

Inhomogeneous big bang nucleosynthesis revisited

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We reanalyze the allowed parameters for inhomogeneous big bang nucleosynthesis in light of the WMAP constraints on the baryon-to-photon ratio η and a recent measurement which has set the neutron lifetime to be $878.5 \pm 0.7 \pm 0.3$ sec. For a set baryon-to-photon ratio η the new lifetime reduces the mass fraction of ${}^4\text{He}$ by 0.0015 but does not significantly change the abundances of other isotopes. This enlarges the region of concordance between ${}^4\text{He}$ and deuterium in the parameter space of η and the IBBN distance scale r_i . The ${}^7\text{Li}$ abundance can be brought into concordance with observed ${}^4\text{He}$ and deuterium abundances by using depletion factors as high as 9.3. The WMAP constraints, however, severely limit the allowed comoving ($T = 100$ GK) inhomogeneity distance scale to $r_i \approx (1.3\text{--}2.6) \times 10^5$ cm.

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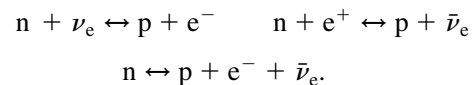
I. INTRODUCTION

Big bang nucleosynthesis (BBN) plays a crucial role in constraining our views on the universe. It is essentially the only probe of physics in the early radiation dominated epoch during the interval from $\sim 1\text{--}10^4$ sec. Moreover, big bang cosmology is currently undergoing rapid evolution based upon high precision determinations of cosmological parameters and improved input physics. Thus, it is important to scrutinize all possible variants of BBN and understand the constraints which modern observations place upon their parameters. This paper summarizes the current status on one such important variant of the big bang, namely, one in which the baryons are spatially inhomogeneously distributed during the epoch of nucleosynthesis. We consider the constraints from WMAP and the latest observed elemental abundances and include recent measurements of thermonuclear reaction rates and the neutron lifetime. We show, that although the parameter space for inhomogeneous big bang nucleosynthesis (IBBN) is significantly limited, interesting regions remain and this paradigm continues to be a viable possibility for the early universe.

II. BACKGROUND

At a high temperature in the early universe, e.g. $T \sim 100$ GK (the corresponding age of the universe is

~ 0.01 sec) baryons mainly exist in the form of free neutrons and protons. During this epoch neutrons and protons are rapidly interconverted via the following weak reactions [1–3]



As long as these reactions are in thermal equilibrium the ratio of neutrons to protons is given by $(n/p) = \exp[-\Delta m/kT]$ where Δm is the mass difference between neutrons and protons. When the universe cools to a temperature $T = 13$ GK these weak reactions fall out of equilibrium. The n/p ratio after that time decreases only due to neutron decay.

When the universe cools to a temperature $T \approx 0.9$ GK the nuclear reaction $n + p \leftrightarrow d + \gamma$ falls out of nuclear statistical equilibrium. Deuterium production becomes significant, leading to the synthesis of increasingly heavier nuclei through a network of nuclear reactions and weak decays. When nearly all free neutrons are incorporated into ${}^4\text{He}$ nuclei, ${}^4\text{He}$ production virtually stops. The final mass fraction can be approximated as

$$Y_p \approx 2(n/p)/[(n/p) + 1]. \quad (1)$$

Detailed final abundances depend most sensitively on the baryon-to-photon ratio η . Observational measurements of ${}^4\text{He}$, deuterium and ${}^7\text{Li}$ abundances can be compared with abundances calculated from BBN to put constraints on η . Constraints can also be put on η from measurements of the Cosmic Microwave Background (CMB). The phys-

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ics of acoustic oscillations in the CMB angular power spectrum has a dependence on various cosmological parameters (See Melchiorri [4] for comprehensive references to methods of CMB calculations), including the baryon density factor $\Omega_b h^2$, and therefore η . Recent CMB measurements using WMAP [5] set $\Omega_b h^2 = 0.0224 \pm 0.0009$ and $\eta = 6.13 \pm 0.25 \times 10^{-10}$. See Lara [6] for references to previous attempts to constrain η using BBN or the CMB.

The final n/p ratio depends on the neutron lifetime, τ_n , which partially determines when weak reactions fall out of equilibrium. The most recent lifetime world average [7] is $\tau_n = 885.7 \pm 0.8$ sec. But a recent measurement by Serebrov *et al.* [8] sets $\tau_n = 878.5 \pm 0.7_{\text{stat}} \pm 0.3_{\text{sys}}$ sec, which differs from the world average by 6 standard deviations. Serebrov *et al.* measured the storage loss rate $\tau_{\text{storage}}^{-1} = \tau_n^{-1} + \tau_{\text{loss}}^{-1}$ of ultracold neutrons in a gravitational trap, and then extrapolated the data down to $\tau_{\text{loss}}^{-1} \rightarrow 0$ to get a value for τ_n^{-1} alone. Their storage loss rate was as much as a factor of two smaller than measurements from previous experiments. So their measurements were much closer to the true neutron decay rate, greatly reducing the systematic error in the measurements. Also, Serebrov *et al.* used the neutron lifetime as a unitarity test of the Cabibbo Kobayashi Maskawa (CKM) matrix. The standard model of particle physics predicts that the CKM matrix is unitary [9,10]. The neutron lifetime by Serebrov *et al.* was in better agreement with this unitarity prediction than the world average lifetime, as shown in Figure 4 of Serebrov *et al.* [8].

Mathews *et al.* [11] applied the measurement $Y_p = 0.2452 \pm 0.0015$ from Izotov *et al.* [12], which is derived from observations of extragalactic HII regions in low metallicity irregular galaxies, to a Standard BBN (SBBN) code [1,13] available for public download [14]. In the results using the world average neutron lifetime, the above 1σ range of Y_p corresponds to a range $\eta = 4.8 \pm 0.8 \times 10^{-10}$, not in agreement with the WMAP results. Mathews *et al.* [11] put in the Serebrov *et al.* lifetime value into the SBBN code [14]. The new lifetime has the effect of lowering Y_p by ≈ 0.0015 for all values of baryon-to-photon ratio η . The 1σ range of η corresponding to $Y_p = 0.2452 \pm 0.0015$ [12] shifts to $\eta = 5.5 \pm 0.9 \times 10^{-10}$. The new range of η is in agreement with the range $\eta = 6.13 \pm 0.25 \times 10^{-10}$ from the WMAP measurements.

This article will discuss how the measured isotope abundances and the WMAP data constrain parameters from an IBBN cosmology when using the Serebrov *et al.* neutron lifetime. Various theories of cosmic phase transitions may lead to the formation of baryon inhomogeneous regions, for example, during a first order quark hadron phase transition [15–17,38] or during the electroweak phase transition [18–20,39]. Inhomogeneities can have planar, cylindrical or spherical symmetry. The evolution of baryon inhomogeneous regions can be modeled using an IBBN

code (See Lara [6] for a comprehensive list of attempts to model baryon inhomogeneities). The inhomogeneous region is broken up into 64 zones. Within each zone s the time evolution of the number density $n(i, s)$ of isotope species i obeys the following equation [6,20,21]

$$\begin{aligned} \frac{\partial n(i, s)}{\partial t} = & n_b(s) \sum_{j,k,l} N_i \left(- \frac{Y^{N_i}(i, s) Y^{N_j}(j, s)}{N_i! N_j!} [ij] \right. \\ & + \frac{Y^{N_k}(k, s) Y^{N_l}(l, s)}{N_k! N_l!} [kl] \left. \right) - 3 \frac{\dot{a}}{a} n(i, s) \\ & + \frac{1}{r^p} \frac{\partial}{\partial r} \left(r^p D_n \frac{\partial \xi}{\partial r} \frac{\partial n(i, s)}{\partial \xi} \right). \end{aligned} \quad (2)$$

The first two terms correspond to nuclear reactions and weak decays that create or destroy isotope i within zone s . The weak reactions that interconvert neutrons and protons go in these terms for instance. The third term corresponds to the expansion of the universe. The final term corresponds to diffusion of isotope i between zones. Neutron diffusion is the most prominent diffusion because of the neutrons' lack of charge. The influence of neutron diffusion on IBBN results is described in detail by Lara [6]. Thermonuclear reaction rates from Angulo *et al.* [22] were used in runs for both Lara [6] and this article.

III. RESULTS

The following results are for a cylindrically symmetric IBBN model with an initial high baryon density in the outermost zones of the model. Parameters used to define the model include R_ρ , the initial ratio of the high to low baryon densities, and f_v , the volume fraction of the high density region. R_ρ is set to 10^6 , and f_v is defined such that the radius of the low density cylindrical core equals 0.925 of the total radius (the high density outer shell then has a thickness equal to 0.075). This model was used by Orito *et al.* [23] and by Lara [6,24] to optimally satisfy the light element abundance constraints and also the inhomogeneous geometry which is calculated in an effective kinetic model of QCD [17]. Final abundance results are presented as contour maps which are dependent on η and the inhomogeneity distance scale r_i , the size of the outermost zone at the initial temperature of 100 GK. For the smallest values of r_i , neutron diffusion homogenizes both neutrons and protons early enough for the resulting abundances to have the same values as abundances from SBBN models.

Note that the code used by Mathews *et al.* [14] uses Bessel function expansions [1] to solve for the rates of the neutron proton interconversion reactions [25]. The code used in this article uses a numerical integration method to solve for those rates [26]. The difference in the methods of calculating the rates can lead to discrepancies in the value of Y_p on the order of 0.001. There are also differences in the calculation of the Coulomb correction and the zero temperature radiative correction between the codes that

affect the results [26,27]. These differences will be discussed and resolved in a subsequent article.

Figure 1 shows both observational constraints for deuterium in an η vs r_i contour map and the CMB constraints $\eta = 6.13 \pm 0.25$ from the WMAP results [5]. The deuterium observational constraints $D/H = 2.78^{+0.44}_{-0.38} \times 10^{-5}$ are the average of measurements of six absorption line systems towards five Quasi Stellar Objects [28]. In the SBBN model the deuterium constraints correspond to $\eta = (5.6 - 6.7) \times 10^{-10}$, encompassing the WMAP constraints. As can be seen in Fig. 1 the results for the smallest values of r_i are equivalent to SBBN results. The bends in the deuterium contour lines are explained in detail in Lara [6]. In models with r_i from ≈ 25000 cm to 10^5 cm, neutron diffusion occurs between weak freeze-out and nucleosynthesis. Contour lines are shifted to lower η (lower deuterium production) because nucleosynthesis is concentrated in regions with large proton density. For r_i from $\approx 10^5$ cm to 2.0×10^6 cm neutron back diffusion does not reach all regions of the model during the time of nucleosynthesis, and contour lines shift instead to higher η (greater deuterium production). At $r_i \approx 2.0 \times 10^6$ cm neutron diffusion peaks at the same time as nucleosynthesis. The deuterium contour lines shift to lower η and the IBBN model is approximately the average of a high baryon density SBBN model and a low density SBBN model. Because of these bends the deuterium constraints are in concordance with the WMAP results for three ranges of distance scale: $r_i \leq 9000$ cm which includes the SBBN case, $r_i = (1.3-2.6) \times 10^5$ cm and $r_i = (2.1-2.9) \times 10^7$ cm.

Figure 2 shows observational constraints for ${}^4\text{He}$ for the cases of both the world average neutron lifetime and the neutron lifetime by Serebrov *et al.*, along with the deute-

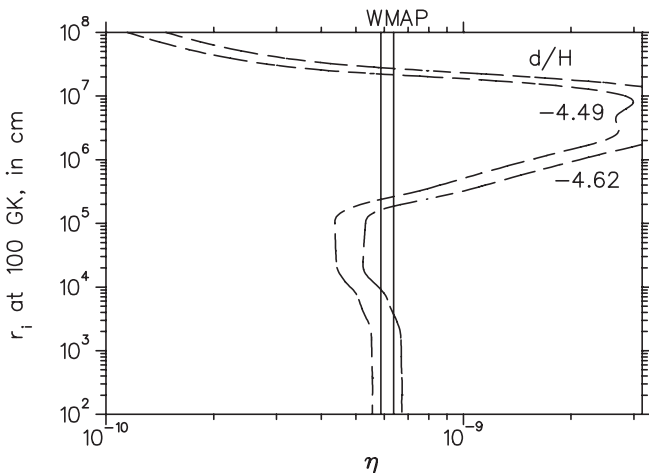


FIG. 1. Contours consistent with the $\pm 2\sigma$ measured abundance ratio D/H of deuterium [28] are shown in long dashed lines. The CMB constraints from the WMAP measurements [5] are also shown as solid lines. The WMAP constraint limits the allowed parameter space to three regions.

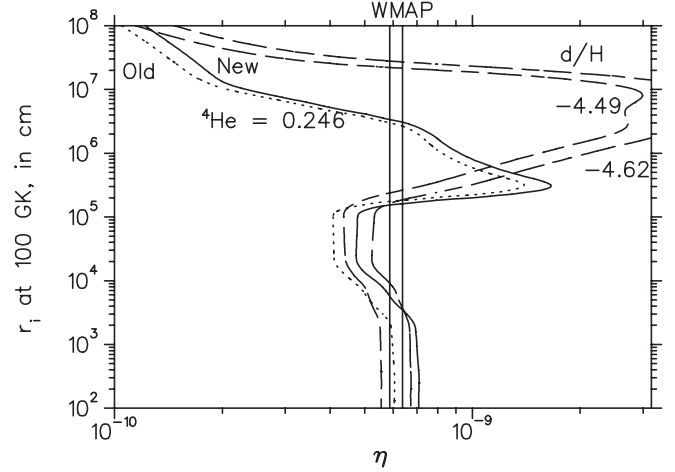


FIG. 2. Constraints from deuterium and CMB observations are shown with the $+2\sigma$ maximum constraint of the ${}^4\text{He}$ mass fraction Y_p by Izotov and Thuan [29]. The contour for the $+2\sigma$ maximum constraint of Y_p is shown by dashed line for the world average neutron lifetime [7] and solid line for the Serebrov *et al.* lifetime [8]. The region to the right of the maximum constraint is excluded. The $+2\sigma$ minimum constraint corresponds to values of η so low it is not necessary to show in this figure. The lifetime effect is to shift the Y_p contour line to a higher η by $\approx 1 \times 10^{-10}$.

rium and WMAP constraints. Figure 2 shows $Y_p = 0.246$, the 2σ maximum constraint of the measurement 0.242 ± 0.002 by Izotov and Thuan [29]. This measurement is more recent than the previously adopted measurement by Izotov *et al.* [12]. If the world average neutron lifetime is used $Y_p = 0.246$ corresponds to $\eta = 6.1 \times 10^{-10}$ for the cases of SBBN and IBBN with the smallest values of r_i . Only a very narrow range $\eta = (5.88-6.1) \times 10^{-10}$ is then permitted for $r_i \leq 2000$ cm. The region of concordance for $r_i = (1.3-2.6) \times 10^5$ cm however is contained within this ${}^4\text{He}$ constraint. Note though that the extent of systematic errors in ${}^4\text{He}$ observations are controversial. Olive and Skillman [30] adopt a conservative approach to accessing uncertainties and report a very large range $0.232 \leq Y_p \leq 0.258$ due to correlated errors. This range should eventually come down as the number of systems with uncorrelated systematic errors increase.

A shorter neutron lifetime means that more neutrons are converted to protons before the onset of nucleosynthesis, and Y_p for a set value of η is smaller. But the lower neutron abundance has no perceptible effect ($< 1\%$) in the final abundances of all other isotopes [11]. Since the neutron lifetime is independent of baryon inhomogeneity, the IBBN run shows the same results. For a set η , Y_p is reduced by 0.0015 while D/H is unchanged, regardless of the value of η or r_i . The contour line of the ${}^4\text{He}$ maximum constraint [29] shifts to higher η by 1×10^{-10} . Using the world average neutron lifetime, the region of concordance between the deuterium and ${}^4\text{He}$ constraints

excludes a range $(4.5 \times 10^3 - 1.3 \times 10^5)$ cm of r_i . The shift in the ${}^4\text{He}$ contour line due to the Serebrov *et al.* lifetime allows for a concordance between deuterium and ${}^4\text{He}$ for all values of $r_i < 6.4 \times 10^5$ cm. The WMAP constraints are in concordance with the deuterium constraints and ${}^4\text{He}$ constraints with the Serebrov *et al.* lifetime for $r_i < 5500$ cm. The region of concordance for $r_i = (1.3 - 2.6) \times 10^5$ cm is still contained within the new ${}^4\text{He}$ constraint. For concordance with the region with $r_i = (2.1 - 2.9) \times 10^7$ cm a maximum value $Y_p = 0.259$ is needed. This value is close to the maximum limit reported by Olive and Skillman, but that maximum limit is expected to diminish as the errors improve.

The primordial abundance of ${}^7\text{Li}$ is in dispute because of disagreement over the calculation of the effective temperatures of metal poor halo stars needed to determine the abundances. Ryan *et al.* [31] and Melendez and Ramirez [32] calculate different values of the effective temperature and different ranges of the ${}^7\text{Li}$ abundance. Figure 3 shows the Ryan *et al.* ${}^7\text{Li}/\text{H} = 1.23^{+0.68}_{-0.32} \times 10^{-10}$ along with the deuterium, WMAP and ${}^4\text{He}$ constraints when using the Serebrov *et al.* lifetime. Figure 4 does the same but instead shows the Melendez & Ramirez constraints ${}^7\text{Li}/\text{H} = 2.34^{+1.64}_{-0.96} \times 10^{-10}$. Regions of concordance between the ${}^4\text{He}$, deuterium, and WMAP constraints are highlighted in yellow (color online).

No region of concordance exists between the ${}^7\text{Li}$ constraints of Ryan *et al.* [31] and the constraints from deuterium and ${}^4\text{He}$ for either the SBBN model or the IBBN model. A depletion factor due to stellar processes is needed

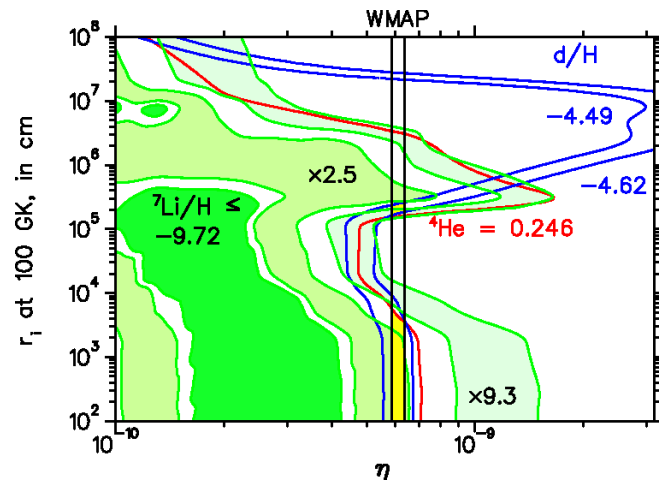


FIG. 3 (color online). (Color Online) Contours for the $\pm 2\sigma$ abundance constraints on ${}^7\text{Li}/\text{H}$ by Ryan *et al.* [31] are shown along with the constraints on deuterium [28], CMB constraint from WMAP [5], and $+2\sigma$ maximum constraint on ${}^4\text{He}$ [29] for the case of $\tau_n = 878.5 \pm 0.7_{\text{stat}} \pm 0.3_{\text{sys}}$ [8]. The region of concordance between WMAP, ${}^4\text{He}$ and deuterium is shown in yellow. The contour lines for ${}^7\text{Li}$ allow for depletion factors from 2.5 to 9.3 to bring all three observational constraints and CMB constraints in concordance with each other.

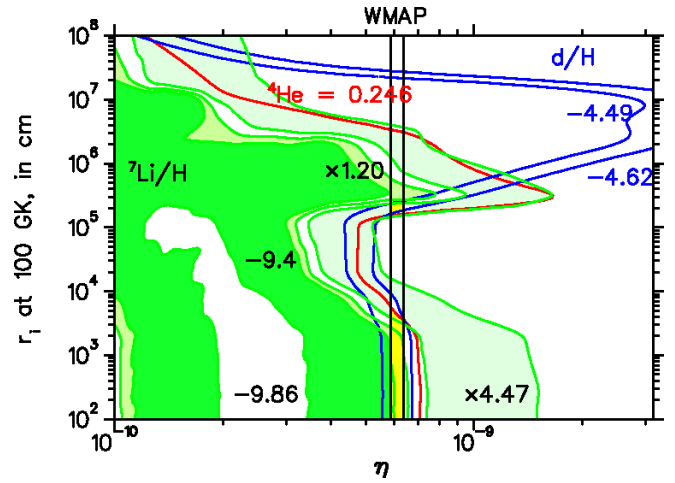


FIG. 4 (color online). (Color Online) Same as in Fig. 3 but with $\pm 2\sigma$ constraints on ${}^7\text{Li}/\text{H}$ by Melendez and Ramirez [32]. A thin region of concordance between the $+2\sigma$ limit of the ${}^7\text{Li}$ constraints and the other constraints exists for $r_i \leq 1000$ cm. A depletion factor of 1.2 improves concordance. A depletion factor of 4.47 would bring these ${}^7\text{Li}$ measurements in concordance with the other constraints for $r_i = (1.3 - 2.6) \times 10^5$ cm.

for concordance. A factor of 2.5 would bring the ${}^7\text{Li}$ constraints into concordance with the WMAP constraints $\eta = 6.13 \pm 0.25 \times 10^{-10}$ for the SBBN case and for the IBBN case with distance scale $r_i \leq 2500$ cm. But for $r_i \approx 25000$ cm nucleosynthesis in high proton density regions leads to considerably increased production of ${}^7\text{Be}$ [6]. The ${}^7\text{Li}$ contour lines shift to lower η than the contour lines for deuterium and ${}^4\text{He}$. Larger depletion factors can then be permitted in the IBBN model. Depletion factors up to 5.9 are permitted using the world average neutron lifetime [6]. The main effect of using the Serebrov *et al.* lifetime is to shift the ${}^4\text{He}$ contour lines to higher η . This shift increases the maximum allowed range for the depletion factor, to 9.3 as shown in Fig. 3. The constraints by Ryan *et al.* with this depletion factor will be in concordance with the narrow region $r_i = (1.3 - 2.6) \times 10^5$ cm permitted by the deuterium, ${}^4\text{He}$, and WMAP constraints.

Figure 4 shows a thin region of concordance $\eta = (5.88 - 6.0) \times 10^{-10}$ between the WMAP constraints and the 2σ limit of the ${}^7\text{Li}$ constraints by Melendez and Ramirez [32]. A depletion factor of 1.2 would bring these ${}^7\text{Li}$ constraints into concordance with the whole range $\eta = 6.13 \pm 0.25 \times 10^{-10}$ of the WMAP constraints for SBBN and for IBBN with $r_i \leq 3000$ cm. Using the world average neutron lifetime a maximum depletion factor of 2.8 of the Melendez & Ramirez constraints is required by this IBBN model [6]. Using the Serebrov *et al.* neutron lifetime, a maximum depletion factor of 4.47 brings the Melendez & Ramirez constraints with the region $r_i = (1.3 - 2.6) \times 10^5$ cm permitted by deuterium, ${}^4\text{He}$, and WMAP constraints.

The results in this article are for a cylindrically symmetric IBBN model with an initial low density core and

high density outer region. Changing the symmetry of the model or changing the values of parameters R_ρ and f_ν have the effect of lessening or amplifying the turns in the contour lines, but the characteristics of the contour lines remain the same qualitatively. Contour lines shift to lower η and then to higher η and then back to lower η as the size of the model increases. The particular model of this article is thus representative of IBBN models in general.

We note that new measurements of the ${}^7\text{Li}$ primordial abundance derived from the ratio (${}^7\text{Li}/{}^6\text{Li}$) measured in the interstellar medium [33,34] support a larger range of permitted depletion factors, consistent with those required here, leaving an enhanced ${}^9\text{Be}$ or ${}^{11}\text{Be}$ abundance as a possible observable signature for the IBBN [35–37].

IV. CONCLUSIONS

We have reanalyzed IBBN in the context of the latest constraints on primordial abundances, input physics, and cosmological parameters. We have shown that some possible IBBN solutions are possible. For example, a cylindrical fluctuation with a comoving radius of $\sim 10^5$ cm fixed at a temperature of 100 GK is still consistent with the D/H,

Y_p and WMAP constraints. Depletion factors of up to 9.3 or 4.47, depending on which measurement is used, can bring the ${}^7\text{Li}$ results into concordance as well. Also any fluctuation up to a radius of $\sim 10^4$ cm is viable. Hence, although the standard homogeneous BBN limit is still a consistent and the simplest solution, IBBN remains a viable possibility should it ever be established that such fluctuations arise from earlier cosmic phase transitions.

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