$

In the evaluation of the transition amplitudes we need the following expressions of scalar products

$$(\epsilon_{+1}^\mu)^*(\tilde{p}_\Delta)(p_N - p_\Delta)_\mu = -\frac{k_L}{\sqrt{2}y},$$

$$(\epsilon_0^\mu)^*(\tilde{p}_\Delta)(p_N - p_\Delta)_\mu = \frac{1}{2M_\Delta y} (\mathbf{k}_\perp^2 - M_\Delta^2 + y^2 M_N^2),$$

$$(\epsilon_{-1}^\mu)^*(\tilde{p}_\Delta)(p_N - p_\Delta)_\mu = \frac{k_R}{\sqrt{2}y}, \quad (\text{C6})$$

where the relation  $\mathbf{p}_{\Delta\perp} = \mathbf{k}_\perp + y\mathbf{p}_{N\perp}$  has been used.

The final results for the transition amplitudes  $N \rightarrow B\pi$  with nucleon helicity  $\lambda = \frac{1}{2}$  are given in Table I. The corresponding results for the vertex functions with nucleon helicity  $\lambda = -\frac{1}{2}$  are given by

TABLE I. Vertex functions for  $N \rightarrow N\pi$  and  $N \rightarrow \pi\Delta$ .

| $\lambda \rightarrow \lambda'$         | $V(N, N\pi)$                           | $V(N, \Delta\pi)$  |
|--|--|--|
| $\frac{1}{2} \rightarrow \frac{3}{2}$  |  | $-ig_{N\Delta\pi} \frac{k_L}{\sqrt{2yy}} (M_\Delta + yM_N)$  |
| $\frac{1}{2} \rightarrow \frac{1}{2}$  | $ig_{NN\pi} \frac{M_N(1-y)}{\sqrt{y}}$ | $ig_{N\Delta\pi} \frac{1}{\sqrt{6yyM_\Delta}} [\mathbf{k}_\perp^2 (2M_\Delta + yM_N) + (M_\Delta + yM_N)^2 (yM_N - M_\Delta)]$ |
| $\frac{1}{2} \rightarrow -\frac{1}{2}$ | $-ig_{NN\pi} \frac{k_R}{\sqrt{y}}$     | $-ig_{N\Delta\pi} \frac{k_R}{\sqrt{6yyM_\Delta}} [\mathbf{k}_\perp^2 + (M_\Delta + yM_N)(yM_N - 2M_\Delta)]$                   |
| $\frac{1}{2} \rightarrow -\frac{3}{2}$ |  | $-ig_{N\Delta\pi} \frac{k_R k_R}{\sqrt{2yy}}$  |

$$V_{\lambda',0}^{\lambda(N,N\pi)}(y, \mathbf{k}_\perp) = (-1)^{1/2-\lambda'} V_{-\lambda'}^{-\lambda(N,N\pi)}(y, \hat{\mathbf{k}}_\perp), \quad (C7)$$

$$V_{\lambda'}^{\lambda(N,\Delta\pi)}(y, \mathbf{k}_\perp) = (-1)^{3/2-\lambda'} V_{-\lambda'}^{-\lambda(N,\Delta\pi)}(y, \hat{\mathbf{k}}_\perp),$$

where  $\hat{\mathbf{k}}_\perp = (k_x, -k_y)$ .

Our calculation for the vertex functions is in agreement with the results of Ref. [49]. However, we found discrepancies with the results reported in Ref. [9]. These differ-

ences in the vertex functions do not affect the splitting functions of Eqs. (57) and (59) which enter in the convolution formulas of parton distributions, but are important for the transverse-momentum dependence of the probability amplitudes  $\phi_{\lambda\lambda'}$  which enter in the convolution formulas for the GPDs.

- [1] R. P. Feynman, *Photon-Hadron Interactions* (W. A. Benjamin, New York, 1972).
- [2] J. D. Sullivan, Phys. Rev. D **5**, 1732 (1972).
- [3] A. W. Thomas, Phys. Lett. B **126**, 97 (1983).
- [4] P. Amaudruz *et al.*, Phys. Rev. Lett. **66**, 2712 (1991); Phys. Lett. B **295**, 159 (1992).
- [5] A. Baldit *et al.* (NA51 Collaboration), Phys. Lett. B **332**, 244 (1994).
- [6] E. A. Hawker *et al.* (E866 Collaboration), Phys. Rev. Lett. **80**, 3715 (1998).
- [7] K. Ackerstaff *et al.* (Hermes Collaboration), Phys. Rev. Lett. **81**, 5519 (1998).
- [8] W. Koepf, L. L. Frankfurt, and M. Strikman, Phys. Rev. D **53**, 2586 (1996).
- [9] J. Speth and W. A. Thomas, Adv. Nucl. Phys. **24**, 83 (1998).
- [10] J. T. Londergan and A. W. Thomas, Prog. Part. Nucl. Phys. **41**, 49 (1998).
- [11] S. Kumano, Phys. Rep. **303**, 183 (1998).
- [12] Xiangdong Ji, J. Phys. G Nucl. Part. Phys. **24**, 1181 (1998).
- [13] A. V. Radyushkin, in *At the Frontier of Particle Physics*, edited by M. Shifman (World Scientific, Singapore, 2001), Vol. 2, p. 1037; hep-ph/0101225.
- [14] K. Goeke, M. V. Polyakov, and M. Vanderhaeghen, Prog. Part. Nucl. Phys. **47**, 401 (2001).
- [15] M. Diehl, Phys. Rep. **388**, 41 (2003).
- [16] Ph. Hägler *et al.* (LHPC and SESAM Collaborations), Phys. Rev. D **68**, 034505 (2003).
- [17] X. Ji, W. Melnitchouk, and X. Song, Phys. Rev. D **56**, 5511 (1997).
- [18] V. Yu. Petrov, P. V. Pobylitsa, M. V. Polyakov, I. Börmig, K. Goeke, and C. Weiss, Phys. Rev. D **57**, 4325 (1998).
- [19] M. Penttinen, M. V. Polyakov, and K. Goeke, Phys. Rev. D **62**, 014024 (2000).
- [20] J. Ossmann, M. V. Polyakov, P. Schweitzer, D. Urbano, and K. Goeke, Phys. Rev. D **71**, 034011 (2005).
- [21] M. Wakamatsu and H. Tsujimoto, Phys. Rev. D **71**, 074001 (2005).
- [22] H. Mineo, S. N. Yang, C. Y. Cheung, and W. Bentz, Phys. Rev. C **72**, 025202 (2005).
- [23] B. Pasquini, M. Traini, and S. Boffi, Phys. Rev. D **71**, 034022 (2005).
- [24] A. V. Radyushkin, Phys. Rev. D **59**, 014030 (1999).
- [25] M. Diehl, Th. Feldmann, R. Jakob, and P. Kroll, Nucl. Phys. **B596**, 33 (2001).
- [26] S. J. Brodsky, M. Diehl, and D. S. Hwang, Nucl. Phys. **B596**, 99 (2001).
- [27] S. Boffi, B. Pasquini, and M. Traini, Nucl. Phys. **B649**, 243 (2003).
- [28] S. Boffi, B. Pasquini, and M. Traini, Nucl. Phys. **B680**, 147 (2004).
- [29] B. Pasquini, M. Pincetti, and S. Boffi, Phys. Rev. D **72**, 094029 (2005).
- [30] G. Parisi and R. Petronzio, Phys. Lett. B **62**, 331 (1976).
- [31] R. L. Jaffe and G. C. Ross, Phys. Lett. B **93**, 313 (1980).
- [32] S. D. Drell, D. J. Levy, and Tung-Mow Yan, Phys. Rev. D **1**, 1035 (1970).
- [33] V. R. Zoller, Z. Phys. C **54**, 425 (1992); **60**, 141 (1993).
- [34] P. J. Mulders, A. W. Schreiber, and H. Meyer, Nucl. Phys. **A549**, 498 (1992).
- [35] W. Melnitchouk and A. W. Thomas, Phys. Rev. D **47**, 3794 (1993).
- [36] W. Melnitchouk, J. Speth, and A. W. Thomas, Phys. Rev. D **59**, 014033 (1999).
- [37] C. Boros and A. W. Thomas, Phys. Rev. D **60**, 074017 (1999).

- [38] P. Faccioli, M. Traini, and V. Vento, Nucl. Phys. **A656**, 400 (1999).
- [39] Ho-Meoyng Choi and Chueng-Ryong Ji, Phys. Rev. D **59**, 074015 (1999).
- [40] S.J. Brodsky and G.P. Lepage, in *Perturbative Quantum Chromodynamics*, edited by A.H. Mueller (World Scientific, Singapore, 1989).
- [41] R. Kreckel, Comput. Phys. Commun. **106**, 258 (1997); physics/9710028.
- [42] M. Diehl, Th. Feldmann, R. Jakob, and P. Kroll, Eur. Phys. J. C **8**, 409 (1999).
- [43] F. Cardarelli, E. Pace, G. Salmè, and S. Simula, Phys. Lett. B **357**, 267 (1995).
- [44] B. Pasquini, M. Traini, and S. Boffi, Phys. Rev. D **65**, 074028 (2002).
- [45] S. Simula, in *Proceedings of the Workshop on the Physics of Excited Baryons, NSTAR, Mainz, 2001*, edited by D. Drechsel and L. Tiator (World Scientific, Singapore, 2001), p. 135.
- [46] B. Pasquini and S. Boffi (to be published).
- [47] F. Antonuccio, S.J. Brodsky, and S. Dalley, Phys. Lett. B **412**, 104 (1997).
- [48] H. Holtmann, A. Szczurek, and J. Speth, Nucl. Phys. **A596**, 631 (1996).
- [49] Z. Dziembowski, H. Holtmann, A. Szczurek, and J. Speth, Ann. Phys. (N.Y.) **258**, 1 (1997).
- [50] J.D. Bjorken and S.D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).
- [51] D. Binosi and L. Theussl, Comput. Phys. Commun. **161**, 76 (2004).