The inside story: Quasilocal tachyons and black holes

Gary T. Horowitz[1] and Eva Silverstein[2]

[1]*Department of Physics, UCSB, Santa Barbara, California 93106, USA*

[2]*SLAC and Department of Physics, Stanford University, Stanford, California 94305-4060, USA*

(Received 20 January 2006; published 15 March 2006)

We analyze the fate of excitations in regions of closed string tachyon condensate, a question crucial for understanding unitarity in a class of black holes in string theory. First we introduce a simple new example of *quasilocal* tachyon condensation in a globally stable AdS/CFT background, and review tachyons' appearance in black hole physics. Then we calculate forces on particles and fields in a tachyon phase using a field theoretic model with spatially localized exponentially growing time-dependent masses. This model reveals two features, both supporting unitary evolution in the bulk of spacetime. First, the growing energy of fields sourced by sets of (real and virtual) particles in the tachyon phase yields outward forces on them, leaving behind only combinations which do not source any fields. Secondly, requiring the consistency of perturbative string theory imposes cancellation of a BRST anomaly, which also yields a restricted set of states. Each of these effects supports the notion of a black hole final state arising from string-theoretic dynamics replacing the black hole singularity.

DOI: 10.1103/PhysRevD.73.064016 PACS numbers: 04.70.Dy

I. INTRODUCTION

There has been recent progress in understanding closed string tachyon condensation and applying it to problems of gravitational interest (e.g. [1–6]). New applications to cosmological and black hole singularities motivate further analysis of the tachyonic phase in order to address basic questions such as unitarity and the process by which black holes explode at the end of Hawking evaporation.

As a time-dependent system, a string background containing closed string tachyon condensation has no *a priori* preferred vacuum state. A string-theoretic version of a Euclidean vacuum is the simplest to control in perturbative string theory [4,7] via its relation to Liouville field theory. However, in many applications, it is important to understand if other vacua are allowed. In this work we begin a detailed study of this question, in the process formulating some simple new examples where tachyon condensation occurs in globally stable string backgrounds.

Most previous discussions have focused on situations where the tachyons are either localized to a small region of space, or occur everywhere at once as in homogeneous cosmologies. In this paper, we examine the intermediate case where a closed string tachyon condenses over a finite region of space. We will refer to this as ''quasilocal tachyons''. This case is of considerable interest for the following reason. Analysis of closed string tachyon condensation yields results indicating that the process smoothly ends spacetime [1–4]. This suggests a perturbative string-catalyzed resolution of spacelike singularities, but raises simple questions about unitarity in string-corrected gravity. For example, if tachyon condensation starts in a finite region of space, what happens to a particle sent into this region? Is the S matrix obtained in the bulk spacetime unitary? Since tachyon condensation is a process which can happen in globally stable systems with asymptotic supersymmetry, and in gravity duals to self contained field theories, these are sharp questions arising in a wide class of backgrounds.

The basic possibilities are twofold:

(1) Multiple states exist in the tachyon condensate phase, entangled with the outside. Since the bulk spacetime does not include the tachyon condensate, this would yield a failure of spacetime unitarity. In the context of AdS/CFT examples, this would mean that the field theory is dual to a system consisting of more than the bulk spacetime string background.

(2) Only a single state is allowed in the tachyon condensate phase in a given system. Evolution in the bulk spacetime remains unitary.

We will present evidence in favor of the second possibility.

We start by introducing a new example of quasilocal tachyon condensation, arising from a moving shell of D3-branes in the gravity dual of gauge theory on a Scherk-Schwarz circle. In this example, at low energies a three-dimensional confining theory is induced time dependently. In the bulk, the region inside the shell is excised by a tachyon condensation process and the resulting spacetime is the AdS soliton [8,9] describing the confined phase of the system. This raises the question of whether information can get stuck in the tachyon condensate phase which replaces the region inside the shell. Similarly, in certain black holes in string theory a tachyon phase replaces the singularity, and the information loss problem can be translated into a question of whether the tachyon condensate phase supports a sector of states entangled with the states in the bulk spacetime.

In order to gain some intuition for the dynamics of particles in the tachyon condensate phase, we next set up and analyze a pure quantum field theory system with some of the same features. Tachyon condensation is described on the worldsheet by a timelike Liouville theory with a semiclassical action deformed by the tachyon vertex operator $\int \mu^2 \mathcal{O} e^{2\kappa X^0}$, where $\mathcal{O}$ is an operator of dimension $\Delta < 2$ and $\kappa^2 = 2 - \Delta$. In the analogue quantum field theory problem, the corresponding deformation of the worldline action constitutes a space-time-dependent mass squared which grows exponentially as a function of time. This suggests that tachyon condensation lifts closed string modes and ultimately spacetime itself, and indeed basic amplitudes in the perturbative string theory are smoothed out by the tachyon term and produce explicit results similar to those of the corresponding field theory analogue [4,7].

In ordinary quantum field theory in Minkowski space, there are two asymptotic regions (in the massless case, past and future null infinity $\mathcal{J}^{\mp}$). As we will see in detail, quantum field theory with a localized region where the particle masses increase sufficiently rapidly with time (and homogeneously in space) yields a new candidate asymptotic region in which free classical particles can get stuck and quantum mechanical wavepackets stop expanding.

This by itself raises the possibility of nontrivial states in the tachyon phase. However, there are two important features of the physics (also captured in the QFT model) which suggest that this is not the case. First, would-be trapped particles generically source other fields. These sourced fields become heavy in the tachyon phase, leading to outward forces on such configurations. This may lead to evacuation of any configurations of (real or virtual) particles in the tachyon phase which source any components of the string field. Secondly, imposing perturbative BRST invariance forces correlations among members of the set of (real or virtual) particles surviving in the tachyon phase (related to the analysis in [10]). Both of these effects are reminiscent of the suggestion [11] for resolving the apparent contradiction between semiclassical calculations and unitarity of black hole evaporation; moreover the former may provide a dynamical mechanism for satisfying the latter.

As a simple illustration, consider a particle inside the shell of D3-branes. It is clear that the particle cannot simply remain inside when the tachyon condenses. This is because the graviton also gets lifted in the tachyon phase [4,7], screening the energy of the particle. Since the total AdS energy must be conserved, the particle must either be forced out classically or quantum mechanically (by pairing up with the negative energy partner of a particle creation event).

In the next section we describe the shell example in some detail, and also review the appearance of tachyons inside black holes in both asymptotically flat and asymptotically AdS spacetimes. In section three we introduce our quantum field theory model for the dynamics of particles and fields in the tachyon phase (which is a generalization of [10,12]) and analyze in detail the behavior of excitations in this model. We note the appearance of a BRST anomaly in the worldline description, which plausibly generalizes to a nontrivial consistency condition in perturbative string theory. The last section contains a discussion of applying these ideas to more realistic black holes such as Schwarzschild, and also some open questions.

II. EXAMPLES OF QUASILOCAL TACHYONS

The first example of quasilocal tachyon condensation appeared in [1]. There, a winding tachyon condensation process splits a Scherk-Schwarz cylinder in two. This can yield decay of handles on Riemann surface target spaces, as well as processes where the surface disconnects into multiple components. Although the discussion in [1] focused on the topology change process in the bulk of spacetime, the fact that the spatial derivatives were required to be small means that the tachyon condensation was important over a finite region of space. Hence this is an example of quasilocal tachyons. However, in this example, it turns out that small inhomogeneities in the tachyon field alone serves to repel most particles from the region.

In this section we introduce several more examples, including ones where the tachyon gradient alone would permit free particles to get trapped inside the tachyonic region. In the next section we will analyze the dynamics of modes sent into the tachyon phase in this class of examples.

A. Shell of D3-branes

Our next example provides a setup where tachyon condensation effects a time-dependent transition between the gravity dual of a compactified field theory on its Coulomb branch and the gravity dual of a confining field theory. The tachyon condensate replaces the region of the spacetime which would correspond to the deep infrared limit of the field theory.

Consider a spherical shell of D3 branes, i.e., the branes are arranged in an S^5 in the six dimensional space transverse to all the branes. One can consider either the asymptotically flat, or asymptotically AdS versions of this example. To use the insights from the AdS/CFT correspondence, we will focus on the asymptotically AdS case. Configurations like this have been discussed in, e.g., [13,14] (see also [15] for a different type of shell). The static metric is $AdS_5 \times S^5$ outside the shell, and ten dimensional flat space inside. Explicitly:

$$ds^2 = h^{-1}(r)[-dt^2 + dx_i dx^i] + h(r)[dr^2 + r^2 d\Omega_5] \tag{2.1}$$

where $i = 1, 2, 3$ and

$$h(r) = \frac{\ell^2}{r^2}(r > R), \qquad h(r) = \frac{\ell^2}{R^2}(r < R) \tag{2.2}$$

As usual, the AdS radius ℓ is related to the number of branes N and string coupling g via $\ell^4 = (4\pi g N) l_s^4$. The constant R is the coordinate radial position of the shell, but the above geometry is in fact independent of R. The proper radius of the shell is always ℓ. We now compactify one of the x_i with period L and put antiperiodic boundary conditions for fermions. This identification causes the geometry to depend on R, as this enters into the size of the circle for small radius. In fact this size grows to infinity at the boundary, decreases linearly as r decreases toward R, and is constant at LR/ℓ everywhere inside the shell $r < R$.

Now imagine that we add a little kinetic energy so the shell slowly contracts. To leading order in small velocities, the metric is given by (2.1) and (2.2) with R a slowly decreasing function of time. Ignoring tachyons, we would eventually form a slightly nonextremal black 3-brane. But the horizon only arises at a very small radius depending the energy we add. Long before this, the radius of the circle reaches the string scale inside and on the shell. This occurs when

$$\frac{LR}{\ell} = l_s \tag{2.3}$$

Just outside the shell the winding tachyon instability should cause the circle to pinch off [1]. This removes the shell, and everything inside. The result is a "bubble of nothing". Unlike the bubble proposed by Witten to describe the decay of the Kaluza-Klein vacuum [16], these bubbles can have nonzero mass and be static, rather than expanding. Inside the shell there is a region of tachyon condensation along a spacelike surface (Fig. 1). The similarity to spacetimes describing black hole evaporation is striking. This example allows us to address questions about how information gets out in a simpler setting, without the complications of Hawking radiation and large curvature. Note that even though the low energy spacetime description is a "bubble of nothing", in string theory the interior should really be thought of as the tachyon condensate.

There is a natural candidate to describe the spacetime after the bubble settles down. This is the product of S^5 and the static AdS soliton [8,9]:

$$ds^2 = \frac{r^2}{\ell^2}[-dt^2 + f d\chi^2 + dy_i dy^i] + \frac{\ell^2}{r^2 f} dr^2 \tag{2.4}$$

where

$$f(r) = 1 - \frac{r_0^4}{r^4} \tag{2.5}$$

and $i = 1, 2$. The radial coordinate satisfies $r \geq r_0$ and $r = r_0$ is the bubble. Regularity at the bubble requires χ to be periodic with period

$$L = \pi\ell^2/r_0 \tag{2.6}$$

The mass of (2.4) was computed in [9] and found to be negative:

$$E_{\rm sol} = -\frac{\pi^3\ell^3 V_2}{16 G_5 L^3} = -\frac{\pi^2}{8}\frac{N^2 V_2}{L^3} \tag{2.7}$$

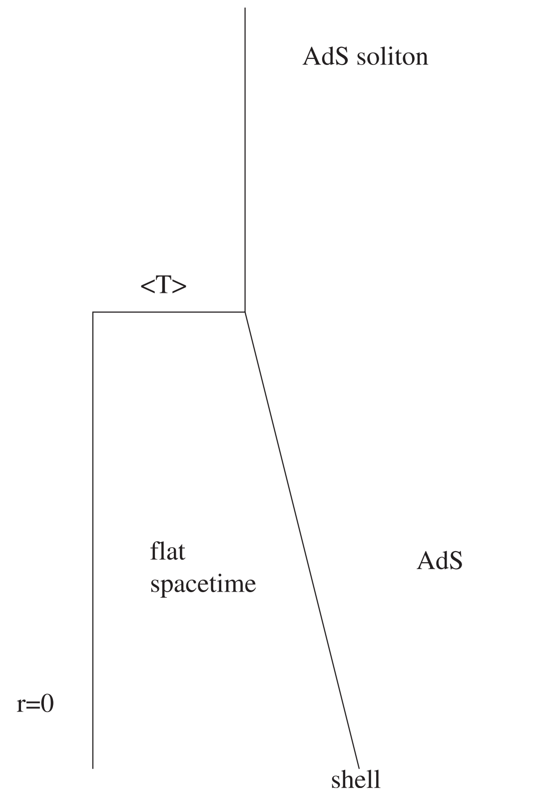

FIG. 1. A shell of D3-branes slowly contracts. The spacetime outside is approx- imately $AdS_5 \times S^5$, while the spacetime inside is approximately flat. The branes are wrapped around a Sherk-Schwarz circle, and when this circle reaches the string scale, the winding tachyons condense. The exterior geometry becomes a bubble which settles down to the AdS soliton (cross S^5). We will be interested in the fate of excitations in the $\langle T\rangle$ region.

where V_2 is the volume of y^i. This negative energy agrees (up to a factor of 3/4) with the Casimir energy of a weakly coupled $\mathcal{N} = 4$ super Yang-Mills theory compactified on a circle of radius L with antiperiodic fermions, though as we will discuss further below, the field theory is expected to confine in the infrared. Since the original spacetime had essentially zero energy, the transition to the bubble produces radiation in the AdS soliton background.

Most of this radiation is not produced immediately when the tachyon condenses. Instead, one first produces a nonstatic bubble, with energy close to zero, which expands out and eventually settles down to the AdS soliton plus radiation. To see this, let us ask what is the size of the χ circle just before it starts to pinch off in the AdS soliton. This is given by the radius of the circle when r is a few times larger than r_0, which is of order $r_0 L/\ell \sim \ell$. In other words, the AdS soliton (2.4) describes a bubble in which the circle pinches off at the AdS radius rather than the string scale.

It is easy to write down time symmetric initial data to describe bubbles where the circle pinches off at various radii. Assuming the same spatial symmetries as the AdS soliton, a general four dimensional metric can be written in the form

$$ds^2 = U(r)d\chi^2 + \frac{dr^2}{U(r)F(r)} + \frac{r^2}{\ell^2}dy_i dy^i \tag{2.8}$$

Since the extrinsic curvature is assumed to vanish, the only restriction is the Hamiltonian constraint:

$$\mathcal{R} = -F'\left(\frac{2U}{r} + \frac{U'}{2}\right) - F\left(\frac{2U}{r^2} + \frac{4U'}{r} + U''\right) = -\frac{12}{\ell^2} \tag{2.9}$$

One can pick $U(r)$ arbitrarily and solve for $F(r)$. Since we want U to vanish at r_0 and asymptotically be r^2/ℓ^2, we can choose for example

$$U(r) = \frac{r^2}{\ell^2} - \frac{r_0^4}{\ell^2 r^2} \tag{2.10}$$

The solution for F turns out to be

$$F(r) = 1 + \frac{b}{3r^4 - r_0^4} \tag{2.11}$$

where b is an arbitrary constant. If $b = 0$, this corresponds to a static slice in the AdS soliton.

Regularity at $r = r_0$ requires that χ be periodically identified with period

$$L = \pi\ell^2\left(r_0^2 + \frac{b}{2r_0^2}\right)^{-1/2} \tag{2.12}$$

Using the freedom in b, we can obtain initial data for any L and bubble location r_0. From (2.12), we simply choose

$$b = 2r_0^2\left(\frac{\pi^2\ell^4}{L^2} - r_0^2\right) \tag{2.13}$$

The total energy of this initial data is easily computed with the result

$$E = \frac{V_2}{16G_5\ell L}\left(\frac{r_0^4 L^2}{\pi\ell^4} - 2\pi r_0^2\right) \tag{2.14}$$

Imagine fixing L and varying r_0. For small r_0 this energy is negative and decreases quadratically in r_0. It reaches a minimum when $r_0 = \pi\ell^2/L$ and then increases, becoming positive and arbitrarily large at large r_0. The minimum corresponds to $b = 0$, consistent with the conjecture that the AdS soliton is the minimum energy solution with these boundary conditions [9].

In our collapsing shell example, the circle pinches off when it reaches the string scale, which corresponds to $r_0 = \ell l_s/L$. So its energy is only slightly negative. It will probably evolve out, oscillate around the AdS soliton and eventually settle down.[1]

Inside the shell, the geometry is similar to a spatially flat collapsing Robertson-Walker universe. However, it is not exactly this, due to retardation effects. Let us specify that the system starts in the ground state in the interior, and we evolve by moving the shell of branes first. Then along a spatially flat surface, the size of the S^1 will be slightly larger at the origin than near the shell. In terms of proper time τ and proper radius ρ inside the shell, the size of the circle is $\mathcal{L} = LR(\tau + \rho - \ell)/\ell$. So along a constant τ surface

$$\frac{d\mathcal{L}}{d\rho} = \frac{L}{\ell}\frac{dR}{d\tau} \tag{2.15}$$

We now make some comments about the dual CFT. With supersymmetric boundary conditions, the initial shell would be described by a point on the Coulomb branch of $D = 4$, $\mathcal{N} = 4$ SYM. However, since the theory is compactified on a circle (of length L) with antiperiodic fermions, the fermions get masses of order $1/L$ at tree level, and since supersymmetry is broken the scalar masses are unprotected. The Coulomb branch is lifted, and the shell cannot stay static. However, the potential on the Coulomb branch is a small effect in our system if we tune the bulk string coupling very small. We are interested in a time-dependent evolution (obtained by tuning parameters and initial conditions as necessary) in which the velocity is small as the circle crosses the string scale in proper size. At very low energies, the theory enters a confining phase of bosonic $D = 3$ Yang-Mills. This is the dual of the AdS soliton; the tachyon condensation excises the would-be IR region of the geometry, reflecting the mass gap. Glueball masses computed from the gravity side are of order $1/L$ [17].

Regarding other tunable parameters in our system: instead of simply changing the shell radius, one can also induce a tachyon transition by making the radius of the circle on which the CFT is compactified decrease with time. In this case, the CFT is living on a time-dependent spacetime (which can be chosen supersymmetric in the far past, for example). If we start in the ground state in the bulk, the information of the changing radius propagates from the boundary to the interior, so that again the Scherk-Schwarz circle in the bulk shrinks later near $r = 0$. This again means that the tachyon turns on first near the shell and later near the origin, which will be important in our analysis of the forces in the problem in Sec. III.

Similar shell examples could be constructed out of D1-D5 branes, or M2 and M5 branes. However, rather than describe these in detail, we move on to examples involving black holes.

B. Asymptotically flat black holes

As discussed in [2], exactly the same type of Scherk-Schwarz winding tachyons appear inside some extended black holes. If a black p-brane carrying RR charge is

[1]Even though the bubble will probably be momentarily at rest when it first forms, it need not have exactly the form of U chosen in (2.10). The initial data above is simply illustrative of the general behavior.

wrapped around a circle, the size of the circle goes to zero at the singularity. Suppose one takes a collection of D-branes and collapses them together to form a charged black brane. We will assume that the branes are wrapped around a circle with antiperiodic boundary conditions. If the initial kinetic energy is very small, the situation will be similar to the shell discussed above. The tachyon instability will set in outside the branes before the horizon is formed, and produce a Kaluza-Klein bubble. Let us assume the initial kinetic energy is large enough to form a horizon. Then the tachyon instability occurs inside the horizon along a space-like surface. This can happen when the curvature is still small so α' corrections are negligible, and the geometry is slowly varying so string creation effects are small. Past this point, the evolution is dominated by the physics of tachyon condensation and no longer given by supergravity. Outside the horizon, Hawking radiation causes the black brane to approach extremality and the size of the circle at the horizon to shrink. When the circle reaches the string scale, tachyon condensation will cause it to pinch off, again producing a bubble. So Hawking radiation eventually causes all (RR charged) black branes to turn into bubbles.

As a specific example, consider the black three-brane

$$ds^2 = H^{-1/2}[-f dt^2 + dx_i dx^i] + H^{1/2}[f^{-1} dr^2 + r^2 d\Omega_5] \tag{2.16}$$

where

$$H(r) = 1 + \frac{\ell^4}{r^4}, \qquad f(r) = 1 - \frac{r_0^4}{r^4} \tag{2.17}$$

If we periodically identify one of the directions along the brane with period L and put antiperiodic boundary conditions, then the tachyon instability arises at a radius r_i when $H(r_i)^{-1/4} L = l_s$ which can lie inside the horizon. We now show that this can also arise when the curvature is small and the geometry is slowly changing. Since we want $L \gg l_s$, $H(r_i) \gg 1$, so near $r = r_i$, $H \approx \ell^4/r^4$ and the geometry is a product of S^5 with radius ℓ and a five dimensional black brane. The circle will reach the string scale when (cf (2.3)) $r_i = \ell l_s/L$. The Ricci curvature is of order $1/\ell^2$, but the Weyl curvature is larger inside the horizon and of order

$$C_{abcd} \sim \frac{r_0^4}{r^4 \ell^2} \tag{2.18}$$

So at the point where the tachyon instability arises, the curvature is of order $r_0^4 L^4/l_s^4 \ell^6$. For any horizon radius r_0, this can clearly be made small by taking $\ell \gg L$. The unit timelike normal to the constant r surfaces is $-f^{1/2} H^{-1/4} \partial/\partial r$. So the rate of change of the size of the circle is

$$\dot{L}_{S^1} = -f^{1/2} H^{-1/4} L \frac{\partial}{\partial r} H^{-1/4} = -\frac{(r_0^4 - r^4)^{1/2} L}{r \ell^2} \tag{2.19}$$

At the point where the tachyon instability begins, this gives $|\dot{L}_{S^1}| < r_0^2 L^2 / l_s \ell^3$ which is again small whenever $\ell \gg L$.

It was shown in [2] that to end up with a static, asymptotically flat bubble there is a restriction on the total charge and L which essentially reduces to $\ell < L$. Thus the condition to get tachyon condensation inside the horizon with small curvature and slow time derivatives ($\ell \gg L$) is incompatible with forming a static bubble outside the horizon. When these black branes evaporate to the point where the circle at the horizon reaches the string scale, the resulting bubble must expand. Even when a static bubble exists as the final endstate, there will be evolution in the bubble before it settles down, just as we saw for the shell above.[2]

C. Asymptotically AdS black holes

Simply dropping the one in the definition of H (2.17) converts the asymptotically flat black brane above into an asymptotically $\mathrm{AdS}_5 \times S^5$ solution. However there is an important difference between the asymptotically flat and asymptotically AdS cases. As we have said, in the asymptotically flat case, Hawking radiation will always cause the size of the circle to shrink at the horizon, so one always evolves to a bubble on the outside. In contrast, AdS acts like a confining box, so a typical black hole will evaporate a small fraction of its mass and quickly come into thermal equilibrium with its own Hawking radiation. Since tachyon condensation can still occur inside, these are examples of eternal black holes with tachyon condensates inside.

Even if one lowers the temperature of the external AdS, one cannot always induce a tachyon transition outside the horizon. For example, the Hawking temperature of the black three-brane is $T \sim r_0/\ell^2$, and its energy is $E_{bb} \sim N^2 T^4 V_2 L$. The circle reaches the string scale at the horizon when $L r_0/\ell = l_s$. This corresponds to a temperature

$$T \sim \frac{l_s}{\ell} \frac{1}{L} \sim \frac{1}{(gN)^{1/4}} \frac{1}{L} \ll \frac{1}{L} \tag{2.20}$$

This is a very low temperature. As discussed in [18], if we lower the temperature of the external AdS, there is a first order phase transition which in the CFT corresponds to a confining/deconfining transition. In the gravity side, the transition is between the AdS soliton and the black brane: One nucleates bubbles of the soliton on the brane. This happens when the free energy of the black brane is equal to the free energy of a gas in the soliton with the same total energy. This occurs when $E_{bb} \sim |E_{\mathrm{sol}}|$ which implies (2.7)

$$N^2 T^4 V_2 L \sim \frac{N^2 V_2}{L^3} \tag{2.21}$$

[2] This does not contradict the condition used in [2] that the area of the static bubble should agree with the area of the spherical cross-section of the horizon. That corresponds to the fact that the S^5 does not change during the evolution of the bubble in the shell example.

That is, when $T \sim 1/L$, which is much higher than the tachyon transition.

As another example, consider the BTZ black hole

$$ds^2 = -\left(\frac{r^2 - r_0^2}{\ell^2}\right)dt^2 + \left(\frac{\ell^2}{r^2 - r_0^2}\right)dr^2 + r^2 d\varphi^2 \quad (2.22)$$

Since global AdS_3 has antiperiodic fermions around the φ circle, any black hole formed from collapse in AdS_3 must have this property. The curvature is constant and set by ℓ so there are no large curvature effects provided $\ell \gg l_s$. The rate of change of the φ circles with respect to proper time inside the horizon is $\dot{r} \approx -r_0/\ell$ so provided $l_s \ll r_0 \ll \ell$ there are no large time derivatives and string creation effects are negligible. Tachyon condensation will occur inside the horizon along the surface $r \sim l_s$. If the black hole can evaporate down to $r_0 \sim l_s$, then tachyon condensation at the horizon will cause the circle to pinch off and one is left with radiation in global AdS_3. However, as we have already mentioned, it is very unlikely that the BTZ black hole will evaporate down this far.

The existence of the equilibrium configuration for black holes in AdS provides a useful simplified version of the problem of Hawking evaporation. If we throw a small amount of energy into the black hole, it radiates a small amount of Hawking radiation and returns to equilibrium. Hence, this can be viewed as an intermediate case between the shell example (with no Hawking radiation) and the asymptotically flat black holes (with large amounts of Hawking radiation).

D. AdS/CFT and unitarity

The examples we have discussed which are embedded in asymptotically AdS geometry benefit from the dual field theory perspective. This guarantees that the full system is unitary. This alone does not guarantee that the unitarity is maintained exclusively in the bulk spacetime; *a priori* it is possible that the tachyon phase supports excitations dual to field theoretic ones. The field theory also guarantees that energy is conserved, and that global symmetries are respected. These quantities are determined by graviton and gauge field behavior at the boundary of AdS. In this work we will focus on the direct gravity-side physics of the tachyon phase, but will return to these constraints from AdS/CFT after some further analysis.

III. PARTICLE AND FIELD DYNAMICS IN THE $\langle T\rangle$ PHASE

In this section, we will gain some intuition for the dynamics of objects sent into the tachyon phase by studying a field theory analogue. As discussed, for example, in [4,7], the tachyon contribution to the semiclassical worldsheet action behaves in some ways like a spacetime-dependent mass squared term for particles in the tachyon background. Full perturbative string computations of the partition function and of the number of pairs of string produced in the time-dependent background yield results identical to those of a field theory with spacetime-dependent masses. This suggests that the corresponding field theory model is a good guide to the dynamics, and we will analyze the forces on particles in the presence of fields with a spacetime-dependent mass. To start, we will discuss free particles, and then generalize to the case of more direct interest in which the particles source fields which are themselves becoming heavy in a spacetime-dependent way.

Consider a scalar field theory on d-dimensional Minkowski space with a spacetime-dependent mass. Its Lagrangian density is

$$\mathcal{L} = -(\partial\phi)^2 - m^2(\vec{x}, x^0)\phi^2 + \mathcal{L}_{\text{interaction}} \quad (3.1)$$

We will be interested in the case where the mass is of the following form, and constant except in a finite region of space

$$m^2(x^0, \vec{x}) = M^2(x^0)f(r) + m_0^2 \quad (3.2)$$

where f has support in a finite region $r < \tilde{L}$, and $M \to \infty$ as $x^0 \to \infty$. A case of particular interest will be one inspired by tachyon condensation, where

$$M_T^2(x^0) = \mu^2 e^{2\kappa x^0} \quad (3.3)$$

but we will consider the problem in more generality. Several important features will depend on whether $M(x^0)$ grows to infinity faster or slower than linearly in x^0.

We will consider both the second quantized description of this system via (3.1) and a first quantized description via the worldline action

$$\mathcal{S}_{\text{wl}} = \int d\tau \sqrt{g_{00}} \frac{1}{2}(-g^{00}(\dot{x}^0)^2 + g^{00}(\dot{\vec{x}})^2 - m^2(x^0, \vec{x})) \quad (3.4)$$

which is a functional of the embedding coordinates $x^\mu(\tau)$ and the worldline metric $g_{00}(\tau)$. Integrating over the worldline metric yields the equivalent form

$$\tilde{\mathcal{S}}_{\text{wl}} = \int dx^0 m(\vec{x}, x^0)\sqrt{1 - \left(\frac{d\vec{x}}{dx^0}\right)^2} \quad (3.5)$$

which is useful classically. In the quantum theory, we may employ a BRST quantization to obtain an action

$$\mathcal{S}_{\text{BRST}} = \int d\tau \left(\frac{1}{2}\dot{x}^\mu \dot{x}_\mu - \frac{1}{2}m^2(x^0, \vec{x}) - \dot{b}c\right) \quad (3.6)$$

in terms of a standard $b - c$ ghost system.

A. Particle mechanics

Let us start by determining the trajectories of free particles, focusing on their behavior in the $r < \tilde{L}$ region. Starting from (3.5) we find the equation of motion

$$\frac{d}{dx^0}\frac{m\frac{d\vec{x}}{dx^0}}{\sqrt{1-(\frac{d\vec{x}}{dx^0})^2}}+\frac{m}{\vec{x}}\sqrt{1-\left(\frac{d\vec{x}}{dx^0}\right)^2}=0 \tag{3.7}$$

To begin, let us take $f(r)$ to be constant in the $r < \tilde{L}$ region. Then in this region the solutions satisfy

$$\frac{m\frac{d\vec{x}}{dx^0}}{\sqrt{1-(\frac{d\vec{x}}{dx^0})^2}}=p \tag{3.8}$$

for constant momentum p. This yields $\frac{d\vec{x}}{dx^0}=\pm p/\sqrt{m^2(x^0,\vec{x})+p^2}$ from which we learn that the distance travelled by a particle of momentum p in the $r < \tilde{L}$ region is

$$|\Delta\vec{x}|=p\int_{t_1}^{t_2}dx^0\frac{1}{\sqrt{m^2(x^0,\vec{x})+p^2}} \tag{3.9}$$

If m grows faster than linearly with x^0 at large x^0, then this distance $|\Delta\vec{x}|$ is finite even if we allow infinite time $t_2 \to \infty$. Otherwise, the total distance diverges as $t_2 \to \infty$ from the large x^0 end of the integral.

In the case of exponential growth (3.3), the distance travelled within the region $r < \tilde{L}$ from time $t_1 = 0$ to $t_2 = \infty$ is

$$|\Delta\vec{x}|=\frac{p}{\omega_0\kappa}\mathrm{ArcSinh}\left(\frac{\omega_0}{\mu}\right) \tag{3.10}$$

where $\omega_0=\sqrt{p^2+m_0^2}$. This grows like $(1/\kappa)\log(\omega_0/\mu)$ for large ω_0. For finite $\tilde{L}$, there is a finite window of frequency

$$\omega_0<\omega_*=\mu\sinh(\tilde{L}\kappa\omega_*/\sqrt{\omega_*^2-m_0^2})$$
$$\sim_{(\omega_*\gg m_0)}\mu\sinh(\tilde{L}\kappa) \tag{3.11}$$

for which a free particle sent into the $r < \tilde{L}$ region at $t = 0$ stays inside this region.

Near $r = \tilde{L}$, $f(r)$ decreases to zero. The gradient term in (3.7) is now important since it is amplified by the increase in the $M(t)$ factor. This prevents particles from entering the region $r < \tilde{L}$ at late time. If $f(r)$ has no local minima in the massive phase, then even particles which enter this region at early time are simply repelled out of the massive phase before $x^0 = \infty$. In our quasilocal tachyon problems, the challenge to unitarity arises in the case that the spatial dependence of the tachyon terms in the worldsheet action does not suffice to classically repel all impinging free particles. For example in the shell discussed in sec. II A, the retardation effects produce a tachyon gradient propelling the particles deeper into the central region.

So far this was purely classical. In quantum mechanics, propagation of particles is described by wave packets which have nontrivial extent in $\vec{x}$. Since wavepackets typically spread out in time, one might have thought that the probability of finding the particle in a finite region $\tilde{L}$ at late time would always go to zero. However this is not the case. Consider sending in a Gaussian wavepacket into the $r < \tilde{L}$ region at time $x^0 = 0$. At large x^0, since the mass is increasing the particle may be described to good approximation by nonrelativistic quantum mechanics. A simple analysis reveals that quantum mechanical wavepackets stop spreading at late times precisely when m grows faster than linearly with x^0.

We have seen that if $m(x^0)$ grows faster than linearly with x^0 at late times, free particles with $\omega_0 < \omega_*$ (3.11) can get stuck in the $r < \tilde{L}$ region. Next we will consider interactions, in particular, coupling our particles to fields which are also gaining mass in the central region.

B. Interactions and field energy

Now consider coupling our particle to a field η which is also getting massive:

$$-\mathcal{L}=(\phi)^2+m^2(\vec{x},x^0)\phi^2+(\eta)^2+m_\eta^2(\vec{x},x^0)\eta^2+\lambda\eta j \tag{3.12}$$

where j is a current of ϕ particles. For definiteness we will consider below the case where η couples to the energy density of ϕ particles. Here

$$m_\eta^2=M_0^2+f(r)M^2(x^0). \tag{3.13}$$

Particles of ϕ source η fields. As the particle propagates into the tachyon phase, it drags its η field along, but this field is getting heavy as well. This contributes to the force on the configuration and must be taken into account in determining whether nontrivial states survive in the tachyon phase.

Let us calculate the energy contained in the η field sourced by a ϕ particle. The field classically is given by solving

$$(\partial_0^2-\vec{\nabla}^2+m_\eta^2(x^0,\vec{x}))\eta=\lambda j(x) \tag{3.14}$$

with a particle source

$$j(x)\sim m(\vec{x},x^0)\delta(\vec{x}-\vec{v}x^0). \tag{3.15}$$

Let us consider a massive slowly moving particle, and neglect the velocity $\vec{v}$ here. The energy contained in the field η is given by

$$E=\int d\vec{x}(\dot{\eta}^2+(\vec{\nabla}\eta)^2+m_\eta^2\eta^2) \tag{3.16}$$

This field energy will depend on how far inside the tachyon phase the particle source sits, and hence will lead to additional forces beyond those obtained in Sec. III A just from the particle mass itself.

Before analyzing this problem explicitly, let us indicate the main point. In the far past, the particle source has a constant mass m_0, and generates a field η scaling like $e^{-M_0 r}/r^{d-3}$ as a function of the distance r from the source. As the time dependence in the masses turn on, the η field set in place by the source particle finds itself on the side of a rapidly growing potential hill, and oscillates rapidly about its minimum at zero. To get an idea for its behavior, consider for simplicity a long wavelength mode of η which is away from its minimum at $\eta = 0$ when the mass begins to rapidly increase. It solves the equation of motion

$$\ddot{\eta} = -m_\eta^2 \eta \tag{3.17}$$

At large x^0 this has solutions

$$\eta \sim \frac{1}{\sqrt{2M(x^0)}} e^{\pm i \int^{x^0} M(t')dt'} \tag{3.18}$$

yielding an energy (3.16) scaling like $M(x^0)$ at large x^0.

In our case of fields sourced by a particle in the massive phase, this effect will yield a contribution to the energy which grows rapidly in time and also increases with increasing distance of the particle in the tachyon phase. Forces resulting from this work to evacuate the tachyon phase of excitations sourcing fields.

Now let us analyze this more quantitatively. We can solve (3.14) in terms of the retarded two point Greens function $G_R(x, y)$. Let us work inside the tachyon phase, where the masses depend only on x^0. Then

$$\begin{aligned}\eta(x) &= i\lambda \int d^d y G_R(\vec{x} - \vec{y}, x^0, y^0) j(y) \\ &= i\lambda \int dy^0 \sqrt{m_0^2 + \mu^2 e^{2\kappa y^0}} G_R(\vec{x}, x^0, y^0)\end{aligned} \tag{3.19}$$

We can write the Greens function G_R in terms of a complete basis of mode solutions $\psi^\pm_{\omega_k}$ satisfying

$$(\partial_0^2 + M^2(x^0))\psi^\pm_{\omega_k}(x^0) = -\omega_k^2 \psi^\pm_{\omega_k}(x^0) \tag{3.20}$$

and

$$\psi^\pm_{\omega_k}(x^0)_0 \psi^\mp_{\omega_k}(x^0) - \psi^\mp_{\omega_k}(x^0)_0 \psi^\pm_{\omega_k}(x^0) = \mp i \tag{3.21}$$

where $\omega_k = \sqrt{k^2 + M_0^2}$ and these modes reduce to the standard Fourier modes in the pretachyonic era:

$$\psi^\pm_{\omega_k}(x^0) \underset{x^0 \to -\infty}{\longrightarrow} \frac{1}{\sqrt{2\omega_k}} e^{\pm i \omega_k x^0} \tag{3.22}$$

The retarded Greens function is

$$\begin{aligned}G_R(x, y) = \theta(x^0 - y^0) \int \frac{d^{d-1}\vec{k}}{(2\pi)^{d-1}} e^{i\vec{k}\cdot(\vec{x}-\vec{y})} (\psi^+_{\omega_k}(x^0)\psi^-_{\omega_k}(y^0) \\ - \psi^-_{\omega_k}(x^0)\psi^+_{\omega_k}(y^0))\end{aligned} \tag{3.23}$$

The factor $\sqrt{m_0^2 + \mu^2 e^{2\kappa y^0}}$ appearing in (3.19) can be separated into two parts; $\sqrt{m_0^2 + \mu^2 e^{2\kappa y^0}} = \mu e^{\kappa y^0} + (\sqrt{m_0^2 + \mu^2 e^{2\kappa y^0}} - \mu e^{\kappa y^0})$. The first term here has no contribution in the far past, while the second term has its main contribution from the $y^0 < 0$ regime where the source particle of mass m_0 sits generating its background η field. Let us consider the second piece, since we are interested in the energy carried by the field generated by the source particle as the field enters the phase of rapidly growing mass. This gives an approximate expression for the η field

$$\begin{aligned}\eta(x) \sim i\lambda \int \frac{d^{d-1}\vec{k}}{(2\pi)^{d-1}} e^{i\vec{k}\cdot\vec{x}} \bigg(\bigg[\int_{-\infty}^0 dy^0 m_0 \psi^-_{\omega_k}(y^0) \bigg] \psi^+_{\omega_k}(x^0) \\ - \bigg[\int_{-\infty}^0 dy^0 m_0 \psi^+_{\omega_k}(y^0) \bigg] \psi^-_{\omega_k}(x^0) \bigg)\end{aligned} \tag{3.24}$$

In the pretachyonic phase $y^0 < 0$, the mode solutions behave approximately as Fourier modes (3.22). Plugging this in and doing the y^0 integral reduces our estimate for the field to

$$\eta(x) \sim \lambda m_0 \int \frac{d^{d-1}\vec{k}}{(2\pi)^{d-1}} \frac{e^{i\vec{k}\cdot\vec{x}}}{\omega_k^{3/2}\sqrt{2}} (\psi^+_{\omega_k}(x^0) + \psi^-_{\omega_k}(x^0)) \tag{3.25}$$

Now at large x^0, the wavefunctions $\psi^\pm_{\omega_k}$ scale as in WKB like

$$\psi^\pm_{\omega_k}(x^0) \underset{x^0 \to \infty}{\longrightarrow} \frac{1}{\sqrt{2M(x^0)}} e^{\pm i \int^{x^0} M(t')dt'} \tag{3.26}$$

Hence the energy in the field (3.16) scales like

$$\begin{aligned}E \sim m_0^2 \lambda^2 M(x^0) \cos^2\bigg(\int^{x^0} M(t')dt' \bigg) \int d^{d-1}\vec{x} f(|\vec{x}|) \\ \times \bigg(\int \frac{d^{d-1}\vec{k}}{(2\pi)^{d-1}} \frac{e^{i\vec{k}\cdot\vec{x}}}{\omega_k^{3/2}} \bigg)^2\end{aligned} \tag{3.27}$$

where in the last step we included the fact that the tachyon phase is quasilocalized, extending over a finite range parameterized by a function $f(|\vec{x}|)$ of finite support (3.2).

As anticipated, this energy grows rapidly with x^0 and increases with increasing extent of the source particle within the tachyon phase. This spatial dependence is particularly strong for low dimensional examples. Again, this effect provides forces outward for combinations of particles which source fields. In the context of string theory, a similar effect will occur for each string mode sourced by particles putatively surviving in the tachyon phase. This may provide a powerful effect pointing toward a full evacuation of this region, though we have not analyzed the full evolution of general particle-field configurations here.

More specifically, an energetically costly configuration of sources and fields can lower its energy by several processes. First, there is an outward force on the configuration of particle and field due to the field energy. The field can redistribute itself to minimize its support in the tachyon phase. It is tempting to make an analogy to flux tubes and expulsion of fields from confining dynamics, but so far our discussion has been purely perturbative. Secondly, quantum mechanically the configuration can also lower its energy by producing pairs of virtual particles which source η with opposite signs. One member of the pair stays inside the tachyon phase $r < \tilde{L}$, pairing with the original particle to form a configuration that does not source η, and the other member of the pair exits the region. More generally, sets of multiple particles which altogether source no fields might survive in the central region. In the case of a field like η which couples to energy, this requires, as just discussed, constituent particles which are virtual (carry negative frequency). In the next subsection we will see that indeed perturbative virtual particles do not decouple at late times in the tachyon phase.

Before turning to that, let us make some remarks on the relation between the field energetics and the AdS/CFT description of the system. As discussed in sec. II D, the AdS/CFT correspondence, when applicable, implies that energy and global symmetry charges are conserved. These are measured at the boundary by the behavior of the graviton and bulk gauge fields. The massing up of the gravitational and gauge fields in the tachyon phase can lead to screening of these charges. So from the AdS/CFT point of view, the energy (and conserved global charges) must end up in the bulk rather than remaining trapped in the tachyon phase. This has an important consequence: If the CFT has a nondegenerate ground state, then there can be no zero energy excitations in the tachyon phase. It must approach a unique state.

In the example discussed in Sec. II A, the dual field theory description of the quasilocal tachyon condensation is a time-dependent transition to a confining dual gauge theory. Although confinement is not explicitly understood in Yang-Mills theory, in such a transition one qualitatively expects the following dynamics. In the confined theory, the gauge-invariant composite glueballs arise at an energy and size scale commensurate with the strong coupling scale of the field theory. In our time-dependent transition, the excitations in the tachyon phase correspond to field theoretic modes at an energy scale below the mass gap. From the dual field theory point of view we expect forces from flux tubes to dynamically force them to shrink toward the size scale of the glueballs in the confining theory. The forces we analyzed in this section, which act to force excitations into the bulk gravitational solution dual to the confining geometry, may provide a gravity-side manifestation of this phenomenon. This effect is similar in some ways to the description of black hole evaporation via hadronization in [18].

C. A BRST anomaly and other subtleties with the S matrix

So far we have studied the classical dynamics of particles and fields in a localized phase of rapidly growing mass. Next we turn to an interesting subtlety with the S matrix in such a system, which translates into a perturbative BRST consistency condition on states in the tachyon region in the string-theoretic case of interest. We will start by explaining the main points and then delve into more of the details.

In the quantum interacting theory, time evolution produces sets of virtual particles which are not individually on shell. In ordinary Minkowski space field theory, a perturbative S matrix can be obtained by extracting on shell perturbative particle poles from Fourier transforms of Greens functions $\int d^d x e^{ip\cdot x} G(x, y_n)$ in the regime of integration where $x^0 \to \pm\infty$. If one considers field theory in a system which lasts for a finite time, the x^0 integration only goes over a finite range, and this quantity has no poles. In our case of interest with a phase of rapidly growing mass as $x^0 \to \infty$, we will see that the new asymptotic region at $x^0 \to \infty$, $r < \tilde{L}$ also does not afford perturbative asymptotic particle poles in the S matrix.

A correlated phenomenon is the following. In a worldline description, by varying the action one can easily show that the saddle point configuration of the path integral for a particle sitting in the tachyon phase has the property that $x^0(\tau)$ reaches infinity at a finite worldline time $\tau = \tau_*$. In the string-theoretic generalization, the string worldsheet in conformal gauge similarly reaches $x^0 = \infty$ in finite worldsheet time $\tau = \tau_*$, which means that the worldsheet has a hole in it.

Worldsheets with holes are not generically BRST invariant; in special circumstances D-brane boundary states render the holes consistent but such D-branes do not generically cancel the anomaly (as the case of the heterotic string makes particularly clear) [19].

One manifestation of this problem is that the worldsheet Hamiltonian fails to be Hermitian. (We will see explicitly below that that the worldline Hamiltonian in our field theory example is not Hermitian for general states [10].) The Hermiticity of the worldsheet Hamiltonian can be restored in the following way (see Fig. 2). On the worldsheet, unitary evolution persists past time τ_* if we map each hole (A) to another hole (A$'$) by a unitary operator on the worldsheet, and continue evolving in the direction indicated in the figure. In real time, this describes perturbative string (a) evolving toward the boundary correlated with another perturbative string (a$'$) (of negative frequency). These two states have equal and opposite frequencies, and generically each is individually off shell, but unitary worldsheet evolution is ensured by the correlation between the holes (A) and (A$'$). Note that this correlation need not be local in space.

This is reminiscent of the black hole final state proposal [11] for solving the information puzzle. In that approach, matter and inner Hawking particles impinge on the singularity in a correlated way. Although it was thought that the final state would be unique, this state was not specified precisely. The proposal involves an $N \times N$ unitary matrix describing a convolution of the correlations and the bulk interactions, and any sufficiently random matrix will do.

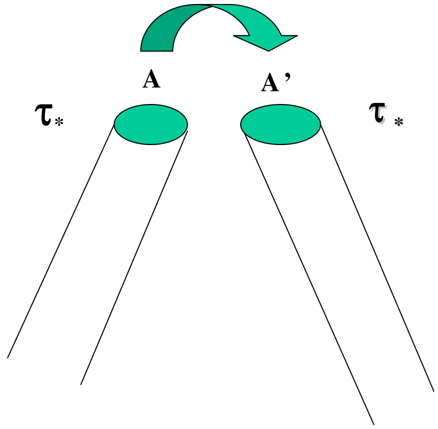

FIG. 2 (color online). In a tachyon condensate phase, the worldsheet of a string sitting in the tachyonic region reaches $x^0 = \infty$ in finite worldsheet time $\tau = \tau_*$. This generically leads to anomalies unless the resulting hole A in the worldsheet is unitarily mapped to a hole A', continuing worldsheet evolution in the directions indicated by the arrows. In more general circumstances the hole A' may be replaced by multiple holes.

Similarly here, imposing cancellation of the BRST anomaly does not uniquely specify the correlations, but just requires some set of correlations described by a unitary map. The simple linear relation indicated above is too simple to model the correlations required in a real black hole, but the same idea may be used to correlate a single worldsheet hole to multiple worldsheet holes.

We now explain in more detail the problems with Hermiticity and BRST invariance. Consider first the worldline description, a framework which also generalizes to the string theory case. The worldline Hamiltonian constraint

$$\hat{H}_{\mathrm{wl}}\Psi = (\partial_\mu \partial^\mu - m^2(x^0, \vec{x}))\Psi \equiv 0 \tag{3.28}$$

constrains particles to lie on the mass shell.

In the case where m grows faster than linearly in x^0, this Hamiltonian has the following property [10]. In the inner product

$$\langle \psi_1 | \psi_2 \rangle \equiv \int dx^0 d^{d-1}\vec{x}\psi_1^*(x^0, \vec{x})\psi_2(x^0, \vec{x}) \tag{3.29}$$

$\hat{H}_{\mathrm{wl}}$ is not self-adjoint on the full set of eigenfunctions of $\hat{H}_{\mathrm{wl}}$.[3] This inner product (3.29) arises both in the first quantized BRST description of the relativistic particle, and in the LSZ prescription for the S matrix.

The failure of Hermiticity arises because of a boundary term

$$\langle \psi_1 | \frac{\partial^2}{\partial (x^0)^2} | \psi_2 \rangle - h.c. = \int d^{d-1}\vec{x}(\psi_{1x^0}^* \psi_2 - \psi_{2x^0}\psi_1^*)|_{x^0=\infty}. \tag{3.30}$$

which can be seen as follows. The eigenfunctions $\psi_{\vec{p},\Delta,\pm}$ which satisfy

$$\hat{H}_{\mathrm{wl}}\psi_{\vec{p},\Delta,\pm} = \Delta\psi_{\vec{p},\Delta,\pm} \tag{3.31}$$

have a WKB form valid at large x^0

$$\psi_{p,\omega_0,\pm}|_{x^0\to\infty} \sim \frac{1}{\sqrt{2}(\Delta + m^2(x^0))^{1/4}} e^{i\vec{p}\cdot\vec{x}} e^{\pm i \int^{x^0} dt\sqrt{\Delta + m(t)^2}} \tag{3.32}$$

Now the combination

$$\langle \psi_{\vec{p}_1,\Delta_1,\pm}\hat{H}_{\mathrm{wl}} | \psi_{\vec{p}_2,\Delta_2,\pm}\rangle - \langle \hat{H}_{\mathrm{wl}}\psi_{\vec{p}_1,\Delta_1,\pm} | \psi_{\vec{p}_2,\Delta_2,\pm}\rangle \tag{3.33}$$

would vanish if $\hat{H}_{\mathrm{wl}}$ were self-adjoint. It reduces to a boundary term of the form (3.30), evaluated at the boundaries $x^0 \to \pm\infty$. For on shell mode solutions, this boundary contribution vanishes; on these solutions the Klein-Gordon inner product is conserved and the contributions from the future and past boundaries cancel.

The total boundary contribution does not cancel in general for modes of different eigenvalue Δ, as explained in this context in [10]. To see this we need some regularization: the boundary contribution at $x^0 \to -\infty$ is not well defined for general modes as the wavefunctions oscillate. Regulating via a rescaling of t by $1 - i\epsilon$ for a small real ϵ, or smearing with a highly peaked distribution in Δ as in [10], kills the contribution from the ordinary Fourier modes in the far past but does not kill the boundary contribution in the far future. In the far future, the boundary contribution (3.30) yields a value of ± 1 for $\psi_j = \psi_{\vec{p},\Delta_j,\pm}$ respectively. Hence if we consider all the independent eigenfunctions, the boundary term fails to cancel for this full collection of modes.

A related point is that in the inner product (3.29), the full space of solutions $\psi_{\vec{p},\Delta,\pm}$ do not satisfy a completeness relation:

$$\int dx^0 \psi_{\vec{p},\Delta_1,+}^*(x^0)\psi_{\vec{p},\Delta_2,+}(x^0) \neq f(\Delta_1, \Delta_2)\delta(\Delta_1 - \Delta_2); \tag{3.34}$$

(for any smooth function f);, in particular, this quantity does not vanish for different eigenvalues Δ. The problem can be seen from the WKB form of the wavefunctions (3.32): for large x^0 these modes all approach the same asymptotic form, leading to a failure of orthogonality in the inner product (3.29) of modes of different eigenvalues Δ. As we will see below, this behavior translates into an absence of poles in Greens functions as the eigenvalue goes on shell $\Delta \to 0$.

In the presence of interactions, this causes problems with worldline BRST invariance. The boundary term

[3] This is mathematically similar to ordinary quantum mechanics with a potential falling off faster than $-x^2$, where the Hamiltonian is again not self-adjoint without extra input [20].

(3.30) violates the worldline BRST symmetry corresponding to worldline time reparameterization. This BRST symmetry is generated by a BRST operator

$$Q_B = c\hat{H}_{\rm wl} \tag{3.35}$$

where c is a Faddeev-Poppov ghost. In the first quantized path integral, the derivation of decoupling of BRST trivial modes depends on the Hermiticity of $\hat{H}_{\rm wl}$ in the inner product (3.29).

The boundary term (3.30) is cancelled in states in which particles impinge on the boundary $r < \tilde{L}$, $x^0 \to \infty$ in correlated combinations with equal contributions from positive and negative frequency. Note that the condition of cancellation of the boundary term (3.30) is a nonlocal condition; it does not require the correlated particles at the boundary to annihilate locally in space. This is consistent with the underlying locality in the field theory, in the same way that EPR correlations are. As discussed above, this aspect is crucial for the application to black hole physics.

Let us discuss this issue from another point of view. In the LSZ prescription for the S matrix in Minkowski space, asymptotic particle states are associated with poles in the Fourier transform of off shell Greens functions with respect to momentum, for example

$$\int d^4x e^{ip\cdot x}\langle T(\phi(x)\phi(y))\rangle|_{p^0\to\pm\sqrt{\vec{p}^2+m_0^2}} \to \frac{\sqrt{Z}}{(p^0)^2-\vec{p}^2-m_0^2+i\epsilon} \tag{3.36}$$

in an ordinary quantum field theory (without a rapidly growing mass as we have here, but instead a constant mass m_0). The poles arise from asymptotic regions $x^0 \to \pm\infty$.

The analogue of this in a more general background is the convolution of the off shell Greens functions with an eigenfunction of $\hat{H} \equiv -\frac{\partial^2}{\partial(x^0)^2} + \frac{\partial^2}{\partial\vec{x}^2} - M^2(x^0)$ (working within the region $r < \tilde{L}$): denote such an eigenfunction $F_\Delta(x^0)e^{i\vec{p}\cdot\vec{x}}$ where $\hat{H}F = \Delta F$. On shell modes have $\Delta = 0$. The new feature of our present case is that no such pole appears from the $x^0 \to \infty$ regime:

$$\int d^4x F_\Delta(x)\langle T(\phi(x)\phi(y))\rangle|_{\Delta\to 0} = \text{finite} \tag{3.37}$$

since in the x^0 integration, the region $x^0 \to \infty$ is exponentially suppressed. In this way, the system behaves similarly to a quantum field theory living on a locally truncated Minkowski space, i.e. a space with time stopped inside the central region. In the latter problem as well, the worldline Hamiltonian is also not Hermitian and no perturbative asymptotic particle states are associated with the central region. Instead, combinations of virtual particles impinge upon the $r < \tilde{L}$, $x^0 \to \infty$ boundary in generic states.

Although this technical analysis applies most directly to the worldline quantum field theory case, similar effects can be expected in the string theory case. One manifestation of the problem is the holes appearing in the worldsheet in the saddle point solutions discussed above. Another is that unitarity relates imaginary parts in loop diagrams arising in the regime $x^0 \to \infty$ to perturbative asymptotic particle states. The shutoff of loop amplitudes in the Euclidean vacuum in the $x^0 \to \infty$ tachyon phase suggests that no imaginary parts will come from this regime. Then as in field theory, perturbative asymptotic string states do not arise in the usual way at $x^0 \to \infty$ in the tachyon phase. As discussed above, cancelling the BRST anomaly leads to intriguing correlations at the would-be singularity reminiscent of [11].

D. Perturbativity?

So far we analyzed two perturbative effects following from the condensation of the perturbative string winding tachyon. The forces coming from the field energy analyzed in sec. III B appear perturbatively, and work toward evacuating any combinations of particles sourcing any components of the string field. Worldsheet BRST invariance is required for perturbative consistency, and is intimately connected with spacetime gauge symmetry.

In the Euclidean vacuum studied in [4,7], the first quantized string amplitudes are self-consistently perturbative and calculate the components of the state in a basis of weakly coupled multistring states in the bulk. In more general states, it is not *a priori* clear if the physics remains perturbative as combinations of (real and virtual) strings approach the singularity. In open string tachyon problems, for example, there are indications of strong coupling physics (confinement) occurring [21], and it is tempting to speculate as many have done that an analogue happens in closed string tachyon condensation. This would provide its own rationale for evacuating the tachyon phase of generic excitations. However, loop vacuum diagrams in the Euclidean vacuum provide concrete evidence for tachyon condensation effectively massing up closed string modes [4], which applies, in particular, to fluctuations of the dilaton. If deformations of the dilaton are indeed massed up, this might provide a mechanism for the system to remain perturbative by freezing the dilaton at its bulk weakly coupled value. Another possible indication of perturbativity is that D-brane probes (whose energy scales inversely with the string coupling in ordinary spacetime string theory) are repelled from winding tachyon phases as seen in [22][4] (though in [24] a two-dimensional background was studied in which nonperturbative objects were conjectured to penetrate a lightlike tachyon wall).

[4]See [23] for discussions of the fate of twisted D-branes in orbifolds.

In any case, the basic tachyon degree of freedom driving the system away from the GR singularity is a perturbative string mode, and as we have seen here a number of important features of the problem are accessible perturbatively.[5]

IV. DISCUSSION

Most work on black holes in string theory, including the present work, focus on theoretical objects which are probably not realistic. It is important to understand which techniques apply in the perhaps more physically relevant case of Schwarzschild black holes. In this section we assess the prospects for applying our methods in these cases, and discuss other open problems stimulated by this work.

A. Schwarzschild black holes

The case of Schwarzschild black holes is of great interest. Inside the horizon, cylinders with spherical cross sections shrink. Topologically stable winding tachyons thus do not appear. However, as discussed in [4,26], the dynamics generating the mass gap in the two-dimensional sigma model on a sphere can behave like a superposition of winding string modes on great circles of the sphere. A more serious challenge in the Schwarzschild case is the rapid velocity with which the spheres shrink.

Inside a large Schwarzschild black hole in d dimensions,

$$ds^2 = -\left[1 - \left(\frac{r_0}{r}\right)^{d-3}\right]dt^2 + \left[1 - \left(\frac{r_0}{r}\right)^{d-3}\right]^{-1}dr^2 + r^2 d\Omega_{d-2} \tag{4.1}$$

there is a $(d-2)$-sphere which starts shrinking rapidly before the spatial curvature of the sphere becomes large. The change in sphere size r with respect to proper time is

$$\dot{r} = -\sqrt{\left(\frac{r_0}{r}\right)^{d-3} - 1} \rightarrow -\left(\frac{r_0}{r}\right)^{(d-3)/2} \quad \text{for } r \ll r_0 \tag{4.2}$$

which can become very rapid for $r_0 \gg r \gg l_s$, i.e. while the sphere is still large.

Starting from the radius r_c at which the d-dimensional curvature is of order $1/l_s^2$,

$$\frac{\dot{r}^2}{r^2}|_{r_c} \equiv \frac{1}{l_s^2} \Rightarrow r_c = (r_0^{d-3} l_s^2)^{1/d-1} \tag{4.3}$$

the time to the crunch is of order string scale. Note that for a large black hole $r_0 \gg l_s$, the spatial curvature of the sphere is still small (the sphere is still huge). More generally, the timescale to the crunch, starting from a given radius r, is

$$\Delta T_{\text{crunch}} = \frac{2}{d-1} r\left(\frac{r}{r_0}\right)^{d-3/2} = \frac{2}{d-1}\left(\frac{r}{r_c}\right)^{d-1/2} l_s \tag{4.4}$$

This rapid velocity causes particle and string production:, in particular, a simple estimate suggests that a Hagedorn density of strings is produced by the time the sphere has shrunk to $r = r_c$. The back reaction of this gas of strings may ultimately behave like a winding tachyon condensate, as suggested also in [27], but this has yet to be controlled. More generally, the rapid shrinking of the sphere can lead to nonadiabaticity for a large class of extended objects, whose spectrum depends on the internal degrees of freedom of the compactification.

This situation is improved considerably in simple models of black hole evaporation. The fact that the null energy condition is violated near the horizon (which is required in order for the area to decrease) causes the spheres to shrink much more slowly near the horizon. To see this, consider the Vaidya metric in four dimensions:

$$ds^2 = -\left(1 - \frac{2GM(v)}{r}\right)dv^2 + 2dvdr + r^2 d\Omega \tag{4.5}$$

This is a solution to Einstein's equation with a null fluid source and has frequently been used to model an evaporating black hole. The unit timelike normal to a constant r surface inside the horizon is

$$n = \left(\frac{2GM(v)}{r} - 1\right)^{-1/2} \frac{\partial}{\partial v} - \left(\frac{2GM(v)}{r} - 1\right)^{1/2} \frac{\partial}{\partial r} \tag{4.6}$$

So the rate of change of the spheres is

$$\dot{r} = -\left(\frac{2GM(v)}{r} - 1\right)^{1/2} \tag{4.7}$$

The velocity will be less than one on a surface of constant r, provided $r < 2GM(v) < 2r$. At infinity, a black hole loses mass at the rate

$$\frac{dM}{dt} \sim -AT^4 \sim -(GM)^{-2} \tag{4.8}$$

So $G^2M^3 \sim t_0 - t$. Since $v = t+$ constant along a surface at large r, we set

$$M(v) = M_0\left(1 - \frac{v}{v_0}\right)^{1/3} \tag{4.9}$$

It now follows that the proper length of the region over which the velocity is less than one is $r^3/G = r(r/l_p)^2$. So even when r is the string scale, the distance from the horizon over which the velocity stays small can be much greater than the string scale.

Since the velocity stays small near the horizon, the arguments of [1,4] now suggest that when the horizon reaches the string scale, it will pinch off, removing the region of large curvature and the singularity. This provides further support for the correspondence principle of [28]. It

[5]Other approaches such as [25] may help determine the degree to which nonperturbative corrections play a role at the singularity.

was argued there that Schwarzschild black holes radiate until the curvature at the horizon reaches the string scale. At this point, the black hole makes a transition to an excited fundamental string. Since the excited string lives in a space which is essentially flat, the sigma model argument provides a dynamical mechanism for the transition from the black hole to the excited string. In order to understand the information flow however, we would need to also control the region deeper inside the black hole where the velocity is still large in order to account for excitations that could putatively be trapped there.

B. Other future directions

1. Cosmological case

In this work we have focused on situations with localized tachyon condensation, or localized regions of exponentially growing mass in the field theory model of particle and field dynamics in the tachyon phase. In the case of spatially delocalized tachyon condensation [4,6] the question of allowed states is also of interest. Although time effectively stops in the case of decay to nothing [4], this alone does not preclude unitarity; ordinary quantum mechanics formulated on a finite time interval is unitary. It is of interest to understand whether multiple states are allowed at a cosmological singularity. The forces we discussed which help to evacuate the tachyon phase in the quasilocal case do not serve this role in a situation with spatially delocalized tachyon condensation. The perturbative BRST anomaly, when applicable, does restrict the allowed states somewhat. Neither of these effects is as powerful in the cosmological case as in the quasilocal case discussed here.

2. Worldsheet analysis

In our analysis we reverted to a field theory model of some of the dynamics in the tachyon phase (generalizing that of [12], which has withstood several tests of its applicability in the full perturbative string theory [4,7]). It would be advantageous to test this quasilocalized version of the field theory model further using full worldsheet string calculations. The prescription suggested in Fig. 2 may require techniques such as those suggested in [19,29,30] which could provide a formalism for describing the consistent states.

One issue for which a full worldsheet analysis is necessary is the backreaction of the large energy density produced when the tachyon condenses. A naive supergravity analysis would indicate that this energy density immediately produces large curvature. However, since the graviton is also becoming effectively massive, supergravity is not a good approximation.

3. Models and constraints

We end with a speculative comment on the possibility of connection to real-world astrophysics. The phenomena discussed in [2] show that the endpoint of Hawking evaporation can result in new types of black hole explosions. It is of interest to translate our growing understanding of string-corrected gravity and singularities to a theory of black hole explosions more generally. The production of primordial black holes small enough to evaporate in our causal past is at best a model dependent proposition, so new effects in black hole evaporation will probably simply serve to mildly constrain model building. Still, it is interesting to contemplate the possibility of "fundamental" origins for astrophysically accessible bursts and jets of energy.[6]

ACKNOWLEDGMENTS

We would like to thank John McGreevy for early collaboration and many useful discussions. We would like to thank D. Marolf, J. Maldacena, V. Schomerus, S. Shenker, A. Strominger, and L. Susskind for useful discussions and O. Aharony for comments on the manuscript. We are supported in part by the DOE under contract No. DE-AC03-76SF00515 and by the NSF under grants No. 9870115 and PHY-0244764.

[6] Amusingly, jets appear both in particle physics hadronization processes and in gamma ray bursts; the relation between black holes and confinement makes it tempting to seek a connection, although the astrophysical jets are probably accounted for by effects of angular momentum.

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