Systematic study of leptonic mixing in a class of $SU_H(2)$ models

K. L. McDonald[*] and B. H. J. McKellar[†]

School of Physics, Research Centre for High Energy Physics, The University of Melbourne, Victoria, 3010, Australia

We perform a systematic analysis of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrices which arise when one assigns the three generations of leptons to the $2 \oplus 1$ representation of a horizontal $SU_H(2)$ symmetry. This idea was explored previously by Kuchimanchi and Mohapatra. However, we assume (i) the neutrino mass matrix results from leptonic couplings to $SU_L(2)$ triplet scalar fields and (ii) hierarchies exist among lepton mass matrix elements which result from couplings to scalar fields with different $SU_H(2)$ charges. Of the 16 candidate PMNS matrices which result it is found that only one is both predictive and possesses a leading order structure compatible with experimental data. The relevant neutrino mass matrix displays the symmetry $L_e - L_\mu - L_\tau$ to leading order and we explore the perturbations required to produce a realistic lepton spectrum. The effective mass in neutrinoless double beta decay is required to lie in the range $\langle m \rangle/(10^{-2} \text{ eV}) \in [0.7, 2.5]$, which is just below current experimental bounds. U_{e3} is nonzero but not uniquely determined.

PACS numbers: 14.60.Pq, 14.60.St

I. INTRODUCTION

The search for a possible connection between the observed families of fermions, the properties possessed by the fermions, and an underlying horizontal symmetry has a long history (see for example [1–20]). In more recent years, the experimental observation of neutrino mixing has lead to an enormous amount of research into a link between the bilarge mixing in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix and a horizontal symmetry. In particular, some recent works [21–23] have attempted to relate the observed values of the PMNS matrix to the structure enforced upon the lepton mass matrices by a $SU_H(2)$ symmetry.

In this paper we perform a systematic analysis of the PMNS mixing matrices that arise when the three lepton families are assigned to the $2 \oplus 1$ representations of $SU_H(2)$, and the neutrino mass matrix results from couplings of the $SU_L(2)$ doublet leptons to $SU_L(2)$ triplet scalar fields.

The main purpose of the study is to attempt to relate the observed PMNS matrix structure to a high energy $SU_H(2)$ symmetry. To prevent the PMNS matrix from becoming too general, we assume that hierarchies exist among contributions to the lepton mass matrices which result from scalars with different horizontal quantum numbers. These hierarchies dictate certain structures upon the lepton mass matrices and thus provide distinct signatures of the horizontal symmetry.

The assumed hierarchies result in 16 different leading order PMNS mixing matrices. The majority of these are found to be incompatible with the experimentally extracted values of the lepton mixing matrix due to, e.g., texture zeros in unwanted locations. Of the remaining matrices

only one is found to have any predictive quality. The identification of this candidate PMNS matrix is the main result of this study.

This mixing matrix results from a neutrino mass matrix possessing the nonstandard lepton number symmetry $L_e - L_\mu - L_\tau$. It predicts $U_{e3} = 0$ and maximal solar mixing to leading order, while the atmospheric mixing angle remains a free parameter. First order perturbations, which result from subdominant lepton mass matrix entries, prove adequate to produce realistic charged lepton and neutrino mass spectra. They also perturb the solar mixing angle to its preferred nonmaximal value and induce nonzero corrections to U_{e3} which are small but not constrained. The experimental data directly constrains the parameters which enter into the effective mass in neutrinoless double beta decay. Consequently it is required to lie in the range,

$$\langle m \rangle/(10^{-2} \text{ eV}) \in [0.7, 2.5], \tag{1}$$

placing it just below current experimental bounds.

The layout of this paper is as follows: In Sec. II we provide some background and motivations for the present investigation. The scalar sector is described in Sec. III while Sec. IV contains the study of candidate lepton mixing matrices. This section contains our main results. Some comments on the scale of $SU_H(2)$ symmetry breaking are offered in Sec. V and we conclude in Sec. VI.

II. BACKGROUND AND MOTIVATIONS

In the present work we shall study the lepton mixing that occurs in models with a $SU_H(2)$ symmetry, under the assumption that the dominant contribution to the neutrino mass matrix results from couplings to $SU_L(2)$ triplets. This could occur in models of the type proposed by Kuchimanchi and Mohapatra (KM) [21,22] by extending the scalar sector to include the necessary $SU_L(2)$ triplet fields. In this case the horizontal symmetry breaking scale

---

*Electronic address: k.mcdonald@physics.unimelb.edu.au
†Electronic address: b.mckellar@physics.unimelb.edu.au

must be large enough, or the relevant coupling constants small enough, to render the seesaw contributions to the neutrino mass matrix contributions subdominant.

Alternatively one may consider a model where the neutrino field N_R, a $SU_H(2)$ doublet of right-chiral neutrinos, is not present. The field N_R was necessary in the KM models to remove the global anomaly. One could instead consider a model with a gauge group $G_{SM} \otimes G'_{SM} \otimes SU_H(2)$, where G_{SM} is the standard model (SM) gauge group, G'_{SM} describes a mirror sector and the horizontal symmetry couples the two sectors. This would allow the groups G_{SM} and G'_{SM} to each possess an odd number of $SU_H(2)$ doublets while the entire theory is free of the global anomaly. In this way mathematical consistency of the model would not necessitate N_R. We note that a $SU_H(2)$ symmetry communicating with a mirror sector is always free from the global anomaly in much the same way as a $SU_H(3)$ which communicates with a mirror sector is free from the $[SU_H(3)]^3$ anomaly [24].

The two model building strategies suggested here to obtain a dominant electroweak triplet contribution to the neutrino mass matrix admittedly represent a further complication of the KM models. We consider, however, the study of the lepton mixing that arises under the assumption of triplet dominance in the KM type models to be of interest for two reasons.

First, the existence of a horizontal $SU_H(2)$ symmetry which manifests itself in the neutrino mass matrix through couplings to $SU_L(2)$ triplets would surely represent a very direct link between the physics beyond the SM and the lepton mixing matrix. This is because the horizontal symmetry may dictate certain structures directly upon the neutrino Majorana mass matrix, providing experimentally accessible information about the nature of higher energy physics. If nature employs a seesaw mechanism to obtain the light observed neutrinos, the link between an operative symmetry and the observed lepton mixing matrix may not be as direct. The seesaw mechanism allows the structure of the heavy Majorana mass matrix to influence the structure of the mass matrix for the light neutrinos, i.e. $M_\nu = -M_D M_R^{-1} M_D^T$, where M_D (M_R) is the Dirac (heavy Majorana) mass matrix. Structures enforced upon M_D and M_R by a symmetry may not be reconstructed as readily by analysis of experimental data as a structure imparted directly upon M_ν.

The second reason is borne out by the structure of the neutrino mass matrix observed when one groups the muon and tauon generations together under the horizontal symmetry. We shall show in Sec. III that the most general neutrino mass matrix in this case takes the form

$$M_\nu = \begin{pmatrix} s_1 & d_1 & d_2 \\ d_1 & t_1 & t_2 \\ d_2 & t_2 & t_3 \end{pmatrix}, \qquad (2)$$

where the entries labeled s, d, and t result from couplings

to $SU_L(2)$ triplets which form singlet, fundamental, and adjoint representations of $SU_H(2)$. To understand our interest in this structure we make a brief digression.

It is known that the observed bilarge mixing may result from a perturbation upon a mixing matrix with the bimaximal form [25–27] (see also [28])

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}. \qquad (3)$$

There are three particularly interesting leading order mass matrices which lead to bimaximal mixing:

(i) The normal hierarchy, $m_3 \gg m_{2,1}$, occurs when

$$M_\nu \approx \frac{\sqrt{\Delta m_A^2}}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}, \qquad (4)$$

where Δm_A^2 is the atmospheric mass-squared difference. This matrix possesses the flavor symmetry L_e and results in maximal mixing in the $2-3$ sector.

(ii) One obtains an inverted hierarchy, $m_2 \approx m_1 \gg m_3$, when

$$M_\nu \approx \sqrt{\frac{\Delta m_A^2}{2}} \begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \qquad (5)$$

This mass matrix produces exactly the bimaximal form of the mixing matrix. It conserves the flavor symmetry $L' = L_e - L_\mu - L_\tau$, though it is not the most general matrix permitted by this symmetry. It has received much attention [29].

(iii) Quasidegenerate neutrinos, $m_1 \approx m_2 \approx m_3$, are obtained when

$$M_\nu \approx m \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}. \qquad (6)$$

Matrices of this type have been considered previously in [30–32]. This matrix possess an $L_\mu - L_\tau$ symmetry, which brought it to the attention of the authors of [33–35].

It is interesting to note a feature which is common to each of the mass matrices (4)–(6). Each of these mass matrices can be obtained by imposing a family symmetry which differentiates in some sense between the electron generation and the mu and tau generations. The matrix (4) arises from a symmetry operative only on the electron neutrino, the L' symmetry of (5) differentiates clearly between ν_e and $\nu_{\mu,\tau}$, while the $L_\mu - L_\tau$ symmetry is operative only on ν_μ and ν_τ.

It is this common feature of these contending leading order neutrino mass matrices that motivates our interest in a Majorana mass matrix with the structure given by (2). We see that if one sets $s_1, d_{1,2} = 0$ in (2) the nonzero entries in

M_ν correspond to those of the normal hierarchy (4). In (2) the entries s_1 and $d_{1,2}$ arise from couplings to different scalars than the one that produces the entries $t_{1,2,3}$. The dominant mass matrix structure may be attributed to the absence, or weaker coupling to, certain scalar representations of the gauge group. Similarly by setting $s_1, t_{1,2,3} = 0$ one obtains a mass matrix with the same zeros as (5) so again this structure may be obtained by consideration of the scalar content of a given model. The quasidegenerate spectrum also can be approximated by (2) if $d_{1,2} = 0$ and $t_3 > t_{1,2}$, though the correlation between the scalar content required to produce (2) and the leading order structure of M_ν is more tenuous in this case.

More details regarding the correlation between the structure of (2) and the scalar content of a given model appear in Sec. III. At this stage we wish merely to make the point that a matrix with this structure can be related to those which reproduce the inverted and normal hierarchies in a simple fashion and that the motivation for the present work stems largely from this observation.

For the present study we shall not need to distinguish between the case where the horizontal symmetry communicates with a mirror sector or that when one includes the states N_R to remove the global anomaly. However, the constraints on the mass of the $SU_H(2)$ gauge bosons will differ in each of these cases and we shall comment on this in Sec. V.

III. SCALAR FIELDS REQUIRED FOR GENERAL LEPTON MASS MATRICES

In this section we obtain the set of scalar fields, forming representations of $\mathcal{G} = SU_L(2) \otimes U_Y(1) \otimes SU_H(2)$, required to produce the most general lepton mass matrices. The fermion content is

$$\Psi = (\psi_{\mu'}, \psi_{\tau'}) \sim (2, -1, 2), \qquad \psi_{e'} \sim (2, -1, 1),$$
$$E_R = (\mu'_R, \tau'_R) \sim (1, -2, 2), \qquad e'_R \sim (1, -2, 1), \quad (7)$$

with the primes indicating weak eigenstates. We have not included any right-chiral neutrino states; however, it should be remembered that one may include these states. As in the original KM model we assign the quarks to trivial representations of $SU_H(2)$. With the above lepton representations the most general charged leptons mass Lagrangian is

$$\mathcal{L}_{M_e} = g\bar{\Psi}\Phi e'_R + g_1\bar{\Psi}\phi v E_R^T + g_2\bar{\psi}_{e'}\phi e'_R$$
$$+ g_3\bar{\psi}_{e'}\Phi\epsilon E_R^T + g_4\bar{\Psi}\chi E_R^T + \text{H.c.}, \quad (8)$$

where ϵ is the two-dimensional antisymmetric tensor and the scalars form the representations

$$\phi \sim (2, 1, 1), \qquad \Phi \sim (2, 1, 2), \qquad \chi \sim (2, 1, 3). \quad (9)$$

We denote the scalar vacuum expectation values (VEV's) as

$$\langle \chi \rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} \frac{1}{\sqrt{2}}\langle\chi_2\rangle & \langle\chi_1\rangle \\ \langle\chi_3\rangle & -\frac{1}{\sqrt{2}}\langle\chi_2\rangle \end{pmatrix}, \quad (10)$$

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}, \qquad \langle \Phi \rangle = \begin{pmatrix} 0 & 0 \\ \kappa_1 & \kappa_2 \end{pmatrix}. \quad (11)$$

The mass Lagrangian becomes

$$\mathcal{L}_{M_e} = \bar{L}'_L M_l L'_R + \text{H.c.} + \dots, \quad (12)$$

where $L' = (e', \mu', \tau')^T$, the dots represent interaction terms, and

$$M_l = \begin{pmatrix} g_2 v & -g_3\kappa_2 & g_3\kappa_1 \\ g\kappa_1 & g_4\frac{\langle\chi_2\rangle}{\sqrt{2}} + g_1 v & g_4\langle\chi_1\rangle \\ g\kappa_2 & g_4\langle\chi_3\rangle & -g_4\frac{\langle\chi_2\rangle}{\sqrt{2}} + g_1 v \end{pmatrix}. \quad (13)$$

To simplify notation we denote this by

$$M_l = \begin{pmatrix} S_2 & D_3 & D_4 \\ D_1 & T_2 + S & T_1 \\ D_2 & T_3 & -T_2 + S \end{pmatrix}, \quad (14)$$

where the entries labeled S, D, and T arise from scalars forming singlet, doublet, and triplet representations of $SU_H(2)$.

The most general neutrino mass Lagrangian is

$$\mathcal{L}_{M_\nu} = f\bar{\Psi}^c\Delta_2\psi_{e'} + f_1\bar{\Psi}^c\Delta_3\Psi + f_2\bar{\psi}^c_{e'}\Delta_1\psi_{e'} + \text{H.c.}, \quad (15)$$

where

$$\Delta_1 \sim (3, 2, 1), \qquad \Delta_2 \sim (3, 2, 2), \qquad \Delta_3 \sim (3, 2, 3). \quad (16)$$

The charge conjugate fermion fields are defined as $\psi^c_{e'} = \epsilon C\psi^*_{e'}$ and $\Psi^c = C\epsilon\Psi^*\epsilon$ where C is the Dirac space charge conjugation matrix. Observe that the Lagrangian $\mathcal{L}_{M_\nu}$ does not contain a term of the type $\bar{\Psi}^c\Delta_1\Psi$. The scalars must acquire VEV's in the electrically neutral direction and we may denote these VEV's as $\langle\Delta^o_1\rangle, \langle\Delta^o_{21}\rangle, \langle\Delta^o_{22}\rangle, \langle\Delta^o_{31}\rangle$ etc.

The Lagrangian $\mathcal{L}_{M_\nu}$ may be rewritten as

$$\mathcal{L}_{M_\nu} = \bar{L}^c_{\nu'} M_\nu L_{\nu'} + \text{H.c.} + \dots, \quad (17)$$

where $L_{\nu'} = (\nu_{e'L}, \nu_{\mu'L}, \nu_{\tau'L})^T$ and

$$M_\nu = \begin{pmatrix} f_2\langle\Delta^o_1\rangle & -f\langle\Delta^o_{22}\rangle & f\langle\Delta^o_{21}\rangle \\ -f\langle\Delta^o_{22}\rangle & -f_1\langle\Delta^o_{31}\rangle & \frac{f_1}{\sqrt{2}}\langle\Delta^o_{32}\rangle \\ f\langle\Delta^o_{21}\rangle & \frac{f_1}{\sqrt{2}}\langle\Delta^o_{32}\rangle & f_1\langle\Delta^o_{33}\rangle \end{pmatrix}.$$

Again, to simplify notation we write

$$M_\nu = \begin{pmatrix} s_1 & d_1 & d_2 \\ d_1 & t_1 & t_2 \\ d_2 & t_2 & t_3 \end{pmatrix}, \quad (18)$$

with the entries labeled s, d, and t resulting from couplings

to $SU_L(2)$ triplet fields which form singlet, doublet and triplet representations of $SU_H(2)$, respectively.

The mass matrices (14) and (18) demonstrate the relationship between the structure of the lepton mass matrices and the scalar content in the class of models we are studying. In the next section we study the leading order PMNS matrices obtained under assumed hierarchies within the matrices (14) and (18).

IV. DETERMINING THE PMNS MIXING MATRIX

Certain assumptions regarding the hierarchy of elements within the lepton mass matrices have been made for the purpose of our analysis. We now discuss these assumptions.

We study the PMNS matrices obtained when one assumes hierarchies of the type $|d| \gg |s| \gg |t|$, $|T| \gg |S| \gg |D|$ etc., where we collectively label all entries resulting from couplings with $SU_H(2)$ singlets, doublets, and triplets in the neutrino (charged lepton) sector by s, d, and t (S, D, and T), respectively. Without an assumption of this type the mass matrices are very general and are unlikely to leave a signature of the horizontal symmetry. The choice of this type of hierarchy rests on the idea that the given leptons may uniformly couple more strongly/weakly to a given scalar representation than others, or that there is a hierarchy among the VEV's of the different representations. It may be unnecessary to include a given scalar in a realistic model if it is found that mass matrix entries arising from this particular scalar are not required to produce realistic mixing. Deviations from this type of hierarchy are considered when it is either physically motivated or presents no significant complication of the overall mixing matrix.

During our analysis we shall retain only two of the three distinctive scalar representations in each of the lepton sectors. The representations absent from the analysis may be considered to contribute subdominantly to a given mass matrix or may not be necessary in model construction. To simplify matters we shall take all mass matrix elements to be real.

The investigations are structured as follows. We begin by obtaining the leading form of the charged lepton mixing matrix U_l, where

$$U_l M_l M_l^\dagger U_l^\dagger = \mathrm{diag}(m_e^2, m_\mu^2, m_\tau^2).$$

The matrix U_l is determined for various assumed hierarchies of the elements of M_l. The neutrino mixing matrix U_ν, where

$$U_\nu M_\nu U_\nu^T = M_\nu^d = \mathrm{diag}(m_1, m_2, m_3),$$

is then determined under the assumed hierarchies of elements of M_ν. The various combinations of the matrices U_l and U_ν are then combined to determine the resulting PMNS matrix U, where $U = U_l U_\nu^\dagger$.

We now present an example analysis for the charged lepton and neutrino sectors. We shall present the details for the case which turns out to be most interesting. The results for the other cases can be found in Table I.

The most general charged lepton mass matrix is

$$M_l = \begin{pmatrix} S_2 & D_3 & D_4 \\ D_1 & T_2 + S & T_1 \\ D_2 & T_3 & -T_2 + S \end{pmatrix}. \qquad (19)$$

We shall refer to the $SU_L(2)$ doublet scalars ϕ, Φ, and χ, respectively, as the singlet, doublet, and triplet scalars. Consider the case when only the singlet and triplet scalars are retained. In this case the mass matrix is

$$M_l = \begin{pmatrix} S_2 & 0 & 0 \\ 0 & T_2 + S & T_1 \\ 0 & T_3 & -T_2 + S \end{pmatrix}. \qquad (20)$$

One cannot take $S \gg T_{1,2,3}$, S_2 as this produces $m_\mu \approx m_\tau$. The size of S_2 relative to $T_{1,2,3}$, S allows two possibilities. If one takes $S_2 \gg T_{1,2,3}$, S then

$$U_l^A = \begin{pmatrix} 0 & c_A & -s_A \\ 0 & s_A & c_A \\ 1 & 0 & 0 \end{pmatrix}, \qquad (21)$$

while for $T_{1,2,3} \gg S$, S_2 or $T_{1,2,3}$, $S \gg S_2$ one has

$$U_l^B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_B & -s_B \\ 0 & s_B & c_B \end{pmatrix}, \qquad (22)$$

where we have adopted the notation $c_X = \cos\theta_X$, $s_X = \sin\theta_X$, and

$$\tan 2\theta_A = \tan 2\theta_B = \frac{2\{T_3(T_2 + S) + T_1(S - T_2)\}}{T_3^2 - T_1^2 - 4T_2 S}. \qquad (23)$$

For the case of $T_{1,2,3} \gg S$, S_2 one also may make the approximation

$$\tan 2\theta_B \approx \frac{2T_2}{T_3 + T_1}. \qquad (24)$$

If one instead retains the triplet and doublet scalars the mass matrix is

$$M_l = \begin{pmatrix} 0 & D_3 & D_4 \\ D_1 & T_2 & T_1 \\ D_2 & T_3 & -T_2 \end{pmatrix}. \qquad (25)$$

Under the hierarchy $T \gg D$ one may write

$$M_l M_l^\dagger = \begin{pmatrix} 0 & 0 & 0 \\ 0 & T_1^2 + T_2^2 & T_2(T_3 - T_1) \\ 0 & T_2(T_3 - T_1) & T_2^2 + T_3^2 \end{pmatrix}$$
$$+ \begin{pmatrix} 0 & D_1 T_2 + D_4 T_1 & D_3 T_3 - D_4 T_2 \\ D_1 T_2 + D_4 T_1 & 0 & 0 \\ D_3 T_3 - D_4 T_2 & 0 & 0 \end{pmatrix}$$
$$+ O(D^2), \tag{26}$$

and treat the second matrix as a perturbation. The zeroth order mixing matrix is

$$U_l^F = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_F & -s_F \\ 0 & s_F & c_F \end{pmatrix}, \tag{27}$$

where

$$\tan 2\theta_F = \frac{2T_2}{T_3 + T_1}. \tag{28}$$

Note that θ_F is equal to $\theta_{A,B}$ in the limit $S \rightarrow 0$ and that U_l^F has the same form as U_l^B. When one includes the perturbation in (26), U_l^F will acquire some corrections. Ultimately, however, the gross structure of U_l in this case will be set by U_l^F. Thus the doublet-triplet case with $T \gg D$ is very similar to the singlet-triplet case with $T \gg S$. The singlet-triplet case with $T \gg S$ allows only 2–3 mixing in the charged lepton sector. The doublet provides more free parameters and allows subdominant mixing to occur between the 2–3 states and the first state.

We now consider an example of the neutrino mixing analysis. We shall now refer to Δ_1, Δ_2, and Δ_3 as the singlet, doublet, and triplet scalars, labelling them by their $SU_H(2)$ transformation properties. The most general mass matrix is

$$M_\nu = \begin{pmatrix} s_1 & d_1 & d_2 \\ d_1 & t_1 & t_2 \\ d_2 & t_2 & t_3 \end{pmatrix}, \tag{29}$$

and we investigate the dominant mixing that results from hierarchies between the elements labeled s, d, and t. As an example we consider the case $d \gg s$. Retaining the singlet and doublet scalars results in the neutrino mass matrix

$$M_\nu = \begin{pmatrix} s_1 & d_1 & d_2 \\ d_1 & 0 & 0 \\ d_2 & 0 & 0 \end{pmatrix}. \tag{30}$$

This matrix has two nonzero eigenvalues

$$\lambda_\pm = \frac{1}{2} \left(s_1 \pm \sqrt{s_1^2 + 4(d_1^2 + d_2^2)} \right), \tag{31}$$

and one zero eigenvalue. Upon defining

$$N_\pm = \frac{1}{\sqrt{d_1^2 + d_2^2 + \lambda_\pm^2}}, \tag{32}$$

one may write the mixing matrix as

$$U_\nu^b = \begin{pmatrix} N_+ \lambda_+ & N_+ d_1 & N_+ d_2 \\ N_- \lambda_- & N_- d_1 & N_- d_2 \\ 0 & \frac{-d_2}{\sqrt{d_1^2 + d_2^2}} & \frac{d_1}{\sqrt{d_1^2 + d_2^2}} \end{pmatrix}. \tag{33}$$

This matrix gives

$$U_\nu^b M_\nu U_\nu^{bT} = \text{diag}(\lambda_+, \lambda_-, 0). \tag{34}$$

If one neglects s_1, i.e. $d \gg s_1$, the nonzero eigenvalues reduce to $\lambda_\pm = \pm\sqrt{d_1^2 + d_2^2}$. In this limit the mass matrix (30) displays the family symmetry $L' = L_e - L_\mu - L_\tau$ discussed earlier. One may define

$$c_c(s_c) = \frac{d_1(d_2)}{\sqrt{d_1^2 + d_2^2}}, \tag{35}$$

and write the mixing matrix as

$$U_\nu^c = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{c_c}{\sqrt{2}} & \frac{s_c}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{c_c}{\sqrt{2}} & \frac{s_c}{\sqrt{2}} \\ 0 & -s_c & c_c \end{pmatrix}. \tag{36}$$

Observe that (36) displays the exact bimaximal form for $d_1 = d_2$.

A. The PMNS mixing matrix

We now present the main results of our study, namely, the PMNS mixing matrices $U = U_l U_\nu^\dagger$ emerging from the hierarchies assumed in the charged lepton and neutrino mass matrices.

The results are presented in Table I, with the columns of this table containing the following information: The first column gives the hierarchy assumed in the charged lepton mass matrix. The second column contains the leading order rotation of the left-chiral charged leptons, U_l. The third column contains the hierarchy assumed in the neutrino mass matrix. The leading order rotation for the neutrinos, U_ν, is presented in the fourth column and some important features of the resulting PMNS matrix are noted in the final column.

The features of a given mixing matrix pointed out in the final column fall into three categories.

 (i) If a given PMNS matrix possesses an entry which is zero or near unity in a location which disagrees with the experimentally observed value this is pointed out to rule out the matrix.

 (ii) When a PMNS matrix is not ruled out but makes no specific predictions we point this out. These matrices are effectively too general to produce any distinct signature of the horizontal symmetry.

(iii) Features of a given PMNS matrix which agree well with experiment are pointed out also. These include $U_{e3} = 0$ and predicted nonzero mixing angles.

When a rotation appears for the first time in the table its form is explicitly shown. We use distinct labels for the

mixing angles in the different cases. Many of these angles are not explicitly shown in the text; however, we wish to make it clear that these angles are in general distinct. The angles not shown correspond to less interesting scenarios and should their exact form be required they may readily be determined.

As noted earlier, the structure of the charged lepton rotations U_l^B and U_l^F are identical, meaning that the PMNS matrices which depend on either U_l^B or U_l^F have the same structure. We thus combine these cases together in Table I.

To discuss the PMNS matrices constructed in Table I we define $U^{Xy} = U_l^X U_\nu^{y\dagger}$. Of the 16 matrices in the table one may discount 12 due to the presence of leading order zero entries in undesirable locations. Two of the remaining PMNS matrices, $U^{Aa'}$ and $U^{Ea'}$, predict only $\theta_{12} \neq 0$ and

TABLE I. Mixing matrices emerging from the hierarchies assumed in the charged lepton and neutrino mass matrices.

Hierarchy in M_l	Charged lepton mixing U_l	Hierarchy in M_ν	Neutrino mixing $U_\nu^\dagger$	Comments on $U = U_l U_\nu^\dagger$
$S_2 \gg T, S$	$U_l^A = \begin{pmatrix} 0 & c_A & -s_A \\ 0 & s_A & c_A \\ 1 & 0 & 0 \end{pmatrix}$	$t \gg s; \; t \gg d$	$U_\nu^{a\dagger} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_a & -s_a \\ 0 & s_a & c_a \end{pmatrix}$	$U_{\tau 2} = U_{\tau 3} = U_{e1} = U_{\mu 1} = 0$
$S_2 \gg T, S$	U_l^A	$s \gg t$	$U_\nu^{a'\dagger} = \begin{pmatrix} 0 & 0 & 1 \\ c_a & s_a & 0 \\ -s_a & c_a & 0 \end{pmatrix}$	only $\theta_{12} \neq 0$, θ_{12} remains general
$S_2 \gg T, S$	U_l^A	$d \gg s; \; d \gg t$	$U_\nu^{c\dagger} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{c_c}{\sqrt{2}} & \frac{c_c}{\sqrt{2}} & -s_c \\ \frac{s_c}{\sqrt{2}} & \frac{s_c}{\sqrt{2}} & c_c \end{pmatrix}$	$U_{\tau 3} = 0$
$S_2 \gg T, S$	U_l^A	$s \gg d$	$U_\nu^{d\dagger} = \begin{pmatrix} 0 & -\frac{\sqrt{d_1^2+d_2^2}}{s_1} & 1 \\ -s_c & s_c & \frac{d_1}{s_1} \\ c_c & s_c & \frac{d_2}{s_1} \end{pmatrix}$	$U_{\tau 1} = 0$
$T \gg S, S_2; \; T, S \gg S_2,$ or $T \gg D$	$U_l^B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_B & -s_B \\ 0 & s_B & c_B \end{pmatrix}$ $U_l^F = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_F & -s_F \\ 0 & s_F & c_F \end{pmatrix}$	$t \gg s; \; t \gg d$	$U_\nu^{a\dagger}$	only $\theta_{23} \neq 0$, θ_{23} remains general
$T \gg S, S_2; \; T, S \gg S_2,$ or $T \gg D$	U_l^B or U_l^F	$s \gg t$	$U_\nu^{a'\dagger}$	$U_{e3} = 1$
$T \gg S, S_2; \; T, S \gg S_2,$ or $T \gg D$	U_l^B or U_l^F	$d \gg s; \; d \gg t$	$U_\nu^{c\dagger}$	$U_{e3} = 0 \; \tan\theta_{12} = 1$
$T \gg S, S_2; \; T, S \gg S_2,$ or $T \gg D$	U_l^B or U_l^F	$s \gg d$	$U_\nu^{d\dagger}$	$U_{e1} = U_{\mu 2} = U_{\mu 3} = U_{\tau 1} = 0 \quad U_{e3} = 1$
$D \gg S$ or $D \gg T,$ with $D_1^2 + D_2^2 \gg D_3^2 + D_4^2$	$U_l^D = \begin{pmatrix} 0 & -s_D & c_D \\ 1 & 0 & 0 \\ 0 & c_D & s_D \end{pmatrix}$	$t \gg s; \; t \gg d$	$U_\nu^{a\dagger}$	$U_{e1} = U_{\mu 2} = U_{\mu 3} = U_{\tau 1} = 0$
$D \gg S$ or $D \gg T,$ with $D_1^2 + D_2^2 \gg D_3^2 + D_4^2$	U_l^D	$s \gg t$	$U_\nu^{a'\dagger}$	$U_{\mu 1} = U_{\mu 2} = U_{e3} = U_{\tau 3} = 0$
$D \gg S$ or $D \gg T,$ with $D_1^2 + D_2^2 \gg D_3^2 + D_4^2$	U_l^D	$d \gg s; \; d \gg t$	$U_\nu^{c\dagger}$	$U_{\mu 3} = 0$
$D \gg S$ or $D \gg T,$ with $D_1^2 + D_2^2 \gg D_3^2 + D_4^2$	U_l^D	$s \gg d$	$U_\nu^{d\dagger}$	$U_{\mu 1} = 0$
$D \gg S$ or $D \gg T,$ with $D_3^2 + D_4^2 \gg D_1^2 + D_2^2$	$U_l^E = \begin{pmatrix} 0 & -s_E & c_E \\ 0 & c_E & s_E \\ 1 & 0 & 0 \end{pmatrix}$	$t \gg s; \; t \gg d$	$U_\nu^{a\dagger}$	$U_{e1} = U_{\mu 1} = U_{\tau 2} = U_{\tau 3} = 0$
$D \gg S$ or $D \gg T,$ with $D_3^2 + D_4^2 \gg D_1^2 + D_2^2$	U_l^E	$s \gg t$	$U_\nu^{a'\dagger}$	only $\theta_{12} \neq 0$, θ_{12} remains general
$D \gg S$ or $D \gg T,$ with $D_3^2 + D_4^2 \gg D_1^2 + D_2^2$	U_l^E	$d \gg s; \; d \gg t$	$U_\nu^{c\dagger}$	$U_{\tau 3} = 0$
$D \gg S$ or $D \gg T,$ with $D_3^2 + D_4^2 \gg D_1^2 + D_2^2$	U_l^E	$s \gg d$	$U_\nu^{d\dagger}$	$U_{\tau 1} = 0$

offer no insight into what the value of θ_{12} should be. Evidently, it is not desirable to consider the near maximal value of θ_{23} observed experimentally as the result of a perturbative correction to these matrices.

The matrices U^{Fa} and U^{Ba} contain θ_{23} as the only non-zero mixing angle. It is possible to construct realistic PMNS matrices as perturbations upon their existing structures. However, these matrices make no prediction as to what value θ_{23} should take and in the absence of an explicit model that generates VEV hierarchies among the VEV's t_{123} or T_{123} they have little predictive power.

The matrices U^{Fc} and U^{Bc} prove to be the most interesting. They possess the same leading order structure, each containing a zero in the $e3$ position. The neutrino mixing matrix U_ν^c results from a mass matrix of the form

$$M_\nu = \begin{pmatrix} 0 & d_1 & d_2 \\ d_1 & 0 & 0 \\ d_2 & 0 & 0 \end{pmatrix}. \qquad (37)$$

This displays the $L' = L_e - L_\mu - L_\tau$ symmetry, leading to an inverted hierarchy with $\Delta m_{12}^2 = 0$. Perturbations are required to produce a splitting of order $\Delta m_{12}^2 \sim 10^{-5}$ eV^2 and a nonmaximal solar mixing angle. These perturbations may come from the charged lepton mass matrix or the neutrino mass matrix.

Consider first the charged lepton sector. The matrices U^{Fc} and U^{Bc} occur when the triplet scalar entries dominate the charged lepton mass matrix,

$$M_l = \begin{pmatrix} 0 & 0 & 0 \\ 0 & T_2 & T_1 \\ 0 & T_3 & -T_2 \end{pmatrix}. \qquad (38)$$

If the singlet entries are included one obtains

$$M_l = \begin{pmatrix} S_2 & 0 & 0 \\ 0 & S+T_2 & T_1 \\ 0 & T_3 & S-T_2 \end{pmatrix}, \qquad (39)$$

so that $m_e = S_2$. This does not alter the solar mixing angle. If one instead includes the doublet entries in M_l,

$$M_l = \begin{pmatrix} 0 & D_3 & D_4 \\ D_1 & T_2 & T_1 \\ D_2 & T_3 & -T_2 \end{pmatrix}, \qquad (40)$$

one obtains $m_e \sim D$. A correction of order $1°$ to the solar mixing angle θ_{12} results, which is much too small to reach the experimentally favored value of $\theta_{12} \sim 34°$ [36].

One can consider the addition of both the singlet and doublet entries to the charged lepton mass matrix. However, the electron mass develops a dependence on both D and S and the mixing angles emerging from the charged lepton sector remain too small under our assumed hierarchies.

If $d_1 \sim d_2$ the neutrino mixing matrix has the bimaximal form. A previous work has studied the forms of the charged lepton mixing matrix which are compatible with the ex-

perimentally observed PMNS matrix values when one assumes bimaximal neutrino mixing [37]. The failure of the charged lepton mass matrices to adequately perturb the bimaximal form of the neutrino mixing matrix in the present analysis reflects the disagreement between the forms of M_l encompassed by the present study and those obtained in [37].

We now consider the addition of perturbations to the neutrino mass matrix. One can readily show that the addition of only s_1 to M_ν does not permit realistic values for Δm_{12}^2 and θ_{12} simultaneously. We shall consider the case where the singlet and triplet entries are included in the neutrino mass matrix. We are interested in the minimum additional number of scalar multiplets required to produce a realistic neutrino spectrum. However, the case where only the triplet entries are included will be readily revealed in the appropriate limit of this more general case. We shall work in the basis where the charged lepton mass matrix has been diagonalized by the matrix U_l^F. We parameterize the neutrino mass matrix as

$$M_\nu = m_0 \begin{pmatrix} 0 & c & -s \\ c & 0 & 0 \\ -s & 0 & 0 \end{pmatrix} + m_0 \begin{pmatrix} -w & 0 & 0 \\ 0 & -x & \frac{y}{\sqrt{2}} \\ 0 & \frac{y}{\sqrt{2}} & z \end{pmatrix}$$

$$\equiv m_0\{M_0 + M_1\}, \qquad (41)$$

where $c = \cos\theta_{23}$ and $s = \sin\theta_{23}$ will turn out to be the cosine and sine of the atmospheric mixing angle. The mass scale of the eigenvalues is set by $m_0 \sim d$ and the entries in M_1 labeled x, y, and z are of order t/m_0 while $w = s_1/m_0$. To first order in the small parameters x, y, z, and w the ratio of mass-squared differences is

$$\frac{\Delta m_{12}^2}{\Delta m_{23}^2} \approx \frac{2(2\kappa - w)}{1 - (2\kappa - w)}, \qquad (42)$$

where we have introduced

$$\kappa = \frac{1}{2}(s^2 z - c^2 x) - \frac{y}{2\sqrt{2}}\sin 2\theta_{23}, \qquad (43)$$

and it will prove useful to define

$$\xi = \frac{1}{2\sqrt{2}}(x+z)\sin 2\theta_{23} - \frac{y}{2}\cos 2\theta_{23}. \qquad (44)$$

The first order mixing matrix takes the form

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}} + f & \frac{1}{\sqrt{2}} - f & \sqrt{2}\xi \\ -\frac{c}{\sqrt{2}} + cf - s\kappa & \frac{c}{\sqrt{2}} + cf - s\kappa & s \\ \frac{s}{\sqrt{2}} - sf - c\kappa & -\frac{s}{\sqrt{2}} - sf - c\kappa & c \end{pmatrix},$$

$$(45)$$

where

$$f = \frac{1}{4\sqrt{2}}(2\kappa + w). \qquad (46)$$

The neutrino mixing angles are

$$\tan\theta_{13} \approx \sqrt{2}\xi, \tag{47}$$

$$\tan\theta_{23} \approx \frac{s}{c}, \tag{48}$$

$$\tan\theta_{12} \approx 1 - \frac{1}{2}(2\kappa + w). \tag{49}$$

Consider first the case where no singlet entry is present in the neutrino mass matrix, namely, the limit $w \to 0$ in M_1. One then has

$$\frac{\Delta m_{12}^2}{\Delta m_{23}^2} \approx \frac{2\kappa}{1 - 2\kappa}, \tag{50}$$

and

$$\tan\theta_{12} \approx 1 + \kappa. \tag{51}$$

The current allowed 3σ ranges for the mass-squared differences are [38]

$$7.1 \le \Delta m_{12}^2/(10^{-5} \text{ eV}^2) \le 8.9,$$
$$1.4 \le \Delta m_{23}^2/(10^{-3} \text{ eV}^2) \le 3.3, \tag{52}$$

and the mixing angles lie in the ranges

$$0.24 \le \sin^2\theta_{12} \le 0.4, \qquad 0.34 \le \sin^2\theta_{23} \le 0.68,$$
$$\sin^2\theta_{13} \le 0.047. \tag{53}$$

Equation (50) requires the single parameter κ to be small (of order 10^{-2}), thereby rendering the required deviation from maximal solar mixing unrealizable in concordance with Eq. (51).

When both the singlet and triplet entries are included as perturbations realistic values for both the ratio of mass-squared differences (42) and the solar mixing angle (49) can be obtained. This results from the fact that the solar mixing angle depends on the linear combination of parameters $(2\kappa + w)$ in (49) while the ratio of mass-squared differences in (42) depends on the linearly independent combination $(2\kappa - w)$. Assuming a maximal value for θ_{23} the experimental range of θ_{12} values are obtained for w and κ in the approximate ranges $|w| \approx [0.18, 0.44]$ and $|\kappa| \approx [0.09, 0.22]$.

This result is similar to that obtained in the work by Leontaris, Rizos, and Psallidas in Ref. [29] (and more recently [39]). In that work lepton mass matrices with leading order structures identical to (37) and (39) were obtained by augmenting the standard model with an anomalous $U(1)$ symmetry.

As the spectrum is inverted, with $m_3 = 0$ eV to zeroth order in the small parameters, one has

$$m_0 \simeq \sqrt{\Delta m_{23}^2},$$

giving the 3σ range

$$m_0/(10^{-2} \text{ eV}) \in [3.7, 5.7]. \tag{54}$$

The effective mass in neutrinoless double beta decay may be expressed as

$$|\langle m \rangle| = |\sum_i U_{ei}^2 m_i| = m_0 w, \tag{55}$$

where $i = 1, 2, 3$ labels the neutrino mass eigenstates. Using the allowed range for w and (54) one obtains

$$\langle m \rangle/(10^{-2} \text{ eV}) \in [0.7, 2.5], \tag{56}$$

which puts the effective neutrinoless double beta decay mass just below current experimental bounds (see e.g. [38]). Thus if the present framework is valid one is likely to observe neutrinoless double beta decay in the next generation of experiments, which will reach an accuracy of $\langle m \rangle \sim$ few $\times 10^{-2}$ eV [40,41]. The mixing matrix element $U_{e3} = \sqrt{2}\xi$ is not uniquely determined in the present framework. Taking the atmospheric mixing angle to be maximal gives $U_{e3} = (x + z)/2$, with the small parameters x and z not sufficiently constrained to allow a unique prediction.

B. Summary

We have found that a realistic lepton spectrum and PMNS matrix are obtained when couplings to the scalars χ and Δ_2 dominate the charged lepton and neutrino mass matrices, respectively. The required fine structure necessitates the inclusion of mass matrix entries resulting from couplings to Δ_1 and Δ_3 in the neutrino sector and either ϕ or Φ in the charged lepton sector. Thus a minimum of five (out of the possible six) scalar multiplets are necessary to reproduce the experimentally observed lepton sector within the present framework. The neutrinos are required to display an inverted hierarchy, with the leading order neutrino mass matrix displaying the nonstandard lepton number symmetry $L' = L_e - L_\mu - L_\tau$. Should the neutrino mass spectrum turn out to be inverted the horizontal symmetry $SU_H(2)$ may explain why nature displays the approximate symmetry L'.

V. SCALARS AND HORIZONTAL GAUGE BOSONS

We wish to make a few remarks regarding the scalar content required for a realistic model and the mass scale of the horizontal gauge bosons. Though it is beyond the scope of the present work to construct and analyze a specific scalar potential, a comment on the scale of the VEV's obtained by the $SU_L(2)$ triplet fields is in order.

It is known that $SU_L(2)$ triplet fields may acquire naturally light VEV's if the scalar potential contains a term linear in the given field [42]. In the case of the SM augmented to include one $SU_L(2)$ triplet field Δ, the field Δ acquires a VEV of order $\mu v^2/M_\Delta^2$, where v denotes the VEV of the SM Higgs field Φ', M_Δ is the mass of the field Δ, and μ denotes the coefficient of the scalar potential term linear in Δ, i.e. $\mu\Delta\Phi'\Phi' \subset V$, where V is the scalar

potential. Thus provided Δ is heavy enough its VEV will be suppressed relative to the electroweak scale.

Within the framework employed in the present study we have seen that a realistic neutrino sector requires three $SU_L(2)$ triplet fields, Δ_1, Δ_2, and Δ_3, which form singlet, fundamental, and triplet representations of $SU_H(2)$, respectively. Evidently we would like to ensure that the potential contains terms linear in these fields to allow them to develop VEV's below the electroweak scale. The minimal scalar content required to produce a realistic charged lepton spectrum consists of the $SU_L(2)$ doublet fields ϕ and χ, which form, respectively, singlet and triplet representations of $SU_H(2)$. With these five fields the potential will contain the terms

$$V \supset \Delta_1(\mu_{11}\phi\phi + \mu_{13}\chi\chi) + \Delta_3(\mu_{31}\phi\chi + \mu_{33}\chi\chi), \tag{57}$$

where the μ's are all dimension-full coupling constants. The above terms, being linear in Δ_1 and Δ_3, should allow these fields to naturally develop subelectroweak scale VEV's. Without the inclusion of the field Φ, which transforms as (2, 2) under $SU_L(2) \otimes SU_H(2)$, the potential does not contain a term linear in Δ_2. The addition of Φ to the scalar spectrum results in additional potential terms, including

$$\Delta_2\mu_{22}\Phi\chi, \tag{58}$$

which may allow Δ_2 to obtain a VEV suppressed relative to the electroweak scale. Although a realistic charged lepton spectrum does not require the field Φ, this field may be necessary to ensure the potential contains an appropriate term linear in Δ_2.

In the work of KM the $SU_H(2)$ symmetry breaking scale is related to the light neutrino masses via the seesaw mechanism, namely, $m_\nu \sim v^2/M_H$, where M_H denotes the scale of horizontal symmetry breaking and v is the electroweak scale. Assuming order one Yukawa couplings constrains M_H to be roughly of order 10^{14} GeV. Consequently the horizontal gauge bosons become unobservably heavy.

In the present work, if a $SU_H(2)$ doublet of right-chiral neutrinos is included to remove the global anomaly, the seesaw contribution to the mass of the light neutrinos is required to be subdominant. Thus the lower bound on the scale of horizontal symmetry breaking becomes even more severe. Note that one only requires $SU_H(2)$ to be broken to $U(1)$ to generate heavy Majorana masses and we have no *a priori* reason to break the $U(1)$ subgroup at the scale M_H. However, it may prove difficult to retain only mild hierarchies among the VEV's of a given scalar multiplet with a nonzero horizontal charge (e.g. among $\langle\chi_1\rangle$, $\langle\chi_2\rangle$, and $\langle\chi_3\rangle$) if this symmetry is broken at a vastly lower scale (see, for example, the discussion in [21]). It is beyond the scope of the present work to pursue this matter further.

The constraints on the horizontal symmetry breaking scale are much less severe if the horizontal symmetry communicates with a mirror sector. In this case the model need not include right-chiral neutrinos and no seesaw induced connection between the mass of the light neutrinos and the scale of horizontal symmetry breaking arises.

The horizontal gauge bosons do not couple directly to quarks so that many of the usual bounds on additional neutral gauge bosons do not apply. The horizontal gauge bosons couple to the charged leptons as follows:

$$-\mathcal{L} = g_H Z'_\mu\{\bar{L}_L\gamma^\mu U_1 L_L + \bar{L}_R\gamma^\mu V_1 L_R\}$$
$$+ \frac{g_H}{\sqrt{2}}Z'_\mu\{\bar{L}_L\gamma^\mu U_2 L_L + \bar{L}_R\gamma^\mu V_2 L_R\} + \text{H.c.},$$

where we use the label Z' generically to denote a horizontal gauge boson and g_H is the horizontal coupling constant. The horizontal charged lepton mixing matrices take the form

$$U_1 = U_l C_1 U_l^\dagger, \qquad U_2 = U_l C_2 U_l^\dagger,$$
$$V_1 = V_l C_1 V_l^\dagger, \qquad V_2 = V_l C_2 V_l^\dagger, \tag{59}$$

where U_l and V_l are the left- and right-chiral charged lepton mixing matrices, respectively,

$$U_l M_l M_l^\dagger U_l^\dagger = \text{diag}(m_e^2, m_\mu^2, m_\tau^2),$$
$$V_l^\dagger M_l^\dagger M_l V_l = \text{diag}(m_e^2, m_\mu^2, m_\tau^2), \tag{60}$$

and

$$C_1 = \text{diag}(0, 1, -1), \qquad C_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}. \tag{61}$$

The matrices $C_{1,2}$ reflect the fact that there is a basis in which only two generations of leptons couple to the horizontal bosons and are proportional to linear combinations of the Pauli matrices.

When the triplet entries dominate the charged lepton mass matrix one has $U_l = U_l^F$ and $V_l = P'U_l^{F\dagger}$ to leading order, with

$$P' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}. \tag{62}$$

In the absence of the field Φ the horizontal gauge bosons only couple to the muon and tauon generations of leptons. If the field Φ is included charged lepton mixing will couple the electron generation to the horizontal gauge bosons. Thus in principle the horizontal gauge bosons could be observed in e^+e^- collisions. However, the hierarchies assumed in the present study mean that the associated mixing angle will be of order $(m_e/m_\mu)^2$ or smaller. To see this, note that under the assumed hierarchies the horizontal mixing matrices will take the symbolic forms

$$U_1, V_1 \sim \begin{pmatrix} \epsilon^2 & \epsilon\delta & \epsilon\delta \\ \epsilon\delta & 1 & \epsilon\delta \\ \epsilon\delta & \epsilon\delta & -1 \end{pmatrix}, \tag{63}$$

and

$$U_2, V_2 \sim \begin{pmatrix} \epsilon^2 & \epsilon\delta & \epsilon\delta \\ \epsilon\delta & \delta^2 & \delta^2 \\ \epsilon\delta & \delta^2 & \delta^2 \end{pmatrix}, \tag{64}$$

where ϵ denotes (different) elements of order m_e/m_μ or smaller, $\delta \sim c_F, s_F$ and $\delta^2 \sim c_F^2, s_F^2, c_F s_F$. Thus the coupling of e^+e^- pairs to the horizontal gauge bosons are suppressed by a factor of $(m_e/m_\mu)^2 \sim 10^{-5}$. The flavor changing vertices coupling electrons and muons are suppressed by the larger factor of (m_e/m_μ). One can use the bound on the lepton number violating decay

$$\tau \to 2\mu + e \tag{65}$$

to constrain the coupling and mass of the horizontal bosons. If we follow [9] and compare the branching ratio for this process to the SM decay

$$\tau \to \mu + 2\nu, \tag{66}$$

we find [43]

$$\frac{g_H^4}{M_{Z'}^4} \le \frac{B(\tau \to 2\mu e)}{B(\tau \to \mu 2\nu)} \left(\frac{m_\mu}{m_e}\right)^2 \frac{g_W^4}{M_W^4}, \tag{67}$$

where g_W is the weak coupling constant, $M_{Z'}$ (M_W) is the horizontal (W) boson mass and we have taken $\delta \sim 1$. Using the current bound $B(\tau \to 2\mu e) < 1.5 \times 10^{-5}$ and $B(\tau \to \mu 2\nu) = 0.1736 \pm 0.0006$ [44] one obtains the weak bound

$$\frac{g_H^4}{M_{Z'}^4} < 3.7 \times \frac{g_W^4}{M_W^4}. \tag{68}$$

This demonstrates the difficulty in obtaining sensitive bounds on the horizontal bosons from processes involving electrons. The horizontal gauge bosons will give rise to additional tauon decay modes of the type $\tau \to \mu +$ invisible, namely,

$$\begin{aligned} \tau \to \mu + Z' &\to \mu + \nu_\mu + \nu_\tau, \\ \tau \to \mu + Z' &\to \mu + \nu'_\mu + \nu'_\tau. \end{aligned} \tag{69}$$

Here ν' denotes mirror neutrinos, recalling that the horizontal gauge bosons couple to the mirror sector [45]. One may obtain bounds on the horizontal bosons by demanding that

$$\begin{aligned} \Gamma_{\text{Extra}}(\tau \to \mu + \text{inv}) &\le \Gamma_{\text{Exp}}(\tau \to \mu + \text{inv}) \\ &- \Gamma_{\text{SM}}(\tau \to \mu + \text{inv}), \end{aligned} \tag{70}$$

where $\Gamma_{\text{Extra}}(\tau \to \mu + \text{inv})$ is the width for the new decays (69), $\Gamma_{\text{Exp}}(\tau \to \mu + \text{inv})$ is the experimentally measured width, and $\Gamma_{\text{SM}}(\tau \to \mu + \text{inv})$ is the SM value. Using the

measured values [44],

$$B(\tau \to \mu + \text{inv}) = 0.1736, \quad \tau = 290.6 \times 10^{-15} \text{ s}, \tag{71}$$

gives

$$\Gamma_{\text{Exp}}(\tau \to \mu + \text{inv}) = 3.93 \times 10^{-10} \text{ MeV}. \tag{72}$$

The SM value may be calculated as [46]

$$\Gamma_{\text{SM}}(\tau \to \mu + \text{inv}) = \frac{G_F^2 m_\tau^5}{192\pi^3} F\left(\frac{m_\mu^2}{m_\tau^2}\right) \times r_{\text{EW}}, \tag{73}$$

where G_F is the Fermi constant, $r_{\text{EW}} = 0.9960$ accounts for the electroweak propagator and radiative corrections [47], and

$$F(x) = 1 - 8x + 8x^3 - x^4 - 12x \ln x. \tag{74}$$

This gives

$$\Gamma_{\text{SM}}(\tau \to \mu + \text{inv}) = 3.90 \times 10^{-10} \text{ MeV}, \tag{75}$$

leading to the upper limit

$$\Gamma_{\text{Extra}}(\tau \to \mu + \text{inv}) \le 3.0 \times 10^{-12} \text{ MeV}. \tag{76}$$

This translates into the more restrictive bound

$$\frac{g_H^2}{M_{Z'}^2} \le 6 \times 10^{-2} \frac{g_W^2}{M_W^2}, \tag{77}$$

which gives $M_{Z'} \gtrsim 320$ GeV for $g_H = g_W$. This bound is low enough to allow the possible observation of horizontal gauge boson contributions to processes like $e^+e^- \to \mu^+\mu^-$ at the Next Linear Collider [48]. As the coupling of the horizontal bosons to $\mu^\pm$ is larger than the couplings to $e^\pm$, the horizontal bosons would create interesting phenomenology at a muon collider. Note also that the matrix V_l appears in the couplings between Z' and the charged leptons. This allows one to experimentally reconstruct the matrix V_l.

VI. CONCLUSION

We have performed a systematic analysis of the PMNS matrices which arise when the three generations of leptons are assigned to the $2 \oplus 1$ representation of the horizontal symmetry $SU_H(2)$, and the neutrino mass matrix results from leptonic couplings to $SU_L(2)$ triplet scalar fields. It was assumed that hierarchies existed among lepton mass matrix elements which result from couplings to scalar fields with different charges under $SU_H(2)$. Of the 16 candidate PMNS matrices which arose in our study it was found that only one was both predictive and possessed a leading order structure compatible with experimental observations. The relevant neutrino mass matrix displayed the symmetry $L_e - L_\mu - L_\tau$ to leading order and emerged when the contribution to the charged lepton mass matrix by a scalar forming a (2, 1, 3) representation of $SU_L(2) \otimes U_Y(1) \otimes SU_H(2)$ dominated and the (3, 2, 2) contributions

to the neutrino mass matrix dominated. This PMNS matrix predicted maximal solar mixing $U_{e3} = 0$ and left the atmospheric mixing angle unconstrained. It also required the neutrinos to display an inverted hierarchy. Perturbations to this leading order structure resulted from contributions to the neutrino mass matrix from additional scalar multiplets. Experimental data directly constrained the parameters which entered into the effective mass in neutrinoless double beta decay. This resulted in the prediction $\langle m \rangle / (10^{-2} \text{ eV}) \in [0.7, 2.5]$, which is just below current experimental bounds.

We also noted that the contribution of $SU_L(2)$ triplet scalar multiplets to the neutrino mass matrix can only dominate if the standard seesaw contribution is subdominant or not present. In the first case an $SU_H(2)$ doublet of right-chiral neutrinos may be included to remove the global anomaly as in the original work of KM. The subdominant nature of the seesaw contribution requires $SU_H(2)$ to be broken at an unobservably large scale. If there are no right-chiral neutrinos the global anomaly may be removed by allowing the horizontal symmetry to communicate with a mirror sector. In this case the scale of horizontal symmetry breaking may be much lower with the hope of observing resonance behavior of the horizontal gauge bosons at e^+e^- colliders operating at TeV energies.

ACKNOWLEDGMENTS

[1] T. Yanagida, Phys. Rev. D **20**, 2986 (1979).
[2] T. Maehara and T. Yanagida, Prog. Theor. Phys. **61**, 1434 (1979).
[3] T. Yanagida, Prog. Theor. Phys. **63**, 354 (1980).
[4] T. Yanagida, Phys. Rev. D **22**, 1826 (1980).
[5] T. Yanagida, Prog. Theor. Phys. **64**, 1103 (1980).
[6] F. Wilczek and A. Zee, Phys. Lett. **70B**, 418 (1977); **72B**, 504(E) (1978).
[7] F. Wilczek and A. Zee, Phys. Rev. Lett. **42**, 421 (1979).
[8] D. R. T. Jones, G. L. Kane, and J. P. Leveille, Nucl. Phys. **B198**, 45 (1982).
[9] D. S. Shaw and R. R. Volkas, Phys. Rev. D **47**, 241 (1993).
[10] A. Pomarol and D. Tommasini, Nucl. Phys. **B466**, 3 (1996).
[11] R. Barbieri, G. R. Dvali, and L. J. Hall, Phys. Lett. B **377**, 76 (1996).
[12] R. Barbieri and L. J. Hall, Nuovo Cimento Soc. Ital. Fis. A **110**, 1 (1997).
[13] D. Falcone, Phys. Rev. D **64**, 117302 (2001).
[14] A. Davidson, T. Schwartz, and R. R. Volkas, J. Phys. G **25**, 1571 (1999).
[15] U. Sarkar and A. K. Ray, Phys. Rev. D **29**, 166 (1984).
[16] A. K. Ray and U. Sarkar, Mod. Phys. Lett. A **16**, 2345 (2001).
[17] Z. Berezhiani and A. Rossi, Nucl. Phys. B, Proc. Suppl. **101**, 410 (2001).
[18] V. Antonelli, F. Caravaglios, R. Ferrari, and M. Picariello, Phys. Lett. B **549**, 325 (2002).
[19] G. D. Kribs, Phys. Rev. D **69**, 111701 (2004).
[20] O. C. Anoka, K. S. Babu, and I. Gogoladze, hep-ph/0512333.
[21] R. Kuchimanchi and R. N. Mohapatra, Phys. Rev. D **66**, 051301 (2002).
[22] R. Kuchimanchi and R. N. Mohapatra, Phys. Lett. B **552**, 198 (2003).
[23] B. Dutta and R. N. Mohapatra, Phys. Rev. D **68**, 056006 (2003).
[24] Z. Berezhiani, hep-ph/0508233.
[25] F. Vissani, hep-ph/9708483.
[26] V. D. Barger, S. Pakvasa, T. J. Weiler, and K. Whisnant, Phys. Lett. B **437**, 107 (1998).
[27] A. J. Baltz, A. S. Goldhaber, and M. Goldhaber, Phys. Rev. Lett. **81**, 5730 (1998).
[28] J. C. Gomez-Izquierdo and A. Perez-Lorenzana, hep-ph/0601223.
[29] S. T. Petcov, Phys. Lett. **110B**, 245 (1982); R. Barbieri, L. J. Hall, D. R. Smith, A. Strumia, and N. Weiner, J. High Energy Phys. 12 1998 (017); A. S. Joshipura and S. D. Rindani, Eur. Phys. J. C **14**, 85 (2000); R. N. Mohapatra, A. Perez-Lorenzana, and C. A. de Sousa Pires, Phys. Lett. B **474**, 355 (2000); Q. Shafi and Z. Tavartkiladze, Phys. Lett. B **482**, 145 (2000); T. Kitabayashi and M. Yasue, Phys. Rev. D **63**, 095002 (2001); A. Aranda, C. D. Carone, and P. Meade, Phys. Rev. D **65**, 013011 (2002); K. S. Babu and R. N. Mohapatra, Phys. Lett. B **532**, 77 (2002); H. J. He, D. A. Dicus, and J. N. Ng, Phys. Lett. B **536**, 83 (2002); H. S. Goh, R. N. Mohapatra, and S. P. Ng, Phys. Lett. B **542**, 116 (2002); G. K. Leontaris, J. Rizos, and A. Psallidas, Phys. Lett. B **597**, 182 (2004); P. H. Frampton, S. T. Petcov, and W. Rodejohann, Nucl. Phys. **B687**, 31 (2004); S. T. Petcov and W. Rodejohann, hep-ph/0409135; W. Grimus and L. Lavoura, J. Phys. G **31**, 683 (2005); G. Altarelli and R. Franceschini, hep-ph/0512202.
[30] P. Binetruy, S. Lavignac, S. T. Petcov, and P. Ramond, Nucl. Phys. **B496**, 3 (1997).
[31] K. S. Babu, E. Ma, and J. W. F. Valle, Phys. Lett. B **552**, 207 (2003).
[32] M. Hirsch, J. C. Romao, S. Skadhauge, J. W. F. Valle, and A. Villanova del Moral, Phys. Rev. D **69**, 093006 (2004).
[33] N. F. Bell and R. R. Volkas, Phys. Rev. D **63**, 013006 (2001).
[34] S. Choubey and W. Rodejohann, Eur. Phys. J. C **40**, 259 (2005).
[35] W. Rodejohann and M. A. Schmidt, hep-ph/0507300.
[36] S. Choubey and W. Rodejohann, Phys. Rev. D **72**, 033016 (2005).

[37] P. H. Frampton, S. T. Petcov, and W. Rodejohann, Nucl. Phys. **B687**, 31 (2004).

[38] J. W. F. Valle, in *Particles, Strings and Cosmology*, edited by K. Choi, AIP Conf. Proc. No. 805 (AIP, New York, 2006), p. 128.

[39] G. K. Leontaris, A. Psallidas, and N. D. Vlachos, hep-ph/0511327.

[40] S. M. Bilenky, C. Giunti, J. A. Grifols, and E. Masso, Phys. Rep. **379**, 69 (2003).

[41] S. M. Bilenky, hep-ph/0509098.

[42] E. Ma and U. Sarkar, Phys. Rev. Lett. **80**, 5716 (1998).

[43] Note that Eqs. (41) and (42) in Ref. [9], obtained using the same procedure as us, contain the masses and coupling constants raised to the second power. These should be raised to the fourth power.

[44] S. Eidelman *et al.* (Particle Data Group), Phys. Lett. B **592**, 1 (2004).

[45] The decay of Z' into mirror neutrinos need not occur if the mirror symmetry is spontaneously broken. To ensure that oscillations of the ordinary neutrinos into mirror neutrinos are at most subdominant on solar system length scales, one requires either very small mass-squared differences between the mirror parity eigenstates or a spontaneously broken mirror symmetry to generate small ordinary-mirror mixing angles.

[46] Y. S. Tsai, Phys. Rev. D **4**, 2821 (1971); **13**, 771(E) (1976).

[47] W. J. Marciano and A. Sirlin, Phys. Rev. Lett. **61**, 1815 (1988).

[48] The Next Linear Collider home page is located at http://www-project.slac.stanford.edu/lc.