

Unquenched Kogut-Susskind quark propagator in lattice Landau gauge QCD

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Quark propagators of the unquenched Kogut-Susskind (KS) fermion obtained from the gauge configurations of the MILC Collaboration are measured after Landau gauge fixing and using the Staple + Naik action. Presence of the $\bar{q}q$ condensates and A^2 condensates in the dynamical mass $M(q)$ and the quark wave function renormalization $Z_\psi(q^2)$ are investigated. We obtain the correlation of the renormalization factor of the running coupling taken at $\mu \sim 6$ GeV and that of the quark wave function renormalization $Z_\psi(q^2)$ of the Staple + Naik action. The mass function $M(q)$ is finite at $q = 0$ and its chiral limit is ~ 0.38 GeV. We compared the results corrected by the scale of the vertex renormalization and the tadpole renormalization with the corresponding values obtained by the Asqtad action without renormalization and observed good agreement. Implication of infrared finite $Z_2(q) = 1/Z_\psi(q^2)$ to the Kugo-Ojima confinement criterion is discussed.

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I. INTRODUCTION

The mechanism of dynamical chiral symmetry breaking and confinement is one of the most fundamental problems of hadron physics. The propagator of dynamical quarks in the infrared region provides information on dynamical chiral symmetry breaking and confinement. In the previous paper [1], we measured gluon propagators and ghost propagators of unquenched gauge configurations obtained with quark actions of Wilson fermions (JLQCD/CP-PACS) and those of Kogut-Susskind (KS) fermions (MILC) in Landau gauge and observed that the configurations of the KS fermions are closer to the chiral limit than those of Wilson fermions.

In the analysis of running coupling obtained from the gluon propagator and the ghost propagator, with use of the operator product expansion (OPE) of the Green function, we observed possible contributions of the quark condensates, A^2 condensates, and gluon condensates in the configurations of the KS fermion [1]. The quark propagator of quenched KS fermion was already measured in [2], and possible contributions of these condensates are reported. Unquenched KS fermion propagator of $20^3 \times 64$ lattice (MILC_c) was measured in [3], but to distinguish the gluon condensates and the quark condensates, it is desirable to measure the quark propagator of larger lattice (MILC_f) and to compare with data of MILC_c. We measure quark propagator of gauge configuration produced by using the Asqtad action 1) MILC_c $20^3 \times 64$, $\beta = 6.76$, and 6.83 and 2) MILC_f $28^3 \times 96$, $\beta = 7.09$, and 7.11 . The paper is organized as follows. In Sec. II we present kinematics of

the staggered fermion on the lattice, in Sec. III we show the renormalization effects, and in Sec. IV numerical results of the mass function and the quark propagator are presented. Finally in Sec. V we give conclusion and discussion. In the appendix we show the algorithm of the inversion to get the quark propagator.

II. KINEMATICS OF THE KS FERMION

The methods of writing the KS fermionic action on the lattice and deriving its propagator are presented in [4]. The KS fermion contains 16 flavor degrees of freedom and it is necessary to reduce the extra degrees of freedom by taking the 4th root of the fermion determinant. The MILC Collaboration improved the fermionic action by fattening the gauge links, including Naik term, and multiplying the tadpole renormalization factor so that the continuum limit can be approached on relatively small size of the lattice [5,6]. Although there are no rigorous justifications, there are indications that taking the 4th root does not yield serious problems.

In [3], a formulation of the KS fermion propagator calculation was presented and the numerical calculation was performed on MILC_c ($20^3 \times 64$ lattices) data [7]. In these papers, the momentum of quarks on the hypercubic lattice is defined by

$$p_\mu = \frac{2\pi n_\mu}{L}, \quad (1)$$

where $n_\mu = 1, 2, \dots, L/4$ and the 16 flavor degrees of freedom $\alpha_\mu = 0, 1$ where $\mu = 1, \dots, 4$, are expressed as

$$k_\mu = p_\mu + \pi\alpha_\mu. \quad (2)$$

In the Asqtad action, the link variable is modified by fattening

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$$U_\mu^{\text{fat}}(x) = c_1 U_\mu(x) + \sum_\nu \left[w_3 S_{\mu\nu}^{(3)}(x) + \sum_\rho \left(w_5 S_{\mu\nu\rho}^{(5)}(x) + \sum_\tau w_7 S_{\mu\nu\rho\tau}^{(7)}(x) \right) \right], \quad (3)$$

where $S_{\mu\nu}^{(3)}$ is the staple contribution

$$S_{\mu\nu}^{(3)}(x) = U_\nu(x) U_\mu(x + \hat{\nu}) U_\nu^\dagger(x + \hat{\mu}) + \text{H.c.} \quad (4)$$

$S^{(5)}$ and $S^{(7)}$ are 5-link and 7-link contributions, respectively. In addition to fattening, so-called Naik term and Lepage term are added. The Naik term is a product of three link variables along one direction [5].

$$U_\mu^{\text{Naik}}(x) = c_N U_\mu(x) U_\mu(x + \hat{\mu}) U_\mu(x + 2\hat{\mu}) \quad (5)$$

and the Lepage term is

$$U_\mu^{\text{Lepage}}(x) = c_L U_\nu(x) U_\nu(x + \hat{\nu}) U_\mu(x + 2\hat{\nu}) \times U_\nu^\dagger(x + \hat{\nu} + \hat{\mu}) U_\nu^\dagger(x + \hat{\mu}). \quad (6)$$

We compare the action including $S^{(3)}$ staple term and the Naik term only (Staple + Naik action), i.e. the same as that of [8], Asq action and Asqtad action.

The Staple + Naik action is

$$S_{\alpha\beta}^{-1}(x, y) = \sum_{\mu=-4}^4 \eta_\mu(x) \text{sign}(\mu) \times \left[\left(c_1 U_\mu(x) + w_3 \sum_{\nu \neq \mu} S_{\mu\nu}^{(3)}(x) \right) \delta_{y, x+\hat{\mu}} + c_N U_\mu^{\text{Naik}}(x) \delta_{y, x+3\hat{\mu}} \right], \quad (7)$$

where $c_1 = 9/32$, $w_3 = 9/64$, and $c_N = -1/24$.

The Asq action contains $S^{(3)}$, $S^{(5)}$, $S^{(7)}$, and Lepage term and the Naik term but no tadpole renormalization factor.

$$S_{\alpha\beta}^{-1}(x, y) = \sum_{\mu=-4}^4 \eta_\mu(x) \text{sign}(\mu) \times \left[\left(c_1 U_\mu(x) + \sum_\nu w_3 S_{\mu\nu}^{(3)}(x) + \sum_\rho \left(w_5 S_{\mu\nu\rho}^{(5)}(x) + \sum_\tau w_7 S_{\mu\nu\rho\tau}^{(7)}(x) \right) + c_L U_\mu^{\text{Lepage}}(x) \right) \delta_{y, x+\hat{\mu}} + c_N U_\mu^{\text{Naik}}(x) \delta_{y, x+3\hat{\mu}} \right], \quad (8)$$

where $c_1 = 5/8$, $w_3 = 1/16$, $w_5 = 1/64$, $w_7 = 1/384$, $c_L = -1/16$, and $c_N = -1/24$.

The full Asqtad action contains the tadpole renormalization factor. Different from [6], we do not absorb one power of u_0 into the quark mass in our calculation of the quark propagator, and use $c_1 = 5/8u_0^{-1}$, $w_3 = 1/16u_0^{-3}$,

$w_5 = 1/64u_0^{-5}$, $w_7 = 1/384u_0^{-7}$, $c_L = -1/16u_0^{-5}$, and $c_N = -1/24u_0^{-3}$.

The inversion of the $S_{\alpha\beta}^{-1}(x, y)$ is performed via conjugate gradient method after preconditioning [9] as shown in the appendix.

We consider

$$\not{D}_\mu(x, y) = \frac{1}{2} \sum_{\mu=1, \dots, 4} \eta_\mu(x) \{ U_\mu^{\text{fat}}(x) \delta_{y, x+\hat{\mu}} + U_\mu^{\text{Naik}}(x) \delta_{y, x+3\hat{\mu}} - U_\mu^{\text{fat}}(x)^\dagger \delta_{y, x-\hat{\mu}} - U_\mu^{\text{Naik}}(x)^\dagger \delta_{y, x-3\hat{\mu}} \} \quad (9)$$

and solve for the bare valence quark mass m_0 ,

$$(\not{D} + m_0)\phi = \rho \quad (10)$$

using the 3×3 color matrix expression to the source term ρ .

The propagator can be expressed in the form [7]

$$S(q) = Z_2(q) \frac{-i\not{q} + \mathcal{M}(q)}{q^2 + \mathcal{M}(q)^2}. \quad (11)$$

Since $\text{tr} \gamma_\mu = 0$, tr over color and flavor yields

$$\text{tr} S(q) = 16N_c \frac{Z_2(q) \mathcal{M}(q)}{q^2 + \mathcal{M}(q)^2} = 16N_c \mathcal{B}(q). \quad (12)$$

On the other hand

$$\begin{aligned} \text{tr}(i\not{q}S) &= \text{tr} Z_2(q) \frac{q^2}{q^2 + \mathcal{M}(q)^2} \\ &= 16N_c q^2 \frac{Z_2(q)}{q^2 + \mathcal{M}(q)^2} \\ &= 16N_c q^2 \mathcal{A}(q). \end{aligned} \quad (13)$$

The dynamical mass of the quark is

$$\mathcal{M}(q) = \frac{\mathcal{B}(q)}{\mathcal{A}(q)} \quad (14)$$

and the quark wave function renormalization is

$$Z_2(q) = \frac{\mathcal{A}(q)^2 q^2 + \mathcal{B}(q)^2}{\mathcal{A}(q)}. \quad (15)$$

We remark that our method of deriving $\mathcal{A}(q)$ and $\mathcal{B}(q)$ from the lattice data $S(q)$ is different from [7], however the results are equivalent.

III. RENORMALIZATION EFFECTS

In the continuum theory, the quark wave function renormalization $Z_\psi(q^2)$ is defined by the renormalized inverse quark propagator $S^{-1}(q)$ as [10,11]

$$S^{-1}(q) = \delta_{ab} Z_\psi(q^2) (i\not{q} + M(q)). \quad (16)$$

On the lattice, renormalized operators must obey the same renormalization condition as the continuum theory.

The quark field renormalization factor is usually defined by the amputated Green function of the vector current.

We define the colorless vector current vertex by using [10]

$$G_\mu(q, p) = \int d^4x d^4y e^{iq \cdot y + ip \cdot x} \langle q(y) \bar{q}(x) \gamma_\mu q(x) \bar{q}(0) \rangle \quad (17)$$

and

$$\Gamma_\mu(q, p) = S^{-1}(q) G_\mu(q, p) S^{-1}(p + q). \quad (18)$$

The general expression of the vertex of the vector current is

$$\Gamma_\mu(q) = \delta_{a,b} \{ g_1(q^2) \gamma_\mu + i g_2(q^2) q_\mu + g_3(q^2) q_\mu \not{A} + i g_4(q^2) [\gamma_\mu, \not{A}] \}, \quad (19)$$

where for 16 flavors

$$g_1(q^2) = \frac{1}{48N_c} \text{tr} \left[\Gamma_\mu(q, p=0) \left(\gamma_\mu - q_\mu \frac{\not{A}}{q^2} \right) \right]. \quad (20)$$

The Ward identity in the renormalized form tells

$$(\Gamma_R)_\mu(q) = -i \frac{\partial}{\partial q^\mu} S_R^{-1}(q). \quad (21)$$

After multiplying both sides by Z_ψ , one obtains

$$Z_V \Gamma_\mu(q) = -i \frac{\partial}{\partial q^\mu} S^{-1}(q), \quad (22)$$

where $Z_V g_1(q^2) = Z_\psi(q^2)$ and $Z_V = 1$ in the continuum limit and $Z_\psi(q^2)/g_1(q^2) = Z_V^{\text{MOM}}(a^{-1}, q^2)$ is expected to be independent of q in the limit of $a^{-1} \rightarrow \infty$.

In [12], we observed that the fluctuation of the ghost propagator in the quenched and unquenched simulations differ significantly.

The renormalization of the Green function for the ghost-gluon coupling is

$$\begin{aligned} \tilde{G}(q, g) &\rightarrow \tilde{z}_3^{-1} G(q, g'), \\ \Gamma_3(q, p, g) &\rightarrow \tilde{z}_1 \Gamma_3(q, p, g'), \\ g &\rightarrow g' = \tilde{z}_1 \tilde{z}_3^{-1/2} \tilde{z}_3^{-1} g. \end{aligned} \quad (23)$$

When a fermion of mass m is coupled and the system is multiplicative renormalizable, the running coupling expressed by the dimensionless structure function of gluon D and that of ghost \tilde{D} and the vertex function Γ_3 :

$$\begin{aligned} \bar{g} \left(\frac{p^2}{\Lambda^2}, \frac{m^2}{\Lambda^2}, g \right) &= g \Gamma_3 \left(\frac{q^2}{\Lambda^2}, \frac{p^2}{\Lambda^2}, \frac{m^2}{\Lambda^2}, g \right) D \left(\frac{q^2}{\Lambda^2}, \frac{m^2}{\Lambda^2}, g \right)^{1/2} \\ &\times \tilde{D} \left(\frac{q^2}{\Lambda^2}, \frac{m^2}{\Lambda^2}, g \right) \end{aligned} \quad (24)$$

is invariant under the change of the cutoff Λ when g is transformed to

$$g \rightarrow \tilde{z}_1^{-1} \tilde{z}_3^{1/2} \tilde{z}_3 g. \quad (25)$$

In the Dyson-Schwinger equation (DSE) approach [13,14] the running coupling g of the ghost-gluon coupling is

$$g(q) = \tilde{Z}_1^{-1} Z_3^{1/2}(\mu^2, q^2) \tilde{Z}_3(\mu^2, q^2) g(\mu), \quad (26)$$

where \tilde{Z}_1 is the ghost vertex renormalization factor, which is taken to be 1 in perturbative QCD (pQCD). On the lattice, however, we took it as a scaling factor that adjusts the right-hand side (rhs) of Eq. (26) at the $q = \mu \sim 6$ GeV fits the value of pQCD [1]. The value of $1/\tilde{Z}_1^2$ is tabulated in Table I.

We observed that running coupling g in the rhs needed to be enhanced by \tilde{Z}_1^{-1} , so that it agrees with the pQCD.

The running coupling of the quark gluon coupling is defined by the dimensionless structure function of gluon D and that of quark D_ψ and the vertex function $\Gamma_{3\psi}$:

$$\begin{aligned} \bar{g} \left(\frac{p^2}{\Lambda^2}, \frac{m^2}{\Lambda^2}, g \right) &= g \Gamma_{3\psi} \left(\frac{q^2}{\Lambda^2}, \frac{p^2}{\Lambda^2}, \frac{m^2}{\Lambda^2}, g \right) D \left(\frac{q^2}{\Lambda^2}, \frac{m^2}{\Lambda^2}, g \right)^{1/2} \\ &\times D_\psi \left(\frac{q^2}{\Lambda^2}, \frac{m^2}{\Lambda^2}, g \right). \end{aligned} \quad (27)$$

When the quark propagator is not calculated consistently using the simplified action as compared to the real action used in producing the gauge fields, one should define the normalization of the quark propagator Z_ψ via other information.

Since at $\mu \sim 6$ GeV, the quark wave function renormalization factor in pQCD is close to 1, we define the scale of $Z_\psi(q^2)$ as

$$Z_\psi(q^2) = \frac{Z_V g_1(\mu^2) \mathcal{A}(q)}{\mathcal{A}(q)^2 q^2 + \mathcal{B}(q)^2}, \quad (28)$$

where $g_1(\mu^2)$ is defined from the running coupling in the following argument. The function $\mathcal{B}(q)$ defines the quark inverse propagator $Z_\psi(q)$ and the dynamical mass $M(q)$ as

$$\mathcal{B}(q) = \mathcal{A}(q) \mathcal{M}(q) \propto \frac{1}{Z_\psi(q^2)} M(q). \quad (29)$$

In the following, we define $M(q) = \mathcal{M}(q)$ and renormalize the quark field as $\sqrt{Z_\psi(\mu^2)} \psi_{\text{bare}} = \psi_R$.

The Ward identity implies that the wave function renormalization can be defined via conserved vector current vertex.

The running coupling of quark-gluon coupling is calculated as

$$g(q) = Z_1^{\psi^{-1}} Z_3^{1/2}(\mu^2, q^2) Z_2(\mu^2, q^2) g(\mu). \quad (30)$$

TABLE I. The $1/\tilde{Z}_1^2$ factor of the unquenched SU(3).

Config.	Heavy	Light	N_f	Comments
MILC _c	1.49(11)	1.43(10)	2 + 1	$\beta_{\text{imp}} = 6.83, 6.76$
MILC _f	1.37(9)	1.41(12)	2 + 1	$\beta_{\text{imp}} = 7.11, 7.09$

At the renormalization point $q = \mu$, we fix $Z_2(\mu^2, \mu^2) = 1$ and $Z_3(\mu^2, \mu^2) = 1$, and thus

$$\tilde{Z}_1 \tilde{Z}_3^{-1} = Z_1^\psi Z_2^{-1}, \quad (31)$$

i.e. $\tilde{Z}_1 = Z_1^\psi$.

In [10], Z_V is found to be almost constant for $q > 5$ GeV, and at $\mu \sim 6$ GeV we identify $Z_V g_1(\mu^2)$ with $Z_1^\psi(\mu^2)$. When quark wave function renormalization factor is measured using the Asqtad action, the running coupling given by Eq. (30) is expected to agree with the pQCD result in a high momentum region. But in the calculation with Staple + Naik action, the wave function renormalization is to be modified by the tadpole renormalization factor u_0 . Further, in the approximate calculation, the Z_1^ψ would be modified as \tilde{Z}_1 , and to give the consistent $g(q)$, additional correction to $Z_2(q)$ would be necessary.

We define $Z_2(q) = 1/Z_\psi(q^2) = Z_2(q)/Z_V g_1(\mu^2)$ and identify $1/Z_V g_1(\mu^2)$ for the Staple + Naik action as the average of the product of vertex renormalization factor and the tadpole renormalization factor, i.e. $\tilde{Z}_1 u_0 = 1/1.38$ for MILC_c and $1/1.36$ for MILC_f.

Thus, the renormalization of Z_ψ on the lattice is defined by the renormalization of the running coupling on the lattice defined at $\mu \sim 6$ GeV.

In the case of Asq action, the renormalization factor $Z_2(q)$ is suppressed by about 10% and the mass function $\mathcal{M}(q)$ is enhanced by about 10% as compared to the Staple + Naik. By inclusion of the tadpole renormalization factor u_0 to the Asq action, the $Z_2(q)$ become consistent with those of Staple + Naik action with correction by $Z_V g_1(\mu^2)$. In [7], the scale of $Z_2(q)$ calculated by the Asqtad action is fixed to 1 at $q = 3$ GeV, but we adopt the bare lattice data $Z_2(q) = \tilde{Z}_2(q)$ and $M(q)$ of Asqtad action to compare with those of Staple + Naik with correction $Z_V g_1(\mu^2)$. Coincidence in the case of $m_0 = 82.2$ MeV is shown in Figs. 1 and 2. In the case of $m_0 = 11.5$ MeV, the $M(q)$ and $Z_2(q)$ of Staple + Naik in the

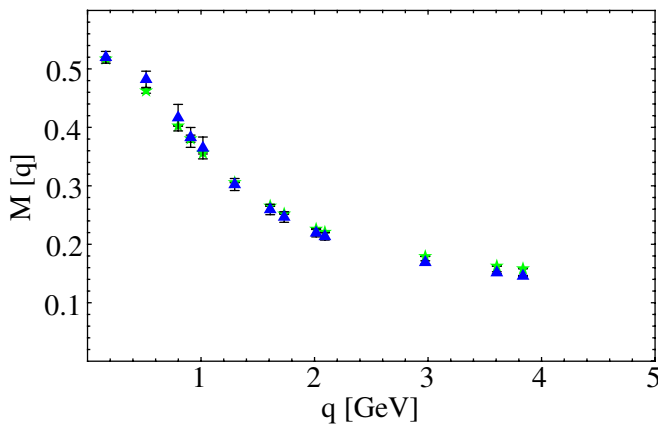


FIG. 1 (color online). The mass function $M(q)$ of Staple + Naik action (stars) and the Asqtad action (triangles) of MILC_c with the bare quark mass $m_0 = 82.2$ MeV.

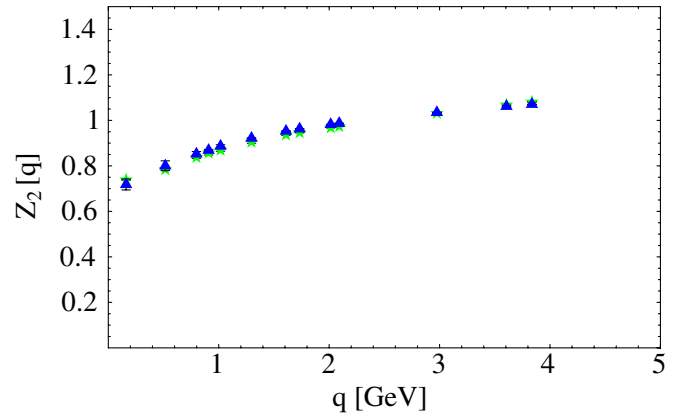


FIG. 2 (color online). The $Z_2(q)$ of Staple + Naik action (stars) and the Asqtad action (triangles) of MILC_c with the bare quark mass $m_0 = 82.2$ MeV.

infrared region are shifted from those of Asqtad slightly, but the difference is about 10% at most, as shown in Figs. 3 and 4, respectively.

IV. ANALYSIS OF $M(q)$ AND $Z_\psi(q^2)$

In pQCD, dynamical mass of a quark is expressed as [2,15,16]

$$M(q) = -\frac{4\pi^2 d_M \langle \bar{q}q \rangle_\mu [\log(q^2/\Lambda_{\text{QCD}}^2)]^{d_M-1}}{3q^2 [\log(\mu^2/\Lambda_{\text{QCD}}^2)]^{d_M}} + \frac{m(\mu^2) [\log(\mu^2/\Lambda_{\text{QCD}}^2)]^{d_M}}{[\log(q^2/\Lambda_{\text{QCD}}^2)]^{d_M}}, \quad (32)$$

where $d_M = \frac{12}{33-2N_f}$. The second term is the contribution of the massive quark.

In this analysis of the lattice data, the quark condensates $-\langle \bar{q}q(\mu) \rangle$ and Λ_{QCD} are the fitting parameters. In the MILC_f lattice, the bare masses are $0.0062/a = 13.6$ MeV and $0.0124/a = 27.2$ MeV for the u - d quarks

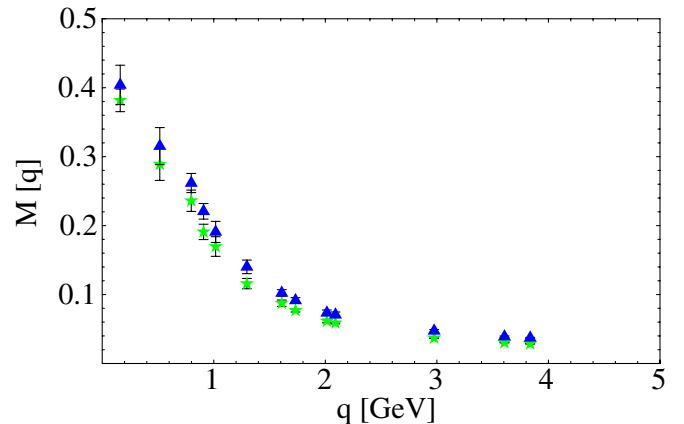


FIG. 3 (color online). The mass function $M(q)$ of Staple + Naik action (stars) and the Asqtad action (triangles) of MILC_c with the bare quark mass $m_0 = 11.5$ MeV.

TABLE II. The parameters c and Λ .

β_{imp}	m_0 (MeV)	c	Λ (GeV)	$c\Lambda$ (GeV)
6.76	11.5	0.44(1)	0.87(2)	0.383
	82.2	0.30(1)	1.45(2)	0.431
6.83	65.7	0.33(1)	1.28(2)	0.420
	82.2	0.30(1)	1.45(2)	0.431
7.09	13.6	0.45(1)	0.82(2)	0.368
	68.0	0.30(1)	1.27(4)	0.381
7.11	27.2	0.43(1)	0.89(2)	0.383
	68.0	0.32(1)	1.23(2)	0.397

and $0.031/a = 68.0$ MeV for the s -quark. In the MILC_c lattice, the corresponding masses are $0.007/a = 11.5$ MeV and $0.040/a = 65.7$ MeV for the u - d quarks and $0.050/a = 82.2$ MeV for the s -quark.

The mass function equation (32) is based on pQCD and cannot fit the data below 2 GeV and we try the phenomenological fit [3,17]

$$M(q) = \frac{c\Lambda^3}{q^2 + \Lambda^2} + m_0, \quad (33)$$

where m_0 is the bare quark mass. Parameters of c and Λ are summarized in Table II. The fitting of the mass function of MILC_c and that of MILC_f are shown in Figs. 5 and 6, respectively.

We observe that as the bare quark mass becomes heavy, c becomes smaller but the product $c\Lambda$ becomes larger. Although the MILC_c configurations of bare mass $m_0 = 82.2$ MeV, with different β agree within errors, the MILC_f configurations of bare mass $m_0 = 68$ MeV show dependence on β . The mass function of $\beta = 7.09$ is smaller than that of $\beta = 7.11$. In the case of $\beta = 7.11$, the chiral limit $M(0)$ is consistent with that of MILC_c and we find $M(0) = 0.37(1)$ GeV. However, in the case of $\beta = 7.09$, $m_0 = 13.6$ MeV, the lowest three momentum points of $M(q)$ are systematically smaller than the other points. The slope of the $\beta = 7.09$ and that of 7.11 are almost the same. A

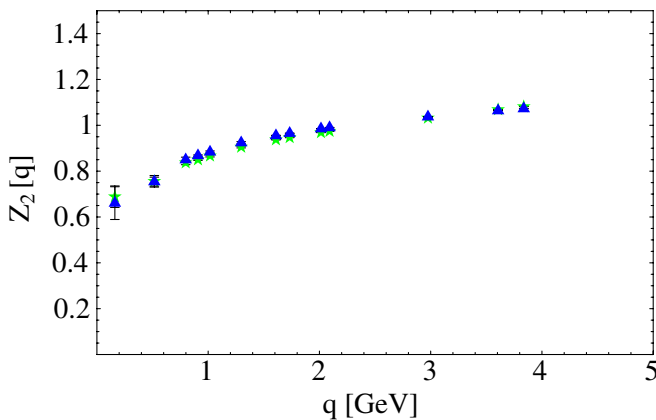


FIG. 4 (color online). The $Z_2(q)$ of Staple + Naik action (stars) and the Asqtad action (triangles) of MILC_c with the bare quark mass $m_0 = 11.5$ MeV.

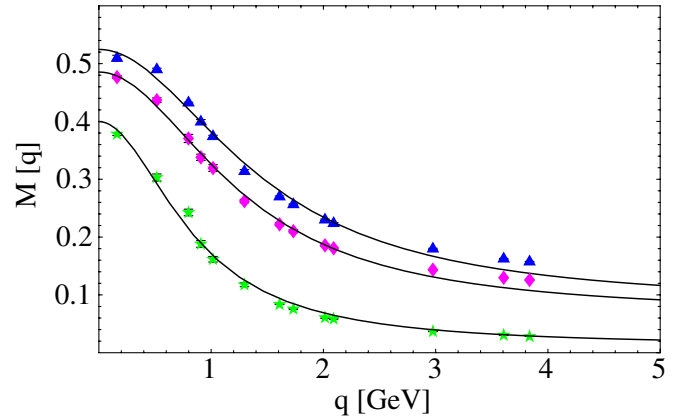


FIG. 5 (color online). The dynamical mass of the MILC_c quark (Staple + Naik) with bear mass $m_0 = 11.5$ MeV (stars), 65.7 MeV (diamonds), and 82.2 MeV (triangles) and the phenomenological fits.

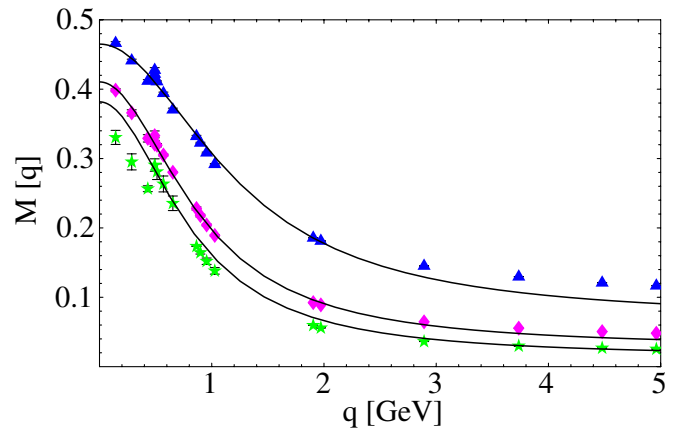


FIG. 6 (color online). Same as Fig. 5 but MILC_f quark with bear mass $m_0 = 13.6$ MeV (stars), 27.2 MeV (diamonds), and 68.0 MeV (triangles) and the phenomenological fits.

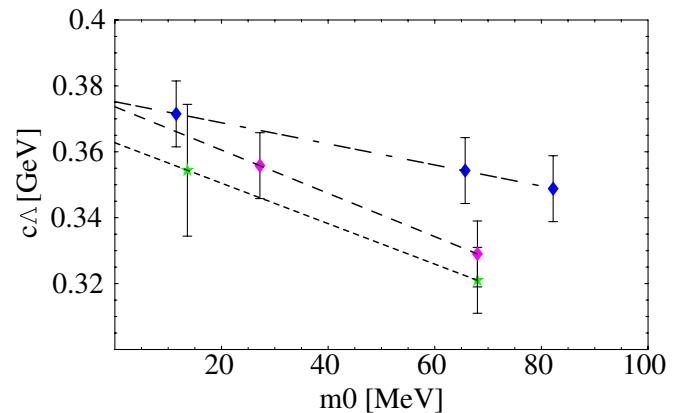


FIG. 7 (color online). The chiral symmetry breaking mass $c\Lambda = M(0) - m_0$ as a function of bare mass and its chiral limit. Dotted line is the extrapolation of MILC_f $\beta = 7.09$, dashed line is $\beta = 7.11$, and the dash-dotted line is that of MILC_c.

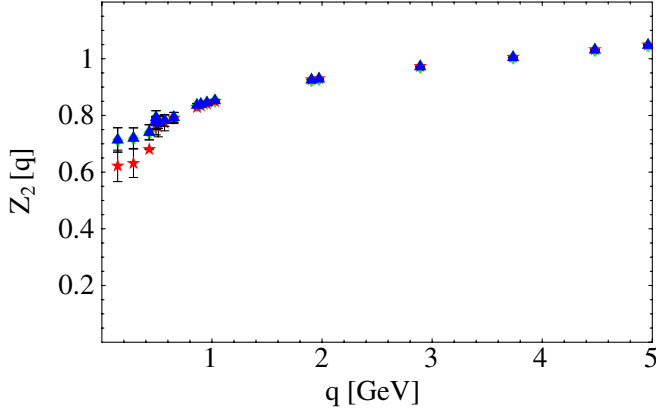


FIG. 8 (color online). The $Z_2(q)$ of MILC_f with bear mass $m_0 = 13.6$ MeV (stars), 27.2 MeV (diamonds), and 68 MeV (triangles).

preliminary analysis of $c\Lambda$ using the Asqtad action suggests that the data of $\beta = 7.09$ and MILC_c obtained by the Staple + Naik are an underestimation, and the discrepancy between MILC_c and MILC_f of about 0.02 GeV remain. We expect that the $M(0)$ in the continuum limit would be about 0.38 GeV, consistent with the value obtained by meromorphic parametrization of lattice data [18]. In a DSE approach [19], deviation of the mass function $M(0)$ from a linear function of $m(\zeta = 19 \text{ GeV})$ where ζ defines the scale of the system is claimed. Although $m(\zeta = 19 \text{ GeV})$ is not identical to m_0 , $M(0) - m(\zeta = 19 \text{ GeV})$ of the DSE increases near the chiral limit in contrast to our naive fitting shown in Fig. 7, which implies that the chiral symmetry effect monotonically decreases as the bare mass increases.

We show the lattice results of $Z_2(q)$ of MILC_f and MILC_c in Figs. 8 and 9, respectively.

The apparent difference in the formulae of [7] and our work are only in the expression and in fact they are equivalent. The $Z_2(q)$ agree with each other.

The $A(q)$ defined as $S(q) = \frac{1}{A(q)\not{q} - B(q)}$ is parametrized in the OPE approach as [20,21]

$$A(q) = 1 + \frac{\pi\alpha_s(\mu^2)\langle A^2 \rangle_\mu}{N_c q^2} - \frac{\pi\alpha_s(\mu^2)\langle F^2 \rangle_\mu}{3N_c q^4} + \frac{3\pi\alpha_s(\mu^2)\langle \bar{q}g\cancel{A}q \rangle_\mu}{4q^4}, \quad (34)$$

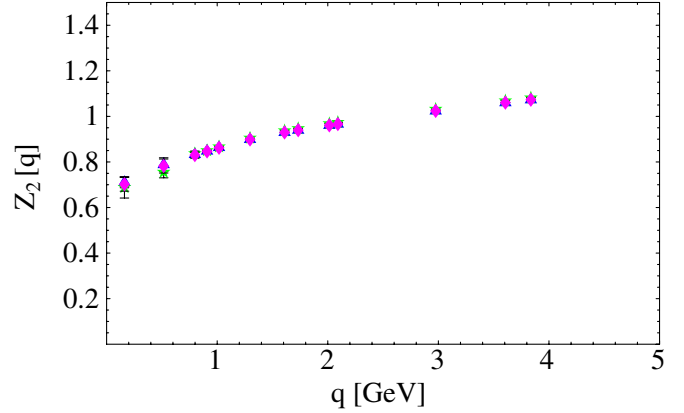


FIG. 9 (color online). Same as Fig. 8 but quark of MILC_c with bear mass $m_0 = 11.5$ MeV (stars), 65.7 MeV (diamonds), and 82.2 MeV (triangles). The last two almost overlap.

where $\langle F^2 \rangle_\mu$ is the gluon condensate, $\alpha_s(\mu^2)\langle A^2 \rangle_\mu$ is the A^2 condensate, and $\langle \bar{q}g\cancel{A}q \rangle_\mu$ is the mixed condensate, which are the fitting parameters.

Similar parametrization was done by [2], in which $\mu = 2 \text{ GeV}$ and $A(q)$ for $q > 1 \text{ GeV}$ is parametrized as

$$A(q) = 1 + \frac{c_1}{q^2} + \frac{c_2}{q^4}. \quad (35)$$

In this parametrization, we obtained $c_1 = 0.25 \text{ GeV}^2$, $c_2 = 0.061 \text{ GeV}^4$ in the case of $m_0 = 27 \text{ MeV}$. Comparing with data of quenched simulation [2] ($c_1 = 0.37 \pm 0.06 \text{ GeV}^2$ and $c_2 = -0.25 \pm 0.04 \text{ GeV}^4$), c_1 is the same order but c_2 is smaller and has opposite sign. The result depends on the choice of the renormalization point μ .

The pQCD result of the wave function renormalization factor $Z_2(q)$ in \overline{MS} scheme up to four loop is given by [22,23]. Orsay group expanded the $\alpha_s^{\overline{MS}}$ in terms of $\alpha_s^{\text{MOM}}(q) = \alpha$ and obtained $Z_\psi^{\text{pert}}(q^2) = 1/Z_2(q)$ as a function of α and fitted the $Z_\psi(q^2)$ of Wilson overlap fermion in the MOM scheme

$$\begin{aligned} (Z_\psi^{\text{pert}}(\mu^2))^{-1} S^{-1}(q) &= (Z_\psi^{\text{pert}}(\mu^2))^{-1} S_{\text{pert}}^{-1}(q) + i\not{p} \frac{d(q^2/\mu^2, \alpha(\mu))}{q^2} \frac{\langle A^2 \rangle_\mu}{4(N_c^2 - 1)} \delta_{ab} + \dots \\ &= i\not{p} \delta_{ab} \left(\frac{Z_\psi^{\text{pert}}(q^2)}{Z_\psi^{\text{pert}}(\mu^2)} + \frac{d(q^2/\mu^2, \alpha(\mu))}{q^2} \frac{\langle A^2 \rangle_\mu}{4(N_c^2 - 1)} \right) + \dots, \end{aligned} \quad (36)$$

where $d(q^2/\mu^2, \alpha(\mu))$ is the solution of the renormalization group equation

$$\left\{ (-\gamma_0 + \gamma_{A^2}^{(0)}) \frac{\alpha(\mu)}{4\pi} + \frac{d}{d \log \mu^2} \right\} d(q^2/\mu^2, \alpha(\mu)) = 0 \quad (37)$$

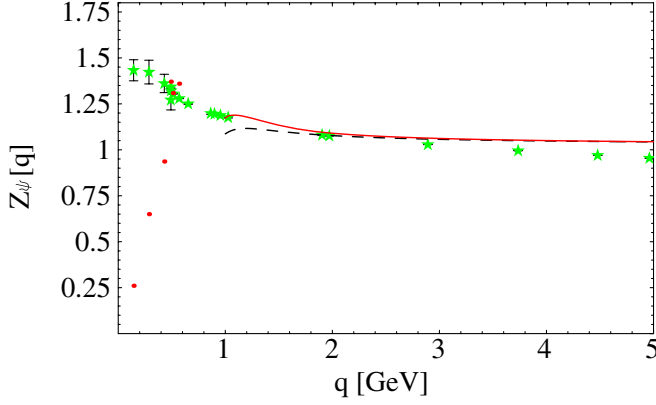


FIG. 10 (color online). The quark wave function renormalization $Z_\psi(q^2)$ of MILC_f bear mass $m_0 = 27.2$ MeV. The solid line is the phenomenological fit as [10] and the dashed line is the pQCD result. The points below 0.6 GeV are $g_1(q^2)$ approximated by the running coupling obtained by the ghost-gluon vertex.

which can be written as

$$d(q^2/\mu^2, \alpha(\mu)) = d(1, \alpha(q)) \left(\frac{\alpha(\mu)}{\alpha(q)} \right)^{(-\gamma_0 + \gamma_{A^2})/\beta_0}. \quad (38)$$

The anomalous dimension of A^2 in the lowest order is [22,24,25] $\gamma_{A^2}^{(0)} = \frac{35}{4} - \frac{2}{3}n_f$ and $\gamma_0 = 0$. We adopt $\langle A^2 \rangle$ as a fitting parameter and calculate

$$\begin{aligned} Z_\psi(q^2) &= \frac{Z_V g_1(\mu^2)}{Z_2(q)} \\ &= Z_\psi^{\text{pert}}(q^2) + \frac{(\frac{\alpha(\mu)}{\alpha(q)})^{(-\gamma_0 + \gamma_{A^2})/\beta_0}}{q^2} \\ &\quad \times \frac{\langle A^2 \rangle_\mu}{4(N_c^2 - 1)} Z_\psi^{\text{pert}}(\mu^2) + \frac{c_2}{q^4}, \end{aligned} \quad (39)$$

where $\alpha(q)$ are data calculated in the $\widetilde{\text{MOM}}$ scheme using the same MILC_f gauge configuration [1].

We fitted the $Z_\psi(q^2)$ of MILC_f, $m = 27.2$ MeV data and choosing $\mu = 2$ GeV and $c_2 = 0$, obtained $\langle A^2 \rangle_\mu = 1.6(3)$ GeV², which is compatible with the Orsay group data 2.4 ± 0.3 GeV² for the Wilson fermion.

In Fig. 10, we show the lattice data of $Z_\psi(q^2) = \frac{Z_V g_1(\mu^2)}{Z_2(q)}$ from the Staple + Naik action with $Z_V g_1(\mu^2) = 1.36$, i.e. the average of the running coupling renormalization factor of MILC_f in the previous analysis [1], and the phenomenological fit $Z_\psi^{\text{fit}}(q^2)$ by Eq. (39). The pQCD result $Z_\psi^{\text{pert}}(q^2)$ above $q = 1$ GeV is shown by the dashed line and the lattice data of $\alpha_s(q)$ below $q = 0.5$ GeV obtained by the ghost-gluon coupling are also plotted. The difference between $\alpha_s(q)$ and $Z_\psi(q^2)$ in the infrared implies that Z_V is strongly momentum dependent in the infrared [10].

The suppression of $Z_\psi(q^2)$ in the infrared suggested by the behavior of $\alpha_s(q)$ could be an artefact. In the case of

Wilson fermion [26], the $Z_\psi(q^2)$ in the continuum limit is infrared finite.

V. CONCLUSION AND DISCUSSION

We measured the quark propagator of the MILC_f lattice ($28^3 \times 96$) of $\beta_{\text{imp}} = 7.09$ and 7.11 and MILC_c lattice ($20^3 \times 64$) of $\beta_{\text{imp}} = 6.76$ and 6.83 using the Staple + Naik action. After the renormalization using the information of the running coupling and the tadpole renormalization, we obtained a good agreement with the lattice bare data of the Asqtad action in the case of MILC_c.

We observed that the denominator of the quark renormalization factor

$$Z_2(q) = \frac{\mathcal{A}(q)^2 q^2 + \mathcal{B}(q)^2}{\mathcal{A}(q)} \quad (40)$$

is a steeply increasing function as q approaches 0 which causes the infrared suppression. Although the function $\mathcal{B}(q)$ in the numerator is also an increasing function of q as q approaches 0, it is almost constant as the bare mass m_0 decreases, in contrast to $\mathcal{A}(q)$ which becomes larger as the bare mass m_0 decreases. In the DS equation approach [18], $\sigma_v(q)$ and $\sigma_s(q)$ correspond to our $\mathcal{A}(q)$ and $\mathcal{B}(q)$, respectively. $\sigma_v(q)$ corresponds to $\frac{\Lambda_{\text{QCD}}}{q^2 + m_0^2}$ and its m_0 dependence is qualitatively consistent with our lattice data. The dominant term of $\sigma_s(q)$ corresponds to $\frac{c\Lambda^3}{(q^2 + m_0^2)(q^2 + \Lambda^2)}$ and its m_0 dependence is weak in the $q \sim 0.1$ GeV region consistent with our lattice data.

The infrared feature of $Z_\psi(q^2) = \frac{1}{Z_2(q)}$ is related to the Kugo-Ojima color confinement criterion:

$$1 + u = \frac{Z_1}{Z_3} = \frac{\tilde{Z}_1}{\tilde{Z}_3} = \frac{Z_{1\psi}}{Z_2} = 0, \quad (41)$$

i.e. the Kugo-Ojima parameter $u = -1$.

In the quark sector, divergence of $Z_2(q)$ or vanishing of $Z_\psi(q^2)$ in the infrared is consistent with the Kugo-Ojima confinement criterion. Infrared finiteness of Z_2 implies that $Z_{1\psi}$ is infrared vanishing, and infrared finite Z_ψ supports the idea that the running coupling freezes to a finite value in the infrared. This situation is similar to that of the gluon sector. The infrared finite lattice data of Z_3 implies that Z_1 is also infrared vanishing. In this case running coupling defined by (26) vanishes unless Z_3 is infrared vanishing. Definition of the running coupling from the triple gluon vertex is more ambiguous than the definition (26) from the ghost-gluon vertex, and we expect that the fluctuation of the ghost propagator in the infrared induces the artefacts in the infrared running coupling.

We compared the Staple + Naik action and the Asqtad action and found that the mass function and the quark propagator are consistent with each other. They are consistent with the results of other group [3,7,27].

We confirmed A^2 condensates which was found by the Orsay group in the running coupling of quenched lattice simulation [28] and observed in the unquenched lattice simulation [1,10] also in the quark propagator. We observed also the indication of the $\bar{q}q$ condensate.

The effect of dynamical chiral symmetry breaking is seen in the mass function $M(q)$. By extrapolation to the chiral limit we obtain $M(0) \sim 0.38$ GeV, which is about 20% larger than [7], but consistent with DS equation approach [18]. An origin of the difference from [7] is their renormalization of the mass function $M(q)$ at 3 GeV to a theoretical value, which we did not adopt. In the definition of the quark mass from the lattice data, one should take into account the mass renormalization by taking into account the lattice constant a dependence of the propagator [26,29–32].

Calculation of the quark propagator of MILC_f using the Asqtad action is underway and the detailed comparison of the Asqtad action data and the DS equation approach will be published in the forthcoming paper.

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APPENDIX: THE LATTICE CALCULATION OF THE KS FERMION PROPAGATOR

The Dirac gamma function of KS fermions are defined as [3]

$$(\tilde{\gamma}_\mu)_{\alpha\beta} = (-1)^{\alpha_\mu} \delta_{\alpha+\zeta^{(\mu)},\beta}, \quad (\text{A1})$$

where

$$\zeta_\nu^{(\mu)} = \begin{cases} 1 & \nu < \mu \\ 0 & \text{otherwise} \end{cases}. \quad (\text{A2})$$

In momentum space the inverse propagator is expressed as

$$S_{\alpha\beta}^{-1}(p, m) = i \sum_\mu (\tilde{\gamma}_\mu)_{\alpha\beta} \left(\frac{9}{8} \sin p_\mu - \frac{1}{24} \sin 3p_\mu \right) + m \bar{\delta}_{\alpha\beta}, \quad (\text{A3})$$

where

$$\bar{\delta}_{\alpha\beta} = \prod_\mu \delta_{\alpha_\mu, \beta_\mu}. \quad (\text{A4})$$

The algorithm for calculating the propagator is as follows. We define the operator divided by the bare mass of

the dynamical quark m as

$$\bar{M} = \left(I + \frac{1}{m} \not{D} \right) \quad (\text{A5})$$

and decompose even sites and odd sites,

$$\bar{M} = \begin{pmatrix} I & \frac{1}{m} \not{D}_{oe} \\ \frac{1}{m} \not{D}_{eo} & I \end{pmatrix} = I - L - U, \quad (\text{A6})$$

where

$$L = \begin{pmatrix} 0 & 0 \\ -\frac{1}{m} \not{D}_{eo} & 0 \end{pmatrix} \quad (\text{A7})$$

and

$$U = \begin{pmatrix} 0 & -\frac{1}{m} \not{D}_{oe} \\ 0 & 0 \end{pmatrix}. \quad (\text{A8})$$

Using the Eisenstat trick, we define

$$\begin{aligned} \tilde{M} &= (I - L)^{-1} \bar{M} (I - U)^{-1} \\ &= (I + L)(I - U - L)(I + U) = I - LU \\ &= \begin{pmatrix} I & 0 \\ 0 & -(\frac{1}{m})^2 \not{D}_{eo} \not{D}_{oe} \end{pmatrix}. \end{aligned} \quad (\text{A9})$$

We note that \tilde{M} is Hermitian and the conjugate gradient method and/or BiCGstab method are applicable.

With use of the definition

$$\frac{1}{m} \rho = \rho' = \begin{pmatrix} \rho'_o \\ \rho'_e \end{pmatrix} \quad (\text{A10})$$

and

$$\phi = \begin{pmatrix} \phi'_o \\ \phi'_e \end{pmatrix}, \quad (\text{A11})$$

we solve

$$\left(I - \frac{1}{m^2} \not{D}_{eo} \not{D}_{oe} \right) \tilde{\phi}_e = \rho'_e - \frac{1}{m} \not{D} \rho'_o \quad (\text{A12})$$

with initial value $\tilde{\phi}_e^{(0)} = \tilde{\rho}'_e$. The odd site solution is $\tilde{\phi}_o = \rho'_o - \frac{\not{D}}{m} \tilde{\phi}_e$.

The decomposition into even sites and odd sites is done by multiplying the projection operator P_e and P_o , and the color source ρ' is taken as a vector of 3×3 matrices.

The convergence condition for the conjugate gradient iterations is

$$\frac{\| \tilde{M} \phi - \rho \|}{\| \rho \|} < \text{a few percent at most}, \quad (\text{A13})$$

where the used norm is maximum norm in the space of site, color, and flavor, and the accuracy gets 10^{-1} higher if L^2 norm is used.

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