What is the discrete gauge symmetry of the minimal supersymmetric standard model

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We systematically study the extension of the supersymmetric standard model (SSM) by an anomaly-free discrete gauge symmetry $\boldsymbol{Z}_N$. We extend the work of Ibáñez and Ross with $N = 2, 3$ to arbitrary values of N. As new fundamental symmetries, we find four $\boldsymbol{Z}_6$, nine $\boldsymbol{Z}_9$, and nine $\boldsymbol{Z}_{18}$. We then place three phenomenological demands upon the low-energy effective SSM: (i) the presence of the μ term in the superpotential, (ii) baryon-number conservation up to dimension-five operators, and (iii) the presence of the seesaw neutrino mass term LH_uLH_u. We are then left with only two anomaly-free discrete gauge symmetries: baryon triality, $\boldsymbol{B}_3$, and a new $\boldsymbol{Z}_6$, which we call proton hexality, $\boldsymbol{P}_6$. Unlike $\boldsymbol{B}_3$, $\boldsymbol{P}_6$ prohibits the dimension-four lepton-number violating operators. This we propose as *the* discrete gauge symmetry of the minimal SSM, instead of R parity.

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I. INTRODUCTION

The action of the standard model (SM) [1,2] is invariant under Poincaré transformations, as well as the gauge group $G_{\rm SM} = SU(3)_C \times SU(2)_W \times U(1)_Y$. When allowing only renormalizable interactions, baryon and lepton number are (accidental) global symmetries of the SM.[1] However, when considering the SM as a low-energy effective theory, $G_{\rm SM}$ allows for nonrenormalizable interactions, which can violate lepton and baryon number. The leading dimension-six operators are suppressed by two powers of an unknown mass scale M, which is unproblematic for proton decay if $M \gtrsim 10^{16}$ GeV; see, however, [7,8].

Enlarging the Poincaré group, the action of the supersymmetric SM (SSM) is invariant under supersymmetry, as well as $G_{\rm SM}$ [9,10]. The *renormalizable* superpotential of the SSM is given by [11–14]

$$\begin{aligned} W = {} & h^E_{ij} L_i H_d \bar{E}_j + h^D_{ij} Q_i H_d \bar{D}_j + h^U_{ij} Q_i H_u \bar{U}_j + \mu H_d H_u \\ & + \lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k + \lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k \\ & + \kappa_i L_i H_u, \end{aligned} \tag{1.1}$$

where we employ the notation of Ref. [15], and $SU(3)_C$ and $SU(2)_W$ indices are suppressed. The fifth, sixth, and eighth terms violate lepton number, and the seventh term violates baryon number. Thus, in the SSM, lepton and baryon number are violated by renormalizable dimension-four interactions. In particular, $LQ\bar{D}$ and $\bar{U}\,\bar{D}\,\bar{D}$ together lead to rapid proton decay. The lower experimental bound on the proton lifetime [16,17] results in the very stringent bounds [13,18,19]

$$\lambda'_{i1j} \cdot \lambda''_{11j} < 2 \times 10^{-27} \left(\frac{M_{\tilde{d}_j}}{100 \text{ GeV}} \right)^2, \qquad i = 1, 2, \quad j \neq 1, \tag{1.2}$$

and the SSM must be considered incomplete. In order to obtain a natural and viable supersymmetric model, we must extend $G_{\rm SM}$, such that at least one of the operators $LQ\bar{D}$ or $\bar{U}\,\bar{D}\,\bar{D}$ is forbidden.[2]

The minimal SSM (MSSM) is conventionally taken as the renormalizable SSM with the superpotential, Eq. (1.1), additionally constrained by the discrete symmetry R parity, $\boldsymbol{R}_p = (-\boldsymbol{1})^{2S+3B+L}$ [22], which acts on the components of the superfields. Here S is spin, B baryon number, and L lepton number. Hence the superpotential of the renormalizable MSSM is given solely by the first line of Eq. (1.1), and baryon and lepton number are conserved. Matter parity ($\boldsymbol{M}_p$) [23] acts on the superfields and leads to the same superpotential as $\boldsymbol{R}_p$. Our working definition of the MSSM shall be the SSM constrained by $\boldsymbol{M}_p$. We return to this in Sec. VI. Another possibility is to extend $G_{\rm SM}$ by baryon triality[3] ($\boldsymbol{B}_3$) [24,25], leading to the R-parity violating MSSM [15].

However, due to the unification of the $G_{\rm SM}$ gauge coupling constants in supersymmetry [27–30], and also the automatic inclusion of gravity in local supersymmetry [31,32], we expect the SSM, and also the MSSM, to be low-energy effective theories, embedded in a more complete theory formulated at the scale of grand unified theories ($M_{\rm GUT} \sim 10^{16}$ GeV) [33], or above. Within the SSM, we must therefore take into account the possible nonrenormalizable operators, which are consistent with $G_{\rm SM}$, within the MSSM, those which are also consistent with $\boldsymbol{M}_p$. In

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[1]When taking into account the sphaleron interactions [3], only $\frac{1}{3}B - L_i$, and $L_i - L_j$ are conserved in the SM. For the effect of sphaleron interactions in supersymmetry see, for example, Refs. [4–6].

[2]For an extensive set of bounds on the products of these operators see Refs. [20,21].

[3]This was originally introduced as baryon parity in [24,25]; however, it is more appropriately called baryon triality [10,26].

particular, we are here interested in the dimension-five baryon- and/or lepton-number violating interactions. In Eq. (6.1), we list the complete set for the SSM [11,12,15,25]; a subset is also present in the MSSM. Even if suppressed by the gravitational scale $M_{\rm grav} = 2.4 \times 10^{18}$ GeV, these operators are potentially dangerous, depending on their flavor structure [11,12,34]. Thus, even though $\boldsymbol{M}_p$ provides the SSM with an excellent candidate for cold dark matter, it has a serious problem with baryon-number violation. When considering the (high-energy) symmetry extension of the SSM, we take into account the effects on the dimension-four and the dimension-five operators.

It is the purpose of this paper to systematically investigate discrete $\boldsymbol{Z}_N$ symmetry extensions of $G_{\rm SM}$ without invoking the existence of new light particles. Since a global discrete symmetry is typically violated by quantum gravity effects [35], we focus on an Abelian discrete *gauge* symmetry (DGS): it is a discrete remnant of a spontaneously broken $U(1)$ gauge symmetry [35,36]. For an explicit Lagrangian see, e.g., Ref. [37]. Assuming the original gauge theory to be anomaly-free, Ibáñez and Ross (IR) determined the constraints on the remnant low-energy and family-independent DGSs [24,25]. They classified all $\boldsymbol{Z}_N$ DGSs for $N = 2, 3$ according to their action on the baryon- and lepton-number violating operators and then determined which are discrete gauge anomaly-free (see the end of Sec. II). They found only two such anomaly-free DGSs which prohibited the dimension-four baryon-number violating operators and allowed the H_dH_u term: matter parity (R_2 in their notation) and baryon triality, $\boldsymbol{B}_3$. The latter has the advantage of also prohibiting the dangerous dimension-five operators.

In this paper, we extend the work of IR to $\boldsymbol{Z}_N$ symmetries with arbitrary values of N. We first determine all family-independent anomaly-free DGSs consistent with the first three terms in Eq. (1.1) (Secs. II, III, and IV). From the low-energy point of view, where heavy and possibly $\boldsymbol{Z}_N$ charged particles do not play a role, this infinite number of anomaly-free DGSs can be rescaled to an equivalent finite set, which we denote as fundamental (Sec. V). We are left with four $\boldsymbol{Z}_6$, nine $\boldsymbol{Z}_9$, and nine $\boldsymbol{Z}_{18}$ new symmetries, beyond the five $\boldsymbol{Z}_{2,3}$ symmetries of IR. Together these 27 fundamental DGSs comprise a complete set. This is one of the main results of this paper. Next, we investigate their effect on the baryon- and lepton-number violating operators (Sec. VI). There is only one DGS which simultaneously allows the H_dH_u term, prohibits all dimension-four baryon- and lepton-number violating operators, prohibits the dimension-five baryon-number violating operators, and *allows* the dimension-five Majorana neutrino mass term LH_uLH_u. This is one of the $\boldsymbol{Z}_6$ symmetries, $R_6^5L_6^2$, in the notation of IR. We shall denote it proton hexality, $\boldsymbol{P}_6$. *This we propose as the DGS of the MSSM.* Every $\boldsymbol{Z}_6$ is isomorphic to a direct product of a $\boldsymbol{Z}_2$ and a $\boldsymbol{Z}_3$ [38], so it is not too surprising that $\boldsymbol{P}_6$ is isomorphic to the direct product of $\boldsymbol{M}_p$ and $\boldsymbol{B}_3$. We then investigate the necessity of heavy fermions in theories with anomaly-free DGSs (Sec. VII), leading to a different conclusion than Ref. [39].

In Secs. II, III, IV, V, VI, and VII we take a bottom-up approach in determining the discrete symmetry. At the CERN LHC, we will hopefully discover supersymmetric fields and their interactions. Through the measured and thus allowed interactions we can infer the discrete symmetry. From this point of view, two discrete symmetries are equivalent, if they result in the same low-energy interactions. In Sec. VIII, we instead investigate the top-down perspective, focusing on the distinct gauge theories leading to low-energy equivalent DGSs. For demonstrational purposes we finally present a gauged $U(1)$ model, which, after spontaneous symmetry breaking, leads to an effective SSM with proton hexality (Sec. IX).

We briefly comment on some related work in the literature. Throughout, we restrict ourselves to family-independent DGSs. For examples of family-dependent DGSs see Refs. [25,40]. We shall, however, in general, allow for the original gauge symmetry to be family dependent. We do not consider discrete R symmetries. For an anomaly-free gauged $U(1)$ R symmetry in a local supersymmetric theory see Refs. [41–43]. This could be broken to a discrete R symmetry. Since R parity is inserted *ad hoc* in the SSM to give the MSSM, there is extensive literature on "gauged" R parity, i.e. where R parity is the remnant of a broken gauge symmetry. Martin has considered R parity as embedded in a $U(1)_{B-L}$ gauge symmetry and classified the possible order parameters in extended gauge symmetries [$SO(10)$, $SU(5)$, $SU(5) \times U(1)$, E_6], which necessarily lead to R parity [44,45]. Babu *et al.* [46] combine DGSs with an attempt to solve the μ problem. Chemtob *et al.* [47] deal with anomaly-free DGSs of the next to MSSM (NMSSM). Although not in our systematic context, some of the anomaly-free DGSs we find are mentioned in the literature explicitly [46] or implicitly [48]. In particular, $\boldsymbol{P}_6$ occurs in Ref. [46], and in Refs. [49,50] a related nonsupersymmetric $\boldsymbol{Z}_6$ is studied.

II. THE LINEAR ANOMALY CONSTRAINTS

In this section, we review the work of IR [24,25] on DGSs. We focus here on constraints arising from the linear $U(1)_X$ anomalies $\mathcal{A}_{CCX}$, $\mathcal{A}_{WWX}$, and $\mathcal{A}_{GGX}$, where we adopt the notation of Ref. [51]. For example, the $SU(3)_C$-$SU(3)_C$-$U(1)_X$ anomaly is denoted as $\mathcal{A}_{CCX}$, and G stands for "gravity." In Sec. IV, we investigate the purely Abelian anomalies, i.e. $\mathcal{A}_{YYX}$, $\mathcal{A}_{YXX}$, and especially the cubic anomaly $\mathcal{A}_{XXX}$.

For the high-energy gauge symmetry, we consider an in general generation-dependent $U(1)_X$ extension of $G_{\rm SM}$, with the chiral superfield charges quantized (i.e. the quotient of any two charges is rational) and normalized to be integers. We assume it is spontaneously broken by the vacuum expectation value (VEV), v, of a scalar field Φ with $U(1)_X$ charge $X_\Phi \equiv N > 1$. The mass scale of the broken symmetry is $M_X = \mathcal{O}(v) \gg M_W$. (We assume here a single field Φ, or a vectorlike pair; cf. Sec. IX.) This leaves a residual, low-energy $\mathbf{Z}_N$ symmetry, which we assume to be generation independent[4] on the SSM chiral superfields [35,37]. In the low-energy theory, we restrict ourselves to the particle content of the SSM, allowing however for additional heavy fermions with masses $\mathcal{O}(M_X)$. To avoid later confusion, we emphasize here that the $U(1)_X$ charge of Φ is not necessarily the same N, which appears in the final $\mathbf{Z}_N$ we obtain when restricting ourselves to the so-called "fundamental" DGSs. We discuss this in more detail in Sec. V.

For the SSM fields, the $\mathbf{Z}_N$ charges q_i are related to the integer $U(1)_X$ charges X_i via a modulo N shift

$$X_i = q_i + m_i N. \tag{2.1}$$

Here the index i labels the SSM particle species and q_i, m_i are integers. Just like the $U(1)_X$ charges, the m_i are, in general, generation *dependent*, whereas the q_i are assumed to be generation independent. We also allow for Dirac and Majorana fermions which become massive at $\mathcal{O}(M_X)$. For the former, two fields with $U(1)_X$ charges X^j_{D1} and X^j_{D2}, respectively, must pair up, resulting in a Dirac mass term, after $U(1)_X$ breaking. The Majorana fields with charge $X^{j'}_{\mathrm{M}}$ can directly form a mass term. The $\mathbf{Z}_N$ invariance of these mass terms requires

$$X^j_{\mathrm{D1}} + X^j_{\mathrm{D2}} = p_j N, \qquad p_j \in \mathbb{Z}, \tag{2.2}$$

$$2 \cdot X^{j'}_{\mathrm{M}} = p'_{j'} N, \qquad p'_{j'} \in \mathbb{Z}. \tag{2.3}$$

The indices j and j' run over all heavy Dirac and Majorana particles, respectively.

Assuming the initial $U(1)_X$ is anomaly-free, IR derive the resulting constraints on the $\mathbf{Z}_N$ charges q_i of Eq. (2.1). From the anomaly cancellation conditions $\mathcal{A}_{CCX} = \mathcal{A}_{WWX} = \mathcal{A}_{GGX} = 0$, we obtain

$$\sum_{i=\mathbf{3},\bar{\mathbf{3}}} q_i = -N \cdot \Big[\sum_{i=\mathbf{3},\bar{\mathbf{3}}} m_i + \sum_{j=\mathbf{3},\bar{\mathbf{3}}} p_j \Big], \tag{2.4}$$

$$\sum_{i=\mathbf{2}} q_i = -N \cdot \Big[\sum_{i=\mathbf{2}} m_i + \sum_{j=\mathbf{2}} p_j \Big], \tag{2.5}$$

$$\sum_{i} q_i = -N \cdot \Big[\sum_{i} m_i + \sum_{j} p_j + \sum_{j'} \frac{1}{2} p'_{j'} \Big]. \tag{2.6}$$

The sums in Eqs. (2.4) and (2.5) run over all color triplets and weak doublets, respectively, i.e. we restrict ourselves to only fundamental representations[5] of $SU(3)_C$ and $SU(2)_W$. As all particles couple gravitationally, we sum over the entire chiral superfield spectrum in Eq. (2.6).

Depending on the charge shifts, m_i, of the low-energy fields, as well as the heavy-fermion particle content, the square brackets in Eqs. (2.4), (2.5), and (2.6) can take on arbitrary integer values. In the case of even N, any half-odd integer is allowed for the square bracket in Eq. (2.6). Hence, we can rewrite them symbolically as

$$\sum_{i=\mathbf{3},\bar{\mathbf{3}}} q_i = N \cdot \mathbb{Z}, \tag{2.7}$$

$$\sum_{i=\mathbf{2}} q_i = N \cdot \mathbb{Z}, \tag{2.8}$$

$$\sum_{i} q_i = N \cdot \mathbb{Z} + \eta \cdot \frac{N}{2} \cdot \mathbb{Z}, \tag{2.9}$$

with $\eta = 0, 1$ for $N =$ odd, even, respectively. From the point of view of the low-energy theory, the various $\mathbb{Z}$'s, including the two in Eq. (2.9), *each* represent an arbitrary and independent integer, which is fixed by the heavy-fermion content and the choice of m_i.

In addition to the anomaly constraints, we obtain constraints on the $U(1)_X$ charges, by requiring a minimal set of interaction terms in the SSM superpotential, which are responsible for the low-energy fermion masses, namely, the first three terms in Eq. (1.1). In Sec. VI we investigate the consequences of additionally imposing $H_d H_u$ invariance. The $\mathbf{Z}_N$ charge equations corresponding to the first three terms of Eq. (1.1) are

$$q_L + q_{H_d} + q_{\bar{E}} = 0 \bmod N, \tag{2.10}$$

$$q_Q + q_{H_d} + q_{\bar{D}} = 0 \bmod N, \tag{2.11}$$

$$q_Q + q_{H_u} + q_{\bar{U}} = 0 \bmod N. \tag{2.12}$$

These are three equations for seven unknowns. We can thus write the family-independent $\mathbf{Z}_N$ charges of the SSM

[4]Note that, due to the *three* nonvanishing mixing angles of the Cabibbo-Kobayashi-Maskawa matrix, one is forced to work with generation-independent discrete charges for the quarks. Concerning the leptons, generation dependence is only possible if one relies on radiatively generated neutrino masses. See Ref. [40].

[5]The contribution of a fermion to an $SU(M)$-$SU(M)$-$U(1)$ anomaly is proportional to the corresponding Dynkin index [52]. Particles constituting higher irreducible representations of $SU(M)$ have a Dynkin index which is an integer multiple of that of the fundamental M-plet [46,50]. Therefore heavy particles in higher irreducible representations need not be considered for our purposes; see Eqs. (2.7), (2.8), and (2.9). Note that in Eqs. (2.4) and (2.5) we do not consider Majorana particles either, because all real representations of $SU(M)$ have a Dynkin index which is an even multiple of that of the fundamental irreducible representation; see Refs. [46,53].

superfields in terms of four independent integers, which we choose as $m, n, p, r = 0, 1, \ldots, N-1$.

$$\begin{aligned} q_Q &= r, \qquad q_{\bar{U}} = -m - 4r, \qquad q_{\bar{D}} = m - n + 2r, \\ q_L &= -n - p - 3r, \qquad q_{\bar{E}} = m + p + 6r, \\ q_{H_d} &= -m + n - 3r, \qquad q_{H_u} = m + 3r. \end{aligned} \tag{2.13}$$

In the following, we make use of the integer normalized hypercharges

$$Y(Q, \bar{U}, \bar{D}, L, \bar{E}, H_d, H_u) = (-1, 4, -2, 3, -6, 3, -3). \tag{2.14}$$

The choice of integers m, n, p in Eq. (2.13) corresponds to the notation of IR. The slightly unusual coefficients for the integer r correspond to the negative normalized hypercharge given in Eq. (2.14), and were chosen for the following charge transformation: To simplify the upcoming calculations, we perform a shift of the integer $\boldsymbol{Z}_N$ charges by their integer hypercharges, such that the resulting charge q'_Q is zero,

$$q_i \to {q_i}' = q_i + Y_i \cdot r. \tag{2.15}$$

In the following, we drop the prime on the charge symbols. This shift in the $\boldsymbol{Z}_N$ charges does *not* change the effect of $\boldsymbol{Z}_N$ on the renormalizable or nonrenormalizable operators of the SSM superpotential or D-terms, since these are all $U(1)_Y$ invariant. It also does not affect the anomaly equations which we consider. However, it does correspond to a change in the underlying $U(1)_X$ gauge theory. The difference can lead to, in principle, observable effects, for example, cross sections which depend on X charges. We return to this change in Sec. VIII.

The choice of charges where $q_Q = 0$ is the basis in which IR work. They show that, in this case, any $\boldsymbol{Z}_N$ symmetry g_N can be expressed in terms of the product of powers of the three (mutually commuting) generators R_N, A_N and L_N [25]:

$$\begin{aligned} g_N = R_N^m \times A_N^n \times L_N^p, &\quad \text{with the exponents} \\ m, n, p = 0, 1, \ldots, N-1. & \end{aligned} \tag{2.16}$$

The charges of the SSM chiral superfields under the three independent $\boldsymbol{Z}_N$ generators are given in Table 1 of Ref. [25]. In terms of the powers m, n, p, the generation-independent $\boldsymbol{Z}_N$ charges of the SSM superfields are[6]

$$\begin{aligned} q_Q &= 0, \qquad q_{\bar{U}} = -m, \qquad q_{\bar{D}} = m - n \\ q_L &= -n - p, \qquad q_{\bar{E}} = m + p, \\ q_{H_d} &= -m + n, \qquad q_{H_u} = m. \end{aligned} \tag{2.17}$$

Note that the integers m, n, p here are the same as in Eq. (2.13). Inserting the charges above into Eqs. (2.7), (2.8), and (2.9), and assuming the SSM light-fermion content, we arrive at the conditions[7]

$$3n = N \cdot \mathbb{Z}, \tag{2.18}$$

$$3(n + p) - n = N \cdot \mathbb{Z}, \tag{2.19}$$

$$3(5n + p - m) - 2n = N \cdot \mathbb{Z} + \eta \cdot \frac{N}{2} \cdot \mathbb{Z}. \tag{2.20}$$

Since all $\mathbb{Z}$'s in Eqs. (2.18), (2.19), and (2.20) stand for arbitrary and independent integers, we can combine these Diophantine equations to obtain a simpler set,

$$3n = N \cdot \mathbb{Z}, \tag{2.21}$$

$$3p - n = N \cdot \mathbb{Z}, \tag{2.22}$$

$$3(m + p) = N \cdot \mathbb{Z} + \eta \cdot \frac{N}{2} \cdot \mathbb{Z}. \tag{2.23}$$

This differs slightly from IR in notation, as we find it more convenient to retain the arbitrary integers $\mathbb{Z}$ on the RHS. These three equations are the basis for our further study. DGSs satisfying all three equations will be called "anomaly-free DGSs," although these constraints are only necessary but not sufficient for complete anomaly freedom of the high-energy theory [39,53].

III. SYMMETRIES ALLOWED BY THE LINEAR CONSTRAINTS

In this section, we go beyond the work of IR and determine the solutions, $(n, p, m; N)$, to the Eqs. (2.21), (2.22), and (2.23) for *general* values of N, not just $N = 2$, 3. We separately consider the two possibilities: either N *is not* or *is* a multiple of 3. We employ the notation

$$(k|N): \Leftrightarrow \; N = 0 \bmod k,$$

$$\neg(k|N): \Leftrightarrow \; N \neq 0 \bmod k.$$

$k \in \mathbb{N}$, where $\mathbb{N}$ is the set of all positive integers including zero.

(1) $\neg(3|N)$: Since $n = 0, 1, \ldots, N-1$, Eq. (2.21) requires $n = 0$. Then Eq. (2.22) similarly gives $p = 0$. Finally, Eq. (2.23) then implies
   - (i) $m = 0$ for odd N. This is the case of the trivial symmetry, the identity.
   - (ii) For even N there are two possibilities, either $m = 0$ (trivial) or $m = \frac{N}{2}$.

   We conclude that the only *nontrivial* anomaly-free DGSs here are

$$g_N = R_N^{N/2}, \qquad N = \text{even}. \tag{3.1}$$

[6] The action of g_N on e.g. the chiral superfields $\bar{D}_i$ is thus given by $\bar{D}_i \to \exp[\frac{2\pi i}{N}(m - n)]\bar{D}_i$.

[7] These equations are r independent; they result by directly plugging Eq. (2.13) into Eqs. (2.7), (2.8), and (2.9). However, when considering the cubic anomaly in Sec. IV, the r dependence does not cancel.

TABLE I. The list of all DGSs satisfying the linear anomaly constraints of Ibáñez and Ross. N' and N'' are defined by $N = 3N' = 9N''$, where $N, N', N'' \in \mathbb{N}$. The $\ell_p = 2$ cases are not listed as they are equivalent to the set of DGSs with $\ell_p = 1$. The last column gives the resulting number of independent nontrivial DGSs, g_N, for fixed N.

$\mathbf{Z}_N$ category		n	p	m	No. of independent g_N
$\neg(3\|N)$	N even	0	0	$\frac{N}{2}$	1
$(3\|N)$	N odd	0	$(0,1)\cdot N'$	$(0,1,2)\cdot N'$	4
	N even	0	$(0,1)\cdot N'$	$(0,1,2,3,4,5)\cdot \frac{N'}{2}$	9
$(9\|N)$	N odd	N'	$(1,4,7)\cdot N''$	$(2,5,8)\cdot N''$	9
	N even	N'	$(1,4,7)\cdot N''$	$(1,4,7,10,13,16)\cdot \frac{N''}{2}$	18

The simplest case with $N = 2$ yields the discrete $\mathbf{Z}_2$ charges: $q_Q = q_L = 0$, $q_{\bar{D}} = q_{\bar{E}} = q_{H_u} = 1$, $q_{\bar{U}} = q_{H_d} = -1$. This charge assignment is, from the low-energy point of view, equivalent to standard matter parity [23]. A reversed hypercharge shift, Eq. (2.15), back to Eq. (2.13) with $r = 1$ yields $q_Q = q_L = q_{\bar{D}} = q_{\bar{U}} = q_{\bar{E}} = 1 \mod 2$, $q_{H_u} = q_{H_d} = 0$.

(2) $(3|N)$: Here we can define an $N' \in \mathbb{Z}$, such that $N \equiv 3N'$. From Eq. (2.21) we obtain $n = 0$, N', or $2N'$:

(i) Focusing first on $n = 0$, we see that $p = \ell_p N'$, for $\ell_p = 0, 1, 2$. Concerning Eq. (2.23), it is again necessary to distinguish between odd and even N. Thus we find a set of anomaly-free DGSs

$$n = 0, \qquad p = \ell_p N', \qquad m = \begin{cases} \ell_m N', & N = \text{odd}, \\ s_m \frac{N'}{2}, & N = \text{even}, \end{cases} \tag{3.2}$$

with $\ell_p, \ell_m = 0, 1, 2$ and $s_m = 0, 1, \ldots, 5$.

(ii) Inserting $n = N'$ into Eq. (2.22), we obtain $p = \frac{N'}{3} + \ell_p N'$, again with $\ell_p = 0, 1, 2$. For $p \in \mathbb{Z}$, we need $(3|N')$ or equivalently $N' \equiv 3N''$, with $N'' \in \mathbb{Z}$. Taking into account Eq. (2.23), we now find

$$n = N', \qquad p = (1 + 3\ell_p)N'', \qquad m = \begin{cases} (2 + 3\ell_m)N'', & N = \text{odd}, \\ (1 + 3s_m)\frac{N''}{2}, & N = \text{even}. \end{cases} \tag{3.3}$$

(iii) Analogously, $n = 2N'$ gives

$$n = 2N', \qquad p = (2 + 3\ell_p)N'', \qquad m = \begin{cases} (1 + 3\ell_m)N'', & N = \text{odd}, \\ (2 + 3s_m)\frac{N''}{2}, & N = \text{even}. \end{cases} \tag{3.4}$$

The class of DGSs given in (iii) need not be investigated any further for it is equivalent to the one in (ii): A $\mathbf{Z}_N$ symmetry with charges q_i is indistinguishable from one with charges $-q_i$; therefore the sets (n, p, m) and $(N - n, N - p, N - m)$ yield equivalent DGSs. As an example, consider the integer p. For every p_2 in Eq. (3.4) we require a p_1 in Eq. (3.3), such that $p_2 \stackrel{!}{=} N - p_1$. Inserting Eqs. (3.3) and (3.4), we obtain $(2 + 3\ell_{p_2})N'' \stackrel{!}{=} (9 - 1 - 3\ell_{p_1})N''$, which is solved for $\ell_{p_1} = 2 - \ell_{p_2} \in \{0, 1, 2\}$. Similarly, the integer m can be treated for even or odd N. Likewise, some DGSs of Eq. (3.2) are not independent of the others.

Table I summarizes the anomaly-free DGSs classified by N and the powers n, p, and m. For example, the two rows with $(3|N)$ correspond to the DGSs of Eq. (3.2). The last column shows the number of independent nontrivial g_N. The 4 in the second row arises because there are three DGSs with $\ell_p = 1$ but only one with $\ell_p = 0$; with $p = 0$, the case $m = 0$ is trivial, whereas $m = N'$ and $m = 2N'$ lead to equivalent DGSs. Similarly, we get nine DGSs instead of 12 for the third row.

IV. THE PURELY ABELIAN ANOMALIES

So far, we have determined the constraints on DGSs arising from the three linear anomaly conditions of Eqs. (2.4), (2.5), and (2.6). Next we consider the three purely Abelian anomalies $\mathcal{A}_{YYX}$, $\mathcal{A}_{YXX}$, and $\mathcal{A}_{XXX}$, respectively.

(1) Analogous to Eqs. (2.4), (2.5), and (2.6), we obtain from $\mathcal{A}_{YYX} = 0$ that

$$\sum_i Y_i^2 q_i = -N\Big[\sum_i Y_i^2 m_i + \sum_j {Y_{\rm D1}^j}^2 p_j\Big]. \tag{4.1}$$

We have used $Y_{\rm D2}^j = -Y_{\rm D1}^j$ and $Y_{\rm M}^j = 0$, as well as Eq. (2.2). Note that each term, unlike those in Eqs. (2.4), (2.5), and (2.6), contains a factor of ${Y_{...}}^2$, which is, in general, different for each field.[8] Recall that we have chosen the hypercharges to be integer for all SSM particles; see Eq. (2.14). Thus the left-hand side (LHS) is integer. However, given

[8] In the case of the non-Abelian linear anomalies $\mathcal{A}_{CCX}$ and $\mathcal{A}_{WWX}$, one encounters a factor proportional to the Dynkin index instead. This is a common factor for all fields provided they are all in the fundamental representation of $SU(3)_C$ and $SU(2)_W$, respectively.

TABLE II. Compatibility of the linear and the cubic anomaly constraints in the case of integer $U(1)_X$ charges for *all* chiral superfields. For each $\boldsymbol{Z}_N$ category, the allowed values of N are given in the far right column. The DGSs are specified by the set (n, p, m), in accordance with Eq. (2.16). We employ the notations $N' \equiv N/3$, $N'' \equiv N/9$, and $N', N'' \in \mathbb{N}$. For special values of N, all linearly allowed DGSs are compatible with the cubic anomaly condition. However, four classes of DGSs within the categories $(3|N)$ (rows 3, 5, 6, 7) are possible for less constrained N.

$\boldsymbol{Z}_N$ category		n	p	m	Possible N
$\neg(3\vert N)$	N even	0	0	$\frac{N}{2}$	$2\cdot\mathbb{N}$
$(3\vert N)$	N odd	0	$(0,1)\cdot N'$	$(0,1,2)\cdot N'$	$9\cdot(2\cdot\mathbb{N}+1)$
		0	N'	N'	$3\cdot(2\cdot\mathbb{N}+1)$
$(3\vert N)$	N even	0	$(0,1)\cdot N'$	$(0,1,2,3,4,5)\cdot\frac{N'}{2}$	$18\cdot\mathbb{N}$
		0	0	$\frac{N}{2}$	$6\cdot\mathbb{N}$
		0	N'	N'	$6\cdot\mathbb{N}$
		0	N'	$5\cdot\frac{N'}{2}$	$6\cdot\mathbb{N}$
$(9\vert N)$	N odd	N'	$(1,4,7)\cdot N''$	$(2,5,8)\cdot N''$	$27\cdot(2\cdot\mathbb{N}+1)$
	N even	N'	$(1,4,7)\cdot N''$	$(1,4,7,10,13,16)\cdot\frac{N''}{2}$	$54\cdot\mathbb{N}$

this normalization, the hypercharges of the heavy fermions need not be integer and the quantity in square brackets need not be in $\mathbb{Z}$. Thus the right-hand side (RHS) can take on *any* value within $\mathbb{Z}$. Therefore Eq. (4.1) poses no constraint.

(2) Now we take $\mathcal{A}_{YXX} = 0$. Analogous to Eq. (4.1), we get

$$\sum_i Y_i {q_i}^2 = -N\Big[\sum_i Y_i m_i(m_i N + 2q_i) - \sum_j Y^j_{\rm D1} p_j(p_j N - 2X^j_{\rm D1})\Big]. \quad (4.2)$$

By considering only the $Y^j_{\rm D1}$, we see that $[\ldots]$ is not necessarily an integer, just as in the previous case. Thus Eq. (4.2) is of no use from the low-energy point of view.[9]

(3) Next, we consider the cubic anomaly $\mathcal{A}_{XXX}$. Here we do not have a mixture of known and unknown charges: We do not know any of the $U(1)_X$ charges. We obtain for the anomaly equation

$$\sum_i {q_i}^3 = -\sum_i (3{q_i}^2 m_i N + 3q_i {m_i}^2 N^2 + {m_i}^3 N^3) - \sum_j (3{X^j_{\rm D1}}^2 p_j N - 3X^j_{\rm D1} {p_j}^2 N^2 + {p_j}^3 N^3) - \frac{1}{8}\sum_{j'} {p'_{j'}}^3 N^3. \quad (4.3)$$

If fractional $X^j_{\rm D1}$ were allowed, again no extraction of a meaningful constraint is feasible, since in this case the right-hand side of Eq. (4.3) is not necessarily of the form $N\cdot\mathbb{Z}$. However, as outlined in Sec. II, we only consider integer X charges here. We shall investigate the case of fractional X charges for the heavy fields in Sec. V, since the difference can be meaningful in cosmology [54–56].

The calculation for the cubic anomaly with only integer charges is similar to the calculation in Sec. III, i.e. it involves many case distinctions. It can be found in the Appendix A. In Table II, we have summarized the results. We show those N, as well as the powers (n, p, m), in the case of only integer X charges, which satisfy the linear anomaly constraints of Sec. III (cf. Table I), as well as the cubic anomaly equation considered here. The main effect of the cubic anomaly constraint is to reduce the (infinite) list of possible DGSs. Considering $N = 3$ for instance, there are four independent g_N symmetries allowed in Table I. However, only one of these, namely, the case where $(n, p, m) = (0, 1, 1)$, complies with Table II. This corresponds to $\boldsymbol{B}_3$, i.e. baryon triality discussed by IR.

Another example is $N = 6$. Here we have nine linearly allowed DGSs, while only three are left after imposing the cubic anomaly constraint: R_6^3, $R_6^2 L_6^2$, and $R_6^5 L_6^2$. The first two are physically equivalent to $\boldsymbol{M}_p$ and $\boldsymbol{B}_3$ from the low-energy point of view. We shall denote $\boldsymbol{P}_6 \equiv R_6^5 L_6^2$, as proton hexality. This is a special discrete symmetry, which we return to in Sec. VI. For $N = 9$ there are $4 + 9$ linearly allowed g_N, of which only four are also consistent with the cubic anomaly condition. $N = 27$ is the first case for $(3|N)$, where the cubic anomaly does not reduce the number of allowed DGSs.

[9]We disagree here with Refs. [24,25] about the reason why $\mathcal{A}_{YYX}$ and $\mathcal{A}_{YXX}$ do not impose useful constraints on $\boldsymbol{Z}_N$ symmetries. It is *not* the (overall) normalization of the Abelian charges, but the fact that these charges are, in general, different for each field.

V. CHARGE RESCALING

So far, we have assumed that hypercharge shifted discrete symmetries, as in Eq. (2.15), are equivalent and *all* chiral superfields have integer $U(1)_X$ charges. However, from the *low-energy* point of view, this latter assumption is too restrictive [39,53]. To see this in our analysis, consider an example from Table II, where $N = 18$. The powers of the elementary discrete gauge group generators, Eq. (2.16), are given by

$$n = 0, \qquad p = 6 \cdot (0, 1), \qquad m = 3 \cdot s_m, \qquad s_m = 0, 1, \ldots, 5, \tag{5.1}$$

which are all multiples of the common factor $F = 3$. The charges of the SSM fields, $q_i + m_i N$, are given in Eq. (2.17) as linear combinations of n, p, and m, and are therefore also all multiples of F, in our example. From the low-energy point of view, with the heavy fields integrated out, such a charge assignment is indistinguishable from a scaled one with charges $(q_i + m_i N)/F$. After the breakdown of $U(1)_X$, the residual DGS is then a $\mathbf{Z}_{N/F}$ instead of a $\mathbf{Z}_N$. However, the $\mathbf{Z}_{N/F}$ does not necessarily satisfy the cubic anomaly, with all integer charges. In our example, we have $N/F = 6$, which, according to Table II, satisfies the cubic anomaly only for very special values of (n, p, m).

This integer rescaling only applies to the charges of the SSM chiral superfields. For the *heavy* fermions, it is typically not possible and leads to fractional charges. From a bottom-up approach, experiments would determine the rescaled DGS group $\mathbf{Z}_{N/F}$. When searching for the possible (low-energy) anomaly-free DGSs, we therefore relax our original assumption of integer charges and instead allow fractional charges for the heavy sector only. We then denote the DGS $\mathbf{Z}_{N/F}$ with the *maximally* rescaled charges as the *fundamental* DGS, i.e. F is the largest common factor of N and all $q_i + m_i N$. In Table III, we present the complete list of fundamental DGSs, obtained from Table II. We see that, after rescaling, the infinite number of DGSs listed in Table II is reduced to a finite set of 27 fundamental $\mathbf{Z}_N$ symmetries: one with $N = 2$, four with $N = 3$, four with $N = 6$, nine with $N = 9$, and nine with $N = 18$.

References [39,53] pointed out that the cubic anomaly constraint is, in general, too restrictive on *low-energy* anomaly-free DGSs due to possible rescalings. Comparing Table II with Table III presents a classification within the SSM of the solutions to this problem. As emphasized earlier, the cubic anomaly constraint is compatible with *all* five classes of linearly allowed DGSs presented in Table I, however only for restricted values of N. Rescaling the charges, and allowing for fractionally charged heavy fermions, eliminates the influence of the $\mathcal{A}_{XXX}$ condition on the fundamental DGSs completely. In other words, *all* linearly allowed *fundamental* DGSs are compatible with the cubic anomaly constraint. Therefore,

TABLE III. All fundamental DGSs satisfying the linear and the cubic anomaly cancellation conditions. The heavy-fermion charges, X^j, are allowed to be fractional. The three underlined DGSs can be realized with only integer heavy-fermion $U(1)_X$ charges.

N	n	p	m	DGSs
2	0	0	1	$\underline{R_2}$
3	0	0	1	R_3
	0	1	(0, 1, 2)	L_3, $\underline{L_3 R_3}$, $L_3 R_3^2$
6	0	0	1	R_6
	0	2	(1, 3, 5)	$L_6^2 R_6$, $L_6^2 R_6^3$, $\underline{L_6^2 R_6^5}$
9	3	1	(2, 5, 8)	$A_9^3 L_9 R_9^2$, $A_9^3 L_9 R_9^5$, $A_9^3 L_9 R_9^8$
	3	4	(2, 5, 8)	$A_9^3 L_9^4 R_9^2$, $A_9^3 L_9^4 R_9^5$, $A_9^3 L_9^4 R_9^8$
	3	7	(2, 5, 8)	$A_9^3 L_9^7 R_9^2$, $A_9^3 L_9^7 R_9^5$, $A_9^3 L_9^7 R_9^8$
18	6	2	(1, 7, 13)	$A_{18}^6 L_{18}^2 R_{18}$, $A_{18}^6 L_{18}^2 R_{18}^7$, $A_{18}^6 L_{18}^2 R_{18}^{13}$
	6	8	(1, 7, 13)	$A_{18}^6 L_{18}^8 R_{18}$, $A_{18}^6 L_{18}^8 R_{18}^7$, $A_{18}^6 L_{18}^8 R_{18}^{13}$
	6	14	(1, 7, 13)	$A_{18}^6 L_{18}^{14} R_{18}$, $A_{18}^6 L_{18}^{14} R_{18}^7$, $A_{18}^6 L_{18}^{14} R_{18}^{13}$

Eq. (4.3) contains only information about whether or not the heavy-fermion $U(1)_X$ charges are fractional or integer. Of the fundamental DGSs listed in Table III, solely $\boldsymbol{M}_p \equiv R_2$, $\boldsymbol{B}_3 \equiv R_3 L_3$, and $\boldsymbol{P}_6 \equiv R_6^5 L_6^2$ are consistent with both the linear and the cubic anomaly conditions, without including fractionally charged heavy particles.

VI. PHYSICS OF THE FUNDAMENTAL DISCRETE GAUGE SYMMETRIES AND THE MSSM

Now that we have found a finite number of fundamental, anomaly-free, low-energy DGSs, we would like to investigate the correspondingly allowed SSM operators. In particular, we study the effect of the 27 fundamental DGSs given in Table III on the crucial baryon- and/or lepton-number violating superpotential and Kähler potential operators [15,25]:

$$\begin{aligned}
\mathcal{O}_1 &= [LH_u]_F, & \mathcal{O}_2 &= [LL\bar{E}]_F,\\
\mathcal{O}_3 &= [LQ\bar{D}]_F, & \mathcal{O}_4 &= [\bar{U}\,\bar{D}\,\bar{D}]_F,\\
\mathcal{O}_5 &= [QQQL]_F, & \mathcal{O}_6 &= [\bar{U}\,\bar{U}\,\bar{D}\,\bar{E}]_F,\\
\mathcal{O}_7 &= [QQQH_d]_F, & \mathcal{O}_8 &= [Q\bar{U}\,\bar{E}\,H_d]_F,\\
\mathcal{O}_9 &= [LH_uLH_u]_F, & \mathcal{O}_{10} &= [LH_uH_dH_u]_F,\\
\mathcal{O}_{11} &= [\bar{U}\bar{D}^*\bar{E}]_D, & \mathcal{O}_{12} &= [H_u{}^*H_d\bar{E}]_D,\\
\mathcal{O}_{13} &= [Q\bar{U}L^*]_D, & \mathcal{O}_{14} &= [QQ\bar{D}^*]_D.
\end{aligned} \tag{6.1}$$

The subscripts F and D denote the F- and D-term of the corresponding product of superfields. Table IV summarizes which operators are allowed for each fundamental anomaly-free DGS. The symbol $\surd$ indicates that an operator is allowed. Thus, for example, matter parity (R_2) allows the operators $[H_d H_u]_F$, but also the dimension-five baryon-

TABLE IV. Physical consequences of the 27 fundamental DGSs. The Higgs Yukawa couplings $LH_d\bar{E}$, $QH_d\bar{D}$, and $QH_u\bar{U}$ are allowed for every DGS we consider by construction. The symbol $\surd$ denotes that the corresponding operator is possible for a given DGS. All anomaly-free fundamental $\mathbf{Z}_9$ and $\mathbf{Z}_{18}$ symmetries forbid the operators listed in the left column.

	R_2	R_3L_3	R_3	L_3	$R_3^2L_3$	$R_6^5L_6^2$	R_6	$R_6^3L_6^2$	$R_6L_6^2$	All $\mathbf{Z}_9$ & $\mathbf{Z}_{18}$
H_dH_u	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	
LH_u		$\surd$								
$LL\bar{E}$		$\surd$								
$LQ\bar{D}$		$\surd$								
$\bar{U}\bar{D}\bar{D}$				$\surd$						
$QQQL$	$\surd$		$\surd$				$\surd$			
$\bar{U}\bar{U}\bar{D}\bar{E}$	$\surd$		$\surd$				$\surd$			
$QQQH_d$				$\surd$						
$Q\bar{U}\bar{E}H_d$		$\surd$								
LH_uLH_u	$\surd$	$\surd$				$\surd$				
$LH_uH_dH_u$		$\surd$								
$\bar{U}\bar{D}^*\bar{E}$		$\surd$								
$H_u{}^*H_d\bar{E}$		$\surd$								
$Q\bar{U}L^*$		$\surd$								
$QQ\bar{D}^*$				$\surd$						

number violating operators $[QQQL]_F$ and $[\bar{U}\bar{U}\bar{D}\bar{E}]_F$, as well as the lepton-number violating operators $[LH_uLH_u]_F$. We have included the bilinear operators LH_u (unlike IR), since, even under the most general complex field rotation [57], they can not be eliminated, when taking into account the corresponding soft-breaking terms [58].

We now demand the existence or absence of certain operators on phenomenological grounds and thus further narrow down our choice of DGSs.

(i) We have not included the term $[\mu H_dH_u]_F$ in the original list leading to Eqs. (2.10), (2.11), and (2.12), since, in principle, it can be generated, e.g. dynamically [59–62]. From a low-energy point of view we must have $\mu \neq 0$, and it must be of order the weak scale [63,64]. There are attempts in the literature to combine the NMSSM or another dynamical mechanism to generate $\mu \neq 0$ with an anomaly-free DGS; see, for example, Ref. [47] or Ref. [46] (and references therein), respectively. This is beyond the scope of this paper. If we explicitly require the $[\mu H_dH_u]_F$ operator in our theory, then, as can be seen from Table IV, all fundamental $\mathbf{Z}_9$ and $\mathbf{Z}_{18}$ symmetries are excluded.

(ii) Concerning proton decay, if we wish to exclude up to dimension-five baryon-number violating operators, we are left with the DGSs: R_3L_3 ($\boldsymbol{B}_3$), $R_3^2L_3$, $R_6^5L_6^2$ ($\boldsymbol{P}_6$), $R_6^3L_6^2$, and $R_6L_6^2$. For R_2 ($\boldsymbol{M}_p$), R_3, or R_6, $QQQL$ and $\bar{U}\bar{U}\bar{D}\bar{E}$ must be suppressed by some mechanism due to the stringent bounds on proton decay; see, e.g., Refs. [34,65]. The DGS L_3 is significantly constrained by the bounds on $\bar{U}\bar{D}\bar{D}$ from heavy nucleon decay [18].

(iii) Now consider neutrino masses. Without right-handed neutrinos, we can generate masses at tree level through the terms LH_uLH_u and LH_u (via mixing with the neutralinos), or via loop diagrams involving $LL\bar{E}$ or $LQ\bar{D}$ [26,66–68]. Hence, the DGSs R_2 ($\boldsymbol{M}_p$), R_3L_3 ($\boldsymbol{B}_3$), and $R_6^5L_6^2$ ($\boldsymbol{P}_6$) can incorporate neutrino masses without right-handed neutrinos.[10] However, right-handed neutrinos can easily be included as heavy Majorana fermions obeying Eq. (2.3). If the corresponding $U(1)_X$ charges allow Dirac neutrino mass terms, we obtain massive light neutrinos via the seesaw mechanism [69–72]. But, in this case, LH_uLH_u must be allowed by the $\boldsymbol{Z}_N$ symmetry as well: invariance of the Dirac mass terms for neutrinos as well as the Majorana mass terms implies a $\boldsymbol{Z}_N$-invariant LH_uLH_u term.

If we combine these phenomenological requirements, we are left with only two DGSs: baryon triality $\boldsymbol{B}_3$, and proton hexality $\boldsymbol{P}_6$. It is remarkable that these discrete symmetries also survived in Sec. V, i.e. they are discrete gauge anomaly-free with *integer* heavy-fermion charges. However, we would like to go a step further. In Sec. I, we defined the MSSM as the SSM restricted by $\boldsymbol{M}_p$. When

[10]It is not possible to generate neutrino masses in the SSM in the case of R_3 or R_6. They allow for the lepton-number violating terms $QQQL$ and $\bar{U}\bar{U}\bar{D}\bar{E}$ but conserve $B - L$.

considering the MSSM as a low-energy effective theory, the dangerous operators $QQQL$ and $\bar{U}\,\bar{U}\,\bar{D}\,\bar{E}$ are *allowed*. This is a highly unpleasant feature of the MSSM. IR already pointed this out as an advantage of the R-parity violating MSSM with $\boldsymbol{B}_3$, which does not suffer this problem. Here we propose a different solution: *We define the MSSM as the SSM which is restricted by proton hexality*, $\boldsymbol{P}_6$. The only phenomenological difference from the conventional MSSM with $\boldsymbol{M}_p$ is with respect to baryon-number violation. However, given the stringent bounds on proton decay, we find this new definition of the MSSM significantly better motivated. Note that, in the language of IR, $\boldsymbol{P}_6$ is a generalized matter parity (GMP).

We conclude this section with some observations:

(1) It is interesting to note that, of the nine fundamental DGSs which allow the $H_d H_u$ term, those with $N = 6$ are each equivalent to the requirement of imposing R_2 (i.e. matter parity) *along with* one of the four fundamental $\boldsymbol{Z}_3$ symmetries. Explicitly one has

$$R_2 \times R_3 L_3 \cong R_6^5 L_6^2, \quad \Longleftrightarrow \quad \boldsymbol{M}_p \times \boldsymbol{B}_3 \cong \boldsymbol{P}_6 \tag{6.2}$$

$$R_2 \times R_3 \cong R_6, \tag{6.3}$$

$$R_2 \times L_3 \cong R_6^3 L_6^2, \tag{6.4}$$

$$R_2 \times R_3^2 L_3 \cong R_6 L_6^2. \tag{6.5}$$

In the first line we have given the corresponding isomorphism in terms of matter parity, baryon triality, and proton hexality. The reason for this is that the Cartesian product of the cyclic groups $\boldsymbol{Z}_2$ and $\boldsymbol{Z}_3$ is isomorphic to $\boldsymbol{Z}_6$, i.e. $\boldsymbol{Z}_2 \times \boldsymbol{Z}_3 \cong \boldsymbol{Z}_6$ [38]. This becomes evident by giving both possible isomorphisms $\boldsymbol{Z}_2 \times \boldsymbol{Z}_3 \rightarrow \boldsymbol{Z}_6$.

$$\begin{aligned} &(0,0) \mapsto 0, \quad &&(0,1) \mapsto 2, \quad &&(0,2) \mapsto 4, \\ &(1,0) \mapsto 3, \quad &&(1,1) \mapsto 5, \quad &&(1,2) \mapsto 1, \end{aligned} \tag{6.6}$$

$$\begin{aligned} &(0,0) \mapsto 0, \quad &&(0,1) \mapsto 4, \quad &&(0,2) \mapsto 2, \\ &(1,0) \mapsto 3, \quad &&(1,1) \mapsto 1, \quad &&(1,2) \mapsto 5. \end{aligned} \tag{6.7}$$

As an example, we calculate the discrete charges in the case of Eq. (6.2). Recalling the relations between q_i and the exponents m, n, and p given in Eq. (2.17), we find for the $\boldsymbol{Z}_2 \times \boldsymbol{Z}_3$ charges, where we compute modulo N [e.g. $q_{\bar{U}} = (-1,-1) = (1,2)$],

$$\begin{aligned} q_Q &= (0,0), \quad & q_{\bar{U}} &= (1,2), \quad & q_{\bar{D}} &= (1,1), \\ q_L &= (0,2), \quad & q_{\bar{E}} &= (1,2), \\ q_{H_d} &= (1,2), \quad & q_{H_u} &= (1,1), \end{aligned} \tag{6.8}$$

and for the $\boldsymbol{Z}_6$ charges

$$\begin{aligned} q_Q &= 0, \quad & q_{\bar{U}} &= 1, \quad & q_{\bar{D}} &= 5, \\ q_L &= 4, \quad & q_{\bar{E}} &= 1, \\ q_{H_d} &= 1, \quad & q_{H_u} &= 5. \end{aligned} \tag{6.9}$$

Both charge assignments are related by the isomorphism of Eq. (6.6). Similarly, the $\boldsymbol{Z}_2 \times \boldsymbol{Z}_3$ and the $\boldsymbol{Z}_6$ charges in Eqs. (6.4) and (6.5) are related by this isomorphism. In the case of Eq. (6.3) we have to apply the isomorphism of Eq. (6.7).

(2) In Ref. [51], a $U(1)_X$ gauge extended SSM was investigated, where all renormalizable MSSM superpotential terms have a total X charge which is an integer multiple of N [cf. Eq. (8.7)]. Then the conditions on the $U(1)_X$ charges were derived, in order to have a low-energy $\boldsymbol{M}_p$ discrete symmetry. In Ref. [73], we derive the corresponding conditions for $\boldsymbol{B}_3$ and $\boldsymbol{P}_6$:

(i) $\boldsymbol{M}_p$: $3X_{Q_1} + X_{L_1} = 2 \cdot \mathbb{Z}$,
$X_{H_d} - X_{L_1} = 2 \cdot \mathbb{Z} - 1$,

(ii) $\boldsymbol{B}_3$: $3X_{Q_1} + X_{L_1} = 3 \cdot \mathbb{Z} \pm 1$,
$X_{H_d} - X_{L_1} = 3 \cdot \mathbb{Z}$,

(iii) $\boldsymbol{P}_6$: $3X_{Q_1} + X_{L_1} = 6 \cdot \mathbb{Z} \pm 2$,
$X_{H_d} - X_{L_1} = 18 \cdot \mathbb{Z} \pm 3$.

(3) Next, we consider domain walls, which pose a severe cosmological problem if they occur [74]. It is commonly held that a spontaneously broken discrete symmetry leads to domain walls. In particular, this is expected to occur in the SSM if the Higgs fields are charged under the $\boldsymbol{Z}_N$ symmetry. In contrast, we do not expect domain walls if the Higgs' discrete charges are zero. However, for this reasoning the first set of charges below Eq. (3.1) ($q_{H_u} = 1$, $q_{H_d} = -1$) implies the existence of domain walls, whereas the second set, standard matter parity ($q_{H_u} = 0$, $q_{H_d} = 0$), does not. As stated in Sec. III, these two symmetries are related by a simple hypercharge shift. They have the same low-energy superpotential and soft terms. Hence the resulting scalar potentials are identical apart from D-term contributions. Therefore the two theories have the same vacuum structure, and either both have or both do not have domain walls.

If the SSM vacuum $\{\upsilon_{H_d}, \upsilon_{H_u}\}$ has zero $\boldsymbol{Z}_N$ charge, then it is unique. If it transforms nontrivially under $\boldsymbol{Z}_N$ then there are up to N distinct ground states $\{\upsilon_{H_d}, \upsilon_{H_u}\}$, $\{\upsilon_{H_d}{}', \upsilon_{H_u}{}'\}$, $\{\upsilon_{H_d}{}'', \upsilon_{H_u}{}''\}$, ..., related by $\boldsymbol{Z}_N$ transformations. In the latter case, however, there are no domain walls, if the $\boldsymbol{Z}_N$ transformation of the vacuum in a given domain can be compensated by a $U(1)_Y$ gauge transformation. Explicitly, we demand that there exists a combined $\boldsymbol{Z}_N + \boldsymbol{Y}$ transformation $\boldsymbol{T}$, such that $\boldsymbol{T}(H_{d,u}) = H_{d,u}$, i.e.

$$\exists\,\alpha(x)\colon\ \exp\left[i\frac{2\pi}{N}\cdot q_{H_{d,u}} + i\alpha(x)\cdot Y_{H_{d,u}}\right]H_{d,u} = H_{d,u}. \tag{6.10}$$

$\alpha(x) \in \mathbb{R}$ is the gauge parameter of $U(1)_Y$. This is equivalent to

$$\frac{2\pi}{N}\cdot q_{H_{d,u}} + \alpha(x)\cdot Y_{H_{d,u}} = 2\pi\cdot I_{d,u}, \quad \text{with} \quad I_{d,u}\in\mathbb{Z}. \tag{6.11}$$

These two equations can be combined to get

$$I_u = \frac{1}{N\cdot Y_{H_d}}\cdot(q_{H_u}\cdot Y_{H_d} - q_{H_d}\cdot Y_{H_u} + N\cdot Y_{H_u}\cdot I_d), \tag{6.12}$$

$$\alpha(x) = \frac{2\pi}{N\cdot Y_{H_d}}\cdot(N\cdot I_d - q_{H_d}). \tag{6.13}$$

The second equation defines the required gauge transformation. We can simplify the first equation, using the hypercharge relation $Y_{H_u} = -Y_{H_d}$,

$$N\cdot(I_u + I_d) = q_{H_d} + q_{H_u}. \tag{6.14}$$

This can only be fulfilled if the $\boldsymbol{Z}_N$ charges of the two Higgs, just like their hypercharges, are the inverse of each other (in the sense of a mod N calculation).[11] This is equivalent to the requirement that the μ-term is allowed by $\boldsymbol{Z}_N$. This is, e.g., the case for $\boldsymbol{M}_p$ canonically, as the Higgs fields are uncharged: $(q_{H_d}, q_{H_u}) = (0, 0)$, $R_2\,(1, 1)$, $\boldsymbol{B}_3\,(2, 1)$, and $\boldsymbol{P}_6\,(1, 5)$. We stress that this argument does not rely on $U(1)_X$ being nonanomalous (cf. Sec. VIII).

VII. THE HEAVY-FERMION SECTOR

An interesting question to ask is as follows: Given a DGS in Table III, do I necessarily need heavy fermions in order to cancel the anomalies? In the case of matter parity, R_2, we can answer the question by considering Eq. (2.23). Here, the left-hand side equals 3, while the right-hand side is $2\cdot\mathbb{Z} + \eta\cdot\mathbb{Z}$. Recalling that the η-term originates from heavy Majorana fermions [cf. Eq. (2.6)], we find that the symmetry R_2 is only possible if we include a heavy-fermion sector, e.g. one right-handed neutrino for each generation.

In the case of the other fundamental DGSs of Table III, let us assume the absence of heavy fermions in what follows. Under this assumption, the anomaly cancellation conditions cannot be satisfied. Inserting the discrete charges of Eq. (2.17) into Eq. (2.6), we obtain

$$13n + 3p - 3m = N\cdot\Big[2m_{H_d} + 2m_{H_u} + \sum_k(6m_{Q_k} + 3m_{\bar{U}_k} + 3m_{\bar{D}_k} + 2m_{L_k} + m_{\bar{E}_k})\Big], \tag{7.1}$$

where k is a generation index. For even N, the right-hand side in Eq. (7.1) is even. However, the left-hand side is odd for the $\boldsymbol{Z}_2$, $\boldsymbol{Z}_6$, and $\boldsymbol{Z}_{18}$ DGSs. Therefore heavy fermions are necessary in these cases.

For the remaining $4 + 9$ $\boldsymbol{Z}_3$ and $\boldsymbol{Z}_9$ symmetries, the RHS of Eq. (7.1) can be both, even or odd. We thus employ the cubic anomaly constraint of Eq. (4.3). For the $\boldsymbol{Z}_9$ symmetries, the RHS of Eq. (4.3) is always a multiple of 27. The LHS of the cubic anomaly condition, given in Eq. (A7), is $-122\cdot 3 + 27\cdot\mathbb{Z}$, which is not a multiple of 27. Thus the fundamental $\boldsymbol{Z}_9$ symmetries also require heavy fermions.

For the four $\boldsymbol{Z}_3$ symmetries, the RHS of Eq. (4.3) is always a multiple of 9. Equation (A5) shows that the LHS of Eq. (4.3) is a multiple of 9 only in the case of the R_3L_3 symmetry. Hence the other three fundamental $\boldsymbol{Z}_3$ symmetries require heavy fermions. But, also, R_3L_3 cannot satisfy the anomaly constraints without a heavy-fermion sector[12]: Although R_3L_3 is neither ruled out by $\mathcal{A}_{GGX} = 0$ nor $\mathcal{A}_{XXX} = 0$ alone, it is in conflict when combining the two conditions; the LHS of Eq. (4.3) for R_3L_3 yields 18, [cf. Eq. (A5)], whereas the RHS is a multiple of 27, as we now show. It is given by

$$-\sum_i(3q_i{}^2 m_i N + 3q_i m_i{}^2 N^2 + m_i{}^3 N^3), \tag{7.2}$$

where i runs over all chiral superfields. The last two terms within the parentheses are multiples of 27, which is not true for the first one. However, evaluating the sum and applying our knowledge of the q_i, we find

$$\sum_i 3q_i{}^2 m_i N = 3N\cdot\Big[2\cdot m_{H_d} + 2\cdot m_{H_u} + \sum_k(3\cdot m_{\bar{U}_k} + 3\cdot m_{\bar{D}_k} + 2\cdot m_{L_k} + 4\cdot m_{\bar{E}_k})\Big], \tag{7.3}$$

where k denotes a generation index. The numerical coefficients inside the brackets are the product of the squared

[11] If the two Higgs do not have opposite $\boldsymbol{Z}_N$ charges, the μ-term is forbidden. This then possibly enables PQ invariance, which allows one to repeat the argument above with $\alpha(x)\cdot Y_{H_{d,u}}$ replaced by $\alpha(x)\cdot Y_{H_{d,u}} + \beta\cdot PQ_{H_{d,u}}$.

[12] Here we disagree with Ibáñez's conclusion in Ref. [39]. See also Ref. [75].

discrete charges and the multiplicity of the particle species. For example, we have three colors of quark fields $\bar{U}_k$ with $q_{\bar{U}_k} = -1$, thus $3 \cdot {q_{\bar{U}_k}}^2 = 3$. We can now adopt the gravity-gravity-$U(1)_X$ anomaly constraint of Eq. (7.1) to rewrite Eq. (7.3). Recalling that $N = 3$, $n = 0$, and $m = p = 1$ for R_3L_3, we get

$$\sum_i 3{q_i}^2 m_i N = -9 \cdot \sum_k (6 \cdot m_{Q_k} - 3 \cdot m_{\bar{E}_k}), \qquad (7.4)$$

also a multiple of 27. This completes our proof.

In conclusion, the 27 fundamental DGSs we have found are *only* anomaly-free with a $U(1)_X$-charged heavy-fermion sector.

VIII. A TOP-DOWN APPROACH

As outlined in Sec. I, we have so far discussed a bottom-up approach to DGSs. However, by definition, a DGS is inherently connected to the anomaly structure of the underlying $U(1)_X$ gauge theory. Here, we consider the DGSs from the latter perspective. We investigate two topics in detail: (i) the definition of the DGSs via the transformation of the superfields (superfieldwise) vs the definition via the transformation of the G_{SM}-invariant operators (operatorwise); (ii) the hypercharge shifts of Eq. (2.15).

At high energies, we start from a $G_{\mathrm{SM}} \times U(1)_X$-invariant Lagrangian, with the X charges scaled to be integers of minimal absolute value. We leave it open at the moment whether $U(1)_X$ is anomalous or not. Below M_X, $U(1)_X$ is assumed to be broken by a single left-chiral flavon superfield Φ (or by two left-chiral superfields Φ, Φ' with opposite X charges; see Sec. IX), which is uncharged under G_{SM}. If in our model, e.g., the operator $L_iL_j\bar{E}_k$ is not $U(1)_X$ invariant, then the nonrenormalizable superpotential[13] operator

$$\Phi^{-(X_{L_i}+X_{L_j}+X_{\bar{E}_k})/X_\Phi} \times L_iL_j\bar{E}_k \qquad (8.1)$$

is. However, due to the cluster decomposition principle (CDP) [76], the Lagrangian exhibits only non-negative integer exponents of the fields [77,78]. Therefore the above term is forbidden if $(X_{L_i} + X_{L_j} + X_{\bar{E}_k})/X_\Phi$ is fractional. After $U(1)_X$ breaking, the operator $L_iL_j\bar{E}_k$ is not generated, since its nonrenormalizable "parent term" is nonexistent. Therefore the constraints of the CDP persist. Whether an operator is allowed or not in the low-energy Lagrangian boils down to whether its overall X charge is an integer multiple of X_Φ. Thus, at low energy, we decompose the X charges as in Eq. (2.1) and the remaining DGS under which the *superfields* transform is a $\mathbf{Z}_{|X_\Phi|}$.

[13]The following arguments in this section proceed analogously for the Kähler potential.

Next consider the operators in the superpotential. Analogous to Eq. (2.1), the overall X charge, X_{total}, of any G_{SM}-invariant product of MSSM chiral superfields satisfies

$$X_{\mathrm{total}} = q_{\mathrm{total}} + m_{\mathrm{total}} \cdot |X_\Phi|,$$
$$\text{with} \quad q_{\mathrm{total}} \equiv \sum q_i, \qquad m_{\mathrm{total}} \equiv \sum m_i. \qquad (8.2)$$

If a certain operator is forbidden by the CDP, then the $|X_\Phi|$th power of this term has $q_{\mathrm{total}} = 0 \bmod (|X_\Phi|)$. However, the superpotential operators are further restricted by G_{SM}. Therefore the $\mathbf{Z}_{|X_\Phi|}$ charges are possibly such that a power smaller than $|X_\Phi|$ suffices to get $q_{\mathrm{total}} = 0 \bmod (|X_\Phi|)$, for *all* superpotential operators. As an example, suppose $|X_\Phi| = 24$ and the superfields obey a $\mathbf{Z}_{24}$. Because of G_{SM}, it may very well be that for *all* operators q_{total} is even. Operatorwise we then have a $\mathbf{Z}_{12}$ instead of a $\mathbf{Z}_{24}$. Furthermore, we can integrate out the heavy particles below their mass scale. When considering only the superfields of the SSM their respective q's could, e.g., be only multiples of 3. The SSM superfields alone then obey a $\mathbf{Z}_{24/3} = \mathbf{Z}_8$ symmetry (cf. Sec. V) and the SSM superfieldwise $\mathbf{Z}_8$ constitutes an SSM-operatorwise $\mathbf{Z}_4$.

We now consider a generation-independent $U(1)_X$ extension of the SSM, which is the high-energy origin of the DGS. We include right-handed neutrinos, $\bar{N}_i$. We demand that for the $U(1)_X$ charge assignments (i) the Yukawa mass terms $QH_d\bar{D}$, $QH_u\bar{U}$, $LH_d\bar{E}$, and $LH_u\bar{N}$ are invariant, and (ii) the anomalies $\mathcal{A}_{CCY}$, $\mathcal{A}_{WWY}$, $\mathcal{A}_{GGY}$, $\mathcal{A}_{CCX}$, $\mathcal{A}_{WWX}$, $\mathcal{A}_{GGX}$, $\mathcal{A}_{YYY}$, $\mathcal{A}_{YYX}$, $\mathcal{A}_{YXX}$, and $\mathcal{A}_{XXX}$ all vanish. We can then express the X charges in terms of two unknowns,

$$\begin{aligned} X_{\bar{D}} &= -X_Q - X_{H_d}, & X_{\bar{U}} &= -X_Q + X_{H_d}, \\ X_L &= -3X_Q, & X_{\bar{E}} &= 3X_Q - X_{H_d}, \\ X_{\bar{N}} &= 3X_Q + X_{H_d}, & X_{H_u} &= -X_{H_d}. \end{aligned} \qquad (8.3)$$

Furthermore, we obtain the well-known result that $U(1)_X$ is necessarily a linear combination of $U(1)_Y$, i.e. hypercharge, and $U(1)_{B-L}$ (see, for example, Refs. [79–81]),

$$X_i = \frac{X_i^{B-L}}{X_Q^{B-L}} \cdot C_1 + \frac{Y_i}{Y_Q} \cdot C_2, \qquad (8.4)$$

where $C_{1,2}$ are free real parameters, such that the X charges are integers, as was required earlier. Equation (8.3) can then be reexpressed in terms of $C_{1,2}$,

$$\begin{aligned} X_Q &= C_1 + C_2, & X_{\bar{D}} &= -C_1 + 2C_2, \\ X_{\bar{U}} &= -C_1 - 4C_2, & X_L &= -3C_1 - 3C_2, \\ X_{\bar{E}} &= 3C_1 + 6C_2, & X_{\bar{N}} &= 3C_1, \\ X_{H_d} &= -3C_2, & X_{H_u} &= 3C_2. \end{aligned} \tag{8.5}$$

For $2C_1 = -5C_2$, we obtain a theory with $SU(5)$-invariant X charges. For $C_1 \neq 0$, the right-handed neutrinos are charged and the seesaw mass term $\bar{N}_i\bar{N}_j$ is forbidden. And, of course, for $C_2 = 0$ we obtain $U(1)_{B-L}$.

At low energy, we performed the hypercharge shift of the DGS, Eq. (2.15). As we argued, this hypercharge shift is irrelevant for the structure of the low-energy superpotentials. From the top-down approach, however, a different choice of C_2 corresponds to a hypercharge shift of the SSM X charges, which in turn corresponds to a hypercharge shift of the corresponding $\mathbf{Z}_N$. How does this change the high-energy theory? The gauge boson and fermionic kinetic terms in the Lagrangian are

$$\mathcal{L} = -\frac{1}{4}F_X^2 - \frac{1}{4}F_Y^2 + \sum \bar{\psi}_k(i\partial_\mu - g_X X_k A_\mu^X - g_Y Y_k A_\mu^Y)\gamma^\mu \psi_k. \tag{8.6}$$

Here $F_{X,Y}^2$ are the squared field strength tensors, and $A_\mu^{X,Y}$ are the corresponding gauge potentials. We see that a simultaneous orthogonal rotation in the fields (A_μ^X, A_μ^Y) and the charges $(g_X X_k, g_Y Y_k)$ leaves the Lagrangian unchanged. But different choices of C_2 in Eq. (8.4), which correspond to hypercharge shifted (not rotated) theories, lead to distinct gauge theories in Eq. (8.6). They differ in their X charges and thus in their scattering cross sections. They are therefore, in principle, experimentally distinguishable at energies $\sqrt{s} = \mathcal{O}(M_X)$. However, at the LHC, we can only determine the low-energy DGS. We *cannot* determine C_2 of Eq. (8.4). When attempting to interpret the LHC results in terms of an underlying unified theory it is important to keep this ambiguity in mind.

Let us now focus on the Φ + SSM sector, i.e. including the flavon field(s). Using the methods of Refs. [51,73], we can compute the total X charge of *any* $G_{\rm SM}$-invariant superpotential term and obtain

$$X_{\rm total}^{\rm SSM} = \mathbb{Z}\cdot(3X_{Q_1} + X_{L_1}) + \mathbb{Z}\cdot(X_{H_d} - X_{L_1}) + \mathbb{Z}\cdot(X_{H_d} + X_{H_u}) + \mathbb{Z}\cdot|X_\Phi|, \tag{8.7}$$

where $\mathbb{Z}$ again denote arbitrary and independent integers. Using Eq. (2.1), this gives

$$q_{\rm total}^{\rm SSM} = \mathbb{Z}\cdot(3q_Q + q_L) + \mathbb{Z}\cdot(q_{H_d} - q_L) + \mathbb{Z}\cdot(q_{H_d} + q_{H_u}) + \mathbb{Z}\cdot|X_\Phi|. \tag{8.8}$$

We have seen that a hypercharge shift of the X charges leads to a *new* $U(1)_X$ gauge theory. Such a shift is however only possible for an originally anomaly-free model (see, e.g., the completely fixed X charges in Ref. [51]) and yields an alternate anomaly-free model. Plugging the X charges of Eq. (8.4) into Eq. (8.7), we find

$$X_{\rm total}^{\rm SSM} = \mathbb{Z}\cdot 3C_1 + \mathbb{Z}\cdot|X_\Phi|, \tag{8.9}$$

of course independent of C_2 and thus of hypercharge. So all the results on the operatorwise DGS coming from $U(1)_X$ are solely determined by C_1 and $|X_\Phi|$. This characteristic, which we demonstrated for a simple example, also holds for all nonanomalous models. This is why we could shift away r in Sec. II. For $C_1 = -C_2$, i.e. $X_Q = 0$, the fieldwise and operatorwise definitions of the DGS coincide.

Equipped with the X charges in Eq. (8.4), we now demonstrate in two examples the emergence of distinct operatorwise and superfieldwise DGSs from the $U(1)_X$.

(i) $C_1 = 1$, $C_2 = 0$, supplemented by a vectorlike pair of flavon superfields, $X_\Phi = 6$, $X_{\Phi'} = -6$. Hence the Yukawa operators have the total X charge $X_{LH_d\bar{E}} = X_{QH_d\bar{D}} = X_{QH_u\bar{U}} = X_{LH_u\bar{N}} = X_{H_dH_u} = 0$, but $X_{LL\bar{E}} = X_{LQ\bar{D}} = X_{\bar{U}\bar{D}\bar{D}} = X_{LH_u} = -3$. To have, e.g., $LL\bar{E}$ generated after $U(1)_X$ breaking would require $\sqrt{\Phi}\cdot LL\bar{E}$, which is not allowed due to the CDP. With Eq. (2.1) we get a superfieldwise $\mathbf{Z}_6$, with $q_Q = 1$, $q_{\bar{D}} = q_{\bar{U}} = 5$, $q_L = q_{\bar{E}} = q_{\bar{N}} = 3$, $q_{H_d} = q_{H_u} = 0$. Plugging these into Eq. (8.8), one finds that any superpotential term has an overall q charge which is an integer multiple of either 3 or 6. Thus the actual DGS of the *operators* is a $\mathbf{Z}_{6/3} = \mathbf{Z}_2$ symmetry. This is matter parity, in fact.

(ii) $C_1 = 2$, $C_2 = 1$ results in $X_Q = 3$, $X_{\bar{D}} = 0$, $X_{\bar{U}} = -6$, $X_L = -9$, $X_{\bar{E}} = 12$, $X_{\bar{N}} = 6$, $X_{H_d} = -3$, $X_{H_u} = 3$, again supplemented by $X_\Phi = 6$, $X_{\Phi'} = -6$. This leads to $q_Q = q_L = q_{H_d} = q_{H_u} = 3$, $q_{\bar{D}} = q_{\bar{U}} = q_{\bar{E}} = q_{\bar{N}} = 0$. The DGS appears to be a $\mathbf{Z}_{6/3} = \mathbf{Z}_2$. However, inserting the charges into Eq. (8.8), we find no DGS whatsoever.

Another example, more elaborate and flavor dependent, is the fourth model in Table 2 in Ref. [82]. It does not cause any DGS after $U(1)_X$ breaking, as our second example. The prefactors of the free parameter q (their notation) are nothing but the usual hypercharges.

The argument that a superfieldwise $\mathbf{Z}_{|X_\Phi|}$ causes an operatorwise $\mathbf{Z}_{|X_\Phi|/N}$ is independent of whether the $U(1)_X$ has anomalies which are canceled via Green-Schwarz [83] or whether the $U(1)_X$ is nonanomalous. The anomalous X charges given in Table 7 of Ref. [51] display a SSM superfieldwise $\mathbf{Z}_{300}$ symmetry, but operatorwise constitute a $\mathbf{Z}_2$, as can be seen by plugging the corresponding discrete charges into Eq. (8.8). *A priori* it is hence not clear whether, e.g., a superfieldwise $\mathbf{Z}_{300}$ gives rise to an operatorwise $\mathbf{Z}_{300}$, $\mathbf{Z}_{150}$, $\mathbf{Z}_{100}$, ..., $\mathbf{Z}_2$ or even $\mathbf{Z}_1$ (trivial).

In summary, from a top-down point of view, hypercharge shifted theories are not equivalent. They are, in principle, experimentally distinguishable by high-energy scattering experiments. If they are anomaly-free, they lead to equivalent low-energy discrete gauge theories and are not distinguishable at the LHC. But even a nonanomalous set and an anomalous set of X charges are equivalent from the low-energy point of view if they lead to the same operatorwise DGS.

IX. A GAUGED P_6 MODEL

In this section, we explicitly present a generation-dependent $U(1)_X$ gauge model, constructed in collaboration with C. A. Savoy and S. Lavignac. $U(1)_X$ is spontaneously broken to proton hexality, $\boldsymbol{P}_6$. We consider this a demonstration of existence, not necessarily an optimized model. Concerning the origin of the needed nonrenormalizable interaction terms, there are several sources imaginable (see, e.g., [84]): Either the terms occur near the string scale or they are generated by integrating out heavy vector-like pairs of $G_{\rm SM}$ charged states (the so-called Froggatt-Nielsen mechanism [85]). Here we adopt the first viewpoint and thus use a simple operator analysis. We assume the $U(1)_X$ breaking superfields to be suppressed by $M_{\rm grav}$; e.g. $Q_1 H_u \bar{U}_1$ derives from $(\Phi_+/M_{\rm grav})^8 \cdot Q_1 H_u \bar{U}_1$.

We first list in Table V the $U(1)_X$ charges of all the chiral superfields in our model. The $G_{\rm SM}$ singlets $\Phi_\pm$ constitute the vectorlike pair of $U(1)_X$ breaking superfields with equal VEVs. The $A_{\dots}$ are $G_{\rm SM}$ singlets as well but do not acquire VEVs; we introduce them solely for the sake of canceling $\mathcal{A}_{GGX}$ and $\mathcal{A}_{XXX}$. All the other (mixed) anomalies vanish within the particle content of the SSM.

The breaking of $U(1)_X$ generates the MSSM Yukawa coupling constants with textures that produce the observed fermionic mass spectrum as well as acceptable mixing matrices. Furthermore, $U(1)_X$ leaves a $\boldsymbol{Z}_{12}$ symmetry as a remnant which, after integrating out the $A_{\dots}$, yields $\boldsymbol{P}_6$:

(i) With

$$\epsilon \equiv \frac{\langle \Phi_\pm \rangle}{M_{\rm grav}} = 0.22, \qquad (9.1)$$

we obtain an effective superpotential which contains the first line of Eq. (1.1) and the mass terms for the left-handed neutrinos $(h^\nu_{ij}/M_\nu \cdot L_i H_u L_j H_u)$, where

$$\boldsymbol{h^U} \sim \begin{pmatrix} \epsilon^8 & \epsilon^4 & \epsilon^2 \\ \epsilon^8 & \epsilon^4 & \epsilon^2 \\ \epsilon^6 & \epsilon^2 & 1 \end{pmatrix}, \quad \boldsymbol{h^D} \sim \epsilon^2 \cdot \begin{pmatrix} \epsilon^4 & \epsilon^2 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & 1 & 1 \end{pmatrix}, \quad \boldsymbol{h^E} \sim \epsilon^2 \cdot \begin{pmatrix} \epsilon^4 & \epsilon^2 & 1 \\ \epsilon^4 & \epsilon^2 & 1 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \qquad (9.2)$$

$$\mu \sim \epsilon^8 \cdot M_\mu, \qquad \boldsymbol{h^\nu} \sim \epsilon^3 \cdot \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \qquad (9.3)$$

To get the μ term and the neutrino masses of the correct order of magnitude, we rely on the existence of intermediate mass scales: $M_\mu \sim 10^8$ GeV (whose necessity has already been anticipated by Refs. [82,86] for anomaly-free Froggatt-Nielsen models without heavy $G_{\rm SM}$ charged matter) and $M_\nu \sim 10^{12}$ GeV. After diagonalization one gets for the masses of the electrically charged SM fermions $m_u{:}m_c{:}m_t \sim \epsilon^8{:}\epsilon^4{:}1$, $m_d{:}m_s{:}m_b \sim \epsilon^4{:}\epsilon^2{:}1$, $m_e{:}m_\mu{:}m_\tau \sim \epsilon^4{:}\epsilon^2{:}1$, $m_\tau{:}m_b{:}m_t \sim \epsilon^2{:}\epsilon^2{:}1$. For the mixing matrices we get an anarchical Maki-Nagakawa-Sakata matrix, which is compatible with experiment (see, e.g., Refs. [87–89]), as well as a Cabibbo-Kobayashi-Maskawa matrix which looks like

$$\boldsymbol{V}^{\rm CKM} \sim \begin{pmatrix} 1 & 1 & \epsilon^2 \\ 1 & 1 & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix}. \qquad (9.4)$$

Thus we have to rely on some moderate fine-tuning among the unknown $\mathcal{O}(1)$ coefficients to be entirely satisfactory.

Furthermore, we get the following mass terms for the heavy fields:

$$\epsilon^6 \cdot M_{\rm grav} A_{\rm D1} A_{\rm D2}, \qquad \epsilon^6 \cdot M_{\rm grav} A'_{\rm D1} A'_{\rm D2}, \qquad \epsilon \cdot M_{\rm grav} A_{\rm M} A_{\rm M}. \qquad (9.5)$$

(ii) After $U(1)_X$ breaking we are left with an overall $\boldsymbol{Z}_{12}$ DGS, since $|X_{\Phi_\pm}| = 6$ and all SSM particles' X charges are integers and the $A_{\dots}$'s X charges half-odd integers. But as can be seen above, the $A_{\dots}$ are quite heavy, so that they all can be integrated out at around $\epsilon^6 M_{\rm grav} \sim 10^{14}$ GeV, leaving the *fundamental* (in the sense of Sec. V) DGS $\boldsymbol{P}_6$.

TABLE V. The $U(1)_X$ charges of all chiral superfields in our model. $\Phi_\pm$ break $U(1)_X$; the $A_{\dots}$ are $G_{\rm SM}$ uncharged heavy particles.

$X_{\Phi_+} = 6,\ X_{\Phi_-} = -6$					
$X_{H_d} = 1,\ X_{H_u} = -49$					
Generation i	X_{Q_i}	$X_{\bar{U}_i}$	$X_{\bar{D}_i}$	X_{L_i}	$X_{\bar{E}_i}$
1	−12	13	−25	40	−77
2	−12	37	−13	40	−17
3	0	49	−13	40	−53
$X_{A_{\rm D1}} = -\frac{27}{2},\ X_{A_{\rm D2}} = -\frac{45}{2},\ X_{A'_{\rm D1}} = \frac{1}{2},\ X_{A'_{\rm D2}} = \frac{71}{2},\ X_{A_{\rm M}} = 3$					

X. SUMMARY

In summary, we have systematically investigated discrete gauge symmetries $\boldsymbol{Z}_N$, for arbitrary values of N. We have classified the anomaly-free theories, depending on whether the necessary (see Sec. VII) heavy fermions are restricted to integer X charges or not. Through a rescaling of the X charges, we have, for a low-energy point of view, reduced this infinite set to a finite fundamental set: All theories related by rescaling lead to the same low-energy superpotential. For this fundamental set we have investigated the phenomenological properties in detail. We have found two outstanding DGSs, the second of them being beyond IR: (i) baryon triality, $\boldsymbol{B}_3$, which allows for low-energy lepton-number violation, but no dimension-five or lower proton decay operators, and (ii) proton hexality, $\boldsymbol{P}_6$. The latter has a renormalizable superpotential which conserves lepton and baryon number and prohibits nonrenormalizable dimension-five proton decay operators. This is one of the main results of this paper and we propose $\boldsymbol{P}_6$ as *the* new discrete gauge symmetry of the MSSM, instead of matter parity. Both baryon triality and proton hexality are free of domain walls.

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APPENDIX: THE CUBIC ANOMALY

In this appendix, we explicitly derive Table II. We thus restrict ourselves to integer charges for *all* chiral superfields [24,25] and investigate the resulting consequences of the cubic anomaly constraint on possible DGSs. Using Eq. (2.17), we can express the LHS of Eq. (4.3) in terms of n, p, and m,

$$\begin{aligned}\text{LHS} = & -n\cdot(13n^2+18np-21nm+18p^2+21m^2)\\ & + p\cdot(-3p^2+9pm+9m^2)+3m^3,\end{aligned} \tag{A1}$$

where we have made use of the fact that there are only three generations in the SSM. Even when disregarding the restrictions on the heavy-particle content arising from the linear constraints, the RHS of Eq. (4.3) *cannot* take on arbitrary integer values. We shall denote it as $\text{RHS} \equiv \text{RHS}_1 + \text{RHS}_2 + \text{RHS}_3$, with a term for each line in Eq. (4.3). We now investigate these terms individually.

(i) RHS_2: Factoring N, we see that the term RHS_2 contributes a multiple of N to the RHS. However, it cannot necessarily take on every possible multiple of N, regardless of what the choice of heavy particles is. For $(3|N)$, we can again write $N = 3N'$ ($N' \in \mathbb{N}$), and rewrite the last term as $p_j^3 N^3 = 3p_j^3 N^2 N'$. We can thus factor $3N$ and therefore the term RHS_2 can take on *at most* values $\in 3N\cdot\mathbb{Z}$. By adding appropriate sets of heavy Dirac particles with simple charges, it is straightforward to show that *any* multiple of $3N$ can be obtained. For DGSs with $\neg(3|N)$, any element $\in N\cdot\mathbb{Z}$ can be obtained.

(ii) RHS_3: For odd N, $p'_{j'}$ has to be even [see Eq. (2.3)], so that the term RHS_3 is an element of $N^3\cdot\mathbb{Z}$. For even N, RHS_3 can take on all values $\in (\frac{N}{2})^3\cdot\mathbb{Z}$.

(iii) RHS_1: The first two terms in RHS_1 are multiples of $3N$, which is included in (i), above. Similarly, the third term is a multiples of N^3 and therefore already included in (ii).

Summarizing, the RHS of Eq. (4.3) can only take on values obeying

$$\text{RHS} = 3N\cdot\mathbb{Z} + \begin{cases} N^3\cdot\mathbb{Z}, & N=\text{odd},\\ (\frac{N}{2})^3\cdot\mathbb{Z}, & N=\text{even},\end{cases} \quad \text{for } (3|N), \tag{A2}$$

$$= \begin{cases} 9N'\cdot\mathbb{Z}, & (3|N), \quad N=\text{odd},\\ \left.\begin{array}{ll} 9\frac{N'}{2}\cdot\mathbb{Z}, & \neg(12|N),\\ 9N'\cdot\mathbb{Z}, & (12|N),\end{array}\right\} & N=\text{even},\end{cases} \tag{A3}$$

where $N' = N/3$, as before. Furthermore

$$\text{RHS} = N\cdot\mathbb{Z} + \left(\frac{N}{2}\right)^3\cdot\mathbb{Z}, \qquad N=\text{even}, \quad \text{for } \neg(3|N). \tag{A4}$$

Now consider the LHS, while taking the linear constraints of Sec. II into account. Again, we investigate the cases $\neg(3|N)$ and $(3|N)$ separately.

(1) $\neg(3|N)$: The DGSs of Eq. (3.1), satisfying the linear constraints, require $n = p = 0$ and $m = \frac{N}{2}$. Thus the LHS becomes $3\cdot(\frac{N}{2})^3$ [cf. Eq. (A1)]. Comparing with Eq. (A4), we see that the cubic anomaly cancellation condition can be satisfied for all anomaly-free DGSs of Table I with $\neg(3|N)$, i.e. the cubic anomaly results in no new constraint.

(2) $(3|N)$: We consider the remaining four categories of Table I in turn.

(i) $(3|N)$, $N = \text{odd}$: Eq. (A3) shows that the RHS must be a multiple of $9N'$. Therefore the LHS must also be a multiple of $9N'$. From the corresponding row in Table I, we see that in this case $n = 0$, $p = \ell_p N'$, and $m = \ell_m N'$. Inserting this into the LHS as given in Eq. (A1) yields

$$\begin{aligned}\text{LHS} = & (-3\ell_p{}^3 + 9\ell_p{}^2\ell_m + 9\ell_p\ell_m{}^2\\ & + 3\ell_m{}^3)\cdot N'^3.\end{aligned} \tag{A5}$$

For the case where $\ell_p = \ell_m$, we can satisfy the condition $(9N'|\text{LHS})$ for all N, which are subsumed in this category, i.e. any $N \in 6 \cdot \mathbb{N} + 3$. The remaining cases of Table I, where $\ell_p \neq \ell_m$, require $(3|N'^2)$, and hence $N = 18 \cdot \mathbb{N} + 9$.

(ii) $(3|N)$, $N =$ even: From Table I we have in this case $n = 0$, $p = \ell_p N'$, and $m = s_m \frac{N'}{2}$. The LHS then becomes

$$\text{LHS} = (-24\ell_p{}^3 + 36\ell_p{}^2 s_m + 18\ell_p s_m{}^2 + 3s_m{}^3) \cdot \left(\frac{N'}{2}\right)^3. \quad \text{(A6)}$$

Because of the form of the RHS for $\neg(12|N)$ [cf. Eq. (A3)], we need $(9\frac{N'}{2}|\text{LHS})$. This leads to three nontrivial possibilities for arbitrary N in this category ($N = 12 \cdot \mathbb{N} + 6$): $[\ell_p = 0 \wedge s_m = 3]$, $[\ell_p = 1 \wedge s_m = 2]$, and $[\ell_p = 1 \wedge s_m = 5]$. All DGSs can satisfy the cubic anomaly constraint if $(3|N'^2)$, hence if $N = 36 \cdot \mathbb{N} + 18$.

Considering the case $(12|N)$ yields exactly the same three sets (ℓ_p, s_m) for nontrivial possible DGSs with arbitrary $N \in 12 \cdot \mathbb{N}$. All DGSs are allowed if $(3|N'^2)$, i.e. for $N = 36 \cdot \mathbb{N}$.

Combining the results for $\neg(12|N)$ and $(12|N)$, we find that for each $N \in 6 \cdot \mathbb{N}$ there are three allowed nontrivial DGSs. Taking $N \in 18 \cdot \mathbb{N}$, *any* DGS satisfying the linear constraints is compatible with the cubic constraint.

(iii) $(9|N)$, $N =$ odd: From Table I we obtain in this case $n = N'$, $p = (1 + 3\ell_p)N''$, and $m = (2 + 3\ell_m)N''$. Inserting this into Eq. (A1) gives

$$\begin{aligned}\text{LHS} = [&-27\ell_p{}^3 + 27\ell_p{}^2(-5 + 3\ell_m) \\ &+ 9\ell_p(-23 + 18\ell_m + 9\ell_m{}^2) \\ &+ (-122 + 18\ell_m - 108\ell_m{}^2 \\ &+ 27\ell_m{}^3)] \cdot N' \cdot N''^2. \end{aligned} \quad \text{(A7)}$$

As 122 is not a multiple of 9, whereas the other coefficients in the square brackets are, $(9N'|\text{LHS})$ [which is necessary due to Eq. (A3)] requires $(9|N''^2)$. Thus we need N to be an odd multiple of 27, i.e. $N = 54 \cdot \mathbb{N} + 27$. For such N, all linearly allowed DGSs are consistent with the cubic anomaly condition.

(iv) $(9|N)$, $N =$ even: From Table I we have in this case $n = N'$, $p = (1 + 3\ell_p)N''$, and $m = (1 + 3s_m)\frac{N''}{2}$. The LHS then becomes

$$\begin{aligned}\text{LHS} = [&-216\ell_p{}^3 + 108\ell_p{}^2(-13 + 3s_m) \\ &+ 18\ell_p(-119 + 18s_m + 9s_m{}^2) \\ &+ (-1291 + 585s_m - 297s_m{}^2 \\ &+ 27s_m{}^3)] \cdot \frac{N'}{2} \cdot \left(\frac{N''}{2}\right)^2. \end{aligned} \quad \text{(A8)}$$

1291 is not a multiple of 9 (it is actually a prime), whereas the remaining coefficients in square brackets are multiples of 9. Therefore the LHS is not a multiple of $9\frac{N'}{2}$ in the case of $\neg(12|N)$, respectively $9N'$ in the case of $(12|N)$ [cf. Eq. (A3)], unless $(9|N''^2)$. Thus the cubic anomaly constraint requires $N \in 54 \cdot \mathbb{N}$ in this category. All linearly allowed DGSs are possible for these values of N.

Table II in Sec. IV summarizes the results of this appendix.

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