Loop cosmological dynamics and dualities with Randall-Sundrum braneworlds

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The discrete quantum geometric effects play an important role in dynamical evolution in the loop quantum cosmology. These effects which are significant at the high energies lead to the quadratic energy density modifications to the Friedmann equation, as in the Randall-Sundrum braneworld scenarios but with a negative sign. We investigate the scalar field dynamics in this scenario and show the existence of a phase of superinflation independent of the inverse scale factor modifications as found earlier. In this regime the scalar field mimics the dynamics of a phantom field and vice versa. We also find various symmetries between the expanding phase, the contracting phase and the phantom phase in the loop quantum cosmology. We then construct the scaling solutions in the loop quantum cosmology and show their dual relationship with those of the Randall-Sundrum cosmology.

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I. INTRODUCTION

The standard model of cosmology has successfully provided us a consistent picture of the evolution of our Universe in various epochs. However, it is expected that when the limit of validity of general relativity is reached, in the regime of very high curvature, the standard Friedmann dynamics shall be modified. It thus has been an attractive idea to seek modifications to the Friedmann equations at the high energy scales. A theory of quantum gravity shall naturally provide us these modifications. Motivated by the string theory, modifications to the Friedmann dynamics in the Randall-Sundrum (RS) braneworld scenarios [1] have been extensively studied [2]. Randall-Sundrum braneworld scenarios are based on the Horova-Witten model [3] where a 3 + 1 dimensional universe is obtained after compactification of a 6 dimensional Calabi-Yau manifold. The bulk spacetime in Randall-Sundrum model is 5 dimensional anti–de Sitter with the extra dimension being spacelike and large. The anti–de Sitter bulk in the Randall-Sundrum scenario leads to the localization of gravity on the brane and the modified Friedmann equation on the brane is of the form $H^2 \propto \rho(1 + \rho/2\sigma)$ where H is the Hubble rate, ρ is the energy density of matter and σ is the brane tension. The ρ^2 modification in the effective Friedmann equation is directly an effect of the existence of a large extra dimension in this model, as it arises by using the Israel junction conditions on the bulk-brane system.

Modifications to the Friedmann dynamics appear also in loop quantum cosmology (LQC) [4] which is the symmetry reduced quantization of homogeneous and isotropic spacetimes based on the loop quantum gravity (LQG) [5]. LQG which is one of the background independent and nonperturbative candidate theories for quantizing Einsteinian gravity in four dimensions predicts that at quantum level, classical spacetime continuum is replaced by a discrete quantum geometry and operators corresponding to length of a curve, area of a surface and volume of an enclosed region have discrete eigenvalues. The geometrical operators in LQC, for example the scale factor and the inverse scale factor, also have a discrete spectrum and the underlying dynamics in loop quantum cosmology is governed by a non-singular discrete quantum difference equation [4]. However recent investigations, pertaining to the study of evolution of the semiclassical states have shown that the discrete quantum dynamics can be very well approximated by an effective modified Friedmann dynamics till scales very close to the Planck scale [6–8]. The modifications to the Friedmann dynamics due to loop quantum effects are of two types. First is based on the modification to the behavior of inverse scale factor below a critical scale factor a_* determined by a half integer parameter j. This parameter arises because inverse scale factor operator is computed by tracing over SU(2) holonomies in an irreducible spin j representation. It turns out that the eigenvalues of the inverse scale factor operator become proportional to the positive powers of scale factor for $a \lesssim a_*$. This change regulates the divergence of energy density for arbitrary matter [9] and changes the classical frictional term to antifrictional in the Klein-Gordon equation for the scalar field in an expanding Universe, thus leading to a phase of superinflation [10]. Various interesting applications have been found, for example resolution of big bang singularity by loop quantum dynamics [11], avoidance of many cosmological [12] and gravitational collapse scenarios [13] by inverse scale factor modifications, increasing the viability of the onset of inflation [10,14,15] nonsingular cyclic models [16], natural trans-Planckian modifications to the frequency dispersion relation [9] etc.

The second type of modification essentially encodes the discrete quantum geometric nature of spacetime, as predicted by the loop quantum gravity, in the Friedmann dynamics. As we would discuss in the next section, this modification arises because the loops on which holonomies are computed have a nonvanishing minimum area given by the eigenvalues of the area operator in LQG [6–8,17,18]. It

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leads to a ρ^2 modification of the Friedmann equation of the form, $H^2 \propto \rho(1 - \rho/\rho_{\rm crit})$ where $\rho_{\rm crit}^{-1} = \alpha\kappa\gamma^2\ell_{\rm P}^2/3$ and ρ in general has modifications due to inverse scale factor for $a < a_*$. Here $\kappa = 8\pi G$ with G being the four dimensional gravitational constant, $\ell_{\rm P} = \sqrt{G}$ is the Planck length [20], $\gamma \approx 0.2375$ is the dimensionless Barbero-Immirzi parameter whose value is set by the black hole thermodynamics in LQG [21] and α is a constant of the order unity determined by the eigenvalues of the area operator. The form of the modification in the Friedmann equation leads to a nonsingular bouncing cosmology [8].

For small values of j parameter or for scale factors $a > a_*$, the modifications to the effective Friedmann dynamics due to change in the behavior of the inverse scale factor can be neglected and only those originating from discreteness effects are important. Working in this setting we would consider the evolution for a Universe composed of a massive scalar field ϕ with a conjugate momentum Π_ϕ and matter component with a constant equation of state $w = p_w/\rho_w$. Our consideration of matter in LQC would be on the similar phenomenological lines as in Ref. [9].

We would aim to investigate various features pertaining to the effective dynamics in this paper. After deriving the necessary effective equations in Sec. II, we would analyze in detail the scalar field dynamics in Sec. III and show that LQC generically leads to a phase of superinflation, independent of the inverse scale factor modifications, when $\rho > \rho_{\rm crit}/2$. Most of the phenomenological applications in LQC are based on the existence of this phase originating from the change in behavior of inverse scale factor for $a < a_*$. However, if value of j is chosen to be small then most of these effects become weak. Our result about the existence of this phase which originates due to discrete quantum effects establishes the robustness of these applications. By considering the dynamical evolution of a massive scalar field and a phantom field in LQC, we would show a peculiar relation between them. In the regime $\rho > \rho_{\rm crit}/2$, a massive scalar field behaves as a phantom field in the standard cosmology, and a phantom field mimics dynamics of an ordinary scalar field. We would then investigate various symmetries between the expanding branch, the contracting branch and the dynamics of a phantom scalar field and show that as in the standard Friedmann cosmology [22,23] and the Randall-Sundrum scenario [23], they have a dual relationship with each other in the effective theory of LQC.

One of the most important questions pertinent for any dynamical evolution is its stability. For that we would derive scaling solutions in LQC which are useful to study the stability properties of cosmological models [24–27]. As has been further established in Ref. [26], these scaling solutions may provide useful links between distinct cosmological scenarios. We explore this avenue in Sec. IV and find that the scaling solutions in LQC have a dual relationship with those in the Randall-Sundrum scenario. We shall note that the scaling solutions in LQC have been constructed earlier [28,29] and correspondences between string-inspired scenarios and loop inspired cosmologies have been found [29], though in the case when only inverse scale factor modifications to the effective dynamics are important and the discrete quantum gravity effects play no role (which may happen if $\rho \ll \rho_{\rm crit}$ in all phases of the cosmological evolution). In this sense the scaling solutions found here would be complimentary to those in Refs. [28,29].

In Sec. V, we summarize the results obtained in this paper and discuss the implications for the duality symmetry between scaling solutions of LQC and Randall-Sundrum scenario. We would discuss the way this relationship can be used for various useful applications and to extract physical predictions, for example, the perturbation spectrum in LQC. Just with its use as a mathematical device this duality can be used to investigate the detailed properties of solutions in LQC given those in Randall-Sundrum scenario. However we would also discuss if it points out to a deeper relationship between two frameworks and its ramifications.

II. EFFECTIVE DYNAMICS IN LOOP QUANTUM COSMOLOGY

LQG is a quantization of gravity based on Ashtekar-Barbero connection variables with the gravitational phase space spanned by SU(2) connection A_a^i and the triad E_i^a on a 3-manifold M (labels a and i denote space and internal indices respectively). In LQC, on imposition of symmetries of the framework, the nontrivial information about the classical phase space gets encoded in variables c related to A_a^i and p related to the triad E_i^a which can have two orientations. On classical solutions (of general relativity) c is given by $c = k + \gamma\dot{a}$, k being curvature index which we would take to be zero in this work. The triad p is related to the scale factor a of the homogeneous and isotropic metric via $|p| = a^2$, where the modulus arises due to two possible orientations of p and in this work we choose the positive one without any loss of generality. In LQG, the connection does not have a corresponding quantum operator hence it is more useful to work with holonomies defined over a loop. The holonomy over an edge ϵ of a loop is defined as $h_i := \cos(\mu c/2) + 2\tau_i \sin(\mu c/2)$ where τ_i are related to the Pauli spin matrices as $\tau_i = -i\sigma_i/2$, and dimensionless μ is related to the physical length λ_ϵ of the edge ϵ as $\lambda_\epsilon = \mu|p|^{1/2}$.

Given this classical phase space structure, we then perform quantization on the lines of LQG by promoting holonomies and triads to quantum operators [30]. It turns out that the triad operator $\hat{p}$ has a discrete eigenvalue spectrum with eigenvalues μ,

$$\hat{p}|\mu\rangle = \frac{4\pi\mu\gamma\ell_{\rm P}^2}{3}|\mu\rangle \tag{1}$$

including the eigenvalue zero. Thus the naive inverse of the triad operator is not densely defined. The eigenvalues of the inverse triad operator are important as they give us information about the way curvature which is proportional to the inverse powers of the scale factor (or equivalently the triad) would behave in LQC. In order to evaluate it we use a classical identity of the Ashtekar-Barbero phase space

$$\frac{1}{\sqrt{|p|}} = \frac{1}{2\pi G\gamma} \operatorname{tr}\left(\sum_i \tau^i h_i\{\hat{h}_i^{-1}, V^{1/3}\}\right). \qquad (2)$$

Here V denotes volume related to p as $V = |p|^{3/2}$. The operator $(\widehat{\sqrt{|p|}})^{-1}$ commutes with the operator $\hat{p}$ and has eigenstates $|\mu\rangle$. It can be shown that its eigenvalue spectrum is bounded on the entire Hilbert space [30]. This implies that the curvature in LQC remains finite and does not blow up, even for the state $|\mu = 0\rangle$ which corresponds to $a = 0$ (the classical big bang).

The Hamiltonian constraint operator is made of the gravitational and the matter part, $\hat{\mathcal{H}} = \hat{\mathcal{H}}_{\rm G} + \hat{\mathcal{H}}_{\rm M}$. The classical gravitational constraint consists of inverse powers of the triad and curvature which are expressed in terms holonomies and their Poisson brackets with positive powers of V (as in Eq. (2)). The quantum operator for the gravitational part of the classical Hamiltonian constraint is given by [30]

$$\begin{aligned}\widehat{\mathcal{H}}_{\rm G} &= \frac{i}{4\pi\kappa\ell_{\rm P}^2\gamma^3\bar{\mu}^3}\sum_{ijk}\epsilon^{ijk}\operatorname{tr}(\hat{h}_i\hat{h}_j\hat{h}_i^{-1}\hat{h}_j^{-1}\hat{h}_k[\hat{h}_k^{-1}, \hat{V}]) \\ &= \frac{6i}{\pi\kappa\ell_{\rm P}^2\gamma^3\bar{\mu}^3}\sin^2\left(\frac{\bar{\mu}c}{2}\right)\cos^2\left(\frac{\bar{\mu}c}{2}\right) \\ &\quad\times\left(\sin\left(\frac{\bar{\mu}c}{2}\right)\hat{V}\cos\left(\frac{\bar{\mu}c}{2}\right) - \cos\left(\frac{\bar{\mu}c}{2}\right)\hat{V}\sin\left(\frac{\bar{\mu}c}{2}\right)\right)\end{aligned} \qquad (3)$$

where we have used the definition of the holonomies. These holonomies are computed over square loops with physical area given by the minimum eigenvalue of the area operator in LQG which is $\alpha\ell_{\rm P}^2$ where α is of the order unity. We have denoted the physical length of an edge of such a loop by $\bar{\mu}|p|^{1/2}$ with the area of the loop $\mathcal{A} = \bar{\mu}^2|p| = \bar{\mu}^2a^2$, then its equality with minimum eigenvalue of area operator in LQG yields

$$\bar{\mu}^2a^2 = \alpha\ell_{\rm P}^2. \qquad (4)$$

Thus quantum geometry acts like a regulator for the size of the loops over which holonomies are evaluated and brings in elements of quantum discreteness inherited from LQG.

Using the relation $V = |p|^{3/2}$, we can also define the volume and the inverse volume operator which lead to a discrete eigenvalues of $\hat{V}$ given by $V_\mu \equiv (4\pi\mu\gamma\ell_{\rm P}^2/3)^{3/2}$ and of $\widehat{V^{-1}}$ given by [10]

$$d_j(\mu) = \left[\frac{4}{3\pi\ell_{\rm P}^2\gamma\bar{\mu}j(j+1)(2j+1)}\sum_{n=-j}^{j} nV_{\mu+2n\bar{\mu}}^{1/2}\right]^6 \qquad (5)$$

which is bounded and implies that physical densities remain finite in LQC. Here $\bar{\mu}$ is given by Eq. (4). The parameter j arises due to tracing over SU(2) holonomies in an irreducible spin j representation. In particular it determines a critical scale factor $a_* = \sqrt{8\pi\gamma j\bar{\mu}/3}\ell_{\rm P}$ below which the eigenvalues of inverse scale factor operator become proportional to the positive powers of scale factor [4]. However, for $a \gtrsim a_*$, the eigenvalues d_j quickly approximate the classical behavior i.e. $d_j \approx a^{-3}$.

The physical states are given by $|\psi\rangle = \sum_\mu \psi(\mu, \phi)|\mu\rangle$, where ϕ denotes matter degrees of freedom which constitute the matter Hamiltonian operator $\hat{\mathcal{H}}_{\rm M}$. The action of these states on $\hat{\mathcal{H}}$ yields the quantum evolution given by the following discrete difference equation

$$\begin{aligned}&\frac{3}{4\kappa\gamma^2\bar{\mu}}[C(\mu+4\bar{\mu})\psi(\mu+4\bar{\mu}, \phi) - 2C(\mu)\psi(\mu, \phi) \\ &\quad + C(\mu-4\bar{\mu})\psi(\mu-4\bar{\mu}, \phi)] + \hat{\mathcal{H}}_{\rm M}\psi(\mu, \phi) = 0\end{aligned} \qquad (6)$$

with

$$C(\mu) = \frac{1}{4\pi\gamma\bar{\mu}\ell_{\rm P}^2}(V_{\mu+\bar{\mu}} - V_{\mu-\bar{\mu}}). \qquad (7)$$

The quantum evolution determined by the above equation has been shown to be nonsingular at the big bang [8,11,30]. At the classical level, $\mathcal{H}_{\rm M}$ in general would contain inverse scale factors which would blow up when $a \longrightarrow 0$ and the evolution would break down. This is cured in LQC due to modification to the behavior of the eigenvalues of the inverse volume operator by Eq. (5) which remain bounded through the whole evolution.

In order to compare the discrete dynamics resulting from the difference equation and the classical theory, one can construct semiclassical states, study their discrete quantum evolution and compute the expectation values of the observables [6–8,17]. These investigations show that for scale factors greater than of the order Planck length we can consider the emergence of a continuous spacetime picture with the dynamics governed by the following effective Hamiltonian constraint which approximates very well the evolution via the difference equation [7,8,17,31]

$$\mathcal{H}_{\rm eff} = -\frac{3}{\gamma^2\kappa\bar{\mu}^2}s_j\sin^2(\bar{\mu}c) + \mathcal{H}_{\rm M} \qquad (8)$$

where s_j is given by [32]

$$s_j = -\frac{3}{4\pi\ell_{\rm P}^2\gamma\bar{\mu}j(j+1)(2j+1)}\sum_{n=-j}^{j} nV_{\mu-2n\bar{\mu}} \qquad (9)$$

with a behavior similar to that of $d_j(\mu)$. For $a > a_*$ it is very well approximated by $s_j \approx a$. Effects due to quantum geometric regulator for the minimum area of loops over which holonomies are computed are manifest in the gravitational part of $\mathcal{H}_{\rm eff}$. The matter Hamiltonian $\mathcal{H}_{\rm M}$ in general includes the modified behavior of the inverse scale factor (Eq. (5)).

We can compare the effective Hamiltonian obtained in LQC with the classical Hamiltonian constraint,

$$\mathcal{H}_{\rm cl} = -\frac{3}{\kappa}\dot{a}^2 a + \mathcal{H}_{\rm M} = -\frac{3}{\gamma^2\kappa}c^2 p^{1/2} + \mathcal{H}_{\rm M}. \quad (10)$$

where we have used the relation $c = \gamma\dot{a}$. In the limit when $a \gg a_*$ and $\bar{\mu}c \ll 1$ it is easily seen that Eq. (8) reduces to Eq. (10). We should note that the modifications in the effective Hamiltonian due to the change in behavior of eigenvalues of the inverse scale factor operator are of significance if $a \lesssim a_*$, where as those due to discrete quantum effects become important whenever $\bar{\mu}c$ is large. Since $\bar{\mu} \propto a^{-1}$ and $c \propto \dot{a}$, the discrete quantum effects become significant at large values of H or $\rho^{1/2}$, strictly speaking when ρ becomes of the order of $\rho_{\rm crit}$ in Eq. (17). The domain in which inverse scale factor modifications can be important is determined solely by the parameter j which determines a_*, whereas the domain in which discrete quantum effects are important depends on the value of energy density. For a general choice of matter configuration it is possible that inverse scale factor modifications and discrete quantum effects are significant in distinct domains. To see this let us consider an example of energy density (ρ_ϕ) sourced by a massless scalar field. Then $\rho_\phi \sim \rho_{\rm crit}$ implies a critical scale factor $a_{\rm crit}$ near which discrete quantum geometric effects become significant. This critical scale factor can be easily computed to be $a_{\rm crit} = (8\pi\alpha\gamma^2/6)^{1/6}\Pi_\phi^{1/3}\ell_{\rm P}$ where Π_ϕ is the scalar field momenta in Planck units. Further we can compute a_* which on using Eq. (4) turns out to be $a_* = (8\pi\alpha^{1/2}\gamma/3)^{1/3}j^{1/3}\ell_{\rm P}$. Comparing the two scales we find that $a_{\rm crit} > a_*$ for $\Pi_\phi > 4j$. Thus for these values of Π_ϕ we have a regime where discrete quantum effects are important and the modifications due to inverse scale factor can be ignored in the dynamics. However we should also note that even for values of Π_ϕ such that $a_{\rm crit} < a_*$, numerical investigations indicate that the discrete quantum effects dominate those due to inverse scale factor, unless we choose an initial matter configuration such that $\bar{\mu}c \ll 1$ through out the evolution [7]. The effects due to inverse scale factor modifications are further weakened when compared to discrete quantum effects if we take into account theoretical arguments which favor a small value of j parameter [33]. For these reasons in this work we would focus exclusively on the modifications due to discrete quantum geometry effects present in the first term of Eq. (8). This choice can be made by considering the matter configuration for a given value of j such that $a_{\rm crit} > a_*$. Then modification due to inverse scale factor can be neglected in (8), especially the matter Hamiltonian $\mathcal{H}_{\rm M}$ and the corresponding expressions for energy density and pressure remain same as classically [9].

In general the matter Hamiltonian, $\mathcal{H}_{\rm M}$, would be composed of a massive scalar field ϕ with a conjugate momentum Π_ϕ, energy density ρ_ϕ and a matter component with a constant equation of state $w = p_w/\rho_w$, and thus the total energy density $\rho = \rho_\phi + \rho_w$. For our case of interest, $a > a_*$, the matter Hamiltonian in the modified dynamics is given by [9]

$$\mathcal{H}_{\rm M} = \frac{1}{2}\frac{\Pi_\phi^2}{p^{3/2}} + p^{3/2}V(\phi) + \rho_w p^{3/2}. \quad (11)$$

Also for this case the energy density and pressure for the scalar field are equal to their classical values [9], i.e.

$$\rho_\phi = \tfrac{1}{2}\dot{\phi}^2 + V(\phi), \qquad p_\phi = \tfrac{1}{2}\dot{\phi}^2 - V(\phi) \quad (12)$$

where we have used the Hamilton's equation $\dot{\phi} = \Pi_\phi/p^{3/2}$. It is then straightforward to find the Klein-Gordon equation using the Hamilton's equations for $\dot{\phi}$ and $\dot{\Pi}_\phi$, which turns out to be of the same form as the one classically and satisfies the stress-energy conservation law, $\dot{\rho}_\phi + 3(\dot{a}/a)(\rho_\phi + p_\phi) = 0$.

The effective Hamiltonian which is now given by

$$\mathcal{H}_{\rm eff} = -\frac{3}{\gamma^2\kappa\bar{\mu}^2}a\sin^2(\bar{\mu}c) + \mathcal{H}_{\rm M} \quad (13)$$

leads via the Hamliton's equation of $\dot{p}$,

$$\dot{p} = \{p, \mathcal{H}_{\rm eff}\} = -\frac{\gamma\kappa}{3}\frac{\partial}{\partial c}\mathcal{H}_{\rm eff} \quad (14)$$

the rate of change of the scale factor

$$\dot{a} = \frac{1}{\gamma\bar{\mu}}\sin(\bar{\mu}c)\cos(\bar{\mu}c). \quad (15)$$

Further, the vanishing of the Hamiltonian constraint implies

$$\sin^2(\bar{\mu}c) = \frac{\kappa\gamma^2\bar{\mu}^2}{3a}\mathcal{H}_{\rm M} \quad (16)$$

which on using Eq. (15) yields

$$H^2 = \frac{\kappa}{3}\rho\left(1 - \frac{\rho}{\rho_{\rm crit}}\right), \qquad \rho_{\rm crit}^{-1} = \alpha\kappa\gamma^2\ell_{\rm P}^2/3 \quad (17)$$

where we have used Eq. (4).

The modifications originating due to discrete quantum effects to the effective Friedmann equation in LQC are thus of the type where a ρ^2 term becomes important in the regime of high energies. This feature holds even at scales less than a_* with ρ now including modifications due to the peculiar behavior of eigenvalues of the inverse scale factor operator. When energy density becomes small compared to $\rho_{\rm crit}$, the modification $\rho/\rho_{\rm crit}$ becomes very small and the effective Friedmann equation reduces to the classical Friedmann equation.

Comparing with the string-inspired Randall-Sundrum scenario, the sign of the correction term is negative which leads to a nonsingular bouncing cosmology [8]. Interestingly, similar Friedmann equation arises in a braneworld scenario which is a modification of the Randall-Sundrum model in the sense that the extra dimension is considered to be timelike [34]. As we would see the correspondence between the effective dynamics in LQC and the Randall-Sundrum braneworlds is much deeper, with the dual relationships between the scaling solutions of both of the scenarios.

III. SCALAR FIELD DYNAMICS

Let us consider the dynamics in the effective theory for a Universe with only a massive scalar field contributing as the matter component. We further work with a generalization that we would allow the scalar field to be of phantom type which has a negative kinetic energy [35]. The energy density and the pressure in Eq. (12) then generalize to

$$\rho_\phi = n\frac{\dot\phi^2}{2} + V(\phi), \qquad p_\phi = n\frac{\dot\phi^2}{2} - V(\phi) \tag{18}$$

where $n = \pm 1$ for the standard scalar field and the phantom field, respectively. The Klein-Gordon equation can then be written as

$$\dot\rho_\phi = -3H(\rho_\phi + p_\phi) = -3nH\dot\phi^2. \tag{19}$$

Using Eq. (17), we can find the rate of change of the Hubble rate

$$\dot H = -\frac{\kappa}{2}n\dot\phi^2\left(1 - 2\frac{\rho}{\rho_{\rm crit}}\right) \tag{20}$$

and

$$\frac{\ddot a}{a} = \frac{\kappa}{3}\left[\rho\left(1 - \frac{\rho}{\rho_{\rm crit}}\right) - \frac{3n}{2}\dot\phi^2\left(1 - 2\frac{\rho}{\rho_{\rm crit}}\right)\right]. \tag{21}$$

It is easy to then see from these equations dynamical features of the effective theory. In the case when $\rho \ll \rho_{\rm crit}$, the dynamical evolution is classical. For the case of a standard scalar field, $\dot H < 0$ and the sign of $\ddot a$ depends on whether or not the potential term dominates over the kinetic term. Whereas for the phantom field, $\dot H > 0$ and $\ddot a$ is positive leading to a phase of superinflation.

The effect of $\rho_{\rm crit}$ term on the scalar field dynamics is very peculiar, especially when $\rho_{\rm crit}/2 < \rho < \rho_{\rm crit}$. In this case, for the standard scalar field $\dot H > 0$ and $\ddot a > 0$ leading to a phase of superinflation, irrespective of the choice of the potential. However, for the phantom field $\dot H < 0$ and the sign of the $\dot a$ term would now depend on the ratio of potential to the kinetic energy, as for the standard scalar field in standard Friedmann cosmology. The behavior of the massive scalar field in the regime when loop quantum modifications are very significant ($\rho_{\rm crit}/2 < \rho < \rho_{\rm crit}$) mimics that of a phantom field and the phantom field in this regime behaves like an ordinary scalar field.

We note that for $a < a_*$, existence of a superinflationary phase in LQC due to modifications pertaining to the inverse scale factor behavior is known [10] and has been investigated in detail [14,15]. It has been established that this phase can yield generic conditions for chaotic inflation to start even when the inflaton is initially at the bottom of the potential. Further, it also leads to distinct signatures in the cosmic microwave background [14]. However, here we have shown that this phase generically exists in LQC even for $a > a_*$ due to discrete quantum geometric effects. This would further increase the range of initial parameters for the onset of inflation. For $a < a_*$, it would lead to an additional superinflation and thus increase the number of e-foldings in the loop modified phase.

In classical as well as the Randall-Sundrum braneworld cosmology, it has been shown that different phases of the scalar field dynamics are related by some symmetry transformations [23]. We would now investigate this issue for the loop cosmology. For that we define a new variable

$$h(\phi) = \frac{\rho}{\rho_{\rm crit} - \rho} \tag{22}$$

such that the Friedmann Eq. (17) can be written as

$$a(\phi)_{,\phi}h(\phi)_{,\phi} = -n\kappa a(\phi)h(\phi) \tag{23}$$

with $a(\phi)$ determined as

$$a(\phi) = \exp\left(-\kappa\int d\phi h(\phi)h_{,\phi}^{-1}\right). \tag{24}$$

An important property of Eq. (23) is that two distinct cosmological models/phases represented by $(a_1(\phi), h_1(\phi), V_1(\phi))$ and $(a_2(\phi), h_2(\phi), V_2(\phi))$, leave Eq. (23) invariant if

$$a_2(\phi) = h_1^\ell(\phi), \qquad h_2(\phi) = a_1^{1/\ell}(\phi) \tag{25}$$

where ℓ is a constant. Cosmological models/phases related via (25) are said to be dual to each other. This property is immensely useful to establish correspondence between different cosmological phases [23] or between different models [25,26]. Here we would study its symmetries for different phases of the scalar field dynamics. In the next section, we would apply this to establish dualities of loop cosmology with braneworld scenarios.

Let us consider an expanding cosmological phase with a standard scalar field, specified by $a_1(\phi)$ and $h_1(\phi)$. Then a simple duality based on Eq. (23) which is given by $h_2(\phi) = a_1(\phi)$ leads to

$$a_2(\phi) = h_1(\phi) = \frac{\rho_1(\phi)}{\rho_{\rm crit} - \rho_1(\phi)} \tag{26}$$

on using Eq. (24). It is then straightforward to find the rate of change of the dual scale factor,

$$\dot{a}_2(\phi) = -\frac{3H_1\dot{\phi}_1^2}{\rho_{\rm crit}}(1 + h_1(\phi))^2 \tag{27}$$

with

$$H_1 = \left(\frac{\kappa\rho_{\rm crit}}{3}\right)^{1/2}\frac{h_1(\phi)^{1/2}}{(1 + h_1(\phi))} \tag{28}$$

and

$$\dot{\phi}_1 = -n\left(\frac{\rho_{\rm crit}}{3\kappa}\right)^{1/2}\frac{h_{1,\phi}}{h_1(\phi)^{1/2}(1 + h_1(\phi))}. \tag{29}$$

Since, $H_1 > 0$ we obtain $\dot{a}_2 < 0$ and thus the duality $a(\phi) \leftrightarrow h(\phi)$ maps an expanding phase to a contracting phase, for the standard scalar field.

Another simple duality transformation is when $a_3(\phi) = h_1^{-1}(\phi)$ and $h_3(\phi) = a_1(\phi)$. In this case it can be checked that $a_3(\phi)$ and $h_3(\phi)$ satisfy the Friedmann equation for the phantom field,

$$a_3(\phi)_{,\phi}h_3(\phi)_{,\phi} = \kappa a_3(\phi)h_3(\phi). \tag{30}$$

It is also easy to verify that

$$\dot{a}_3(\phi) = \frac{3H_1\dot{\phi}_1^2}{\rho_{\rm crit}}\frac{(1 + h_1(\phi))^2}{h_1(\phi)^2} \tag{31}$$

so that if $a_1(\phi)$ corresponds to an expanding (contracting) branch then $a_3(\phi)$ also denotes an expanding (contracting) branch. Thus, this duality transformation maps an expanding branch for the standard scalar field to an expanding branch of a phantom field. Further, since $h(\phi)$ is greater (less) than unity for ρ greater (less) than $\rho_{\rm crit}/2$, this duality maps orbits $h_1(\phi) \gtrless 1$ belonging to the superinflationary phase for the standard scalar field to the orbits $a_3(\phi) \lesseqgtr 1$ for the phantom field. Similarly, the contracting branch of the standard scalar field and the phantom branch are dual to each other under the map, $a_2(\phi) = a_3^{-1}(\phi)$, $h_2(\phi) = h_3(\phi)$.

IV. SCALING SOLUTIONS AND DUALITIES WITH RANDALL-SUNDRUM BRANEWORLDS

A significant issue concerning the effective dynamics in LQC is its stability. For that it is useful to construct the scaling solutions as they provide insights on the asymptotic behavior of solutions and can also serve to establish symmetries between distinct cosmologies [26]. In a model when only modifications due to eigenvalues of the inverse scale factor operator are important, scaling solutions have been obtained [28,29]. For the case of our interest, these modifications can be neglected and hence our scaling solutions would be different from those in Ref. [28,29]. We would consider the energy density with contributions from a massive scalar field and matter with a fixed equation of state w. In order to study the scaling solutions it is useful to define new variables:

$$x := (\dot{\phi}^2/2\rho)^{1/2}, \qquad y := (V/\rho)^{1/2} \tag{32}$$

and

$$\Omega := -\frac{\kappa^{-1/2}}{(1 - \rho/\rho_{\rm crit})^{1/2}}\frac{V_{,\phi}}{V}. \tag{33}$$

Then the set of dynamical equations composed of Eq. (17), the stress-energy conservation law

$$\dot{\rho}_w + 3\frac{\dot{a}}{a}(\rho_w + p_w) = 0 \tag{34}$$

and the Klein-Gordon Eq. (19) can be casted into dynamical equations in (x, y, Ω),

$$\frac{dx}{dN} = -3x + (3/2)^{1/2}\Omega y^2 + (3/2)xB(x, y, \Omega) \tag{35}$$

$$\frac{dy}{dN} = -(3/2)^{1/2}\Omega xy + (3/2)yB(x, y, \Omega) \tag{36}$$

$$\frac{d\Omega}{dN} = -\sqrt{6}x\Omega^2 V\left[\frac{V_{,\phi\phi}}{V_{,\phi}^2} - 1\right] - \frac{3\Omega}{2}B(x, y, \Omega) \times \frac{\rho/\rho_{\rm crit}}{(1 - \rho/\rho_{\rm crit})} \tag{37}$$

where $N := \log a$ and

$$B(x, y, \Omega) := 2x^2 + (1 + w)(1 - x^2 - y^2). \tag{38}$$

As before we also express the Friedmann Eq. (17) in the form (23), with $h(\phi)$ now given as

$$h(\phi) = \exp\left((x^2 + y^2)\int\frac{d\rho}{\rho(1 - \rho/\rho_{\rm crit})}\right) \tag{39}$$

where we have used $x^2 + y^2 = \rho_\phi/\rho$. Also the scale factor $a(\phi)$ is determined by Eq. (24) with the above $h(\phi)$.

The scaling solutions can then be obtained by solving

$$\frac{dx}{dN} = \frac{dy}{dN} = \frac{d\Omega}{dN} = 0.$$

It is straightforward to check that for the case when $\rho_w \gg \rho_\phi$, the critical point is given by

$$x_c = \left(\frac{3}{2}\right)^{1/2}\frac{1 + w}{\Omega}, \qquad y_c = \frac{1}{\Omega}\left(\frac{3(1 - w^2)}{2}\right)^{1/2} \tag{40}$$

with the constraint

$$\rho\frac{\rho_{,\phi\phi}}{\rho_{,\phi}^2} + \frac{1}{2}\frac{\rho/\rho_{\rm crit}}{(1 - \rho/\rho_{\rm crit})} = 1. \tag{41}$$

Further, when $\rho_\phi \gg \rho_w$ the critical point is

$$x_c = \Omega/\sqrt{6}, \qquad y_c = \left(1 - \frac{\Omega^2}{6}\right)^{1/2} \tag{42}$$

with the same constraint as Eq. (41). Interestingly, if Ω is treated as a constant then the form of the Eqs. (35)–(37) is identical to the one obtained in the standard FRW cosmology [24]. This important feature has been noted earlier in the context of modified gravity scenarios in the string-inspired cosmologies [25,26]. Because of this correspondence with the classical equations we expect that the critical points for our case belongs to the set of critical points in the classical theory. This turns out to be true if we recall that for classical theory, the corresponding variables defined as

$$x_{\rm cl} = \left(\frac{\kappa\dot{\phi}^2}{6H^2}\right)^{1/2}, \qquad y_{\rm cl} = \left(\frac{\kappa V}{3H^2}\right)^{1/2}, \qquad \Omega_{\rm cl} = -\left(\frac{V_{,\phi}}{\kappa^{1/2}V}\right) \tag{43}$$

do indeed lead to the critical points given by Eq. (40) and (42) for the cases when $\rho_w \gg \rho_\phi$ and $\rho_\phi \gg \rho_w$ respectively [24]. The stability analysis of Ref. [24] then implies that both of the critical points (40) and (42) are attractors if the former satisfies $\Omega^2 > 3(1+w)$ and latter satisfies $\Omega^2 < 3(1+w)$.

The potential for the scaling solutions can be determined by integrating Eq. (41) and using Eq. (32), which turns out to be

$$V(\phi) = y_c^2 \rho_{\rm crit} {\rm sech}^2(-\kappa^{1/2}\Omega\phi/2). \tag{44}$$

On integrating Eqs. (24) and (39) we can obtain $a(\phi)$ and $h(\phi)$ for the two scaling solutions

$$a(\phi) = \cosh^{2\Omega^{-2}(x_c^2+y_c^2)^{-1}}(-\kappa^{1/2}\Omega\phi/2) \tag{45}$$

$$h(\phi) = \rho_{\rm crit}^{(x_c^2+y_c^2)} {\rm csch}^{2(x_c^2+y_c^2)}(-\kappa^{1/2}\Omega\phi/2). \tag{46}$$

Once we have found the set $(a(\phi), h(\phi), V(\phi))$ for the scaling solutions in LQC, we can then find a model with a dual scaling solutions using Eq. (23). Such dualities have been studied for modified gravity scenarios earlier and in Ref. [26] it was established that if a cosmological model with a modified Friedmann equation of the form $H^2 = (\kappa/3)\rho(\phi)G_1(\phi)$ has a scaling solution $(a_1(\phi), b_1(\phi), V_1(\phi))$, then a necessary and sufficient condition for different cosmological model with $H^2 = (\kappa/3)\rho(\phi)G_2(\phi)$ and scaling solution $(a_2(\phi), b_2(\phi), V_2(\phi))$ to satisfy Eq. (25) and to be the dual is that

$$G_1^{1/2}(\phi) = -\frac{\ell f}{G_2^{1/2}(\phi)} \tag{47}$$

with

$$f = (x_c^2 + y_c^2)^2\Omega_1\Omega_2. \tag{48}$$

In order to find the dual of the scaling solution in LQC, we first note that $G_1(\phi) = 1 - \rho/\rho_{\rm crit}$ which using Eq. (44) and (32) gives

$$(1 - \rho/\rho_{\rm crit})^{1/2} = \tanh(-\kappa^{1/2}\Omega_1\phi/2). \tag{49}$$

Interestingly, the corresponding function $G_2(\phi)$ for the Randall-Sundrum scenario is [26]

$$(1 + \rho/2\sigma)^{1/2} = \coth(-\kappa^{1/2}\Omega_2\phi/2) \tag{50}$$

which on using Eq. (47) and further choosing $\Omega_1 = \Omega_2 = \Omega$ implies that the scaling solution of the effective theory of LQC and the Randall-Sundrum scenario are dual to each other if $\ell = -1/(x_c^2 + y_c^2)^2\Omega^2$ and $\rho_{\rm crit}$ is identified with the brane tension σ,

$$\rho_{\rm crit} = \left(\frac{\alpha\kappa\gamma^2\ell_{\rm P}^2}{3}\right)^{-1} = 2\sigma. \tag{51}$$

In the Randall-Sundrum braneworld scenarios σ plays the role of a critical energy density scale near and above which the effects due to extra dimensions become significant and the standard Friedmann dynamics is modified. In the effective theory of LQC, the analog of brane tension is $\rho_{\rm crit}$ which contains the fundamental loop parameter γ. Since both $\rho_{\rm crit}$ and σ play the same role in the modified Friedmann dynamics originating from LQC and the string-inspired scenarios, respectively, the above correspondence leads to the dual relationship at the level of scaling solutions between the two scenarios.

V. SUMMARY

In this work we have analyzed the effective dynamics in LQC when effects due to discrete quantum geometric modifications are important and those from the change in behavior of the eigenvalues of the inverse scale factor operator are negligible. We have shown that in this case the Friedmann equations gets a ρ^2 modification similar to the Randall-Sundrum scenario but with a negative sign (which also arises in a modified Randall-Sundrum model with a timelike extra dimension). The ρ^2 term is also present if the modifications to the inverse scale factor for $a < a_*$ are included.

The modification to the Friedmann equation has various interesting properties. We have shown that it leads to a generic phase of superinflation when $\rho > \rho_{\rm crit}/2$. This phase is independent of the one earlier studied [10,14,15] which originates due to change in behavior of the kinetic term in the Klein-Gordon equation for $a < a_*$. Existence of this additional phase of superinflation in the very early Universe would increase the number of e-foldings originating in the loop quantum modified phase and assist onset of conventional inflation. Further, an ordinary scalar field behaves as a phantom field and the vice versa in this regime. We have also found various symmetries linking the expanding and the contracting branch with the phantom field dynamics in the effective theory.

We have obtained scaling solutions in the effective theory which give us valuable information about the stability of the dynamics and can also be used to find symmetries between distinct cosmological models. We find that the scaling solutions in the effective theory are dual to those of the Randall-Sundrum scenario if the critical density arising in LQC is identified with the brane tension. Scaling solutions for effective dynamics in LQC with arbitrary j but without any discrete quantum geometric effects have been obtained earlier and dualities with standard cosmology and the string-inspired scenarios have also been noted [28,29]. The scaling solutions obtained here and those in Ref. [28,29] thus belong to complimentary domains of the effective theory. It would be interesting to consider effective dynamics with discrete quantum corrections for scale factors below a_* and obtain the scaling solutions and the associated dualities with other modified gravity scenarios.

It is very interesting to see that two distinct quantization schemes for gravity yield ρ^2 modification to the Friedmann equation at high energies. Though the effective theory of LQC is dual to the Randall-Sundrum braneworlds in the sense described above we shall remember that both models yield different predictions for the early Universe. For example, the phase of superinflation in the very early Universe and of a nonsingular bounce in a contracting Universe which are the features generically present in LQC, are absent in the Randall-Sundrum scenario. Let us now point out various uses of this duality symmetry. The first and perhaps the most straightforward use is to apply this duality to classify the similarities and differences in various dynamical solutions between LQC and Randall-Sundrum braneworlds. Such a classification is important to compare physical results predicted by scenarios which predict modifications to the standard Friedmann dynamics. Detailed comparison of dynamical solutions between Randall-Sundrum scenario, Dvali-Gabadadze-Porrati (DGP) braneworlds [36] and Cardassian cosmology [37] has been done earlier [26]. Thus with the duality established between LQC and Randall-Sundrum scenario, we can further explore the connection of LQC with Cardassian and DGP models, compare the dynamical properties and obtain insights into links between seemingly unrelated and distinct cosmological models.

An interesting use of duality symmetry can be to explore the detailed dynamical properties of LQC by using the established results in braneworld scenarios. For example, the duality relation can be used to calculate the spectral index of scalar perturbations in the effective theory in LQC. For this let us recall that recently the duality relation between inflationary and cyclic models was used to confirm the the value of spectral index in cyclic model, given the value in inflationary models [23]. In LQC, a full fledged calculation of spectral index by including inhomogeneities and reducing the symmetry of the framework is yet to be undertaken and the effective theory with inhomogeneities is not known. However, a calculation on the above lines starting from the spectral index in Randall-Sundrum cosmology and obtaining the one in LQC would provide a first estimate of what we may obtain by a detailed analysis. Such an exercise would also provide a test for the duality relation found in this paper as the calculation of spectral index by using duality symmetry should confirm with the result obtained from the effective theory obtained after including inhomogeneities in the quantum theory.

We should note that in LQC, the evolution is generically nonsingular and a flat Universe bounces at scales close to Planck length [8] whereas it has remained an outstanding problem to obtain generic nonsingular bouncing solutions in string-inspired cosmologies. It has been shown that duality relation can be used to relate singular cosmological background with nonsingular one [26]. This may help in shedding some light on construction of nonsingular bouncing models in string-inspired cosmologies by using the solutions in LQC. Further since LQC is based on Dirac's method of canonical quantization, another the use of this duality can be to establish a similar construction for braneworld scenarios.

The existence of an exact duality between the scaling solutions of LQC and the Randall-Sundrum scenario comes as a surprise since they are derived from two very different unrelated approaches to quantize gravity. At high energies both of the scenarios predict a ρ^2 modification to the Friedmann equation (although with a different sign) but the origin of this modification is the existence of an extra dimension in Randall-Sundrum scenario and the discrete quantum geometry in LQC. Note that the spacetime geometry in string-inspired Randall-Sundrum scenario is continuous whereas LQC is based on a four dimensional quantization of spacetime. Existence of any symmetry relationship between such two significantly distinct models to describe nature of the very early Universe is very nontrivial and perhaps more than a mere coincidence. The duality symmetry between Randall-Sundrum model and LQC suggest that some of the effects originating due to existence of extra dimensions in a continuous spacetime bulk might be mimicked by the quantum geometric nature of a four dimensional spacetime. This signifies the nontrivial and deep nature of this duality symmetry. We should here emphasize that both string theory and LQG, the underlying theories on which Randall-Sundrum scenario and LQC are, respectively, based, are still far from being complete theories. In fact they suffer from complimentary problems: lack of a nonperturbative background independent treatment in string theory and little insights on the way to obtain a semiclassical perturbative description in LQG. Therefore any relation like above, if proved as a deep link between two theories by future investigations, might prove immensely useful in gaining insights on the resolution of these problems [38] and may also provide a new paradigm for a complete theory of quantum gravity which may include both stringy and loopy ideas.

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