Localization of 4D gravity on pure geometrical thick branes

Nandinii Barbosa-Cendejas* and Alfredo Herrera-Aguilar†

Instituto de Física y Matemáticas, Universidad Michoacana de San Nicolás de Hidalgo, Edificio C-3, Ciudad Universitaria, C.P. 58040, Morelia, Michoacán, México

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We consider the generation of thick brane configurations in a pure geometric Weyl integrable 5D spacetime which constitutes a non-Riemannian generalization of Kaluza-Klein (KK) theory. In this framework, we show how 4D gravity can be localized on a scalar thick brane which does not necessarily respect reflection symmetry, generalizing in this way several previous models based on the Randall-Sundrum (RS) system and avoiding both, the restriction to orbifold geometries and the introduction of the branes in the action by hand. We first obtain a thick brane solution that preserves 4D Poincaré invariance and breaks Z_2-symmetry along the extra dimension which, indeed, can be either compact or extended, and supplements brane solutions previously found by other authors. In the noncompact case, this field configuration represents a thick brane with positive energy density centered at $y = c_2$, whereas pairs of thick branes arise in the compact case. Remarkably, the Weylian scalar curvature is nonsingular along the fifth dimension in the noncompact case, in contraposition to the RS thin brane system. We also recast the wave equations of the transverse traceless modes of the linear fluctuations of the classical background into a Schrödinger's equation form with a volcano potential of finite bottom in both the compact and the extended cases. We solve Schrödinger equation for the massless zero mode $m^2 = 0$ and obtain a single bound wave function which represents a stable 4D graviton. We also get a continuum gapless spectrum of KK states with $m^2 > 0$ that are suppressed at $y = c_2$ and turn asymptotically into plane waves.

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I. INTRODUCTION

In the present work we shall consider the formation of thick branes in a particular generalization of Kaluza-Klein theory in which the Riemannian structure of spacetime is enlarged into a Weylian affine manifold where vector lengths may be not preserved along parallel transportation. More precisely, a Weyl geometry is an affine manifold specified by (g_{MN}, ω_M), where g_{MN} is the metric tensor and ω_M is "gauge" vector involved in the definitions of the affine connections of the manifold. The particular type of gauge geometries in which the gauge vector is the gradient of a scalar function is called conformally Riemann or Weyl integrable spacetime, since a conformal transformation maps a Riemann geometry into a Weyl integrable one. If laws of physics were invariant under conformal transformations, the Weyl scalar function would be unobservable. However, since this is not the case, this scalar field cannot be discarded in principle by a convenient gauge choice; moreover, it is enough to dynamically break the conformal invariance of a given Weyl integrable theory to transform the Weyl scalar function into an observable field. Thus, in this scheme, a fundamental role in the generation of thick brane configurations is ascribed to the scalar Weyl field, which is not a bulk field.

On the other hand, the fact that we could live in a higher dimensional spacetime with extended extra dimensions turns out to be completely compatible with present time gravitational experiments. An interesting picture arises in such scenarios: from the point of view of an observer located at a 3-brane in which matter is confined, gravity is essentially 4-dimensional, however, the world can be higher dimensional with infinite extra dimensions and gravity can propagate in all of them. Multidimensional spacetimes with large extra dimensions turned out to be very useful when addressing several problems of high energy physics like the cosmological constant, dark matter and the mass hierarchy problem [1–4] as well as the recent nonsupersymmetric string model realization of the standard model at low energy with no extra massless matter fields [5]. The striking success of these higher dimensional scenarios motivated several generalizations in various directions including thick brane configurations [6–8]. These configurations were generalized in the framework of Weyl geometries for Z_2-symmetric manifolds in [9]. Moreover, localization of 4D gravity on thick branes that break reflection symmetry was presented in [10] for a fixed self-interacting potential of the scalar field ω ($U = \lambda e^{2\omega}$).

In this paper we keep working in a manifold endowed with Weyl structure and present the realization of such a scenario on thick brane solutions made out of scalar matter with a self-interacting potential endowed with an arbitrary parameter ξ: $U = \lambda e^{(1-16\xi)\omega}$, enlarging the class of potentials for which 4D gravity can be localized.

Thus, we begin by considering a 5-dimensional Weyl gravity model in which geometrical thick branes arise naturally without the necessity of introducing them by hand in the action of the theory. In order to obtain solutions which describe such configurations and respect 4D Poincaré invariance, we implement a conformal transformation to pass from the Weyl frame to the Riemann one,

*E-mail: nandinii@ifm.umich.mx
†E-mail: herrera@ifm.umich.mx

where the Weylian affine connections become Christoffel symbols and the field equations are simpler, solve these equations and return to the Weyl frame to analyze the physics of the solution. In what follows we shall refer to this method as the conformal technique. Thus, in this way we obtain a solution that represents a localized function which does not necessarily respect reflection Z_2-symmetry and allows for both compact and noncompact manifolds in the extra dimension. By looking at the energy density of the scalar field of these solutions we interpret the field configuration as thick branes. The structure of these brane configurations depends on the topology of the extra dimension and the value of the parameter $p(\xi)$. We investigate as well the fluctuations of the metric around the classical background solution to understand whether 4D gravity can be described in our setup. We show that this is the case since the analog quantum mechanical problem with a volcano potential for the transverse traceless sector of the fluctuations of the metric yields a continuum and gapless spectrum of Kaluza-Klein (KK) states with a stable zero mode that corresponds to the 4D graviton. We finally make our conclusions. Let us start by considering a pure geometrical Weyl action in five dimensions. This non-Riemannian generalization of the Kaluza-Klein theory is given by

$$S_5^W = \int_{M_5^W} \frac{d^5x\sqrt{|g|}}{16\pi G_5} e^{(3/2)\omega}[R + 3\tilde{\xi}(\nabla\omega)^2 + 6U(\omega)], \quad (1)$$

where M_5^W is a Weyl manifold specified by the pair (g_{MN}, ω), g_{MN} being the metric and ω a Weyl scalar function. The Weylian Ricci tensor reads

$$R_{MN} = \Gamma^A_{MN,A} - \Gamma^A_{AM,N} + \Gamma^P_{MN}\Gamma^Q_{PQ} - \Gamma^P_{MQ}\Gamma^Q_{NP},$$

where

$$\Gamma^C_{MN} = \begin{Bmatrix} C \\ MN \end{Bmatrix} - \frac{1}{2}(\omega_{,M}\delta^C_N + \omega_{,N}\delta^C_M - g_{MN}\omega^{,C})$$

are the affine connections on M_5^W, $\{^{\ C}_{MN}\}$ are the Christoffel symbols and M, $N = 0, 1, 2, 3, 5$; the constant $\tilde{\xi}$ is an arbitrary coupling parameter, and $U(\omega)$ is a self-interaction potential for the scalar field ω. This action is of pure geometrical nature since the scalar field that couples to gravity is precisely the scalar function ω that enters in the definition of the affine connections of the Weyl manifold and, thus, cannot be discarded in principle from our consideration. Apart from the self-interaction potential, the action (1) is invariant under Weyl rescalings

$$g'_{MN} \to \Omega^{-2}g_{MN}, \qquad \omega' \to \omega + \ln\Omega^2, \\ \tilde{\xi}' \to \tilde{\xi}/(1 + \partial_\omega \ln\Omega^2)^2, \quad (2)$$

where Ω^2 is a smooth function on M_5^W. Thus, from these relations it follows that the potential must undergo the transformation $U' \to \Omega^2 U$ in order to keep such an invariance. Thus, $U(w) = \lambda e^{\omega}$, where λ is a constant parameter, is the form of the potential which preserves the scale invariance of the Weyl manifold (1). When this invariance is broken, the Weyl scalar field transforms from a geometrical object into a physically observable matter degree of freedom which, in turn, generates the thick brane configurations.

Since we are looking for a solution to the theory (1) with 4-dimensional Poincaré invariance we shall consider the line element in the form

$$ds_5^2 = e^{2A(y)}\eta_{mn}dx^m dx^n + dy^2, \quad (3)$$

where $e^{2A(y)}$ is the warp factor depending on the extra coordinate y, and m, $n = 0, 1, 2, 3$. Thus, the 5-dimensional stress-energy tensor is given by its 4-dimensional and pure 5-dimensional components

$$T_{mn} = \frac{3}{8\pi G_5}e^{2A}[A'' + 2(A')^2]\eta_{mn}, \qquad T_{55} = \frac{6(A')^2}{8\pi G_5}, \quad (4)$$

where the comma denotes derivatives with respect to the fifth coordinate y.

II. SOLUTIONS TO THE SYSTEM

Since we shall use the conformal technique to find solutions to our system, we perform the conformal transformation $\hat{g}_{MN} = e^{\omega}g_{MN}$, mapping the Weylian action (1) into the Riemannian one

$$S_5^R = \int_{M_5^R} \frac{d^5x\sqrt{|\hat{g}|}}{16\pi G_5}[\hat{R} + 3\xi(\hat{\nabla}\omega)^2 + 6\hat{U}(\omega)], \quad (5)$$

where $\xi = \tilde{\xi} - 1$, $\hat{U}(\omega) = e^{-\omega}U(\omega)$ and all hatted magnitudes and operators are defined in the Riemann frame. In this frame we have a theory which describes 5-dimensional gravity minimally coupled to a scalar field which possesses a self-interaction potential. After this transformation, the line element (3) yields the Riemannian metric

$$\hat{d}s_5^2 = e^{2\sigma(y)}\eta_{nm}dx^n dx^m + e^{\omega(y)}dy^2, \quad (6)$$

where $2\sigma = 2A + \omega$. Further, by following [9] we introduce the new variables $X = \omega'$ and $Y = 2A'$ and get the following pair of coupled field equations from the action (5)

$$X' + 2YX - \frac{3}{2}X^2 = \frac{1}{\xi}\frac{d\hat{U}}{d\omega}e^{-\omega}, \\ Y' + 2Y^2 - \frac{3}{2}XY = \left(\frac{1}{\xi}\frac{d\hat{U}}{d\omega} + 4\hat{U}\right)e^{-\omega}. \quad (7)$$

In general, it is not trivial to fully integrate these field equations. Under some assumptions, it is straightforward to construct several particular solutions to them. However, quite often such solutions lead to expressions of the dynamical variables that are too complicated for an analytical treatment in closed form.

As pointed out in [9], this system of equations can be easily solved if one uses the condition $X = kY$, where k is an arbitrary constant parameter which is not allowed to adopt the value $k = 1$ because the system would be incompatible. It turns out that this restriction leads to a Riemannian potential of the form $\hat{U} = \lambda e^{(4k\xi/1-k)\omega}$. Thus, under these conditions, both field equations in (7) reduce to a single differential equation

$$Y' + \frac{4-3k}{2}Y^2 = \frac{4\lambda}{1-k}e^{[(4k\xi/1-k)-1]\omega}. \tag{8}$$

Those authors noticed as well that by choosing the parameter $\xi = (1-k)/(4k)$ (while leaving the parameter k arbitrary) the exponential function of the right hand side of (8) disappears and the equation can be easily solved. This case corresponds to having a fixed self-interaction potential of the form $U = \lambda e^{2\omega}$ in the Weyl frame which, indeed, breaks the invariance under Weyl rescalings. It is interesting to note that after solving Eq. (8) with such a simplification, the obtained solution

$$\omega(y) = b \ln[\cosh(ay)]$$

does not allow the value $k = 4/3$ (apart from $k = 1$) since the constants involved in it read

$$a = \sqrt{\frac{4-3k}{1-k}2\lambda} \quad \text{and} \quad b = \frac{2}{4-3k}.$$

In this paper we shall consider another truncation that leads to a further simplification and to a simple solution of the Eq. (8). This can be done by setting $k = 4/3$, while leaving ξ arbitrary, allowing to have a self-interaction potential

$$U = \lambda e^{(1-16\xi)\omega}$$

in the Weyl frame. In this sense our solution supplements the solution obtained in [9,10]. This potential breaks the invariance under Weyl scaling transformations for arbitrary $\xi \neq 0$ and also transforms the geometrical scalar field ω into an observable one [11]. Thus, after imposing the condition $k = 4/3$, the second term in the left hand side of the Eq. (8) vanishes, yielding

$$Y' + 12\lambda e^{-p\omega} = 0 \quad \text{or} \quad \omega'' + 16\lambda e^{-p\omega} = 0, \tag{9}$$

where $p = 1 + 16\xi$.

By solving the latter equation for ω and integrating the relation $2A' = 3\omega'/4$ one gets the following solution

$$\omega = \frac{2}{p}\ln\left[\frac{\sqrt{-8\lambda p}}{c_1}\cosh(c_1(y-c_2))\right], \qquad e^{2A} = \left[\frac{\sqrt{-8\lambda p}}{c_1}\cosh(c_1(y-c_2))\right]^{3/2p}, \tag{10}$$

where c_1 and c_2 are arbitrary integration constants, and we have set to one a constant that multiplies the warp factor.

By looking at the solution, we see that for $p < 0$ it constitutes a localized object which does not necessarily preserve the reflection Z_2-symmetry ($y \to -y$) along the fifth dimension and breaks it through nontrivial values of the shift parameter c_2. Thus, the 5-dimensional spacetime is not restricted to be an orbifold geometry, allowing for a more general type of manifolds. The extra coordinate can be compact or extended depending on the signs of the constants p and λ, and the real or imaginary character of the parameter c_1 which, indeed, characterizes the width of the warp factor $\Delta \sim 1/c_1$. Let us consider the cases of physical interest:

(A) $\lambda > 0$, $p < 0$, $c_1 > 0$. In this case the domain of the fifth coordinate is $-\infty < y < \infty$; thus, we have a noncompact manifold in the extra dimension. It turns out that in this case the warp factor is concentrated near $y = c_2$ and represents a smooth localized function of width Δ which remarkably reproduces the metric of the RS model in the thin brane limit, namely, when $c_1 \to \infty$, $p \to -\infty$ keeping $c_1/p = \beta$ finite.
The energy density μ of the scalar matter is given by the null-null component of the stress-energy tensor:

$$\mu(y) = \frac{-9c_1^2}{32p\pi G_5}\left[\frac{\sqrt{-8\lambda p}}{c_1}\cosh(c_1(y-c_2))\right]^{3/2p} \times \left[1 + \frac{3-2p}{2p}\tanh^2(c_1(y-c_2))\right]. \tag{11}$$

It represents a thick brane with positive energy density [12] centered at $c_2 = 2$. In Fig. 1 we display the function μ together with its thin brane limit, it shows a positive maximum at $y = c_2$ and a mini-

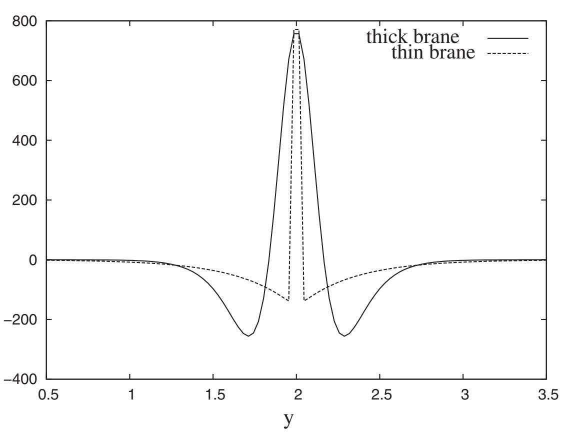

FIG. 1. The qualitative behavior of the scalar energy density function μ for the noncompact case A) and its ‘‘rescaled’’ thin brane limit. The thick brane with positive energy density is centered at $c_2 = 2$ and has $c_1 = 4$.

mum at each side of the maximum, vanishing as y approaches infinity.

The 5-dimensional curvature scalar adopts the form

$$R_5 = \frac{-6c_1^2}{p}\left[1 + \frac{15-8p}{8p}\tanh^2(c_1(y-c_2))\right], \tag{12}$$

and is always bounded, thus, we have a 5-dimensional manifold that is nonsingular, in opposition to the RS model, where the 5-dimensional curvature scalar is singular.

(B) $\lambda > 0$, $p > 0$, $c_1 = iq_1$. In this case we get a compact manifold along the extra dimension with $-\pi \le q_1(y - c_2) \le \pi$. The expressions for the warp factor and the scalar field in this compact case read

$$e^{2A(y)} = \left[\frac{\sqrt{8\lambda p}}{q_1}\cos(q_1(y-c_2))\right]^{3/2p}, \qquad \omega = \frac{2}{p}\ln\left[\frac{\sqrt{8\lambda p}}{q_1}\cos(q_1(y-c_2))\right]. \tag{13}$$

The corresponding energy density of the scalar matter is given by

$$\mu(y) = \frac{9q_1^2}{32p\pi G_5}\left[\frac{\sqrt{8\lambda p}}{q_1}\cos(q_1(y-c_2))\right]^{3/2p} \times\left[1 + \frac{2p-3}{2p}\tan^2(q_1(y-c_2))\right]. \tag{14}$$

The shape of the scalar energy density μ is plotted in Fig. 2 for $p = 1/2$ and $p = 3/8$. The structure of the thick branes depends on the value of the parameter $p(\xi)$. Thus, for instance, when $3/(2p) = 2n$, $(n = 2, 3, 4, 5, \ldots)$, the shape of μ corresponds to the case $p = 3/8$, a pair of smooth thick branes of the form displayed in Fig. 1; whereas for $3/(2p) = 2n - 1$, $(n = 2, 4, 5, 6, \ldots)$, the structure of μ is similar to the case $p = 1/2$ and here we have one thick brane of the form plotted in Fig. 1, together with another "inverted brane", a fact that reflects the change of sign in the warp factor when $y = \pm\frac{\pi}{2q_1} + c_2$. Actually, these branes live in different disconnected regions of the Weyl manifold due to the fact that the 5d curvature scalar is singular at the points $y = \pm\frac{\pi}{2q_1} + c_2$:

$$R_5 = \frac{6q_1^2}{p}\left[1 + \frac{8p-15}{8p}\tan^2(q_1(y-c_2))\right], \tag{15}$$

where plausibly we have null scalar energy densities.

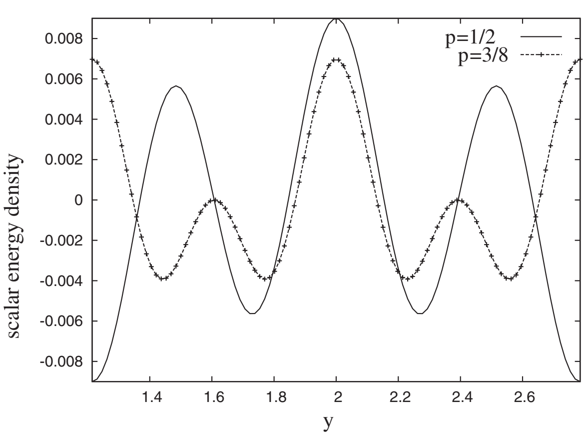

FIG. 2. The shape of the energy density function μ for the compact case B) when $p = 1/2$ and $p = 3/8$ with $c_2 = 2$ and $c_1 = 4$. Thick branes are separated at $y = \pm\frac{\pi}{2q_1} + c_2$ due to the singular character of the Weyl manifold at this points.

Other cases of physical interest are contained in A) and B), namely, the discrete cases $\lambda > 0$, $p < 0$, $c_1 < 0$ with $p = -3/(4n)$ and $\lambda < 0$, $p < 0$, $c_1 < 0$ (or $c_1 > 0$) with $p = -3/(8n)$, where $n \neq 0$, $n \in N$, are contained in the noncompact case A), whereas the cases $\lambda > 0$, $p > 0$, $c_1 = -iq_1$ with $p = 3/(4n)$ and $\lambda < 0$, $p > 0$, $c_1 = \pm iq_1$ with $p = 3/(8n)$ are included in the case B). The remaining possible values of these parameters lead to unphysical situations in which the warp factor and the scalar energy density are singular at certain values of the fifth dimension y and, hence, do not represent localized functions.

III. FLUCTUATIONS OF THE METRIC

Let us turn to study the metric fluctuations h_{mn} of the metric (3) given by the perturbed line element

$$ds_5^2 = e^{2A(y)}[\eta_{mn} + h_{mn}(x, y)]dx^m dx^n + dy^2. \tag{16}$$

Even if one cannot avoid considering fluctuations of the scalar field when treating fluctuations of the background metric, in [6] it was shown that the transverse traceless modes of the metric fluctuations decouples from the scalar sector and hence, can be approached analytically.

By following this method, we perform the coordinate transformation $dw = e^{-A}dy$, which leads to a conformally flat metric and to the following wave equation for the transverse traceless modes h_{mn}^T of the metric fluctuations

$$(\partial_w^2 + 3A'\partial_w + \Box^\eta)h_{mn}^T = 0. \tag{17}$$

This equation supports a massless and normalizable 4D graviton given by $h_{mn}^T = C_{mn}e^{imx}$, where C_{mn} are constant parameters and $m^2 = 0$.

In [3] it was proved useful to recast Eq. (17) into Schrödinger's equation form. In order to accomplish this, we adopt the following ansatz for the transverse traceless modes of the fluctuations $h_{mn}^T = e^{imx}e^{-3A/2}\Psi_{mn}(w)$ and get

$$[\partial_w^2 - V(w) + m^2]\Psi = 0, \tag{18}$$

where we have dropped the subscripts in Ψ, m is the mass of the KK excitation, and the potential reads

$$V(w) = \tfrac{3}{2}\partial_w^2 A + \tfrac{9}{4}(\partial_w A)^2. \qquad (19)$$

For the particular noncompact case A) we have found two particular cases ($p = -1/4$ and $p = -3/4$) for which we can invert the coordinate transformation $dw = e^{-A}dy$ and explicitly express y in terms of w; although the first case is more involved, it is qualitatively equivalent to the second one in the sense that their potential presents a similar behavior. For the sake of simplicity we shall consider just the simplest case $p = -3/4$. Thus, the function $A(w)$ adopts the form

$$A(w) = \ln[c_1/\sqrt{c_1^4(w-w_0)^2 + 6\lambda}]$$

which yields the following quantum mechanical potential

$$V(w) = \frac{3c_1^4[5c_1^4(w-w_0)^2 - 12\lambda]}{4[c_1^4(w-w_0)^2 + 6\lambda]^2}. \qquad (20)$$

In the Schrödinger equation, the spectrum of eigenvalues m^2 parameterizes the spectrum of graviton masses that a 4-dimensional observer located at w_0 sees. It turns out that for the zero mode $m^2 = 0$, this equation can be solved. The only normalizable eigenfunction reads

$$\Psi_0 = q[c_1^4(w-w_0)^2 + 6\lambda]^{-3/4},$$

where q is a normalization constant. This function represents the lowest energy eigenfunction of the Schrödinger Eq. (18) since it has no zeros. This fact allows for the existence of a 4D graviton with no instabilities from transverse traceless modes with $m^2 < 0$. In addition to this massless mode, there exists a tower of higher KK modes with positive $m^2 > 0$.

It turns out that a similar situation takes place in the compact case B). Remarkably, the coordinate transformation $dw = e^{-A}dy$ can be inverted for $p = 3/8$ yielding

$$\cos(q_1(y-c_2)) = \pm q_1/\sqrt{q_1^2 + 9\lambda^2(w-w_0)^2},$$

i.e., decompactifying the fifth dimension and pushing to infinity the singularities (an inverse effect takes place when one compactifies the radial coordinate r in the Schwarzschild and Kerr solutions [13], see [14]). This mathematical fact implies that we actually have two disconnected regions in the Weyl manifold: the region $-\frac{\pi}{2} \le q_1(y-c_2) \le \frac{\pi}{2}$ is separated from the region $\frac{\pi}{2} \le q_1(y-c_2) \le \frac{3\pi}{2}$ (since we can shift the domain of the compact dimension to $-\frac{\pi}{2} \le q_1(y-c_2) \le \frac{3\pi}{2}$) by the physical singularities located at $y = \pm(\pi/2q_1) + c_2$ (recall that the curvature scalar is singular at these points). Each one of these regions leads to

$$A(w) = \ln\{3\lambda/[q_1^2 + 9\lambda^2(w-w_0)^2]\}$$

and, hence, to the following potential

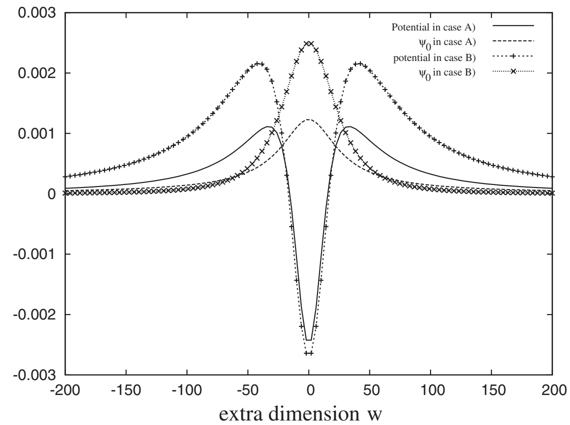

FIG. 3. The shape of the potential $V(w)$ and the zero mode wave function Ψ_0 centered at $w_0 = 0$ for cases A) and B).

$$V(w) = -\frac{27\lambda^2[q_1^2 - 36\lambda^2(w-w_0)^2]}{[q_1^2 + 9\lambda^2(w-w_0)^2]^2} \qquad (21)$$

and the wave function corresponding to the zero mode

$$\Psi_0 = k[q_1^2 + 9\lambda^2(w-w_0)^2]^{-3/2},$$

where k is a constant.

In Fig. 3 we display the shape of these potentials and their respective zero mode wave functions. Both potentials have the volcano form: a well of finite bottom and positive barriers at each side that vanish asymptotically. The wave functions are lumps localized around w_0. These facts imply that we have only one gravitational bound state (the massless one) and a continuous and gapless spectrum of massive KK states with $m^2 > 0$ in both cases A) and B).

Thus, we have obtained Weylian thick brane generalizations of the RS model with no reflection symmetry imposed in which the 4D effective theory possesses an energy spectrum quite similar to the spectrum of the thin wall case, in particular, 4D gravity turns out to be localized at a certain value of the fifth dimension in both cases A) and B).

IV. CONCLUDING REMARKS

We considered the formation of thick brane configurations in a geometric Weyl integrable manifold. We used the conformal technique to obtain a solution which preserves 4D Poincaré invariance and, in particular, represents a smooth localized function characterized by the width parameter $\Delta \sim 1/c_1$ and the constant c_2 which breaks the Z_2-symmetry along the extra dimension; both of these parameters are integration constants of the relevant field Eq. (9), in contraposition to the solutions obtained in [9,10], where the width parameter $\Delta \sim 1/a(\lambda, k)$ depends on the coupling constant of the potential $U(\omega)$ and the constant k. Our field configurations correspond to thick brane generalizations of the RS model which do not restrict the 5-dimensional spacetime to be an orbifold geometry, a fact that can be useful in approaching several issues like the cosmological constant problem, black hole physics and holographic ideas, where there is a relationship between the position in the extra dimension and the mass scale [15]. These thick branes supplement previously found solutions with a new family in which the self-interacting potential is endowed with an arbitrary parameter ξ: $U = \lambda e^{(1-16\xi)\omega}$, enlarging the class of potentials for which 4D gravity can be localized.

In the noncompact case A), the scalar energy density μ can be interpreted as a generic thick brane with positive energy density centered at $y = c_2$ and accompanied by a small amount of negative energy density at each side; the corresponding warp factor reproduces the metric of the RS model in the thin brane limit, even if the matter content of the theory does not correspond to the same brane configuration. A remarkable fact is that in this case, the scalar curvature of the Weyl integrable manifold turns out to be completely regular in the extra dimension. In the compact case B) the situation is different: we have several pairs of thick brane configurations disconnected by physical singularities. The structure of these branes depends on the value of the parameter $p(\xi)$. In a special case ($p = 3/8$) we managed to perform a coordinate transformation which makes the metric conformally flat, decompactifies the fifth dimension and simultaneously pushes the singularities of the manifold to infinity!

We wrote the wave equations of the transverse traceless modes of the linear fluctuations of the metric into the Schrödinger's equation form for both cases A) and B). The analog quantum mechanical potential involved in it represents a volcano potential with finite bottom: a negative well located between two finite positive barriers that vanish when $w \to \pm\infty$. It turned out that for the massless zero modes ($m^2 = 0$) the Schrödinger equation can be solved in both cases. As a result of this fact, in each case we obtained an analytic expression for the lowest energy eigenfunction of the Schrödinger equation which represents a single bound state and allows for the existence of a stable 4D graviton since there are no tachyonic modes with $m^2 < 0$. Apart from these massless states, we also got a continuum and gapless spectrum of massive KK modes with positive $m^2 > 0$ that are suppressed at $y = c_2$ and turn asymptotically into continuum plane waves in both cases A) and B), as in [4,6,10].

The shape of the analog quantum mechanical potential and the localization of 4D gravity on thick branes with a continuum and gapless spectrum of massive KK modes are quite similar to those obtained by [4,6,7].

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[11] In fact, the choice of the parameter $\xi = 1/16$ leads to a constant self-interacting potential $U = \lambda$ in the original Weyl frame, a case which is not present in the setup

considered by [9,10].
[12] As in [10], we initially thought that we had one thick brane with positive energy density and two branes with negative energy density, however, after taking the thin brane limit, we realize that we have a single brane since their parts cannot be separated at all.
[13] We are really grateful to Professor U. Nucamendi for drawing our attention to this effect and the correponding reference.
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