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Abelian embedding formulation of the Stueckelberg model and its power-counting renormalizable extension

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We elucidate the geometry of the polynomial formulation of the non-Abelian Stueckelberg mechanism. We show that a natural off-shell nilpotent Becchi-Rouet-Stora-Tyutin (BRST) differential exists allowing to implement the constraint on the σ field by means of BRST techniques. This is achieved by extending the ghost sector by an additional U(1) factor (Abelian embedding). An important consequence is that a further BRST-invariant but not gauge-invariant mass term can be written for the non-Abelian gauge fields. As all versions of the Stueckelberg theory, also the Abelian embedding formulation yields a nonpower-counting renormalizable theory in $D = 4$. We then derive its natural power-counting renormalizable extension and show that the physical spectrum contains a physical massive scalar particle. Physical unitarity is also established. This model implements the spontaneous symmetry breaking in the Abelian embedding formalism.

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I. INTRODUCTION

The Stueckelberg formalism [1,2] allows for a gauge-invariant mass term for non-Abelian vector bosons without the need to introduce physical scalar fields in the classical action. The main disadvantage of the non-Abelian Stueckelberg mechanism is the fact that it yields a nonpower-counting renormalizable theory. In particular the Stueckelberg mass term

$$\frac{1}{2}m^2\,\mathrm{Tr}\Big[\Big(A_\mu - \frac{i}{g}\Omega\partial_\mu\Omega^\dagger\Big)^2\Big], \tag{1}$$

with $\Omega = \exp(igT_a\varphi_a(x))$ an element of the non-Abelian gauge group G, contains an infinite number of interaction vertices involving the fields $\varphi_a(x)$.

There have been some attempts in the literature aiming at a polynomial formulation of the Stueckelberg mechanism [3,4]. It is hoped that a polynomial interaction could help in establishing a consistent subtraction scheme for the definition of the Stueckelberg theory at the quantum level.

In Ref. [4] it has been pointed out that a polynomial action implementing the Stueckelberg construction can be derived from an interpolating action which reproduces for different choices of its parameters the Stueckelberg theory, the Higgs model as well as an embedding of the Higgs model which includes additional physical scalar fields. The construction makes use of a BRST-like (on-shell nilpotent) symmetry involving a pair of ghost-antighost fields which are singlet under the non-Abelian gauge transformations. An extension of this approach has been used in [5] in order to propose a model for massive gauge bosons without fundamental scalars.

In this paper we elucidate the geometry of the polynomial formulation of the Stueckelberg theory. We show that the requirement of polynomiality of the Stueckelberg interaction can be formulated by means of a truly off-shell nilpotent BRST symmetry. This leads to an Abelian embedding implementing the σ model constraint by means of an additional U(1) pair of ghost-antighost fields. These fields play the role of the G-singlet ghost-antighost fields proposed in [4]. Moreover it turns out that an Abelian gauge connection B_μ can be introduced and given a mass without violating the BRST invariance. B_μ can be chosen to be a free massive U(1) field.

The BRST invariants of this theory are particularly interesting in the case of the group SU(2). For this group a polynomial composite vector field can be constructed which transforms as a connection under the BRST differential (but not under the SU(2) gauge transformations). The rather surprising consequence is the possibility to generate a new polynomial BRST-invariant but not gauge-invariant mass term for the non-Abelian gauge fields.

As all known versions of the Stueckelberg mechanism, the Abelian embedding model is not power-counting renormalizable. We then study an extension thereof which is both power-counting renormalizable and physically unitary. Its physical spectrum is analyzed by BRST techniques and shown to contain the three physical polarizations of the massive gauge fields as well as a physical scalar particle. We prove by cohomological techniques that this theory is indeed physically unitary to all orders in the perturbative expansion and give the whole set of counterterms of the model.

This theory provides an alternative implementation of the spontaneous symmetry breaking. Since it is power-counting renormalizable, one could conjecture that it is physically equivalent to the Higgs model, i.e. that it yields the same physical S-matrix elements. The check of the physical equivalence in the perturbative expansion is an interesting question which deserves to be further investigated.

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The paper is organized as follows. In Sec. II we briefly review the standard formulation of the Stueckelberg model based on the use of a flat connection for the gauge group G and discuss how the Higgs model can be derived as the power-counting renormalizable extension of the flat connection version of the Stueckelberg theory. In Sec. III we develop the Abelian embedding formalism for the Stueckelberg model. The additional BRST-invariant but not gauge-invariant mass term that can be written for $G = \mathrm{SU}(2)$ is discussed in Sec. IV. In Sec. V we move to the analysis of a physically unitary and power-counting renormalizable extension of the Abelian embedding formalism. Power-counting renormalizability is established as a consequence of a set of functional identities defining the theory. The physical spectrum is constructed in Sec. VI. Conclusions are finally given in Sec. VII.

II. FLAT CONNECTION FORMULATION OF THE STUECKELBERG MODEL

For the sake of definiteness we consider the gauge group $G = \mathrm{SU}(2)$. We follow the derivation given in Ref. [6] (for a review of the standard Stueckelberg mechanism see also [2]). The (global SU(2)-symmetric) Yang-Mills action in the Proca gauge is

$$S = \int d^4x \left(-\tfrac{1}{4} G_{a\mu\nu} G_a^{\mu\nu} + m^2 \,\mathrm{Tr}[A_\mu A^\mu]\right), \tag{2}$$

where $A_\mu = \tau_a A_{a\mu}$. τ_a are the Pauli matrices and $G_{a\mu\nu}$ is the field strength

$$G_{a\mu\nu} = \partial_\mu A_{a\nu} - \partial_\nu A_{a\mu} + g f_{abc} A_{b\mu} A_{c\nu} \tag{3}$$

with $f_{abc} = 2\epsilon_{abc}$. Let us now perform an operator-valued SU(2) local transformation

$$A'_\mu = \Omega^\dagger A_\mu \Omega + \frac{i}{g} \Omega^\dagger \partial_\mu \Omega \tag{4}$$

with $\Omega \in \mathrm{SU}(2)$. Then one gets the Stueckelberg action

$$S = \int d^4x \bigg(-\frac{1}{4} G_{a\mu\nu} G_a^{\mu\nu} + \frac{m^2}{g^2} \mathrm{Tr}[(g\Omega^\dagger A_\mu \Omega + i\Omega^\dagger \partial_\mu \Omega)(g\Omega^\dagger A_\mu \Omega + i\Omega^\dagger \partial_\mu \Omega)] \bigg). \tag{5}$$

S is invariant under the local SU(2) left transformations

$$A'_\mu = U_L A_\mu U_L^\dagger + \frac{i}{g} U_L \partial_\mu U_L^\dagger, \qquad \Omega' = U_L \Omega. \tag{6}$$

Ω is the Stueckelberg field [1,2]. The matrix Ω can be parametrized in terms of three independent fields ϕ_a as follows:

$$\Omega = \frac{g}{2m}(\phi_0 \cdot 1 + i\tau_a \phi_a) \tag{7}$$

with the constraint

$$\phi_0^2 + \phi_a^2 = \frac{4m^2}{g^2}. \tag{8}$$

Equation (8) allows to express ϕ_0 in terms of the fields ϕ_a

$$\phi_0 = \sqrt{\frac{4m^2}{g^2} - \phi_a^2}. \tag{9}$$

Therefore, as a consequence of Eq. (9), the action S in Eq. (5) contains an infinite number of interaction vertices and the theory is not renormalizable by power counting (in $D = 4$). Physical unitarity of the Stueckelberg model in the Landau gauge has been discussed in detail in [7].

By setting

$$\Phi = \frac{\sqrt{2}m}{g} \Omega v_+ = \frac{1}{\sqrt{2}} \begin{pmatrix} i\phi_1 + \phi_2 \\ \phi_0 - i\phi_3 \end{pmatrix}, \tag{10}$$

with $v_+^T = (0 \quad 1)$ the Stueckelberg mass term reduces to

$$\int d^4x (D_\mu \Phi)^\dagger (D^\mu \Phi). \tag{11}$$

By dropping the constraint on the field ϕ_0 one obtains the Higgs model [8–10]. In contrast with the Stueckelberg model, the Higgs model is power-counting renormalizable in $D = 4$. In the Higgs model ϕ_0 becomes an independent field. As is well known, in addition to the gauge-invariant term in Eq. (11) power-counting renormalizability in $D = 4$ allows for two further invariants depending on Φ, namely $\int d^4x \Phi^\dagger \Phi$ and $\int d^4x (\Phi^\dagger \Phi)^2$. Their coefficients can be chosen in such a way that spontaneous symmetry breaking is triggered by the tree-level potential and consequently ϕ_0 acquires a nonvanishing vacuum expectation value (vev) v. The resulting action depends on an additional parameter λ which controls the strength of the quartic Higgs self-interaction:

$$S_\lambda = \int d^4x \bigg(-\frac{1}{4} G_{a\mu\nu} G_a^{\mu\nu} + (D_\mu \Phi)^\dagger (D^\mu \Phi) - \lambda \Big(\Phi^\dagger \Phi - \frac{v^2}{2} \Big)^2 \bigg). \tag{12}$$

After the shift $\phi_0 = \sigma + v$, which amounts to the redefinition

$$\Phi \to \Phi + \frac{v}{\sqrt{2}} v_+, \tag{13}$$

one obtains a theory for massive non-Abelian gauge bosons which contains an additional physical scalar particle described by the physical Higgs field σ. The Stueckelberg model can be formally obtained by taking the limit $\lambda \to \infty$ in the action (12), yielding the constraint

$$\Phi^\dagger \Phi - \frac{v^2}{2} = 0. \tag{14}$$

This coincides with Eq. (8) by setting $m = \frac{gv}{2}$.

III. ABELIAN EMBEDDING FORMULATION OF THE STUECKELBERG MODEL

In Ref. [4] it has been pointed out that a polynomial action implementing the Stueckelberg mechanism can be derived from an interpolating action which reproduces for different choices of its parameters the Stueckelberg theory, the Higgs model as well as an embedding of the Higgs model which includes additional physical scalar fields. The construction makes use of a BRST-like (on-shell nilpotent) symmetry involving a pair of ghost-antighost fields which are singlet under the non-Abelian gauge transformations.

It is the purpose of this section to obtain a polynomial formulation of the Stueckelberg action based on a truly off-shell nilpotent BRST symmetry. We perform a R_ξ-gauge-fixing of the action

$$S_0 = \int d^4x(-\tfrac{1}{4}G_{a\mu\nu}G_a^{\mu\nu} + (D_\mu\Phi)^\dagger D^\mu\Phi) \tag{15}$$

in the BRST formalism and obtain the gauge-fixed action

$$\begin{aligned} S_0' = S_0 + \int d^4x\Big(&\frac{\xi}{2}B_a^2 - B_a(\partial A_a + \xi g v\phi_a) \\ &+ \bar\omega_a[\partial^\mu(D_\mu\omega)_a + \xi g^2 v(\sigma + v)\omega_a \\ &+ \xi g^2 v\epsilon_{abc}\phi_b\omega_c]\Big). \end{aligned} \tag{16}$$

ω_a are the non-Abelian ghost fields, $\bar\omega_a$ the corresponding antighosts. B_a are the Nakanishi-Lautrup multiplier fields. ξ is the gauge parameter.

S_0' is invariant under the following BRST differential

$$\begin{gathered} sA_{a\mu} = (D_\mu\omega)_a = \partial_\mu\omega_a + gf_{abc}A_{b\mu}\omega_c, \\ s\omega_a = -\tfrac{1}{2}gf_{abc}\omega_b\omega_c, \qquad s\Phi = ig\omega_a\tau_a\Phi, \\ s\phi_0 = -g\omega_a\phi_a, \qquad s\phi_a = g(\omega_a\phi_0 + \epsilon_{abc}\phi_b\omega_c), \\ s\bar\omega_a = B_a, \qquad sB_a = 0. \end{gathered} \tag{17}$$

At this point we wish to implement the constraint in Eq. (14)

$$\Phi^\dagger\Phi - \frac{v^2}{2} = \frac{1}{2}\sigma^2 + v\sigma + \frac{1}{2}\phi_a^2 = 0 \tag{18}$$

by means of BRST techniques. The simplest possibility is to introduce an antighost field $\bar c$ transforming under s as follows:

$$s\bar c = \Phi^\dagger\Phi - \frac{v^2}{2}. \tag{19}$$

Since the constraint in Eq. (18) is gauge invariant, $s^2\bar c = 0$. One should also introduce the ghost c corresponding to the antighost $\bar c$, which we pair in a BRST doublet [11–13] with a scalar field X as follows

$$sX = vc, \qquad sc = 0. \tag{20}$$

Although it is not strictly necessary, it is tempting to consider c as the Abelian ghost of a U(1) connection B_μ, so that one might also set

$$sB_\mu = \partial_\mu c. \tag{21}$$

Then the original BRST symmetry is embedded in a larger differential with an Abelian component given by Eqs. (20) and (21).

We remark that in the embedding theory the quartic potential in S_λ in Eq. (12) is s-exact since

$$-\int d^4x\lambda\Big(\Phi^\dagger\Phi - \frac{v^2}{2}\Big)^2 = s\Big[\int d^4x\Big(-\lambda\bar c\Big(\Phi^\dagger\Phi - \frac{v^2}{2}\Big)\Big)\Big]. \tag{22}$$

By adding to the action S_0' in Eq. (16) the following term

$$\begin{aligned} S_{\rm constr} &= \int d^4x s\Big(\frac{1}{v}X\Box\bar c\Big) \\ &= \int d^4x\Big[-\bar c\Box c + \frac{1}{2v}X\Box(\sigma^2 + 2v\sigma + \phi_a^2)\Big], \end{aligned} \tag{23}$$

the constraint of the Stueckelberg model is reproduced in the way suggested in Ref. [4]. The nonrenormalizability by power counting is induced by the interaction vertices $\frac{1}{2v}X\Box(\sigma^2 + \phi_a^2)$ in the right-hand side (rhs) of Eq. (23).

By looking at Eqs. (20) and (21) it is also clear that one can add a kinetic term and a mass term for B_μ without violating the BRST symmetry:

$$S_{\rm U(1)} = \int d^4x\Big(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M^2\Big(B_\mu - \frac{1}{v}\partial_\mu X\Big)^2\Big), \tag{24}$$

where

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{25}$$

With the choice of Eq. (24) B_μ is a free massive U(1) gauge field.

IV. FURTHER BRST-INVARIANT MASS TERMS

We now go back to Eq. (15) and consider the current linearly coupled to the gauge fields $A_{a\mu}$:

$$j_a^\mu = -ig\partial^\mu\Phi^\dagger\tau_a\Phi + ig\Phi^\dagger\tau_a\partial^\mu\Phi. \tag{26}$$

We evaluate its variation under a gauge transformation $\delta\Phi = ig\alpha_a\tau_a\Phi$ with gauge parameters $\alpha_a(x)$:

$$\delta j_a^\mu = -2g^2\Phi^\dagger\Phi\partial^\mu\alpha_a + gf_{abc}j_b^\mu\alpha_c. \tag{27}$$

This is not the transformation of a gauge connection due to the appearance of the factor $\Phi^\dagger\Phi$ in front of the gradient of α_a. This factor can be compensated by the Abelian antighost field as follows. We consider the composite vector field

$$\tilde F_a^\mu = j_a^\mu + 2g^2\bar c\partial^\mu\omega_a \tag{28}$$

and compute its BRST variation:

$$
\begin{aligned}
s\tilde{F}^{\mu}_a &= sj^{\mu}_a + 2g^2\Big(\Phi^\dagger\Phi - \frac{v^2}{2}\Big)\partial^\mu\omega_a - 2g^2\bar{c}\partial^\mu\Big(-\frac{g}{2}f_{abc}\omega_b\omega_c\Big) \\
&= -2g^2\Phi^\dagger\Phi\partial^\mu\omega_a + gf_{abc}j^{\mu}_b\omega_c + 2g^2\Big(\Phi^\dagger\Phi - \frac{v^2}{2}\Big)\partial^\mu\omega_a + 2g^2\bar{c}gf_{abc}\partial^\mu\omega_b\omega_c \\
&= -v^2g^2\partial^\mu\omega_a + gf_{abc}(\tilde{F}^{\mu}_b - 2g^2\bar{c}\partial^\mu\omega_b)\omega_c + 2g^2\bar{c}gf_{abc}\partial^\mu\omega_b\omega_c = -v^2g^2\partial^\mu\omega_a + gf_{abc}\tilde{F}^{\mu}_b\omega_c. \qquad (29)
\end{aligned}
$$

The above equation allows us to derive a vector field which transforms as a connection under s by properly rescaling $\tilde{F}^{\mu}_a$: by setting

$$
\begin{aligned}
F^{\mu}_a &= -\frac{1}{g^2v^2}\tilde{F}^{\mu}_a \\
&= -\frac{1}{g^2v^2}(-ig\partial^\mu\Phi^\dagger\tau_a\Phi + ig\Phi^\dagger\tau_a\partial^\mu\Phi \\
&\quad + 2g^2\bar{c}\partial^\mu\omega_a), \qquad (30)
\end{aligned}
$$

we get

$$
sF^{\mu}_a = \partial^\mu\omega_a + gf_{abc}F^{\mu}_b\omega_c. \qquad (31)
$$

By Eq. (31) one can use F^{μ}_a in order to generate a new polynomial BRST-invariant (but not gauge-invariant) mass term for A^{μ}_a, given by

$$
\tfrac{1}{2}m^2(A^{\mu}_a - F^{\mu}_a)^2. \qquad (32)
$$

This term is absent in the standard flat connection formulation of the Stueckelberg theory.

As a final point we remark that for an arbitrary gauge group G with generators T_a Eq. (27) becomes

$$
\delta j^{\mu}_a = -g^2\Phi^\dagger\{T_a, T_b\}\Phi\partial^\mu\alpha_b + gf_{abc}j^{\mu}_b\alpha_c. \qquad (33)
$$

From the above equation we see that in order to apply the compensation mechanism based on the Abelian antighost $\bar{c}$ the anticommutator $\{T_a, T_b\}$ has to be proportional to δ_{ab} times the identity matrix.

V. A POWER-COUNTING RENORMALIZABLE EXTENSION OF THE ABELIAN EMBEDDED STUECKELBERG MODEL

In this section we discuss a mechanism for obtaining a power-counting renormalizable theory of massive gauge bosons from the Abelian embedding formulation of the Stueckelberg model. We will require that the BRST differential controlling the theory is off-shell nilpotent. Moreover we wish to formulate the theory without higher derivatives. For that purpose we set now

$$
sX_1 = vc, \qquad sc = 0, \qquad s\bar{c} = \Phi^\dagger\Phi - \frac{v^2}{2} - vX_2, \qquad sX_2 = 0. \qquad (34)
$$

The gauge-fixed action obtained from S_λ is

$$
\begin{aligned}
S'_\lambda = S_\lambda + \int d^4x\Big(&\frac{\xi}{2}B_a^2 - B_a(\partial A_a + \xi g v\phi_a) \\
&+ \bar{\omega}_a[\partial^\mu(D_\mu\omega)_a + \xi g^2 v(\sigma + v)\omega_a \\
&+ \xi g^2 v\epsilon_{abc}\phi_b\omega_c]\Big). \qquad (35)
\end{aligned}
$$

To this action we add

$$
\begin{aligned}
S_{\mathrm{constr},X_2} &= \int d^4x\Big[s\Big(\frac{X_1 + X_2}{v}(\Box + 2\lambda v^2)\bar{c}\Big) - \frac{v^2}{2\rho}X_2^2\Big] \\
&= \int d^4x\Big(-\bar{c}(\Box + 2\lambda v^2)c + \frac{1}{v}(X_1 + X_2) \\
&\quad \times(\Box + 2\lambda v^2)\Big(\frac{1}{2}\sigma^2 + v\sigma + \frac{1}{2}\phi_a^2 - vX_2\Big) \\
&\quad - \frac{v^2}{2\rho}X_2^2\Big). \qquad (36)
\end{aligned}
$$

We also introduce the antifields for $A_{a\mu}$, ω_a, σ, ϕ_a, $\bar{c}$, which we denote by $A^*_{a\mu}$, ω^*_a, σ^*, ϕ^*_a, $\bar{c}^*$. The complete action is finally given by

$$
\begin{aligned}
\Gamma^{(0)} = S'_\lambda + S_{\mathrm{constr},X_2} + \int d^4x\Big[&A^*_{a\mu}(D^\mu\omega)_a - g\sigma^*\omega_a\phi_a \\
&+ g\phi^*_a(\omega_a(\sigma + v) + \epsilon_{abc}\phi_b\omega_c) \\
&- \frac{1}{2}\omega^*_a g f_{abc}\omega_b\omega_c \\
&+ \bar{c}^*\Big(\frac{1}{2}\sigma^2 + v\sigma + \frac{1}{2}\phi_a^2 - vX_2\Big)\Big]. \qquad (37)
\end{aligned}
$$

We can assign a ghost number to the fields and antifields of the theory. $A_{a\mu}$, ϕ_a, σ, B_a, X_1, X_2, $\bar{c}^*$ have ghost number zero, $A^*_{a\mu}$, ϕ^*_a, σ^*, $\bar{\omega}_a$, $\bar{c}$ have ghost number -1, ω_a, and c ghost number $+1$ while ω^*_a has ghost number -2.

We notice that $\Gamma^{(0)}$ is separately invariant under the BRST differential s_0 given by

$$
\begin{aligned}
&s_0A_{a\mu} = (D_\mu\omega)_a = \partial_\mu\omega_a + gf_{abc}A_{b\mu}\omega_c, \\
&s_0\omega_a = -\tfrac{1}{2}gf_{abc}\omega_b\omega_c, \qquad s_0\sigma = -g\omega_a\phi_a, \\
&s_0\phi_a = g(\omega_a(\sigma + v) + \epsilon_{abc}\phi_b\omega_c), \qquad (38) \\
&s_0\bar{\omega}_a = B_a, \qquad s_0B_a = 0, \\
&s_0\bar{c} = s_0c = s_0X_1 = s_0X_2 = 0
\end{aligned}
$$

and under the BRST differential s_1 given by

$$s_1 X_1 = v c, \qquad s_1 c = 0,$$
$$s_1 \bar{c} = \Phi^\dagger \Phi - \frac{v^2}{2} - v X_2, \qquad s_1 X_2 = 0, \tag{39}$$
$$s_1 A_{a\mu} = s_1 \sigma = s_1 \phi_a = s_1 \omega_a = s_1 \bar{\omega}_a = s_1 B_a = 0.$$

$\Gamma^{(0)}$ fulfills the following functional identities

(i) the non-Abelian ghost equation

$$\frac{\delta \Gamma^{(0)}}{\delta \bar{\omega}_a} = \partial^\mu \frac{\delta \Gamma^{(0)}}{\delta A_a^{\mu *}} + \xi g v \frac{\delta \Gamma^{(0)}}{\delta \phi_a^*} \tag{40}$$

(ii) the Abelian ghost equation

$$\frac{\delta \Gamma^{(0)}}{\delta \bar{c}} = -(\square + 2\lambda v^2) c \tag{41}$$

(iii) the Abelian antighost equation

$$\frac{\delta \Gamma^{(0)}}{\delta c} = (\square + 2\lambda v^2) \bar{c} \tag{42}$$

(iv) the B-equation

$$\frac{\delta \Gamma^{(0)}}{\delta B_a} = \xi B_a - \partial A_a - \xi g v \phi_a \tag{43}$$

(v) the X_1-equation

$$\frac{\delta \Gamma^{(0)}}{\delta X_1} = \frac{1}{v}(\square + 2\lambda v^2) \frac{\delta \Gamma^{(0)}}{\delta \bar{c}^*} \tag{44}$$

(vi) the X_2-equation

$$\frac{\delta \Gamma^{(0)}}{\delta X_2} = \frac{1}{v}(\square + 2\lambda v^2) \frac{\delta \Gamma^{(0)}}{\delta \bar{c}^*} - (\square + 2\lambda v^2)(X_1 + X_2) - \frac{v^2}{\rho} X_2 - v \bar{c}^* \tag{45}$$

(vii) the Slavnov-Taylor (ST) identity

$$\mathcal{S}(\Gamma^{(0)}) = \int d^4x \left(\frac{\delta \Gamma^{(0)}}{\delta A_a^{\mu *}} \frac{\delta \Gamma^{(0)}}{\delta A_{a\mu}} + \frac{\delta \Gamma^{(0)}}{\delta \sigma^*} \frac{\delta \Gamma^{(0)}}{\delta \sigma} + \frac{\delta \Gamma^{(0)}}{\delta \phi_a^*} \frac{\delta \Gamma^{(0)}}{\delta \phi_a} + \frac{\delta \Gamma^{(0)}}{\delta \omega_a^*} \frac{\delta \Gamma^{(0)}}{\delta \omega_a} + \frac{\delta \Gamma^{(0)}}{\delta \bar{c}^*} \frac{\delta \Gamma^{(0)}}{\delta \bar{c}} + B_a \frac{\delta \Gamma^{(0)}}{\delta \bar{\omega}_a} + v c \frac{\delta \Gamma^{(0)}}{\delta X_1} \right) = 0. \tag{46}$$

By virtue of Eq. (42) invariance of $\Gamma^{(0)}$ under s_0 is recovered by projecting Eq. (46) at order zero in powers of c while invariance of $\Gamma^{(0)}$ under s_1 is obtained by projecting Eq. (46) at order one in powers of c.

The choice of gathering both invariances into a single ST identity equipped with the grading in c proves useful in the renormalization of the model, as is discussed in the Sec. V C.

A. Power-counting rules

In Appendix A we give the propagators of the model. Diagonalization of the quadratic part in the fields of $\Gamma^{(0)}$ is achieved by setting

$$B'_a = B_a - \frac{1}{\xi}(\partial A_a + \xi g v \phi_a), \qquad \sigma' = \sigma - X_1 - X_2. \tag{47}$$

The corresponding UV mass dimensions of the fields and external sources can be summarized as follows. $A_{a\mu}$, σ', X_1, X_2, ϕ_a, $\bar{\omega}_a$, ω_a, $\bar{c}$, and c have dimension one, B'_a has dimension two. $\bar{c}^*$, $A^*_{a\mu}$, ϕ^*_a, σ^*, and ω^*_a have dimension two.

All interaction vertices in $\Gamma^{(0)}$ with the exception of

$$\frac{1}{v}(X_1 + X_2) \square \left(\frac{1}{2}\sigma^2 + \frac{1}{2}\phi_a^2 \right) \tag{48}$$

have UV dimension ≤ 4. We remark for future use that the interaction vertices depend on X_1, X_2 only via the combination $X_1 + X_2$.

B. Power-counting renormalizability

In this section we show that the model is indeed power-counting renormalizable, despite the fact that it contains the vertices in Eq. (48). We impose Eqs. (40)–(45) on the 1-PI vertex functional Γ:

$$\frac{\delta \Gamma}{\delta \bar{\omega}_a} = \partial^\mu \frac{\delta \Gamma}{\delta A_a^{\mu *}} + \xi g v \frac{\delta \Gamma}{\delta \phi_a^*}, \tag{49}$$

$$\frac{\delta \Gamma}{\delta \bar{c}} = -(\square + 2\lambda v^2) c, \tag{50}$$

$$\frac{\delta \Gamma}{\delta c} = (\square + 2\lambda v^2) \bar{c}, \tag{51}$$

$$\frac{\delta \Gamma}{\delta B_a} = \xi B_a - \partial A_a - \xi g v \phi_a, \tag{52}$$

$$\frac{\delta \Gamma}{\delta X_1} = \frac{1}{v}(\square + 2\lambda v^2) \frac{\delta \Gamma}{\delta \bar{c}^*}, \tag{53}$$

$$\frac{\delta \Gamma}{\delta X_2} = \frac{1}{v}(\square + 2\lambda v^2) \frac{\delta \Gamma}{\delta \bar{c}^*} - (\square + 2\lambda v^2)(X_1 + X_2) - \frac{v^2}{\rho} X_2 - v \bar{c}^*. \tag{54}$$

The above set of functional equations hold together with the ST identity

$$\mathcal{S}(\Gamma) = \int d^4x\Big(\frac{\delta\Gamma}{\delta A_a^{\mu *}}\frac{\delta\Gamma}{\delta A_{a\mu}} + \frac{\delta\Gamma}{\delta\sigma^*}\frac{\delta\Gamma}{\delta\sigma} + \frac{\delta\Gamma}{\delta\phi_a^*}\frac{\delta\Gamma}{\delta\phi_a} + \frac{\delta\Gamma}{\delta\omega_a^*}\frac{\delta\Gamma}{\delta\omega_a} + \frac{\delta\Gamma}{\delta\bar{c}^*}\frac{\delta\Gamma}{\delta\bar{c}} + B_a\frac{\delta\Gamma}{\delta\bar{\omega}_a} + vc\frac{\delta\Gamma}{\delta X_1}\Big) = 0. \tag{55}$$

We develop Γ according to the loop order as follows

$$\Gamma = \sum_{j=0}^{\infty}\hbar^{(j)}\Gamma^{(j)}. \tag{56}$$

From Eq. (53) we get

$$\frac{\delta\Gamma^{(j)}}{\delta X_1} = \frac{1}{v}(\Box + 2\lambda v^2)\frac{\delta\Gamma^{(j)}}{\delta\bar{c}^*}, \qquad j \geq 1 \tag{57}$$

and therefore $\Gamma^{(j)}$ depends on X_1 only via the combination

$$\bar{c}^* + \frac{1}{v}(\Box + 2\lambda v^2)X_1. \tag{58}$$

From Eq. (54) we get

$$\frac{\delta\Gamma^{(j)}}{\delta X_2} = \frac{1}{v}(\Box + 2\lambda v^2)\frac{\delta\Gamma^{(j)}}{\delta\bar{c}^*}, \qquad j \geq 1 \tag{59}$$

which implies that $\Gamma^{(j)}$ depends on X_2 only via the combination

$$\bar{c}^* + \frac{1}{v}(\Box + 2\lambda v^2)X_2. \tag{60}$$

Equation (58) together with Eq. (60) yields that the dependence of $\Gamma^{(j)}$ on X_1, X_2 is only via

$$\widehat{\bar{c}^*} = \bar{c}^* + \frac{1}{v}(\Box + 2\lambda v^2)(X_1 + X_2). \tag{61}$$

Moreover from Eq. (49) we have

$$\frac{\delta\Gamma^{(j)}}{\delta\bar{\omega}_a} = \partial^\mu\frac{\delta\Gamma^{(j)}}{\delta A_a^{\mu *}} + \xi g v\frac{\delta\Gamma^{(j)}}{\delta\phi_a^*}, \qquad j \geq 1, \tag{62}$$

i.e. $\Gamma^{(j)}$ depends on $\bar{\omega}_a$ only via the combinations

$$\widehat{A^*_{a\mu}} = A^*_{a\mu} - \partial_\mu\bar{\omega}_a, \qquad \widehat{\phi_a^*} = \phi_a^* + \xi g v\bar{\omega}_a. \tag{63}$$

From Eqs. (50) and (51) we get

$$\frac{\delta\Gamma^{(j)}}{\delta\bar{c}} = 0, \qquad \frac{\delta\Gamma^{(j)}}{\delta c} = 0, \qquad j \geq 1, \tag{64}$$

and thus $\Gamma^{(j)}$ does not depend on $\bar{c}$, c. From Eq. (52) we obtain

$$\frac{\delta\Gamma^{(j)}}{\delta B_a} = 0, \tag{65}$$

hence $\Gamma^{(j)}$ does not depend on B_a.

Therefore we can restrict the analysis of the divergences of the theory to the 1-PI Green functions depending on $A_{a\mu}, \phi_a, \sigma, \omega_a, A^*_{a\mu}, \phi^*_a, \sigma^*, \omega^*_a, \bar{c}^*$ (those depending on at least one of X_1, X_2, $\bar{\omega}_a$ can be obtained by functional differentiation of Eqs. (57), (59), and (62) respectively). In all these amplitudes X_1 and X_2 are exchanged within 1-PI graphs in the combination $X = X_1 + X_2$. The latter is associated to the propagator

$$\begin{aligned}\Delta_{XX} &= \Delta_{X_1X_1} + \Delta_{X_2X_2} \\ &= -\frac{i}{p^2 - 2\lambda v^2} + \frac{i}{p^2 - (2\lambda + \frac{1}{\rho})v^2} \\ &= \frac{iv^2}{\rho(p^2 - 2\lambda v^2)(p^2 - (2\lambda + \frac{1}{\rho})v^2)}\end{aligned} \tag{66}$$

which falls off for $p^2 \to \infty$ as $1/(p^2)^2$. This means that X has UV dimension zero and thus the vertex in Eq. (48) still obeys the power-counting renormalizability bounds. Moreover, since $A_{a\mu}$, ϕ_a, σ, ω_a, $A^*_{a\mu}$, ϕ^*_a, σ^*, ω^*_a, $\bar{c}^*$ have positive dimension, only a finite number of counterterms is needed in order to remove all the divergences of the theory.

C. Structure of the counterterms

We assume that divergences have been recursively subtracted up to order $n - 1$ in the loop expansion and that the ST identity holds up to order n. We assume as well that the set of functional equations (49)–(54) is fulfilled up to order n. The nth order ST identity reads for the symmetrically regularized nth order vertex functional $\Gamma_R^{(n)}$

$$\delta\Gamma_R^{(n)} = -\sum_{j=1}^{n-1}(\Gamma^{(j)}, \Gamma^{(n-j)}), \tag{67}$$

where δ is the linearized ST operator

$$\begin{aligned}\delta = \int d^4x\Big[&(D_\mu\omega)_a\frac{\delta}{\delta A_{a\mu}} - g\omega_a\phi_a\frac{\delta}{\delta\sigma} + g(\omega_a(\sigma + v) + \epsilon_{abc}\phi_b\omega_c)\frac{\delta}{\delta\phi_a} - \frac{1}{2}gf_{abc}\omega_b\omega_c\frac{\delta}{\delta\omega_a} + B_a\frac{\delta}{\delta\bar{\omega}_a} + vc\frac{\delta}{\delta X_1} \\ &+ \Big(\frac{1}{2}\sigma^2 + v\sigma + \frac{1}{2}\phi_a^2 - vX_2\Big)\frac{\delta}{\delta\bar{c}} + \frac{\delta\Gamma^{(0)}}{\delta A_a^\mu}\frac{\delta}{\delta A^*_{a\mu}} + \frac{\delta\Gamma^{(0)}}{\delta\phi_a}\frac{\delta}{\delta\phi_a^*} + \frac{\delta\Gamma^{(0)}}{\delta\sigma}\frac{\delta}{\delta\sigma^*} + \frac{\delta\Gamma^{(0)}}{\delta\omega_a}\frac{\delta}{\delta\omega_a^*} + \frac{\delta\Gamma^{(0)}}{\delta\bar{c}}\frac{\delta}{\delta\bar{c}^*}\Big]\end{aligned} \tag{68}$$

and the bracket in the rhs of Eq. (67) is given by

$$(X, Y) = \int d^4x \Big[\frac{\delta X}{\delta A_a^{*\mu}} \frac{\delta Y}{\delta A_{a\mu}} + \frac{\delta X}{\delta \phi_a^*} \frac{\delta Y}{\delta \phi_a} + \frac{\delta X}{\delta \sigma^*} \frac{\delta Y}{\delta \sigma} + \frac{\delta X}{\delta \omega_a^*} \frac{\delta Y}{\delta \omega_a} + \frac{\delta X}{\delta \bar{c}^*} \frac{\delta Y}{\delta \bar{c}} \Big]. \tag{69}$$

Since the divergences have been recursively subtracted up to order $n - 1$, the rhs of Eq. (67) is finite. Thus, as a consequence of Eq. (67), the divergent part $\Gamma^{(n)}_{R,\mathrm{div}}$ of $\Gamma^{(n)}_R$ must obey the linearized ST identity

$$\delta \Gamma^{(n)}_{R,\mathrm{div}} = 0. \tag{70}$$

In order to solve Eq. (70) it is useful to decompose δ according to the degree induced by the counting operator for the δ-invariant variable $\hat{\bar{c}}^*$ in Eq. (61). Then δ can be written as

$$\delta = \delta_0 + \delta_1, \tag{71}$$

where δ_0 preserves the number of $\hat{\bar{c}}^*$'s and δ_1 increases it by one. The explicit action of the differentials δ_0, δ_1 on the variables of the model is given in Eqs. (B4) and (B5).

The most general solution to Eq. (70) of dimension ≤ 4 and subject to the constraints in (57), (59), (62), (64), and (65) is derived in Appendix B. It is given by

$$\begin{aligned} \Gamma^{(n)}_{R,\mathrm{div}} = {}& d_1 \int d^4x G_{\mu\nu a} G_a^{\mu\nu} + d_2 \int d^4x (D_\mu \Phi)^\dagger (D^\mu \Phi) + d_3 \int d^4x \Big(\Phi^\dagger\Phi - \frac{v^2}{2}\Big) + d_4 \int d^4x \Big(\Phi^\dagger\Phi - \frac{v^2}{2}\Big)^2 \\ & + d_5 \int d^4x \delta_0(\hat{\phi}_a^* \phi_a) + d_6 \int d^4x \delta_0(\sigma^*\sigma) + d_7 \int d^4x \delta_0(\widehat{A^*_{a\mu}A^\mu_a}) + d_8 \int d^4x \delta_0(\omega_a^*\omega_a) + d_9 \int d^4x \hat{\bar{c}}^* \\ & + d_{10} \int d^4x \hat{\bar{c}}^* \Big(\Phi^\dagger\Phi - \frac{v^2}{2}\Big) + d_{11} \int d^4x \hat{\bar{c}}^{*2} + \int d^4x \hat{\bar{c}}^* [(d_6 - d_5)\sigma^2 + (d_6 - 2d_5) v\sigma], \end{aligned} \tag{72}$$

where $d_1, \ldots, d_{11}$ parameterize the nth loop overall local divergences. After the recursive subtraction has been performed, the nth order local divergences $\Gamma^{(n)}_{R,\mathrm{div}}$ are removed by adding the nth order counterterms $-\Gamma^{(n)}_{R,\mathrm{div}}$. The ST identity is preserved by this subtraction.

We notice that one can always add to the resulting nth order vertex functional

$$\Gamma^{(n)} = \Gamma^{(n)}_R + (-\Gamma^{(n)}_{R,\mathrm{div}})$$

a functional of the same form as in Eq. (B13) with finite coefficients $a_1, \ldots, a_{11}$ while preserving the nth order ST identity and the functional equations (49)–(54). These ambiguities have to be fixed by a suitable choice of normalization conditions. A convenient set of normalization conditions is given at the end of Sec. VI.

VI. PHYSICAL UNITARITY

In this section we address the issue of Physical Unitarity. We first discuss the tree-level approximation and then move to the analysis of the renormalized theory.

A. Tree level

The ST identity in Eq. (46) yields by projection at order zero in powers of c the following functional identity:

$$\begin{aligned} \mathcal{S}_0(\Gamma^{(0)}) = \int d^4x \Big(& \frac{\delta\Gamma^{(0)}}{\delta A_a^{\mu*}} \frac{\delta\Gamma^{(0)}}{\delta A_{a\mu}} + \frac{\delta\Gamma^{(0)}}{\delta\sigma^*} \frac{\delta\Gamma^{(0)}}{\delta\sigma} + \frac{\delta\Gamma^{(0)}}{\delta\phi_a^*} \frac{\delta\Gamma^{(0)}}{\delta\phi_a} \\ & + \frac{\delta\Gamma^{(0)}}{\delta\omega_a^*} \frac{\delta\Gamma^{(0)}}{\delta\omega_a} + B_a \frac{\delta\Gamma^{(0)}}{\delta\bar{\omega}_a} \Big) = 0. \end{aligned} \tag{73}$$

Moreover, projection of Eq. (46) at order one in powers of c yields

$$\mathcal{S}_1(\Gamma^{(0)}) = \int d^4x \Big(\frac{\delta\Gamma^{(0)}}{\delta\bar{c}^*} \frac{\delta\Gamma^{(0)}}{\delta\bar{c}} + vc \frac{\delta\Gamma^{(0)}}{\delta X_1} \Big) = 0. \tag{74}$$

The functional identity in Eq. (73) is generated by the invariance of $\Gamma^{(0)}$ under the BRST differential s_0 in Eq. (38), the functional identity in Eq. (74) is generated by the invariance of $\Gamma^{(0)}$ under the BRST differential s_1 in Eq. (39).

Correspondingly there are two conserved asymptotic charges Q_0 and Q_1 associated with Eq. (73) and (74), respectively. They act as follows on the fields of the theory ($[,]_+$ denotes the anticommutator):

$$\begin{aligned} &[Q_0, A_{a\mu}] = \partial_\mu \omega_a, \qquad [Q_0, \omega_a]_+ = 0, \\ &[Q_0, \phi_a] = gv\omega_a, \qquad [Q_0, \bar{\omega}_a]_+ = B_a, \\ &[Q_0, B_a] = 0, \qquad [Q_0, \sigma] = 0, \qquad [Q_0, X_1] = 0, \\ &[Q_0, X_2] = 0, \qquad [Q_0, \bar{c}]_+ = 0, \qquad [Q_0, c]_+ = 0, \end{aligned} \tag{75}$$

and

$$\begin{aligned} [Q_1, A_{a\mu}] &= [Q_1, \phi_a] = [Q_1, \sigma] = [Q_1, B_a] = [Q_1, \omega_a]_+ \\ &= [Q_1, \bar{\omega}_a]_+ = 0, \\ &[Q_1, X_1] = vc, \qquad [Q_1, X_2] = 0, \\ &[Q_1, \bar{c}]_+ = v\sigma - vX_2, \qquad [Q_1, c]_+ = 0. \end{aligned} \tag{76}$$

We characterize the physical Hilbert space $\mathcal{H}_{\mathrm{phys}}$ as the space

$$\mathcal{H}_{\mathrm{phys}} = \mathcal{H}_0 \cap \mathcal{H}_1, \tag{77}$$

where

$$\mathcal{H}_0 = \frac{\ker Q_0}{\mathrm{Im} Q_0} \quad \text{and} \quad \mathcal{H}_1 = \frac{\ker Q_1}{\mathrm{Im} Q_1}. \tag{78}$$

That is, $\mathcal{H}_{\rm phys}$ is the intersection of the quotient spaces [14–17] associated with the two conserved BRST charges Q_0 and Q_1.

In the sector spanned by σ, X_1, X_2 the mass eigenstates are $\sigma' = \sigma - X_1 - X_2$, X_1 and X_2. σ' and X_1 have mass $p^2 = 2\lambda v^2$, X_2 has mass $p^2 = (2\lambda + \frac{1}{\rho})v^2$. $\bar{c}$ and c have mass $p^2 = 2\lambda v^2$. $\bar{\omega}_a$, ω_a have mass $p^2 = \xi(gv)^2$, ϕ_a and the longitudinal component ∂A_a of $A_{a\mu}$ have mass $p^2 = \xi(gv)^2$.

We first construct $\mathcal{H}_0$. From Eq. (75) we see that $\mathcal{H}_0$ contains $\bar{c}$, c, X_1, X_2, and σ'. Moreover the only modes belonging to $\mathcal{H}_0$ in the sector spanned by $A_{a\mu}$, ϕ_a, B_a, $\bar{\omega}_a$, and ω_a are the three transverse components (in the four-dimensional sense) of $A_{a\mu}$, i.e. those whose polarization vector $\epsilon_\mu(p)$ fulfills

$$\epsilon_\mu(p) p^\mu = 0 \quad \text{at } p^2 = (M_A^{(0)})^2 = (gv)^2. \tag{79}$$

In the above equation $M_A^{(0)}$ stands for the tree-level mass of $A_{a\mu}$. Indeed we find (in the momentum space representation)

$$[Q_0, \epsilon_\mu(p) A_a^\mu(p)] = -i\epsilon_\mu(p) p^\mu \omega_a = 0. \tag{80}$$

Moreover $\bar{\omega}_a$ and B_a are Q_0-doublets [11–17]:

$$[Q_0, \bar{\omega}_a]_+ = B_a, \qquad [Q_0, B_a] = 0 \tag{81}$$

and hence they are not in $\mathcal{H}_0$. The ghost ω_a is also paired into a Q_0-doublet with the longitudinal polarization $\rho_\mu(p)$ of $A_{a\mu}$ (i.e. such that $\rho_\mu(p)p^\mu = 1$ at $p^2 = \xi(M_A^{(0)})^2$):

$$[Q_0, \rho_\mu(p) A_a^\mu(p)] = -i\rho_\mu(p) p^\mu \omega_a. \tag{82}$$

From the above equation we see that ω_a and $\rho_\mu(p)A_a^\mu(p)$ are not in $\mathcal{H}_0$. Finally ϕ_a does not belong to the kernel of Q_0 and thus it is outside $\mathcal{H}_0$.

We now characterize $\mathcal{H}_1$. Since by Eq. (76)

$$[Q_1, \sigma'] = -vc \tag{83}$$

we get that σ' is not in $\mathcal{H}_1$. By Eq. (76) we also see that X_1 is not in $\mathcal{H}_1$ while X_2 belongs to $\mathcal{H}_1$. For any finite value of ρ the Q_1-invariant combination $\sigma' + X_1$ is Q_1-exact since

$$[Q_1, \bar{c}]_+ = v\sigma - vX_2 = v(\sigma' + X_1). \tag{84}$$

Therefore $\sigma' + X_1$ does not belong to $\mathcal{H}_1$. From the above equation we also see that $\bar{c}$ is not in $\mathcal{H}_1$. Furthermore c is not in $\mathcal{H}_1$ since it forms a Q_1-doublet with $\frac{1}{v}X_1$. This implies that the only mode in $\mathcal{H}_1$ in the sector spanned by X_1, X_2, σ, $\bar{c}$, c is X_2. Its mass is given by

$$m_{X_2} = \left(2\lambda + \frac{1}{\rho}\right)v^2. \tag{85}$$

From Eq. (76) we get that $A_{a\mu}$, ϕ_a, $\bar{\omega}_a$, ω_a, B_a are also in $\mathcal{H}_1$.

By taking into account the above construction of $\mathcal{H}_0$ and $\mathcal{H}_1$ we conclude according to Eq. (77) that $\mathcal{H}_{\rm phys}$ is spanned by the transverse polarizations of $A_{a\mu}$ in Eq. (79) and by the scalar X_2.

B. Higher orders

The analysis of the physical states in the renormalized theory follows a similar path. By Eq. (51) the ST identity in Eq. (55) can be projected at order zero in powers of c yielding

$$\mathcal{S}_0(\Gamma) = \int d^4x \left(\frac{\delta\Gamma}{\delta A_a^{\mu *}} \frac{\delta\Gamma}{\delta A_{a\mu}} + \frac{\delta\Gamma}{\delta\sigma^*} \frac{\delta\Gamma}{\delta\sigma} + \frac{\delta\Gamma}{\delta\phi_a^*} \frac{\delta\Gamma}{\delta\phi_a} + \frac{\delta\Gamma}{\delta\omega_a^*} \frac{\delta\Gamma}{\delta\omega_a} + B_a \frac{\delta\Gamma}{\delta\bar{\omega}_a} \right) = 0. \tag{86}$$

Moreover, the projection of Eq. (55) at order one in powers of c gives

$$\mathcal{S}_1(\Gamma) = \int d^4x \left(\frac{\delta\Gamma}{\delta\bar{c}^*} \frac{\delta\Gamma}{\delta\bar{c}} + vc \frac{\delta\Gamma}{\delta X_1} \right) = 0. \tag{87}$$

These are the renormalized ST identities associated with the BRST differentials s_0 and s_1, respectively.

By taking into account global SU(2) invariance and Eq. (51) we derive the action of the conserved asymptotic charges Q_0 and Q_1 associated with Eq. (86) and (87) on the fields of the theory:

$$\begin{gathered}
[Q_0, A_{a\mu}] = \Gamma_{\omega_b A^*_{a\mu}} \omega_b, \qquad [Q_0, \omega_a]_+ = 0, \\
[Q_0, \phi_a] = \Gamma_{\omega_b \phi_a^*} \omega_b, \qquad [Q_0, \bar{\omega}_a]_+ = B_a, \\
[Q_0, B_a] = 0, \qquad [Q_0, \sigma] = 0, \qquad [Q_0, X_1] = 0, \\
[Q_0, X_2] = 0, \qquad [Q_0, \bar{c}]_+ = 0, \qquad [Q_0, c]_+ = 0,
\end{gathered} \tag{88}$$

and

$$\begin{gathered}
[Q_1, A_{a\mu}] = [Q_1, \phi_a] = [Q_1, \sigma] = [Q_1, B_a] = [Q_1, \omega_a]_+ \\
= [Q_1, \bar{\omega}_a]_+ = 0, \\
[Q_1, X_1] = vc, \qquad [Q_1, X_2] = 0, \\
[Q_1, \bar{c}]_+ = \Gamma_{\sigma\bar{c}^*}\sigma + \Gamma_{X_1\bar{c}^*}X_1 + \Gamma_{X_2\bar{c}^*}X_2, \\
[Q_1, c]_+ = 0.
\end{gathered} \tag{89}$$

The shorthand notations $\Gamma_{\omega_b A^*_{a\mu}}$, $\Gamma_{\omega_b\phi_a^*}$, $\Gamma_{\sigma\bar{c}^*}$, $\Gamma_{X_1\bar{c}^*}$, and $\Gamma_{X_2\bar{c}^*}$ stand for the two-point 1-PI Green functions

$$
\begin{aligned}
\Gamma_{\omega_b A^*_{a\mu}} &= \left.\frac{\delta^2\Gamma}{\delta\omega_b(-p)\delta A^*_{a\mu}(p)}\right|_{\zeta=0},\\
\Gamma_{\omega_b \phi^*_a} &= \left.\frac{\delta^2\Gamma}{\delta\omega_b(-p)\delta \phi^*_a(p)}\right|_{\zeta=0},\\
\Gamma_{\sigma\bar c^*} &= \left.\frac{\delta^2\Gamma}{\delta\sigma(-p)\delta \bar c^*(p)}\right|_{\zeta=0},\\
\Gamma_{X_1\bar c^*} &= \left.\frac{\delta^2\Gamma}{\delta X_1(-p)\delta \bar c^*(p)}\right|_{\zeta=0},\\
\Gamma_{X_2\bar c^*} &= \left.\frac{\delta^2\Gamma}{\delta X_2(-p)\delta \bar c^*(p)}\right|_{\zeta=0},
\end{aligned}
\tag{90}
$$

where ζ is a collective notation for all the fields and external sources of the theory. It is also useful to introduce the scalar form factor $G(p^2)$ for $\Gamma_{\omega_b A^*_{a\mu}}$ by setting

$$
\Gamma_{\omega_b A^*_{a\mu}} = i p_\mu \delta^{ab} G(p^2). \tag{91}
$$

Again the physical Hilbert space $\mathcal{H}_{\rm phys}$ is defined as the intersection of the quotient spaces $\mathcal{H}_0$ and $\mathcal{H}_1$ associated with the conserved charges Q_0 and Q_1.

We study first $\mathcal{H}_0 = \ker Q_0/{\rm Im} Q_0$. From Eq. (88) we get that σ, X_1, X_2 $\bar c$, and c belong to $\mathcal{H}_0$. In the sector spanned by $A_{a\mu}$, ϕ_a, B_a, $\bar\omega_a$, ω_a the analysis proceeds as in the standard treatment given in [14–17]. From the first of Eqs. (88) we obtain that the transverse polarizations $\epsilon_\mu(p)$ of $A_{a\mu}$ (i.e. those obeying

$$
\epsilon_\mu(p)p^\mu = 0 \quad \text{at } p^2 = M_A^2, \tag{92}
$$

where M_A^2 is the renormalized mass of the gauge bosons $A_{a\mu}$) are in $\mathcal{H}_0$. This follows since

$$
\begin{aligned}
[Q_0, \epsilon_\mu(p)A^\mu_a(p)] &= -i\epsilon_\mu(p)\Gamma_{\omega_b A_a^{*\mu}}\omega_b\\
&= \epsilon_\mu(p)p^\mu G(p^2)\omega_a = 0.
\end{aligned}
\tag{93}
$$

In the above equation we have used Eqs. (91) and (92). Equation (86) together with Eqs. (49) and (52) ensures [14–17] that the unphysical modes described by ∂A_a, ϕ_a, $\bar\omega_a$, and ω_a have a common mass M_ξ located at the solution of the equation

$$
\Gamma_{\omega_b\bar\omega_a} = ip^\mu\Gamma_{\omega_b A^*_{\mu a}} + \xi g v \Gamma_{\omega_b\phi^*_a} = 0. \tag{94}
$$

The longitudinal polarization $\rho_\mu(p)$, obeying

$$
\rho_\mu(p)p^\mu = 1 \quad \text{at } p^2 = M_\xi^2, \tag{95}
$$

forms a Q_0-doublet with ω_a:

$$
\begin{aligned}
[Q_0, \rho_\mu(p)A^\mu_a(p)] &= -i\rho_\mu(p)\Gamma_{\omega_b A^*_{\mu a}}\omega_b\\
&= \rho_\mu(p)p^\mu G(p^2)\omega_a.
\end{aligned}
\tag{96}
$$

Thus $\rho_\mu(p)A^\mu_a(p)$ and ω_a do not belong to $\mathcal{H}_0$. ϕ_a is not in the kernel of Q_0 and hence it is outside $\mathcal{H}_0$. Finally $\bar\omega_a$ and B_a form a Q_0-doublet

$$
[Q_0, \bar\omega_a]_+ = B_a \tag{97}
$$

and consequently they are not in $\mathcal{H}_0$. We conclude that $\mathcal{H}_0$ is spanned by X_1, X_2, $\bar c$, c, σ and the three transverse polarizations of $A_{a\mu}$.

The analysis of $\mathcal{H}_1 = \ker Q_1/{\rm Im} Q_1$ at the quantum level requires to discuss the mixing in the sector spanned by σ, X_1, X_2. The relevant two-point functions are controlled by Eqs. (53) and (54). One gets

$$
\Gamma_{\sigma X_1} = \frac{1}{v}(-p^2 + 2\lambda v^2)\Gamma_{\sigma\bar c^*}, \tag{98}
$$

$$
\Gamma_{\sigma X_2} = \frac{1}{v}(-p^2 + 2\lambda v^2)\Gamma_{\sigma\bar c^*}, \tag{99}
$$

$$
\begin{aligned}
\Gamma_{X_1X_1} &= \frac{1}{v}(-p^2 + 2\lambda v^2)\Gamma_{X_1\bar c^*}\\
&= \left(\frac{1}{v}\right)^2(-p^2 + 2\lambda v^2)^2\Gamma_{\bar c^*\bar c^*},
\end{aligned}
\tag{100}
$$

$$
\begin{aligned}
\Gamma_{X_1X_2} &= \frac{1}{v}(-p^2 + 2\lambda v^2)\Gamma_{X_2\bar c^*}\\
&= \frac{1}{v}(-p^2 + 2\lambda v^2)\left[\frac{1}{v}(-p^2 + 2\lambda v^2)\Gamma_{\bar c^*\bar c^*} - v\right],
\end{aligned}
\tag{101}
$$

$$
\begin{aligned}
\Gamma_{X_2X_2} &= \frac{1}{v}(-p^2 + 2\lambda v^2)\Gamma_{X_2\bar c^*} - \frac{v^2}{\rho} + (p^2 - 2\lambda v^2)\\
&= \frac{1}{v}(-p^2 + 2\lambda v^2)\left[\frac{1}{v}(-p^2 + 2\lambda v^2)\Gamma_{\bar c^*\bar c^*} - 2v\right]\\
&\quad - \frac{v^2}{\rho},
\end{aligned}
\tag{102}
$$

where we have used the fact that, again as a consequence of Eqs. (53) and (54),

$$
\begin{aligned}
\Gamma_{X_1\bar c^*} &= \frac{1}{v}(-p^2 + 2\lambda v^2)\Gamma_{\bar c^*\bar c^*},\\
\Gamma_{X_2\bar c^*} &= \frac{1}{v}(-p^2 + 2\lambda v^2)\Gamma_{\bar c^*\bar c^*} - v.
\end{aligned}
\tag{103}
$$

The determinant of the two-point function matrix Γ_2 in the sector spanned by σ, X_1, X_2 is

$$
\det\Gamma_2 = \frac{1}{\rho}(p^2 - 2\lambda v^2)^2(\Gamma^2_{\sigma\bar c^*} - (\rho + \Gamma_{\bar c^*\bar c^*})\Gamma_{\sigma\sigma}) = 0. \tag{104}
$$

Therefore the masses of the particles in this sector are located at

$$
p^2 = 2\lambda v^2 \tag{105}
$$

and at the solution of the equation

$$
\Gamma^2_{\sigma\bar c^*}(p^2) - (\rho + \Gamma_{\bar c^*\bar c^*}(p^2))\Gamma_{\sigma\sigma}(p^2) = 0. \tag{106}
$$

We denote the solution to Eq. (106) by

$$p^2 = \bar{M}^2. \tag{107}$$

We notice the appearance in Eqs. (100)–(102) of the combination $(p^2 - 2\lambda v^2)^2$. Its coefficient must be zero in order to ensure that the asymptotic states are described by pure Klein-Gordon fields (no dipole components). Remarkably, from Eqs. (100)–(102) this requirement can be fulfilled by imposing the single normalization condition

$$\Gamma_{\bar{c}^*\bar{c}^*}|_{p^2=2\lambda v^2} = 0. \tag{108}$$

The above normalization condition is compatible with the symmetries of the theory. It can be imposed order by order in the loop expansion by exploiting the δ-invariant

$$\int d^4x \hat{\bar{c}}^{*2}, \tag{109}$$

which can be freely added to the nth order effective action while preserving all the functional identities of the model.

Next we decompose the two-point function $\Gamma_{\sigma\sigma}$ into its tree-level contribution and the quantum correction $\Sigma_{\sigma\sigma}$ as follows

$$\Gamma_{\sigma\sigma}(p^2) = p^2 - 2\lambda v^2 + \Sigma_{\sigma\sigma}(p^2). \tag{110}$$

It is convenient to use the δ-invariant

$$\int d^4x \left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^2 \tag{111}$$

in order to enforce recursively, order by order in the loop expansion, the normalization condition

$$\Sigma_{\sigma\sigma}|_{p^2=2\lambda v^2} = 0. \tag{112}$$

The analysis of the states spanned by σ, X_1, X_2 can be done by studying the eigenstates of the two-point matrix

$$\Gamma_2 = \begin{pmatrix} \Gamma_{\sigma\sigma} & \Gamma_{\sigma X_1} & \Gamma_{\sigma X_2} \\ \Gamma_{X_1\sigma} & \Gamma_{X_1X_1} & \Gamma_{X_1X_2} \\ \Gamma_{X_2\sigma} & \Gamma_{X_2X_1} & \Gamma_{X_2X_2} \end{pmatrix} \tag{113}$$

at $p^2 = 2\lambda v^2$ and at $p^2 = \bar{M}^2$, respectively.

We first describe the asymptotic states at $p^2 = 2\lambda v^2$. We introduce a vector $\underline{\varphi}_\sharp$ collecting the fields σ, X_1, X_2 (at $p^2 = 2\lambda v^2$) by setting

$${}^T\underline{\varphi}_\sharp = (\sigma_\sharp, X_{1\sharp}, X_{2\sharp}). \tag{114}$$

The subscript $\sharp$ means that σ, X_1, X_2 are taken at $p^2 = 2\lambda v^2$. The solutions of the equation

$$\Gamma_2|_{p^2=2\lambda v^2}\underline{u} = 0, \tag{115}$$

where we have set ${}^T\underline{u} = (u_\sigma, u_{X_1}, u_{X_2})$, parametrize the asymptotic states at $p^2 = 2\lambda v^2$ on the basis spanned by the components of $\varphi_\sharp$. The field corresponding to the vector $\underline{u}$ is thus

$$\varphi_\sharp(\underline{u}) = \underline{u} \cdot \underline{\varphi}_\sharp = u_\sigma\sigma_\sharp + u_{X_1}X_{1\sharp} + u_{X_2}X_{2\sharp}. \tag{116}$$

From Eqs. (98)–(102) and by taking into account Eq. (108) and (112) we get that there are two independent solutions to Eq. (115):

$${}^T\underline{u}_1 = (1, 0, 0), \qquad {}^T\underline{u}_2 = (0, 1, 0) \tag{117}$$

so that

$$\varphi_\sharp(\underline{u}_1) = \sigma_\sharp, \qquad \varphi_\sharp(\underline{u}_2) = X_{1\sharp}. \tag{118}$$

$\underline{u}_1$ and $\underline{u}_2$ allow to introduce a projector $\Pi_{2\lambda v^2}$ on the mass eigenstates at $p^2 = 2\lambda v^2$. $\Pi_{2\lambda v^2}$ acts on any vector ${}^T\underline{w} = (w_1, w_2, w_3)$ as follows:

$$\Pi_{2\lambda v^2}(\underline{w}) = (\underline{u}_1 \cdot \underline{w})\underline{u}_1 + (\underline{u}_2 \cdot \underline{w})\underline{u}_2. \tag{119}$$

Correspondingly the action on $\varphi_\sharp(\underline{w})$ is given by

$$\begin{aligned}\Pi_{2\lambda v^2}(\varphi_\sharp(\underline{w})) &= (\underline{u}_1 \cdot \underline{w})\varphi_\sharp(\underline{u}_1) + (\underline{u}_2 \cdot \underline{w})\varphi_\sharp(\underline{u}_2) \\ &= w_1\sigma_\sharp + w_2 X_{1\sharp}.\end{aligned} \tag{120}$$

From Eq. (89) we see that $\varphi_\sharp(\underline{u}_2) = X_{1\sharp}$ does not belong to $\mathcal{H}_1$ while $\varphi_\sharp(\underline{u}_1) = \sigma_\sharp$ does. Moreover from Eq. (89) we also obtain that c is not in $\mathcal{H}_1$ since it forms a Q_1-doublet with $\frac{1}{v}X_{1\sharp}$. Furthermore $\sigma_\sharp$ is Q_1-exact also at the quantum level. Indeed from Eq. (89) the action of Q_1 on $\bar{c}$ reads

$$[Q_1, \bar{c}]_+ = \Gamma_{\sigma\bar{c}^*}\sigma_\sharp + \Gamma_{X_1\bar{c}^*}X_{1\sharp} + \Gamma_{X_2\bar{c}^*}X_{2\sharp}. \tag{121}$$

Since $\bar{c}$ has support at $p^2 = 2\lambda v^2$, we need to apply the operator $\Pi_{2\lambda v^2}$ to the rhs in order to project it on the subspace of asymptotic states at $p^2 = 2\lambda v^2$. By Eq. (120) we obtain

$$\begin{aligned}[Q_1, \bar{c}]_+ &= \Pi_{2\lambda v^2}(\Gamma_{\sigma\bar{c}^*}\sigma_\sharp + \Gamma_{X_1\bar{c}^*}X_{1\sharp} + \Gamma_{X_2\bar{c}^*}X_{2\sharp}) \\ &= \Gamma_{\sigma\bar{c}^*}\sigma_\sharp + \Gamma_{X_1\bar{c}^*}X_{1\sharp}.\end{aligned} \tag{122}$$

The second term in the second line of the above equation vanishes at $p^2 = 2\lambda v^2$ as a consequence of the first of Eqs. (103). Then one is left with

$$[Q_1, \bar{c}]_+ = \Gamma_{\bar{c}^*\sigma}\sigma_\sharp. \tag{123}$$

Equation (123) implies that $\sigma_\sharp$ is Q_1-exact provided that

$$\Gamma_{\bar{c}^*\sigma}|_{p^2=2\lambda v^2} \neq 0. \tag{124}$$

If Eq. (124) is fulfilled, σ does not belong to $\mathcal{H}_1$. We notice that the condition in Eq. (124) is verified at tree level since

$$\Gamma^{(0)}_{\bar{c}^*\sigma} = v$$

and can be recursively preserved at the quantum level by making use of the δ-invariant

$$\int d^4x \hat{\bar{c}}^*\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right). \tag{125}$$

Moreover, Eq. (124) together with Eq. (112) implies that the solution of Eq. (106) cannot coincide with $p^2 = 2\lambda v^2$. This implies that the solution of

$$\Gamma_2|_{p^2=\bar{M}^2}\tilde{u} = 0 \tag{126}$$

at $p^2 = \bar{M}^2$ (asymptotic state at $p^2 = \bar{M}^2$) is Q_1-invariant. This can be proven as follows. We denote by $\sigma_\flat, X_{1\flat}, X_{2\flat}$ the fields σ, X_1, X_2 at $p^2 = \bar{M}^2$. Then the solution to Eq. (126) is associated to the field

$$\varphi_\flat(\tilde{u}) = \tilde{u}_\sigma \sigma_\flat + \tilde{u}_{X_1} X_{1\flat} + \tilde{u}_{X_2} X_{2\flat}. \tag{127}$$

This is Q_1-invariant, since by Eq. (89) $[Q_1, \sigma_\flat] = 0$, $[Q_1, X_{2\flat}] = 0$, and also $[Q_1, X_{1\flat}] = 0$, due to the fact that by Eq. (41) c has support at $p^2 = 2\lambda v^2$. It cannot be Q_1-exact since the only scalar G-singlet field with negative ghost number is $\bar{c}$, which by Eq. (51) has support at $p^2 = 2\lambda v^2$. $\varphi_\flat(\tilde{u})$ in Eq. (127) is the physical mode which is described at tree level by the field X_2.

From Eq. (89) we also see that $A_{a\mu}$, ϕ_a, B_a, ω_a, $\bar{\omega}_a$ are in $\mathcal{H}_1$. Therefore $\mathcal{H}_1$ is spanned by $A_{a\mu}$, ϕ_a, B_a, ω_a, $\bar{\omega}_a$ and the mode in Eq. (127).

By taking into account the above characterization of $\mathcal{H}_0$ and $\mathcal{H}_1$ we conclude that the space $\mathcal{H}_{\rm phys}$ in Eq. (77) contains the three transverse polarization modes of the gauge field $A_{a\mu}$ and a scalar particle with mass $p^2 = \bar{M}^2$ given by the solution of Eq. (106).

At this point we are in a position to provide the physical interpretation of the parameters $a_1, \ldots, a_{11}$ in Eq. (B13). a_1 is associated with the finite renormalization of the gauge coupling constant, a_2 with that of the mass of the non-Abelian gauge bosons. a_3 has to be used to impose the normalization condition (absence of σ tadpole)

$$\frac{\delta\Gamma}{\delta\sigma}\bigg|_{\zeta=0}(p^2 = 0) = 0. \tag{128}$$

Analogously a_9 allows to set the normalization condition (absence of X_1 and X_2 tadpoles)

$$\frac{\delta\Gamma}{\delta X_1}\bigg|_{\zeta=0}(p^2 = 0) = \frac{\delta\Gamma}{\delta X_2}\bigg|_{\zeta=0}(p^2 = 0) = 0. \tag{129}$$

a_4 is associated with the normalization condition on the two-point function $\Gamma_{\sigma\sigma}$ at zero momentum and is used to enforce Eq. (112). a_5, a_6, a_7, a_8 are associated to finite field redefinitions of ϕ_a, σ, $A_{a\mu}$, ω_a, respectively. By Eq. (106) a_{10} controls the finite renormalization of the mass of the physical scalar mode. Finally the freedom on the choice of a_{11} is used in order to impose Eq. (108), which guarantees the absence of dipole fields in the asymptotic states in the sector spanned by σ, X_1, X_2.

VII. CONCLUSIONS

A polynomial formulation of the Stueckelberg mechanism has been derived by making use of an off-shell nilpotent BRST symmetry. This symmetry is related to a natural Abelian embedding of the Stueckelberg action. The antighost field of the U(1) symmetry is responsible for the implementation of the Stueckelberg constraint. Moreover we have shown that a mass term for the additional U(1) gauge connection B_μ can be introduced in a BRST-invariant way.

We have proven that for the gauge group SU(2) a composite vector field transforming as a connection under the BRST differential (but not under the SU(2) gauge transformations) can be obtained by using the Abelian antighost field. This allows us to generate a new polynomial BRST-invariant but not gauge-invariant mass term for the non-Abelian gauge fields. We have given a sufficient condition for the existence of this type of mass term for a general gauge group G.

The Abelian embedded Stueckelberg model discussed in this paper is not power-counting renormalizable. We have shown that there is a natural theory which extends it to a power-counting renormalizable model. The resulting theory is physically unitary and contains in the physical sector the three physical polarizations of the massive gauge fields as well as a physical scalar particle.

The existence of the conserved charge Q_0 shows that the spontaneous symmetry breaking mechanism is implemented in the model. Since the theory is power-counting renormalizable, one could conjecture that it is physically equivalent to the Higgs model, i.e. that it yields the same physical S-matrix elements. The check of the conjectured equivalence in the full perturbative expansion is an interesting question which deserves to be further investigated.

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APPENDIX A: PROPAGATORS

By setting $B'_a = B_a - \frac{1}{\xi}(\partial A_a + \xi g v \phi_a)$ the propagators for B'_a, $A_{a\mu}$, and ϕ_a are diagonal:

$$\Delta_{B'_a B'_b} = \frac{i}{\xi}\delta_{ab}, \qquad \Delta_{\phi_a\phi_b} = \frac{i}{p^2 - \xi(gv)^2}\delta_{ab},$$
$$\Delta_{A_{a\mu}A_{b\nu}} = i\bigg(\frac{1}{-p^2 + (gv)^2}T^{\mu\nu} + \frac{1}{-\frac{p^2}{\xi} + (gv)^2}\bigg)\delta_{ab}. \tag{A1}$$

Moreover we set $\sigma = \sigma' + X_1 + X_2$. Then the propagators for X_1, X_2, and σ' are

$$\Delta_{\sigma'\sigma'} = \frac{i}{p^2 - 2\lambda v^2}, \qquad \Delta_{X_1X_1} = -\frac{i}{p^2 - 2\lambda v^2},$$
$$\Delta_{X_2X_2} = \frac{i}{p^2 - (2\lambda + \frac{1}{\rho})v^2}. \tag{A2}$$

In the ghost sector

$$\Delta_{\bar{\omega}_a\omega_b} = \frac{i}{-p^2+\xi(gv)^2}\delta_{ab}, \qquad \Delta_{\bar{c}c} = \frac{i}{p^2-2\lambda v^2}. \tag{A3}$$

The remaining off-diagonal mixed propagators are all zero.

APPENDIX B: ANALYSIS OF THE COHOMOLOGY OF δ IN THE ACTIONLIKE SECTOR

The nilpotent linearized ST operator δ acts as follows on the fields and the antifields of the model:

$$\begin{gathered}\delta A_{a\mu} = (D_\mu\omega)_a, \qquad \delta\sigma = -g\omega_a\phi_a, \qquad \delta\phi_a = g(\omega_a(\sigma+v)+\epsilon_{abc}\phi_b\omega_c), \qquad \delta X_1 = vc, \qquad \delta X_2 = 0,\\ \delta\bar{c} = \frac{1}{2}\sigma^2+v\sigma+\frac{1}{2}\phi_a^2-vX_2, \qquad \delta c = 0, \qquad \delta\bar{\omega}_a = B_a, \qquad \delta B_a = 0, \qquad \delta\omega_a = -\frac{1}{2}gf_{abc}\omega_b\omega_c,\\ \delta\widehat{A^*_{a\mu}} = \frac{\delta\Gamma_0}{\delta A_{a\mu}}, \qquad \delta\sigma^* = \frac{\delta\Gamma_0}{\delta\sigma}, \qquad \delta\hat{\phi}^*_a = \frac{\delta\Gamma_0}{\delta\phi_a}, \qquad \delta\hat{\bar{c}}^* = 0, \qquad \delta\omega^*_a = \frac{\delta\Gamma_0}{\delta\omega_a}\end{gathered} \tag{B1}$$

where Γ_0 is given by

$$\Gamma_0 = S_\lambda + \int d^4x\Big(\widehat{A^*_{a\mu}}(D^\mu\omega)_a + \sigma^*(-g\omega_a\phi_a) + g\hat{\phi}^*_a(\omega_a(\sigma+v)+\epsilon_{abc}\phi_b\omega_c) - \frac{1}{2}\omega^*_a g f_{abc}\omega_b\omega_c + \hat{\bar{c}}^*\Big(\frac{1}{2}\sigma^2+v\sigma+\frac{1}{2}\phi_a^2-vX_2\Big)\Big). \tag{B2}$$

It is convenient to decompose δ according to the degree induced by the counting operator for $\hat{\bar{c}}^*$:

$$\delta = \delta_0 + \delta_1, \tag{B3}$$

where δ_0 preserves the number of $\hat{\bar{c}}^*$'s and δ_1 increases it by one. δ_0 is given by

$$\begin{gathered}\delta_0 A_{a\mu} = (D_\mu\omega)_a, \qquad \delta_0\sigma = -g\omega_a\phi_a, \qquad \delta_0\phi_a = g(\omega_a(\sigma+v)+\epsilon_{abc}\phi_b\omega_c), \qquad \delta_0 X_1 = vc, \qquad \delta_0 X_2 = 0,\\ \delta_0\bar{c} = \frac{1}{2}\sigma^2+v\sigma+\frac{1}{2}\phi_a^2-vX_2, \qquad \delta_0 c = 0, \qquad \delta_0\bar{\omega}_a = B_a, \qquad \delta_0 B_a = 0, \qquad \delta_0\omega_a = -\frac{1}{2}gf_{abc}\omega_b\omega_c,\\ \delta_0\widehat{A^*_{a\mu}} = \frac{\delta\Gamma_0}{\delta A_{a\mu}}, \qquad \delta_0\sigma^* = \frac{\delta\Gamma_0}{\delta\sigma}\Big|_{\hat{\bar{c}}^*=0}, \qquad \delta_0\hat{\phi}^*_a = \frac{\delta\Gamma_0}{\delta\phi_a}\Big|_{\hat{\bar{c}}^*=0}, \qquad \delta_0\hat{\bar{c}}^* = 0, \qquad \delta_0\omega^*_a = \frac{\delta\Gamma_0}{\delta\omega_a}.\end{gathered} \tag{B4}$$

δ_1 is zero on all variables but σ^* and $\hat{\phi}^*_a$:

$$\delta_1\sigma^* = (\sigma+v)\hat{\bar{c}}^*, \qquad \delta_1\hat{\phi}^*_a = \phi_a\hat{\bar{c}}^*. \tag{B5}$$

We now derive the most general solution to the equation

$$\delta\Sigma = 0, \tag{B6}$$

where Σ is at most of dimension 4 in the fields, the antifields, and their derivatives and fulfills the same identities as $\Gamma^{(j)}$ in Eqs. (57), (59), (62), (64), and (65). Since $\hat{\bar{c}}^*$ has dimension two, the expansion of Σ in powers of $\hat{\bar{c}}^*$ stops at the second order term

$$\Sigma = \Sigma_0 + \Sigma_1 + \Sigma_2, \tag{B7}$$

where Σ_j contains j $\hat{\bar{c}}^*$'s. Thus Eq. (B6) is equivalent to the coupled set of equations

$$\begin{gathered}\delta_0\Sigma_0 = 0, \qquad \delta_0\Sigma_1 + \delta_1\Sigma_0 = 0,\\ \delta_0\Sigma_2 + \delta_1\Sigma_1 = 0.\end{gathered} \tag{B8}$$

The solution to the first of the above equations is known [11,12,18–20] and can be written in terms of eight independent parameters $a_1, \ldots, a_8$

$$\begin{aligned}\Sigma_0 = {}& a_1\int d^4x G_{\mu\nu a}G^{\mu\nu}_a + a_2\int d^4x (D_\mu\Phi)^\dagger(D^\mu\Phi) + a_3\int d^4x\Big(\Phi^\dagger\Phi - \frac{v^2}{2}\Big) + a_4\int d^4x\Big(\Phi^\dagger\Phi-\frac{v^2}{2}\Big)^2\\ &+ a_5\int d^4x\,\delta_0(\hat{\phi}^*_a\phi_a) + a_6\int d^4x\,\delta_0(\sigma^*\sigma) + a_7\int d^4x\,\delta_0(\widehat{A^*_{a\mu}}A^\mu_a) + a_8\int d^4x\,\delta_0(\omega^*_a\omega_a).\end{aligned} \tag{B9}$$

Evaluation of $\delta_1\Sigma_0$ gives

$$\begin{aligned}\delta_1\Sigma_0 &= \int d^4x\hat{\bar{c}}^*[2(a_6-a_5)g\sigma\omega_a\phi_a + (a_6-2a_5)gv\omega_a\phi_a]\\ &= \delta_0\Big(\int d^4x(-\hat{\bar{c}}^*[(a_6-a_5)\sigma^2+(a_6-2a_5)v\sigma])\Big).\end{aligned} \tag{B10}$$

Therefore the second of Eqs. (B8) is solved by

$$\Sigma_1 = \int d^4x \hat{\bar{c}}^* [(a_6 - a_5)\sigma^2 + (a_6 - 2a_5)v\sigma] + a_9 \int d^4x \hat{\bar{c}}^* + a_{10} \int d^4x \hat{\bar{c}}^* \Big(\Phi^\dagger \Phi - \frac{v^2}{2}\Big), \quad \text{(B11)}$$

where the terms in the second line of Eq. (B11) are δ_0-invariant. a_9, a_{10} are free parameters. Obviously $\delta_1 \Sigma_1 = 0$ and thus the last of Eqs. (B8) reduces to $\delta_0 \Sigma_2 = 0$. By power counting

$$\Sigma_2 = a_{11} \int d^4x \hat{\bar{c}}^{*2}, \quad \text{(B12)}$$

where a_{11} is again a free parameter. Finally we get that the most general solution to Eq. (B6), compatible with Eqs. (57), (59), (62), (64), and (65), is

$$\Sigma = a_1 \int d^4x G_{\mu\nu a} G_a^{\mu\nu} + a_2 \int d^4x (D_\mu \Phi)^\dagger (D^\mu \Phi) + a_3 \int d^4x \Big(\Phi^\dagger \Phi - \frac{v^2}{2}\Big) + a_4 \int d^4x \Big(\Phi^\dagger \Phi - \frac{v^2}{2}\Big)^2 + a_5 \int d^4x \delta_0 (\hat{\phi}_a^* \phi_a) + a_6 \int d^4x \delta_0 (\sigma^* \sigma) + a_7 \int d^4x \delta_0 (\widehat{A^*_{a\mu}} A_a^\mu) + a_8 \int d^4x \delta_0 (\omega_a^* \omega_a) + a_9 \int d^4x \hat{\bar{c}}^* + a_{10} \int d^4x \hat{\bar{c}}^* \Big(\Phi^\dagger \Phi - \frac{v^2}{2}\Big) + a_{11} \int d^4x \hat{\bar{c}}^{*2} + \int d^4x \hat{\bar{c}}^* [(a_6 - a_5)\sigma^2 + (a_6 - 2a_5)v\sigma]. \quad \text{(B13)}$$

We notice that the fact that the dependence of Σ on X_2 is only via the combination $\hat{\bar{c}}^*$ prevents the appearance of further δ-invariants with dimension ≤ 4 like

$$\int d^4x X_2 \Big(\Phi^\dagger \Phi - \frac{v^2}{2}\Big), \qquad \int d^4x X_2^2 \Big(\Phi^\dagger \Phi - \frac{v^2}{2}\Big), \qquad \int d^4x \delta(\sigma^* X_2). \quad \text{(B14)}$$

[1] E. C. G. Stückelberg, Helv. Phys. Acta **11**, 225 (1938).
[2] H. Ruegg and M. Ruiz-Altaba, Int. J. Mod. Phys. A **19**, 3265 (2004).
[3] N. Dragon, T. Hurth, and P. van Nieuwenhuizen, Nucl. Phys. B, Proc. Suppl. **56**, 318 (1997).
[4] A. A. Slavnov, Phys. Lett. B **620**, 97 (2005).
[5] A. A. Slavnov, hep-th/0601125.
[6] R. Ferrari, M. Picariello, and A. Quadri, Phys. Lett. B **611**, 215 (2005).
[7] R. Ferrari and A. Quadri, J. High Energy Phys. 11 (2004) 019.
[8] P. W. Higgs, Phys. Lett. **12**, 132 (1964).
[9] T. W. B. Kibble, Phys. Rev. **155**, 1554 (1967).
[10] G. 't Hooft, Nucl. Phys. **B35**, 167 (1971).
[11] O. Piguet and S. P. Sorella, *Algebraic Renormalization: Perturbative Renormalization, Symmetries and Anomalies*, Lect. Notes Phys. Vol. M28 (Springer-Verlag, Berlin, 1995), p. 1.
[12] G. Barnich, F. Brandt, and M. Henneaux, Phys. Rep. **338**, 439 (2000).
[13] A. Quadri, J. High Energy Phys. 05 (2002) 051.
[14] G. Curci and R. Ferrari, Nuovo Cimento Soc. Ital. Fis. A **35**, 273 (1976).
[15] C. Becchi, A. Rouet, and R. Stora, Phys. Lett. B **52**, 344 (1974).
[16] T. Kugo and I. Ojima, Phys. Lett. B **73**, 459 (1978).
[17] C. Becchi, in *Proceedings of Relativity, Groups, and Topology, Les Houches, 1983*, p. 787.
[18] G. Barnich, F. Brandt, and M. Henneaux, Commun. Math. Phys. **174**, 93 (1995).
[19] G. Barnich, F. Brandt, and M. Henneaux, Commun. Math. Phys. **174**, 57 (1995).
[20] G. Barnich and M. Henneaux, Phys. Rev. Lett. **72**, 1588 (1994).