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D-brane dynamics in a plane wave background

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By using the Dirac-Born-Infeld action we study the dynamics of Dp-brane propagating in the NS5-near-horizon plane wave background. We study systematically D-brane embedding in this pp-wave background, and analyze the equations of motion for various auxiliary fields. We further discuss the motion of the probe Dq-brane in the presence of source Dp-branes in this plane wave background.

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I. INTRODUCTION

Study of string theory in a time dependent background is a challenging topic which is believed to be able to answer questions in early universe cosmology. One of the most studied subjects in this direction is the decaying phenomena of the unstable D-brane in the presence of a tachyonic mode. The condensation of this tachyon leads to a more stable brane configuration or a complete annihilation (in case of brane-antibrane pair). In the conformal field theory language, it corresponds to studying the boundary conformal field theory of the D-brane by a marginal deformation. The recent proposal of Sen marks the spatially homogeneous decay of the unstable D-brane by a deformation of the open string world sheet by an exact marginal rolling tachyon background [1–3]. This process can also be realized by the localization of the S-brane in a timelike direction. In the study of the time dependent solutions in string theory, the recent observations reveal that the Dirac-Born-Infeld action captures, surprisingly well, many aspects of the decay of unstable D-branes [4–7]. More recently a geometric tachyon has been prosed in [8] that is the decay of the Dp-brane into the throat of a stack of NS5-branes. It has been observed that not only the decay process resembles that of the rolling tachyon of the open string models, but also one could learn even more far reaching consequences by studying in general the time dependent dynamics of the D-branes in curved backgrounds. Hence one attempts naturally to make more progress in the understanding of the physics of D-branes in curved backgrounds.

In the recent past string theory in plane-wave background [9] has also been a topic of intense discussion. String theory in this background has been shown to be exactly solvable in the light-cone gauge and it provides a perfect laboratory for testing the celebrated AdS/CFT duality beyond the supergravity regime. String theory in the pp-wave background that arises as the Penrose limit of certain near-horizon geometries is known to provide a holographic description of certain sectors of dual field theory [10]. Study of D-branes in this background is interesting and has been investigated by using various techniques in the past, see for example [11–17]. One of the interesting pp-backgrounds is obtained in the Penrose limit of the near-horizon region of a stack of NS5-branes (the linear dilaton background) [18]. It is one of the simplest pp-wave backgrounds with constant NS-NS flux and was shown to very closely resemble the flat space. D-brane solutions in this background have been studied in [19]. While the perturbative spectrum seems to be close enough to the flat space, the inclusion of nonperturbative objects like D-branes drastically changes the situation. For example, the space-time supersymmetry seems to be lost in the presence of D-branes in the pp-wave background of linear dilaton geometry.

The rest of the paper is organized as follows. In Sec. II we give a very brief review of the near-horizon geometry of the NS5-branes and the description of the Dp-brane in its pp-wave background. In Sec. III we describe the Dirac-Born-Infeld (DBI) action of the Dp-brane in general background, and the nature of the equations of motion. Then we study various embeddings of the D-branes in pp-wave background. In particular, we discuss two types of branes, namely, the "longitudinal" branes (both (u, v) directions along the brane world volume) and the "transversal" branes (with one of the light-cone directions (u) along the brane). We suppose that the world volume fields depend on u only and derive solutions for them. We further generalize the situation by turning on appropriate gauge fields on the world volume of the branes. In Sec. IV, we study the relative motion of the Dq-brane in the pp-wave background in the presence of other Dp-brane sources. We argue that how the embedding of branes changes the relative motion of various D-branes. We propose a particular kind of embedding where the brane motion resembles that of the flat space-time. In Sec. V we present our conclusions.

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II. GENERAL DISCUSSION OF Dp-BRANES IN THE NEAR-HORIZON LIMIT OF NS5-BRANES AND THEIR pp WAVE LIMIT

Nongravitational theory such as the little string theory (LST) is an interesting examples of nonlocal theory. This theory arises on the world volume of the NS5-brane when one takes the decoupling limit: $g_s \to 0$ with fixed α'. To learn about the high energy spectrum of this theory, which is not yet fully understood, one generally advocates in terms of the dual field theory language, namely, in terms of the string propagation in linear dilaton background. To probe the high energy regime, the Penrose limit of the space-time has been very useful and intuitive. Let us recall some basic facts about the linear dilaton background and its associated Penrose limit.

The string frame metric, 3-form H, and dilaton of the NS5-brane background are given by

$$ds^2 = -dt^2 + dy_5^2 + H(r)(dr^2 + r^2(d\theta^2 + \cos^2\theta d\psi^2 + \sin^2\theta d\phi^2))$$

$$H = N\epsilon_3, \qquad e^{2\phi} = g_s^2 H(r) \qquad H(r) = 1 + \frac{Nl_s^2}{r^2},$$

where ϵ_3 is the volume form on the transverse S_3, N is the NS5-brane charge, and $H(r)$ is the harmonic function in the transverse directions of the NS5-branes. The near-horizon geometry corresponds to the limit $r \to 0$ which removes the 1 in $H(r)$ and, on rescaling the time ($t = \sqrt{N} l_s \tilde{t}$), leads to the linear dilaton background

$$ds^2 = Nl_s^2\left(-d\tilde{t}^2 + \frac{dr^2}{r^2} + \cos^2\theta d\psi^2 + d\theta^2 + \sin^2\theta d\phi^2\right) + dy_5^2.$$

The Penrose limit is then taken with respect to the null geodesic along the equator ($\theta = 0$) of the transverse S^3 resulting in the following expressions for the metric and NS-NS 3-form [18],

$$ds^2 = 2dudv - \mu^2(z_1^2 + z_2^2)(du)^2 + \sum_{a=1}^{8} dz^a dz^a, \qquad B_{12} = 2\mu u. \tag{2.1}$$

To construct the Dp-brane in this particular background, one could in principle follow various methods. The one that was adopted in [19] was to write an ansatz for a particular Dp-brane solution in the NS5-brane near-horizon pp-wave background and then to solve the type IIB field equations of motion along with the Bianchi identities. The supersymmetry variations revealed the absence of any Killing spinors, and hence the solutions are nonsupersymmetric. But a careful analysis of the world sheet study revealed that the D-branes in the NS5-near-horizon pp-wave background preserve as much supersymmetry as the flat space. The contradiction was resolved by showing that all Fourier modes of the allowed supersymmetry parameters depend on the world sheet coordinate σ, while a local description in terms of the space-time variable is blind to the extension of the string.

Recently in the study of open string tachyon condensation, the Dirac-Born-Infeld action for D-branes has been very useful in understanding the intriguing aspects of underlying physics. Hence one wonders whether approaching the problem from this viewpoint helps us in understanding the Dp-branes more in the pp-wave background. We would like to stress that the aim of this paper is not to look for supersymmetric solutions in this plane wave background, rather to study the properties of already known D-brane solutions from an effective field theory point of view. We consider various brane embedding in this pp-wave background and study their dynamics.

Before going to the next section, where we discuss the DBI action for a probe brane in the NS5-pp wave background, we would like make a brief review of the Penrose limit on the probe. The effect of the Penrose limit on the dynamics of probe branes has been investigated in [20]. It was observed that the Penrose limit is essentially taking the large tension limit of the probe brane. Below we give an outline following [20] the analysis of the world volume dynamics of the Dp-brane in Penrose limit. Let us consider the Dp-brane in general background

$$I_p[g, B, C] = -\tau_p\left[\int d^{p+1}\sigma e^{-\Phi}\sqrt{-\det(g_{\mu\nu} + \mathcal{F}_{\mu\nu})} + \int e^{\mathcal{F}} \wedge C\right], \tag{2.2}$$

where

$$\tau_p = \frac{1}{(2\pi\alpha')^{(p+1)/2} k_p}, \tag{2.3}$$

where k_p is a constant that depends on string coupling constant $g_s = e^{-\Phi(\infty)}$. We also have

$$\mathcal{F}_{\mu\nu} = \alpha' F_{\mu\nu} + B_{\mu\nu}, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \qquad B_{\mu\nu} = B_{MN}\partial_\mu X^M \partial_\nu X^N, \qquad C = \sum C_{(k)} \tag{2.4}$$

Now we set $\alpha' = \Omega^2\alpha'$. The Dp-brane action is given by

$$I_p[\Omega^{-2}g, \Omega^{-2}B, \Omega^{-k}C] = \frac{1}{k_p(2\pi\alpha')^{(p+1)/2}}\left(\int d^{p+1}\sigma e^{\Phi}\sqrt{\Omega^{-2}(g + B) + \alpha' F} + \left[\sum_k e^{\Omega^{-2}B + \alpha' F} \wedge \Omega^{-k} C_k\right]\right). \tag{2.5}$$

Next step is to adopt the coordinates for the Penrose limit and take $\Omega \ll 1$, the Dp-brane action is expanded in the following way:

$$I_p[\Omega^{-2}g, \Omega^{-2}B, \Omega^{-k}C] = I_p[\bar{g}, \bar{B}, \bar{C}] + O(\Omega), \tag{2.6}$$

where the bar valued quantities are the fields in the Penrose limit. Hence one sees that the Dp-branes in the large tension limit propagate in the Penrose limit of the associated space-time.

III. Dp-BRANE PROBE IN NS5- NEAR-HORIZON pp-WAVE BACKGROUND

Now we come to the main objective of the present paper, namely, we start to study the dynamics of probe Dp-brane in the NS5-near-horizon plane wave background. Recall that the action for a Dp-brane in generic background has the form

$$S_p = -\tau_p \int d^{p+1}\sigma e^{-\Phi}\sqrt{-\det\mathbf{A}}, \qquad \mathbf{A}_{\mu\nu} = \gamma_{\mu\nu} + F_{\mu\nu}, \tag{3.1}$$

where τ_p is the Dp-brane tension, $\Phi(X)$ is dilaton, and $\gamma_{\mu\nu}$, $\mu, \nu = 0, \ldots, p$ is embedding of the metric to the world volume of the Dp-brane

$$\gamma_{\mu\nu} = g_{MN}\partial_\mu X^M \partial_\nu X^N, \qquad M, N = 0, \ldots, 9 \tag{3.2}$$

In (3.1) the form $F_{\mu\nu}$ is defined as

$$F_{\mu\nu} = b_{MN}\partial_\mu X^M \partial_\nu X^N + \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{3.3}$$

The equation of motion for X^K can be easily determined from (3.1) and take the form

$$\begin{aligned} &\partial_K[e^{-\Phi}]\sqrt{-\det\mathbf{A}} + \tfrac{1}{2}e^{-\Phi}(\partial_K g_{MN}\partial_\mu X^M \partial_\nu X^N \\ &+ \partial_K b_{MN}\partial_\mu X^M \partial_\nu X^N)(\mathbf{A}^{-1})^{\nu\mu}\sqrt{-\det\mathbf{A}} \\ &- \partial_\mu[e^{-\Phi}g_{KM}\partial_\nu X^M (\mathbf{A}^{-1})^{\nu\mu}_S\sqrt{-\det\mathbf{A}}] \\ &\quad - \partial_\mu[e^{-\Phi}b_{KM}\partial_\nu X^M (\mathbf{A}^{-1})^{\nu\mu}_A\sqrt{-\det\mathbf{A}}] = 0, \end{aligned} \tag{3.4}$$

where the symmetric and antisymmetric part, respectively, of the matrix $(\mathbf{A}^{-1})^{\mu\nu}$ are given by:

$$\begin{aligned} (\mathbf{A}^{-1})^{\nu\mu}_S &= \tfrac{1}{2}((\mathbf{A}^{-1})^{\nu\mu} + (\mathbf{A}^{-1})^{\mu\nu}), \\ (\mathbf{A}^{-1})^{\nu\mu}_A &= \tfrac{1}{2}((\mathbf{A}^{-1})^{\nu\mu} - (\mathbf{A}^{-1})^{\mu\nu}). \end{aligned} \tag{3.5}$$

Finally, we should also determine the equation of motion for the gauge field A_μ:

$$\partial_\nu[e^{-\Phi}(\mathbf{A}^{-1})^{\nu\mu}_A\sqrt{-\det\mathbf{A}}] = 0. \tag{3.6}$$

Now we are going to study the time dependent dynamics of the probe Dp-brane in the plane wave background of the linear dilaton geometry that, as discussed in the previous section, takes the form

$$ds^2 = 2dudv - \mu^2(z_1^2 + z_2^2)du^2 + \sum_{i=1}^{2} dz_i^2 + dx^2 + dy_5^2. \tag{3.7}$$

together with a nonzero NS–NS two form field

$$B_{12} = 2\mu u \tag{3.8}$$

Recall that u is the time coordinate. We will now solve the equations of motion for Dp-branes embedded in this background. It is clear that their properties will strongly depend on the embedding of these Dp-branes in the pp-wave background. Since we are interested in the time dependent case, we fix the world volume time coordinate σ^0 to be equal to the target space coordinate u. From the structure of the background metric it is natural to consider two cases. The first case corresponds to a Dp-brane that wraps the v direction as well and we denote this Dp-brane as $(u, v, p-1)$. This means that it is also extended in $p-1$ spatial dimensions including some dimensions from y^p. Since the metric obtained above corresponds to the massless geodesic moving at constant r it is natural to presume that the probe Dp-brane is located at some fixed x. We also presume that z^i coordinates are transverse to the world volume of the Dp-brane. The second case corresponds to the probe Dp-branes that are not extended in the v direction, but the u direction is along the world volume of the brane.

A. $(u, v, p-1)$-branes

In this case we propose the following gauge fixing:

$$u = \sigma^0 \equiv t, \qquad v = \sigma^p, \qquad \sigma^a = y^a, \qquad a = 1, \ldots, p-1. \tag{3.9}$$

Consequently the matrix $\mathbf{A}$ of Eq. (3.1) is given by

$$\mathbf{A} = \begin{pmatrix} -\mu^2 Z_i Z^i + \partial_0 Z_i \partial_0 Z^i + (\partial_0 X)^2 + \partial_0 Y^r \partial_0 Y_r & 0 & 1 \\ 0 & \delta_{ab} & 0 \\ 1 & 0 & 0 \end{pmatrix} \tag{3.10}$$

with $r, s = p, \ldots, 5$ and $i = 1, 2$. Now looking at the form of the metric and dilaton in the pp-wave limit we see that they are functions of z^i only and consequently the equations of motion for X and Y^r take the forms

$$-\partial_0[e^{-\Phi}g_{XX}\partial_0 X(\mathbf{A}^{-1})^{00}_S\sqrt{-\det\mathbf{A}}] = 0, \tag{3.11}$$

Since $(\mathbf{A}^{-1})^{00} = 0$ we see that the equation of motion are obeyed for any X. The same also holds for Y^r since

$$\partial_0[e^{-\Phi}g_{rr}\partial_0 Y^r(\mathbf{A}^{-1})_S^{00}\sqrt{-\det\mathbf{A}}] = 0 \tag{3.12}$$

We should also solve the equation of motion for U, V, and Y^p. Since once again the metric does not depend on these variables we get

$$\partial_0[\sqrt{-\det\mathbf{A}}] = 0 \tag{3.13}$$

that is again obeyed trivially. On the other hand the equations of motion for V and Z^i are obeyed automatically because of the vanishing $(\mathbf{A}^{-1})^{00}$.

In summary, we see that for the Dp-brane of the type $(u, v, p-1)$ the equations of motion do not restrict the possible form of these Dp-branes. Similar observations were made in [14] for Dp-branes in a maximally supersymmetric pp-wave background.

Before we proceed to the more general ansatz we consider the possibility of having world volume modes that depend on v only. Consequently the matrix $\mathbf{A}$ takes the form

$$\mathbf{A} = \begin{pmatrix} -\mu^2 Z_i Z^i & 0 & 1 \\ 0 & \delta_{ab} & 0 \\ 1 & 0 & g_{IJ}\partial_p X^I \partial_p X^J \end{pmatrix} \tag{3.14}$$

Then we get that the determinant is equal to

$$\det\mathbf{A} = -\mu^2 Z_i Z^i(g_{IJ}\partial_v X^I \partial_v X^J) - 1, \tag{3.15}$$

where we have used the gauge fixing $v = \sigma_p$.

As usual, the equations of motion for X and Y^r take the form

$$\begin{aligned} &\partial_v[g_{xx}\partial_v X(\mathbf{A}^{-1})_S^{vv}\sqrt{-\det\mathbf{A}}] = 0, \\ &\partial_v[g_{rs}\partial_v Y^s(\mathbf{A}^{-1})_S^{vv}\sqrt{-\det\mathbf{A}}] = 0, \end{aligned} \tag{3.16}$$

which can be solved with the ansatz $\partial_v X = \partial_v Y^s = 0$. On the other hand, the equation of motion for Z^i is equal to

$$\frac{\mu^2 Z_i(\partial_v Z^j \partial_v Z_j)}{\sqrt{-\det\mathbf{A}}} - \partial_v\left[\frac{\partial_v Z^i(\mu^2 Z^j Z_j)}{\sqrt{-\det\mathbf{A}}}\right] = 0 \tag{3.17}$$

Let us propose the ansatz for Z^1 and Z^2 in the following form

$$Z^1 = R\cos(kv), \qquad Z^2 = R\sin(kv) \tag{3.18}$$

which implies

$$\det\mathbf{A} = -1 - \mu^2 k^2 R^4 \tag{3.19}$$

and the equation above takes the form

$$\frac{\mu^2 Z_i k^2 R^2}{\sqrt{-\det\mathbf{A}}} - \partial_v^2 Z \frac{\mu^2 R^2}{\sqrt{-\det\mathbf{A}}} = 0 \tag{3.20}$$

which implies that $k = 0$ and, consequently, $Z^1 = R$, $Z^2 = 0$. On the other hand, let us presume that $Z^2 = 0$ (this solves the equation of motion for Z^2 but for $Z^1 = f(v)$. Then $\det\mathbf{A}$ is not a constant and we should check that the equations of motion for U and V are also obeyed. The equation of motion for U takes the form

$$\partial_v\left[(\mu^2 Z_1^2 - \mu^2 Z_1^2)\frac{1}{\sqrt{-\det\mathbf{A}}}\right] = 0 \tag{3.21}$$

and hence it is again obeyed. On the other hand, the equation of motion for V takes the form

$$\partial_v\left[\frac{1}{\sqrt{-\det\mathbf{A}}}\right] = 0 \tag{3.22}$$

which implies

$$\sqrt{-\det\mathbf{A}} = K, \tag{3.23}$$

where K is a constant. Now from this equation we get the differential equation for Z^1 in the form

$$Z_1 dZ_1 = \frac{\sqrt{K^2-1}}{\mu}dv \tag{3.24}$$

which has the solution

$$\frac{Z_1^2}{2} + C = \frac{\sqrt{K^2-1}}{\mu^2}v \tag{3.25}$$

with some constant C. On the other hand, the equation of motion for Z^1 in case of constant $\det\mathbf{A}$ takes the form

$$\mu^2 Z_1(\partial_v Z^1)^2 - \mu^2\partial_v^2 Z^1(\partial_v Z_1)^2 - 2\mu^2(\partial_v Z^1)^2\partial_v^2 Z^1 = 0. \tag{3.26}$$

If we insert the ansatz given above we get

$$\frac{\mu^2(K^2-1)}{Z} - 3\frac{\mu^2(K^2-1)^2}{Z^5} = 0 \tag{3.27}$$

and we see that the equation of motion is obeyed for $K = 1$ which also implies $\partial_v Z = 0$.

B. More general ansatz

In this section we generalize the solution given above to the case when some modes depend on u, v as well. First we eliminate some world volume fields where the metric explicitly does not depend on them. More precisely, the equation of motion for X takes the form

$$\partial_\mu[e^{-\Phi}g_{xx}\partial_\nu X(\mathbf{A}^{-1})^{\nu\mu}\sqrt{-\det\mathbf{A}}] = 0 \tag{3.28}$$

which can be solved with $\partial_\mu X = 0$. At the same time we will solve the equation of motion for Y^r with the same ansatz. In summary, X and Y^r will be considered as constant. In order to simplify calculation further we will restrict in this paper to the study of the $D1$ and $D3$-probes in the given background and we will study them separately.

C. (u, v)-$D1$ brane

We will consider a $D1$-brane that is extended in u, v directions. In this case the nonzero components of the matrix $\mathbf{A}$ takes the form

$$\begin{aligned}\mathbf{A}_{uu} &= -\mu^2 Z_i Z^i + (\partial_u Z^i)^2,\\ \mathbf{A}_{vv} &= (\partial_v Z^i)^2,\\ \mathbf{A}_{uv} &= 1 + \partial_u Z^i \partial_v Z_i + \partial_u A_v\\ &\quad + 2\mu u(\partial_u Z^1 \partial_v Z^2 - \partial_u Z^2 \partial_v Z^1),\\ \mathbf{A}_{vu} &= 1 + \partial_v Z^i \partial_u Z_i - \partial_u A_v\\ &\quad + 2\mu u(\partial_v Z^1 \partial_u Z^2 - \partial_v Z^2 \partial_u Z^1),\end{aligned} \tag{3.29}$$

where we presume that all free fields on the world volume of the $D1$-brane are constant.

We start to solve the equation of motion for U and V. The equation of motion for U takes the form

$$\partial_u[(\mu^2 Z_i Z^i (\mathbf{A}^{-1})_S^{uu} + 2(\mathbf{A}^{-1})_S^{vu})\sqrt{-\det\mathbf{A}}] - \partial_v[(\mu^2 Z_i Z^i (\mathbf{A}^{-1})_S^{uv} + 2(\mathbf{A}^{-1})_S^{vv})\sqrt{-\det\mathbf{A}}] = 0. \tag{3.30}$$

Similarly, the equation of motion for V takes the form

$$\partial_u[(\mathbf{A}^{-1})_S^{uu}\sqrt{-\det\mathbf{A}}] + \partial_v[(\mathbf{A}^{-1})_S^{uv}\sqrt{-\det\mathbf{A}}] = 0. \tag{3.31}$$

Then we get following equation of motion for Z^1

$$-\mu^2 Z^1 (\mathbf{A}^{-1})^{uu}\sqrt{-\det\mathbf{A}} - \partial_\mu[\partial_\nu Z^1 (\mathbf{A}^{-1})_S^{\nu\mu}\sqrt{-\det\mathbf{A}}] - \partial_\mu[b_{12}\partial_\nu Z^2 (\mathbf{A}^{-1})_A^{\nu\mu}\sqrt{-\det\mathbf{A}}] = 0 \tag{3.32}$$

and clearly the same for Z^2. Let us now propose the ansatz for Z^1 and Z^2 as follows

$$Z_1 = R\cos(k(u+v)), \qquad Z_2 = R\sin(k(u+v)). \tag{3.33}$$

Looking at the form of the equation given above we see that in the case of nonzero A_v they take very complicated forms thanks to the explicit time dependence of b_{12}. For that reason we restrict ourselves to the case when $\partial_\mu A_v = 0$. Then we get that $(\mathbf{A}^{-1})_A = 0$. Explicitly

$$\mathbf{A} = \begin{pmatrix} -\mu^2 R^2 + k^2 R^2 & 1 + k^2 R^2 \\ 1 + k^2 R^2 & k^2 R^2 \end{pmatrix} \tag{3.34}$$

and hence

$$\det\mathbf{A} = -1 - R^2 k^2 (2 + \mu^2 R^2). \tag{3.35}$$

Since now $(\mathbf{A}^{-1})$, $\det\mathbf{A}$, $Z_i Z^i$ are constant it is easy to see that the equations of motion (3.30) and (3.31) are trivially satisfied. On the other hand, the equation of motion for Z^i takes the form (using the fact that $\partial_\mu Z^1 = -kR\sin(k(u+v))$)

$$\frac{Z_1}{\sqrt{-\det\mathbf{A}}}(\mu^2 k^2 R^2 - 4k^2 + \mu^2 R^2 k^2) = 0. \tag{3.36}$$

This however implies that $k = 0$ and we get $Z^1 = R =$ const and $Z^2 = 0$.

As the next possibility we will consider the following ansatz for the Z^1 and Z^2

$$Z^1 = R\cos(k(u-v)), \qquad Z^2 = R\sin(k(u-v)) \tag{3.37}$$

which implies following form of the matrix $\mathbf{A}$

$$\mathbf{A} = \begin{pmatrix} -\mu^2 R^2 + R^2 k^2 & 1 - k^2 R^2 \\ 1 - k^2 R^2 & k^2 R^2 \end{pmatrix} \tag{3.38}$$

and hence

$$\det\mathbf{A} = -1 - k^2 R^2 (\mu^2 R^2 - 2). \tag{3.39}$$

Again, the nontrivial equation of motion corresponds to $Z^{1,2}$ and takes the form [using the fact that $\partial_u Z^1 = -kR\sin(k(u-v))$, $\partial_v Z^1 = kR\sin(k(u-v))$]

$$\frac{2Zk^2}{\sqrt{-\det\mathbf{A}}}k^2(1 - \mu^2 R^2) = 0. \tag{3.40}$$

Now we see that the above equation has two solutions. One with $k = 0$, and other with arbitrary k but with $R = \mu^{-1}$.

D. $(u, v, 2)$-$D3$ brane

We now consider a $D3$-brane where additional spatial components span y subspace. More precisely, we define the embedding of this $D3$-brane as

$$U = \sigma^0 = u, \qquad V = \sigma^3 = v, \qquad y^1 = \sigma^1, \qquad y^2 = \sigma^2. \tag{3.41}$$

Then the embedding coordinates are X, Y^r, $r = 3, 4, 5$, and Z^i, $i = 1, 2$. We also switch on the gauge field $A_{2,3}$ where, following results given in previous subsection, we take $A_v = 0$. Using $SO(2)$ symmetry in the subspace spanned by y^1, y^2we will switch on the component A_1 only and we will presume that this field depends on u only. Again the equations of motion for X, Y^r will be solved with the ansatz X, Y^r constant. As in the previous subsection we take the ansatz for Z^i modes as

$$Z^1 = R\cos(k(u+v)), \qquad Z^2 = R\sin(k(u+v)). \tag{3.42}$$

We again get

$$\mathbf{A} = \begin{pmatrix} -\mu^2 R^2 + k^2 R^2 & \partial_u A_1 & 0 & 1 + k^2 R^2 \\ -\partial_u A_1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 + k^2 R^2 & 0 & 0 & k^2 R^2 \end{pmatrix}. \tag{3.43}$$

Then the determinant takes the form

$$\det\mathbf{A} = -\mu^2 k^2 R^4 - 1 - 2k^2 R^2 + (\partial_u A_1)^2 k^2 R^2. \tag{3.44}$$

First we start to solve the equation of motion for A_μ. These equations take the form

$$\partial_\mu[(\mathbf{A}^{-1})_A^{\nu\mu}\sqrt{-\det\mathbf{A}}] = 0. \tag{3.45}$$

We see that all equations of motion are solved where $\mathbf{A}$ is constant. This implies that

$$\partial_u A_1 \equiv n = \text{const}. \tag{3.46}$$

Generally, n could take any real value however it is well known that it is proportional to the number of fundamental strings.

Now we have that the matrix $\mathbf{A}$ and its inverse is constant. This implies that we could proceed as in the previous section. However, we should check that the term containing the target space b_{12} in the equations of motion vanishes. In fact, this term is

$$\partial_\mu[b_{12}\partial_\nu(\mathbf{A}^{-1})_A^{\nu\mu}\sqrt{-\det\mathbf{A}}] \tag{3.47}$$

which for Z that depends on u, v gives

$$\partial_u[b_{12}(\partial_u Z^2(\mathbf{A}^{-1})_A^{uu} + \partial_v Z^2(\mathbf{A}^{-1})_A^{vu})\sqrt{-\det\mathbf{A}}] + \partial_v[b_{12}(\partial_u Z^2(\mathbf{A}^{-1})_A^{uv} + \partial_v Z(\mathbf{A}^{-1})_A^{vv})\sqrt{-\det\mathbf{A}}] = 0 \tag{3.48}$$

using the fact that $(\mathbf{A}^{-1})_A^{uu} = (\mathbf{A}^{-1})_A^{vv} = (\mathbf{A}^{-1})^{uv} = 0$. Then we can really proceed as in the previous subsection and the equation of motion for Z^1 gives

$$\frac{Z_1}{\sqrt{-\det\mathbf{A}}}(-2 + n^2 + \mu^2R^2)k^2 = 0. \tag{3.49}$$

so that we once again obtain the condition $k = 0$.

E. $(u, \varnothing, p)$-brane

Now let us discuss the case, when the Dp-branes do not wrap the v-direction. We call them $(u, \varnothing, p)$-branes. In this case, the static gauge has the form

$$U = \sigma^0 \equiv u, \qquad \sigma^a = y^a, \qquad a = 1, \ldots, p \tag{3.50}$$

and hence the embedding coordinates are

$$V, \qquad X, \qquad Z^i, \qquad Y^r, \qquad r = p + 1, \ldots, 5. \tag{3.51}$$

In this subsection we will consider the more general case when $p = 1$ and $p = 3$. Then the nonzero components of the matrix $\mathbf{A}$ are

$$\mathbf{A}_{uu} = -\mu^2 Z_i Z^i + 2\partial_u V + (\partial_u X)^2 + (\partial_u Z^i)^2 + (\partial_u Y^i), \quad \mathbf{A}_{ab} = \delta_{ab}. \tag{3.52}$$

Consequently we get

$$\det\mathbf{A} = -\mu^2 Z_i Z^i + 2\partial_u V + (\partial_u X)^2 + (\partial_u Z^i)^2 + (\partial_u Y^i)^2. \tag{3.53}$$

Again we start to solve the equation of motion. For $U = \sigma^0$ we get

$$\partial_u\left[\frac{(-\mu^2 Z_i Z^i + \partial_u V)}{\sqrt{\mu^2 Z_i Z^i - 2\partial_u V - (\partial_u X)^2 - \partial_u Z^i\partial_u Z_i - \partial_u Y^i\partial_u Y_i}}\right] = 0. \tag{3.54}$$

The equations of motion for $Y^a = \sigma^a$ imply

$$\partial_u[g_{ab}(\mathbf{A}^{-1})^{bu}\sqrt{-\det\mathbf{A}}] = 0 \tag{3.55}$$

using the fact that all modes depend on t only. The equations of motion for X imply

$$\partial_u\left[\frac{\partial_u X}{\sqrt{\mu^2 Z_i Z^i - 2\partial_u V - (\partial_u X)^2 - \partial_u Z^i\partial_u Z_i - \partial_u Y^i\partial_u Y_i}}\right] = 0 \tag{3.56}$$

which in turn specify the conserved momentum P_x:

$$\frac{\partial_u X}{\sqrt{\mu^2 Z_i Z^i - 2\partial_u V - (\partial_u X)^2 - \partial_u Z^i\partial_u Z_i - \partial_u Y^i\partial_u Y_i}} = P_x. \tag{3.57}$$

In the same way we obtain conserved momenta P_r

$$\frac{\partial_u Y^r}{\sqrt{\mu^2 Z_i Z^i - 2\partial_u V - (\partial_u X)^2 - \partial_u Z^i\partial_u Z_i - \partial_u Y^i\partial_u Y_i}} = P_r. \tag{3.58}$$

Finally, the equation of motion for V is equal to

$$\partial_u\left[\frac{1}{\sqrt{-\det\mathbf{A}}}\right] = 0. \tag{3.59}$$

In other words we get that $\det\mathbf{A} = \text{const}$. This also implies $\partial_u V = \text{const}$ and hence the Eq. (3.54) again implies $Z^i Z_i = \text{const}$. Finally, the equation of motion for Z_i is equal to

$$\partial_u^2 Z^i + \mu^2 Z_i^2 = 0 \tag{3.60}$$

which again has the solution

$$Z^1 = R_0 \sin\mu u, \qquad Z^2 = R_0 \cos\mu u. \tag{3.61}$$

In summary, we obtain following dependence of the world volume fields

$$X = V_x u + X_0, \qquad V = V_v u + V_0, \qquad Y^r = V_r u + Y_0^r \tag{3.62}$$

together with the time dependence of Z^i given in (3.61).

IV. *D*-BRANE MOTION

In this section, we would like to analyze the motion of the probe Dq-brane in the presence of another (or a stack of) Dp-brane in this particular pp-wave background. We assume that the dimension of the background brane is always greater than that of the probe. This is a good approximation when the mass of the higher dimensional brane is bigger than the mass of the lower dimensional brane. To perform the analysis we start by writing down the classical solution of a Dp-brane in a pp-wave background, and try to probe a Dq-brane. The metric, dilaton, and the NS-NS 3-form flux of such a configuration is given by [19]

$$\begin{aligned} ds^2 &= H_p^{-1/2}(x^a)\Big[2dudv - \mu^2\sum_{i=1}^{2} z_i^2 du^2 + \sum_{\alpha=1}^{p-1}(dx^\alpha)^2\Big] \\ &\quad + H_p^{1/2}(x^a)\Big[\sum_{i=1}^{2}(dz^i)^2 + \sum_{a=p+3}^{8}(dx^a)^2\Big], \\ B_{12} &= 2\mu u, \\ A^{(p+1)} &= \pm\Big(\frac{1}{H(x^a)} - 1\Big)du\wedge dv\wedge dx^1\wedge dx^2\wedge\ldots \\ &\quad \wedge dx^{p-1} e^{2\phi} = H_p(x^a)^{(3-p)/2}, \\ H_p &= 1 + Ng_s\Big(\frac{l_s}{r}\Big)^{7-p} \equiv 1 + \frac{\lambda_p}{r^{7-p}}, \end{aligned} \tag{4.1}$$

where

$$r^2 = \sum_{i=1}^{2} z_i z^i + \sum_{a=p+3}^{8} x_a x^a. \tag{4.2}$$

The harmonic function written in the last line for $p = 7$, is given by

$$H_7 = 1 - Ng_s \log(r/l). \tag{4.3}$$

We will study the motion of the probe Dq-brane, where $q < p$, $p - q = 2k$, $k = 0, 1, 2$ in this background. For simplicity we consider the case when all the gauge fields are set to zero. In general the action for such a Dq-brane probe is given by

$$S_= -\tau_q\int d^{q+1}\sigma e^{-\Phi}\sqrt{-\det\mathbf{A}} + S_{WZ} \tag{4.4}$$

where

$$\begin{aligned} &\mathbf{A}_{\mu\nu} = g_{\mu\nu} + F_{\mu\nu}, \qquad \gamma_{\mu\nu} = g_{MN}\partial_\mu X^M\partial_\nu X^N, \\ &M, N = 0,\ldots,9, \qquad F_{\mu\nu} = b_{MN}\partial_\mu X^M\partial_\nu X^N. \end{aligned} \tag{4.5}$$

We propose the static gauge in the form

$$\sigma^0 = u, \qquad \sigma^q = v, \qquad \sigma^i = x^i, \qquad i = 1,\ldots,q-1 \tag{4.6}$$

and try to see whether they solve the appropriate equations of motion. Generally we presume that the world volume fields depend on u, v only. Then the nonzero components of the matrix $\mathbf{A}$ are

$$\begin{aligned} \mathbf{A}_{00} &= -H_p^{-1/2}(\mu^2 z_i z^i + \partial_0 Y^r\partial_0 Y_r) \\ &\quad + H^{1/2}(\partial_0 Z_i\partial_0 Z^i + \partial_0 X^a\partial_0 X_a) \\ \mathbf{A}_{0q} &= H^{-1/2} + H^{-1/2}\partial_0 Y^r\partial_q Y_r \\ &\quad + H^{1/2}(\partial_0 Z^i\partial_q Z_i + \partial_0 X^a\partial_q X_a) \\ &\quad + \mu U(\partial_0 Z^1\partial_q Z^2 - \partial_0 Z^2\partial_q Z^2) \\ \mathbf{A}_{q0} &= H^{-1/2} + H^{-1/2}\partial_0 Y^r\partial_q Y_r \\ &\quad + H^{1/2}(\partial_q Z^i\partial_0 Z_i + \partial_0 X^a\partial_q X_a) \\ &\quad - \mu U(\partial_0 Z^1\partial_q Z^2 - \partial_0 Z^2\partial_q Z^2), \\ \mathbf{A}_{ij} &= g_{ij} = H^{-1/2}\delta_{ij}, \\ \mathbf{A}_{qq} &= H^{-1/2}\partial_q Y^r\partial_q Y_r + H^{1/2}(\partial_q Z^i\partial_q Z_i + \partial_q X^a\partial_q X_a), \end{aligned} \tag{4.7}$$

where $r, s = q,\ldots,p-1$. Since the metric does not depend on Y^r then the equations of motion for them take the form

$$\partial_\mu[e^{-\Phi}g_{rs}\partial_\nu Y^s(\mathbf{A}^{-1})_S^{\nu\mu}\sqrt{-\det\mathbf{A}}] = 0 \tag{4.8}$$

which can be solved with $Y^r = \text{const}$. The problem seems rather complicated thanks to the dependence of the metric on x, z. Since we are interested in the properties of the Dq-brane as a probe it is natural to restrict to the dependence of all modes on σ^0 only. In this case the nonzero components of the matrix $\mathbf{A}$ are

$$\begin{aligned} \mathbf{A}_{00} &= -H_p^{-1/2}(\mu^2 z_i z^i + \partial_0 Y^r\partial_0 Y_r) + H^{1/2}(\partial_0 Z_i\partial_0 Z^i \\ &\quad + \partial_0 X^a\partial_0 X_a), \\ \mathbf{A}_{0q} &= H^{-1/2} = \mathbf{A}_{q0}, \qquad \mathbf{A}_{ij} = H^{-1/2}\delta_{ij}, \qquad \mathbf{A}_{qq} = 0. \end{aligned} \tag{4.9}$$

This, however, implies that $\det\mathbf{A} = -H_p^{-(q+1)/2}$. Let us start to solve the equation of motion for U that is give by

$$\partial_0[H_p^{(p-q-4)/4}] = 0. \tag{4.10}$$

This result is trivially satisfied for $p - q = 4$ while for $p - q = 2$ this is obeyed for $\partial_0 X^a = \partial_0 Z^i = 0$.

As the next step we consider the equation of motion for V which for our ansatz takes the form

$$\partial_0[e^{-\Phi}g_{VU}(\mathbf{A}^{-1})^{00}\sqrt{-\det\mathbf{A}}] = 0 \tag{4.11}$$

which is satisfied since $(\mathbf{A}^{-1})^{00} = 0$. Finally, the equation of motion for X^a takes the form

$$\begin{aligned} &\partial_{X^a}e^{-\Phi}\sqrt{-\det\mathbf{A}} + \frac{1}{2}\partial_{X^a}g_{MN}\partial_\mu X^M\partial_\nu X^N(\mathbf{A}^{-1})^{\nu\mu}\sqrt{-\det\mathbf{A}} \\ &\quad = \lambda_p(p-7)\frac{X^a}{R^{9-p}}H_p^{(p-q-8)/4}(p+q-2) = 0 \end{aligned} \tag{4.12}$$

using the fact that $(\mathbf{A}^{-1})^{00} = 0$ and also

$$\partial_{X^a} g_{MN} \partial_\mu X^M \partial_\nu X^N (\mathbf{A}^{-1})^{\nu\mu} = \frac{1}{2H^{3/2}} \frac{\delta H_p}{\delta X^a}(q+1) \times \frac{H_p^{-q/2}}{\sqrt{-\det \mathbf{A}}}, \tag{4.13}$$
$$\partial_{X^a} e^{-\Phi} = \frac{p-3}{4} H_p^{(p-7)/4} \frac{\delta H_p}{\delta X^a}.$$

It is clear that the same equation of motion holds for Z^i as well thanks to the fact that the variation $\frac{\delta g_{UU}}{\delta Z^i}$ is proportional to $(\mathbf{A}^{-1})^{00}$ in the equation of motion and this vanishes. Let us then concentrate on the equation above and discuss its properties for various values of p and q and for limits $X^a \to 0$ or $X^a \to \infty$. Generally, for $X^a \to 0$ (for $Z^i = 0$, $X^b = 0$, $b \neq a$) we have

$$\lim_{X^a\to 0} \frac{X^a}{R^{9-p}} H_p^{(p-q-8)/4} \sim \lim_{X^a \to 0} (X^a)^{(1/4)[(p-8)(p-3)-q(p-7)]}. \tag{4.14}$$

On the other hand, the limit $X^a \to \infty$ gives

$$\lim_{X^a\to \infty} \frac{X^a}{R^{9-p}} H_p^{(p-q-8)/4} \sim \lim_{X^a \to \infty} \frac{1}{(X^a)^{9-p}} \tag{4.15}$$

which goes to zero for all $p < 7$. Let us discuss the situations case by case.

(i) $p = 6$
In this case $q = 2, 4$ since by presumption the Dq-brane wraps the u, v directions and, hence has to be at least two dimensional. For $q = 2, 4$ we get that (4.14) blows up so that the point $X^a = 0$ cannot be a solution of the equation of motion. Then the only possibility is to consider the configuration where all $X^a \to \infty$. In other words the Dq-brane cannot form a bound state with the $D6$-brane. This result can be compared with the analysis performed in [21] where it was also shown that in the near-horizon region of $D6$-brane the potential diverges.

(ii) $p = 5$
Now $q = 3, 1$. Then the exponent on X^a is equal to $\frac{-3+q}{2}$ and hence we again get the expression (4.14) diverges for $q = 1$ while it is constant for $q = 3$. In any case the $D3$-brane or $D1$-brane cannot approach the $D5$-brane in this particular configuration.

(iii) $p = 4$
Now in the limit $X^a \to 0$, we have $(X^a)^{(-4+3q)/4}$, which vanishes for $q = 2$. Hence the $D2$-brane can approach the $D4$-brane.

(iv) $p = 3$
In the limit $X^a \to 0$, one has $(X^a)^q$, which for $q = 1$ goes to zero and hence the $D1$-brane can approach the $D3$-brane.

As the final example we will study the probe Dp-brane in the background of the Dp-brane in the NS5-brane pp wave. In this case we should take the Wess-Zumino term (WZ) into account which takes the form

$$S_{\mathrm{WZ}} = -\tau_p \int A^{(p+1)} = -q_p \tau_1 \int d^{p+1}\sigma \left(\frac{1}{H_p} - 1\right), \tag{4.16}$$

where q_p takes values ± 1 according to the case whether the Dp-brane probe corresponds to the Dp-brane or anti-Dp-brane. The presence of this WZ term only changes the equations of motion for X^a and Z^i and we get

$$\lambda_p(p-7)\frac{X^a}{R^{9-p}H_p^2}(2p-2) - \lambda_p(p-7)\frac{q_p X^a}{R^{9-p}H_p^2} = \lambda_p(p-7)\frac{X^a}{R^{9-p}H_p^2}(2p-2-q_p) = 0 \tag{4.17}$$

Then for $X^a \to 0$ the upper expression is proportional to $(X^a)^{6-p}$ which goes to zero for $p < 6$ and hence the only solution of the equation of motion corresponds to $X^a = Z^i = 0$.

Alternative embedding

In this section we mention a possibility of an alternative embedding of the probe Dq-brane and examine the motion of the probe brane described in the previous section. Once again, we start with the action (4.4). However, we propose the static gauge in the following form

$$U = \sigma^0 + \sigma^q, \qquad V = \sigma^0 - \sigma^q, \qquad \sigma^i = X^i, \qquad i = 1, \ldots, q-1 \tag{4.18}$$

and try to see whether they solve the appropriate equations of motion. We will also presume that the world volume fields depend on σ^0 only. Then the matrix $\mathbf{A}$ takes the form

$$\begin{aligned} \mathbf{A}_{00} &= -H_p^{-1/2}(\mu^2 Z_i Z^i + 2 + \partial_0 Y^r \partial_0 Y_r) + H_2^{1/2}((\partial_0 Z^i)^2 + (\partial_0 X^a)^2), \\ \mathbf{A}_{0q} &= \mathbf{A}_{q0} = -H_p^{-1/2}\mu^2 Z_i Z^i, \\ \mathbf{A}_{ij} &= H^{-1/2}\delta_{ij}, \\ \mathbf{A}_{qq} &= -H_p^{-1/2}\mu^2 Z_i Z^i - 2H_p^{-1/2} \end{aligned} \tag{4.19}$$

so that the determinant is equal to

$$\det \mathbf{A} = -H_p^{-(q+1)/2}[4 + (\mu^2 Z_i Z^i + 2)((\partial_0 Y^r)^2 + H_p(\partial_0 X^a)^2 + H_p(\partial_0 Z^i)^2)]. \tag{4.20}$$

We again start with the equation of motion for U which now takes the form

$$-4\partial_0\left[\frac{H_p^{(p-q-4)/4}(\mu^2 Z_i Z^i + 2)}{\sqrt{4 + (\mu^2 Z_i Z^i + 2)((\partial_0 Y^r)^2 + H_p(\partial_0 X^a)^2 + H_p(\partial_0 Z^i)^2)}}\right] = 0. \tag{4.21}$$

This equation has the form of the energy conservation equation and in fact it can be interpreted as a conservation of the world volume energy [22]. We will use this equation later. The equation of motion for V takes the form

$$\partial_0\left[\frac{H_p^{(p-q-4)/4}}{\sqrt{4 + (\mu^2 Z_i Z^i + 2)((\partial_0 Y^r)^2 + H_p(\partial_0 X^a)^2 + H_p(\partial_0 Z^i)^2)}}\right] = 0. \tag{4.22}$$

Now comparing the above two equations we get the condition

$$\partial_0[\mu^2 Z_i Z^i + 2] = 0. \tag{4.23}$$

To find the solution of the above differential equation, let us consider the equation of motion for Z^i. In this case the situation is more complicated since we have a dilaton and metric component that are Z^i dependent. However, since the variation of g_{uu} with respect to Z^i contains a linear term Z^i, we will solve the equation of motion for Z^i in terms of $Z^i = 0$, which is clearly a solution of the equation of motion.

Let us now consider the equation of motion for Y^r that takes the form

$$\partial_0[e^{-\Phi} g_{rs}\partial_0 Y^r(\mathbf{A}^{-1})^{00}\sqrt{-\det\mathbf{A}}] = 0. \tag{4.24}$$

The solution of this equation of motion is given by the constant expression under the right bracket that corresponds to the constant momenta conjugate to Y^r. If we could in principle express $\partial_0 Y^r$ with this momenta and insert it into the square root we will consider the simpler case when $P_r = 0$ in order not to complicate the expressions further.

Now we come to the analysis of the dynamics of the modes X^a. Using the manifest $SO(9-p)$ invariance of the space R^{9-p} transverse to the background Dp-brane we can restrict ourselves to the study of the dynamics in the two-dimensional plane, say x^7, x^8, where we introduce coordinates

$$X^7 = R\cos\theta, \qquad X^8 = R\sin\theta. \tag{4.25}$$

Now we could again in principle solve the equation of motion for R, θ directly, however we would rather use the equation of motion for U that has the form of the equation of the conservation of the energy. In other word, this equation implies

$$\frac{H_p^{(p-q-4)/4}}{\sqrt{4 + 2(H_p(\dot{R}^2 + R^2\dot{\theta}^2)}} = \frac{E}{\tau_q}, \tag{4.26}$$

where $\dot{(\ldots)} = \partial_0(\ldots)$. Again, since the action does not depend explicitly on θ it turns out that the momentum conjugate to P_θ is a constant. For simplicity we restrict ourselves to the case of $P_\theta = 0$. Now the equation above implies

$$\frac{\dot{R}^2}{2} + H_p^{-1} - \frac{\tau_q^2}{4E^2}H_p^{(p-q-4)/2-1} = 0, \tag{4.27}$$

which corresponds to the motion of the particle with zero energy in the potential of the form

$$V(R, E) = H_p^{-1} - \frac{\tau_q^2}{4E^2}H_p^{(p-q-4)/2-1}. \tag{4.28}$$

Now let us try to solve the above equation. However, we can see that solving the equation above in full generality is very hard. What we can do of course is to take the following two simple possibilities. $q = p-2 \Rightarrow p-q = 2$ or $q = p-4 \Rightarrow p-q = 4$ [the Bogomol'nyi-Prasad-Sommerfield bound (BPS) case]. In the second case the differential equation above takes the very simple form

$$dRR^{(p-7)/2} = \pm\left(\sqrt{\frac{\tau_q^2}{2E^2} - 2}\sqrt{\lambda_p}\right)d\sigma^0, \tag{4.29}$$

where we have taken the near-horizon approximation where $\frac{\lambda_p}{R^{7-p}} \gg 1$. Now Eq. (4.29) can be easily solved with the following (for $p \neq 5$)

$$R^{(p-5)/2} = \frac{p-5}{2}\left(\left(\sqrt{\lambda_p}\sqrt{\frac{\tau_q^2}{2E^2} - 2}\right)\sigma^0 + C_0\right). \tag{4.30}$$

From this result we see that for $p = 6$, the $D2$-brane cannot reach the world volume of the $D6$-brane in the same way as in flat space-time. For $p = 5$, however, the equation above has the solution

$$R = R_0\exp\left(-\left(\sqrt{\lambda_p}\sqrt{\frac{\tau_q^2}{2E^2} - 2}\right)\sigma^0\right), \tag{4.31}$$

where we have chosen the $-$ sign in the exponential function to find the solution that describes the $D1$-brane that approaches the $D5$-brane. Finally, we cannot consider $p < 5$ in this case since we then have $q < 1$. However, we have presumed that the Dq-brane is two dimensional at least.

Let us now consider the first case when $p - q = 2$. Then the Eq. (4.27) implies the following bound on R

$$R^{7-p} > \frac{\lambda_p}{\frac{\tau_q^2}{4E^2} - 1}. \tag{4.32}$$

In other words the $D(p\text{-}2)$-brane that approaches the Dp-brane from $r = \infty$ can reach the minimal distance $R_{\min}^{7-p} = \frac{\lambda_p}{\tau_q^2/4E^2-1}$. In other words the Dq-brane that moves only radially cannot reach the world volume of the Dp-brane.

V. CONCLUSION

In this paper we have discussed the dynamics of the Dp-branes in the NS5-near-horizon pp-wave background. We consider various embedding of the D-branes in this background to understand the relevant physics out of it. We consider the longitudinal and the transversal branes and study their dynamics. We observe that for the branes which wrap both u, v directions, the equations of motion derived from the DBI action do not restrict the possible form of the D-branes. We further analyze the properties of the solutions by turning on additional world volume gauge fields. Finally we study the motion of a probe Dq-brane in the presence of source Dp-branes in this plane wave background. By assuming that the background branes are heavier than the probe, we solve the equations of motion derived from the DBI action of such probe. Once again we have begun with the assumption that the world volume fields depend on both u and v. We explain how the gauge fixing plays a crucial role in the study of the brane motion. By taking various examples, we have also shown the interesting trajectory of the probe branes falling into source branes in the pp-wave background of the linear dilaton geometry. In a particular gauge fixing, we solve the time dependent equation of motion of the radial component, when the probe and the background branes are very close to each other. We find out, in particular, an exponential solution of the radial mode, when the $D1$-branes approach the $D5$-branes in the pp-wave background. For $p - q = 2$, we find that there is a minimum distance beyond which the Dq-brane, which moves only radially, cannot fall into the Dp-branes. One could possibly study some properties of the branes by using other world sheet techniques.

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