Nonperturbative gluon pair production from a constant chromo-electric field via the Schwinger mechanism in arbitrary gauge

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We study the nonperturbative production of gluon pairs from a constant SU(3) chromo-electric background field via the Schwinger mechanism. We fix the covariant background gauge with an arbitrary gauge parameter α. We determine the transverse momentum distribution of the gluons, as well as the total probability of creating pairs per unit space time volume. We find that the result is independent of the covariant gauge parameter α used to define arbitrary covariant background gauges. We find that our nonperturbative result is both gauge invariant and gauge parameter α independent.

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I. INTRODUCTION

The chromo-electric flux tube model of particle production [1] which is based on earlier work in QED by Heisenberg and Euler, Schwinger and Weiskopf [2], has been a very useful tool for understanding aspects of particle production in Heavy Ion Colliders such as the RHIC [3] at Brookhaven. One of the main stumbling blocks for making more realistic models which include backreaction has been the problem of finding approximations to QCD which are not only gauge invariant but *also* independent of the gauge fixing parameter α. Although the background field gauge leads to an action which is gauge invariant, the action still depends on the gauge parameter α. In perturbation theory it is known that at the one-loop and two loop level choosing the background Feynman gauge ($\alpha = 1$) simplifies calculations and leads to the same result as a resummation of the Feynman graphs which give a gauge invariant and α independent result [4]. Also some recent results using Pinch techniques [5] suggest that the background Feynman gauge result may be the correct choice to all orders in perturbation theory in the sense of leading to Schwinger Dyson equations that are gauge invariant.

In a recent paper, Nayak and van Nieuwenhuizen [6] calculated the rate for pair production of gluons and p_T distributions from a constant chromo-electric field E^a (a $= 1, \ldots .8$) via the Schwinger mechanism in the background Feynman-t'Hooft gauge (α=1). They found that the results for the p_T distribution of the gluons produced depends on two Casimir/gauge invariant quantities E^aE^a and $[d_{abc}E^aE^bE^c]^2$ in SU(3). This dependence on these two gauge invariant quantities also occurs in quark-antiquark production as found by Nayak in [7]. In the case of quark-antiquark production there is no dependence on the gauge parameter and hence the result found in [7] is trivially gauge invariant and gauge parameter independent. However, in the case of gluon pairs there is a gauge parameter dependence coming from the gauge fixing term and hence one needs to show that the results obtained for case of gluon pairs for $\alpha = 1$ gauge in [6] is also valid in any arbitrary covariant gauge parameter α. In this paper we prove that the nonperturbative result obtained in the paper [6] for the gluon pair case in $\alpha = 1$ gauge, is the correct gauge invariant and gauge parameter α independent result. Thus we find that for this nonperturbative process, the background Feynman-t'Hooft gauge *does* give the correct gauge invariant *and* α independent result. Now that we have an effective action that leads to gauge invariant results for constant external fields, we can use this action with arbitrary time (and space) dependent background fields that obey the generalized Yang-Mills equations and thus study the backreaction problem in analogy to what was done by Cooper and Mottola for the Electric Field problem [8]. This will be the subject of a future paper.

II. BACKGROUND FIELD METHOD OF QCD AND SCHWINGER MECHANISM IN PURE GAUGE THEORY

In the background field method of QCD the gauge field is the sum of a classical background field and the quantum gluon field:

$$A_\mu^a \to A_\mu^a + Q_\mu^a \tag{1}$$

where in the right hand side A_μ^a is the classical background field and Q_μ^a is the quantum gluon field. The gauge field Lagrangian density is given by

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F_{\mu\nu}^a[A+Q]F^{\mu\nu a}[A+Q]. \tag{2}$$

The background gauge fixing is given by [9]

$$D_\mu[A]Q^{\mu a} = 0, \tag{3}$$

where the covariant derivative is defined by

$$D_\mu^{ab}[A] = \delta^{ab}\partial_\mu + gf^{abc}A_\mu^c. \tag{4}$$

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The gauge fixing Lagrangian density is

$$\mathcal{L}_{\rm gf} = -\frac{1}{2\alpha}[D_\mu[A]Q^{\mu a}]^2 \tag{5}$$

where α is any arbitrary gauge parameter, and the corresponding ghost contribution is given by

$$\mathcal{L}_{\rm ghost} = \bar{\chi}^a D_\mu^{ab}[A]D^{\mu,bc}[A+Q]\chi^c = \bar{\chi}^a K^{ab}[A,Q]\chi^b. \tag{6}$$

Now adding Eqs. (2), (5), and (6) we get the Langrangian density for gluons interacting with a classical background field

$$\mathcal{L}_{\rm gluon} = -\frac{1}{4}F^a_{\mu\nu}[A+Q]F^{\mu\nu a}[A+Q] - \frac{1}{2\alpha}[D_\mu[A]Q^{\mu a}]^2 + \bar{\chi}^a K^{ab}[A,Q]\chi^b. \tag{7}$$

To discuss gluon pair production at the one-loop level one considers just the part of this Lagrangian which is quadratic in quantum fields. This quadratic Lagrangian is invariant under a restricted class of gauge transformations. The quadratic Lagrangian for a pair of gluons interacting with a background field A^a_μ is given by

$$\mathcal{L}_{\rm gg} = \frac{1}{2}Q^{\mu a}M^{ab}_{\mu\nu}[A]Q^{\nu b} \tag{8}$$

where

$$M^{ab}_{\mu\nu}[A] = \eta_{\mu\nu}[D_\rho(A)D^\rho(A)]^{ab} - 2gf^{abc}F^c_{\mu\nu} + \left(\frac{1}{\alpha}-1\right)[D_\mu(A)D_\nu(A)]^{ab} \tag{9}$$

with $\eta_{\mu\nu} = (-1,+1,+1,+1)$.

For our purpose we write

$$M^{ab}_{\mu\nu}[A] = M^{ab}_{\mu\nu,\alpha=1}[A] + \alpha'[D_\mu(A)D_\nu(A)]^{ab} \tag{10}$$

where $\alpha' = (\frac{1}{\alpha}-1)$. The matrix elements for the gauge parameter α=1 is given by

$$M^{ab}_{\mu\nu,\alpha=1}[A] = \eta_{\mu\nu}[D_\rho(A)D^\rho(A)]^{ab} - 2gf^{abc}F^c_{\mu\nu} \tag{11}$$

which was studied in [6]. In this approximtaion the ghost Lagrangian density (which we will discuss separately) is given by

$$\mathcal{L}_{\rm ghost} = \bar{\chi}^a D^{ab}_\mu[A]D^{\mu,bc}[A]\chi^c = \bar{\chi}^a K^{ab}[A]\chi^b \tag{12}$$

The vacuum-to-vacuum transition amplitude in pure gauge theory in the presence of a background field A^a_μ is given by:

$$_+<0|0>^A_- = \int[dQ][d\chi][d\bar{\chi}]e^{i(S+S_{gf}+S_{\rm ghost})}. \tag{13}$$

For the gluon pair part this can be written as

$$_+<0|0>^A_- = \frac{Z[A]}{Z[0]} = \frac{\int[dQ]e^{i\int d^4x Q^{\mu a}M^{ab}_{\mu\nu}[A]Q^{\nu b}}}{\int[dQ]e^{i\int d^4x Q^{\mu a}M^{ab}_{\mu\nu}[0]Q^{\nu b}}} = e^{iS^{(1)}_{eff}} \tag{14}$$

where $S^{(1)}_{\rm eff}$ is the one-loop effective action. The nonperturbative real gluon production is related to the imaginary part of the effective action $S^{(1)}_{\rm eff}$ which is physically due to the instability of the QCD vacuum in the presence of the background field. The above equation can be written as

$$_+<0|0>^A_- = \frac{Z[A]}{Z[0]} = \frac{{\rm Det}^{-1/2}M^{ab}_{\mu\nu}[A]}{{\rm Det}^{-1/2}M^{ab}_{\mu\nu}[0]} = e^{iS^{(1)}_{\rm eff}} \tag{15}$$

which gives

$$S^{(1)}_{\rm eff} = -i{\rm Ln}\frac{({\rm Det}[{\rm M}^{\rm ab}_{\mu\nu}[{\rm A}]])^{-1/2}}{({\rm Det}[{\rm M}^{\rm ab}_{\mu\nu}[{\rm A}]])^{-1/2}} = \frac{i}{2}Tr[{\rm Ln}M^{ab}_{\mu\nu}[A] - {\rm Ln}M^{ab}_{\mu\nu}[0]]. \tag{16}$$

The trace Tr contains an integration over d^4x and a sum over color and Lorentz indices. To the above action, we need to add the ghost action. The ghost action is gauge independent and eliminates the unphysical gluon degrees of freedom. The one-loop effective action for the ghost in the background field A^a_μ is given by

$$S^{(1)}_{\rm ghost} = -i{\rm Ln}(DetK) = -iTr\int_0^\infty \frac{ds}{s}[e^{is[K[0]+i\epsilon]} - e^{is[K[A]+i\epsilon]}] \tag{17}$$

where $K^{ab}[A]$ is given by (12). Since the total action is the sum of the gluon and ghost actions, the gauge parameter dependent part proportional to $(\frac{1}{\alpha}-1)$ can be evaluated as an addition to the $\alpha=1$ result and discussed seperately from the $\alpha=1$ calculation done earlier. In what follows we will assume when discussing the $\alpha=1$ calculation that we have included the ghost contribution.

The nonperturbative gluon pair production per unit volume per unit time is related to the imaginary part of this effective action via

$$\frac{dN}{dtd^3x} \equiv {\rm Im}\mathcal{L}_{\rm eff} = \frac{{\rm Im}S^{(1)}_{\rm eff}}{d^4x}. \tag{18}$$

This is the general formulation of Schwinger mechanism in pure gauge theory where $M^{ab}_{\mu\nu}[A]$ is given by Eq. (10) and $M^{ab}_{\mu\nu,\alpha=1}[A]$ is given by Eq. (11).

III. SCHWINGER MECHANISM IN PURE GAUGE THEORY IN ARBITRARY COVARIANT α GAUGE

Using the above formalism the Schwinger mechanism for gluon pair production was studied in [6] in $\alpha=1$ gauge. In this α =1 gauge the final expression for the number of nonperturbative gluon (pair) production per unit time per unit volume and per unit transverse momentum from constant chromo-electric field E^a is given by [6]

$$\frac{dN_{g,g}}{dtd^3xd^2p_T} = \frac{1}{4\pi^3}\sum_{j=1}^{3}|g\lambda_j|\mathrm{Ln}[1 + e^{-(\pi p_T^2)/(|g\lambda_j|)}]. \tag{19}$$

where

$$\lambda_1^2 = \frac{C_1}{2}[1 - \cos\theta], \qquad \lambda_2^2 = \frac{C_1}{2}\left[1 + \cos\left(\frac{\pi}{3} - \theta\right)\right],$$
$$\lambda_3^2 = \frac{C_1}{2}\left[1 + \cos\left(\frac{\pi}{3} + \theta\right)\right], \tag{20}$$

and

$$\cos^3\theta = -1 + 6C_2/C_1^3. \tag{21}$$

They depend only on the two Casimir/gauge invariants for SU(3)

$$C_1 = E^aE^a, C_2 = [d_{abc}E^aE^bE^c]^2, \tag{22}$$

where $a, b, c = 1, \ldots, 8$ are the color indices of the adjoint representation of the gauge group SU(3). In this paper we will check if this result remains same for any arbitrary gauge parameter α.

In the arbitrary gauge parameter case, we have to evaluate the one-loop effective action (Eq. (16)) which is given by

$$S_{\mathrm{eff}}^{(1)} = \frac{i}{2}Tr[\mathrm{Ln}M_{\mu\nu}^{ab}[A] - \mathrm{Ln}M_{\mu\nu}^{ab}[0]], \tag{23}$$

where $M_{\mu\nu}^{ab}[A]$ is given by Eq. (10) and $M_{\mu\nu,\alpha=1}^{ab}[A]$ is given by Eq. (11). To evaluate the trace in Eq. (23) we can evaluate the trace of

$$\mathrm{Tr\,Ln}[M_{\mu\lambda}^{ab}[A]\eta^{\lambda\nu}] = \mathrm{Tr\,Ln}[M_{\mu\lambda,\alpha=1}^{ab}[A]\eta^{\lambda\nu} + \alpha'[D_\mu(A)D^\nu(A)]^{ab}], \tag{24}$$

where we added $\eta^{\lambda\nu}$ inside the trace in $\mathrm{Tr\,Ln}[M_{\mu\nu}^{ab}[A]]$ because this cancels against the free part $\mathrm{Tr\,Ln}[M_{\mu\nu}^{ab}[0]]$ in Eq. (23). The above equation can be split into the $\alpha = 1$ part (which was studied in[6]) and a gauge parameter α dependent part as follows

$$\mathrm{Tr\,Ln}[M_{\mu\lambda}^{ab}[A]\eta^{\lambda\nu}] = \mathrm{Tr\,Ln}[M_{\mu\lambda,\alpha=1}^{ab}[A]\eta^{\lambda\nu}] + \mathrm{Tr\,Ln}[\delta^{ab}\delta_\mu^\nu + \alpha' M_\mu^{-1ac\lambda}{}_{,\alpha=1}[A] \times [D_\lambda(A)D^\nu(A)]^{cb}]. \tag{25}$$

We follow the $\alpha = 1$ case [6] and assume that the constant chromo-electric field is along the z-axis (the beam direction) and we choose the gauge $A_0^a = 0$ so that $A_3^a = -E^a\hat{x}^0$. The color indices (a = 1, ...8) are arbitrary. Now using the relation

$$[D_\mu, D_\nu]^{ab} = -gf^{abc}F_{\mu\nu}^c \tag{26}$$

we find that

$$[D_\nu(A)M_{\alpha=1}^{\nu\mu}[A]]^{ab} = [D^2(A)D^\mu(A)]^{ab} \tag{27}$$

and

$$[M_{\alpha=1}^{\mu\nu}[A]D_\nu(A)]^{ab} = [D^\mu(A)D^2(A)]^{ab} \tag{28}$$

where $M_{\mu\nu,\alpha=1}^{ab}[A]$ is given by Eq. (11). We can use these identities to rewrite Eq. (25) in a very convenient fashion which will allow us to show the α independence of the result. First we multiply $M_{\mu\nu,\alpha=1}^{-1ab}[A]$ from the left in Eq. (28) and obtain

$$[M_{\mu\lambda,\alpha=1}^{-1}[A]M_{\alpha=1}^{\lambda\nu}[A]D_\nu(A)]^{ab} = [M_{\mu\lambda,\alpha=1}^{-1}[A]D^\lambda(A)D^2(A)]^{ab} \tag{29}$$

which gives

$$D_\mu^{ab}(A) = [M_{\mu\lambda,\alpha=1}^{-1}[A]D^\lambda(A)D^2(A)]^{ab} \tag{30}$$

Now multiplying by $[\frac{1}{D^2(A)}]^{bc}$ on the right in the above equation we get

$$\left[D_\mu(A)\frac{1}{D^2(A)}\right]^{ab} = [M_{\mu\lambda,\alpha=1}^{-1}[A]D^\lambda(A)]^{ab}. \tag{31}$$

Multiplying $D_\nu^{ab}(A)$ from the right in the above equation we get

$$\left[D_\mu(A)\frac{1}{D^2(A)}D^\nu(A)\right]^{ab} = [M_{\mu\lambda,\alpha=1}^{-1}[A]D^\lambda(A)D^\nu(A)]^{ab}. \tag{32}$$

Using Eq. (32) in Eq. (25) we get

$$\mathrm{Tr\,Ln}[M_\mu^{ab\nu}[A]] = \mathrm{Tr\,Ln}[M_{\mu,\alpha=1}^{ab\nu,}[A]] + \mathrm{Tr\,Ln}\left[\delta^{ab}\delta_\mu^\nu + \alpha'\left[D_\mu(A)\frac{1}{D^2(A)}D^\nu(A)\right]^{ab}\right] \tag{33}$$

Now let us evaluate the trace of the last term

$$\mathrm{Tr\,Ln}\left[\delta^{ab}\delta^{\nu}_{\mu}+\alpha'\left[D_\mu(A)\frac{1}{D^2(A)}D^\nu(A)\right]^{ab}\right]=\mathrm{Tr}\left[\alpha'\left[D_\mu(A)\frac{1}{D^2(A)}D^\nu(A)\right]^{ab}\right]-\mathrm{Tr}\left[\frac{\alpha'^2}{2}\left[D_\mu(A)\frac{1}{D^2(A)}D^\nu(A)\right]^{ab}\right]+\mathrm{Tr}\left[\frac{\alpha'^3}{3}\left[D_\mu(A)\frac{1}{D^2(A)}D^\nu(A)\right]^{ab}\right]-\mathrm{Tr}\left[\frac{\alpha'^4}{4}\left[D_\mu(A)\frac{1}{D^2(A)}D^\nu(A)\right]^{ab}\right]+\mathrm{Tr}\left[\frac{\alpha'^5}{5}\left[D_\mu(A)\frac{1}{D^2(A)}D^\nu(A)\right]^{ab}\right]+\ldots. \quad (34)$$

Summing the series we obtain

$$\mathrm{Ln}\,[1+\alpha']\mathrm{Tr}\left[\left[D_\mu(A)\frac{1}{D^2(A)}D^\nu(A)\right]^{ab}\right] \quad (35)$$

The trace Tr equals to

$$\mathrm{Tr}\;=\mathrm{tr}_{xy}\,\mathrm{tr}_{ab}\,\mathrm{tr}_{\mu\nu} \quad (36)$$

where

$$\mathrm{tr}\,[\mathcal{O}]_{xy}=\int d^4x<x|\mathcal{O}|x>. \quad (37)$$

Using the cyclic properties of the full trace (Tr) we then obtain:

$$\mathrm{Tr}\left[D_\mu(A)\frac{1}{D^2(A)}D^\nu(A)\right]^{ab}=\mathrm{Tr}\left[\left[D^\nu(A)D_\mu(A)\frac{1}{D^2(A)}\right]^{ab}\right]=8\int d^4x. \quad (38)$$

Using this in Eq. (35) we find

$$\mathrm{Tr\;Ln}\left[\delta^{ab}\delta^{\nu}_{\mu}+\alpha'\left[D_\mu(A)\frac{1}{D^2(A)}D^\nu(A)\right]^{ab}\right]=8\mathrm{Ln}[1+\alpha']\int d^4x=-8\mathrm{Ln}[\alpha]\int d^4x \quad (39)$$

In case of free gluons ($M^{ab}_{\mu\nu}[0]$) we get:

$$\mathrm{Tr\;Ln}\left[\delta^{ab}\delta^{\nu}_{\mu}+\alpha'\left[\partial_\mu\frac{1}{\partial^2}\partial^\nu\delta^{ab}\right]\right]=8\,\mathrm{Ln}[1+\alpha']\int d^4x=-8\,\mathrm{Ln}[\alpha]\int d^4x \quad (40)$$

Hence from Eqs. (23), (33), (39), and (40) we get:

$$S^{(1)}_{\mathrm{eff}}=\frac{i}{2}\mathrm{Tr}[\mathrm{Ln}\,M^{ab\nu}_{\mu}[A]-\mathrm{Ln}\,M^{ab\nu}_{\mu}[0]]=\frac{i}{2}\mathrm{Tr}\,[\mathrm{Ln}\,M^{ab\nu,}_{\mu\alpha=1}[A]-\mathrm{Ln}\,M^{ab\nu,}_{\mu\alpha=1}[0]]=S^{(1)}_{eff,\alpha=1}. \quad (41)$$

Hence the gauge parameter dependence on α exactly cancels from the interacting part and the free part and we get a gauge parameter independent (and gauge invariant!) result for gluon production which is the same as that obtained in the α=1 gauge.

IV. CONCLUSIONS

We have studied nonperturbative gluon (pair) production from a constant chromo-electric field with arbitrary color via the Schwinger mechanism in a class of covariant background gauges described by the gauge parameter α by directly evaluating the path integral. We find that the nonperturbative gluon production rate and its p_T distribution are independent of the gauge parameter α and hence the result is both gauge invariant and gauge parameter α independent. This result will allow us to use the effective two gluon action in the Feynman-t'Hooft gauge as a starting point for studying gluon pair production with backreaction.

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[1] A. Casher, H. Neuberger, and S. Nussinov, Phys. Rev. D **20**, 179 (1979).

[2] W. Heisenberg and H. Euler, Z. Phys. **98**, 714 (1936); J. Schwinger, Phys. Rev. **82**, 664 (1951); V. Weisskopf, kong. Dans. Vid. Selsk. Math-fys. Medd. **XIV** No. 6 (1996); [A. I. Miller, *Early Quantum Electrodynamics: A Source Book* (Cambridge University Press, Cambridge, England, 1994)]; J. Schwinger, Phys. Rev. **82**, 664 (1951).

[3] G. Baym, Phys. Lett. B **138**, 18 (1984); K. Kajantie and T. Matsui, Phys. Lett. B **164**, 373 (1985); K. J. Eskola and M. Gyulassy, Phys. Rev. C **47**, 2329 (1993); G. C. Nayak and V. Ravishankar, Phys. Rev. D **55**, 6877 (1997); Phys. Rev. C **58**, 356 (1998); R. S. Bhalerao and G. C. Nayak, Phys. Rev. C **61**, 054907 (2000); G. C. Nayak, Phys. Lett. B **442**, 427 (1998); J. High Energy Phys. 02 (1998) 005; Y. Kluger, J. M. Eisenberg, B. Svetitsky, F. Cooper, and E. Mottola, Phys. Rev. Lett. **67**, 2427 (1991); Phys. Rev. D **45**, 4659 (1992); F. Cooper, E. Mottola, and G. C. Nayak,

Phys. Lett. B **555**, 181 (2003); G.C. Nayak and W. Greiner, hep-th/0001009; D. Dietrich, G.C. Nayak, and W. Greiner, Phys. Rev. D **64**, 074006 (2001); hep-ph/0009178; J. Phys. G **28**, 2002 (2001); G.C. Nayak, D. Dietrich, and W. Greiner, hep-ph/0104030; D. Dietrich, Phys. Rev. D **68**, 105005 (2003); Phys. Rev. D **70**, 105009 (2004); hep-ph/0408343; Phys. Rev. D **71**, 045005 (2005).

[4] J. Cornwall and J. Papavassiliou, Phys. Rev. D **40**, 3474 (1989); S. Hashimoto, J. Kodaira, Y Yasui, and K Sasaki, Phys. Rev. D **50**, 7066 (1994); J. Papavassiliou, Phys. Rev. D **51**, 856 (1995); J. Papavassiliou, Phys. Rev. D **62**, 045006 (2000).

[5] J. Papavassiliou, J. Phys. G **30**, 203 (2004).

[6] G. Nayak and P. van Nieuwenhuizen Phys. Rev. D **71**, 125001 (2005).

[7] G. C. Nayak, Phys. Rev. D **72**, 125010 (2005).

[8] F. Cooper and E. Mottola, Phys. Rev. D **40**, 456 (1989); Y. Kluger, J. M. Eisenberg, B. Svetitsky, F. Cooper, and E. Mottola, Y. Kluger, J. M. Eisenberg, B. Svetitsky, F. Cooper, and E. Mottola, Phys. Rev. D **45**, 4659 (1992); F. Cooper, J. M. Eisenberg, Y. Kluger, E. Mottola, and B. Svetitsky , Phys. Rev. D **48**, 190 (1993); F. Cooper, J. Dawson, Y. Kluger, and H. Shepard, Nucl. Phys. **A566**, 395c (1994).

[9] G. 't Hooft, Nucl. Phys. **B62**, 444 (1973.