Tribimaximal neutrino mixing from a supersymmetric model with A_4 family symmetry

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In the supersymmetric seesaw model of neutrino masses, augmented by the non-Abelian discrete tetrahedral symmetry A_4, a specific pattern of neutrino mixing is *automatically generated* if one of the three heavy singlet neutrino superfields acquires a nonzero vacuum expectation value. This pattern turns out to be exactly that of tribimaximal mixing, i.e. $\sin^2\theta_{23} = 1/2$, $\sin^2\theta_{12} = 1/3$, and $\sin^2\theta_{13} = 0$, in good agreement with data.

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In the well-known canonical seesaw mechanism [1], three heavy singlet Majorana neutrinos N_i ($i = 1, 2, 3$) are added to the standard model of elementary particles, so that

$$\mathcal{M}_\nu^{(e,\mu,\tau)} = -\mathcal{M}_D \mathcal{M}_N^{-1} \mathcal{M}_D^T, \tag{1}$$

where $\mathcal{M}_D$ is the 3×3 Dirac mass matrix linking the observed neutrinos ν_α ($\alpha = e, \mu, \tau$) to N_i, and $\mathcal{M}_N$ is the Majorana mass matrix of N_i. Consider now its diagonalization, i.e.

$$\mathcal{M}_\nu^{(e,\mu,\tau)} = U_{\alpha i} \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U_{j\beta}^T. \tag{2}$$

Present neutrino-oscillation data have determined the absolute values of $U_{\alpha i}$ to a large extent, as well as the two differences of the absolute squares of the three masses [2]. Theoretically, the obvious challenge is to find a simple and natural understanding of these results.

In the following, it will be shown that in the context of supersymmetry, augmented by the non-Abelian discrete symmetry A_4 [3], a specific three-parameter form of $\mathcal{M}_\nu^{(e,\mu,\tau)}$ is automatically generated if one of the three N_i superfields acquires a nonzero vacuum expectation value. This results in a specific $U_{\alpha i}$ which turns out to be exactly that of the so-called tribimaximal mixing of Harrison, Perkins, and Scott [4], i.e.

$$U_{\alpha i} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}. \tag{3}$$

In terms of the usual neutrino-oscillation parameters, this means that

$$\sin^2\theta_{23} = \tfrac{1}{2}, \qquad \sin^2\theta_{12} = \tfrac{1}{3}, \qquad \sin^2\theta_{13} = 0, \tag{4}$$

in good agreement with data [2].

The non-Abelian finite group A_4 is the symmetry group of the even permutation of four objects. It is also the symmetry group of the regular tetrahedron, one of five perfect geometric solids which was identified by Plato with the Greek element "fire" [5]. There are 12 group elements and four irreducible representations: $\underline{1}$, $\underline{1}'$, $\underline{1}''$, and 3. Let $a_{1,2,3}$ and $b_{1,2,3}$ transform as 3 under A_4, then [6]

$$a_1b_1 + a_2b_2 + a_3b_3 \sim \underline{1}, \tag{5}$$

$$a_1b_1 + \omega^2 a_2b_2 + \omega a_3b_3 \sim \underline{1}', \tag{6}$$

$$a_1b_1 + \omega a_2b_2 + \omega^2 a_3b_3 \sim \underline{1}'', \tag{7}$$

$$(a_2b_3, a_3b_1, a_1b_2) \sim \underline{3}, \tag{8}$$

$$(a_3b_2, a_1b_3, a_2b_1) \sim \underline{3}, \tag{9}$$

where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$.

Under A_4, the lepton doublets (ν_i, l_i) transform as $\underline{3}$ and the charged-lepton singlets l_i^c as $\underline{1}$, $\underline{1}'$, $\underline{1}''$, with three Higgs doublets (ϕ_i^0, ϕ_i^-) transforming as $\underline{3}$. Assuming equal $\langle\phi_i^0\rangle = v$, the charged-lepton mass matrix linking l_i to l_j^c is then given by [3]

$$\mathcal{M}_l = U_L \begin{pmatrix} h_e & 0 & 0 \\ 0 & h_\mu & 0 \\ 0 & 0 & h_\tau \end{pmatrix} \sqrt{3} v, \tag{10}$$

where

$$U_L = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}. \tag{11}$$

In the neutrino sector, the three singlets N_i transform as 3 under A_4 with one Higgs doublet (η^+, η^0) transforming as 1. Hence

$$\mathcal{M}_D = U_L^\dagger \begin{pmatrix} m_D & 0 & 0 \\ 0 & m_D & 0 \\ 0 & 0 & m_D \end{pmatrix}, \tag{12}$$

and

$$\mathcal{M}_N = \begin{pmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{pmatrix}. \tag{13}$$

The resulting $\mathcal{M}_\nu$ in the (e, μ, τ) basis is then given by

$$\mathcal{M}_\nu = U_L^\dagger \begin{pmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & m_0 \end{pmatrix} U_L^* = \begin{pmatrix} m_0 & 0 & 0 \\ 0 & 0 & m_0 \\ 0 & m_0 & 0 \end{pmatrix}, \tag{14}$$

where $m_0 = -m_D^2/M$. This is the starting point of the two original A_4 models [3,7]. Since all three neutrinos have the same absolute mass, there is actually no mixing in this case. Whereas small radiative perturbations can result in a realistic mass matrix [5,7,8], $U_{\alpha i}$ is not completely predicted in this approach.

Here it is proposed that $\mathcal{M}_N$ is actually of the form

$$\mathcal{M}_N = \begin{pmatrix} A & 0 & 0 \\ 0 & B & C \\ 0 & C & B \end{pmatrix}. \tag{15}$$

The justification of this in terms of the superpotential of N_i will be discussed in detail later. For now, just consider the resulting 3×3 Majorana neutrino mass matrix in the (e, μ, τ) basis. Using Eqs. (1), (11), (12), and (15), $\mathcal{M}_\nu$ is then given by

$$\frac{-m_D^2}{3A(B^2 - C^2)} \begin{pmatrix} B^2 - C^2 + 2AB - 2AC & B^2 - C^2 - AB + AC & B^2 - C^2 - AB + AC \\ B^2 - C^2 - AB + AC & B^2 - C^2 - AB - 2AC & B^2 - C^2 + 2AB + AC \\ B^2 - C^2 - AB + AC & B^2 - C^2 + 2AB + AC & B^2 - C^2 - AB - 2AC \end{pmatrix}. \tag{16}$$

This matrix is a special form of the four-parameter matrix proposed in Ref. [9], i.e.

$$\mathcal{M}_\nu = \begin{pmatrix} a + (2d/3) & b - (d/3) & c - (d/3) \\ b - (d/3) & c + (2d/3) & a - (d/3) \\ c - (d/3) & a - (d/3) & b + (2d/3) \end{pmatrix}, \tag{17}$$

with

$$a = \frac{-m_D^2(B^2 - C^2 + 2AB)}{3A(B^2 - C^2)}, \quad b = c = \frac{-m_D^2(B^2 - C^2 - AB)}{3A(B^2 - C^2)}, \qquad d = \frac{m_D^2 C}{B^2 - C^2}. \tag{18}$$

As promised, it is exactly diagonalized by Eq. (3), i.e.

$$\mathcal{M}_\nu = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix} \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \times \begin{pmatrix} \sqrt{2/3} & -\sqrt{1/6} & -\sqrt{1/6} \\ \sqrt{1/3} & \sqrt{1/3} & \sqrt{1/3} \\ 0 & -\sqrt{1/2} & \sqrt{1/2} \end{pmatrix}, \tag{19}$$

with

$$m_1 = \frac{-m_D^2}{B + C}, \qquad m_2 = \frac{-m_D^2}{A}, \qquad m_3 = \frac{m_D^2}{B - C}. \tag{20}$$

Since there are three independent parameters (A, B, C), it is clear that the three neutrino masses may be chosen arbitrarily to fit the data. In other words, this model predicts $U_{\alpha i}$ but not $m_{1,2,3}$.

To obtain $\mathcal{M}_N$ of Eq. (15), consider the most general superpotential of N_i invariant under A_4 up to quartic terms, i.e.

$$W = \frac{1}{2} m_N (N_1^2 + N_2^2 + N_3^2) + f N_1 N_2 N_3 + \frac{\lambda_1}{4M_{\rm Pl}} (N_1^4 + N_2^4 + N_3^4) + \frac{\lambda_2}{2M_{\rm Pl}} (N_2^2 N_3^2 + N_3^2 N_1^2 + N_1^2 N_2^2), \tag{21}$$

where $M_{\rm Pl} = 1.2 \times 10^{19}$ GeV is the Planck mass. To preserve the supersymmetry of the complete theory at this high scale, a solution must exist for which the minimum of the resulting scalar potential

$$V = \left| m_N N_1 + f N_2 N_3 + \frac{\lambda_1}{M_{\rm Pl}} N_1^3 + \frac{\lambda_2}{M_{\rm Pl}} N_1 (N_2^2 + N_3^2) \right|^2 + \left| m_N N_2 + f N_3 N_1 + \frac{\lambda_1}{M_{\rm Pl}} N_2^3 + \frac{\lambda_2}{M_{\rm Pl}} N_2 (N_3^2 + N_1^2) \right|^2 + \left| m_N N_3 + f N_1 N_2 + \frac{\lambda_1}{M_{\rm Pl}} N_3^3 + \frac{\lambda_2}{M_{\rm Pl}} N_3 (N_1^2 + N_2^2) \right|^2 \tag{22}$$

is zero. The only solution which has ever been assumed up to now is $\langle N_{1,2,3} \rangle = 0$, for which $\mathcal{M}_N$ is indeed given by Eq. (13). However, there is another natural solution, i.e.

$$\langle N_{2,3} \rangle = 0, \qquad \langle N_1 \rangle^2 = \frac{-m_N M_{\rm Pl}}{\lambda_1}. \tag{23}$$

In that case, the mass term corresponding to the shifted field $N_1' \equiv N_1 - \langle N_1 \rangle$ in W becomes

$$m_N + \frac{3\lambda_1 \langle N_1 \rangle^2}{M_{\rm Pl}} = -2m_N, \tag{24}$$

and $N_2 N_3$ has the mass term $f\langle N_1 \rangle$, whereas N_2^2 and N_3^2 have the mass term

$$m_N + \frac{\lambda_2 \langle N_1 \rangle^2}{M_{\rm Pl}} = m_N \left(1 - \frac{\lambda_2}{\lambda_1}\right). \tag{25}$$

In other words, Eq. (15) is *automatically generated* with

$A = -2m_N$, $B = (1 - \lambda_2/\lambda_1)m_N$, and $C = f\langle N_1\rangle$ which is of order A and B if f is of order $|\lambda_1 m_N/M_{\rm Pl}|^{1/2}$.

Since the superpotential also contains the term

$$h[(\nu_1 N_1 + \nu_2 N_2 + \nu_3 N_3)\eta^0 - (l_1 N_1 + l_2 N_2 + l_3 N_3)\eta^+], \tag{26}$$

the soft term

$$-h\langle N_1\rangle(\nu_1\eta^0 - l_1\eta^+) \tag{27}$$

must be added to allow (ν_1, l_1) and (η^+, η^0) to remain massless at this high scale. This is of course fine tuning, but once it is done, it is protected by the exact R-parity and supersymmetry of the residual theory. It is analogous to the usual situation in the minimal supersymmetric standard model, where the term $(\nu_i\eta^0 - l_i\eta^+)$ is allowed by all its gauge symmetries, but simply forbidden by the imposition of R-parity, i.e. whatever the allowed term is, a term is added to cancel it exactly.

It should be noted that the symmetry being broken at the large scale is A_4. Because of the explicit trilinear term $N_1N_2N_3$ in the superpotential, there is no additional discrete symmetry involved in the breaking. In other words, the concept of R-parity does not appear at this point. Below the breaking scale, with the addition of the above-mentioned soft term, the concept of R-parity emerges for the first time, and applies only to the superfields of the minimal supersymmetric standard model. It does not apply to the N superfields because they have all been integrated away. This is perfectly consistent with an effective supersymmetric field theory at the electroweak scale with Majorana neutrino masses.

If soft terms which break A_4 are simply added to the Majorana mass matrix of N_i, the same model below the seesaw scale can be obtained. There are, however, two important differences. One is that whereas this procedure may be used to obtain any pattern that is desired, the procedure advocated here will only result in the particular pattern shown. The other is that the two models have different interactions above the seesaw scale. Even though they are experimentally indistinguishable at present energies, they are at least theoretically distinct.

To avoid having three Higgs doublet superfields (ϕ_i^0, ϕ_i^-) and their three partners at the electroweak scale, this model can be modified by having just one $(\phi^0, \phi^-) \sim \underline{1}$ under A_4, but with the addition of three heavy singlets ζ_i which transform as $\underline{3}$ under A_4. The Yukawa coupling terms in the charged-lepton sector are then given by [10,11]

$$\frac{h_{ijk}}{\Lambda}(\nu_i\phi^- - l_i\phi^0)l_j^c\zeta_k. \tag{28}$$

To decouple ζ_i from N_i, an extra Z_4 symmetry is assumed, under which the only nontrivial transformations are $\zeta \sim i$ and $l^c \sim -i$. Consider then the superpotential

$$W_\zeta = \frac{1}{2}m_\zeta(\zeta_1^2 + \zeta_2^2 + \zeta_3^2) + \frac{\lambda_3}{4M_{\rm Pl}}(\zeta_1^4 + \zeta_2^4 + \zeta_3^4) + \frac{\lambda_4}{2M_{\rm Pl}}(\zeta_2^2\zeta_3^2 + \zeta_3^2\zeta_1^2 + \zeta_1^2\zeta_2^2), \tag{29}$$

where m_ζ breaks Z_4 softly. The resulting scalar potential is

$$V_\zeta = \left|m_\zeta\zeta_1 + \frac{\lambda_3}{M_{\rm Pl}}\zeta_1^3 + \frac{\lambda_4}{M_{\rm Pl}}\zeta_1(\zeta_2^2 + \zeta_3^2)\right|^2 + \left|m_\zeta\zeta_2 + \frac{\lambda_3}{M_{\rm Pl}}\zeta_2^3 + \frac{\lambda_4}{M_{\rm Pl}}\zeta_2(\zeta_3^2 + \zeta_1^2)\right|^2 + \left|m_\zeta\zeta_3 + \frac{\lambda_3}{M_{\rm Pl}}\zeta_3^3 + \frac{\lambda_4}{M_{\rm Pl}}\zeta_3(\zeta_1^2 + \zeta_2^2)\right|^2, \tag{30}$$

which has the desired solution

$$\langle\zeta_1\rangle = \langle\zeta_2\rangle = \langle\zeta_3\rangle = \frac{-m_\zeta M_{\rm Pl}}{\lambda_3 + 2\lambda_4}, \tag{31}$$

for which the supersymmetry is unbroken.

Other realizations of Eq. (3) also exist [12–15]. They can be classified according to the four parameters (a, b, c, d) of Eq. (17) as follows. In Ref. [12], it is proposed that $b = c = 0$. In Ref. [13], the case $a = 0$ and $b = c$ is discussed. In Ref. [14], the conditions are $b = c$ and $d^2 = 3b(b - a)$. Here and in Ref. [15], Eq. (17) is reduced by only $b = c$.

(All these examples are based on A_4 except the last one, which is based on the Coxeter group B_4, which is also the symmetry group of the hyperoctahedron [16].) What sets the present model apart from all others is the automatic generation of Eq. (15), using the hitherto unrecognized possibility of Eq. (23).

In conclusion, it has been shown how the tribimaximal mixing pattern of neutrinos can be derived in the supersymmetric seesaw model with A_4 symmetry. The spontaneous breaking of A_4 through the nonzero vacuum expectation value of one of the three heavy singlet neutrino superfields automatically generates the desired neutrino mass matrix. Below the seesaw scale, the model is identical to that of the minimal supersymmetric standard model, but with arbitrary nonzero Majorana neutrino masses which mix tribimaximally [17].

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