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Evaporation induced traversability of the Einstein-Rosen wormhole

S. Krasnikov

The Central Astronomical observatory at Pulkovo, Russia

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Suppose the Universe comes into existence (as classical spacetime) already with an empty spherically symmetric macroscopic wormhole present in it. Classically the wormhole would evolve into a part of the Schwarzschild space and thus would not allow any signal to traverse it. I consider semiclassical corrections to that picture and build a model of an evaporating wormhole. The model is based on the assumption that the vacuum polarization and its backreaction on the geometry of the wormhole are weak. The lack of information about the era preceding the emergence of the wormhole results in appearance of three parameters which—along with the initial mass—determine the evolution of the wormhole. For some values of these parameters the wormhole turns out to be long-lived enough to be traversed and to transform into a time machine.

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I. INTRODUCTION

The question as to whether there are traversable wormholes in the Universe is at present among the most important problems of classical relativity. The reason is that in the course of its evolution a spacetime with such a wormhole is apt to develop the Cauchy horizon [1]. At one time it was believed that closed timelike curves must lurk beyond the horizon and it was commonplace to tie existence of wormholes with possibility of time machines. Later it has become clear that the two phenomena are not directly connected—the spacetime *always* can be extended through the Cauchy horizon in infinitely many ways, all these extensions being equal (since *none* of them is globally hyperbolic), and always some of them are causal [2]. Nevertheless, the fact remains that having a wormhole one can (try to) force the spacetime to choose between a number of continuations and we have no idea as to the criteria of the choice.[1] That is the existence of traversable wormholes would possibly imply the existence of an unknown classical (though nonlocal) law governing the evolution of the Universe.

The process of emergence of the classical spacetime from what precedes it is not clear yet (to say the least). So, it is well possible that the whole problem is spurious and there are no traversable wormholes just because they have never appeared in the first place. In principle, one can speculate that there is a mechanism suppressing formation of any topological “irregularity“ at the onset of the classical universe. However, at present nothing suggests the existence of such a mechanism and it seems reasonable to pose the question: assuming a wormhole did appear in the end of the Planck era, what would happen with it? Would it last for long enough to threaten global hyperbolicity?

Traditionally in searching for traversable wormholes one picks a stationary (and hence traversable) wormhole and looks for matter that could support it. However, in none of the hitherto examined wormholes the required matter looks too realistic. In some cases it is phantom matter with a prescribed equation of state [5], in some others—classical scalar field [6]. True, two wormholes are known [7,8] the matter content of which is less exotic in that it at least obeys the weak energy condition (all necessary [1,9] violations of the latter being provided by the vacuum polarization). However, the first of them has the throat $67l_{\rm Pl}$ wide and therefore, being nominally a wormhole, can scarcely be called traversable. The second is macroscopic (arbitrarily large, in fact), but needs some classical matter. Though this matter does satisfy the weak energy condition, nothing at the moment is known about how realistic it is in other respects. In this paper I take a different approach: first, I fix the initial form and the matter content of the wormhole trying to choose them as simple as possible (the hope is that the simpler is the model the better are the chances that it reflects general properties of the real wormholes). Then I subject it to the (semiclassical) Einstein equations

$$G_{\mu\nu} = 8\pi T^{c}_{\mu\nu} + 8\pi T_{\mu\nu}$$

(here $T_{\mu\nu}$ is the expectation value of the quantum stress-energy tensor and $T^{c}_{\mu\nu}$ is the contribution of the classical matter) and trace the evolution of this presumably realistic wormhole testing it for traversability.

The wormhole under consideration—I shall denote it $M_{\rm wh}$—comes into being in the end of the Planck era as the Schwarzschild space with mass m_0 (to be more precise, as a three-dimensional subspace $\mathcal{S}$ thereof), hence the name Einstein-Rosen wormhole. The form of $\mathcal{S}$ is defined by trans-Planckian physics that gives birth to the wormhole. I set three conditions on $\mathcal{S}$, of which only one seems to lead

[1] The Cauchy horizons are also expected inside the black holes, but we are protected from whatever is beyond them by the event horizons. At the same time a wormhole enables a mad scientist with finite resources to destroy the Universe, as is romantically put by Wald [3]. For discussion on quantum effects that may, or may not save the Universe, see [4].

to noticeable loss in generality. Each of the allowed $\mathcal{S}$ is characterized by three numbers—κ_R, κ_L, and ϖ. For a given mass ϖ is related to the minimal possible radius of $\mathcal{S}$ and, when ϖ is fixed, $\kappa_{R(L)}$ loosely speaking measures the delay between the end of Planck era near the throat and in the remote parts of the right (left) asymptotically flat region (I mostly consider an "interuniverse wormhole" [10], i.e. a spacetime with two asymptotically flat regions connected by a throat; an "intrauniverse wormhole" is constructed in Section IV by identifying parts of these regions, correspondingly a new parameter d—the distance between the mouths—appears).

The wormhole is taken to be empty: $T^{c}_{\mu\nu} = 0$ (for reasons of simplicity again). Hence, classically it would be just (a part of) the Schwarzschild space M_S, which is a standard of nontraversability [1]. But the Schwarzschild black hole, as is well known, evaporates, that is quantum effects in M_S give rise to a nonzero vacuum stress-energy tensor $\mathring{T}_{\mu\nu}$. So, by the Einstein equations M_{wh} is anything but M_S. Determination of its real geometry is, in fact, a longstanding problem, see e.g. [11] for references and [12] for some discussion on its possible relation to the wormholes. In this paper I make no attempts to solve it. It turns out that to study traversability of a wormhole all one needs to know is the metric in the immediate vicinity of the apparent horizons and, fortunately, for wormholes with the proper values of ϖ this—simpler—problem can be solved separately.

To that end I make a few assumptions based on the idea that quantum effects are relatively weak. Roughly, I assume that the system (Einstein equations + quantum field equations) has a solution M_{wh} with the geometry resembling that of the Schwarzschild space—and coinciding with the latter on $\mathcal{S}$—and with $T_{\mu\nu}$ close to that of the conformal scalar field in the Unruh vacuum (what exactly the words "resembling" and "close" mean in this context is specified in Section II C). Though the above-mentioned assumptions are quite usual and on the face of it seem rather innocent, in *some* situations, as we shall see, they cannot be true (which on the second thought is not surprising—one does not expect the vacuum polarization to be weak near the singularity, or in the throat at the moment of its maximal expansion). Therefore the consideration will be restricted to the class of wormholes with $\varpi \in (1, \frac{\sqrt{5}+1}{2})$.

Throughout the paper the Planck units are used: $G = c = \hbar = 1$ and the mass m_0 is supposed to be large in these units.

II. THE MODEL AND THE ASSUMPTIONS

A. The Schwarzschild spacetime

We begin with recapitulating some facts about the Schwarzschild space, which will be needed later.

The eternal (though nonstatic) spherically symmetric empty wormhole is described by the Schwarzschild metric, which we shall write in the form

$$\mathrm{d}s^2 = -\mathring{F}^2(u,v)\mathrm{d}u\mathrm{d}v + \mathring{r}^2(u,v)(\mathrm{d}\theta^2 + \cos^2\theta\mathrm{d}\phi), \quad (1)$$

where

$$\mathring{F}^2 = 16m_0^2x^{-1}e^{-x}, \qquad \mathring{r} = 2m_0x \quad (2)$$

and the function $x(u, v)$ is defined implicitly by the equation

$$uv = (1-x)e^x. \quad (3)$$

It is easy to check that the following relations hold

$$\mathring{r}_{,v} = -\frac{2m_0u}{xe^x}, \quad (4a)$$

$$\mathring{r}_{,u} = -\frac{2m_0v}{xe^x} = 2m_0\frac{x-1}{ux}, \quad (4b)$$

$$\mathring{r}_{,uv} = -2m_0\frac{e^{-x}}{x^3}, \quad (4c)$$

$$\mathring{\varphi}_{,u} = -\frac{1}{2}(\ln x + x)_{,u} = -\frac{1+x}{2x}x_{,u} \qquad \text{where } \mathring{\varphi} \equiv \ln\mathring{F}. \quad (4d)$$

In the Unruh vacuum the expectation value of the stress-energy tensor of the conformal scalar field has the following structure:

$$4\pi\mathring{T}_{vv} = \tau_1\mathring{r}_{,u}^{-2}, \qquad 4\pi\mathring{T}_{uu} = \tau_2\mathring{r}_{,u}^2m_0^{-4},$$
$$4\pi\mathring{T}_{uv} = \tau_3m_0^{-2}, \qquad 4\pi\mathring{T}_{\theta\theta} = 4\pi\cos^{-2}\theta\mathring{T}_{\phi\phi} = \tau_4m_0^{-2}$$

(all remaining components of $\mathring{T}_{\mu\nu}$ are zero due to the spherical symmetry), where τ_i are functions of x, but not of u, v, or m_0, separately. What is known about $\tau_i(x)$ supports the idea that in the Planck units they are small. In particular, $|\tau_i(1)| \lesssim 10^{-3}$ and K defined in (5a) is $\approx 5\times10^{-6}$ as follows from the results of [13,14], see the appendix. At the horizons $\mathring{h}$, which in this case are the surfaces $x = 1$,

$$\mathring{r}_{,u}^2\mathring{T}_{vv}\Big|_{\mathring{h}} = \frac{\tau_1(1)}{4\pi} = -\frac{\mathring{F}^4(1)K}{16m_0^4}, \qquad K \equiv -\frac{\tau_1(1)e^2}{64\pi}, \quad (5a)$$

$$\mathring{r}_{,u}^{-2}\mathring{T}_{uu}\Big|_{\mathring{h}} = \frac{\tau_2(1)}{4\pi m_0^4}, \quad (5b)$$

$$|\mathring{T}_{uv}|\Big|_{\mathring{h}} = \frac{\tau_3(1)}{4\pi m_0^2} \ll \frac{\mathring{F}^2}{64\pi m_0^2}. \quad (5c)$$

B. The geometry of the Einstein-Rosen wormhole

The wormhole M_{wh} being discussed is a spacetime with the metric

$$\mathrm{d}s^2 = -F^2(u,v)\mathrm{d}u\mathrm{d}v + r^2(u,v)(\mathrm{d}\theta^2 + \cos^2\theta\mathrm{d}\phi). \quad (6)$$

To express the idea that the wormhole is "initially Schwarzschildian" we require that there should be a surface $\mathcal{S}$ such that F, r, and their first derivatives are equal, on $\mathcal{S}$, to $\mathring{F}$, $\mathring{r}$, and their derivatives, respectively. The surface is subject to the following three requirements:

(a) It is spacelike between the horizons, i.e. at $x < 1$;

(b) For the points of $\mathcal{S}$ with $x > 1$ and $u > v$ the dependence $u(v)$ is a smooth positive function without maximums. The same must hold also with v and u interchanged;

(c) Far from the wormhole (i.e. at $r \gg m_0$) $\mathcal{S}$ must be just a surface of constant Schwarzschild time, that is it must be given by the equation $u/v = $ const. Thus (see Fig. 1) for any point $p \in \mathcal{S}$ with $r(p) \gg m_0$

$$\begin{aligned} v(p) > u(p) &\Rightarrow u(p) = -\kappa_R v(p), \\ v(p) < u(p) &\Rightarrow v(q) = -\kappa_L u(q), \end{aligned} \qquad \kappa_R, \kappa_L > 0.$$

Condition (a) restricts substantially the class of wormholes under examination, in contrast to (b), which is of minor importance and can be easily weakened, if desired. The idea behind (c) is that far from the wormhole the Schwarzschild time becomes the "usual" time and that the Planck era ended—by that usual time—simultaneously in different regions of the Universe. Though, remarkably, (c) does not affect the relevant geometrical properties of M_{wh}, it proves to be very useful in their interpretation. In particular, it enables us to assign in an intuitive way the "time" T to any event p' near the throat of the wormhole. Namely, p' happens at the moment when it is reached by the photon emitted in the end of Planck era from the point p (or p'') located in the left (respectively, right) asymptotically flat region. The distance from this point to the wormhole—when it is large enough—is approximately $2m_0 \ln u^2(p')\kappa_L$ (respectively, $\approx 2m_0 \ln v^2(p')\kappa_R$). Taking this distance to be the measure of the elapsed time from the end of the Planck era we define

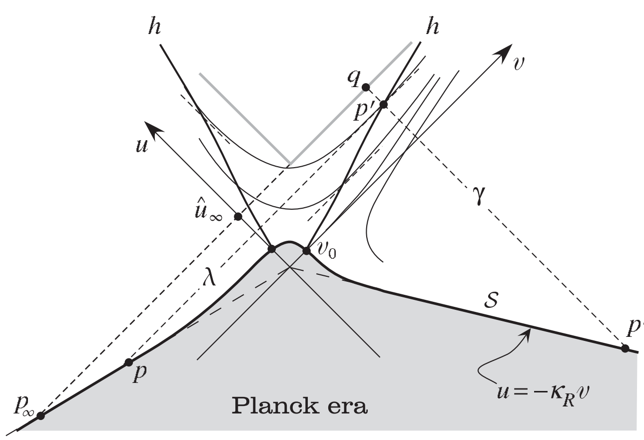

FIG. 1. The section $\phi = \theta = 0$ of the Einstein-Rosen wormhole. The thin solid lines are surfaces $r = $ const. The gray angle is the event horizon.

$$T_L(p') = 2m_0 \ln u^2(p')\kappa_L, \qquad T_R(p') = 2m_0 \ln v^2(p')\kappa_R \tag{7}$$

(even though ∇T is null, as is with "advanced" and "retarded" time in the Schwarzschild case). Note that as long as we consider the two asymptotically flat regions as different and totally independent (i.e. up to Section IV) there is no relation between κ_R and κ_L, nor is there a preferred value for either of them.

Among other things the choice of $\mathcal{S}$ fixes the coordinates u and v up to a transformation

$$u \mapsto u' = Cu, \qquad v \mapsto v' = C^{-1}v. \tag{8}$$

To fix this remaining arbitrariness and thus to make formulas more compact we require

$$u_0 = v_0,$$

where u_0 and v_0 are the coordinates of the intersections of $\mathcal{S}$ with the coordinate axes, see Fig. 1. Though no reasons are seen to think that wormholes with some particular values of v_0 are more common than with any other, we restrict our consideration to those with

$$1 < \varpi < \tfrac{\sqrt{5}+1}{2}, \quad \text{where } \varpi \equiv e\eta/v_0^2, \qquad \eta \equiv 16\pi K m_0^{-2}.$$

As we shall see below the wormholes with smaller ϖ may be nontraversable, while those with larger ϖ evaporate too intensely and cannot be studied within our simple model. To summarize, we have four independent parameters m_0, ϖ, and $\kappa_{R(L)}$, all values of which are considered equally possible as long as $m_0 \gg 1$, $\varpi \in (1, \frac{\sqrt{5}+1}{2})$, and $\kappa_{R(L)} > 0$.

Our subject will be the (right, for definiteness) horizon, by which I understand the curve h lying in the (u, v)-plane and defined by the condition

$$r_{,v}\Big|_h = 0. \tag{9}$$

By (4a) $r_{,v}$ is negative in all points of $\mathcal{S}$ with $u > 0$ and vanishes in the point $(0, v_0)$. In this latter point the horizon starts. h cannot return to $\mathcal{S}$, because there are no more points $r_{,v} = 0$ on $\mathcal{S}$ [by condition (b)]. Neither can it have an end point, being a level line of the function with nonzero [by condition (12a) imposed below] gradient. So, h goes from $\mathcal{S}$ to infinity dividing the plane (u, v) above $\mathcal{S}$ into two parts: $r_{,v}$ is strictly negative to the left of h and strictly positive to the right. So the horizon exists and is unique. The physical meaning of h is that its each small segment shows where the event horizon would pass if the evolution stopped at this moment. The metric in that case would be just the Schwarzschild metric with mass

$$m(v) \equiv \tfrac{1}{2} r(h(v)). \tag{10}$$

The fact that h can be parametrized by v, as is implied in this expression, will become obvious below. Alternatively the horizon can be parametrized by m.

Notation. From now on we shall write $\hat{f}$ for the restriction of a function $f(u, v)$ to h. In doing so we view $\hat{f}$ as a function of v or m depending on which parametrization is chosen for h (this is a—slight—abuse of notation, because strictly speaking $\hat{f}(v)$ and $\hat{f}(m)$ are different functions, but no confusion must arise). Partial derivatives are, of course, understood to act on f, not on $\hat{f}$. Thus, for example,

$$m = \frac{1}{2}\hat{r}, \qquad \frac{\partial}{\partial v}\hat{r}_{,u} = r_{,uv}(h(v)),$$
$$\hat{\varphi}_{,uv}(m) = \varphi_{,uv}(h(m)), \qquad \text{etc.}$$

In conformity with this notation the function $v \to u$ whose graph is h will be denoted by $\hat{u}(v)$, while $\hat{u}(m)$ is a shorthand notation for $\hat{u}(v(m))$.

Traversability of the wormhole is determined by the fact that $\hat{u}(m)$ tends to $\hat{u}_\infty > v_0$ as $m \to 1$ (what happens at smaller m is, of course, beyond the scope of this paper). Indeed, consider a null geodesic λ given by $u = u(p')$, where $p' \in h$. In our model $\hat{r}_{,vv}$ is strictly positive [see Eq. (26) below] and hence λ intersects h once only. As we have just discussed $r_{,v}$ is negative in all points of λ preceding p' and is positive afterwards. So, r reaches in p' its minimum on λ. That is the photon emitted in $p = \lambda \cap \mathcal{S}$ passes in p' the throat of the wormhole[2] and *escapes to infinity.* As we move from p to the left the same reasoning applies to all photons as long as their u-coordinates are small enough to enforce the intersection of h and λ. The boundary of this region is generated by the points p_∞ with

$$u(p_\infty) = \hat{u}_\infty \equiv \sup_{m\in(1,m_0)} \hat{u}(m)$$

(as we shall see the supremum is provided, in fact, by $m = 1$). Correspondingly, we define the *closure time*—the moment when the wormhole ceases to be traversable for a traveler wishing to get from the left asymptotically flat region to the other one:

$$T_{\rm L}^{\rm cl} \equiv 2m_0 \ln \hat{u}_\infty^2 \kappa_L.$$

Similarly is defined the *opening time* $T_{\rm L}^{\rm op} \equiv 2m_0 \ln \hat{u}_0^2 \kappa_L$. So, the time of traversability of a wormhole is

$$T_{\rm L}^{\rm trav} = T_{\rm L}^{\rm cl} - T_{\rm L}^{\rm op} = 4m_0 \ln\frac{\hat{u}_\infty}{\hat{u}_0}. \qquad (11)$$

Thus the goal of the paper is essentially to estimate $\hat{u}_\infty/\hat{u}_0$.

Remark 1. The fact that $r \geq r(p')$ for all points of λ, guarantees that within our model no photon from the singularity $r = 0$ will come out of the wormhole. So, in spite of evaporation and the weak energy condition violations involved, the wormhole fits in with the (weak) cosmic censorship conjecture.

[2] I call it a throat just because it is the narrowest place on the photon's way, but, since λ is orthogonal to the sphere $u, v =$ const through p', this term is in agreement with what is proposed in [15].

C. Weak evaporation assumption

The physical assumption lying in the heart of the whole analysis is the "evaporation stability" of the Einstein-Rosen wormhole, i.e. I assume that there is a solution of the system (Einstein equations + field equations) which starts from $\mathcal{S}$ and has the following property: the geometry in a small neighborhood of any point p is similar to that in a point $\mathring{p}$ of the Schwarzschild space with mass $\mathring{m}$ (of course $\mathring{p}$ and $\mathring{m}$ depend on p), while the stress-energy tensor in p is small and close to $\mathring{T}_{\mu\nu}(\mathring{m}, x(\mathring{p}))$.

More specifically I require of $M_{\rm wh}$ that to the future of $\mathcal{S}$

$$r_{,uv} < 0 \qquad (12a)$$

[cf. (4c)] and

$$\nabla r \neq 0. \qquad (12b)$$

The latter means that at $\mathcal{S}$ the throat of the wormhole is already contracting and that later contraction does not pass into expansion.

The requirement that $T_{\mu\nu}$ in a point $p \in h$ is close to $\mathring{T}_{\mu\nu}(m, 1)$ is embodied in the assumption that the relations (5) are valid when the sign $^\circ$ is removed and m_0 is replaced with m:

$$\hat{r}_{,u}^{\,2}\hat{T}_{vv} = -\tfrac{K}{16}\hat{F}^4 m^{-4}, \qquad 0 < K \ll 1; \qquad (13)$$

$$4\pi\hat{r}_{,u}^{-2}\hat{T}_{uu} = cm^{-4}, \qquad c \ll 1; \qquad (14)$$

$$4\pi|\hat{T}_{vu}| \ll \tfrac{1}{16}\hat{F}^2 m^{-2}. \qquad (15)$$

I also assume that outside the horizon

$$T_{uu} \geq 0. \qquad (16)$$

In the Schwarzschild case this inequality is known to hold at $x \approx 1$, see (A8). Elster's results ($\mathring{T}_{uu} \sim \mu + p_r + 2s$ in notation of [14]) make it obvious that (16) holds also at $x > 1.5$. It is still possible, of course, that $\mathring{T}_{uu}$ by whatever reasons changes its sign somewhere[3] between 1 and 1.5, however, even if (16) breaks down the results established below are not affected unless the violation is so strong that it changes the sign of the relevant *integral*, see (49). Finally, I assume that

$$|T_{\theta\theta}| \ll \tfrac{1}{2\pi} r |r_{,vu}| F^{-2}. \qquad (17)$$

Again, the corresponding inequality in the Schwarzschild case—it is $2\tau_4 \ll m_0^2/x$, see (2) and (4c)—holds both on the horizon and at large x, see Eqs. (A4) and (A2). And, again, we actually do not need (17) to be true *pointwise*.

[3] This hopefully can be verified numerically. Some arguments against this possibility can be found in [16].

The smallness of the relevant integral [see Eq. (34)] would suffice.

Remark 2. All these assumptions are local in the sense that to check their validity an observer in a point p does not need to know anything beyond a small vicinity of p. For, the requirement that the metric in this vicinity is (approximately) (1) fixes the coordinates up to the transformation (8) and the assumptions are invariant under that transformation.

D. Groundless apprehensions

Now that the model is built finding out whether the Einstein-Rosen wormhole is traversable becomes a matter of mathematics. But traversability of wormholes, let alone the evolution of the black hole horizons, are long being investigated and both theories have arguments that seem to enable one to answer in the negative even without solving any equations. In this subsection I point the holes in two of these conceivable arguments.

1. Quantum inequalities

From (13) it is seen that the weak energy condition is violated in some *macroscopic* region $\mathcal{V}$ around the throat of the wormhole. At the same time the energy density ρ measured by a free falling observer—whose proper time we denote by t—obeys in $\mathcal{V}$ the inequality [17]

$$\int_{t_1}^{t_2} \rho(t)\mathrm{d}t \lesssim |t_1 - t_2|^{-3}, \qquad \text{when} \quad |t_1 - t_2| \lesssim m_0. \tag{18}$$

The combination of these two properties in a few occasions (note that the global structure of the spacetime is irrelevant here, it need not be a wormhole) led to quite impressive estimates. Thus, in particular, it was found in [18] that in the Alcubierre bubble and in the Krasnikov tube there are three-surfaces Ξ and unit timelike vector fields $\boldsymbol{u}$ such that

$$\int_{\Xi} T_{\mu\nu}u^\mu u^\nu \mathrm{d}^3V \approx -10^{32}M_{\mathrm{Galaxy}}. \tag{19}$$

The figure in the right-hand side is so huge that both spacetimes were dismissed as "unphysical." So, is there not any danger of that kind in our case?

The answer is negative by at least two reasons. First, we explore not the capabilities of a hypothetical advanced civilization (as is usual in discussing the above-mentioned spacetimes), but a natural phenomenon. And there is a vital interpretational difference between these two situations. Indeed, in the former case the fact that a physical quantity has a presumably unrealistic value can be used as a ground for ruling the corresponding solution out as unphysical or unfeasible. But in the case at hand the situation—once the assumptions about the initial data, the values of the parameters, and the other constituents of the model are admitted reasonable—is *reverse*. If calculations yield (19), this would not signify that the spacetime is unphysical. On the contrary, it would mean that huge values of the integral may occur in physically appropriate situations and thus cannot serve as sign of unfeasibility of a spacetime.

Second, the estimates like (19) do not follow from (18) automatically. Additional assumptions are necessary and the approximate equality

$$\frac{\max|G_{\hat{k}\hat{l}}(p)|}{\max|R_{\hat{i}\hat{j}\hat{m}\hat{n}}(p)|} = \frac{8\pi\max|T_{\hat{k}\hat{l}}(p)|}{\max|R_{\hat{i}\hat{j}\hat{m}\hat{n}}(p)|} \approx 1 \qquad p \in \mathcal{V} \tag{20}$$

is among them [19]. At first glance, violation of this equality would signify some unnatural fine-tuning (note that $\approx$ can be understood quite liberally, the difference in 5–10 orders being immaterial). In fact, however, this is not the case: Eq. (20) corresponds to the situation in which the geometry of $\mathcal{V}$ is defined mostly by its (exotic) matter content, while the contribution to the curvature of the Weyl tensor is neglected. But in four dimensions this is not always possible. For example, Eq. (20) breaks down, in *any* nonflat empty region (the numerator vanishes there but the denominator does not). And the Einstein-Rosen wormhole is just another example. Loosely speaking, the Schwarzschild spacetime is a wormhole by itself. In making it traversable exotic matter is needed not to shape the spacetime into a wormhole, but only to keep the latter ajar.

2. The gap between the horizons

The model built above is not entirely new. The behavior of the apparent horizon in very similar assumptions was studied back in 1980s (see, i.e. [11] for some review). The spacetime under consideration, though, was not the wormhole M_{wh}, but the black hole originating from gravitational collapse (such a spacetime is not a wormhole, nor is it empty). The general consensus (see though [20]) was that the backreaction results only in the shift of the event horizon to a radius smaller than $2m$ by $\delta \sim m^{-2}$, which is physically negligible [21]. To see why such an overwhelmingly small δ does not make wormholes nontraversable note that δ is the shift in radius and *not the distance* between the horizons.[4] That is $\delta = r(q) - r(p')$, see Fig. 1. Clearly this quantity has nothing to do with traversability of the wormhole.

III. THE EVOLUTION OF THE HORIZON

The Einstein equations for the metric (6) read

$$4\pi T_{vu} = \frac{F^2}{4r^2} + (rr_{,vu} + r_{,v}r_{,u})r^{-2}, \tag{21}$$

[4]The event horizon is a null surface and there is no such thing as the (invariant) distance between a point and a null surface. Consider, for example, the surface $t = x + \delta$ in the Minkowski plane. Is it far from, or close to the origin of coordinates? The answer is: neither. Simply by an appropriate coordinate transformation one can give *any* value to δ.

$$4\pi T_{vv} = (2r_{,v}\varphi_{,v} - r_{,vv})r^{-1}, \tag{22}$$

$$4\pi T_{uu} = (2r_{,u}\varphi_{,u} - r_{,uu})r^{-1} \tag{23}$$

$$= -\frac{F^2}{r}\left(\frac{r_{,u}}{F^2}\right)_{,u}, \tag{23'}$$

$$4\pi T_{\theta\theta} = -\frac{2r^2}{F^2}(r_{,vu}/r + \varphi_{,vu}). \tag{24}$$

On the horizon the left-hand side in (21) can be neglected by (15), while $r_{,v}$ vanishes there by definition and we have

$$\hat{r}_{,vu} = -\frac{\hat{F}^2}{8m}. \tag{25}$$

Equations (22) and (13) give

$$\hat{r}_{,vv} = \frac{\pi K \hat{F}^4}{2m^3 \hat{r}_{,u}^2}. \tag{26}$$

Likewise, (23) and (14) result in

$$\hat{r}_{,uu} = 2\hat{r}_{,u}\hat{\varphi}_{,u} - 2c\hat{r}_{,u}^2 m^{-3}. \tag{27}$$

Finally, Eqs. (24) and (17) yield

$$\varphi_{,vu} = -r_{,vu}/r. \tag{28}$$

A. $\hat{u}$ as function of m

Our aim in this subsection is to find the function $\hat{u}(m)$. To this end we, first, use Eqs. (25)–(28) to find a system of two ordinary differential equations defining $\hat{u}(m)$ [these are Eqs. (30) and (36), below]. Then for wormholes with

$$\varpi > 1, \tag{29a}$$

$$\varpi < \tfrac{\sqrt{5}+1}{2}, \tag{29b}$$

we integrate the system and obtain a simple explicit expression for $\hat{u}$.

The horizon can be parametrized by v, or by m (as was already mentioned), or finally by u. The relations between the three parametrizations are given by the obvious formulas:

$$2\frac{dm}{du} = \frac{d\hat{r}}{du} = \hat{r}_{,u} \tag{30}$$

and

$$\frac{dv}{du} = -\frac{\hat{r}_{,vu}}{\hat{r}_{,vv}}, \tag{31}$$

of which the former follows right from the definitions (9) and (10) and the latter from the fact that $0 = d\hat{r}_{,v} = \hat{r}_{,vu}du + \hat{r}_{,vv}dv$ on h. These formulas enable us to write down

$$\frac{d}{dm}\hat{r}_{,u} = \frac{du}{dm}\left(\frac{\partial}{\partial u} + \frac{dv}{du}\frac{\partial}{\partial v}\right)\hat{r}_{,u} = 2\hat{r}_{,u}^{-1}\left(\hat{r}_{,uu} - \frac{\hat{r}_{,vu}^2}{\hat{r}_{,vv}}\right). \tag{32}$$

Using (27) and the relation

$$\frac{\hat{r}_{,vu}^2}{\hat{r}_{,u}^2\hat{r}_{,vv}} = \frac{m}{32\pi K},$$

which follows from Eqs. (25) and (26), one can rewrite (32) as

$$\hat{r}_{,u}^{-1}\frac{d\hat{r}_{,u}}{dm} = 4\frac{\hat{\varphi}_{,u}}{\hat{r}_{,u}} - 4cm^{-3} - \frac{m}{16\pi K}. \tag{33}$$

To assess the first term in the right-hand side consider the segment λ of the null geodesic $u = \text{const}$ between a pair of points $p \in \mathcal{S}$, $p' \in h$. Below I write for brevity $\bar{r}$, $\bar{x}_{,u}$, etc. for $r(p)$, $x_u(p)$, etc. (note that in this notation $\bar{u} = \hat{u}$). By (28) and (4d)

$$\begin{aligned}\hat{\varphi}_{,u} &= \varphi_{,u}(p') = \varphi_{,u}(p) + \int_\lambda \varphi_{,uv}dv \\ &= -\frac{1+\bar{x}}{2\bar{x}}\bar{x}_{,u} - \int_\lambda \frac{r_{,uv}}{r}dv.\end{aligned} \tag{34}$$

The sign of $r_{,uv}$ is constant by (12a), while r—as was shown in Section II B it monotonically falls on λ—varies from $\bar{r}$ to $2m$. Thus,

$$\begin{aligned}\hat{\varphi}_{,u} &= \left(\frac{1}{2m_*} - \frac{1+1/\bar{x}}{4m_0}\right)\bar{r}_{,u} - \frac{1}{2m_*}\left(\bar{r}_{,u} + \int_\lambda r_{,uv}dv\right) \\ &= \left(\frac{1}{2m_*} - \frac{1+1/\bar{x}}{4m_0}\right)\bar{r}_{,u} - \frac{1}{2m_*}\hat{r}_{,u}, \qquad m \leqslant m_* \leqslant \frac{1}{2}\bar{r}.\end{aligned} \tag{35}$$

Substituting this in (33) and neglecting the terms $\sim m_*^{-1}$, m^{-3} in comparison with the last term we finally get

$$\hat{r}_{,u}^{-1}\frac{d\hat{r}_{,u}}{dm} = \frac{2\xi\bar{x}_{,u}}{\hat{r}_{,u}} - \frac{m}{16\pi K}, \qquad \xi \equiv \left(\tfrac{2m_0}{m_*} - 1 - \tfrac{1}{\bar{x}}\right),$$
$$\hat{r}_{,u}(m_0) = -2\frac{m_0 v_0}{e} \tag{36}$$

(the last equation follows from (4b) and serves as a boundary condition for the differential equation). Introducing new notations

$$\mu \equiv \frac{m}{m_0}, \qquad y(\mu) \equiv \exp\frac{1-\mu^2}{2\eta}$$

one readily finds the solution of this equation:

$$\begin{aligned}\hat{r}_{,u}(\mu) &= -2\frac{m_0 v_0}{e}[1 + \Xi(\mu)]y(\mu), \\ \Xi &\equiv \frac{e}{v_0}\int_\mu^1 \frac{\xi}{y}\left(\frac{\bar{x}-1}{\bar{u}\,\bar{x}}\right)d\mu,\end{aligned} \tag{37}$$

and, correspondingly, [the first equality is an obvious consequence of Eq. (30)]

$$\hat{u}(m) = 2\int_{m_0}^{m}\frac{dm}{\hat{r}_{,u}} = A(\mu)\frac{e}{v_0}\int_{\mu}^{1}\frac{d\zeta}{y(\zeta)}. \qquad (38)$$

Here $A(\mu)$ is an unknown function bounded by

$$[\max_{(\mu,1)}(1+\Xi)]^{-1} \leqslant A(\mu) \leqslant [\min_{(\mu,1)}(1+\Xi)]^{-1}. \qquad (39)$$

In the remainder of this subsection I demonstrate that $|\Xi| < 1$, which implies, in particular, that $\hat{u}(m)$ monotonically falls and therefore $\hat{u}_\infty$ is just $\hat{u}(1)$. To simplify the matter the further consideration will be held separately for small and for large $\hat{u}$.

The case $\hat{u} < v_0$.—On this part of h it is possible that $\lambda \cap \mathcal{S}$ consists of one, two, or three points. But one of them always lies between the horizons and it is this point that we take to be the point p that enters (35) and thus (38). We then are ensured that $\bar{x} < 1$ and $\bar{v} < v_0$. By (3) it follows

$$(1-\bar{x})/\bar{u} = \bar{v}e^{-\bar{x}} < v_0,$$
$$\bar{x} > 1-(1-\bar{x})e^{\bar{x}} = 1-\bar{u}\,\bar{v} > 1-v_0^2$$

and therefore (recall that $\eta \ll 1$ and by (29a) so is v_0)

$$\frac{|\bar{x}-1|}{\bar{u}\,\bar{x}} < 2v_0. \qquad (40)$$

Now note that by (35) at $\bar{x} < 1$

$$1 \leqslant \frac{1}{\bar{x}} \leqslant \frac{m_0}{m_*} \leqslant \frac{1}{\mu}$$

and hence

$$0 < \xi = (\tfrac{m_0}{m_*}-1) + (\tfrac{m_0}{m_*}-\tfrac{1}{\bar{x}}) \leqslant 2\frac{1-\mu}{\mu}. \qquad (41)$$

Consequently,

$$|\Xi| \leqslant \frac{2e}{v_0}\int_{\mu}^{1}\frac{1-\zeta}{\zeta y(\zeta)}\frac{|\bar{x}-1|}{\bar{u}\,\bar{x}}d\zeta < 4e\int_{\mu}^{1}\frac{1-\zeta}{\zeta y(\zeta)}d\zeta. \qquad (42)$$

To proceed let us write down the following equality obtained by integrating by parts

$$\int_{\mu}^{1}\frac{1-\zeta}{\zeta y(\zeta)}d\zeta = \eta\Big[-N + \exp -\frac{1}{2\eta}\int_{\mu}^{1}\Big(\frac{2}{\zeta^3}-\frac{1}{\zeta^2}\Big) \times \exp\frac{\zeta^2}{2\eta}d\zeta\Big],$$

$$N \equiv \frac{1-\mu}{\mu^2}\exp\frac{-1+\mu^2}{2\eta}.$$

Note that the integrand in the right-hand side monotonically grows at $1/m_0 \leqslant \zeta < 1$ (i.e. as long as the wormhole remains macroscopic). So, splitting when necessary (i.e. when $\mu < 1-100\eta$) the range of integration by the point $\zeta = 1-100\eta$ and replacing the integrand on either interval by its maximum we obtain the following estimate (recall that $100\eta \ll 1$)

$$e^{-1/2\eta}\int_{\mu}^{1}\Big(\frac{2}{\zeta^3}-\frac{1}{\zeta^2}\Big)e^{\zeta^2/2\eta}d\zeta \leqslant \Big(\frac{2}{\mu^3}-\frac{1}{\mu^2}\Big)e^{(\mu^2-1)/2\eta}\Big|_{\mu=1-100\eta} + 100\eta \approx e^{-100} + 100\eta.$$

So, taking into consideration that N is positive,

$$Z \equiv \int_{\mu}^{1}\frac{1-\zeta}{\zeta y}d\zeta \leqslant e^{-100}\eta + 100\eta^2, \qquad \forall m > 1. \qquad (43)$$

Substituting which in (42) yields $|\Xi| \ll 1$ and hence $A(\mu) = 1$. Correspondingly,

$$\hat{u} = \frac{e}{v_0}\int_{\mu}^{1}\frac{d\zeta}{y(\zeta)}.$$

This expression is valid on the whole segment $\hat{u} < v_0$, i.e. at $\mu \geqslant \mu_*$

$$\mu_*\colon \hat{u}(\mu_*) = v_0.$$

To find μ_* we employ the formula (see, e.g., [22])

$$\int_0^{\mu/\sqrt{2\eta}} e^{\zeta^2}d\zeta = \frac{\sqrt{\eta/2}}{\mu}\exp\frac{\mu^2}{2\eta},$$

which is valid (asymptotically) at small η

$$v_0 = \hat{u}(\mu_*) = \frac{\exp 1-\frac{1}{2\eta}}{v_0}\int_{\mu_*}^{1}\exp\frac{\zeta^2}{2\eta}d\zeta = \frac{\exp 1-\frac{1}{2\eta}\eta}{v_0}\Big(\exp\frac{1}{2\eta}-\tfrac{1}{\mu_*}\exp\frac{\mu_*^2}{2\eta}\Big) = \frac{e\eta}{v_0}\Big(1-\tfrac{1}{\mu_*}\exp\frac{\mu_*^2-1}{2\eta}\Big).$$

So,

$$\tfrac{1}{\mu_*}\exp\frac{\mu_*^2-1}{2\eta} = 1-\varpi^{-1}. \qquad (44)$$

The case $\hat{u} > v_0$.—Now $\bar{x} > 1$ and instead of (40) we have

$$\frac{\bar{x}-1}{\bar{u}\,\bar{x}} < \frac{1}{v_0} = \frac{\varpi}{e\eta}v_0,$$

and instead of (41)

$$|\xi| \leqslant \frac{1-\mu}{\mu}+\frac{1}{\mu}.$$

Substituting these inequalities in (37) and neglecting the contribution of the segment $(\mu_*, 1)$ in Ξ gives

$$|\Xi| \leqslant \frac{\varpi}{\eta}\int_{\mu}^{\mu_*} y^{-1}\Big(\frac{1-\zeta}{\zeta}+\frac{1}{\zeta}\Big)d\zeta \leqslant \frac{\varpi}{\eta}Z + \frac{\varpi}{\eta}\int_{\mu}^{\mu_*}\frac{d\zeta}{\zeta y}.$$

The first term can be neglected by (43) and we have

$$|\Xi| \leqslant \frac{\varpi}{\eta}\int_{1/m_0}^{\mu_*}\exp\frac{\zeta^2-1}{2\eta}\zeta^{-1}\mathrm{d}\zeta = \frac{\varpi}{2\eta}\exp\frac{-1}{2\eta}\int_{1/(2\eta m_0^2)}^{\mu_*^2/(2\eta)} e^{\zeta}\zeta^{-1}\mathrm{d}\zeta = \frac{\varpi}{\mu_*^2}\exp\frac{\mu_*^2-1}{2\eta} = \varpi - 1$$

(the last equality follows from (44) and the last but one—from the fact that $\int_a^b e^{\zeta}\zeta^{-1}\mathrm{d}\zeta \sim e^b/b$ at large b). Thus on this segment of the horizon

$$\hat{u} = v_0 + A(\mu)\frac{e}{v_0}\int_{\mu}^{\mu_*}\frac{\mathrm{d}\zeta}{y(\zeta)}, \qquad \frac{1}{\varpi} \leqslant A \leqslant \frac{1}{2-\varpi}.$$

Whence, in particular,

$$\hat{u}_\infty \geqslant v_0 + \frac{e}{v_0\varpi}\eta(1-\varpi^{-1}) = v_0(2-\varpi^{-1}), \tag{45a}$$

$$\hat{u}_\infty \leqslant v_0 + \frac{1}{2-\varpi}\frac{e}{v_0}\eta(1-\varpi^{-1}) = v_0\left(1+\frac{\varpi-1}{2-\varpi}\right) = \frac{v_0}{2-\varpi}. \tag{45b}$$

We see that $\hat{u}_\infty > v_0$ and thus the wormhole in study proves to be traversable. Depending on the value of ϖ its time of traversability [see (11), (29), and (45)] varies from

$$T_{\mathrm{L}}^{\mathrm{trav}} = 0 \qquad \text{at } \varpi = 1 \tag{46}$$

to

$$T_{\mathrm{L}}^{\mathrm{trav}} = \alpha m_0, \qquad 1.3 \leqslant \alpha \leqslant 3.8 \qquad \text{at } \varpi = \tfrac{\sqrt{5}+1}{2}. \tag{47}$$

It should be emphasized that the upper bound on $T_{\mathrm{L}}^{\mathrm{trav}}$ restricts *not* the traversability time of empty wormholes (nothing in our analysis suggests that this time is restricted at all), but the traversability time of the *wormholes obeying* (29); it says not that the time the wormhole is open is less than $4m_0$, but only that to exceed that time a wormhole would have to have so large ϖ that our model cannot describe it. To see why it happens and why the condition (29) has to be imposed we need to examine the form of the horizon in more detail.

B. $\hat{u}$ as function of v

To relate m with v let us, first, combine Eqs. (30) and (31) and substitute Eqs. (25) and (26) into the result:

$$\frac{\mathrm{d}v}{\mathrm{d}m} = -2\hat{r}_{,u}^{-1}\frac{\hat{r}_{,vu}}{\hat{r}_{,vv}} = \frac{\hat{r}_{,u}m^2}{2\pi K}\hat{F}^{-2},$$

or, equivalently,

$$\frac{\mathrm{d}v}{\mathrm{d}\mu^3} = \frac{8m_0}{3\eta}\frac{\hat{r}_{,u}}{\hat{F}^2}. \tag{48}$$

Now let γ be a segment of a null geodesic $v = v(p')$ from $p'' \in \mathcal{S}$ to $p' \in h$. By (23') on γ

$$\left(\frac{r_{,u}}{F^2}\right)_{,u}\mathrm{d}u = -\frac{4\pi r}{F^2}T_{uu}\mathrm{d}u = -\frac{4\pi r}{r_{,u}^2}T_{uu}\left(\frac{r_{,u}}{F^2}\right)\mathrm{d}r$$

and hence

$$\frac{\hat{r}_{,u}}{\hat{F}^2}(p') = \frac{r_{,u}}{F^2}(p'')\cdot\exp\left\{\int_\gamma\left(\ln\frac{r_{,u}}{F^2}\right)_{,u}\mathrm{d}u\right\} = -\frac{v}{8m_0}\exp\left\{-\int_\gamma\frac{4\pi r T_{uu}\mathrm{d}r}{r_{,u}^2}\right\} \tag{49}$$

(the factor at the exponent is reduced with the use of the first equalities in Eqs. (2) and (4b), which are valid in p''). γ does not intersect the left horizon and therefore $r_{,u}$ is negative in each of its points. So, the integration in (49) is performed in the sense of decreasing r. By (16) it follows then

$$\frac{\hat{r}_{,u}}{\hat{F}^2}(v) \leqslant -\frac{v}{8m_0}. \tag{50}$$

Substituting which in (48) we finally obtain

$$v(\mu) \geqslant v_0\exp\left\{\frac{1}{3\eta}(1-\mu^3)\right\} \tag{51}$$

and, in particular,

$$v_\infty \equiv v(m=0) \geqslant v_0\exp\frac{1}{3\eta}.$$

The latter formula enables one, among other things, to bound from below the time of evaporation [in the sense of (7)]

$$T_{\mathrm{R}}^{\mathrm{evap}} = 2m_0\ln(v_\infty^2\kappa_R) \geqslant T_{\mathrm{R}}^{\mathrm{op}} + \frac{m_0^3}{12\pi K}.$$

Let us check now that our model is self-consistent in that the condition (12b) does hold in M_{wh}. To this end note that it is equivalent to the condition that the left and right horizons do not intersect, for which it is sufficient that

$$\hat{u}(\mu) < v(\mu). \tag{52}$$

Clearly, this condition holds for all $\hat{u} \leqslant v_0$, that is for all $\mu \geqslant \mu_*$. At the same time $\mu < \mu_*$ implies [the first inequality follows from (44)]

$$\mu^3 - 1 < 3\eta\ln(1-\varpi^{-1}) < 3\eta\ln(2-\varpi). \tag{53}$$

It is the last inequality in this chain that we need (29b) for. Combining (45b), (51), and (53) we finally see that

$$\hat{u}/v < \hat{u}_\infty/v \leqslant \frac{1}{2-\varpi}\exp\frac{\mu^3-1}{3\eta} < 1,$$

i.e. (52) is satisfied and the horizons do not intersect.

Remark 3. By the coordinate transformation $(u, v) \to (r, \tilde{v})$, where $\tilde{v} \equiv 4m_0\ln v$, one could cast the metric into

$$\mathrm{d}s^2 = -F^2 r_{,u}^{-1}\mathrm{d}v(-r_{,v}\mathrm{d}v + \mathrm{d}r) + r^2(\mathrm{d}\theta^2 + \cos^2\theta\mathrm{d}\phi)$$
$$= \frac{F^2 v}{8r_{,u}m_0}\left[\frac{1}{2m_0} v r_{,v}\mathrm{d}\tilde{v}^2 - 2\mathrm{d}r\mathrm{d}\tilde{v}\right] + r^2(\mathrm{d}\theta^2 + \cos^2\theta\mathrm{d}\phi).$$

So, if the integral in (49) is neglected and the relation (50) becomes therefore an equality (as in the Schwarzschild case), the metric takes the form

$$\mathrm{d}s^2 = -(1 - 2m_V/r)\mathrm{d}\tilde{v}^2 + 2\mathrm{d}r\mathrm{d}\tilde{v} + r^2(\mathrm{d}\theta^2 + \cos^2\theta\mathrm{d}\phi),$$
$$m_V \equiv r\frac{2m_0 - v r_{,v}}{4m_0}.$$

In the vicinity of the horizon this, in fact, is the Vaidya metric, because

$$4m_0 m_{V,u} = 2m_0 r_{,u} - v(r_{,u}r_{,v} + r r_{,uv})\Big|_h = 2m_0\hat{r}_{,u} + \tfrac{1}{4}v\hat{F}^2 = 0$$

[the second equality follows from (25)] and hence

$$m_V = m_V(v) = m(v).$$

IV. TRAVERSABILTY

A photon arriving to the wormhole (in the "left universe") after T_L^cl will never traverse it. At the same time photons with $u < v_0$, i.e. with $T_\mathrm{L} < T_\mathrm{L}^\mathrm{op}$ (such photons exist, unless $\mathcal{S}$ is spacelike, which is uninteresting) cannot traverse it either: on their way to the wormhole they get into the Planck region, their afterlife is veiled in obscurity. And the traversability time $T_\mathrm{L}^\mathrm{trav}$ turns out to be rather small, see (47). For the wormholes in discussion it is only $\sim 2m_0$, which is of the order of minutes even for giant black holes which presumably can be found in the centers of galaxies. And for a stellar mass wormhole it measures only a few microseconds. It may appear that so small $T_\mathrm{L}^\mathrm{trav}$ make the Einstein-Rosen wormholes useless in "interuniverse communicating" even for an advanced civilization. This, however, is not so by the reason mentioned in footnote 4. Indeed, consider a spaceship moving in the left asymptotically flat region towards the wormhole. Suppose, at T_L^op it is at the distance $l \gg m_0$ from the mouth and moves so fast that reaches the mouth (i.e. the vicinity of the left horizon) at $T_\mathrm{L} \approx T_\mathrm{L}^\mathrm{cl}$. Then neglecting the terms $\sim m_0/l$ and $\sim \hat{u}_\infty/l$ it is easy to find that the travel takes $\Delta\tau \approx 2\sqrt{lm_0}$ by the pilot's clock. Thus if T_L^op is large enough, the pilot may have plenty of time to send a signal through the wormhole.

Now let us consider the intrauniverse wormholes. To transform our model into one describing such a wormhole we first enclose the throat in a surface $\mathcal{T}: \ r = r_M \gg 2m_0$. This surface is a disjoint union of two cylinders $\mathbb{S}^2 \times \mathbb{R}^1$, one of which lies in the left asymptotically flat region and the other in the right:

$$\mathcal{T} = \mathcal{T}_L \cup \mathcal{T}_R, \qquad \mathcal{T}_{L(R)}: \ r = r_M, \qquad v \lessgtr 0.$$

We shall consider the spacetime outside $\mathcal{T}$ (which is, correspondingly, a disjoint union of two asymptotically flat regions M_L and M_R) as flat. This, of course, is some inexactness, but not too grave—in reality the space far enough from a gravitating body *is* more or less flat. Let us fix Cartesian coordinates in $M_{L(R)}$ so that the t-axes are parallel to the generators of $\mathcal{T}$ and the x_1-axes—to the line $t = \phi = \theta = 0$. The x_1-coordinates of the points of $\mathcal{T}$ are understood to lie within the range $[-r_M, r_M]$ and $\mathcal{S}$ must be the surface $t = 0$. Now an intrauniverse wormhole is obtained by the standard procedure: one removes the regions $x_1 > d/2$ from M_L and $x_1 < d/2$ from M_R and identify the points with the same coordinates on their boundaries (the three-surfaces $x_1 = -d/2$ and $x_1 = d/2$, respectively). The resulting spacetime, see Fig. 2, is the Minkowski space in which the interiors of two cylinders (their boundaries are $\mathcal{T}_L$ and $\mathcal{T}_R$) are replaced by a connected region, so that, for example, a photon intersecting $\mathcal{T}_L$ at a moment $t_\mathrm{in} \in (T_\mathrm{L}^\mathrm{op}, T_\mathrm{L}^\mathrm{cl})$ emerges from $\mathcal{T}_R$ at some $t_\mathrm{out}(t_\mathrm{in})$.

Now note that it would take only d for the photon to return to $\mathcal{T}_L$. So M_wh is causal if and only if

$$t < t_\mathrm{out}(t) + d, \qquad \forall t \in (T_\mathrm{L}^\mathrm{op}, T_\mathrm{L}^\mathrm{cl}).$$

By changing κ_L to κ'_L—all other parameters being fixed—one shifts the interval $(T_\mathrm{L}^\mathrm{op}, T_\mathrm{L}^\mathrm{cl})$ and the graph of $t_\mathrm{out}(t)$ to the right by $\approx 2m_0\ln(\kappa'_L/\kappa_L)$, see (7). So, if κ'_L is taken to be sufficiently large the inequality breaks down. Which means that irrespective of the values of m_0, d, ϖ, and κ_R, the intrauniverse Einstein-Rosen wormholes with sufficiently large κ_L are time machines.

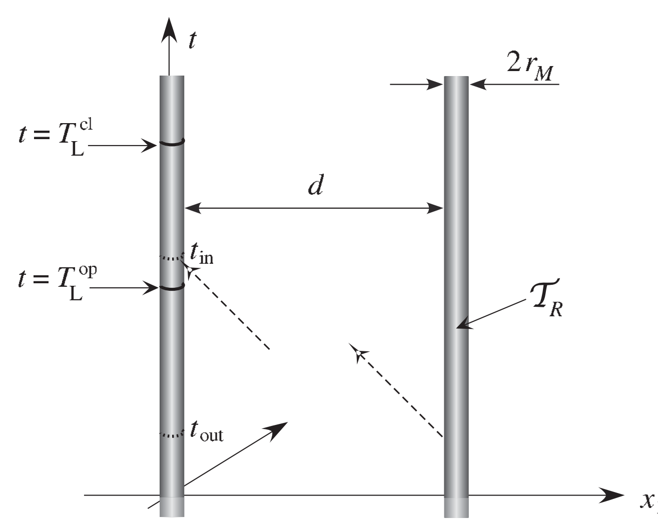

FIG. 2. The two dashed lines depict the world line of the same photon.

V. CONCLUSIONS

We have studied the evolution of the spherically symmetric empty wormhole, or to put it otherwise the backreaction of the Hawking radiation on the (approximately) Schwarzschild metric. A few simplifying assumptions were made, which physically, reduced to the idea that the metric and the vacuum polarization around each observer remain approximately those of the Schwarzschild black hole. It turns out that such a wormhole is characterized by three parameters in addition to the initial mass and the distance between the mouths. The explicit calculations within this model have shown that *for a macroscopic time interval*—its duration is determined by those parameters—*the wormhole is traversable*.

None of the assumptions made in this paper looks too wild, so its results can be regarded as evidence for possibility of natural "transient" wormholes. Obviously the existence of such wormholes would be of enormous significance, the implications ranging from a process generating highly collimated flashes to causality violation (or at least violation of the strong cosmic censorship conjecture).

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APPENDIX

In this appendix I extract the relevant estimates on τ_i from the results obtained in [13,14]. At large r the radial pressure $\mathring{T}^\theta_\theta = \mathring{T}^\phi_\phi$ equals (see Eqs. (2.6), (4.8), (5.5), and (6.21) of [13]) to

$$\mathring{T}^{\,\theta}_{\,\theta} \approx \tfrac{\lambda}{16} K m_0^{-4} x^{-4},$$

where

$$0 < \lambda \leq 27, \qquad K = \frac{9}{40 \cdot 8^4 \pi^2}. \tag{A1}$$

Correspondingly,

$$\tau_4 = 16\pi m_0^4 x^2 \mathring{T}^\theta_\theta \approx \pi\lambda K x^{-2}, \qquad \text{at large } x. \tag{A2}$$

Near the horizon $\mathring{T}^\theta_\theta$ was found (numerically) in [14], where it was denoted by p_t:

$$0 < \mathring{T}^\theta_\theta \approx -\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}x}\mathring{T}^\theta_\theta \lesssim 2 \times 10^{-6} m_0^{-4} \tag{A3}$$

[the value of the derivative will be needed in (A5)], whence

$$\tau_4(1) \lesssim 10^{-4}. \tag{A4}$$

Further, for the conformal field the trace $\mathring{T}^a_a$ is defined by the conformal anomaly and in the Schwarzschild space

$$T \equiv \mathring{T}^a_a = \tfrac{m_0^{-4}}{3840\pi^2} x^6 \approx 3 \times 10^{-5} m_0^{-4} x^6,$$

$$T'\Big|_{x=1} \approx 2 \times 10^{-4} m_0^{-4},$$

see, e.g., Eq. (4.8) in [13]. So, for the quantity $Y \equiv T - \mathring{T}^\theta_\theta - \mathring{T}^\phi_\phi$ one finds

$$Y\Big|_{x=1} \approx 3 \times 10^{-5} m_0^{-4}, \qquad Y'\Big|_{x=1} \approx 10^{-4} m_0^{-4}. \tag{A5}$$

In coordinates t, r^*

$$t = 2m_0 \ln(-v/u), \qquad r^* = 2m_0 \ln(-vu),$$

which are used in [13,14], the Schwarzschild metric (1) takes the form

$$\mathrm{d}s^2 = \tfrac{x-1}{x}(-\mathrm{d}t^2 + \mathrm{d}r^{*2}) + \mathring{r}^2(\mathrm{d}\theta^2 + \cos^2\theta \mathrm{d}\phi)$$

and one has

$$\mathring{T}_{uv} = \tfrac{4m_0^2}{vu}(\mathring{T}_{r^*r^*} - \mathring{T}_{tt}) = -\tfrac{4m_0^2 e^{-x}}{x}(\mathring{T}^{r^*}_{r^*} + \mathring{T}^t_t)$$
$$= -\tfrac{4m_0^2 e^{-x}}{x} Y\Big|_{x=1} \approx -4 \times 10^{-5} m_0^{-2}.$$

From whence it follows

$$|\tau_3(1)| \approx 5 \times 10^{-4}. \tag{A6}$$

Likewise,

$$\mathring{T}_{vv} = \tfrac{4m_0^2}{v^2}(\mathring{T}_{tt} + \mathring{T}_{r^*r^*} + 2\mathring{T}_{tr^*})$$
$$= \tfrac{(x-1)}{x}\left(\tfrac{4m_0^2}{xe^x}\right)^2 \mathring{r}_{,u}^{-2}(-\mathring{T}^t_t + \mathring{T}^{r^*}_{r^*} + 2\mathring{T}^{r^*}_t).$$

At the horizon $x = 1$ and $G(1) = H(1) = Q = 0$ (see [[13] section 2] for the definitions of the relevant functions). So the only contribution to $\mathring{T}^a_b$ comes from its divergent part $T^{(2)a}_b$:

$$T^{(2)a}_b = \frac{K}{4m_0^4 x(x-1)} E^a_b,$$

where I defined

$$E^t_t = -E^{r^*}_t = E^t_{r^*} = -E^{r^*}_{r^*} = 1.$$

Thus

$$\mathring{T}_{vv}|_{x=1} = -16e^{-2} K \mathring{r}_{,u}^{-2}. \tag{A7}$$

Finally,

$$\begin{aligned}
\mathring{T}_{uu} &= \frac{x}{x-1}\mathring{r}^2_{,u}(-\mathring{T}^t_t + \mathring{T}^{r^*}_{r^*} - 2\mathring{T}^{r^*}_t) = \frac{x}{x-1}\mathring{r}^2_{,u}\Big(\frac{2x}{r^2(x-1)}(H+G) - Y\Big) \\
&= \frac{x}{x-1}\mathring{r}^2_{,u}\Big(\frac{2x}{r^2(x-1)}\cdot\frac{1}{2}\int_{2m_0}^{r}[(r'-m_0)T + (r'-3m_0)(T-2Y)]\mathrm{d}r' - Y\Big) \\
&= \frac{x}{x-1}\mathring{r}^2_{,u}\Big(\frac{1}{x(x-1)}\int_1^x\Big[\Big(x'-\frac{1}{2}\Big)T + \Big(x'-\frac{3}{2}\Big)(T-2Y)\Big]\mathrm{d}x' - Y\Big) \\
&= \frac{x}{x-1}\mathring{r}^2_{,u}\Big(\frac{4}{x(x-1)}\int_1^x(x'-1)\mathring{T}^\theta_\theta\mathrm{d}x' + \frac{1}{x(x-1)}\int_1^x Y\mathrm{d}x' - Y\Big) \\
&\to \frac{x}{x-1}\mathring{r}^2_{,u}\Big(\frac{2(x-1)}{x}\mathring{T}^\theta_\theta + \Big(\frac{1}{x}-1\Big)Y + \frac{x-1}{2x}Y'\Big) \to \frac{1}{2}\mathring{r}^2_{,u}Y'\Big|_{x=1} \approx 10^{-4}m_0^{-4}\mathring{r}^2_{,u}
\end{aligned} \tag{A8}$$

and, correspondingly,

$$\tau_2(1) \approx 10^{-3}. \tag{A9}$$

Remark 4. To avoid confusion note that our coordinates u and v *differ* from those used [13]. The latter—let us denote them u_{CF} and v_{CF}—are related to the former by

$$u_{\mathrm{CF}} = -4m_0\ln(-u), \qquad v_{\mathrm{CF}} = 4m_0\ln v.$$

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