# Destruction of small-scale dark matter clumps in the hierarchical structures and galaxies

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A mass function of small-scale dark matter clumps is calculated in the standard cosmological scenario with an inflationary-produced primordial fluctuation spectrum and with a hierarchical clustering. We take into account the tidal destruction of clumps at early stages of structure formation starting from a time of clump detachment from the Universe expansion. Only a small fraction of these clumps, ~0.1%–0.5%, in each logarithmic mass interval $\Delta \log M \sim 1$ survives the stage of hierarchical clustering. The surviving clumps can be disrupted further in the galaxies by tidal interactions with stars. We performed the detailed calculations of the tidal destruction of clumps by stars in the Galactic bulge and halo and by the Galactic disk itself. It is shown that the Galactic disc provides the dominant contribution to the tidal destruction of small-scale clumps outside the bulge. The results obtained are crucial for calculations of the dark matter annihilation signal in the Galaxy.

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## I. INTRODUCTION

One of the promising indirect manifestation of the dark matter (DM) particles is their possible annihilation in the Galactic halo [1]. A local annihilation rate is proportional to the square of the number density of DM particles. Therefore, annihilation proceeds more efficiently in the dense DM substructures of the Galactic halo. Both analytical calculations [2–5] and numerical simulations [6–8] with the inflationary-produced adiabatic density fluctuations predict the existence of DM clumps in the Galactic halo. The enhancement of the annihilation signal due to the presence of substructures in the Galactic halo depends on the fraction of the most dense small-scale clumps [4,9]. The most essential characteristics of clumps for calculations of DM annihilation in the Galactic halo are the minimum mass and distribution function of clumps. At the same time the tidal destruction of clumps [4] strongly influences the number density of clumps in the Galaxy.

The small-scale clumps [2–4] are formed only if the corresponding density fluctuations are large enough. The inflation models predict the power-law primordial fluctuation spectrum with a power index $n_p \approx 1.0$. The small-scale clumps are formed earlier than the larger ones and captured by the larger clumps in the process of a hierarchical clustering in the expanding Universe. Eventually all clumps consist in part of the smaller ones and of the free DM particles. An effective index of the density perturbation power spectrum $n \rightarrow -3$ at small-scales (when mass inside the perturbation $M \rightarrow 0$). This means that a gravitational clustering of small-scale structures proceeds very fast. As a result the formation of new clumps and their capture by the larger ones are nearly simultaneous processes.

A convenient formalism, which describes statistically this hierarchical clustering, is the Press-Schechter theory [10] and its extensions, in particular, the "excursion set" formalism developed by Bond *et al.* [11] (for a clear introduction see [12]). However, this theory does not include an important process of the tidal destruction of small clumps inside the bigger ones. This process has been taken into account in our previous work [4], where it was demonstrated that only a small fraction of the small-scale clumps survives the tidal destruction in the hierarchical clustering. Nevertheless, even this small fraction of survived small-scale clumps is enough to dominate the DM annihilation rate for the most reasonable spectra of primordial fluctuations.

A mass distribution of small-scale clumps survived in the hierarchical structuring was derived in [4]:

$$\xi_{\rm int}\frac{dM}{M} \simeq 0.01(n+3)\frac{dM}{M}, \tag{1}$$

where $M$ is a clump mass, $n$ is a power index of density perturbations at a mass scale $M$. The distribution function $\xi_{\rm int}$ is a mass fraction of DM in the form of clumps in the logarithmic mass interval $d\log M$.

The minimal mass of DM clumps $M_{\rm min}$ is determined by the leakage of DM particles from the growing density fluctuations (the diffuse leakage and free streaming) and depends on the properties of DM particles [3,13–18]. The existing estimates of $M_{\rm min}$ for neutralino DM are substantially different, from $M_{\rm min} \sim 10^{-12} M_\odot$ in [19] to $M_{\rm min} \sim (10^{-7} - 10^{-6})M_\odot$ in [15–17]. In [4] we performed detailed calculations of a DM particle diffusion and free streaming in the kinetic equation approach. For the case of a neutralino considered as a pure bino, we obtained for the minimal mass of DM clumps

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$$M_{\min} = 1.5 \times 10^{-8}\left(\frac{m_\chi}{100\ \mathrm{GeV}}\right)^{-15/8}\left(\frac{\tilde{M}}{1\ \mathrm{TeV}}\right)^{-3/2} \times \left(\frac{g_*}{10}\right)^{-15/16}\left(\frac{\Lambda^*}{83}\right)^3 M_\odot, \quad (2)$$

where $\tilde{M}^2 = \tilde{m}^2 - m_\chi^2$, with $m_\chi$ being a neutralino mass, $\tilde{m}$ is a sfermion mass, and $\Lambda^*$ has only a logarithmic dependence on $\tilde{M}$ and $m_\chi$. In the considered range of parameters $\Lambda^* \simeq 83$. Our value of $M_{\min}$ agrees reasonably well with [15–17] and strongly disagrees with [19] for the reasons explained in [4].

Because of uncertainties in the SUSY parameters, a numerical value of $M_{\min}$ is not exactly predicted. With our choice of SUSY parameters [4], $M_{\min} \sim 10^{-8} M_\odot$ is of the Moon-scale mass. With the other choice of SUSY parameters [17], $M_{\min} \sim 10^{-6} M_\odot$ is of the Earth-scale mass. In the numerical simulations [20], $M_{\min} \sim 10^{-6} M_\odot$ was actually assumed by putting the corresponding cutoff in the initial density perturbations.

The very interesting numerical simulations of the formation of small-scale DM clumps with a mass larger than the Earth mass have been performed recently in [20]. There is a direct correspondence of this simulation with the earlier theoretical calculations:

(i) The density profile of large-scale clumps is influenced by the hierarchical clustering of the smaller ones. The new important result of numerical simulations [20] is a resolution of the density profile of the isolated minimal mass clumps, $M_{\min} \sim 10^{-6} M_\odot$. The clumps of minimal mass are formed directly from the isolated fluctuations and their density profile is not influenced by the hierarchical clustering. The internal density profile of small-scale clumps in these simulations is proved to be the same as in the theoretical calculations [21] performed for the isolated density fluctuations. The agreement between the theory and numerical simulations for the predicted internal density profile of clumps, $\propto r^{-\beta}$, is fairly good within the involved uncertainties: $\beta = 1.7$–$1.8$ in [21], and $\beta = 1.5$–$2.0$ in [20] respectively.

(ii) The numerical simulations [20] agree rather well with the shape of theoretically derived mass function of small-scale clumps [4] but with the different normalizations.

A tidal destruction in the Galaxy of the Earth-size clumps from simulations [20] has been recently considered in [22–24]. The results are rather controversial. Authors of [22,23] conclude that all the Earth-mass clumps are destructed in tidal interactions with stars in the Galaxy, while in [24] this result was not confirmed under a different assumption on the star number density.

In this paper we present the alternative and independent calculations for all processes of the tidal destruction of small-scale clumps: (i) in the hierarchical clustering, (ii) by stars from the stellar bulge, (iii) by stars from the halo and (iv) by the Galactic disc. The last of these processes is the most effective. We also describe a new method for calculations of clump destruction in the hierarchical clustering, which is a more general (valid for the arbitrary spectra of primeval fluctuations) and formally more transparent than the earlier one in [4].

Our calculations of the tidal destruction of clumps by stars in the Galaxy are quite different from [22–24] by both methods and results. While in the references above only the tidal destructions in collisions of clumps with the individual stars were studied, we found that dominant effect is provided by the destruction of clumps in the collective gravitational field of the Galactic disc. As a result we predict that only 17% of the Earth-mass clumps survived the tidal destruction at the position of the Sun. This result is crucial for the rate of DM annihilation in the Galaxy.

The paper is organized as it follows: In Sec. II we describe a new method for calculation of the small-scale clump destruction in the hierarchical clustering. We calculate a mass function of survived clumps and compare it with a corresponding one from numerical simulations. In Sec. III the tidal destruction of clumps by the Galactic disk is considered. In Sec. IV the lifetime of clumps in the central stellar bulge and stellar halo spheroid is calculated. In Sec. V we discuss the obtained results.

We perform our calculations for the standard cosmological model with a matter density parameter $\Omega_m = 0.3$, a cosmological constant term $\Omega_\Lambda = 1 - \Omega_m \simeq 0.7$ and the Hubble constant $h = 0.7$.

## II. DESTRUCTION OF CLUMPS IN HIERARCHICAL CLUSTERING

The process of hierarchical clustering and tidal destruction of DM clumps can be outlined in the following way. The DM clumps of minimal mass are formed first in the expanding Universe. The clumps of larger mass, which host the smaller ones are formed later, and so on. Some parts of the clumps are destroyed tidally in the gravitational field of their host clumps.

In this section we study the destruction of DM clumps in the process of hierarchical structuring long before the final galaxy formation. At small mass scales the hierarchical clustering is a fast and rather complicated nonlinear process. We use a simplified model which nevertheless takes into account the most important features of hierarchical clustering.

To describe the formation of clumps we will use the model of spherical collapse [12] in flat cosmology without the $\Lambda$-term. This assumption is well justified at early times of clumps formation when the $\Lambda$-term is negligible in comparison with the matter density. In this model a formation time of clump with an internal density $\rho$ is $t = (\kappa\rho_{eq}/\rho)^{1/2} t_{eq}$, where $\kappa = 18\pi^2$ and $\rho_{eq} = \rho_0(1 + z_{eq})^3$

is a cosmological density at the time of matter-radiation equality $t_{eq}$, $1 + z_{eq} = 2.35 \times 10^4 \Omega_m h^2$ and $\rho_0 = 1.9 \times 10^{-29} \Omega_m h^2$ g cm$^{-3}$. The index eq here and throughout below refers to quantities at the time of matter-radiation equality $t_{eq}$.

The DM clumps of mass $M$ can be formed from density fluctuations of different peak-height $\nu = \delta_{eq}/\sigma_{eq}(M)$, where $\sigma_{eq}(M)$ is the fluctuation dispersion on a mass-scale $M$ at the time $t_{eq}$. A mean internal density of clump $\rho$ is fixed at the time of clump formation and according to [12] equals $\rho = \kappa \rho_{eq}[\nu \sigma_{eq}(M)/\delta_c]^3$, where $\delta_c = 3(12\pi)^{2/3}/20 \simeq 1.686$.

The tidal destruction of clumps is most effective at the early epochs of the Galactic halo formation, when the host density profiles are not finally established. The tidal interaction of clumps is a complicated process and depends on many factors: a clump formation history, host density profile, an existence of different substructures inside the host, orbital parameters of individual clumps in the hosts, etc. Only in numerical simulations can all these factors be taken into account properly. In this paper we use a simplified approach by calculating an energy gain per each tidal interaction and a number of tidal interactions per dynamical time in the hosts.

An internal energy of self-gravitating object increases in tidal interactions. This energy increase was calculated e.g. in [25] for the case of a star globular cluster in a spherical galaxy. By using the model of tidal heating from [25], we determine now a survival time (or a time of tidal destruction) $T$ of some chosen small-scale clump due to the tidal heating inside of a larger mass host clump. The motion of a clump would be rather complicated in the case of a fast hierarchical clustering of hosts. During a dynamical time in the host $t_{dyn} \simeq 0.5(G\rho_h)^{-1/2}$, where $\rho_h$ is a mean internal density of the host, the chosen small-scale clump can belong to several successively destructed hosts. We will consider a typical clump orbit inside the host and assume for simplicity in this section the isothermal internal density profile of the clump.

A clump trajectory in the host experiences successive turns accompanied by the "tidal shocks" [25,26]. For the considered small-scale clump with a mass $M$ and radius $R$, the corresponding internal energy increase after a single tidal shock is

$$\Delta E \simeq \frac{4\pi}{3} \gamma_1 G \rho_h M R^2, \qquad (3)$$

where a numerical factor $\gamma_1 \sim 1$. Let us denote the number of tidal shocks per dynamical time $t_{dyn}$ by $\gamma_2$. A corresponding rate of clump internal energy growth is $\dot{E} = \gamma_2 \Delta E/t_{dyn}$. A clump is destroyed in the host if its internal energy increase due to tidal shocks exceeds a total energy $|E| \simeq GM^2/2R$. As a result, for a typical time $T = T(\rho, \rho_h)$ of the tidal destruction of a small-scale clump with density $\rho$ inside a more massive host with a density $\rho_h$ we obtain:

$$T^{-1}(\rho, \rho_h) = \dot{E}/|E| \simeq 4\gamma_1 \gamma_2 G^{1/2} \rho_h^{3/2} \rho^{-1}. \qquad (4)$$

It turns out that a resulting mass function of small-scale clumps (see in the section below) depends rather weakly on the value of $\gamma_1 \gamma_2$.

During the lifetime of an individual small-scale clump, it can sequentially inhabit many host clumps of larger mass. After the tidal disruption of the first lightest host, a small-scale clump becomes a constituent part of a heavier one, etc. The process of hierarchical transition of a small-scale clump from one host to another occurs almost continuously in time up to the final host formation, where the tidal interaction becomes inefficient.

A corresponding mass fraction of small-scale clumps with mass $M$ escaping the tidal destruction in hierarchical clustering (or probability of clump survival) is given by the exponent function $e^{-J}$ with

$$J \simeq \sum_h \frac{\Delta t_h}{T(\rho, \rho_h)}. \qquad (5)$$

Here $\Delta t_h$ is a difference of formation times $t_h$ of two successive hosts, and summation is over all clumps of intermediate mass-scales, which successively host the chosen small-scale clump of a mass $M$. By changing the summation to integration in (5) we obtain

$$J(\rho, \rho_f) = \int_{t_1}^{t_f} \frac{dt_h}{T(\rho, \rho_h)} \simeq \gamma \frac{\rho_1 - \rho_f}{\rho} \simeq \gamma \frac{\rho_1}{\rho}, \qquad (6)$$

where

$$\gamma = 2\gamma_1 \gamma_2 \kappa^{1/2} G^{1/2} \rho_{eq}^{1/2} t_{eq} \simeq 14(\gamma_1 \gamma_2/3), \qquad (7)$$

and $t_1$, $t_f$, $\rho_1$, and $\rho_f$ are, respectively, the formation times and internal densities of the first and final hosts. It can be seen from (6) that the first host provides a major contribution to the tidal destruction of small-scale clumps, especially if the first host density $\rho_1$ is close to $\rho$, and consequently $e^{-J} \ll 1$.

A mass function of small-scale clumps (i.e. a differential mass fraction of DM in the form of clumps survived in hierarchical clustering) can be expressed as

$$\xi \frac{dM}{M} d\nu = dM d\nu (2/\pi) e^{-\nu^2/2} \int_0^{\nu} d\nu_1 e^{-\nu_1^2/2} \times \int_{t_1(\nu_1)}^{t_0} d\tilde{t} \left| \frac{\partial^2 F(M, \tilde{t})}{\partial M \partial \tilde{t}} \right| e^{-J[\rho(\nu), \rho(\tilde{t})]}. \qquad (8)$$

In this expression $t_0$ is the present age of the Universe and $F(M, t)$ is the mass fraction of unconfined clumps (i.e. clumps not belonging to the more massive hosts) with a mass smaller than $M$ at time $t$. According to [12], the mass fraction of unconfined clumps is $F(M, t) = \mathrm{erf}(\delta_c/[\sqrt{2}\sigma_{eq}(M) D(t)])$, where erf$(x)$ is the error function and $D(t)$ is the growth factor normalized by $D(t_{eq}) = 1$. An upper integration limit $t_0$ in (8) is not crucial and may be extrapolated to infinity because a main contribution to the tidal destruction of clumps is provided by the early formed hosts at the first steps of hierarchical clustering.

Two processes respond for a time evolution of the fraction $\partial F(M, t)/\partial M dM$ of unconfined clumps in the mass interval $dM$: (i) the formation of new clumps and (ii) the capture of smaller clumps into the larger ones. Both of these processes are equally effective at the time when $\partial^2 F/(\partial M \partial t) = 0$. To take into account the confined clumps (i.e. clumps in the hosts) we need only the 2nd process (ii) for the fraction $\partial F(M, t)/\partial M$. Nevertheless, in (8) the fraction $\partial F(M, t)/\partial M$ is influenced by both processes. This is not accurate at a typical formation time of a clump with a mass $M$ when clump density is comparable with the density of hosts. Fortunately, for this time the exponent in (8) is very small, $e^{-J} \ll 1$, as can be seen from (6) and (7). Respectively, an uncertain contribution from the process (i) to the integral (8) is also very small. Meanwhile, only the process (ii) dominates in the integration region where the exponent $e^{-J}$ is not small. For this reason (8) provides a suitable approximation for the mass fraction of clumps survived in the hierarchical clustering.

Finally, we transform the distribution function (8) to the following form:

$$\xi \frac{dM}{M} d\nu \simeq \frac{1}{\sqrt{2\pi}} e^{-\nu^2/2} y(\nu) d\nu \frac{d \log\sigma_{eq}(M)}{dM} dM. \qquad (9)$$

Here the numerically calculated function $y(\nu)$ depends rather weakly on the parameter $\gamma$ from (7) and is shown in the Fig. 1. By deriving (9), we take into account that $\sigma(M)$ is a slowly varying function of clump mass $M$. For the same reason, providing an integration in (8) we use the dependence of $t_1(\rho_1)$ only on the variable $\nu$ by neglecting the dependence on $M$. Physically the rising of $y(\nu)$ with $\nu$ corresponds to a more effective survival of high-density clumps (i.e. with large values of $\nu$) with respect to the low-density ones (with small values of $\nu$). Integrating (9) over

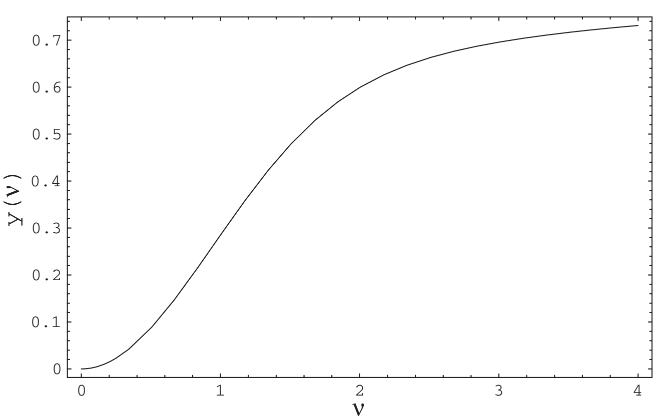


FIG. 1. Numerically calculated function $y(\nu)$ from (9).

$\nu$, we obtain

$$\xi_{int} \frac{dM}{M} \simeq 0.017(n + 3) \frac{dM}{M}. \qquad (10)$$

This mass function is in a reasonable agreement with the similar one from (1) in the our earlier work [4]. An effective power-law index $n$ in (10) is given by

$$n = -3\left[1 + 2\frac{\partial \log\sigma_{eq}(M)}{\partial \log M}\right] \qquad (11)$$

and depends very weakly on $M$. At the small mass scales one has approximately $n \simeq n_p - 4$. Equation (10) implies that for the suitable values of $n$ only a small fraction of clumps, about 0.1%–0.5%, survives the stage of hierarchical tidal destruction in the each logarithmic mass interval $\Delta \log M \sim 1$. A simple $M^{-1}$ shape of the mass function (10) is in a very good agreement with the corresponding one obtained recently in the numerical simulations [20].

It must be stressed that a physical meaning of the survived clump distribution function $\xi d\nu dM/M$ is different from the similar one for the unconfined clumps, given by the Press-Schechter mass function $\partial F/\partial M$. For comparison, the Press-Schechter mass function of unconfined clumps [12] is

$$\xi_{PS}(t) \frac{dM}{M} = \frac{2\delta_c}{\sqrt{2\pi}\sigma_{eq}^2 D(t)} \frac{d\sigma_{eq}}{dM} \exp\left[-\frac{\delta_c^2}{2\sigma_{eq}^2 D^2(t)}\right] dM, \qquad (12)$$

where $\sigma_{eq} = \sigma_{eq}(M)$ The mass function of clumps survived in a hierarchical clustering (10) is several times less than the Press-Schechter mass function (12) at a mean time of clump formation with $\sigma_{eq}(M)D(t) \simeq \delta_c$.

In further calculations we will use an interpolation fitting of the fluctuation dispersion $\sigma_{eq}(M)$ from [27] (see also [28]):

$$\sigma_{eq}(M) \simeq \frac{2 \times 10^{-4}}{\sqrt{f_s(\Omega_\Lambda)}} \left(\frac{k}{k_{h0}}\right)^{(n_p-1)/2} \left[\log\left(\frac{k}{k_{eq}}\right)\right]^{3/2}, \qquad (13)$$

where the wave vector $k \propto M^{-1/3}$, respectively $k_{eq}$ and $k_{h0}$, correspond to a mass inside the cosmological horizon at the moments $t_{eq}$ and $t_0$, $n_p$ is a primordial perturbation index, and $f_s(\Omega_\Lambda) = 1.04 - 0.82\Omega_\Lambda + 2\Omega_\Lambda^2$. It must be noted that interpolation (13) is valid only for the small-scale clumps, with $M \leq 10^3 M_\odot$. The analysis of the WMAP data of the CMB anisotropy [29] reveals a power-law spectrum of initial perturbations with $n_p = 0.99 \pm 0.04$ in a good agreement with the canonical inflation value $n_p = 1.0$. However, when the data from 2dF galaxy power-spectrum and Ly-$\alpha$ are included in the analysis, the best-fit favors in a softer spectrum with $n_p = 0.96 \pm 0.02$. Nevertheless, the recent observations do not exclude even the values $n_p = 1.1$.

Note that a differential number density of small-scale clumps in the Galactic halo $n(M)dM \propto dM/M^2$ from (10) coincides not only with a similar one from the recent numerical simulations of small-scale clumps [20] but is also very close to that obtained in the numerical simulations for large-scale clumps with mass $M \geq 10^6 M_\odot$. See Fig. 2 for a comparison.

Strictly speaking, our calculations are not valid for large-scale clumps because of their continuing tidal destruction in the halo up to the present epoch $t_0$ and the accretion of the additional large-scale clumps into the halo from the intergalactic space. Nevertheless, our approach remains valid even for the large-scale clumps in the narrow mass range, where the power-law perturbation spectrum can be used as a rather good approximation.

In Fig. 2, a differential number density of small-scale clumps from (10) is shown by the solid line. As it was noted above, the region of validity for this curve is $M \leq 10^3 M_\odot$. For larger masses an extrapolation is shown (right part of the solid line). The corresponding mass functions from numerical simulations can be parametrized in the form $\xi(M)dM/M = AM^{1-\lambda}dM$. The constant $A$ can be determined by fixing a power-law index $\lambda$ and a fraction $\varepsilon$ of the halo mass in the form of clumps with mass from $M_{\min} \simeq 10^6 M_\odot$ to $M_{\max} \simeq 10^{10} M_\odot$:

$$\varepsilon = \int_{M_{\min}}^{M_{\max}} AM^{1-\lambda}dM. \tag{14}$$

With this parametrization, a mass function of large-scale DM clumps may be expressed as

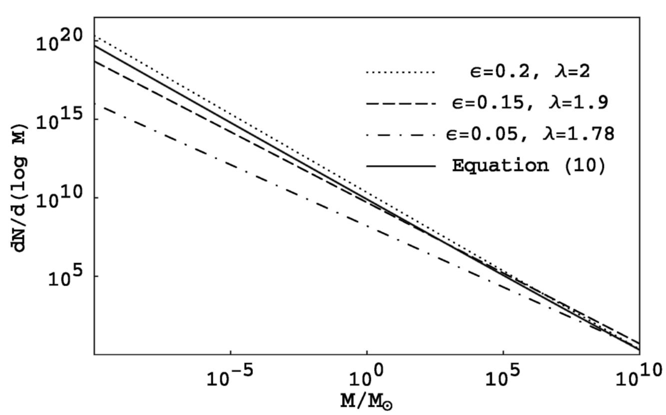


FIG. 2. A differential number of small clumps in the Galaxy from (10) for $n_p = 1.0$ is shown by the solid line. These calculations are valid only for small-scale clumps with $M < 10^3 M_\odot$ and extrapolated to the larger masses (the right part of the solid line). Other curves are the corresponding number densities of large-scale clumps with $M > 10^6 M_\odot$ from numerical simulations (see details in the text) for different values of parameters $\varepsilon$ and $\lambda$ from (15). The left parts of these curves are the extrapolations to small masses.

$$\xi(M)\frac{dM}{M} = \varepsilon\frac{dM}{M}\begin{cases}\frac{(2-\lambda)M^{2-\lambda}}{M_{\max}^{2-\lambda}-M_{\min}^{2-\lambda}}, & \lambda \neq 2;\\ \log^{-1}(\frac{M_{\max}}{M_{\min}}), & \lambda = 2.\end{cases} \tag{15}$$

In Fig. 2, a differential number density of large-scale clumps $(M_{\rm H}/M)\xi(M)dM/M$ from (15) is shown for different values of $\lambda$ and $\varepsilon$ taken from various numerical simulations: $\varepsilon \simeq 0.2$, $\lambda = 2$ from [6]; $\varepsilon \simeq 0.15$, $\lambda = 1.9$ from [30], and $\varepsilon \simeq 0.05$, $\lambda = 1.78$ from [31]. Observations of the Galactic halo lensing [32] give a smaller clump fraction value, $\varepsilon \simeq 0.02$. One can see in Fig. 2 a reasonable agreement between the extrapolation of our calculations and the corresponding numerical simulations of the large-scale clumps.

## III. DESTRUCTION OF CLUMPS BY DISK

Crossing the Galactic disc, a clump can be tidally destructed by the collective gravitational field of stars in the disc. This phenomenon is similar to the destruction of a globular cluster by the "tidal shocking" in the Galactic disc [33].

The rate of energy gain per unit mass due to the tidal shocking was calculated in [33]:

$$\frac{d\tilde{E}}{dt} = \frac{4g_m^2(\Delta z)^2}{T_c v_{z,c}^2}. \tag{16}$$

Here $g_m$ is the maximum gravitational acceleration acquired by the constituent DM particle of the clump moving in the gravitational field of the disk, $\Delta z =$ is a vertical (perpendicular to the disk plane) distance of a DM particle from the clump center, $T_c$ is an orbital period of clump in the halo, $v_{z,c}$ is a vertical velocity of disk crossing. In (16) the two crossings of disks by a globular cluster during the orbital period $T_c$ are assumed, while only one disk crossing is typical in the case of elongated orbits of DM clumps in the halo (see below).

A surface mass of the Galactic disk can be approximated by a simple exponential law [34]

$$\sigma_s(r) = \frac{M_d}{2\pi r_0^2}e^{-r/r_0}, \tag{17}$$

with $M_d = 8 \times 10^{10} M_\odot$ and $r_0 = 4.5$ kpc. The maximum gravitational acceleration during the disk crossing is

$$g_m(r) = 2\pi G\sigma_s(r). \tag{18}$$

Following to [21], we use a power-law parametrization of the internal density profile of DM clumps:

$$\rho_{\rm int}(r) = \frac{3-\beta}{3}\rho\left(\frac{r}{R}\right)^{-\beta}, \tag{19}$$

where $\rho$ and $R$ is, respectively, a mean internal density and radius of clump, $\beta = 1.7$–$1.8$ and we put $\rho_{\rm int}(r) = 0$ at $r > R$. The corresponding power-law profile of small-scale clumps with $\beta \simeq 1.5 - 2$ has been recently found in numerical simulations [20]. For this profile a total (kinetic and potential) energy of a clump is given by

$$|E| = \frac{3-\beta}{2(5-2\beta)} \frac{GM^2}{R}. \quad (20)$$

Integrating (16) over the clump volume with the profile (19), one can obtain a total rate of energy gain by clump $dE/dt$ and then a time of clump destruction by the disk:

$$t_d = \frac{|E|}{\dot{E}} = \frac{2(5-\beta)}{3(5-2\beta)} \frac{GT_c \rho v_{z,c}^2}{g_m^2}. \quad (21)$$

Note that the adiabatic correction for the disk shocking (see e.g. [35]) is very small in the case of DM clumps and may be neglected.

To estimate the tidal shocking effect produced by different parts of the Galactic disk at radial distance $r$, let us consider at first a toy halo model by assuming the circular orbits of DM clumps. Then a disk crossing velocity $v_{z,c}$ equals to a circular velocity:

$$v_{z,c} = v_{\rm rot}(r) = \left[\frac{GM_{\rm H}(r)}{r}\right]^{1/2}, \quad (22)$$

where $M_{\rm H}(r)$ is a halo mass inside the sphere of radius $r$. Using for a clump orbital period $T_c = 2\pi r/v_{\rm rot}(r)$, one finally finds

$$t_d = \frac{(5-\beta)}{3\pi(5-2\beta)} \frac{r_0^4 \rho M_{\rm H}^{1/2}(r) r^{1/2}}{G^{1/2} M_d^2} e^{2r/r_0}. \quad (23)$$

Comparison of a clump destruction time $t_d$ from (23) with the Universe age $t_0$ shows that all clumps with the internal density $\rho < 2 \times 10^{-22}$ g cm$^{-3}$ are effectively destructed within the radius $r < 15$ kpc from the Galactic center. In particular, the major part of the Moon-mass clumps with $M = 2 \times 10^{-8} M_\odot$, $n_p = 1$ and $\nu = 2$ do not survive inside the central 15 kpc.

However, in the real Galactic halo the DM clumps have an elongated orbits in general. These orbits cross the stellar Galactic disk only *once* during the orbital period. Therefore, we must introduce a factor 2 in (21) for these orbits. At the same time it is much more important that clumps with elongated orbits have the longer orbital periods $T_c$ than in the previous toy model. This elongation of orbits significantly increases the probability of clump survival.

As an example let us consider the Galactic halo model with an isotropic velocity distribution. This model is appropriate for the halo formed by the hierarchical clustering of clumps. In this model, according to [36], the energy distribution function $f(E)$ of DM particles is related with the density profile of the halo $\rho_{\rm H}(r)$ as

$$\rho_{\rm H}(r) = 2^{5/2}\pi \int_{U(r)}^0 \sqrt{E - U(r)} f(E) dE, \quad (24)$$

$$f(E) = \frac{1}{2^{3/2}\pi^2} \frac{d}{dE} \int_{r(E)}^\infty \frac{dr}{\sqrt{E - U(r)}} \frac{d\rho_{\rm H}(r)}{dr}, \quad (25)$$

where $U(r)$ is a gravitational potential energy and function $r = r(E)$ is defined by the equation $U[r(E)] = E$.

We suppose here for simplicity a pure isothermal density profile of the halo

$$\rho_{\rm H}(r) = \frac{1}{4\pi} \frac{v_{\rm H}^2}{Gr^2}, \quad (26)$$

where $v_{\rm H} = (GM_{\rm H}/R_{\rm H})^{1/2}$ is the halo rotational velocity and $R_{\rm H}$ is the Galactic halo radius. At $r > R_{\rm H}$ we put $\rho_{\rm H}(r) = 0$. In the absence of an analytical model of the finite isothermal sphere, we construct a simplified model which approximates the isothermal sphere in the inner region, at $r \ll R_{\rm H}$. A gravitational potential energy $U(r)$ corresponding to the density profile (26) is

$$U(r) = m v_{\rm H}^2 [\log(r/R_{\rm H}) - 1], \quad (27)$$

where $m$ is a mass of DM particle. The radial motion of a particle with mass $m$ and angular momentum $L$ in the spherical potential obeys the equation

$$\dot{r}^2 = \frac{2}{m}[E - U(r)] - \frac{L^2}{m^2 r^2}. \quad (28)$$

By introducing the dimensionless variables

$$s = \frac{r}{R_{\rm H}}, \qquad x = \frac{E}{m v_{\rm H}^2}, \qquad y = \frac{L^2}{R_{\rm H}^2 m^2 v_{\rm H}^2}, \quad (29)$$

the equation for the turning points, $\dot{r}^2 = 0$, in the potential (27) can be written as

$$\frac{y}{s^2} = 2(x - \log s + 1). \quad (30)$$

The derivatives of the left and right sides of this equation are equal at $s = y^{1/2}$. Respectively, the roots $s_{\rm min}(x, y)$ and $s_{\rm max}(x, y)$ of (30) satisfy the condition $s_{\rm min}(x, y) < y^{1/2} < s_{\rm max}(x, y)$. A condition for the existence of the solution of (30) is $x \geq (\log y - 1)/2$. The equality in this condition corresponds to the circular orbit with $s_{\rm min} = s_{\rm max}$. From (28) one can determine the orbital period:

$$T_c(x, y) = 2\frac{R_{\rm H}}{v_{\rm H}} \int_{s_{\rm min}}^{s_{\rm max}} \frac{ds'}{\sqrt{2(x - \log s' + 1) - y/s'^2}}. \quad (31)$$

In the following we will solve (30) and find the orbital period $T_c(x, y)$ from (31) numerically. Denoting $p = \cos\theta$, where $\theta$ is an angle between the radius-vector $\vec{r}$ and the particle velocity $\vec{v}$, we have

$$y = 2(1 - p^2)s^2(x - \log s + 1). \quad (32)$$

We find from (25) the distribution function of particles with an energy $x < -1$ by using the density profile (26) with a cutoff at $r = R_{\rm H}$:

$$f(x) = \frac{1}{2^{5/2}\pi^3 e} \frac{v_{\rm H}^{1/2}}{Gm^{3/2}R_{\rm H}^2} F(x), \tag{33}$$

where

$$F(x) = \sqrt{2\pi} e^{-2x} {\rm erf}[\sqrt{-2(x+1)}] + \frac{e^2}{\sqrt{-(x+1)}}. \tag{34}$$

Note that the isotropic distribution function (33) reproduces the density profile (26) only in the inner halo region, at $r \ll R_{\rm H}$. The assumed isotropy of particle distribution (i.e. an independence of distribution function on the particle angular momentum $L$) is violated near the boundary of the halo, at $r \simeq R_{\rm H}$. Nevertheless, the distribution function (33) is adequate for our purpose because a tidal destruction of clumps by the Galactic disk takes place only in the inner halo region, at $r \ll R_{\rm H}$. At the same time, in the considered model the clumps on the outer orbits at $r \simeq R_{\rm H}$ provide only small contribution to the halo density at $r \ll R_{\rm H}$. Neglecting these outer clumps, we can define the probability of clump survival in tidal destruction by the Galactic disk (or the fraction of survived clumps) as a function of radius $r = sR_{\rm H}$ in the following form:

$$P_d(r) = \frac{\int_0^1 dp \int_{\log s - 1}^{-1} dx \sqrt{x - \log s + 1} F(x) e^{-t_0/t_d}}{\int_{\log s - 1}^{-1} dx \sqrt{x - \log s + 1} F(x)}. \tag{35}$$

Here $t_d$ is from (21) but with an additional factor 2 (one disk crossing per orbital period) and with the replacements $T_c \Rightarrow T_c(x, y)$ from (31), $g_m \Rightarrow g_m(r_{\rm min})$, $v_{z,c} \Rightarrow v(r_{\rm min})$, where $r_{\rm min} = s_{\rm min} R_{\rm H}$, $s_{\rm min}$ is the minimal root of (30) and $v(r) = \sqrt{2[E - U(r)]/m}$. See in Figs. 4–6 the resulting probabilities of clump survival in the Galaxy (or fractions of clumps survived the tidal destruction).

Note that the probability of clump survival $P_d(r)$ in (35) is an approximate expression averaged over an angle between the plane of the Galactic disk and the clump orbit plane. In the real Galactic halo there must be some anisotropy in clump distribution with respect to the disk plane. For example, the clumps with orbits in the Galactic disk plane are destructed more efficiently than ones outside the Galactic plane.

## IV. DESTRUCTION OF CLUMPS BY STARS

The internal energy increase of a clump during a single star flyby is

$$\Delta E = \frac{1}{2} \int d^3 r \rho_{\rm int}(r)(v_z - \tilde{v}_z)^2, \tag{36}$$

where $v_z$ is a velocity increase of constituent DM particle inside a clump in the direction of axis $z$ and $\tilde{v}_z$ is a similar one for a clump center-of-mass. The axis $z$ is directed along the line connecting a clump center-of-mass with a star at the moment of a star's closest approach. In the impulse approximation, by neglecting the internal motion of DM particles in a clump during the star encounter and assuming the straight line orbit of a star (see e.g. [25]) we have

$$v_z - \tilde{v}_z \simeq \frac{\partial v_z}{\partial l} \Delta l = \frac{\partial v_z}{\partial l} r \cos\psi, \tag{37}$$

where $l$ is the distance of a star closest approach to a DM clump and $\psi$ is a polar angle in the spherical coordinates.

Let us take $v_{\rm rel}$ as the relative velocity of a star with respect to a DM clump. In the approximation of a rectilinear motion, an angle $\phi$ between the line connecting a clump center-of-mass and $\vec{v}_{\rm rel}$ evolves as

$$\frac{d\phi}{dt} = -\frac{v_{\rm rel}}{l} \cos^2\phi. \tag{38}$$

Changing a variable $t$ to $\phi$ in the Newton equation of motion, one gets

$$\frac{dv_z}{d\phi} = -\frac{Gm_*}{v_{\rm rel} l} \cos\phi, \tag{39}$$

where $m_*$ is a typical star mass. After integration of this equation we obtain

$$v_z = \frac{2Gm_*}{v_{\rm rel} l}. \tag{40}$$

Now by integrating (36) over a clump volume with a density profile $\rho_{\rm int}(r)$ from (19), we find in the case of $l > R$:

$$\Delta E = \frac{2(3-\beta)}{3(5-\beta)} \frac{G^2 M R^2 m_*^2}{v_{\rm rel}^2 l^4}. \tag{41}$$

The opposite case $l < R$ was considered e. g. in [4]. It is easily verified that the maximum internal energy increase occurs for a star flyby with $l \simeq R$.

At this step we must distinguish two cases: (i) clump destruction during a single star flyby and (ii) clump destruction after numerous star collisions. In the first case a threshold for clump destruction is achieved at $\Delta E = |E|$, where a total energy of clump $E$ is given by (20). From the equality $\Delta E = |E|$ one finds the maximal impact parameter $l_*$ for a single flyby destruction

$$\left(\frac{l_*}{R}\right)^4 = \frac{4(5-2\beta)}{3(5-\beta)} \frac{Gm_*^2}{MRv_{\rm rel}^2} \sim \left(\frac{V}{v_{\rm rel}}\right)^2 \left(\frac{m_*}{M}\right)^2, \tag{42}$$

where $V \simeq (GM/R)^{1/2}$ is a velocity dispersion of DM particles in the clump. The fraction $l_*/R$ as a function of clump mass $M$ is shown in Fig. 3. Note that condition $l_*/R > 1$ is satisfied for clumps of the smallest mass. A total rate of clump destruction by stars in the case of $l_*/R > 1$ is given by

$$t_*^{-1} = \frac{\dot{E}}{|E|} = \pi l_*^2 n_* v_{\rm rel} + \frac{\dot{E}(l > l_*)}{|E|}. \tag{43}$$

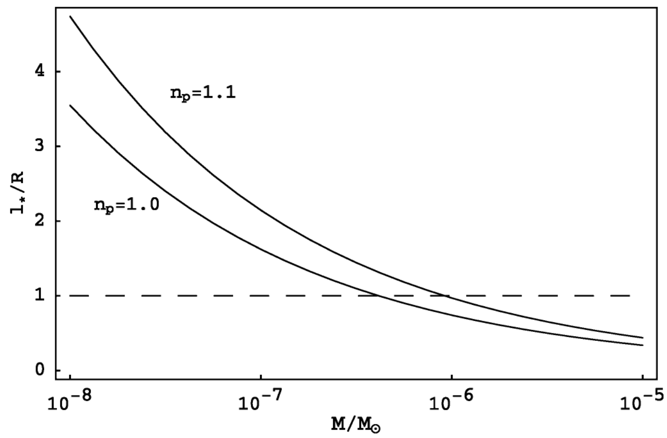


FIG. 3. The fraction $l_*/R$ according to (42) as a function of clump mass $M$ at the distance from the Galactic center $r = 2$ kpc, $\nu = 2$ and $n_p = 1.0$ and $n_p = 1.1$, respectively.

where $n_*$ is a number density of stars and

$$\dot{E}(l > l_*) = 2\pi \int_{l_*}^{\infty} \Delta E(l) n_* v_{\rm rel} l dl. \tag{44}$$

After integration in (44) with $l_*$ from (42), we find that the second term in (43) is equal to the first one. Thus, the resulting time of clump destruction in the case of $l_*/R > 1$ is

$$t_* = \frac{1}{2\pi l_*^2 v_{\rm rel} n_*} = \frac{1}{4\pi n_* m_*}\left[\frac{3(5-\beta)}{(5-2\beta)}\frac{M}{GR^3}\right]^{1/2}. \tag{45}$$

We see from (45) that the time of clump destruction by stars does not depend on $v_{\rm rel}$ in the case of $l_*/R > 1$.

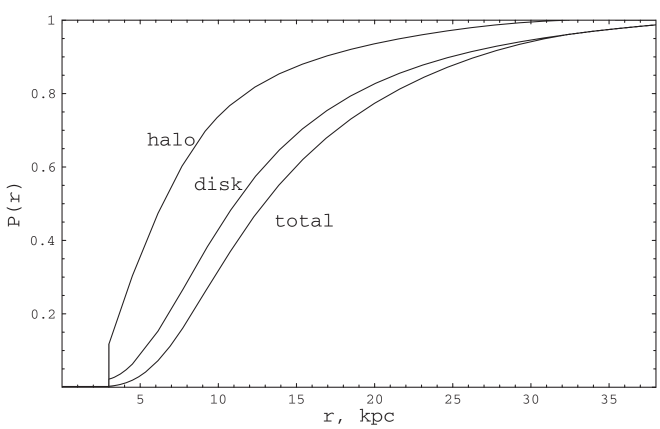


FIG. 4. The fraction of clumps with mass $M = 2 \times 10^{-8} M_\odot$ and peak-height $\nu = 2$ survived a tidal destruction in the Galactic disc $P_d$, in the Galactic halo $P_{\rm H}$ and the resulting total fraction $P_{\rm tot} = P_{\rm H} P_d$ as a function of distance from the Galactic center. The cutoff at $r < 3$ kpc is due to the destruction of clumps inside the bulge.

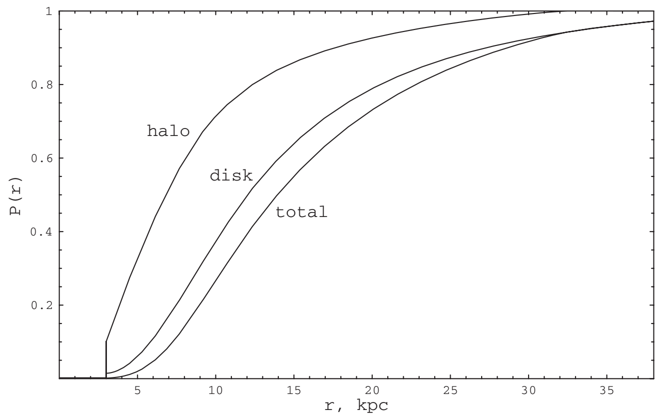


FIG. 5. The same as the Fig. 4 but for $M = 10^{-6} M_\odot$.

Similarly, the time of clump destruction by stars in the case of $l_* < R$ is

$$t_* = \frac{3(5-\beta)}{8\pi(5-2\beta)}\frac{v_{\rm rel} M}{GRm_*^2 n_*}. \tag{46}$$

### A. Destruction of clumps in the bulge

The bulge is central spheroidal subsystem of the Galaxy. Following [37], we approximate the radial number density distribution of stars in the bulge in the radial range $r =$ 1–3 kpc as

$$n_{b,*}(r) = (\rho_b/m_*)\exp[-(r/r_b)^{1.6}], \tag{47}$$

where $\rho_b = 8 M_\odot/{\rm pc}^3$ and $r_b = 1$ kpc. By using (47) together with (45) or (46) it can be shown that inside the bulge, at $r \leq 3$ kpc, all small-scale clumps with $M \geq 10^{-8} M_\odot$ are tidally destructed during the Hubble time, i.e. $t_* \ll t_0$. Therefore, there is an empty cavity in clump distribution in the Galactic center with a radius $r \simeq 3$ kpc as it can be shown in Figs. 4–6.

What is the fate of the core of a tidally destructed clump? Let us consider the scaling of destruction time $t_*$ in dependence on a varying clump radius $r$ and mass

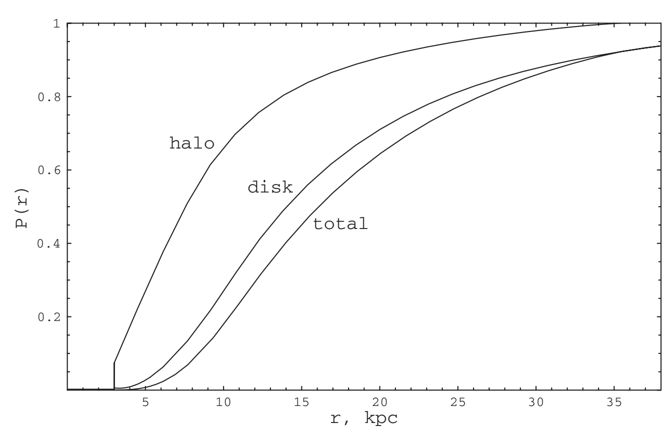


FIG. 6. The same as the Fig. 4 but for $M = 10^{-3} M_\odot$.

$M(r) \propto r^{3-\beta}$. According to (45) or (46), a time of clump destruction is scaled, respectively, as $t_* \propto r^{-\beta/2}$ or $t_* \propto r^{2-\beta}$. In order for the core to survive, it is necessary that $t_* \to 0$ at $r \to 0$. This is possible only if the internal density profile $\rho(r) \propto r^{-\beta}$ is rather steep, $\beta > 2$. Meanwhile, both the theoretical models and numerical simulations predict $\beta < 2$, and therefore, the core does not survive during the tidal destruction of DM clump.

### B. Destruction of clumps in the halo

The radial number density distribution of stars in the Galactic halo (outside the Galactic disk) at radii $r > 3$ kpc can be approximated as

$$n_{h,*}(r) = (\rho_\odot/m_*)(r_\odot/r)^3, \tag{48}$$

where $\rho_\odot = 10^{-4} M_\odot/\mathrm{pc}^3$ and $r_\odot = 8.5$ kpc. The stellar density profile in the Galactic halo is rather poorly known, and so (48) must be considered only as an upper limit [24]. We will describe the distribution of clumps in the halo the same way as in the Sec. III. To take into account a varying number density of stars $n_{h,*}(r)$, we made an averaging of the rate of clump destruction by stars $t_*^{-1}$ along the orbital trajectory during the orbital period:

$$\langle t_*^{-1}(x, y)\rangle = \frac{2R_\mathrm{H}}{v_\mathrm{H} T_c(x, y)} \int_{s_\mathrm{min}}^{s_\mathrm{max}} \frac{ds' t_*^{-1}}{\sqrt{2(x - \log s' + 1) - y/s'^2}}. \tag{49}$$

In this expression the dimensionless variables $x$ and $y$ are from (29), a clump orbital period $T_c(x, y)$ is from (31) and destruction time of clump $t_*$ is from (45) or (46) with the replacements $n_s \Rightarrow n_{h,*}(r)$ and $v_\mathrm{rel} \Rightarrow \sqrt{2[E - U(r)]/m}$. The averaging procedure (49) is in fact the integration of energy gain rate $\int \dot{E} dt$ along the clump orbit.

The resulting probability of clump survival in tidal destruction by the Galactic halo stars $P_\mathrm{H}(r)$ is defined by a similar expression as (35) but with a replacement $e^{-t_0/t_d} \Rightarrow e^{-t_0 \langle t_*^{-1}(x,y)\rangle}$, where $\langle t_*^{-1}(x, y)\rangle$ is from (49).

The results of numerical calculations of the tidal destruction of DM clumps by different Galactic components are summarized in Figs. 4–6. These calculations were performed for DM clumps originating from fluctuations with the peak-height $\nu = 2$.

Correspondingly, 32%, 27%, and 18% of clumps survive the destruction by the Galactic disk tidal shocking at the Sun position, $r_\odot = 8.5$ kpc, for clump masses $M = 2 \times 10^{-8} M_\odot$, $M = 10^{-6} M_\odot$, and $M = 10^{-3} M_\odot$. The Galactic disk destroys clumps even outside its boundary, at $r > 15$ kpc, because some of DM clumps with the extended orbits intersect the Galactic disk in the inner part of the halo. The destruction of clumps by the Galactic disk becomes inefficient at $r > 40$ kpc. The respective fractions of clumps of the same masses surviving the tidal destruction by stars in the Galactic halo (outside the Galactic disk) are 66%, 63%, and 57%. The final fractions of clumps of the same masses survived the tidal destruction both by the Galactic disk and stars in the Galactic halo $P(r_\odot) = P_\mathrm{H}(r_\odot) P_d(r_\odot)$ are 21%, 17%, and 10%, respectively.

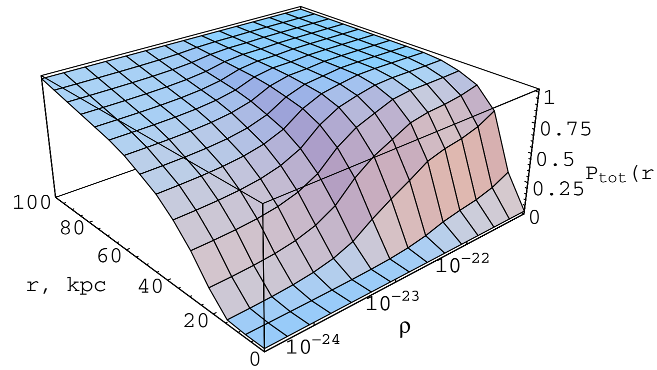


FIG. 7 (color online). The survived fraction of small-scale clumps $P_\mathrm{tot}(r)$ in the Galactic halo inside the radial distance $r \leq 100$ kpc. A mean internal density of clump $\rho$ is in g cm$^{-3}$.

In the Fig. 7 the fraction of survived small-scale clumps in the Galactic halo is shown in dependence on a mean internal density of small-scale clumps.

## V. CONCLUSIONS

We calculated the number density distribution of small-scale DM clumps in the Galactic halo in dependence on a clump mass $M$, radius $R$ (expressed through the fluctuation peak-height $\nu$) and radial distance $r$ to the Galactic center. These calculations were performed by taking into account the tidal destruction of clumps in the early hierarchical clustering and later in the Galaxy.

Calculations of the distribution function of small-scale clumps are carried out, following [4], in the framework of the standard cosmological model and the hierarchical model of structure formation. The primeval power spectrum of density perturbations $P(k) \propto k^{n_p}$ is taken from the inflation models with $n_p \simeq 1$ (the Harrison-Zeldovich spectrum). In this model the small-scale clumps are formed earlier than the bigger ones. The minimal mass $M_\mathrm{min}$ of clumps is determined by the free streaming of DM particles from a growing fluctuation. The value of $M_\mathrm{min}$ is a model dependent quantity. For neutralino as DM particle, the minimal mass $M_\mathrm{min}$ is given by (2), and it is the Moon-scale mass.

In the process of hierarchical clustering, the small clumps are captured by the bigger ones, and so on. Thus the hierarchical structure is formed, when all clumps consist in part the smaller ones and the free DM particles. Some part of DM clumps are tidally disrupted in the gravitational field of the bigger host clumps. In this scenario we calculated the differential distribution of the survived clumps given by (9) as a function of two independent parameters: e.g. a clump mass $M$ and fluctuation peak-height $\nu$ (or a clump mass $M$ and radius $R$). The corresponding integral mass function is given by (10), where the small factor $\xi_{\rm int} \simeq 0.017(n+3)$ gives the mass fraction of clumps survived the tidal destruction in the hierarchical structuring.

The predicted differential number density of small clumps $\xi(M)(\rho_{\rm H}/M)dM/M$ is very close to our previous calculations [4], and both are in a good agreement with the recent results of numerical simulations [20].

Our calculations are valid only for small-scale clumps with masses $M \leq 10^3 M_\odot$. The physics of larger mass clumps is rather different. For large-scale clumps the dynamical friction, tidal stripping and accretion of new clumps into the halo proceed in a different way. Nevertheless, the calculated mass function is in good agreement with a mass function of the large clumps (obtained in the numerical simulations) in the intermediate mass range (see in Fig. 2).

The mutual tidal destruction of small-scale DM clumps is effective only at the early stage of hierarchical clustering. At later stages the DM clumps are additionally destructed by stars and by the collective gravitational field of the Galactic disc. In the Galaxy at radial distance $r \leq$ 3 kpc all small-scale clumps are destructed by stars in the central bulge. At radial distances in the range $r =$ 3–40 kpc the DM clumps are destructed by stars from the halo and by the tidal shocking in the Galactic disk. The latter provides the major contribution to the tidal destruction of clumps outside the bulge. Only 21%, 17%, and 10% of clumps survive the tidal destruction near the Sun position for clump masses $M = 2 \times 10^{-8} M_\odot$, $M = 10^{-6} M_\odot$, and $M = 10^{-3} M_\odot$ respectively. Our results on the tidal destruction of clumps differ from [22–24] with the intermediate conclusions. At radial distances $r >$ 40 kpc the destruction of clumps by the Galactic disk becomes inefficient, and the number density of clumps is determined only by the early epoch of hierarchical clustering.

The tidal destruction of clumps by the Galactic disk and stars affects the annihilating signal mainly in the central region of the Galaxy where destructions are most effective. Therefore, a growing fraction of survived clumps $P(r)$ smooths the anisotropy of the awaited annihilation signal at the Sun position. A local annihilation rate is proportional to the clumps number density and, respectively, to $P(r)$. For example, at the position of the Sun the 17% of clumps survive, and so the local annihilation rate is more than 5 times less in comparison with the $P = 1$ case.

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