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Possible new source of T and CP violation in neutrino oscillations

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A model is presented to illustrate that vacuum neutrino oscillations can be essentially T and CP invariant up to a certain energy but strongly T and CP noninvariant at much higher energies. Detailed model results for the vacuum probabilities $P(\nu_\mu \to \nu_e)$ and $P(\nu_e \to \nu_\mu)$ are given, which may be relevant to proposed long-baseline neutrino-oscillation experiments.

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I. INTRODUCTION

One of the main goals of neutrino-oscillation physics in the coming decennia will be to determine (or constrain) the violation of time-reversal (T) invariance and charge-conjugation-parity (CP) invariance in the lepton sector; see, e.g., Refs. [1,2] for a comprehensive report and recent review article. With three-flavor neutrino oscillations being solely due to mass differences [3], the ultimate source of this T and CP violation would be the complex Dirac phase δ in the unitary mixing matrix (here, denoted X) between weak-interaction states and mass states [4].

Another possible contribution to neutrino oscillations may come from Lorentz-noninvariant Fermi-point-splitting effects [5–7]. (The Fermi-point-splitting mechanism of neutrino oscillations has a direct motivation from condensed-matter physics [8–10], but there have been many other suggestions for alternative mechanisms; see, e.g., Ref. [11] for an extensive list of references.) With Fermi-point splittings present, there is then a new unitary mixing matrix (Y) between weak-interaction states and Fermi-point states. If there are both mass differences and Fermi-point splittings in the neutrino sector, the relevant mixing matrix for neutrino oscillations is between weak-interaction states and *propagation* states, where the neutrino propagation is affected simultaneously by mass and Fermi point. This mixing matrix (Z) is determined, in part, by the matrices X and Y of the mass and Fermi-point sectors, respectively.

The crucial point, now, is that the Fermi-point-splitting matrix Y may have mixing angles (χ_{ij}) and complex Dirac phase (ω) completely different from those of the mass-sector matrix X (usually, denoted θ_{ij} and δ). In particular, there is the possibility that *all* parameters χ_{ij} and ω are nonvanishing, or even maximal. This would then correspond to a new source of T (and CP) violation effects in neutrino oscillations. The goal of the present article is to illustrate this possibility with a relatively simple model.

A potential new source of leptonic T and CP violation is all the more interesting as neutrinos may play a decisive role in the creation of the observed matter-antimatter asymmetry of the universe [12–14]. The suggestion is that neutrinos would in some way be responsible for the creation of a net lepton number L at very high temperature ($T \gg M_W \approx 10^2$ GeV), which, at the electroweak scale ($T \sim M_W$), is partially transformed by sphaleron processes into a net baryon number B [15–19]. Even though it will be difficult to relate the ultrahigh-energy CP violation needed for leptogenesis to any T and CP violation of neutrino-oscillation experiments at relatively low energies [13] and the fundamental mechanism of electroweak $B + L$ violation at high temperatures ($T \gtrsim M_W$) is not fully understood [19], the topic of leptonic T and CP violation can be expected to play an important role in a discussion of the physics of the early universe.

The outline for the remainder of this article is as follows. In Sec. II, we describe the model. In Sec. III, we give model results for vacuum oscillation probabilities in the so-called "golden channel," $\nu_e \leftrightarrow \nu_\mu$. In Sec. IV, we present concluding remarks.

II. MODEL

A. General remarks

In a previous article [7], we have considered a simple three-flavor neutrino-oscillation model with both mass-square differences (Δm^2_{ij}) and timelike Fermi-point splittings ($\Delta b_0^{(ij)}$). The mixing of the mass sector was taken to be bi-maximal and the one of the Fermi-point-splitting sector trimaximal, with all complex phases vanishing. The model had furthermore a hierarchy of Fermi-point splittings ($b_0^{(1)} = b_0^{(2)} \neq b_0^{(3)}$) which parallels the hierarchy of mass squares ($m_1^2 = m_2^2 \neq m_3^2$). For the physics motivation of this type of model (e.g., quantum phase transitions in superfluids), see Refs. [8–10] and references therein. As to the expected energy scale of neutrino Fermi points, there are speculations [9,20] but no firm predictions.

The present article extends the previous one by presenting results on the appearance probability $P_{\mu e} \equiv P(\nu_\mu \to \nu_e)$ from a generalized model with the same mass hierarchy as the model of Ref. [7] but with equidistant Fermi-point splittings ($b_0^{(2)} - b_0^{(1)} = b_0^{(3)} - b_0^{(2)}$) and one nonvan-

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ishing complex phase ($\omega = \pi/4$). In addition, we will consider the case of relatively strong Fermi-point-splitting effects compared to mass-difference effects, whereas Ref. [7] focused on relatively weak splitting effects. For this purpose, we introduce a new parametrization (with nonnegative dimensionless parameters ρ and τ) which makes a straightforward comparison between different long-baseline neutrino-oscillation experiments possible. Relatively weak or strong Fermi-point-splitting effects then correspond to $\tau \ll 1$ or $\tau \gtrsim 1$, respectively. The behavior of $P_{\mu e}(\rho, \tau)$ turns out to be quite complicated for $\tau \gtrsim 1$.

For this generalized model with complex phase $\omega = \pi/4$, we also give the model probability of the time-reversed process, $\nu_e \to \nu_\mu$. It will be seen that the generalized model has a rather interesting phenomenology with stealthlike characteristics in certain cases and strong time-reversal noninvariance in others.

In this article, we mainly speak about possible T-violating effects in neutrino oscillations from Fermi-point splitting. Whether or not there are corresponding CP-violating effects depends on the (unknown) physics responsible for the Fermi-point splittings, i.e., whether or not there is CPT invariance. Depending on the Fermi-point splittings of the right-handed "antineutrinos" compared to those of the left-handed "neutrinos," there may or may not be CP violation in addition to the T violation of the model considered ($\sin\omega \neq 0$); see Sec. 4 of Ref. [6] for further details. For the rest of this article, we take an agnostic point of view on the CPT invariance of Fermi-point splitting, and focus on the manifest T violation from the presence of complex phases in the Hamiltonian.

B. Specifics

Setting $\hbar = c = 1$ and writing $p \equiv |\mathbf{p}|$ for the (large) neutrino momentum, the Hamiltonian of the generalized version of the model of Ref. [7] contains three terms in the $(\nu_e, \nu_\mu, \nu_\tau)$ flavor basis,

$$H \supset p\mathbb{1} + X \cdot D_m \cdot X^\dagger + D_{\alpha\beta} \cdot Y \cdot D_{b_0} \cdot Y^\dagger \cdot D_{\alpha\beta}^\dagger, \quad (1)$$

with diagonal matrices

$$D_m \equiv \mathrm{diag}(m_1^2/(2p), m_2^2/(2p), m_3^2/(2p)), \quad (2a)$$

$$D_{b_0} \equiv \mathrm{diag}(b_0^{(1)}, b_0^{(2)}, b_0^{(3)}), \quad (2b)$$

$$D_{\alpha\beta} \equiv \mathrm{diag}(\exp[\mathrm{i}\alpha], \exp[-\mathrm{i}(\alpha + \beta)], \exp[\mathrm{i}\beta]), \quad (2c)$$

and $SU(3)$ matrices

$$X \equiv M_{32}(\theta_{32}) \cdot M_{13}(\theta_{13}, \delta) \cdot M_{21}(\theta_{21}), \quad (3a)$$

$$Y \equiv M_{32}(\chi_{32}) \cdot M_{13}(\chi_{13}, \omega) \cdot M_{21}(\chi_{21}), \quad (3b)$$

in terms of the basic matrices

$$M_{32}(\vartheta) \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\vartheta & \sin\vartheta \\ 0 & -\sin\vartheta & \cos\vartheta \end{pmatrix}, \quad (4a)$$

$$M_{21}(\vartheta) \equiv \begin{pmatrix} \cos\vartheta & \sin\vartheta & 0 \\ -\sin\vartheta & \cos\vartheta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (4b)$$

$$M_{13}(\vartheta, \varphi) \equiv \begin{pmatrix} \cos\vartheta & 0 & e^{i\varphi}\sin\vartheta \\ 0 & 1 & 0 \\ -e^{-i\varphi}\sin\vartheta & 0 & \cos\vartheta \end{pmatrix}. \quad (4c)$$

The following dimensionless parameters are chosen in the mass sector:

$$R_m \equiv \Delta m_{21}^2/\Delta m_{32}^2 \equiv (m_2^2 - m_1^2)/(m_3^2 - m_2^2) = 0, \quad (5a)$$

$$\theta_{21} = \theta_{32} = \pi/4, \qquad \theta_{13} = 0, \qquad \delta = 0, \quad (5b)$$

in the Fermi-point-splitting sector:

$$R \equiv R_{b_0} \equiv \frac{\Delta b_0^{(21)}}{\Delta b_0^{(32)}} \equiv \frac{b_0^{(2)} - b_0^{(1)}}{b_0^{(3)} - b_0^{(2)}} \in (-\infty, \infty), \quad (5c)$$

$$\chi_{21} = \chi_{32} = \chi_{13} = \pi/4, \qquad \omega \in [0, 2\pi), \quad (5d)$$

and for the relative complex phases between mass and Fermi-point sectors:

$$\alpha = \beta = 0. \quad (5e)$$

In addition to the dimensionless parameters R and ω, there are two dimensionful model parameters relevant to neutrino oscillations,

$$\Delta m_{31}^2 \equiv m_3^2 - m_1^2 > 0, \quad \Delta b_0^{(31)} \equiv b_0^{(3)} - b_0^{(1)} > 0, \quad (6)$$

which have been taken positive. Remark that the mass-sector parameters (5a) and (5b) are not unrealistic (with $\sin\theta_{13} = 0$, the chosen value of δ is, in fact, irrelevant) but the sign of Δm_{31}^2 is still undetermined experimentally [1,2].

For high-energy neutrino oscillations over a travel distance L, two dimensionless parameters can be defined as follows ($E_\nu \sim p$):

$$\rho \equiv \frac{2E_\nu \hbar c}{L|\Delta m_{31}^2|c^4} \approx 1.5786\left(\frac{E_\nu}{10\ \mathrm{GeV}}\right)\left(\frac{10^3\ \mathrm{km}}{L}\right) \times \left(\frac{2.5 \times 10^{-3}\ \mathrm{eV}^2/c^4}{|\Delta m_{31}^2|}\right), \quad (7a)$$

$$\tau \equiv \frac{L|\Delta b_0^{(31)}|}{\hbar c} \approx 5.0671\left(\frac{L}{10^3\ \mathrm{km}}\right)\left(\frac{|\Delta b_0^{(31)}|}{10^{-12}\ \mathrm{eV}}\right), \quad (7b)$$

with $\hbar$ and c temporarily reinstated. These two dimensionless parameters, together with R and ω, completely determine the oscillation probabilities, at least for the simple model considered and with matter effects neglected [21].

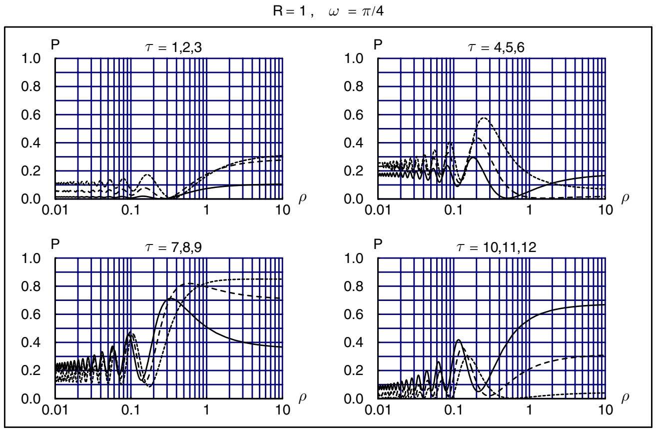

FIG. 1 (color online). Numerical model results for the vacuum neutrino-oscillation probability $P_{\mu e} \equiv P(\nu_\mu \to \nu_e)$ as a function of the dimensionless parameters ρ and τ, defined by Eqs. (7a) and (7b). The model, described in Sec. II B, has Fermi-point-splitting ratio $R = 1$ and complex phase $\omega = \pi/4$. Shown are constant-τ slices of $P(\rho, \tau) \equiv P_{\mu e}(\rho, \tau)$, where the solid, long-dashed, and short-dashed curves correspond to $\tau = 1, 2, 0$ (mod 3), respectively.

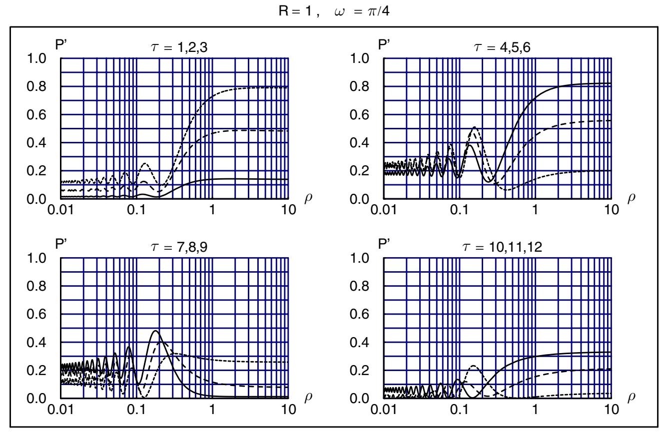

FIG. 2 (color online). Same as Fig. 1 but for the time-reversed process with probability $P' \equiv P(\nu_e \to \nu_\mu)$. If CPT invariance holds, P' also corresponds to $P(\overline{\nu}_\mu \to \overline{\nu}_e)$.

III. RESULTS

The model defined by Eqs. (1)–(6), for $R = 1$ and $\omega = \pi/4$, gives the vacuum probability $P_{\mu e} \equiv P(\nu_\mu \to \nu_e)$ shown in Fig. 1 as a function of the parameters ρ and τ from Eqs. (7a) and (7b). For $\rho \to \infty$ (high neutrino energies at fixed $\Delta m^2_{31} \times L$), this model is similar to the pure Fermi-point-splitting model studied previously [5,6], for which $P_{\mu e}$ is known exactly [22]. For $\rho \to 0$ (low energies), the $P_{\mu e}$ behavior can also be understood analytically [23].

These last remarks explain the observed stealthlike behavior of $P_{\mu e}(\rho, \tau)$ at certain special values of τ, with the appearance probability being nonzero only for a relatively small range of energies. An example would be given by the case of $\tau \approx 12$ in Fig. 1. For a given nonzero value of $\Delta b_0^{(31)}$, the particular appearance probability $P_{\mu e}$ would be nearly shut off at the corresponding distance $L \approx 12/|\Delta b_0^{(31)}|$, reappearing, however, at generic values of L [24].

The model with $R = 1$ and $\omega = \pi/4$ can be expected to have T violation for large enough neutrino energy. (The corresponding pure mass-difference model, relevant at low energies, does not have T violation, in particular, because $\sin\theta_{13}$ vanishes.) The probability of the $\nu_e \to \nu_\mu$ process is shown in Fig. 2 and the difference with Fig. 1, for $\rho \gtrsim 0.2$ and generic values of τ, indeed signals time-reversal noninvariance.

If CPT invariance holds true (cf. Sec. II A), the curves of Fig. 2 also apply to $P(\overline{\nu}_\mu \to \overline{\nu}_e)$ and the difference with Fig. 1 signals CP noninvariance. If, on the other hand, CPT invariance is violated maximally, the probability $P(\overline{\nu}_\mu \to \overline{\nu}_e)$ is given by the curves of Fig. 1 and there is only T violation, with $P(\nu_\mu \to \nu_e) = P(\overline{\nu}_\mu \to \overline{\nu}_e) \neq P(\nu_e \to \nu_\mu)$.

IV. CONCLUSION

Figures 1 and 2 show strong time-reversal (T) noninvariance of vacuum neutrino oscillations $\nu_\mu \leftrightarrow \nu_e$ at high energies, which traces back to the large complex phase ω of the model considered, together with the large mixing angles χ_{ij} and ratio R in the Fermi-point-splitting sector [25]. (As mentioned in the last paragraph of Sec. II A, there may or may not be a corresponding CP violation, depending on whether or not the Fermi-point splittings respect CPT.) In other words, this T (and CP) violation would primarily take place *outside* the mass sector and show up at the high end of the neutrino energy spectrum [26].

A neutrino factory with broad energy spectrum $E_\nu \approx$ 10–50 GeV and several detectors at baselines L up to 12 800 km would be the ideal machine, in principle, to establish such strong T (and CP) violation in high-energy neutrino oscillations [1,27–29]. Perhaps nearer in the future, (redesigned) superbeam experiments such as NOνA [30] can also look for possible new sources of CP violation. High-energy cosmic neutrinos might provide additional information; cf. Ref. [11].

As mentioned in the introduction, *any* new source of T (and CP) violation in neutrino oscillations, especially at the high end of the neutrino energy spectrum, may be relevant to the physics of the early universe. Moreover, this new T (and CP) violation could be related to the emergence of the standard model from a nonrelativistic underlying theory. The search for new sources of T and CP violation in high-energy neutrino oscillations is therefore an important task of future superbeam and neutrino-factory experiments.

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[20] Consider, for example, an emergent-physics scenario [10] with $N_F = 3$ fermion families and two cutoff scales: $E_{\rm UV} \sim 10^{42}$ GeV corresponding to the fundamental, Lorentz-violating fermionic theory and $E_{\rm c} \sim 10^{13}$ GeV corresponding to the compositeness scale of the standard model gauge bosons. (The approximate numerical values of these cutoff scales are based on the gauge coupling constants measured at LEP and the corresponding renormalization group equations.) The idea, now, is that the ultrahigh-energy Lorentz violation *reenters* at ultralow energy $(E_c)^2/E_{\rm UV} \sim 10^{-7}$ eV and that this energy scale determines the Lorentz noninvariance in the neutrino sector ($b_0^{(i)} \neq 0$). Reentrance effects in superfluid ^{3}He-A have been discussed by G. E. Volovik, JETP Lett. **73**, 162 (2001) [Pis'ma Zh. Eksp. Teor. Fiz. **73**, 182 (2001)].
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[22] The notation of the neutrino dispersion relation in Refs. [5,6] differs from the one used here and in Ref. [7], but the different sign of $b_0^{(f)}$ can be compensated by a sign change of the complex phase ϵ. For example, setting $\epsilon = -\omega$ in the exact model probability $P(\nu_\mu \to \nu_e)$ from Eq. (11e) in Ref. [6] reproduces the numerical $\rho \to \infty$ results for $R = 1$, $\omega = \chi_{21} = \chi_{32} = \pi/4$, and $\chi_{13} = \arctan\sqrt{1/2} \approx \pi/5$ (these numerical results resemble those of Fig. 1 for $\chi_{13} = \pi/4$).
[23] Degenerate perturbation theory for $\rho \to 0$ and fixed τ (arbitrary R and ω) gives $P_{\mu e} \sim (4R^2/S)\sin^2[\sqrt{S}\tau/(8 + 8R)]$ with $S \equiv 4 + 4R + 9R^2$.
[24] For a case study and further model results, see an earlier version of this article, F. R. Klinkhamer, hep-ph/0509111.
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