Magnetized black holes and black rings in the higher dimensional dilaton gravity

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In this paper we consider magnetized black holes and black rings in the higher dimensional dilaton gravity. Our study is based on exact solutions generated by applying a Harrison transformation to known asymptotically flat black hole and black ring solutions in higher dimensional spacetimes. The explicit solutions include the magnetized version of the higher dimensional Schwarzschild-Tangherlini black holes, Myers-Perry black holes, and five-dimensional (dipole) black rings. The basic physical quantities of the magnetized objects are calculated. We also discuss some properties of the solutions and their thermodynamics. The ultrarelativistic limits of the magnetized solutions are briefly discussed and an explicit example is given for the D-dimensional magnetized Schwarzschild-Tangherlini black holes.

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I. INTRODUCTION

In recent years the higher dimensional gravity is attracting much interest. Apart from the fact that the higher dimensional gravity is interesting in its own right, the increasing amount of works devoted to the study of the higher dimensional spacetimes is inspired from the string theory and the brane-world scenario with large extra dimensions [1–4]. This scenario suggests a possibility of unification of the electroweak and Planck scales at the TeV scale. A striking prediction in this scenario is the formation of higher dimensional black holes smaller than the size of the extra dimensions at accelerators [5,6].

Some solutions of the higher dimensional classical general relativity have been known for some time. These include the higher dimensional analogs of the Schwarzschild and Reissner-Nordstrom solution found by Tangherlini [7] and the higher dimensional generalization of the Kerr solution found by Myers and Perry [8]. As one should expect and as it was confirmed by recent investigations, the gravity in higher dimensions exhibits much richer dynamics than in four dimensions. An interesting development in the black hole studies is the discovery of the black ring solutions of the five-dimensional Einstein equations by Emparan and Reall [9,10]. These are asymptotically flat solutions with an event horizon of topology $S^2 \times S^1$ rather than the much more familiar S^3 topology. Moreover, it was shown in [10] that both the black hole and the black ring can carry the same conserved charges, the mass and a single angular momentum, and therefore there is no uniqueness theorem in five dimensions. Since Emparan and Reall's discovery many explicit examples of black ring solutions were found in various gravity theories [11–21]. Elvang was able to apply the Hassan-Sen transformation to the solution [10] to find a charged black ring in the bosonic sector of the truncated heterotic string theory [11]. A supersymmetric black ring in five-dimensional minimal supergravity was derived in [12] and then generalized to the case of concentric rings in [13,14]. A static black ring solution of the five-dimensional Einstein-Maxwell gravity was found by Ida and Uchida in [22]. In [23] Emparan derived "dipole black rings" in the Einstein-Maxwell-dilaton (EMD) theory in five dimensions. In this work Emparan showed that the black rings can exhibit a novel feature with respect to the black holes. Black rings can also carry nonconserved charges which can be varied continuously without altering the conserved charges. This fact leads to continuous nonuniqueness. The thermodynamics of the dipole black rings, within the quasilocal counterterm method, was discussed by Astefanesei and Radu [24]. Static and asymptotically flat black ring solutions in five-dimensional EMD gravity with arbitrary dilaton coupling parameter α were presented in [25]. A systematical derivation of the asymptotically flat static black ring solutions in five-dimensional EMD gravity with an arbitrary dilaton coupling parameter was given in [26]. In the same paper and in [27], the author systematically derived new type static and rotating black ring solutions which are not asymptotically flat.

In the present paper we study higher dimensional black holes and black rings immersed in external magnetic fields within the framework of EMD gravity. The interest in studying black holes under the influence of external fields has a long history. In 1976 Ernst [28] applied a Harrison transformation [29] to the Schwarzschild solution to obtain a static black hole in the Melvin universe [30,31]. The Ernst-Schwarzschild solution was subsequently discussed by many authors [32–38]. The Ernst result was generalized to more complicated metrics as the Kerr-Newman metrics [39,40]. The investigation of the magnetized Kerr-Newman metrics resulted in finding interesting astrophysical effects, such as charge accretion and flux expulsion from extreme black holes [39,41–45]. The flux expulsion was also studied in Kaluza-Klein and string theories [46]. Five-dimensional black holes in external electromagnetic fields were discussed by Aliev and Frolov [47] and by Ida and Uchida in [22]. In [47] the authors use Wald's test field approach [48] while the discussion in [22] is based on exact

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solutions. Magnetized static and rotating black holes in arbitrary dimensions as well as magnetized rotating black rings in five dimensions were recently studied by Ortaggio in [49] within the D-dimensional Einstein-Maxwell (EM) gravity. The discussion is based on exact solutions found by applying a Harrison transformation to known exact black hole and black ring solutions. In this work Ortaggio also discussed some properties of the magnetized black holes and black rings as well as their thermodynamics and gave the ultrarelativistic limit of the magnetized D-dimensional Schwarzschild solution. Here we generalize the results of [49] in the presence of the dilaton field nonminimally coupled to the electromagnetic field.

The paper is organized as follows. In the first section we systematically derived the Harrison transformation for the EMD equations in D-dimensional spacetimes with relevant symmetries. Then in the subsequent sections we apply the Harrison transformation to known black hole and black ring solutions to obtain their magnetized versions. We also discuss some properties of the magnetized solutions as well as their thermodynamics. The last section is devoted to a summary of the results. In Appendix B we present the explicit expressions of the ultrarelativistic limits of some of the magnetized solutions.

II. BASIC EQUATIONS AND HARRISON TRANSFORMATION

The EMD gravity in D-dimensional spacetimes is described by the action

$$S = \frac{1}{16\pi}\int d^D x\sqrt{-g}(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - e^{-2\alpha\varphi}F^{\mu\nu}F_{\mu\nu}). \tag{1}$$

The field equations derived from this action are

$$R_{\mu\nu} = 2\partial_\mu\varphi\partial_\nu\varphi + 2e^{-2\alpha\varphi}\left[F_{\mu\rho}F_\nu^\rho - \frac{g_{\mu\nu}}{2(D-2)}F_{\beta\rho}F^{\beta\rho}\right], \tag{2}$$

$$\nabla_\mu\nabla^\mu\varphi = -\frac{\alpha}{2}e^{-2\alpha\varphi}F_{\nu\rho}F^{\nu\rho}, \tag{3}$$

$$\nabla_\mu[e^{-2\alpha\varphi}F^{\mu\nu}] = 0. \tag{4}$$

We consider spacetimes admitting a spacelike, hypersurface-orthogonal Killing vector which we denote by η. In adapted coordinates in which $\eta = \partial/\partial y$, the spacetime metric can be written in the form

$$ds^2 = e^{2u}dy^2 + e^{-2u/(D-3)}h_{ij}dx^i dx^j, \tag{5}$$

where h_{ij} is a $(D-1)$-dimensional metric with Lorentz signature. Both u and h_{ij} depend on the coordinates x^i only. The electromagnetic field is taken in the form

$$F = dA_y \wedge dy. \tag{6}$$

The potential A_y depends on x^i only. In terms of the potentials u, A_y, and φ the field equations read

$$\mathcal{D}_i\mathcal{D}^i u = -2\frac{D-3}{D-2}e^{-2\alpha\varphi-2u}h^{ij}\mathcal{D}_iA_y\mathcal{D}_jA_y, \tag{7}$$

$$\mathcal{D}_i\mathcal{D}^i\varphi = -\alpha e^{-2\alpha\varphi-2u}h^{ij}\mathcal{D}_iA_y\mathcal{D}_jA_y, \tag{8}$$

$$\mathcal{D}_i(e^{-2\alpha\varphi-2u}\mathcal{D}^iA_y) = 0, \tag{9}$$

$$R(h)_{ij} = \frac{D-2}{D-3}\partial_i u\partial_j u + 2\partial_i\varphi\partial_j\varphi + 2e^{-2\alpha\varphi-2u}\partial_iA_y\partial_jA_y. \tag{10}$$

Here $\mathcal{D}_i$ and $R(h)_{ij}$ are the coderivative operator and Ricci tensor with respect to the metric h_{ij}. These equations can be derived from the action

$$S = \int d^{D-1}x\sqrt{|h|}\left[R(h) - \frac{D-2}{D-3}h^{ij}\partial_i u\partial_j u - 2h^{ij}\partial_i\varphi\partial_j\varphi - 2e^{-2\alpha\varphi-2u}h^{ij}\partial_iA_y\partial_jA_y\right]. \tag{11}$$

In order to unveil the symmetries of the action (11) we introduce the symmetric matrix

$$P = e^{(\alpha_D-1)u}e^{-(\alpha_D+1)\varphi_D}\begin{pmatrix} e^{2u+2\alpha_D\varphi_D} + (1+\alpha_D^2)\Psi_D^2 & -\sqrt{1+\alpha_D^2}\Psi_D \\ -\sqrt{1+\alpha_D^2}\Psi_D & 1 \end{pmatrix}, \tag{12}$$

where

$$\alpha_D = \sqrt{\frac{D-2}{2(D-3)}}\alpha, \qquad \varphi_D = \sqrt{\frac{2(D-3)}{(D-2)}}\varphi, \qquad \Psi_D = \sqrt{\frac{2(D-3)}{(D-2)}}A_y. \tag{13}$$

The action (11) can be written in the form of a σ-model action

$$S = \int d^{D-1}x\sqrt{|h|}\left[R(h) + \frac{1}{2(1+\alpha_D^2)}\times\frac{(D-2)}{(D-3)}h^{ij}Sp(\mathcal{D}_iP\mathcal{D}_jP^{-1})\right]. \tag{14}$$

Clearly, the action is invariant under the group $GL(2, R)$ where the natural group action is

$$P \rightarrow GPG^T. \tag{15}$$

In fact, the matrix P parametrizes the coset $GL(2, R)/SO(2)$. A similar σ model was previously discussed in [50]. However, in [50], the target space is parametrized by 3×3 matrices while here we give the target space parametrization in terms of 2×2 matrices.

In what follows we will be interested in a particular subgroup of $SL(2, R) \subset GL(2, R)$ which gives the Harrison transformation, namely, the subgroup consisting of the matrices

$$H = \begin{pmatrix} 1 & 0 \\ b & 1 \end{pmatrix}. \tag{16}$$

The Harrison transformation generates new solutions from known ones which have the same $(D-1)$-dimensional metric h_{ij} and the new matrix

$$P' = HPH^T. \tag{17}$$

In explicit form the new potentials are given by

$$e^{2u'} = \Lambda^{-2/(1+\alpha_D^2)} e^{2u}, \tag{18}$$

$$e^{-2\varphi'_D} = \Lambda^{2\alpha_D/(1+\alpha_D^2)} e^{-2\varphi_D}, \tag{19}$$

$$\Psi'_D = \Lambda^{-1}\left[\Psi_D + \frac{b}{\sqrt{1+\alpha_D^2}}(e^{2u+2\alpha_D\varphi_D} + (1+\alpha_D^2)\Psi_D^2)\right], \tag{20}$$

where

$$\Lambda = b^2 e^{2u+2\alpha_D\varphi_D} + (1 + b\sqrt{1+\alpha_D^2}\Psi_D)^2. \tag{21}$$

In other words, the old metric

$$ds^2 = e^{2u}dy^2 + g_{ij}dx^i dx^j \tag{22}$$

is transformed to the new one

$$ds'^2 = e^{2u'}dy^2 + \Lambda^{[2/(D-3)(1+\alpha_D^2)]} g_{ij}dx^i dx^j. \tag{23}$$

In the particular case $\alpha = 0$, we obtain the Harrison transformation in the Einstein-Maxwell gravity discussed in [49]. For $D = 4$ we recover the Harrison transformation in the four-dimensional EMD gravity [51].

III. DILATON-MELVIN SOLUTION

In this section we derive and briefly comment on the dilaton-Melvin solution which plays the role of background for all magnetized objects we consider in this work. The dilaton-Melvin solution in D dimensions was first found in [52] by solving the corresponding equations. In order to derive this solution here we apply the Harrison transformation to the D-dimensional flat spacetime presented in appropriate coordinates

$$ds^2 = -dt^2 + dz_1^2 + dz_2^2 + \cdots + dz_{D-3}^2 + d\rho^2 + \rho^2 d\phi^2. \tag{24}$$

The Harrison transformation with respect to the Killing vector $\partial/\partial\phi$ then generates the dilaton-Melvin solution

$$\begin{aligned} ds^2 &= \Lambda^{[2/(D-3)(1+\alpha_D^2)]}[-dt^2 + dz_1^2 + dz_2^2 + \cdots dz_{D-3}^2 + d\rho^2] + \Lambda^{-2/(1+\alpha_D^2)}\rho^2 d\phi^2, \\ e^{-2\alpha\varphi} &= \Lambda^{2\alpha_D^2/(1+\alpha_D^2)}, \\ A_\phi &= \Lambda^{-1}\sqrt{\frac{D-2}{2(D-3)}}\frac{b}{\sqrt{1+\alpha_D^2}}\rho^2, \\ \Lambda &= 1 + b^2\rho^2. \end{aligned} \tag{25}$$

If $\alpha = 0$ this solution is the D-dimensional EM Melvin solution, whose properties were discussed in [53] (and in [54–56] for $D = 4$). The properties of the D-dimensional dilaton-Melvin solution are similar. The geometry is a warped product of a $(D-2)$-dimensional Minkowski spacetime and a noncompact 2-dimensional space M_2 with a metric

$$dl^2 = \Lambda^{[2/(D-3)(1+\alpha_D^2)]} d\rho^2 + \Lambda^{-2/(1+\alpha_D^2)}\rho^2 d\phi^2. \tag{26}$$

The circumference of the circles $\rho = \text{const}$ at first increases and then monotonically decreases to zero as $\rho \rightarrow \infty$. The dilaton field φ is divergent at $\rho \rightarrow \infty$, but the scalar invariants tend to zero for $\rho \rightarrow \infty$, for example,

$$R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} \sim \rho^{-4\{[(D-3)(1+\alpha_D^2)+2]/(D-3)(1+\alpha_D^2)\}}, \tag{27}$$

therefore the geometry is well behaved there. Moreover, it can be checked that the curvature scalars are everywhere regular. As an example we present the Kretchmann scalar:

$$\begin{aligned} R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} &= \frac{16b^4}{(D-3)^2(1+\alpha_D^2)^2}\Lambda^{-4\{[(D-3)(1+\alpha_D^2)+1]/(D-3)(1+\alpha_D^2)\}}K_1 \\ &\quad + \frac{16b^4}{(1+\alpha_D^2)^2}\frac{D-2}{(D-3)^2}\Lambda^{-2\{[(D-3)(1+\alpha_D^2)+2]/(D-3)(1+\alpha_D^2)\}}K_2, \end{aligned} \tag{28}$$

where

$$K_1 = \left[(3D-8) - (D-2)\frac{1-\alpha_D^2}{1+\alpha_D^2}b^2\rho^2\right]^2, \qquad (29)$$

$$K_2 = \frac{2}{(D-3)(1+\alpha_D^2)^2}\left(\frac{b^2\rho^2}{\Lambda}\right)^2 + \left[1 - 2\frac{b^2\rho^2}{\Lambda}\right]^2 + \left[1 - \frac{2}{1+\alpha_D^2}\frac{b^2\rho^2}{\Lambda}\right]^2. \qquad (30)$$

IV. DILATON-SCHWARZSCHILD-MELVIN SPACETIMES

Let us consider the D-dimensional, spherically symmetric Schwarzschild-Tangherlini black hole spacetimes given by the metric [7]

$$ds_D^2 = -\lambda(r)dt^2 + \frac{dr^2}{\lambda(r)} + r^2 d\Omega_{D-2}^2, \qquad (31)$$

where

$$\lambda(r) = 1 - \frac{\mu}{r^{D-3}} \qquad (32)$$

and $d\Omega_{D-2}^2$ is the line element of the unit $(D-2)$-dimensional sphere. The parameter $\mu > 0$ is related to the black hole mass via the relation[1]

$$M = \frac{\mu(D-2)}{16\pi}\Omega_{D-2}. \qquad (33)$$

It is convenient to present the line element $d\Omega_{D-2}^2$ in the form

$$d\Omega_{D-2}^2 = \cos^2\theta d\Omega_{D-4}^2 + d\theta^2 + \sin^2\theta d\phi^2 \qquad (34)$$

or

$$ds_D^2 = -\lambda(r)dt^2 + \frac{dr^2}{\lambda(r)} + r^2\cos^2\theta d\Omega_{D-4}^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2. \qquad (35)$$

The Killing vector $\partial/\partial\phi$ is spacelike and hypersurface orthogonal, and, therefore, we can consider the Harrison transformation associated with it. Since the Schwarzschild-Tangherlini is characterized with the trivial dilaton and electromagnetic field we find

$$\Lambda = 1 + b^2 r^2 \sin^2\theta. \qquad (36)$$

[1] $\Omega_{(D-2)}$ is the area of the unit $(D-2)$-dimensional sphere.

The new solution generated by the Harrison transformation is

$$\begin{aligned} ds'^2 &= \Lambda^{2/(D-3)(1+\alpha_D^2)}\Big[-\lambda(r)dt^2 + \frac{dr^2}{\lambda(r)} + r^2\cos^2\theta d\Omega_{D-4}^2 + r^2 d\theta^2\Big] + \Lambda^{-2/(1+\alpha_D^2)} r^2\sin^2\theta d\phi^2, \\ e^{-2\alpha\varphi'} &= \Lambda^{2\alpha_D^2/(1+\alpha_D^2)}, \\ A'_\phi &= \Lambda^{-1}\sqrt{\frac{D-2}{2(D-3)}}\frac{b}{\sqrt{1+\alpha_D^2}} r^2\sin^2\theta. \end{aligned} \qquad (37)$$

This solution reduces to that of the EM gravity [49] for $\alpha = 0$.

The constant b introduced by the Harrison transformation parametrizes the strength of the magnetic field. In order to find the relation between the parameter b and the asymptotic magnetic field B let us consider the invariant

$$\frac{1}{2}F_{\mu\nu}F^{\mu\nu} = 2\frac{(D-2)}{(D-3)}\frac{b^2}{1+\alpha_D^2}[\lambda(r)\sin^2\theta + \cos^2\theta] \times \Lambda^{[2(D-4)/(D-3)(1+\alpha_D^2)]-4}. \qquad (38)$$

This invariant takes the constant value B^2 at the "axis" $\theta = 0$

$$B^2 = 2\frac{(D-2)}{(D-3)}\frac{b^2}{1+\alpha_D^2} \qquad (39)$$

which gives the relation between the asymptotic magnetic field and the parameter b.

The magnetized solution (37) has a single horizon located where $\lambda(r) = 0$, i.e. $r_h = \mu$. As in the case of EM gravity the location of the horizon is not affected by the value of the dilaton and magnetic field. The horizon is regular and the spacetime can be extended across the horizon by the standard techniques. The curvature invariants diverge at $r = 0$ indicating the presence of a curvature singularity there. As an illustrative example we may consider the Ricci scalar curvature

$$R = \frac{4b^2}{1+\alpha_D^2}\Lambda^{-2\{[1+(D-3)(1+\alpha_D^2)]/(D-3)(1+\alpha_D^2)\}} \times \left[\frac{D-2}{D-3}\frac{\alpha_D^2}{1+\alpha_D^2}r^2\sin^2\theta + \frac{D-4}{D-3}\right] \times \left[1 - \frac{\mu}{r^{D-3}}\sin^2\theta\right]. \qquad (40)$$

For $r \to \infty$ the solution tends to the D-dimensional dilaton-Melvin solution (25), which can be obtained by setting $\mu = 0$. The connection between the coordinates of (25) and those of (37) is given by

$$\rho = r\sin\theta, \tag{41}$$

$$(z_1^2 + z_2^2 + \cdots + z_{D-3}^2)^{1/2} = r\cos\theta, \tag{42}$$

$$d(r\cos\theta)^2 + (r\cos\theta)^2 d\Omega_{D-4}^2 = dz_1^2 + dz_2^2 + \cdots + dz_{D-3}^2. \tag{43}$$

The solution (37) can be interpreted as a black hole in the dilaton-Melvin background (or dilaton-Melvin Universe). As should be expected, the background deforms the black hole horizon and its geometry is now different from that of the round $(D-2)$-dimensional sphere of radius r_h. The geometry of the horizon is given by the line element

$$ds_h^2 = \Lambda_h^{2/(D-3)(1+\alpha_D^2)}[r_h^2\cos^2\theta d\Omega_{D-4}^2 + r_h^2 d\theta^2] + \Lambda_h^{-2/(1+\alpha_D^2)} r_h^2 \sin^2\theta d\phi^2, \tag{44}$$

where

$$\Lambda_h = 1 + b^2 r_h^2 \sin^2\theta. \tag{45}$$

A good illustrative measure of the departure form sphericity is the Ricci scalar curvature of the horizon

$$R_h = \frac{\Lambda_h^{-2/(D-3)(1+\alpha_D^2)}}{r_h^2}\bigg[(D-2)(D-3) + \frac{4b^2r_h^2}{1+\alpha_D^2}\frac{\Lambda_h^{-1}}{(D-3)}[D\cos^2\theta - (D-3)\sin^2\theta] - \frac{4b^4r_h^4}{1+\alpha_D^2}\frac{\Lambda_h^{-2}}{(D-3)}\bigg[2 + \frac{1}{1+\alpha_D^2}\frac{D^2-5D+8}{D-3}\bigg] \times \sin^2\theta\cos^2\theta\bigg]. \tag{46}$$

As can be seen the Ricci scalar curvature differs from that of the round $(D-2)$-dimensional sphere of radius r_h, $R_{D-2} = (D-2)(D-3)/r_h^2$. The background deforms the horizon but preserves the horizon area since the Harrison transformation leaves the determinant of the horizon metric invariant. Therefore the horizon area of the magnetized black hole is that of the Schwarzschild-Tangherlini black hole

$$\mathcal{A}_h = \Omega_{D-2} r_h^{D-2}. \tag{47}$$

The horizon temperature can be found by Euclideanizing the metric and the result is

$$T = \frac{1}{2\pi}\frac{(D-3)\mu}{r_h^{D-2}} = \frac{1}{4\pi}\frac{D-3}{r_h} \tag{48}$$

and is the same as for the Schwarzschild-Tangherlini black hole.

It is interesting to compute the magnetic flux across a portion Σ of the horizon. The flux is found to be

$$\Phi = \oint_{\partial\Sigma} A'_\phi = \frac{B\pi r_h^2\sin^2\theta}{1 + \frac{(D-3)}{2(D-2)}(1+\alpha_D^2)B^2 r_h^2 \sin^2\theta}. \tag{49}$$

For weak magnetic fields the flux is proportional to the magnetic field strength, while it tends to zero for $B \to \infty$. There is a maximum of the magnetic flux for intermediate values of B which is a consequence of the concentration of the magnetic field under its self-gravity.

In order to compute the black hole mass we use the quasilocal formalism approach (see Appendix A). Following this approach we decompose the metric into the form

$$ds'^2 = -N^2 dt^2 + \chi_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \tag{50}$$

where

$$N^2 = \Lambda^{2/(D-3)(1+\alpha_D^2)}\lambda(r), \tag{51}$$

$$N^i = 0, \tag{52}$$

$$\chi_{ij}dx^i dx^j = \Lambda^{2/(D-3)(1+\alpha_D^2)}\bigg[\frac{dr^2}{\lambda(r)} + r^2\cos^2\theta d\Omega_{D-4}^2 + r^2 d\theta^2\bigg] + \Lambda^{-2/(1+\alpha_D^2)} r^2 \sin^2\theta d\phi^2. \tag{53}$$

Further we consider the $(D-2)$-dimensional surfaces S_t^r with the unit spacelike normal $n_r = \sqrt{\chi_{rr}}\frac{\partial}{\partial r}$ and metric

$$\sigma_{ab}dx^a x^b = \Lambda^{2/(D-3)(1+\alpha_D^2)}[r^2\cos^2\theta d\Omega_{D-4}^2 + r^2 d\theta^2] + \Lambda^{-2/(1+\alpha_D^2)} r^2\sin^2\theta d\phi^2. \tag{54}$$

The extrinsic curvature is

$$k = -\frac{(D-2)\sqrt{\lambda(r)}}{r}\Lambda^{-1/(D-3)(1+\alpha_D^2)} = \Lambda^{-1/(D-3)(1+\alpha_D^2)}\tilde{k}, \tag{55}$$

where $\tilde{k}$ is the extrinsic curvature for the Schwarzschild-Tangherlini solution.

The natural background is obviously the dilaton-Melvin spacetime for which

$$k_0 = -\frac{(D-2)}{r}\Lambda^{-1/(D-3)(1+\alpha_D^2)} = \Lambda^{-1/(D-3)(1+\alpha_D^2)}\tilde{k}_0. \tag{56}$$

Taking into account that

$$\sqrt{\sigma}N = \sqrt{\lambda(r)} r^{D-2}\Lambda^{1/(D-3)(1+\alpha_D^2)}\sqrt{\det\Omega_{D-2}} = \Lambda^{1/(D-3)(1+\alpha_D^2)}\sqrt{\tilde{\sigma}}\tilde{N} \tag{57}$$

we find

$$E(r) = \frac{(D-2)\Omega_{D-2}}{8\pi} r^{D-3}\sqrt{\lambda(r)}[1-\sqrt{\lambda(r)}]. \qquad (58)$$

These explicit calculations show that the contribution of the background cancels out and as a final result we obtain

$$M = \lim_{r\to\infty} E(r) = \frac{\mu(D-2)\Omega_{D-2}}{16\pi}, \qquad (59)$$

which is exactly the Schwarzschild-Tangherlini black hole mass. In this way we see that the mass of the black hole is not affected by the background. This a general result for the mass and angular momentum of black holes in the dilaton-Melvin background and it is proven in Appendix A. Moreover, we have shown that all thermodynamical quantities of the black hole remain the same independently of the external magnetic field. Moreover, as shown in Appendix A the physical Euclidean action of the Schwarzschild-Tangherlini solution and its magnetized version coincide:

$$I_P = \tilde{I}_P. \qquad (60)$$

Therefore the black hole thermodynamics is not affected by the background just as in the four-dimensional case [57].

V. DILATON-MYERS-PERRY-MELVIN SPACETIMES

The Myers-Perry black holes [8] are generalizations of the four-dimensional Kerr solution to higher dimensions. These solutions are described in different forms depending on whether the spacetime is even or odd dimensional. Here we consider the odd dimensional case. The even dimensional case is treated in the same way. In spacetime with an odd number of dimensions the Myers-Perry solution with one of the spin parameters set to zero (say $a_1 = 0$) is given by

$$ds^2 = -dt^2 + \sum_{i=2}^{(D-1)/2}(r^2+a_i^2)(d\mu_i^2 + \mu_i^2 d\phi_i^2) + \frac{\mu r^2}{\Pi F}\left(dt + \sum_{i=2}^{(D-1)/2} a_i\mu_i^2 d\phi_i\right)^2 + \frac{\Pi F}{\Pi - \mu r^2}dr^2 + r^2 d\mu_1^2 + r^2\mu_1^2 d\phi_1^2, \qquad (61)$$

where

$$F = 1 - \sum_{i=2}^{(D-1)/2}\frac{a_i^2\mu_i^2}{a_i^2+\mu_i^2}, \qquad (62)$$

$$\Pi = r^2\prod_{i=2}^{(D-1)/2}(r^2+a_i^2), \qquad (63)$$

$$\sum_{i=1}^{(D-1)/2}\mu_i^2 = 1. \qquad (64)$$

The Myers-Perry solution (in odd dimensions) admits $(D-1)/2$ commuting spacial Killing vectors $\partial/\partial\phi_i$. What is important for us is that the Killing vector $\partial/\partial\phi_1$ is hypersurface orthogonal since we have set $a_1 = 0$. Therefore, we can apply the Harrison transformation associated with this Killing vector to the Myers-Perry solution. Doing so we find

$$\begin{aligned}
ds'^2 &= \Lambda^{2/(D-3)(1+\alpha_D^2)}\left[-dt^2 + \sum_{i=2}^{(D-1)/2}(r^2+a_i^2)(d\mu_i^2+\mu_i^2 d\phi_i^2) + \frac{\mu r^2}{\Pi F}\left(dt + \sum_{i=2}^{(D-1)/2}a_i\mu_i^2 d\phi_i\right)^2 + \frac{\Pi F}{\Pi-\mu r^2}dr^2 + r^2 d\mu_1^2\right] + \Lambda^{-2/(1+\alpha_D^2)}r^2\mu_1^2 d\phi_1^2,\\
A'_{\phi_1} &= \Lambda^{-1}\left(\frac{D-2}{2(D-3)}\right)^{1/2}\frac{b}{\sqrt{1+\alpha_D^2}}r^2\mu_1^2,\\
e^{-2\alpha\varphi'} &= \Lambda^{2\alpha_D^2/(1+\alpha_D^2)},
\end{aligned} \qquad (65)$$

where

$$\Lambda = 1 + b^2 r^2 \mu_1^2.$$

The outer event horizon is determined as the largest (real) root of $g^{rr} = 0$. In explicit form the equation for the horizon is given by

$$\Pi - \mu r^2 = r^2\left[\prod_{i=2}^{(D-1)/2}(r^2+a_i^2) - \mu\right] = 0, \qquad (66)$$

and coincides with that for the Myers-Perry solution. As it is known this equation has a positive root independent of the magnitude of a_i for[2] $D \geq 7$, i.e. there exist black holes with arbitrary angular momentum for $D \geq 7$. In five dimensions, however, there is an upper bound for the angular momentum. In the limit $r \to \infty$ we obtain the dilaton-

[2]In general, for even and odd dimensions, we have $D \geq 6$.

Melvin background with asymptotic magnetic field

$$B = \left[2\frac{(D-2)}{(D-3)}\right]^{1/2}\frac{b}{\sqrt{1+\alpha_D^2}}. \tag{67}$$

Therefore, the solution can be interpreted as a rotating black hole in the dilaton-Melvin background.

The geometry of the horizon is described by the line element

$$ds_h^2 = \Lambda_h^{2/(D-3)(1+\alpha_D^2)}\left[\sum_{i=2}^{(D-1)/2}(r_h^2+a_i^2)(d\mu_i^2+\mu_i^2 d\phi_i^2) + \frac{\mu r_h^2}{\Pi_h F_h}\left(\sum_{i=2}^{(D-1)/2} a_i\mu_i^2 d\phi_i\right)^2 + r_h^2 d\mu_1^2\right] + \Lambda_h^{-2/(1+\alpha_D^2)} r_h^2\mu_1^2 d\phi_1^2, \tag{68}$$

where the subscript h means that the corresponding quantity is evaluated on the horizon. As in the static case, this metric describes the distorted $(D-2)$-dimensional sphere. The area of the horizon is

$$\mathcal{A}_h = \Omega_{D-2} r_h \prod_{i=2}^{(D-1)/2}(r_h^2+a_i^2) \tag{69}$$

which coincides with that of the Myers-Perry solution. This is, as we have already mentioned, a consequence of the fact that the Harrison transformation preserves the volume element of the horizon metric.

In order to compute mass and the angular momenta of the black hole we use the quasilocal formalism given in Appendix A. The contribution of the background cancels out and the mass, angular momenta, and horizon angular velocity are the same as for the Myers-Perry solution:

$$M = \frac{\mu(D-2)}{16\pi}\Omega_{D-2}, \tag{70}$$

$$J_i = \frac{\mu a_i}{8\pi}\Omega_{D-2}, \tag{71}$$

$$\omega_i = \frac{a_i}{r_h^2+a_i^2}. \tag{72}$$

We also find that the temperature is not affected by the background and is given by

$$T = \frac{1}{2\pi}\sum_{i=2}^{(D-1)/2}\frac{r_h}{r_h^2+a_i^2}. \tag{73}$$

The same is true for the Euclidean action, $I_P = \tilde{I}_P$ (see Appendix A). As in the static case, although the background deforms the horizon it does not affect the black hole thermodynamics. This seems to be a consequence of the fact that the vector potential is parallel to the nonrotating Killing vector and the magnetic field and the rotation do not couple. In a more general case when the Harrison transformation is associated with a rotating Killing vector one should expect that the external magnetic field will influence the black hole thermodynamics via the coupling with the rotation, as in four dimensions [57].

VI. BLACK RINGS IN DILATON-MELVIN BACKGROUND

The black ring metric is given by [10]

$$ds_5^2 = -\frac{F(y)}{F(x)}\left(dt + C(\nu,\lambda)\mathcal{R}\frac{1+y}{F(y)}d\psi\right)^2 + \frac{\mathcal{R}^2}{(x-y)^2}F(x)\left[-\frac{G(y)}{F(y)}d\psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)}d\phi^2\right], \tag{74}$$

where

$$F(x) = 1+\lambda x, \qquad G(x) = (1-x^2)(1+\nu x), \tag{75}$$

and

$$C(\nu,\lambda) = \sqrt{\lambda(\lambda-\nu)\frac{1+\lambda}{1-\lambda}}. \tag{76}$$

The coordinates x and y vary within the ranges

$$-1 \le x \le 1, \qquad -\infty < y \le -1, \tag{77}$$

and the parameters λ and ν within

$$0 < \nu \le \lambda < 1. \tag{78}$$

In order to avoid conical singularities at $y=-1$ and $x=-1$ the angular variables must be identified with periodicity

$$\Delta\psi = \Delta\phi = 2\pi\frac{\sqrt{1-\lambda}}{1-\nu}. \tag{79}$$

To avoid a conical singularity at $x=1$ the parameters λ and ν must be related as

$$\lambda = \frac{2\nu}{1+\nu^2}. \tag{80}$$

With these choices, the solution has a regular horizon of topology $S^2\times S^1$ at $y=-1/\nu$ and ergosurface of the same topology at $y=-1/\lambda$. Asymptotic spatial infinity is at $x\to y\to -1$. The static solution is obtained for $\lambda=\nu$ instead of (80). The black ring metric admits three Killing vectors $\partial/\partial t$, $\partial/\partial\psi$, and $\partial/\partial\phi$. The Killing vector $\partial/\partial\phi$ is spacelike and hypersurface orthogonal and the Harrison transformation associated with it gives the following EMD solution:

$$ds'^2 = \Lambda^{1/(1+\alpha_5^2)}\left[-\frac{F(y)}{F(x)}\left(dt + C(\nu,\lambda)\mathcal{R}\frac{1+y}{F(y)}d\psi\right)^2 + \frac{\mathcal{R}^2}{(x-y)^2}F(x)\left(-\frac{G(y)}{F(y)}d\psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)}\right)\right] + \Lambda^{-2/(1+\alpha_5^2)}\mathcal{R}^2\frac{G(x)}{(x-y)^2}d\phi^2, \quad (81)$$

$$A'_\phi = \frac{\sqrt{3}}{2}\frac{\Lambda^{-1}b}{\sqrt{1+\alpha_5^2}}\mathcal{R}^2\frac{G(x)}{(x-y)^2}, \quad (82)$$

$$e^{-2\alpha\varphi'} = \Lambda^{2\alpha_5^2/(1+\alpha_5^2)}, \quad (83)$$

with

$$\Lambda = 1 + b^2\mathcal{R}^2\frac{G(x)}{(x-y)^2}. \quad (84)$$

The metric has a horizon at $y = -1/\nu$ with topology $S^2 \times S^1$ and an ergosurface at $y = -1/\lambda$ with the same topology. The external fields do not affect the location of the horizon and the ergosurface. Although the external fields deform the horizon and the ergosurface, their topology remains the same while the geometry is distorted. For $x, y \to -1$ the solution tends to the five-dimensional dilaton-Melvin background with magnetic field

$$B = \frac{1-\nu}{\sqrt{1-\lambda}}\frac{\sqrt{3}}{2}\frac{b}{\sqrt{1+\alpha_D^2}}. \quad (85)$$

This can be seen by performing the coordinate transformation

$$r\cos\theta = \sqrt{\frac{1-\lambda}{1-\nu}}\mathcal{R}\frac{\sqrt{y^2-1}}{x-y}, \quad (86)$$

$$r\sin\theta = \sqrt{\frac{1-\lambda}{1-\nu}}\mathcal{R}\frac{\sqrt{1-x^2}}{x-y}, \quad (87)$$

$$\tilde{\psi} = \frac{1-\nu}{\sqrt{1-\lambda}}\psi, \quad (88)$$

$$\tilde{\phi} = \frac{1-\nu}{\sqrt{1-\lambda}}\phi. \quad (89)$$

The solution then can be interpreted as a rotating black ring in the dilaton-Melvin background.

As in the previous cases, the background does not affect the black ring thermodynamics and the physical quantities characterizing the magnetized black ring are the same as for the neutral black ring solution:

$$M = \frac{3\pi}{4}\mathcal{R}^2\frac{\lambda}{1-\nu}, \quad (90)$$

$$J_\psi = \frac{\pi\mathcal{R}^3}{2}\frac{\sqrt{\lambda(\lambda-\nu)(1+\lambda)}}{(1-\nu)^2}, \quad (91)$$

$$\mathcal{A}_h = 8\pi^2\mathcal{R}^3\frac{\sqrt{\nu^3\lambda(1-\lambda^2)}}{(1-\nu^2)(1+\nu)}, \quad (92)$$

$$\omega_\psi = \frac{1}{\mathcal{R}}\sqrt{\frac{\lambda-\nu}{\lambda(1+\lambda)}}, \quad (93)$$

$$T = \frac{1+\nu}{4\pi\mathcal{R}}\sqrt{\frac{1-\lambda}{\nu\lambda(1+\lambda)}}. \quad (94)$$

Here ω_ψ is the angular velocity of the horizon.

VII. DIPOLE BLACK RINGS IN DILATON-MELVIN BACKGROUND

The dipole black rings are solutions of the EMD gravity equations given by [23]

$$ds_5^2 = -\frac{F(y)}{F(x)}\left(\frac{H(x)}{H(y)}\right)^{1/(1+\alpha_5^2)}\left(dt + C(\nu,\lambda)\mathcal{R}\frac{1+y}{F(y)}d\psi\right)^2 + \frac{\mathcal{R}^2F(x)}{(x-y)^2}(H(x)H^2(y))^{1/(1+\alpha_5^2)} \times\left[-\frac{G(y)}{F(y)H^{3/(1+\alpha_5^2)}(y)}d\psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)H^{3/(1+\alpha_5^2)}(x)}d\phi^2\right], \quad (95)$$

$$e^{-2\alpha\varphi} = \left(\frac{H(x)}{H(y)}\right)^{2\alpha_5^2/(1+\alpha_5^2)}, \quad (96)$$

$$A_\phi = \frac{\sqrt{3}}{2\sqrt{1+\alpha_5^2}}C(\nu,-\mu)\mathcal{R}\frac{1+x}{H(x)}. \quad (97)$$

The functions $F(x)$, $G(x)$, and $C(\lambda,\nu)$, the range of the coordinates x, y and the parameters λ, ν are the same as in the case of the neutral black ring. The function $H(x)$ is given by

$$H(x) = 1 - \mu x, \quad (98)$$

where $0 \leq \mu < 1$.

The possible conical singularities at $x = -1$ and $y = -1$ are avoided by setting

$$\Delta\psi = \Delta\phi = 2\pi \frac{(1+\mu)^{3/2(1+\alpha_5^2)}}{1-\nu}\sqrt{1-\lambda}. \tag{99}$$

The avoidance of the conical singularity at $x = 1$ simultaneously with (99) is achieved only if

$$\frac{1-\lambda}{1+\lambda}\left(\frac{1+\mu}{1-\mu}\right)^{3/(1+\alpha_5^2)} = \left(\frac{1-\nu}{1+\nu}\right)^2. \tag{100}$$

The solution has a regular outer horizon of topology $S^2 \times S^1$ at $y = -1/\nu$. There is also an inner horizon at $y = -\infty$. The metric can be continued beyond this horizon to positive values of y until $y = 1/\mu$ which is a curvature singularity. The extremal limit when the two horizons coincide is achieved for $\nu = 0$. In addition there is an ergosurface with ring topology at $y = -1/\lambda$.

The dipole black rings carry local magnetic charge [23] given by

$$\mathcal{Q} = \frac{\sqrt{3}\mathcal{R}}{2\sqrt{1+\alpha_5^2}} \frac{(1+\mu)^{(2-\alpha_5^2)/2(1+\alpha_5^2)}}{(1-\nu)} \times \sqrt{\frac{\mu(\mu+\nu)(1-\lambda)}{1-\mu}}. \tag{101}$$

Therefore the dipole rings are specified by the three physical quantities M, J, and $\mathcal{Q}$. The local charge is independent of the mass M and the angular momentum J and is a classically continuous parameter. This implies infinite classical nonuniqueness in five dimensions.

The Harrison transformation generates the following new EMD solution:

$$\begin{aligned} ds_5'^2 = {} & \Lambda^{1/(1+\alpha_5^2)}\left[-\frac{F(y)}{F(x)}\left(\frac{H(x)}{H(y)}\right)^{1/(1+\alpha_5^2)}\left(dt + C(\nu,\lambda)\mathcal{R}\frac{1+y}{F(y)}d\psi\right)^2 + \frac{\mathcal{R}^2F(x)}{(x-y)^2}(H(x)H^2(y))^{1/(1+\alpha_5^2)} \right. \\ & \left. \times\left(-\frac{G(y)}{F(y)H^{3/(1+\alpha_5^2)}(y)}d\psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)}\right)\right] + \Lambda^{-2/(1+\alpha_5^2)}\frac{\mathcal{R}^2G(x)}{(x-y)^2}\left(\frac{H(y)}{H(x)}\right)^{2/(1+\alpha_5^2)}d\phi^2, \end{aligned} \tag{102}$$

$$A'_\phi = \Lambda^{-1}\left[A_\phi + \frac{\sqrt{3}b}{2\sqrt{1+\alpha_5^2}}\left[\frac{\mathcal{R}^2G(x)}{(x-y)^2}\left(\frac{H(y)}{H(x)}\right)^2 + \frac{4}{3}(1+\alpha_5^2)A_\phi^2\right]\right], \tag{103}$$

$$e^{-2\alpha\varphi'} = \left(\frac{H(x)}{H(y)}\right)^{2\alpha_5^2/(1+\alpha_5^2)}\Lambda^{2\alpha_5^2/(1+\alpha_5^2)}, \tag{104}$$

where

$$\Lambda = b^2\frac{\mathcal{R}^2G(x)}{(x-y)^2}\left(\frac{H(y)}{H(x)}\right)^2 + \left(1 + \frac{2b}{\sqrt{3}}\sqrt{1+\alpha_5^2}A_\phi\right)^2. \tag{105}$$

To avoid conical singularities at $x = -1$ and $y = -1$ the angular coordinates must have periodicity given by (99). The balance between the forces in the ring will be achieved when, in addition, there are no conical singularities at $x = 1$. Since the ring is carrying a local magnetic charge there will be an additional force caused by the coupling between the local charge and the external magnetic field. This force manifests itself by the presence of the external magnetic field strength (via the parameter b) in the equilibrium condition

$$\frac{1-\lambda}{1+\lambda}\left(\frac{1+\mu}{1-\mu}\right)^{3/(1+\alpha_5^2)}\Lambda^{-3/(1+\alpha_5^2)}|_{x=1} = \left(\frac{1-\nu}{1+\nu}\right)^2. \tag{106}$$

After performing the coordinate transformation (86) one can show that for $x \to y \to -1$ the solution asymptotes the five-dimensional dilaton-Melvin solution with asymptotic magnetic field

$$B = (1+\mu)^{-3/2(1+\alpha_5^2)}\frac{(1-\nu)}{\sqrt{1-\lambda}}\frac{\sqrt{3}}{2}\frac{b}{\sqrt{1+\alpha_D^2}}. \tag{107}$$

The magnetized dipole ring solution has a regular outer horizon of topology $S^2 \times S^1$ at $y = -1/\nu$ and an ergosurface at $y = -1/\lambda$ with the same topology. There is also an inner horizon at $y = -\infty$. The metric can be continued beyond this horizon to positive values of y until $y = 1/\mu$ which is a curvature singularity. The extremal limit when the two horizons coincide is achieved for $\nu = 0$.

The external magnetic field does not affect the values of the mass, angular momentum, and the horizon area and they are the same as for the seed solution (see Appendix A)

$$M = \frac{3\pi\mathcal{R}^2}{4}\frac{(1+\mu)^{3/(1+\alpha_5^2)}}{1-\nu}\left(\lambda + \frac{1}{1+\alpha_5^2}\frac{\mu(1-\lambda)}{1+\mu}\right), \tag{108}$$

$$J_\psi = \frac{\pi\mathcal{R}^3}{2}\frac{(1+\mu)^{9/2(1+\alpha_5^2)}}{(1-\nu)^2}\sqrt{\lambda(\lambda-\nu)(1+\lambda)}, \quad (109)$$

$$\mathcal{A}_h = 8\pi^2\mathcal{R}^3\frac{(1+\mu)^{3/(1+\alpha_5^2)}}{(1-\nu)^2(1+\nu)} \times \nu^{3\alpha_5^2/2(1+\alpha_5^2)}(\mu+\nu)^{3/2(1+\alpha_5^2)}\sqrt{\lambda(1-\lambda^2)}. \quad (110)$$

The same is true also for the angular velocity of the horizon and the temperature

$$\omega_\psi = \frac{1}{\mathcal{R}}(1+\mu)^{-3/2(1+\alpha_5^2)}\sqrt{\frac{\lambda-\nu}{\lambda(1+\lambda)}}, \quad (111)$$

$$T = \frac{1}{4\pi\mathcal{R}}\frac{\nu^{(2-\alpha_5^2)/2(1+\alpha_5^2)}(1+\nu)}{(\mu+\nu)^{3/2(1+\alpha_5^2)}}\sqrt{\frac{1-\lambda}{\lambda(1+\lambda)}}. \quad (112)$$

The local charge of the magnetized ring is

$$\mathcal{Q} = \frac{\sqrt{3}\mathcal{R}}{2\sqrt{1+\alpha_5^2}}\frac{(1+\mu)^{(2-\alpha_5^2)/2(1+\alpha_5^2)}}{(1-\nu)} \times \sqrt{\frac{\mu(\mu+\nu)(1-\lambda)}{1-\mu}}\Lambda^{-1/2(1+\alpha_5^2)}|_{x=1}. \quad (113)$$

The Euclidean action of the magnetized solution is independent of the magnetic field and coincides with that of the seed solution, $I_P = \tilde{I}_P$ [note, however, that the balance condition (106) does involve the strength of the external magnetic field]. At first sight, it seems strange that the thermodynamics is not affected by the external magnetic field. One might expect that the coupling between the local magnetic charge and the external magnetic field would affect the thermodynamics. In fact, a similar phenomena is well known in the classical statistical physics—the external magnetic field does not affect the classical partition function (the so-called Van Leeuwen's theorem for the nonexistence of diamagnetism in classical physics [58]).

It is interesting to consider the static limit of the magnetized dipole ring solution. This limit is obtained for $\lambda = \nu$. In the static case the conditions for the absence of conical singularities are reduced to

$$\Delta\psi = \Delta\phi = 2\pi\frac{(1+\mu)^{3/2(1+\alpha_5^2)}}{\sqrt{1-\lambda}} \quad (114)$$

and

$$\left(\frac{1+\mu}{1-\mu}\right)^{3/(1+\alpha_5^2)}\Lambda^{-3/(1+\alpha_5^2)}|_{x=1} = \left(\frac{1-\lambda}{1+\lambda}\right). \quad (115)$$

Equation (115) can be solved to determine B as a function of the parameters λ and μ. In other words the external magnetic field can always be chosen such that to cancel the conical singularity and to support the static ring in equilibrium. Therefore, there exist static dipole black rings with regular horizons in the external magnetic field. This is possible because of the coupling between the external magnetic field B and the local charge $\mathcal{Q}$. The magnetized static dipole black rings provide infinite examples of regular, static black holes with horizon topology different from that of the Schwarzschild-Tangherlini black holes, but with the same mass and asymptotics.

VIII. CONCLUSION

In this paper we presented explicit solutions describing magnetized black holes and black rings in the higher dimensional dilaton gravity. The basic physical quantities of the magnetized black objects were calculated and some of their properties were discussed. In particular we have shown that the external magnetic field deforms the black holes horizon but it does not change the horizon area. Moreover, we have shown that the external magnetic field does not affect the thermodynamics of the black objects. This seems to be related to the fact that the electromagnetic potential is parallel to the nonrotating Killing vector which means that there is no coupling between the rotations and the magnetic field. In the more general case of the rotating Killing vector we expect that the external field will influence the black hole thermodynamics as in the four dimensions [57]. The general case, however, requires more sophisticated mathematical techniques and we will address this question in a future work. We also discussed briefly the ultrarelativistic limits of the magnetized solutions and gave an explicit example for the D-dimensional Schwarzschild-Tangherlini solution. The ultrarelativistic limits of the magnetized black holes and black rings might be useful in the theoretical study of the black hole production in the near-future accelerators.[3]

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APPENDIX A: QUASILOCAL FORMALISM

Here we briefly discuss the quasilocal formalism in EMD gravity [59]. The spacetime metric can be decomposed into the form

$$ds^2 = -N^2dt^2 + \chi_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \quad (A1)$$

where N is the lapse function and N^i is the shift vector.

This decomposition means that the spacetime is foliated by spacelike surfaces Σ_t of metric $\chi_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$,

[3] Who knows ...?

labeled by a time coordinate t with a unit normal vector $u^\mu = -N\delta_0^\mu$. A timelike vector v^μ, satisfying $v^\mu \nabla_\mu t = 1$, is decomposed into the lapse function and shift vector as $v^\mu = N u^\mu + N^\mu$. The spacetime boundary consists of the initial surface Σ_i ($t = t_i$) and the final surface Σ_f ($t = t_f$) and a timelike surface $\mathcal{B}$ to which the vector u^μ is tangent. The surface $\mathcal{B}$ is foliated by $(D-2)$-dimensional surfaces S_t^r, of metric $\sigma_{\mu\nu} = \chi_{\mu\nu} - n_\mu n_\nu$, which are intersections of Σ_t and $\mathcal{B}$. The unit spacelike outward normal to S_t^r, n_μ is orthogonal to u^μ.

In order to have a well-defined variational principle we must supplement the action (1) with the corresponding boundary term:

$$S = \frac{1}{16\pi}\int d^D x\sqrt{-g}(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - e^{-2\alpha\varphi}F^{\mu\nu}F_{\mu\nu}) + \frac{1}{8\pi}\int_{\Sigma_i}^{\Sigma_f} K\sqrt{\chi}d^{D-1}x - \frac{1}{8\pi}\int_{\mathcal{B}}\Theta\sqrt{\gamma}d^{D-1}x. \qquad \text{(A2)}$$

K is the trace of the extrinsic curvature $K^{\mu\nu}$ of $\Sigma_{t_{i,f}}$ and Θ is the trace of the extrinsic curvature $\Theta^{\mu\nu}$ of $\mathcal{B}$, given by

$$K_{\mu\nu} = -\frac{1}{2N}\left(\frac{\partial\chi_{\mu\nu}}{\partial t} - 2D_{(\mu}N_{\nu)}\right), \qquad \text{(A3)}$$

$$\Theta_{\mu\nu} = -\gamma_\mu^\alpha\nabla_\alpha n_\nu, \qquad \text{(A4)}$$

where ∇_μ and D_ν are the covariant derivatives with respect to the metrics $g_{\mu\nu}$ and χ_{ij}, respectively.

The quasilocal energy and angular momentum are given by

$$E = \frac{1}{8\pi}\int_{S_t^r}\sqrt{\sigma}\left[N(k - k_0) + \frac{n_\mu p^{\mu\nu}N_\nu}{\sqrt{\chi}}\right]d^{D-2}x + \frac{1}{4\pi}\int_{S_t^r}A_0(\hat{\Pi}^j - \hat{\Pi}_0^j)n_j d^{D-2}x, \qquad \text{(A5)}$$

$$J_i = -\frac{1}{8\pi}\int_{S_t^r}\frac{n_\mu p_i^\mu}{\sqrt{\chi}}\sqrt{\sigma}d^{D-2}x - \frac{1}{4\pi}\int_{S_t^r}A_i\hat{\Pi}^j n_j d^{D-2}x. \qquad \text{(A6)}$$

Here $k = -\sigma^{\mu\nu}D_\nu n_\mu$ is the trace of the extrinsic curvature of S_t^r embedded in Σ_t. The momentum variable p^{ij} conjugated to χ_{ij} is given by

$$p^{ij} = \sqrt{\chi}(\chi^{ij}K - K^{ij}). \qquad \text{(A7)}$$

The quantity $\hat{\Pi}^j$ is defined by

$$\hat{\Pi}^j = -\frac{\sqrt{\sigma}}{\sqrt{\chi}}\sqrt{-g}e^{2\alpha\varphi}F^{0j}. \qquad \text{(A8)}$$

The quantities with the subscript "0" are those associated with the background. Detailed discussion of the quasilocal formalism can be found in [59].

Let us denote by a tilde all quantities which refer to the seed solution. Then the Harrison transformation gives a new solution which is characterized with the following quantities:

$$N = \Lambda^{1/(D-3)(1+\alpha_D^2)}\tilde{N}, \qquad \text{(A9)}$$

$$N^{\hat{i}} = \tilde{N}^{\hat{i}}, \qquad \text{(A10)}$$

$$N^y = \tilde{N}^y = 0, \qquad \text{(A11)}$$

$$\chi_{ij}dx^i dx^j = \Lambda^{2/(D-3)(1+\alpha_D^2)}\tilde{\chi}_{\hat{i}\hat{j}}dx^{\hat{i}}dx^{\hat{j}} + \Lambda^{-2/(1+\alpha_D^2)}\tilde{\chi}_{yy}dy^2, \qquad \text{(A12)}$$

$$\sigma_{ab}dx^a dx^b = \Lambda^{2/(D-3)(1+\alpha_D^2)}\tilde{\sigma}_{\hat{a}\hat{b}}dx^{\hat{a}}dx^{\hat{b}} + \Lambda^{-2/(1+\alpha_D^2)}\tilde{\sigma}_{yy}dy^2, \qquad \text{(A13)}$$

where $\hat{i}$ and $\hat{a}$ take the same values as i and a except for $i = y$ and $a = y$. It is easy to see that

$$\chi = \Lambda^{2/(D-3)(1+\alpha_D^2)}\tilde{\chi}, \qquad \text{(A14)}$$

$$\sigma = \tilde{\sigma}. \qquad \text{(A15)}$$

For the cases considered in this paper we have

$$k = \Lambda^{-1/(D-3)(1+\alpha_D^2)}\tilde{k}, \qquad \text{(A16)}$$

$$k_0 = \Lambda^{-1/(D-3)(1+\alpha_D^2)}\tilde{k}_0, \qquad \text{(A17)}$$

$$K_i^\mu = \Lambda^{-1/(D-3)(1+\alpha_D^2)}\tilde{K}_i^\mu. \qquad \text{(A18)}$$

Taking into account these results and the fact that in our case $A_0 = 0$ and $\hat{\Pi}^j = 0$ we find

$$E = \frac{1}{8\pi}\int_{S_t^r}\sqrt{\sigma}\left[N(k - k_0) + \frac{n_\mu p^{\mu\nu}N_\nu}{\sqrt{\chi}}\right]d^{D-2}x = \frac{1}{8\pi}\int_{S_t^r}\sqrt{\tilde{\sigma}}\left[\tilde{N}(\tilde{k} - \tilde{k}_0) + \frac{\tilde{n}_\mu\tilde{p}^{\mu\nu}\tilde{N}_\nu}{\sqrt{\chi}}\right]d^{D-2}x = \tilde{E}, \qquad \text{(A19)}$$

$$J_i = -\frac{1}{8\pi}\int_{S_t^r}\frac{n_\mu p_i^\mu}{\sqrt{\chi}}\sqrt{\sigma}d^{D-2}x = -\frac{1}{8\pi}\int_{S_t^r}\frac{\tilde{n}_\mu\tilde{p}_i^\mu}{\sqrt{\tilde{\chi}}}\sqrt{\tilde{\sigma}}d^{D-2}x = \tilde{J}_i. \qquad \text{(A20)}$$

Therefore the Harrison transformation leaves the quasilocal mass and angular momenta of the seed solution unchanged. In the same way one can show that the Euclidean action [i.e. the Euclideanized version of (A2)] of the magnetized solutions with respect to the dilaton-Melvin background I_P coincide with that of the corresponding seed solutions (with respect to the Minkowski background)

$$I_P = \tilde{I}_P. \qquad \text{(A21)}$$

It is worth noting that the Euclideanization of the dipole black rings is subtle. In fact, as shown in [24], we are forced to work with a complex geometry. Nevertheless, the final result gives a real action.

APPENDIX B: ULTRARELATIVISTIC LIMITS

The ultrarelativistic limit of the Schwarzschild-Tangherlini spacetime is obtained via the Aichelburg-Sexl procedure, i.e. by boosting the Schwarzschild-Tangherlini black hole to the speed of light [60]. Performing a Lorentz boost in the z_1 direction and taking the limit $V \to 1$ while keeping the ratio $p = M/\sqrt{1-V^2}$ fixed we obtain the ultrarelativistic limit of the Schwarzschild-Tangherlini spacetime

$$ds^2 = 2dudv + dz_2^2 + \cdots + dz_{D-3}^2 + d\rho^2 + \rho^2 d\phi^2 + \mathcal{H}\delta(u)du^2, \tag{B1}$$

where

$$u = \frac{z_1 - t}{\sqrt{2}}, \qquad v = \frac{z_1 + t}{\sqrt{2}}, \tag{B2}$$

$$\mathcal{H} = -8\sqrt{2}p\ln\rho \qquad (D = 4), \tag{B3}$$

$$\mathcal{H} = \frac{16\pi\sqrt{2}p}{(D-4)\Omega_{D-3}(z_2^2 + z_3^2 + \cdots + z_{D-3}^2 + \rho^2)^{(D-4)/2}} \quad (D > 4). \tag{B4}$$

The ultrarelativistic limit of the dilaton-Schwarzschild-Melvin solution can be found by applying the Harrison transformation[4] to the ultrarelativistic limit of the Schwarzschild-Tangherlini spacetime and the result is

$$ds^2 = \Lambda^{2/(D-3)(1+\alpha_D^2)}[2dudv + dz_2^2 + \cdots + dz_{D-3}^2 + d\rho^2] + \Lambda^{-2/(1+\alpha_D^2)}\rho^2 d\phi^2 + \Lambda^{2/(D-3)(1+\alpha_D^2)}\mathcal{H}\delta(u)du^2. \tag{B5}$$

The dilaton field and the magnetic field are invariant under the Lorentz boost in the z_1 direction and remain unchanged. The metric (B5) represents an impulsive gravitational wave propagating in the dilaton-Melvin background (Universe) along the z_1 direction. The impulsive wave front corresponds to the null hypersurface $u = 0$. On the impulsive wave front the metric is given by

$$ds^2 = \Lambda^{2/(D-3)(1+\alpha_D^2)}[dz_2^2 + \cdots + dz_{D-3}^2 + d\rho^2] + \Lambda^{-2/(1+\alpha_D^2)}\rho^2 d\phi^2. \tag{B6}$$

This metric does not depend on time and, therefore, the impulsive wave is nonexpanding. In addition the explicit form of the metric shows that the impulsive wave front is curved (for $b \neq 0$).

The ultrarelativistic limits of the other magnetized solutions can be found in the same manner by applying the Harrison transformation to the ultrarelativistic limits of their seed solutions (see [61–63]).

[4] This is possible, since, in our cases, the Aichelburg-Sexl limit and the Harrison transformation commute as one can see.

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